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Robust Nonlinear Position Control with Extended State Observer for Single-Rod Electro-Hydrostatic Actuator

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Abstract: In the existing literature, studies on position controller design using only position feedback, considering the disturbances for single-rod electro-hydrostatic actuators (EHAs), have not been reported. Herein, we propose a robust nonlinear position control with an extended state observer (ESO) for single-rod EHAs. A new EHA model that consists of position, velocity, and acceleration with an internal state variable is developed. Instead of the separated port pressure dynamics, the acceleration dynamics were defined. The external disturbance, model, and input function uncertainties were lumped into a disturbance. An ESO is developed to estimate the disturbance, as well as the position, velocity, and acceleration. In practice, it is difficult to accurately estimate the disturbance because it includes the external disturbance, system dynamics, and input function uncertainty. The poor estimation performance may degrade the position tracking performance, but a high gain cannot be used to suppress the estimation error because of the measurement noise amplification. To resolve this problem, a robust nonlinear position controller is developed via a backstepping procedure. In the controller, a nonlinear gain is implemented to sufficiently suppress position tracking without the use of a high gain. The stability of the closed-loop system is mathematically proven using the input-to-state stability. The proposed method is simple and suitable for real-time control.

Keywords: electro-hydrostatic actuator; position measurement; state estimation; position control

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1. Introduction

In various industries, electro-hydraulic systems (EHSs) have been widely used as actuators because of their high power density, flexibility, and high stiffness, compared to their electrical counterparts [1]. Valve-controlled hydraulic systems are widely used, owing to their simplicity, but they have several problems such as low energetic efficiency (because of throttling losses), maintenance load, limited installation space, high weight, and environmental pollution (due to the leakage of the working fluid) [2]. To overcome these problems, electro-hydrostatic actuators (EHAs) have been developed by integrating an electric motor instead of a valve in valve-controlled hydraulic system. In EHAs, integrating the electric motor can lead to several advantages, such as smaller size, higher energy efficiency, and faster response due to the higher stiffness compared to valve-controlled hydraulic actuators. However, it is difficult to control the EHSs because the compressibility of the hydraulic fluid and the complex flow properties result in higher nonlinearities in their dynamics.

Various control methods have been proposed to improve the control performance of EHSs. A linear control method has been designed for hydraulic systems [1]. In [3,4], control methods using variable structure control (VSC) were developed for controlling the EHS. For disturbance compensation, a robust adaptive tracking control method was designed [5]. Control methods using input-output linearization have been developed to avoid the nonlinearities of EHSs [6–8]. A nonlinear control method was recently developed using the flatness property to improve the position-tracking performance [9]. The EHS

model is in the form of strict feedback. Thus, various backstepping control methods have been developed using the EHS model properties [10–12]. An adaptive sliding mode control scheme was designed to resolve the nonlinearity problem of changes in the EHA [13]. In [14], a nonlinear adaptive position control method was developed based on a sliding mode back-stepping design method for the EHA. Although they improved the control performance, these methods require both full-state feedback and system parameter information. Thus, nonlinear controllers with observers have been designed to improve the control performance of the EHS using only output feedback [15,16]. However, external disturbances (i.e., load force and friction) and parameter uncertainties were not considered in previous methods. To resolve these problems, several control methods have been developed to address external disturbances and parameter uncertainties. In [17,18], nonlinear control methods using singular perturbation theory were proposed for robustness against the effective bulk modulus uncertainty. Adaptation control methods have been established to compensate for uncertainties in EHS parameters [19–21]. Although these methods are robust against parameter uncertainties, they still require a full state. Furthermore, to the best of our knowledge, studies regarding position controller design using only position feedback, considering the disturbances for single-rod EHAs, have not been reported.

