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# Decision-Making with Fuzzy Soft Matrix Using a Revised Method: A Case of Medical Diagnosis of Diseases

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**Abstract:** In this paper, we study the matrix representation of fuzzy soft sets, complement of fuzzy soft sets, product of fuzzy soft matrices and the application of fuzzy soft matrices in medical diagnosis presented by Lavanya and Akila. Additionally, a new method (max-min average) based on fuzzy reference function is introduced instead of the max-product method by Lavanya and Akila to extend Sanchez's technique for decision making problems in medical diagnosis. Using the same data by Lavanya and Akila, the result shows that the new method gives more information about the medical status of the patients being considered in relation to a set of diseases.

**Keywords:** soft set; fuzzy soft set; fuzzy soft matrix; membership function; reference function; membership value

**MSC:** 16S36; 16Y99; 20N20



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## 1. Introduction

Zadeh [1] was the first to introduce the theory of fuzzy sets (FS). In 1999, Molodtsov [2] initiated soft set (SS) theory to explain further the notion of fuzzy set theory. Maji et al. [3] developed the theory again and presented the operations of soft sets. Maji et al. studied the notions of soft sets, fuzzy soft sets (FSS) and intuitionistic fuzzy soft sets [3–5].

Ali et al. [6] pointed out some inadequacies in the operations of soft sets defined in Maji et al. [3], and some new operations were introduced by them. In addition, in 2009, Ali et al. [7] put forward some algebraic structures associated with the newly defined operations on soft sets. After the notable start of soft sets, many researchers have contributed to the speedy progress of the notions relating to soft sets. Sezer and Ataguv [8] introduced soft vector spaces. In 2014, Çağman [9] initiated a new approach in soft set theory.

As earlier mentioned, the notion of fuzzy soft sets was introduced by Maji et al. [3], but Nas and Shabir [10] initiated the study of algebraic structures associated with fuzzy soft sets. Roy and Maji studied fuzzy soft sets in decision making problems (see more [11,12]).

Lee [13], in 2000, put forward the concept of bipolar-valued fuzzy sets. In 2014, Abdullah [14] used Lee's idea in a decision making problem. Shabir and Nas [15] introduced the idea of bipolar soft sets, and later Nas and Shabir [16] studied the notion of f-bipolar soft sets and their algebraic structures and their applications.

Çağman and Enginoğlu [17] proposed soft matrices (SMs) to make the computation with the operations of soft sets initiated by Maji et al. [3] easier and also put forward the soft max-min decision making method to solve decision making problems. Yang and Chenli [18] initiated fuzzy soft matrix (FSM) by introducing matrix representation of FSSs and applied it to problems associated with decision making.

The concept of complement of a fuzzy soft set (CFSS) was introduced by Maji et al. [3]. More on the properties of fuzzy soft set can be found in the work of Ruban and Saradah [19]. Baruah [20–22] put forward two ways to represent a FS: fuzzy membership function and fuzzy reference function. In this context, he introduced again CFSS. Neog and Sut [23,24] re-presented the definition of FSS, fuzzy soft complement matrix and put forward FSM and its applications. More works on fuzzy matrix and fuzzy soft matrix can be found in [25–27].

The theory of FSMs has been widely applied in decision making problems in the area of medical diagnosis, agriculture among others (see [5,17,28–31] for some more details). Decision-making in a fuzzy context has also been studied via intuitionistic fuzzy soft sets (see [32–36] for some applications).

In this paper, we study the Lavanya and Akila’s technique of medical diagnosis using fuzzy soft complement matrix initiated by Neog and Sut [23]. The limitation with these earlier techniques is that they only point out the extent to which an attribute is exhibited. That is, they can only give the degree of possibility of an entity exhibiting an attribute but cannot show the degree to which it exhibits the opposite. In this paper, the new method proposed points out the extent to which an attribute is exhibited and the extent to which its opposite is exhibited. In particular, an application of FSM using a revised method based on fuzzy reference function to extend Sanchez’s technique for decision making problems in medical diagnosis is presented. The paper is organized by presenting basic definitions of FSSs in Section 2. In Section 3, we present the concept of FSMs. An application of the revised method for fuzzy soft matrices in medical diagnosis is presented in Section 4. Sections 5 and 6 conclude the paper.

## 2. Preliminaries

In this section, we recall basic definitions of FSSs.

