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An Application of Neutrosophic Set to Relative Importance Assignment in AHP

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Abstract: The paper addresses a new facet of problem regarding the application of AHP in the real world. There are occasions that decision makers are not certain about relative importance assignment in pairwise comparison. The decision makers think the relative importance is among a set of scales, each of which is associated with a different possibility degree. A Discrete Single Valued Neutrosophic Number (DSVNN) with specified degrees of truth, indeterminacy, and falsity is employed to represent each assignment by taking into account all possible scales according to the decision maker's thought. Each DSVNN assignment is transformed into a crisp value via a deneutrosophication using a similarity-to-absolute-truth measure. The obtained crisp scales are input to a pairwise comparison matrix for further analysis. The proposed neutrosophic set-based relative importance assignment is another additional novelty of the paper, which is different from all prior studies focusing only on the definition of measurement scales. The presented assignment emulates the real-world approach of decision making in human beings which may consider more than one possibility. It is also shown herein that the single and crisp relative importance assignment in the original AHP by Saaty is just a special case of the proposed methodology. The sensitivity analysis informs that when decision makers have neither absolute truth nor falsity about a scale, the proposed methodology is recommended for obtaining reliable relative importance scale. The applicability of the proposed methodology to the real-world problem is shown through the investment in equity market.

Keywords: fuzzy and neutrosophic AHP; neutrosophic set; relative importance assignment; discrete single valued neutrosophic number (DSVNN); deneutrosophication; similarity measure; comparison matrix

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1. Introduction

Among the most popular Multi-Criteria Decision Making (MCDM) methods is Analytical Hierarchy Process (AHP) [1,2]. Vaidya and Kumar reviewed 150 publications, published in international journals between 1983 and 2003, and concluded that the AHP technique was useful for solving, selecting, evaluating, and making decisions [3]. Achieving a consensus decision despite the large number of decision makers is another advantage of the AHP [4]. The method is based on the concept of pairwise comparisons. The pairwise comparison determines how much more one criterion contributes to the decision goal with respect to the other for each pair of criteria. The relative importance scale is invented in linguistic terms with their respective associated numbers for numerical computation.

The lack of clearly defined boundary, the vital feature of fuzziness [5], in the linguistic terms of relative importance scale leads to the application of fuzzy set to AHP [6]. The fuzzy set [7,8], however, considers only truth of belongingness. The falsity of belongingness was appended to the fuzzy set, which has been referred to as intuitionistic fuzzy set

[9]. Nevertheless, the intuitionistic fuzzy set still lacks of the indeterminacy of belongingness. To resolve the drawbacks of fuzzy and intuitionistic fuzzy sets, neutrosophic set was proposed [10]. The neutrosophic set considers truth, indeterminacy, and falsity of belongingness and is thus more consistent with reality. In other words, truth, indeterminacy, and falsity correspond to membership, non-membership, and hesitancy, respectively. Accordingly, neutrosophic set allows paraconsistent, dialetheist, and incomplete information to be characterized in subsets which can be used to distinguish between relativity and absoluteness [11].

As an extension and further development, the neutrosophic set was applied to AHP [12,13]. The application is to redefine the numerical value for each linguist term using neutrosophic set. The relative importance scale is specifically referred to as neutrosophic scale. The crisp values of the scales are obtained using either score or accuracy function. Triangular neutrosophic sets with pre-determined degrees of truth, indeterminacy, and falsity are used [12]. There are different score and accuracy functions, e.g., [14–16]. They are used for ranking neutrosophic set. The results from such application are depicted in Table 1.

Table 1. Definition of neutrosophic scale [12].

Relative Importance to Goal	Saaty Scale	Crisp Scale by Score Function (Self-Calculation)	Crisp Scale by Accuracy Function (Self-Calculation)
Equally important/influential/preferable	1	0.56	0.94
Slightly important/influential/preferable	3	0.96	2.53
Strongly important/influential/preferable	5	4.59	5.34
Very strongly important/influential/preferable	7	7.09	7.61
Absolutely important/influential/preferable	9	10.13	10.13
Sporadic values between two close scales	2	0.86	1.76
	4	2.78	3.98
	6	4.84	6.19
	8	5.55	6.45

It should be noticed from Table 1 that the new definition of relative importance scale using neutrosophic sets creates a refinement of Saaty scale. Similar idea of defining relative importance scale using neutrosophic sets was proposed [13]. However, there are no predetermined truth, indeterminacy, and falsity degree in each scale. The decision maker is allowed to specify the degrees.

Once the relative importance scale is defined, another crucial step in the AHP is the assignment of relative importance to form a pairwise-comparison matrix. There are occasions that decision makers are not certain about the relative importance. For example, a decision maker is not sure whether a criterion is equally important/influential/preferable to or slightly more important/influential/preferable than another one with respect to goal contribution. The decision maker cannot decisively select just only one specific relative importance scale from those two. The question is how to realize the thought of the decision maker about more than one possibility of scale.