In this paper, we propose a robust nonlinear position control with an extended state observer (ESO) for single-rod EHAs. A new EHA model that consists of position, velocity, and acceleration with an internal state variable is proposed. The acceleration dynamics were defined instead of the separated port-pressure dynamics. The external disturbance, model, and input function uncertainties were lumped into a single disturbance. An ESO is developed to estimate the disturbance as well as the position, velocity, and acceleration. In practice, it is difficult to accurately estimate the disturbance because the disturbance includes the external disturbance, the system dynamics, and the input function uncertainty. Furthermore, poor estimation performance may degrade the position tracking performance, but the high gain to suppress the estimation error cannot be used because of the measurement noise amplification. To resolve this problem, a robust nonlinear position controller is developed via a backstepping procedure. In the controller, a nonlinear gain is implemented to sufficiently suppress position tracking without the use of a high gain. The stability of the closed-loop system is mathematically proven using the input-to-state stability (ISS). Only a nominal value of the input function is required in the proposed method. Thus, the proposed method is robust against the model uncertainties and the external disturbance. The proposed method is simple and suitable for real-time control. The performance of the proposed method is validated via simulations.

2. System Model and Problem Formulation

2.1. Electro-Hydrostatic Actuator Model

The structure of the single-rod electro-hydrostatic actuator is shown in Figure 1. The mathematical model of the EHA dynamics is represented as follows [16]:

$$\begin{aligned}\dot{x}_p &= x_v \\ \dot{x}_v &= \frac{1}{m}[-kx_p - bx_v + S_A P_A - S_B P_B - d_L + d_F] \\ \dot{P}_A &= \frac{\beta}{V_A(x_p)}[D_p \omega_m - S_A x_v - C_p P_A + C_p P_B] \\ \dot{P}_B &= \frac{\beta}{V_B(x_p)}[-D_p \omega_m + S_B x_v + C_p P_A - C_p P_B]\end{aligned}\quad (1)$$

where x_p is the piston position [m], x_v is the piston velocity [m/s], P_A and P_B are the pressures of both chambers [N/m²], ω_m is the rotational speed of the electric motor [rad/s], m is the total mass of the piston and load [kg], k is the load spring constant [N/m], b is the viscous damping coefficient [N/m/s], d_L is the load force [N], d_F is the friction force [N], D_p is the volumetric capacity of the pump [m³/rad], β_e is the effective bulk modulus of the

system $[\text{N}/\text{m}^2]$, C_p is the leakage coefficient of the pump $[\text{m}^5/\text{Ns}]$, and $V_A(x_p) = V_{A0} + S_A x_p$ and $V_B(x_p) = V_{B0} - S_B x_p$ are the volumes of chambers A and B $[\text{m}^3]$, respectively. In the EHA (1), x_p is physically bounded as:

$$-x_{p\lim} \leq x_p \leq x_{p\lim} \quad (2)$$

where $x_{p\lim}$ is a positive constant. The ranges of $V_A(x_p)$ and $V_B(x_p)$ are defined as:

$$\begin{aligned} V_{A0} - S_A x_{p\lim} &= V_{A\min} \leq V_A(x_p) \leq V_{A\max} = V_{A0} + S_A x_{p\lim} \\ V_{B0} - S_B x_{p\lim} &= V_{B\min} \leq V_B(x_p) \leq V_{B\max} = V_{B0} + S_B x_{p\lim}. \end{aligned} \quad (3)$$

Figure 1. Structure of the single-rod electro-hydrostatic actuator.

For simplification, (1) can be rewritten as:

Assumption 1. The derivatives of the load force F_L and friction F_F with respect to time exist and are bounded.

2.2. Problem Formulation

In this paper, it is assumed that only the position is measurable. Now we define the new state variables z and x as:

$$\begin{aligned} z &= V_A P_A + V_B P_B \\ x &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_p \\ x_v \\ \dot{x}_v \end{bmatrix}. \end{aligned} \quad (4)$$

From (4) the relationships can be obtained as:

$$\begin{aligned} x_p &= x_1 \\ x_v &= x_2 \\ P_A &= \frac{V_B k x_1 + V_B b x_2 + V_B m x_3 + S_B z + V_B d_L - V_B d_F}{S_A V_B + S_B V_A} \\ P_B &= \frac{-V_A k x_1 - V_A b x_2 - V_A m x_3 + S_A z - V_A d_L + V_B d_F}{S_A V_B + S_B V_A}. \end{aligned} \quad (5)$$

