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Fuzzy Evaluation Model of Bank APP Performance Based on Circular Economy Thinking

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Abstract: As the environment of the Internet of Things (IoT) gradually becomes common and mature, various smart application (APP) platforms have sprung up, making what we are doing more convenient, more economical and more efficient. Then, this paper used a bank APP as the research background to discuss issues related to smart APPs. Obviously, through the bank APPs, customers can complete their transfer and payment for various expenses at home, eliminating the inconvenience of going out, which not only can alleviate traffic congestion as well as reduce carbon emissions but also can save the manpower expenditure costs for banks. Consequently, improving APP performance and increasing the number of users of an APP is a very important issue. Therefore, this paper proposed an APP performance index to evaluate the performance of a bank APP. This APP performance index is to evaluate the performance of the APP through the time interval of customers' access to the APP. The shorter the time interval is, the greater the number of users within a unit time is. In addition, based on cost considerations and effectiveness, the sample size n is usually not too large in practice, in order to make decisions quickly and accurately in a short time. Since the fuzzy testing model based on the confidence interval can be integrated with the past accumulated experience of data experts, the testing accuracy can be leveled up under the condition of small-sized samples. Accordingly, a fuzzy evaluation model was proposed to evaluate whether the performance of the bank APP can reach the required level, and this model was also regarded as a basis for decision-making to determine whether to improve the bank APP. At the same time, we can grasp the opportunities for improvement, achieve the effect of cost reduction, energy saving and carbon reduction, and further move towards the goal of innovative and intelligent management.

Keywords: circular economy thinking; bank App; APP performance index; confidence interval; fuzzy evaluation model

1. Introduction

Many studies have pointed out that in the face of increasingly serious global warming issues, circular economy (CE) thinking is attracting more and more attention from companies and governments [1–5]. Based on the concept of circular economy, a lot of scholars have been engaged in research on related topics, such as product design and manufacturing, supplier selection, and service operation management [6–8]. Lin et al. [9] indicated that with the gradual popularity and maturity of the environment of the Internet of Things (IoT), various smart APPs have sprung up, making what we are doing more convenient, economical and efficient. Numerous scholars have suggested that the cloud platform built by the IoT can collect customers' data and come up with various data analysis models to improve the performance of APPs, so as to increase customers' willingness to use the APPs [10–13]. The effective data analysis and application can accelerate the development of smart APPs as well as move towards the goal of innovative and smart management [14–16].

Next, this paper will take a bank APP as an example to further illustrate the application of smart APPs. Apparently, customers can transfer money and pay various fees at home through the APP built by the bank, eliminating the inconvenience of going out. Not only can the APP ease traffic jams and reduce carbon emissions, but it also can save labor costs for the bank. As mentioned earlier, a good-quality APP can increase customers' willingness to use the APP platform. This paper will establish an APP performance evaluation model so as to improve the service performance of the APP, and at the same time to raise customers' satisfaction with the use of the APP as well as increase the number of APP users. According to Ross [17] and Kim et al. [18], the number of customers entering the APP was a Poisson process. Chen and Yang [19] considered that the time between two customers entering the APP was distributed as the exponential distribution. Thus, an operating performance index was proposed. Li et al. [20] revised the operating performance index as a more concise ratio operating performance index according to the concept that the index is a simple and unitless management tool. Therefore, based on the concept of Li et al. [20], this paper defines an APP performance index as an evaluation tool for the APP performance.

Plenty of studies have revealed that gathering the time interval of customers' access to the APP is more effective than the number of customers' arrivals within the time unit [19,21,22]. Hence, this paper evaluates the APP performance through the time intervals of customers' entering the APP. The shorter the time interval is, the higher the number of users within the time unit is. As mentioned above, this approach can reach effects, including convenience, cost reduction, energy saving and carbon reduction. In addition, based on cost considerations and effectiveness, in order to make decisions quickly and accurately in a short time, the sample size n is usually not too large in practice; consequently, the accuracy of statistical inference will be affected [23,24]. Since the fuzzy testing model based on the confidence interval can be incorporated into the past accumulated experience of data experts [25,26], the testing accuracy can be improved under the condition of small-sized samples. Therefore, this paper will derive the confidence interval of the App performance index to construe a fuzzy number and a membership function of the APP performance index. Next, a fuzzy evaluation model is proposed to evaluate whether the performance of the bank APP can reach the required level and regarded as a basis of determining whether the bank APP needs to improve.