**Definition 1** ([2]). A pair  $(\beta, D)$  is called a SS over  $V$  if and only if  $\beta$  is a mapping of  $D$  into the set of all subsets of the set  $V$ .

**Definition 2** ([3]). A pair  $(\beta, X)$  is called FSS over  $V$  where  $\beta : X \rightarrow \beta(V)$  is a mapping from  $X$  into  $\beta(V)$ , where  $\beta(V)$  is the set of all fuzzy subsets of  $V$ .

**Definition 3** ([24]). Let  $V$  be a universe and  $D$  a set of attributes. Then, the pair  $(V, D)$  denotes the power set of all FSSs on  $V$  with attributes from  $D$  and is called a fuzzy soft class.

**Definition 4** ([23]). A FSS  $(\beta, X)$  over  $V$  is said to be null FSS denoted by  $\tilde{\phi}$  if for all  $\varepsilon \in X$ ,  $\beta(\varepsilon)$  is the null FS  $\phi$  of  $V$ , where  $\phi(x) = 0 \forall x \in V$ .

**Remark 1.** A FSS said to be absolute FSS denoted by  $\tilde{X}$  if  $\forall \varepsilon \in X$ ,  $\beta(\varepsilon)$  is the absolute fuzzy set  $V$ . The complement FSS of  $(\beta, X)$  denoted by  $(\beta, X)^c$  is defined by  $(\beta, X)^c = (\beta^c, \neg X)$  where  $\beta^c : \neg X \rightarrow \beta(V)$  is a mapping given by  $\beta^c(x) = [\beta(x)]^c$ , for all  $x \in X$ .

**Definition 5** ([24]). Let  $V = \{c_1, \dots, c_m\}$  be the universal set and  $D = \{e_1, \dots, e_n\}$ . Then, FSS  $(\beta, D)$  can be expressed in matrix form as  $X = [x_{ij}]_{m \times n}$ ,  $i = 1, \dots, m$ ,  $j = 1, \dots, n$  and  $x_{ij} = (\sigma_{j1}(c_i), \sigma_{j2}(c_i))$  where  $\sigma_{j1}(c_i)$  and  $\sigma_{j2}(c_i)$  represent the fuzzy membership function and fuzzy reference function, respectively of  $c_i$  in the fuzzy set  $\beta(e_j)$  so that  $\delta_{ij} = \sigma_{j1}(c_i) - \sigma_{j2}(c_i)$  gives membership value of  $c_i$ .

The set of all  $(m \times n)$ -fuzzy soft matrices is denoted by  $FSM_{m \times n}$ .

**Definition 6** ([24]). Let  $X = [x_{ij}]_{m \times n}$ , where  $x_{ij} = (\sigma_{j1}(c_i), \sigma_{j2}(c_i))$  be a FSM. Then, the membership value corresponding to  $X$  is defined as

$$MV(X) = [\delta_{(X)ij}]_{m \times n}, \text{ where } \delta_{(X)ij} = \sigma_{j1}(c_i) - \sigma_{j2}(c_i), \forall i = 1, \dots, m \text{ and } j = 1, \dots, n.$$

**Definition 7 ([28]).** Given  $X, Y \in FSM_{m \times n}$ , where  $X = [x_{ij}]_{m \times n}, x_{ij} = (\sigma_{j1}(c_i), \sigma_{j2}(c_i))$  and  $Y = [y_{ij}]_{m \times n}, y_{ij} = (\tau_{j1}(c_i), \tau_{j2}(c_i))$ . Then,

$$\begin{aligned} X + Y &= Z \\ &= (c_{ik})_{m \times n} \\ &= \{\vee(\sigma_{j1}(c_i), \tau_{j1}(c_i)), \wedge(\sigma_{j2}(c_i), \tau_{j2}(c_i))\}. \end{aligned}$$

If  $\sigma_{j2}(c_i) = \tau_{j2}(c_i) = 0$  for all  $i, j$  then

$$\begin{aligned} X + Y &= Z \\ &= (c_{ik})_{m \times n} \\ &= \{\vee(\sigma_{j1}(c_i), \tau_{j1}(c_i)), \wedge(0, 0)\} \\ &= \{\vee(\sigma_{j1}(c_i), \tau_{j1}(c_i)), (0)\}. \end{aligned}$$

**Definition 8 ([31]).** Let  $X = [(x_{ij}, 0)] \in FSM_{m \times n}$ . Then  $X^c$  is called the fuzzy soft complement matrix of  $X$  if  $X = [(1, x_{ij})]_{m \times n}$  for all  $x_{ij} \in [0, 1]$ .