This paper presents a methodology to answer such a question. The problem and its solution of representing the preferential uncertainty in assigning relative importance are considered and introduced here. The assignment of relative importance is proposed to be the application of a neutrosophic set. Conceptually, all possibilities of relative importance scale according to decision maker thought are considered. The employed scales follow the definition by Saaty. The decision maker assigns a set of scales each of which is associated with the truth, indeterminacy, and falsity degree to indicate the relative importance. The assigned set is thus corresponding to a Discrete Single Valued Neutrosophic Number (DSVNN), i.e., Single Valued Neutrosophic Number (SVNN) with discrete membership

[17]. The deneutrosophication is later applied to the DSVNN, which results in a crisp scale. The obtained crisp scale is then input to the pairwise comparison matrix. It is emphasized that the present work is different from the aforementioned studies in that the present work applies the neutrosophic set to the relative importance assignment while the past studies focus on the new scale definitions. The contribution of this paper thus does not contradict the argument by Saaty regarding the fuzzifying AHP [18].

The study contributions can be summarized as follows:

1. The problem of preferential uncertainty in relative importance assignment for AHP is considered;
2. Propose DSVNN as a model of assignment; and
3. Illustrate the applications of DSVNN for such a purpose.

After this introduction, theoretical backgrounds related to neutrosophic set are described. The application of neutrosophic set theory to relative importance assignment in AHP is later explained. Illustrative examples are employed to clarify the concept and implementation. Finally, conclusions are drawn.

2. Preliminaries

2.1. Neutrosophic Set

Let U be an universe of discourse, then the neutrosophic set A is defined as $\widetilde{A} = \left\{\left\langle x : T_A(x), I_A(x), F_A(x), x \in U \right\rangle\right\}$, where the functions T, I, F: $U \to \left]^{-}0,1^{+}\right[$ define, respectively, the degree of membership (or truth), the degree of indeterminacy, and the degree of non-membership (or falsity) of the element $x \in U$ to the set $\widetilde{A}$ with the condition $^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}$ [11].

2.2. Single Valued Neutrosophic Set (SVNS)

Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS, $\widetilde{A}$, in X is characterized by a truth–membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity–membership function $F_A(x)$, for each point $x \in X, T_A(x), I_A(x), F_A(x) \in [0,1]$. Therefore, a SVNS $\widetilde{A}$ can be written a $\widetilde{A} = \left\{\left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle, x \in X\right\}$[17].

2.3. Discrete Single Valued Neutrosophic Number (DSVNN)

Let X be a space of points (objects) with generic elements in X denoted by x. A DSVNN, $\widetilde{A}$, in X is $\widetilde{A} = \sum_{i=1}^{N} \left\langle T_A(x_i), I_A(x_i), F_A(x_i) \right\rangle / x_i$ with $x_i \in X$. $X = \{x_1, \ldots, x_N\}$ is a discrete fuzzy set support. A DSVNN is thus a special neutrosophic set on the real number set R[17].

2.4. Similarity Measure

Similarity measure s for SVNS(X) is a real function on universe X such that $s : \mathrm{SVNX}(X) \times \mathrm{SVNX}(X) \to [0,1]$ and satisfies the following properties [19,20]

1. $0 \le s\left(\widetilde{A}, \widetilde{B}\right) \le 1$; $\forall \widetilde{A}, \widetilde{B} \in \mathrm{SVNS}(X)$
2. $s\left(\widetilde{A}, \widetilde{B}\right) = s\left(\widetilde{B}, \widetilde{A}\right)$; $\forall \widetilde{A}, \widetilde{B} \in \mathrm{SVNS}(X)$
3. $s\left(\widetilde{A}, \widetilde{B}\right) = 1$ if and only if $\widetilde{A} = \widetilde{B}$; $\forall \widetilde{A}, \widetilde{B} \in \mathrm{SVNS}(X)$

4. If $\widetilde{A} \subset \widetilde{B} \subset \widetilde{C}$, then $s(\widetilde{A},\widetilde{B}) \geq s(\widetilde{A},\widetilde{C})$ and $s(\widetilde{B},\widetilde{C}) \geq s(\widetilde{A},\widetilde{C})$; $\forall \widetilde{A},\widetilde{B},\widetilde{C} \in \mathrm{SVNS}(X)$

The four properties above cannot cope with the case where the similarity between the total affirmation and the total denial of the belongingness of an element to a given neutrosophic set is zero. Consequently, [21] there was a proposal to add another property to cover such a case. The proposed fifth property is

5. $s(\widetilde{A},\widetilde{B}) = 0$, if $\widetilde{A} = \{\langle x,1,0,0\rangle\}$ and $\widetilde{B} = \{\langle x,0,0,1\rangle\}$; $\forall \widetilde{A},\widetilde{B} \in \mathrm{SVNS}(X)$

which gives the sufficient condition for which the similarity between $\widetilde{A}$ and $\widetilde{B}$ will be zero.