In summary, the purpose of this research is to propose a fuzzy evaluation model that can take into account both timeliness and accuracy. It can not only evaluate the performance of the bank APP but also grasp the opportunities for improvement at the same time, in order to achieve the effect of cost reduction, energy saving and carbon reduction and then move towards the goal of innovative and intelligent management. Finally, to facilitate the application of readers and the industry, we use an application example to illustrate the application of the fuzzy testing method proposed in this paper.

The rest of the article is arranged as follows: Section 2 defines the App performance index. Section 3 shows the statistical hypothesis testing model based on the confidence interval. Section 4 proposes a fuzzy evaluation model for the performance index. Section 5 presents an application example demonstrating the applicability of the proposed approach. Section 6 provides conclusions. Last but not least, Section 7 describes limitations and possible future research directions.

2. APP Performance Index

According to Chen and Yang [18] and Li et al. [19], the number of customers ($N(t)$) entering the APP is a Poisson distribution with rate λ, denoted by $N(t) \sim Poisson(\lambda t)$. Then, the probability density function (*p.d.f.*) is given by:

$$p(N(t)=n)=\frac{e^{-\lambda t}(\lambda t)^n}{n!},\ n = 0, 1, 2, 3, \ldots \tag{1}$$

Let T_j denote the elapsed time from the $(j-1)^{st}$ customer entering the APP until the occurrence of the j^{th} customer. The sequence $\{T_1,\ldots,T_j,\ldots,T_n,\ldots\}$ is called the sequence of interval times. Therefore:

$$F_{T_j}(t)=1-p\{T_j>t\}=1-exp\{-\lambda t\} \tag{2}$$

and:

$$f_{T_j}(t)=\frac{\partial F_{T_j}(t)}{\partial t}=\lambda\times exp\{-\lambda t\}. \tag{3}$$

Let $\tau = 1/\lambda$ be the mean time between two customers continuously entering the APP, then:

$$f_{T_j}(t)=\frac{1}{\tau}\times exp\left\{-\frac{t}{\tau}\right\},\ t\geq 0. \tag{4}$$

In this case, the sequence of interval time T_j is independent and an identically distributed exponential random variable with mean τ. If at least N_0 customers are required to enter the APP for operation within a time unit, then it is equivalent to request $T_j \leq U$, where $U = 1/N_0$ denotes the maximum expected interval times of two customers continuously entering the APP. On this basis, the sequence of interval time T_j belongs to the smaller-the-better performance characteristic. Based on Li et al. [19], the APP performance index A_{PI} can defined as:

$$A_{PI}=\frac{U}{\tau}. \tag{5}$$

Let $q = p(T_j > U)$ represent the rate of non-compliance with performance requirements, then:

$$q = p(T_j > U) = exp\{-A_{PI}\}. \tag{6}$$

Obviously, the performance index A_{PI} and the rate of non-compliance with performance requirements q have a one-to-one mathematical relationship. In fact, the partial differentiation for q is negative. Therefore, q is the monotonically decreasing function of A_{PI} and shown in Figure 1.

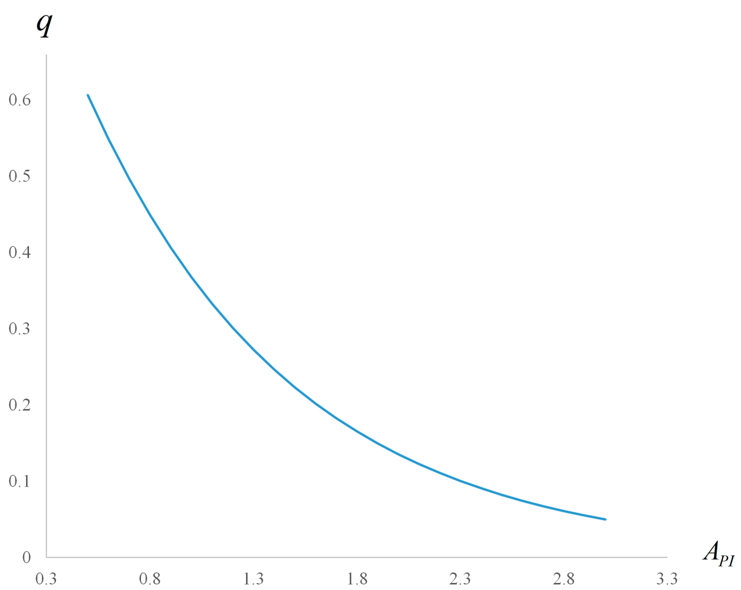

Figure 1. Various values of performance index A_{PI} and their corresponding values of *q*.