**Definition 9 ([24]).** Let  $X = [x_{ij}]_{m \times n}, x_{ij} = (\sigma_{j1}(c_i), \sigma_{j2}(c_i))$  and  $Y = [y_{jk}]_{n \times p}, y_{jk} = (\tau_{j1}(c_i), \tau_{j2}(c_i))$ . The product (composition) of  $X$  and  $Y, X \cdot Y$  is defined as,

$$\begin{aligned} X \cdot Y &= (c_{ij})_{m \times p} \\ &= \{\vee [\wedge(\sigma_{j1}(c_i), \tau_{j1}(c_i))], \wedge [\vee(\sigma_{j2}(c_i), \tau_{j2}(c_i))]\}_{m \times p}. \end{aligned}$$

If  $\sigma_{j2}(c_i) = \tau_{j2}(c_i) = 0$  for all  $i, j$ , it implies that

$$\begin{aligned} X \cdot Y &= (c_{ij})_{m \times p} \\ &= \{\vee [\wedge(\sigma_{j1}(c_i), \tau_{j1}(c_i))], \wedge [\vee(0, 0)]\} \\ &= \{\vee [\wedge(\sigma_{j1}(c_i), \tau_{j1}(c_i))], 0\} \\ &= \{\vee [\wedge(\sigma_{j1}(c_i), \tau_{j1}(c_i))]\}. \end{aligned}$$

**Example 1.** Let

$$X = \begin{pmatrix} (0.2, 0.0) & (0.5, 0.0) & (0.9, 0.0) \\ (0.3, 0.0) & (0.7, 0.0) & (0.6, 0.0) \\ (0.6, 0.0) & (0.1, 0.0) & (0.4, 0.0) \end{pmatrix} \text{ and } Y = \begin{pmatrix} (0.4, 0.0) & (0.7, 0.0) & (0.6, 0.0) \\ (0.1, 0.0) & (0.5, 0.0) & (0.2, 0.0) \\ (0.3, 0.0) & (0.2, 0.0) & (0.8, 0.0) \end{pmatrix}.$$

$$X + Y = \begin{pmatrix} (0.4, 0.0) & (0.7, 0.0) & (0.9, 0.0) \\ (0.3, 0.0) & (0.7, 0.0) & (0.6, 0.0) \\ (0.6, 0.0) & (0.2, 0.0) & (0.8, 0.0) \end{pmatrix}, \quad X \cdot Y = \begin{pmatrix} (0.3, 0.0) & (0.5, 0.0) & (0.8, 0.0) \\ (0.3, 0.0) & (0.5, 0.0) & (0.6, 0.0) \\ (0.4, 0.0) & (0.6, 0.0) & (0.6, 0.0) \end{pmatrix}.$$

$$X^c = \begin{pmatrix} (1.0, 0.2) & (1.0, 0.5) & (1.0, 0.9) \\ (1.0, 0.3) & (1.0, 0.7) & (1.0, 0.6) \\ (1.0, 0.6) & (1.0, 0.1) & (1.0, 0.4) \end{pmatrix}, \quad Y^c = \begin{pmatrix} (1.0, 0.4) & (1.0, 0.7) & (1.0, 0.6) \\ (1.0, 0.1) & (1.0, 0.5) & (1.0, 0.2) \\ (1.0, 0.3) & (1.0, 0.2) & (1.0, 0.8) \end{pmatrix}.$$

$$X^c + Y^c = \begin{pmatrix} (1.0, 0.2) & (1.0, 0.5) & (1.0, 0.6) \\ (1.0, 0.1) & (1.0, 0.5) & (1.0, 0.2) \\ (1.0, 0.3) & (1.0, 0.1) & (1.0, 0.4) \end{pmatrix}.$$

Product operation for FSMs based on the reference function in Definition 9 was introduced and applied by Neog and Sut [24], Sarala and Rajkumari [30] and Lavanya and Akila [37]. As a matter of fact, Definition 9 is not appropriate to represent max-min product since it chooses row and row by the way it was indexed. Hence, a revision of it is made in the following section, and a new method for fuzzy soft matrices based on the reference function is presented.