To satisfy the five required properties of similarity measure, a novel similarity measure function is proposed [21]

$$s(\widetilde{A},\widetilde{B}) = 1 - \frac{1}{2N}\sum_{i=1}^{N}\left[|T_A(x_i) - T_B(x_i)| + max\{|I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\}\right] \quad (1)$$

Consider the case where $\widetilde{A} = \{\langle x,1,0,0\rangle\}$ and $\widetilde{B} = \{\langle x,0,0,1\rangle\}$, which represents the total affirmation and total denial of the belongingness, respectively. According to the definition in Equation (1),

$$s(\widetilde{A},\widetilde{B}) = 1 - \frac{1}{2}\left[|1-0| + max\{|0-0|, |0-1|\}\right] = 0 \quad (2)$$

which satisfies the property (v). Clearly, $s(\widetilde{A},\widetilde{B})$ according to the definition in Equation (1) satisfies all required properties of similarity measure as specified by (i)–(v) above.

2.5. Deneutrosiphication

The purpose of deneutrosophication is to convert a neutrosophic set to a single real number which represents the real output [22]. The deneutrosophication process consists of two steps:

1. Synthesization: It is the transformation f_{NF} applied to convert a neutrosophic set $\widetilde{A} = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$ into a fuzzy set $\overline{A}$. Accordingly, it is a mapping:

$$f_{NF}(T_A(x), I_A(x), F_A(x)): [0,1] \times [0,1] \times [0,1] \to [0,1] \quad (3)$$

A synthesization based on the similarity measure in Equation (1) is proposed [21] as follows. Let $\widetilde{A} = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$ be a neutrosophic set. Its equivalent fuzzy membership set is defined as $\overline{A} = \{\langle x, \mu_A(x)\rangle, x \in X\}$, where $\mu_A(x) := s((T_A(x), I_A(x), F_A(x)), (1,0,0))$. Consequently,

$$\mu_A(x) = 1 - \frac{1}{2}\left[|T_A(x) - 1| + max\{I_A(x), F_A(x)\}\right] \quad (4)$$

Since $\mu_A(x) \in [0,1], \forall x \in X$, the derived membership function (4) satisfies the property of the membership function of fuzzy set. The fundamental notion of using the similarity measure for the synthesization is that <1,0,0> indicates the total affirmation of belongingness or the full belongingness. The level of belongingness in fuzzy sets is reflected by the membership degree. The full belongingness is corresponding to membership degree of 1. With different mixtures of degrees in truth, indeterminacy, and falsity, the level

of belongingness varies. A mixture that results in a similarity very close to a full belongingness implies a high level of belongingness and is therefore corresponding to a high membership degree. In another extreme case, a mixture of <0,0,1> that indicates the complete falsity of belongingness yields the similarity of zero, i.e., the zero-membership degree, according to

$$\mu_A(x) = 1 - \frac{1}{2}\left[|0-1| + max\{0,1\}\right] = 0 \tag{5}$$

For a DSVNN $\widetilde{A} = \sum_{i=1}^{N} \langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle / x_i$, the membership function, according to Equation (1), is

$$\mu_A(x_i) = 1 - \frac{1}{2}\left[|T_A(x_i) - 1| + \max\{I_A(x_i), F_A(x_i)\}\right] \tag{6}$$

2. Defuzzification: From the synthesization in the first step, a fuzzy set $\overline{A} = \{\langle x, \mu_A(x) \rangle, x \in X\}$ is obtained, where the membership function is derived using the definition (4). Defuzzification is a process of converting a fuzzy set into a single crisp value. Many methods have been proposed in literature to perform defuzzification. The commonly used defuzzification methods are weighted average method, centroid method, and mean-max method [23]. Following the centroid method [24], the defuzzified value is calculated by the formula

$$A = \frac{\int \mu_A(x) x dx}{\int \mu_A(x) dx} \tag{7}$$

In case of DSVNN $\widetilde{A} = \sum_{i=1}^{N} \langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle / x_i$, a Discrete Fuzzy Set (DFS) $\overline{A} = \sum_{i=1}^{N} \mu_A(x_i) / x_i$ is obtained, in which the membership function is computed using the definition (6). The defuzzified value following the centroid method is

$$A = \frac{\sum_{i=1}^{N} x_i \mu_A(x_i)}{\sum_{i=1}^{N} \mu_A(x_i)} \tag{8}$$

In summary, the deneutrosiphication of a neutrosophic set results in a crisp value according to Equation (7) for a continuous neutrosophic set or Equation (8) for a discrete neutrosophic set. According to the presented concept in the synthesization step, the deneutrosopication is based on a similarity-to-absolute-truth measure.