3. Statistical Hypothesis Testing Based on Confidence Interval

Suppose the required performance value of the index A_{PI} is A_0, then we consider the problem of the hypothesis tests, the null hypothesis $H_0 : A_{PI} = A_0$ against the alternative hypothesis $H_1 : A_{PI} \neq A_0$, at a desired level of significance level α. Let $T_1, \ldots, T_j, \ldots, T_n$ be random variables of an independent identity distribution (*i.i.d.*) with the exponential distribution. Then, we propose an unbiased estimator of A_{PI} as follows:

$$A_{PI}^* = \frac{U}{\tau^*}, \tag{7}$$

where $\tau^* = (n-1)^{-1} \sum_{j=1}^{n} T_j$ is the estimator of mean τ. Furthermore, let:

$$W = \frac{(n-1) A_{PI}}{A_{PI}^*}, \tag{8}$$

then is distributed as $G(n,1)$, and the probability density function of W is displayed as follows:

$$f_W(w) = \frac{1}{\Gamma(n)} \times w^{n-1} \times exp\{-w\}, \quad w \geq 0. \tag{9}$$

Obviously:

$$p\left(G_{\alpha/2}(n,1) \leq W \leq G_{1-\alpha/2}(n,1)\right) = 1-\alpha$$

$$\Rightarrow p\left(G_{\alpha/2}(n,1) \leq \frac{(n-1) A_{PI}}{A_{PI}^*} \leq G_{1-\alpha/2}(n,1)\right) = 1-\alpha$$

$$\Rightarrow p\left(\frac{G_{\alpha/2}(n,1)}{n-1} A_{PI}^* \leq A_{PI} \leq \frac{G_{1-\alpha/2}(n,1)}{n-1} A_{PI}^*\right) = 1-\alpha, \tag{10}$$

where $G_{\alpha/2}(n, 1)$ is the lower $\alpha/2$ quintile of $G(n,1)$, and $G_{1-\alpha/2}(n, 1)$ is the lower $1-\alpha/2$ quintile of $G(n,1)$. Therefore, the lower confidence limit of A_{PI} can be expressed as follows:

$$LA_{PI} = \frac{G_{\alpha/2}(n,1)}{n-1} A_{PI}^{*} \tag{11}$$

Similarly, the upper confidence limit of A_{PI} can be represented as follows:

$$UA_{PI} = \frac{G_{1-\alpha/2}(n,1)}{n-1} A_{PI}^{*} . \tag{12}$$

Let $\left(t_1,\ldots,t_j,\ldots,t_n\right)$ be the observed value of $\left(T_1,\ldots,T_j,\ldots,T_n\right)$, then the observed value of A_{PI}^{*} is expressed as follows:

$$A_{PI0}^{*} = \frac{U}{\tau_0^{*}}, \tag{13}$$

where $\tau_0^{*} = \left(n-1\right)^{-1} \sum_{j=1}^{n} t_j$ is the observed value of τ^{*}. Therefore, the observed value of LA_{PI} and UA_{PI} can be demonstrated separately as follows:

$$LA_{PI0} = \frac{G_{\alpha/2}(n,1)}{n-1} A_{PI0}^{*} ; \tag{14}$$

$$UA_{PI0} = \frac{G_{1-\alpha/2}(n,1)}{n-1} A_{PI0}^{*} . \tag{15}$$

As noted above, the statistical testing rules with significance level α can be listed as follows:

(1) If $UA_{PI0} < A_0$, then reject H_0 and conclude that $A_{PI} < A_0$.
(2) If $LA_{PI0} > A_0$, then reject H_0 and conclude that $A_{PI} > A_0$.
(3) If $LA_{PI0} \le A_0 \le UA_{PI0}$, then do not reject H_0 and conclude that $A_{PI} = A_0$.