From (1) and (5), we obtain the dynamics of z and x_3 as:

$$\begin{aligned}\dot{z} &= (kx_1 + bx_2 + mx_3 + d_L - d_F - \beta S_A + \beta S_B)x_2 \\ \dot{x}_3 &= \frac{1}{m}[-kx_2 - bx_3 + S_A \dot{P}_A - S_B \dot{P}_B - \dot{d}_L + \dot{d}_F] \\ &= f(z, x, \dot{d}_L, \dot{d}_F) + \underbrace{\beta D_p \left(\frac{S_A}{V_A(x_1)} + \frac{S_B}{V_B(x_1)} \right)}_{g(x_1)} u\end{aligned}\quad (6)$$

where u is the control input and is defined as $u = \omega_m$. With (4) and (5), the EHA dynamics (1) are transformed into:

$$\begin{aligned}\dot{z} &= (kx_1 + bx_2 + mx_3 + d_L - d_F - \beta S_A + \beta S_B)x_2 \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= f(z, x, \dot{d}_L, \dot{d}_F) + g(x_1)u.\end{aligned}\quad (7)$$

Remark 1. In (7), z is the internal state variable for the control of x_1 . Generally, repetitive signals, i.e., sinusoidal signals or constant values are used for the desired position. Thus, the zero dynamics of z are bounded. Consequently, only controls of x_1 , x_2 , and x_3 are required for position tracking control. In (7), x_1 , x_2 , and x_3 are controlled by the proposed control method.

Remark 2. In $g(x_1)$, D_p , S_A , S_B , $V_A(x_1)$, and $V_B(x_1)$ can be easily known because they are mechanical fixed parameters. The effective bulk modulus β is also a known and fixed parameter. However, in practice, β is difficult to be exactly known. Furthermore, due to temperature variations and air entrapment, β may vary during the operation of the hydraulic system [17].

The uncertainty and nominal value of $g(x_1)$ are defined as $\Delta g(x_1)$ and g_0 so that $g(x_1) = g_0 + \Delta g(x_1)$. Because $V_A(x_1)$, and $V_B(x_1)$ are bounded, $\Delta g(x_1)$ is bounded.

Assumption 2. Among all parameters, only the nominal value of $g(x_1)$ is known.

Let us define the lumped disturbance d as $d = f(z, x, \dot{d}_L, \dot{d}_F) + \Delta g(x_1)u$. The dynamics of x are obtained as:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= g_0 u + d.\end{aligned}\quad (8)$$

The main goal in the controller design is the position tracking control using only x_1 feedback and g_0 information.

3. Controller Design

3.1. Nonlinear Damping Position Controller

For the controller design, we use the following assumption as follows:

Assumption 3. x_1 , x_2 , and x_3 are available. The estimated disturbance, $\hat{d}$ is bounded.

x_1 , x_2 , x_3 , and d are estimated by the observer that will be designed in the next subsection. The tracking error $e = [e_1 \ e_2 \ e_3]^T$ is defined as:

$$\begin{aligned} e_1 &= x_1 - x_{1_d} \\ e_2 &= x_2 - x_{2_d} \\ e_3 &= x_3 - x_{3_d} \end{aligned} \quad (9)$$

where $x_{1_d} = x_{p_d}$ is the desired position, and both the desired velocity, x_{2_d} and desired acceleration, x_{3_d} are yet to be defined. The tracking error dynamics are:

$$\begin{aligned} \dot{e}_1 &= e_2 + x_{2_d} - \dot{x}_{1_d} \\ \dot{e}_2 &= e_3 + x_{3_d} - \dot{x}_{2_d} \\ \dot{e}_3 &= g_0 u + d - \dot{x}_{3_d}. \end{aligned} \quad (10)$$