3. Proposed Methodology

The application of DSVNN to the assignment of relative importance scale is described in this section. The steps are as follows:

1. A decision maker would like to assign the relative contribution to the goal $\widetilde{c}_{ij}$ of the *i*-th criterion compared to the *j*-th one;
2. The decision maker is not certain which single scale is suitable for the relative importance among possible scales $r_1, \ldots, r_{Nij}$. Each scale $r_q \ (q = 1, \ldots, N_{ij})$ takes just

one value from {1,2,3,4,5,6,7,8,9}. The value depends on the level of relative importance. In other words, the scales follow those defined by Saaty, Table 2;

3. In addition to the scale assignment, the decision maker has the degrees of truth $T_{ij}(r_q)$, indeterminacy $I_{ij}(r_q)$, and falsity $F_{ij}(r_q)$ for the scale r_q;
4. The relative importance $\widetilde{c}_{ij}$ is then represented by a DSNN $\widetilde{c}_{ij} = \sum_{q=1}^{Nij} \langle T_{ij}(r_q), I_{ij}(r_q), F_{ij}(r_q) \rangle / r_q$;
5. The DSNN $\widetilde{c}_{ij}$ undergoes the deneutrosiphication process according to Section 2.5 to obtain the corresponding crisp value c_{ij} which builds up a comparison matrix;
6. The relative importance c_{ji} is obtained from

$$c_{ji} = \frac{1}{c_{ij}} \tag{9}$$

7. The relative importance of the u-th alternative compared to the v-th one with respect to the s-th sub-criterion/criterion contribution $\widetilde{a}_{uv,s}$ can be modeled follows the steps 1–6 above.

Table 2. Definition of relative importance scale following Saaty [1].

Scale	Relative Importance to Goal
1	Equally important/influential/preferable
3	Slightly important/influential/preferable
5	Strongly important/influential/preferable
7	Very strongly important/influential/preferable
9	Absolutely important/influential/preferable
2 4 6 8	Sporadic values between two close scales

The proposed methodology is summarized in Figure 1.

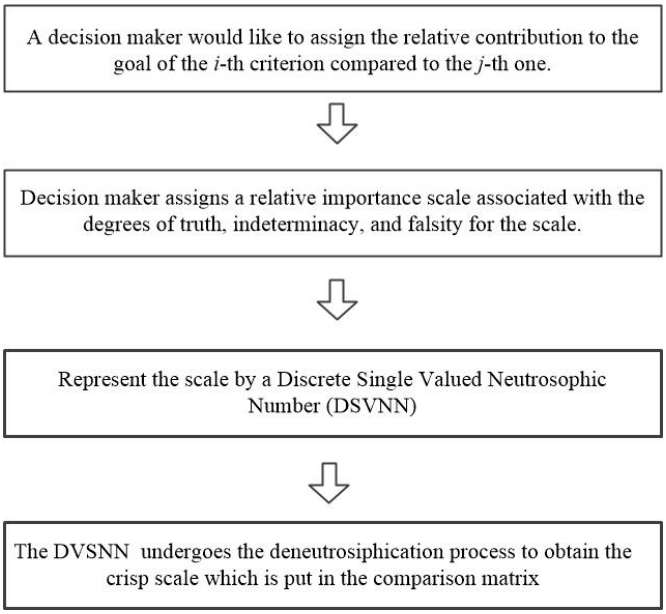

Figure 1. Summary of proposed methodology.

The computational algorithm according to the proposed methodology is summarized as follows.

1. For each comparison between the *i*-th criterion to the *j*-th criterion, select possible scales $r_1,\ldots,r_{Nij}$. Each scale $r_q\ (q=1,\ldots,N_{ij})$ takes just one value from {1,2,3,4,5,6,7,8,9} which is the set of relative importance scales following Saaty.
2. Assign the degrees of truth $T_{ij}(r_q)$, indeterminacy $I_{ij}(r_q)$, and falsity $F_{ij}(r_q)$ for each scale r_q. The relative importance $\tilde{c}_{ij}$ is then represented by a DSNN

$$\tilde{c}_{ij}=\sum_{q=1}^{Nij}\left\langle T_{ij}\left(r_q\right),I_{ij}\left(r_q\right),F_{ij}\left(r_q\right)\right\rangle/r_q \tag{10}$$

3. Compute the membership level $\mu_{ij}\left(r_q\right)$ of the scale r_q according to the formula (6), i.e.,

$$\mu_{ij}\left(r_q\right)=1-\frac{1}{2}\left[\left|T_{ij}\left(r_q\right)-1\right|+max\left\{I_{ij}\left(r_q\right),F_{ij}\left(r_q\right)\right\}\right] \tag{11}$$