4. Fuzzy Evaluate Model for Performance Index A_{PI}

With the gradual maturity and stability of the IoT environment and big data analytics technology, fast and accurate analysis and decision-making models have been developed via data, which seems to be the trend of business development. Not only can the models meet the needs of enterprises to pursue rapid response, but they also can help the industry move towards the goal of intelligent and innovative management. Nevertheless, under such a situation, the sample size is usually relatively small, which often leads to a decrease in the accuracy of evaluations. Various studies have pointed out that the fuzzy test based on the confidence interval can be derived from the accumulated data in the past, so the accuracy of evaluations can be improved in the state of small samples. Hence, this paper will apply the confidence interval derived in Section 2 to develop the fuzzy evaluation model of performance index A_{PI}

Based on Li et al. [20], the *α-cuts* of triangular shaped fuzzy number $\tilde{A}_{PI}$ can be obtained below:

$$\tilde{A}_{PI}\left[\alpha\right] = \begin{cases} \left[A_{PI1}(\alpha), A_{PI2}(\alpha)\right], 0.01 \le \alpha \le 1 \\ \left[A_{PI1}(0.01), A_{PI2}(0.01)\right], 0 \le \alpha \le 0.01 \end{cases}, \tag{16}$$

where:

$$A_{PI1}(\alpha) = \frac{G_{\alpha/2}(n,1)}{n-1} A_{PI0}^{*} ; \tag{17}$$

$$A_{PI2}(\alpha) = \frac{G_{1-\alpha/2}(n,1)}{n-1}\ A^*_{PI0}\,. \tag{18}$$

Thus, the triangular shaped fuzzy number of A_{PI} is $\tilde{A}_{PI} = (A_L, A_M, A_R)$, where:

$$A_L = \frac{G_{0.005}(n,1)}{n-1}\ A^*_{PI0}\,; \tag{19}$$

$$A_M = \frac{G_{0.5}(n,1)}{n-1}\ A^*_{PI0}\,; \tag{20}$$

$$A_R = \frac{G_{0.995}(n,1)}{n-1}\ A^*_{PI0}\,. \tag{21}$$

Furthermore, the membership function of $\tilde{A}_{PI}$ is:

$$\eta(x) = \begin{cases} 0 & if\ \ x < A_L \\ \alpha' & if\ \ A_L \le x < A_M \\ 1 & if\ \ x = A_M \\ \alpha'' & if\ \ A_M < x \le A_R \\ 0 & if\ \ A_R < x \end{cases}, \tag{22}$$

where α' and α'' are determined by:

$$G_{\alpha'/2}(n,1) = (n-1)x/A^*_{PI0} \tag{23}$$

and:

$$G_{1-\alpha''/2}(n,1) = (n-1)x/A^*_{PI0}\,. \tag{24}$$

According to Equation (22), we can construct a graph containing membership function $\eta(x)$ with vertical line $x = A_0$ as illustrated in Figure 2.

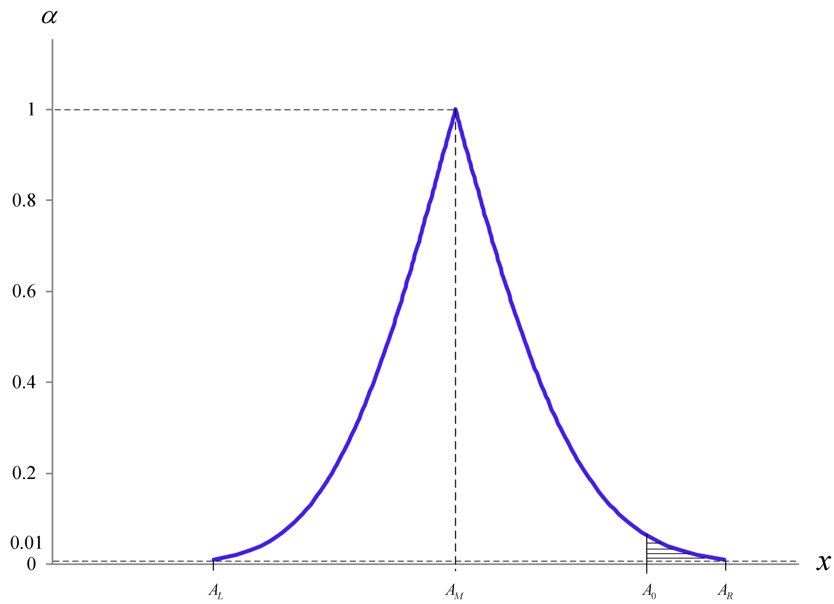

Figure 2. The membership function $\eta(x)$ with vertical line $x = A_0$.