For the position tracking, the controller is designed as:

$$\begin{aligned} x_{2_d} &= -k_1 e_1 + \dot{x}_{1_d} \\ x_{3_d} &= -k_2 e_2 + \dot{x}_{2_d} \\ u &= \underbrace{\frac{1}{g_0} (-k_3 e_3 + \dot{x}_{3_d} - \hat{d})}_{u_a} + \underbrace{\frac{1}{g_0} (k_d(\hat{e}_1, \hat{d}) e_3)}_{u_b}. \end{aligned} \quad (11)$$

where control gains, $k_1, k_2, k_3, k_{d_1}, k_{d_2}, v_1$ and v_2 are positive, $\hat{e}_1 = \hat{x}_1 - x_{1_d}$ is the estimated output tracking error, $\hat{x}_1$ is the estimation of x_1 , and $k_d(\hat{e}_1, \hat{d}) = \left(k_{d_1} \sqrt{\hat{e}_1^2 + v_1} + k_{d_2} \sqrt{\hat{d}^2 + v_2} \right)$ is the nonlinear damping gain.

Remark 3. In (11), u_a is used for stabilization and the other part u_b is used to suppress the output tracking error as the estimation error of the disturbance is increased. In practice, because the external disturbances and system dynamics are lumped in the disturbance, it may be difficult to accurately estimate d . As long as the disturbance estimation error $\tilde{d}$ increases, the estimation error, estimated disturbance $\hat{d}$, and estimated output tracking error $\hat{x}_1$ may also increase. Thus, the nonlinear damping gain $k_d(\hat{e}_1, \hat{d})$ is designed to suppress the tracking error when $\hat{e}_1$ and $\hat{d}$ are large.

Theorem 1. Suppose that the EHA dynamics (8). Under Assumptions 1–3, if the control law (11) is used to the EHA dynamics (8), then the tracking error has the following ISS properties:

$$\begin{aligned} |e_1(t)| &\leq \exp\left(-\frac{k_1}{2}t\right) |e_1(0)| + \frac{2}{k_1} \sup_{0 \leq \tau \leq t} e_2(\tau) \\ |e_2(t)| &\leq \exp\left(-\frac{k_2}{2}t\right) |e_2(0)| + \frac{2}{k_2} \sup_{0 \leq \tau \leq t} e_3(\tau) \\ |e_3(t)| &\leq \exp\left(-\frac{k_3}{2}t\right) |e_3(0)| + \frac{2}{k_3} \sup_{0 \leq \tau \leq t} \sigma(\tau). \end{aligned} \quad (12)$$

where $\sigma = \frac{|d - \hat{d}|}{0.5k_3 + k_d(\hat{e}_1, \hat{d})}$.

Proof. With control law (11), the tracking error dynamics (10) become:

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 + e_2 \\ \dot{e}_2 &= -k_2 e_2 + e_3 \\ \dot{e}_3 &= -k_3 e_3 - k_d(\hat{e}_1, \hat{d}) e_3 + d - \hat{d}. \end{aligned} \quad (13)$$

From (13), we have:

$$\begin{aligned}\frac{d}{dt}\left(\frac{e_1^2}{2}\right) &= -k_1 e_1^2 + e_1 e_2 \\ &\leq -\frac{k_1}{2} e_1^2 - \frac{k_1}{2} |e_1| \left(|e_1| - \frac{2}{k_1} |e_2|\right) \\ \frac{d}{dt}\left(\frac{e_2^2}{2}\right) &= -k_2 e_2^2 + e_2 e_3 \\ &\leq -\frac{k_2}{2} e_2^2 - \frac{k_2}{2} |e_2| \left(|e_2| - \frac{2}{k_2} |e_3|\right).\end{aligned}\quad (14)$$

Then, the following results are derived as:

$$\begin{aligned}|e_1(t)| &\leq \exp\left(-\frac{k_1}{2}t\right)|e_1(0)| + \frac{2}{k_1} \sup_{0 \leq \tau \leq t} e_2(\tau) \\ |e_2(t)| &\leq \exp\left(-\frac{k_2}{2}t\right)|e_2(0)| + \frac{2}{k_2} \sup_{0 \leq \tau \leq t} e_3(\tau).\end{aligned}\quad (15)$$