4. Compute the final relative importance scale c_{ij} from Equation (8), i.e.,

$$c_{ij}=\frac{\sum_{q=1}^{Nij}r_q\mu_{ij}\left(r_q\right)}{\sum_{q=1}^{Nij}\mu_{ij}\left(r_q\right)} \tag{12}$$

5. The relative importance c_{ji} is obtained from

$$c_{ji}=\frac{1}{c_{ij}} \tag{13}$$

4. Illustrative Examples

4.1. Assignment with Certainty

In this example, the proposed methodology is applied to the case of AHP with the assignment of single and crisp scale, i.e., the original AHP of Saaty. The criteria that measure the failure risk of IT project are composed of people-related aspect, process-related aspect, technology-related aspect, and external-factor aspect [25]. Let C_1, C_2, C_3, and C_4 represent the criterion of the people-related aspect, the process-related aspect, the technology-related aspect, and the external-factor aspect, respectively. A decision maker would like to compare the relative contribution to the goal of the first criterion over the second one. The decision maker is certain that the first criterion is strongly influential to the failure risk over the second one. Based on Table 2, the relative importance is equal to 5.

Next, closer inspection of the section illustrates that the proposed methodology yields the same result as in the crisp scale of Saaty. Since the decision maker has certainty in assigning the relative importance, the degrees of truth, indeterminacy, and falsity are:

$$T_{c_{12}}\left(r_1\right)=1,\ I_{c_{12}}\left(r_1\right)=0,\ \text{and}\ F_{c_{12}}\left(r_1\right)=0 \tag{14}$$

In addition, the strongly influential level of the first criterion over the second one is equivalent to the scale $r_1=5$ according to Table 2. The DSNN representation of the assignment is thus

$$\widetilde{c}_{12} = \langle 1,0,0 \rangle / 5 \tag{15}$$

Applying Equation (6) yields

$$\mu_{C12}(r_1) = 1 - \frac{1}{2}\left[|1-1| + \max\{0,0\}\right] = 1 \tag{16}$$

The defuzzified value according to the formula (8) is

$$c_{12} = \frac{5 \times 1}{1} = 5 \tag{17}$$

Obviously, the assignment of relative importance using neutrosophic set gives the same relative importance of 5. It can be said that the proposed methodology treats the relative importance assignment by Saaty as a special case.

4.2. Assignment with Uncertainty

This example considers the same problem and criteria as those of 4.1. The criteria that measure the failure risk of IT project are composed of people-related aspect, process-related aspect, technology-related aspect, and external-factor aspect [25]. Let C1, C2, C3, and C4 represent the criterion of the people-related aspect, the process-related aspect, the technology-related aspect, and the external-factor aspect, respectively.

The decision maker wants to compare the relative contribution to the goal of the first criterion over the third one. However, the decision maker cannot decisively select only one specific scale to characterize the relative importance. The decision maker thinks there are two possibilities with different degrees of truth, indeterminacy, and falsity, which may fit the relative importance, namely strongly influential and very strongly influential. The detail of the scale assignment is given in Table 3.

Table 3. The description of scale assignment for the first criterion over the third one.

Possibility	Level	Scale	Truth Degree	Indeterminacy Degree	Falsity Degree
r_1	Strongly influential	5	0.7	0.2	0.3
r_2	Very strongly influential	7	0.6	0.3	0.2

The DSNN representation of the assignment is

$$\widetilde{c}_{13} = \langle 0.7,0.2,0.3 \rangle / 5 + \langle 0.6,0.3,0.2 \rangle / 7 \tag{18}$$

The membership function is

$$\mu_{C13}(r_1) = 1 - \frac{1}{2}\left[|0.7-1| + \max\{0.2,0.3\}\right] = 0.7 \tag{19}$$

$$\mu_{C13}(r_2) = 1 - \frac{1}{2}\left[|0.6-1| + \max\{0.3,0.2\}\right] = 0.65 \tag{20}$$

The defuzzified value is

$$c_{13} = \frac{0.7 \times 5 + 0.65 \times 7}{0.7 + 0.65} = 5.96 \tag{21}$$

The number of possible scales needs not be the same for different pairs of relative importance comparison. In the following, an extreme case of three possible scales is shown. The example of three scales is intended only for an illustrative purpose and may

not occur in reality. In this exemplified case, the decision maker thinks there are three possibilities that may fit the relative importance of the first criterion compared to the fourth one. The detail of the scale assignment is shown in Table 4.

Table 4. The description of scale assignment for the first criterion over the fourth one.