In Figure 2, the y-axis is α and x-axis is x, where $x = G_{\alpha/2}(n,1)/(n-1)\ A_{PI0}^{*}$ for $x \le A_M$ and $x = G_{1-\alpha/2}(n,1)/(n-1)\ A_{PI0}^{*}$ for $x > A_M$.

Let set A_T be the area in the graph of $\eta(x)$, such that:

$$A_T = \{(x,\alpha) \mid A_{PI1}(\alpha) \le x \le A_{PI2}(\alpha), 0 \le \alpha \le 1\}. \tag{25}$$

Since the calculation of A_T is complicated, this study uses d_T to replace A_T for practical application according to the method proposed by Chen [21]. As d_T is the length of the bottom of A_T, then $d_T = A_R - A_L$ can be expressed as follows:

$$d_T = \left(\frac{G_{0.995}(n,1)}{n-1} - \frac{G_{0.005}(n,1)}{n-1} \right) A_{PI0}^{*} \tag{26}$$

On the other hand, let set A_S be the area in the graph of $\eta(x)$ but to the right of the vertical line $x = A_0$, then:

$$A_S = \{(x,\alpha) \mid A_0 \le x \le A_{PI2}(\alpha), 0 \le \alpha \le a_0\}, \tag{27}$$

where:

$$A_0 = \begin{cases} A_{PI1}(a_0)\ if\ A_0 \le A_M \\ A_{PI2}(a_0)\ if\ A_0 > A_M \end{cases}. \tag{28}$$

Similarly, when d_S is the length of the bottom of A_S, then $d_S = A_R - A_0$ can be calculated as follows:

$$d_S = \begin{cases} 0\ if\ A_0 > UA_{PI0} \\ \dfrac{G_{0.995}(n,1)}{n-1} A_{PI0}^{*} - A_0\ if\ LA_{PI0} \le A_0 \le UA_{PI0} \\ 1\ if\ A_0 < LA_{PI0} \end{cases}. \tag{29}$$

Thus, the value of d_S/d_T can be calculated as follows:

$$d_S/d_T = \begin{cases} 0\ if\ A_0 > UA_{PI0} \\ \dfrac{G_{0.995}(n,1) A_{PI0}^{*} - (n-1) A_0}{(G_{0.995}(n,1) - G_{0.005}(n,1)) A_{PI0}^{*}}\ if\ LA_{PI0} \le A_0 \le UA_{PI0} \\ 1\ if\ A_0 < LA_{PI0} \end{cases}. \tag{30}$$

Next, according to Lee et al. [27] let $0 < \phi < 0.5$ and revise the method of Li et al. [19] to formulate the fuzzy evaluation rules as follows:

(1) If $d_S/d_T < \phi$, then reject H_0 and conclude that $A_{PI} < A_0$.
(2) If $d_S/d_T > 1-\phi$, then reject H_0 and conclude that $A_{PI} > A_0$.
(3) If $\phi \le d_S/d_T \le 1-\phi$, then do not reject H_0 and conclude that $A_{PI} = A_0$.

5. Application Example

In this section, we use an application example to demonstrate the fuzzy evaluation method for the bank APP proposed in Section 4. Suppose that a bank APP evaluates its performance with a goal set at the $A_{PI} = 5$ ($A_0 = 5$) against the alternative hypothesis $H_1: A_{PI} \ne 5$. This is equivalent to the following hypotheses:

Null Hypothesis $H_0: A_{PI} = 5$

versus

Alternative Hypothesis $H_1 : A_{PI} \neq 5$.