From (15) it is guaranteed that the relationships between e_1 and e_2 , and e_2 and e_3 have ISS property. The dynamics of e_3^2 are:

$$\begin{aligned}\frac{d}{dt}\left(\frac{e_3^2}{2}\right) &= -k_3 e_3^2 - k_d(\hat{e}_1, \hat{d}) e_3^2 + (d - \hat{d}) e_3 \\ &\leq -\frac{k_3}{2} e_3^2 - \left(\frac{k_3}{2} + k_d(\hat{e}_1, \hat{d})\right) |e_3| (|e_3| - \sigma).\end{aligned}\quad (16)$$

Then,

$$|e_3(t)| \leq \exp\left(-\frac{k_3}{2}t\right)|e_3(0)| + \frac{2}{k_3} \sup_{0 \leq \tau \leq t} \sigma(\tau).\quad (17)$$

(17) shows that the relationship between e_3 and σ has the ISS property. From (15) and (17), the ISS property of the overall tracking error system is (12). $\square$

Remark 4. In (12), it is observed that σ gets smaller simultaneously when $\hat{d}$ in the denominator of σ becomes larger. In other words, the nonlinear damping in (11) increases so that the effects of $|d - \hat{d}|$ to e_3 can be sufficiently suppressed. Consequently, from the ISS property of the overall tracking error system (12), we can observe that the nonlinear damping gain $k_d(\hat{e}_1, \hat{d})$ helps to suppress $|e_1|$.

3.2. Extended State Observer Design

In the previous subsection, the controller was designed using Assumption 3. In this subsection, the ESO is designed to estimate x_1 , x_2 , x_3 , and d . Let us define x_4 as $x_4 = d$. We define the extended state x_a as:

$$x_a = [x_1 \quad x_2 \quad x_3 \quad x_4]^T.\quad (18)$$

The dynamics of d can be defined as:

$$\dot{d} = \delta.\quad (19)$$

The estimated states, $\hat{x}$, and the estimated extended states, $\hat{x}_a$, are defined as:

$$\begin{aligned}\hat{x} &= [\hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3]^T \\ \hat{x}_a &= [\hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3 \quad \hat{x}_4]^T.\end{aligned}\quad (20)$$

The ESO is developed as:

$$\dot{\hat{x}}_a = A_o \hat{x}_a + L(x_1 - \hat{x}_1) \quad (21)$$

where $A_o = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $B_o = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $L = [l_1 \quad l_2 \quad l_3 \quad l_4]^T$ is the observer gain matrix. The estimation errors of the states and the extended states, $\tilde{x}$ and $\tilde{x}_a$ are defined as:

$$\begin{aligned}\tilde{x} &= \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \\ x_3 - \hat{x}_3 \end{bmatrix} \\ \tilde{x}_a &= \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \end{bmatrix} = \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \\ x_3 - \hat{x}_3 \\ x_4 - \hat{x}_4 \end{bmatrix}.\end{aligned}\quad (22)$$

The dynamics of $\tilde{x}_a$ are:

$$\dot{\tilde{x}}_a = (A_o - LC_a)\tilde{x}_a + B_d\delta \quad (23)$$

where $B_d = [0 \quad 0 \quad 0 \quad 1]^T$ and $C_a = [1 \quad 0 \quad 0 \quad 0]$.

Assumption 4. The states are bounded, i.e., $\|x\|_2 \leq B_x$ where B_x is an unknown constant.

In most systems, all states are physically bounded [22]. Thus Assumption 4 is reasonable. The derivative of $\Delta g(x_1)$ is the function of x_2 , thus, it is also bounded. Under Assumptions 1 and 4, there exists an upper boundedness of δ , $|\dot{d}| = |\dot{x}_4| = |\delta| \leq \delta_{\max}$.

Proposition 1. Let us consider the dynamics of $\tilde{x}_a$ (23). If the observer gains are chosen such that the roots of:

$$s^4 + l_1 s^3 + l_2 s^2 + l_3 s + l_4 = 0. \quad (24)$$

are in the left-half plane, then $\tilde{x}_a$ is uniformly ultimately bounded.