Possibility	Level	Scale	Truth Degree	Indeterminacy Degree	Falsity Degree
r_1	Equally influential	1	0.4	0.6	0.2
r_2	Slightly influential	3	0.5	0.4	0.1
r_3	Strongly influential	5	0.4	0.5	0.2

Following the same procedure as above, one obtains

$$\widetilde{c}_{14} = \langle 0.4,0.6,0.2\rangle/1 + \langle 0.5,0.4,0.1\rangle/3 + \langle 0.4,0.5,0.2\rangle/5 \tag{22}$$

$$\mu_{C14}(r_1) = 1 - \frac{1}{2}\left[|0.4-1| + \max\{0.6,0.2\}\right] = 0.4 \tag{23}$$

$$\mu_{C14}(r_2) = 1 - \frac{1}{2}\left[|0.5-1| + \max\{0.4,0.1\}\right] = 0.55 \tag{24}$$

$$\mu_{C14}(r_3) = 1 - \frac{1}{2}\left[|0.4-1| + max\{0.5,0.2\}\right] = 0.45 \tag{25}$$

$$c_{14} = \frac{0.4\times 1 + 0.55\times 3 + 0.45\times 5}{0.4 + 0.55 + 0.45} = 3.07 \tag{26}$$

It can be noticed from the membership function formula that the membership degree is deteriorated by the indeterminacy and falsity degrees.

The preceding illustrative examples show that the neutrosophic set and the fuzzy set has a relationship through the similarity measure as defined by Equation (6). Given the truth, indeterminacy, and falsity degree of a neutrosophic set, the corresponding membership level for fuzzy set can be obtained from that relationship. The membership level is thus extensively expressed in terms of truth, indeterminacy, and falsity possibility. In other words, the fuzzy set is characterized by three possibilities of belongingness levels.

4.3. Determination of Criteria Weights

This example shows the complete application of a neutrosophic set to relative importance assignment in AHP. It was shown that the AHP could be used as a means of determining the criterion weights [26]. Correspondingly, the AHP will be used for such a purpose herein. Consider again the same criteria as in all previous examples. The relative importance scales of the 1st criterion compared to the 2nd, 3rd, and 4th ones take the values as obtained in Equation (17), Equation (21), and Equation (26), respectively. The scales are summarized again below:

$$c_{12} = 5 \tag{27}$$

$$c_{13} = 5.96 \tag{28}$$

$$c_{14} = 3.07 \tag{29}$$

Suppose the decision make assigns the remaining elements in the comparison matrix to be:

$$\widetilde{c}_{32} = \langle 0.7,0.4,0.2\rangle / 1 + \langle 0.3,0.3,0.1\rangle / 2 \tag{30}$$

$$\widetilde{c}_{42} = \langle 1,0,0\rangle / 2 \tag{31}$$

$$\widetilde{c}_{43} = \langle 1,0,0\rangle / 2 \tag{32}$$

The deneutrosophication of these elements yields

$$\mu_{C32}(r_1) = 1 - \frac{1}{2}\left[|0.7-1| + \max\{0.4,0.2\}\right] = 0.65 \tag{33}$$

$$\mu_{C32}(r_2) = 1 - \frac{1}{2}\left[|0.3-1| + \max\{0.3,0.1\}\right] = 0.5 \tag{34}$$

$$c_{32} = \frac{0.65\times 1 + 0.5\times 2}{0.65 + 0.5} = 1.44 \tag{35}$$

$$c_{42} = 2 \tag{36}$$

$$c_{43} = 2 \tag{37}$$

Consequently, the deneutrosophicated comparison matrix is

$$\boldsymbol{C} = \begin{bmatrix} 1 & 5 & 5.96 & 3.07 \\ 1/5 & 1 & 1/1.44 & 1/2 \\ 1/5.96 & 1.44 & 1 & 1/2 \\ 1/3.07 & 2 & 2 & 1 \end{bmatrix} \tag{38}$$

It is noted that the application of the deneutrosophication/defuzzification process to neutrosophic/fuzzy comparison matrix before determining criterion weights is one of standard and highly referred approaches in the context of fuzzy AHP, see e.g., [27]. Afterwards, the weight determination readily follows the original AHP by Saaty. The notion of transforming to crisp comparison matrix before determining criterion weights has been also adopted by previous applications of neutrosophic set to AHP [12,13].

According to the original AHP by Saaty, the criterion weights are obtained from solving the eigenvalue problem:

$$\boldsymbol{C}\boldsymbol{\Psi} = \lambda\boldsymbol{\Psi} \tag{39}$$

The vector of criterion weights $\mathbf{W} = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}^T$ is obtained from the normalization of the right principal eigenvector $\boldsymbol{\Psi}_{max}$ corresponding to the principal eigenvector λ_{max} of $\boldsymbol{C}$.

The consistency of the pairwise comparison through $\boldsymbol{C}$ is measured through the consistency ratio *CR* of which the definition is

$$CR = \frac{CI}{RI} \tag{40}$$

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{41}$$

RI is the random consistency index which is the function of *n* (Table 5).