Let $\left(T_1,\ldots,T_j,\ldots,T_{225}\right)$ be random variables of an *i.i.d.* with the exponential distribution. Based on the observed value $\left(t_1,\ldots,t_j,\ldots,t_{225}\right)$ and set $U = 10$, then we have:

$$\tau_0^* = (n-1)^{-1}\sum_{j=1}^{n} t_j = (225-1)^{-1}\sum_{j=1}^{225} t_j = 2.33$$

$$A_{PI0}^* = \frac{U}{\tau_0^*} = \frac{10}{2.33} = 4.29.$$

In fact, we have $G_{0.005}(250,1) = 188.24$, $G_{0.5}(250,1) = 224.67$, and $G_{0.995}(250,1)\,265.51$. Therefore, the observed values of A_L, A_M, and A_R can be individually calculated as follows:

$$A_L = \frac{G_{0.005}(n,1)}{n-1} A_{PI0}^* = \frac{188.24}{224} \times 4.29 = 3.60;$$

$$A_M = \frac{G_{0.5}(n,1)}{n-1} A_{PI0}^* = \frac{224.67}{224} \times 4.29 = 4.30;$$

$$A_R = \frac{G_{0.995}(n,1)}{n-1} A_{PI0}^* = \frac{265.51}{224} \times 4.29 = 5.08.$$

Therefore, the triangular shaped fuzzy number of A_{PI} is $\tilde{A}_{PI} = (3.61,\ 4.30,\ 5.08)$ and the membership function of $\tilde{A}_{PI}$ can be exhibited as follows:

$$\eta(x) = \begin{cases} 0 & if \ \ x < 3.61 \\ \alpha' & if \ \ 3.61 \le x < 4.30 \\ 1 & if \ \ x = 4.30 \\ \alpha'' & if \ \ 4.30 < x \le 5.08 \\ 0 & if \ \ 5.08 < x \end{cases}$$

where α' and α'' are determined by:

$$G_{\alpha'/2}(225,1) = 52.21x$$

and:

$$G_{1-\alpha''/2}(225,1) = 52.21x\ .$$

Thus, the graph of membership function $\eta(x)$ with vertical line $x = 5$ is presented in Figure 3. In Figure 3, the y-axis is α and x-axis is x, where $x = G_{\alpha/2}(225,1)\times 4.29/224$ for $x \le 4.30$ and $x = G_{1-\alpha/2}(225,1)\times 4.29/224$ for $x > 4.30$.

According to the above-mentioned, the values of $d_T = A_R - A_L$ and $d_S = A_R - A_0$ can be calculated as follows:

$$d_T = \left(\frac{G_{0.995}(225,1)}{224} - \frac{G_{0.005}(225,1)}{224}\right) A_{PI0}^* = \left(\frac{265.51}{224} - \frac{188.24}{224}\right) \times 4.29 = 1.48;$$

$$d_S = \frac{G_{0.995}(225,1)}{224} A_{PI0}^* - A_0 = \frac{265.51}{224} \times 4.29 - 5 = 0.08.$$

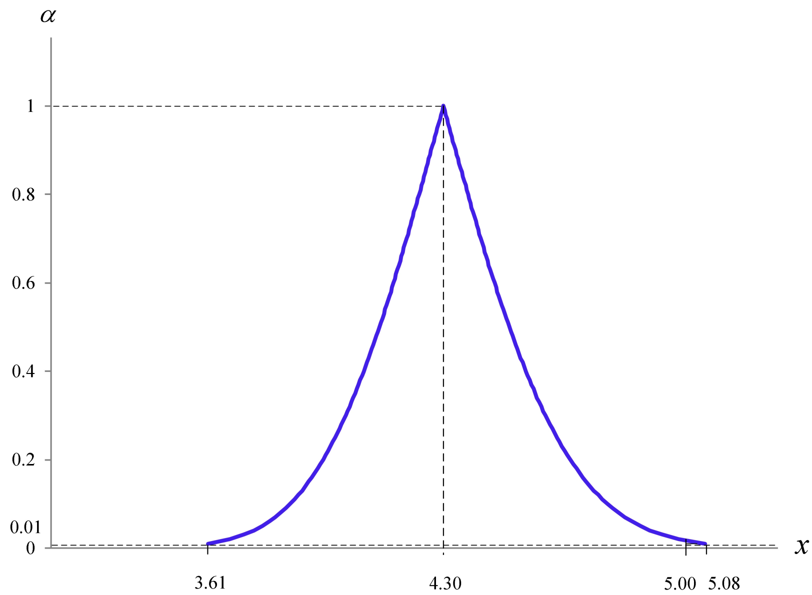

Figure 3. The membership function $\eta(x)$ with vertical line $x=5$.