Proof. We define the Lyapunov function V_o as:

$$V_o = \tilde{x}_a^T P_o \tilde{x}_a. \quad (25)$$

where P_o is positive definite such that $(A_a - LC_a)^T P_o + P_o (A_a - LC_a) = -I$. The derivative of V_o with respect to time is derived as:

$$\begin{aligned}\dot{V}_o &= \tilde{x}_a^T [(A_a - LC_a)^T P_o + P_o (A_a - LC_a)] \tilde{x}_a + 2\tilde{x}_a^T P_o B_d \delta \\ &\leq -\|\tilde{x}_a\|_2^2 + 2\delta_{\max} \|P_o\|_2 \|\tilde{x}_a\|_2 \\ &\leq -\|\tilde{x}_a\|_2 (\|\tilde{x}_a\|_2 - 2\lambda_{\max}(P_o) \delta_{\max})\end{aligned}\quad (26)$$

where $\lambda_{\max}(P_o)$ is the maximum eigenvalue of P_o .

For $\|\tilde{x}_a\|_2 \geq 2\lambda_{\max}(P_o)\delta_{\max}$,

$$\dot{V}_o \leq -\|\tilde{x}_a\|_2^2. \quad (27)$$

Thus $\tilde{x}_a$ is uniformly ultimately bounded. $\square$

3.3. Analysis of the Closed-Loop Stability

In this subsection, the stability of the closed-loop system is examined. In (11), $\hat{x}$ is used instead of x . Thus, the actual control law is:

$$\begin{aligned} \hat{x}_{2_d} &= -k_1 \hat{e}_1 + \dot{x}_{1_d} \\ \hat{x}_{3_d} &= -k_2 \hat{e}_2 + \dot{x}_{2_d} \\ \hat{u} &= \frac{1}{g_0} (-k_3 \hat{e}_3 + \hat{x}_{3_d} - \hat{d} - k_d(\hat{e}_1, \hat{d})\hat{e}_3). \end{aligned} \quad (28)$$

Note that $\hat{u}$ is used to avoid confusion between (11) and (28). Thus, $\hat{u}$ (28) is applied to the EHS, i.e., $\hat{u} = \omega$. With (28), the tracking error dynamics (10) become:

$$\dot{e} = A_e e + B_e \zeta \quad (29)$$

where $A_e = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 1 \\ 0 & 0 & -k_3 \end{bmatrix}$, $B_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and $\zeta(x, \hat{x}) = k_3 e_3 + \hat{u} + d - \dot{x}_{1_d}$. From (23) and (29), the closed-loop system is:

$$\begin{aligned} \dot{e} &= A_e e + B_e \zeta \\ \dot{\hat{x}}_a &= A_o \tilde{x}_a + B_d \delta. \end{aligned} \quad (30)$$

Theorem 2. Let us consider the system (8). Under Assumptions 1, 2 and 4, if the output feedback controller (21) and (28) is implemented in the system (8), the following ISS properties for the tracking error hold:

$$\begin{aligned} |e_1(t)| &\leq \exp\left(-\frac{k_1}{2}t\right)|e_1(0)| + \frac{2}{k_1} \sup_{0 \leq \tau \leq t} e_2(\tau) \\ |e_2(t)| &\leq \exp\left(-\frac{k_2}{2}t\right)|e_2(0)| + \frac{2}{k_2} \sup_{0 \leq \tau \leq t} e_3(\tau) \\ |e_3(t)| &\leq \exp\left(-\frac{k_3}{2}(t)\right)|e_3(0)| + \sup_{0 \leq \tau \leq t} k_{\sigma_1}(k_{\hat{x}} + 1)\|\tilde{x}(\tau)\| \end{aligned} \quad (31)$$

where $k_{\sigma_1} = \frac{1}{0.5k_3 + k_d(\hat{e}_1, \hat{d})}$.