Table 5. Random consistency index (*RI*) [25].

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

The pairwise comparison is considered to be sufficiently consistent if $CR \le 0.1$, otherwise the pairwise comparison matrix $\boldsymbol{C}$ needs to be revised [28].

The obtained criterion weight vector is $\boldsymbol{W} = \begin{bmatrix} 0.5876 & 0.0979 & 0.1124 & 0.2021 \end{bmatrix}^T$ while $CR = 0.01$ which implies that the comparison matrix of the criteria is almost fully consistent.

5. Investment in Equity Market: A Real-World Application

The evaluation and selection of equities (stocks and shares) are complex decision processes, which involve a large number of environmental variables and considerations arising from individual psychology and experience. [29] Criteria for the evaluation and selection of equities depend in the first place on the type of stock in question and the investment environment. To construct such a set of criteria, the performance factors identified to be empirically important in equity markets by Nagy and Obenberger [30] are employed. These include in particular: (1) expected earnings, (2) image, (3) track record, (4) quantity and quality of accounting information, (5) risk, and (6) economic indicators. Items (1)–(5) are specific to the individual firm, while (6) is indicative of broader economic activity and its effects. The description of criteria is indicated in Table 6.

Table 6. Fundamental equity investment criteria [30].

Criterion	Description
Image	Perceived market image of firm
Indicators	Key economic indicators
Earnings	Acceptable level of expected earnings
Track	Business track record of firm
Risk	Acceptable level of investment risk
Account	Reliable accounting information

Let the image, indicators, earnings, track, risk, and account be defined as the criterion c_1 to c_6, respectively. The comparison between each criterion is carried out as follows:

$$\widetilde{c}_{12} = \langle 0.5,0.2,0.4 \rangle / 2 + \langle 0.9,0.2,0.2 \rangle / 3 \tag{42}$$

$$\widetilde{c}_{15} = \langle 0.3,0.1,0.6 \rangle / 1 + \langle 0.8,0.2,0.1 \rangle / 2 \tag{43}$$

$$\widetilde{c}_{16} = \langle 0.5,0.2,0.5 \rangle / 3 + \langle 0.9,0.3,0.3 \rangle / 4 \tag{44}$$

$$\widetilde{c}_{34} = \langle 0.8,0.2,0.3 \rangle / 1 + \langle 0.5,0.3,0.3 \rangle / 2 \tag{45}$$

$$\widetilde{c}_{35} = \langle 0.3,0.1,0.6 \rangle / 3 + \langle 0.9,0.3,0.2 \rangle / 4 \tag{46}$$

$$\widetilde{c}_{36} = \langle 0.5,0.3,0.4 \rangle / 3 + \langle 0.8,0.2,0.1 \rangle / 4 \tag{47}$$

$$\widetilde{c}_{45} = \langle 0.9,0.2,0.4 \rangle / 5 + \langle 0.5,0.3,0.5 \rangle / 6 \tag{48}$$

$$\widetilde{c}_{46} = \langle 0.6,0.2,0.4 \rangle / 3 + \langle 0.4,0.2,0.2 \rangle / 4 \tag{49}$$

$$\widetilde{c}_{56} = \langle 0.5,0.2,0.4 \rangle / 1 + \langle 0.7,0.1,0.2 \rangle / 2 \tag{50}$$

The decision maker is certain about the remaining scales which are thus crisp values. The values of the crisp scales are given in the comparison matrix $\boldsymbol{C}$ below. Based on the DSNN scales from (42) to (50), the deneutrosophicated scales according to the computational algorithm are summarized in Table 7.

Table 7. Resulting deneutrosophicated scales.

Comparison	DSNN Scale	T	I	F	μ	Deneutrosophicated Scale
1 to 2	2	0.5	0.2	0.4	0.55	2.61
	3	0.9	0.2	0.2	0.85	
1 to 5	1	0.3	0.1	0.6	0.35	1.70
	2	0.8	0.2	0.1	0.8	
1 to 6	3	0.5	0.2	0.5	0.5	3.62
	4	0.9	0.3	0.3	0.8	
3 to 4	1	0.8	0.2	0.3	0.75	1.44
	2	0.5	0.3	0.3	0.6	
3 to 5	3	0.3	0.1	0.6	0.35	3.70
	4	0.9	0.3	0.2	0.8	
3 to 6	3	0.5	0.3	0.4	0.55	3.59
	4	0.8	0.2	0.1	0.8	
4 to 5	5	0.9	0.2	0.4	0.75	5.40
	6	0.5	0.3	0.5	0.5	
4 to 6	3	0.6	0.2	0.4	0.6	3.50
	4	0.4	0.2	0.2	0.6	
5 to 6	1	0.5	0.2	0.4	0.55	1.58
	2	0.7	0.1	0.2	0.75	