Thus, the value of d_S/d_T can be calculated as follows:

$$d_S/d_T = \frac{0.08}{1.48} = 0.05.$$

Based on Li et al. [20], take $\phi = 0.2$, and since $d_S/d_T = 0.05 < \phi$, then reject and conclude that $A_{PI} < 5$. According to the statistical test, the upper limit of index A_{PI} is $UA_{PI0} = 5.08 > A_0 = 5$, so H_0 is not rejected, and $A_{PI} > A_0 = 5$ is concluded. As a matter of fact, the value of A^*_{PI0} is only 4.29 (A^*_{PI0} = 4.29), which is far less than 5. Based on the above-stated, it is obvious that the fuzzy test is more reasonable than the statistical test.

6. Conclusions

In this paper, we proposed an APP performance index to evaluate the performance of a bank APP. This performance index and the rate of non-compliance with performance requirements q have a one-to-one mathematical relationship. The rate of non-compliance with performance q is the monotonically decreasing function because the partial differentiation for q is negative. In order to solve the problem that small samples easily increase the evaluation errors, we proposed a fuzzy evaluation method using the APP performance index as a decision-making basis of improvement. Obviously, this approach is a conventional measurement method which can be used during the confidence interval of the APP performance index and then can be used to construct a fuzzy membership function for a fuzzy evaluation. Furthermore, the advantage of this approach is not only to lower the risk of misjudgment caused by sampling errors but also to enhance the accuracy of the evaluation. In addition, this study presented a simple calculation model to obtain the approximate values of d_S and d_T, which makes a valuable contribution in practice. We further took an application example to illustrate the proposed fuzzy evaluation method. This example demonstrated that from a practical perspective the proposed approach derived more reasonable results than the statistical method. Based on the above-mentioned, the method proposed in this paper can not only meet the needs of enterprises to pursue rapid response but also help the industry achieve the effects of cost reduction, energy saving and carbon reduction, and then it moves towards the goal of innovative and intelligent management. In addition, the aspects in which the fuzzy evaluation method is superior to the statistical testing method include: (1) it can be integrated into the expert experience of the past accumulated data, and the accuracy of the evaluation can be maintained when the sample size is small; (2) the applied small samples can answer the enterprises' need for quick response.

7. Limitations and Future Research

The fuzzy evaluation model of the bank App performance proposed by this study is applicable to exponential process distribution and belongs to the smaller-the-better performance characteristic. In contrast, this model is not applicable to other distributions, such as normal distributions. Normal distributions of the smaller-the-better performance characteristic characteristics that cannot be included in this research could be a topic for future research.

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Nomenclature

$N(t)$	The number of customers entering the APP
λ	The rate of Poisson distribution
T_j	The elapsed time from the $(j-1)^{st}$ customer entering the APP until the occurrence of the j^{th} customer
$\{T_1,\ldots,T_j,\ldots,T_n,\ldots\}$	The sequence of interval times
τ	The mean time between two customers continuously entering the APP
N_0	at least customers number are required to enter the APP for operation within a time unit
U	$1/N_0$
A_{PI}	The APP performance index
q	The rate of non-compliance with performance requirements
A_0	The required performance value of the index A_{PI}
H_0	Null hypothesis
H_1	Alternative hypothesis
α	Significance level
A_{PI}^*	An unbiased estimator of A_{PI}
τ^*	The estimator of mean τ
$f_W(w)$	The probability density function of W

LA_{PI}	The lower confidence limit of A_{PI}
UA_{PI}	The upper confidence limit of A_{PI}
$(t_1,\ldots,t_j,\ldots,t_n)$	The observed value of $(T_1,\ldots,T_j,\ldots,T_n)$
A^*_{PI0}	The observed value of A^*_{PI}
τ^*_0	The observed value of τ^*
LA_{PI0}	The observed value of LA_{PI}
UA_{PI0}	The observed value of UA_{PI}
$\tilde{A}_{PI}[\alpha]$	The *α-cuts* of the triangular shaped fuzzy number $\tilde{A}_{PI}$
$\tilde{A}_{PI}$	The triangular shaped fuzzy number of A_{PI}
$\eta(x)$	The membership function of $\tilde{A}_{PI}$
A_T	The area in the graph of $\eta(x)$
A_S	The area in the graph of $\eta(x)$ but to the right of the vertical line $x = A_0$
d_T	$A_R - A_L$
d_S	$A_R - A_0$

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