Proof. In Proposition 1, it was proven that $\tilde{x}_a$ is uniformly ultimately bounded. In (12) of Theorem 1, it was shown that the tracking errors possess a cascade nature. Because the control input (28) that uses the estimation states, $\hat{x}$ is implemented in the e_3 subsystem of e , and it is sufficient to investigate the behavior of the e_3 subsystem owing to the cascade nature. In u (11) and $\hat{u}$ (28), different desired velocities and actual torques are used respectively. However, the same desired position x_{1_d} is used in both u (11) and $\hat{u}$ (28). Thus, there exists $k_{\hat{x}} > 0$ such that:

$$|\hat{u}(\hat{x}, x_{1_d}) - u(x, x_{1_d})| \leq k_{\hat{x}}\|\hat{x} - x\|. \quad (32)$$

With $\hat{u}$, the dynamics of e_3 in (10) become:

$$\dot{e}_3 = -k_3 e_3 - k_d(\hat{e}_1, \hat{d})e_3 + d - \hat{d} + \hat{u} - u. \quad (33)$$

Equation (33) can be obtained as:

$$\frac{d}{dt} \left(\frac{e_3^2}{2} \right) \leq -\frac{k_3}{2} e_3^2 + \left(\frac{k_3}{2} + k_d(\hat{e}_1, \hat{d}) \right) |e_3| (|e_3| - \sigma_1) \quad (34)$$

where $\sigma_1 = \frac{|d-\hat{d}| + \hat{u} - u}{0.5k_3 + k_d(\hat{e}_1, \hat{d})}$. From (32),

$$\begin{aligned} \sigma_1 &= \frac{|d-\hat{d}| + \hat{u} - u}{0.5k_3 + k_d(\hat{e}_1, \hat{d})} \\ &\leq \frac{\|\tilde{x}\| + k_{\tilde{x}} \|\hat{x} - x\|}{0.5k_3 + k_d(\hat{e}_1, \hat{d})} \\ &\leq k_{\sigma_1} (k_{\tilde{x}} + 1) \|\tilde{x}\|. \end{aligned} \quad (35)$$

where $k_{\sigma_1} = \frac{1}{0.5k_3 + k_d(\hat{e}_1, \hat{d})}$. Then,

$$|e_3(t)| \leq \exp\left(-\frac{k_3}{2}(t)\right) |e_3(0)| + \frac{2}{k_3} \sup_{0 \leq \tau \leq t} k_{\sigma_1} (k_{\tilde{x}} + 1) \|\tilde{x}(\tau)\|. \quad (36)$$

Thus, the overall tracking error system has the ISS properties of a serial interconnected system (31). $\square$

Remark 5. As long as the disturbance increases, the estimation error $\tilde{x}$, estimated disturbance $\hat{d}$, and estimated output tracking error e_1 may also increase. It is observed that k_{σ_1} decreases simultaneously when k_d in the denominator of k_{σ_1} increases. Thus, the tracking error is sufficiently suppressed. From the ISS properties of the serial interconnected system (31), we see that a high observer gain in the ESO is not necessary for a small output tracking error.

Figure 2 shows a block diagram of the proposed method. The ESO estimates x_1, x_2, x_3 , and d using only the output feedback x_p . The controller then generates the control input ω using the estimated states and the desired position x_{pd} .

Figure 2. Block diagram of the proposed method.

4. Simulations

The performance of the proposed method was evaluated using simulations. In this paper, it was assumed that only x_1 and g_0 are known. Thus, only x_1 and g_0 were used for the proposed method in these simulations. The EHA and control parameters used are listed in Table 1. For comparison, two control methods were tested, i.e., the proposed method and the following PID controller:

$$u = k_P(x_{1d} - x_1) + k_I \int_0^t (x_{1d} - x_1) dt + k_D(\dot{x}_{1d} - \dot{x}_1) + k_f \dot{x}_1 \quad (37)$$

where $k_P = 5000$, $k_I = 100$, $k_D = 100$, and $k_{ff} = 20$ are the control gains. These control gains were chosen with the consideration of the effects of P, I, and D tuning listed in Table 2 [23]. The control gains were tuned to obtain the best position tracking performances

in both methods. In the simulations, the following load force and friction were used as external disturbances.