All the deneutrosophicated and crisp scales are input together in the comparison matrix $\boldsymbol{C}$ below.

$$\boldsymbol{C} = \begin{bmatrix} 1 & 2.61 & 1/2 & 1/2 & 1.70 & 3.62 \\ 1/2.61 & 1 & 1/3 & 1/3 & 1/2 & 1/2 \\ 2 & 3 & 1 & 1.44 & 3.70 & 3.59 \\ 2 & 3 & 1/1.44 & 1 & 5.40 & 3.50 \\ 1/1.70 & 2 & 1/3.70 & 1/5.40 & 1 & 1.58 \\ 1/3.62 & 2 & 1/3.59 & 1/3.50 & 1/1.58 & 1 \end{bmatrix} \quad (51)$$

Since $CR = 0.04$ is less than 0.1, the comparison matrix of the criteria is therefore almost fully consistent.

Apart from the illustrated application, the proposed methodology can also be used for other real-world decision-making problems.

6. Sensitivity Analysis

Since the similarity measure for determining the membership level (6) is the key factor in the defuzzification, its sensitivity analysis is therefore interesting. A numerical simulation is carried out by uniformly generating the grids of the truth, indeterminacy, and falsity degree within an interval [0,1]. Each sample is then used in the computation of the membership function according to (6). The cases of full and non-belongingness are considered first. The results are shown in Table 8 to Table 9.

Table 8. The truth, indeterminacy, and falsity degree for full belongingness.

Truth Degree	Indeterminacy Degree	Falsity Degree	μ_A
1	0	0	1

Table 9. The truth, indeterminacy, and falsity degree for non-belongingness.

Truth Degree	Indeterminacy Degree	Falsity Degree	μ_A
0	0	1	0
0	1	0	0

Table 8 shows that the full belongingness happens when the truth degree is from 0.9 to 1 and the remaining degrees are within an interval of 0 to 0.1. Table 9 shows that the non-belongingness happens when truth degree is 0. For other combinations of truth, indeterminacy, and falsity degree, the level of belongingness has values that are not close to 0 or 1. Examples of such combinations are shown in Table 10. From the sensitivity analysis, useful information can be drawn:

1. When decision makers have absolute indeterminacy about a scale, i.e., indeterminacy degree of 1, their selections of that scale have no effect on final result.
2. When decision makers have neither absolute truth nor falsity about a scale, the proposed methodology is recommended for obtaining final relative importance scale.
3. The full belongingness happens when the truth degree is equal to 1 and the non-belongingness takes place when the falsity degree is equal to 1.

The information above is corresponding to reality which implies the reliability of the proposed methodology.

Table 10. Examples of truth, indeterminacy, and falsity degree combination for non-zero and non-one belongingness levels.

Truth Degree	Indeterminacy Degree	Falsity Degree	μ_A
0.6	0	0.8	0.4
0.6	0.2	0.8	0.4
0.6	0.4	0.8	0.4
0.6	0.6	0.8	0.4
0.6	0.8	0	0.4
0.6	0.8	0.2	0.4
0.6	0.8	0.4	0.4
0.6	0.8	0.6	0.4
0.6	0.8	0.8	0.4
0.6	0	0.6	0.5
0.6	0.2	0.6	0.5
0.6	0.4	0.6	0.5
0.6	0.6	0	0.5
0.6	0.6	0.2	0.5
0.6	0.6	0.4	0.5
0.6	0.6	0.6	0.5
0.6	0	0.4	0.6
0.6	0.2	0.4	0.6
0.6	0.4	0	0.6
0.6	0.4	0.2	0.6
0.6	0.4	0.4	0.6

7. Conclusions

The problem of assigning scale to characterize relative importance in pairwise comparison was addressed. Decision makers are uncertain which single and crisp scales should be assigned to pairwise comparison. The decision makers think there is more than one scale that may possibly fit the relative importance. To take into account all possibilities of scale according to the decision maker's thought, the application of a neutrosophic set is introduced. According to the proposed methodology, each assignment is represented by a Discrete Single Valued Neutrosophic Number (DSVNN). A DSVNN not only represents an all-possible thoughts scale but also takes into account the degrees of truth, indeterminacy, and falsity for each possibility. Each DSVNN assignment is transformed into a crisp value via a deneutrosophication using a similarity-to-absolute-truth measure. The resulting deneutrosophicated values then form a crisp comparison matrix for further analysis. The single and crisp relative importance assignment in the original AHP of Saaty is considered a special case of the proposed neutrosophic set-based methodology. The sensitivity analysis informs that when decision makers have neither absolute truth nor falsity about a scale, the proposed methodology is recommended for obtaining reliable relative importance scale. The applicability of the proposed methodology to the real-world problem is shown through the investment in the equity market.

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