- Load force: $d_L = 200$
- Friction: $d_F = \left(\alpha_0 + \alpha_1 \exp\left(-\frac{x_v}{v_0}\right)^2 \right) \text{sgn}(x_v) + \alpha_2 x_v$.

Table 1. Parameters.

Parameter	Value	Parameter	Value
k	4000	b	100
m	80	S_A	7.94226×10^{-5}
S_B	5.43251×10^{-5}	D_p	8.43521×10^{-8}
C_p	2×10^{-13}	β_e	1.8×10^9
k_1	200	k_2	400
k_3	400	k_{d_1}	100
k_{d_2}	100	v_1	1
v_2	1	l_1	5.0265×10^3
l_2	9.4748×10^6	l_3	7.9376×10^9
l_4	2.4937×10^{12}		

Table 2. Effects of P, I, and D tuning.

Closed-Loop Response	Rise Time	Overshoot	Settling Time	Steady-State Error	Stability
Increasing K_p	Decrease	Increase	Small increase	Decrease	Degrade
Increasing K_I	Small decrease	Increase	Increase	Large decrease	Degrade
Increasing K_D	Small decrease	Decrease	Decrease	Minor change	Improve

The friction model used in the simulations includes the stiction, viscous damping, Coulomb friction, and Stribeck effect [24].

4.1. Sinusoidal Desired Position Tracking

The sinusoidal desired position $x_{1_d} = 15 \sin(0.5\pi t)$ depicted in Figure 3 was used. The estimation performances and errors of the ESO are shown in Figures 4 and 5. The estimated states and disturbances satisfactorily follow the actual states and disturbances. The peaking phenomenon occurred because of the nonzero initial velocity at the desired position. However, the estimated states and disturbances converged rapidly to the actual states and disturbances. The position tracking errors of the PID controller and the proposed method are shown in Figure 6. The PID controller had a larger position-tracking error than the proposed method. At the outset, a large position tracking error appeared because of the nonzero initial velocity of the desired position. In the proposed method, the increased nonlinear damping gain suppressed the position tracking error. Thus, the proposed method was able to reduce the position-tracking error. Figure 7 shows the control input of the PID controller and the proposed method. At the outset, a slightly larger input was used in the proposed method compared to the PID controller because the nonlinear damping increased to suppress the position tracking error. After the position tracking error was sufficiently suppressed at 0.9 s, the control input of the proposed method was similar to that of the PID controller.

Figure 3. Desired position, x_{1d} .

(a) Position, x_1

(b) Velocity, x_2

(c) Acceleration, x_3

(d) Disturbance, d

Figure 4. Estimation performances of the proposed method.

(a) Position estimation error

(b) Velocity estimation error

(c) Acceleration estimation error

(d) Disturbance estimation error

Figure 5. Estimation errors of the proposed method. **Figure 6.** Position tracking error of the PID controller and the proposed method, $x_{1d} - x_1$.

Figure 7. Control input of the PID controller and the proposed method, ω_m .

4.2. Step Desired Position Tracking

The step desired position depicted in Figure 8 was used. The position tracking performances of the PID controller and the proposed method are shown in Figure 9. Due to step change in the desired position, large overshoots were observed in both methods. Thus, the large oscillation appeared in the PID controller. On the other hand, the position tracking error rapidly converged to zero due to the nonlinear damping effect of the proposed method.

Figure 8. Desired position, x_{1d} .

(a) Position tracking

(b) Position tracking error

Figure 9. Position tracking performance of the PID controller and the proposed method.

5. Conclusions

Robust nonlinear position control with an ESO was developed for single-rod EHAs. A new EHA model was established for the controller design. The external disturbance, model, and input function uncertainties were lumped into a single disturbance. The ESO

was developed to estimate the disturbance as well as the position, velocity, and acceleration. A robust nonlinear position controller was developed with the nonlinear damping gain via a backstepping procedure. The stability of the closed-loop system was mathematically proven using the ISS. The performance of the proposed method was validated using simulations. It was shown that the position tracking error was reduced by the proposed method using only position feedback. Furthermore, we see that the proposed method is robust against the model uncertainties and the external disturbance in the simulation results.

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