### 3. New Fuzzy Soft Matrices Operation

In what follows,  $\sigma_{ij}(c_1)$  will denote the first component of the entry in the  $i$ -th row and  $j$ -th column of the fuzzy soft matrix.

**Definition 10** (Revised Max-Product). Let  $X = [x_{ij}]_{m \times n}, x_{ij} = (\sigma_{ij}(c_1), \tau_{ij}(c_2))$  and  $Y = [y_{jk}]_{n \times p}, y_{jk} = (\sigma_{jk}(c_1), \tau_{jk}(c_2))$ . The product (composition) of  $X$  and  $Y, X \cdot Y$  is defined as

$$\begin{aligned} X \cdot Y &= (c_{ik})_{m \times p} \\ &= \{ \vee [\wedge_{j=1}^n (\sigma_{ij}(c_1), \sigma_{jk}(c_1))], \wedge [\vee_{j=1}^n (\tau_{ij}(c_2), \tau_{jk}(c_2))] \}_{m \times p}. \end{aligned}$$

If  $\tau_{ij}(c_2) = \tau_{jk}(c_2) = 0$  for all  $i, j, k$ , it implies that

$$\begin{aligned} X \cdot Y &= (c_{ik})_{m \times p} \\ &= \{ \vee [\wedge_{j=1}^n (\sigma_{ij}(c_1), \sigma_{jk}(c_1))], \wedge [\vee(0, 0)] \} \\ &= \{ \vee [\wedge_{j=1}^n (\sigma_{ij}(c_1), \sigma_{jk}(c_1))], 0 \}. \end{aligned}$$

In what follows, the new operation on fuzzy soft matrix is introduced.

**Definition 11.** Let  $X = [x_{ij}]_{m \times n}, x_{ij} = (\sigma_{ij}(c_1), \tau_{ij}(c_2))$  and  $Y = [y_{jk}]_{n \times p}, y_{jk} = (\sigma_{jk}(c_1), \tau_{jk}(c_2))$ . Then, the revised method for fuzzy soft composition of  $X$  and  $Y, X \varphi Y$  with fuzzy membership function and fuzzy reference function is defined as

$$X \varphi Y = \left\{ \text{Max}_j \left[ \frac{(\sigma_{ij}(c_1) + \sigma_{jk}(c_1))}{2} \right], \text{Min}_j \left[ \frac{(\tau_{ij}(c_2) + \tau_{jk}(c_2))}{2} \right] \right\}.$$

If  $\tau_{jk}(c_2) = \tau_{ij}(c_2) = 0 \forall i, j, k$  then,

$$X \varphi Y = \left[ \text{Max}_j \left\{ \frac{1}{2} (\sigma_{ij}(c_1) + \sigma_{jk}(c_1)) \right\}, 0 \right].$$

**Remark 2.** In order to obtain the  $\{a_{st}\}$  entries in the matrix  $X \varphi Y$ , we do

$$\left[ \text{Max} \left\{ \frac{x_{s1} + y_{1t}}{2}, \frac{x_{s2} + y_{2t}}{2}, \frac{x_{s3} + y_{3t}}{2}, \dots, \frac{x_{sn} + y_{nt}}{2} \right\}, 0 \right].$$

**Example 2.** Let

$$\begin{aligned} X &= \begin{pmatrix} (0.2, 0) & (0.5, 0) & (0.9, 0) \\ (0.3, 0) & (0.7, 0) & (0.6, 0) \\ (0.6, 0) & (0.1, 0) & (0.4, 0) \end{pmatrix} \text{ and } Y = \begin{pmatrix} (0.4, 0) & (0.7, 0) & (0.6, 0) \\ (0.1, 0) & (0.5, 0) & (0.2, 0) \\ (0.3, 0) & (0.2, 0) & (0.8, 0) \end{pmatrix}. \\ X \varphi Y &= \begin{pmatrix} (0.60, 0) & (0.55, 0) & (0.85, 0) \\ (0.45, 0) & (0.60, 0) & (0.70, 0) \\ (0.50, 0) & (0.65, 0) & (0.60, 0) \end{pmatrix}. \end{aligned}$$

### 4. Application of New Fuzzy Soft Matrices Operation in Medical Diagnosis

In this section, an application of fuzzy soft matrices to medical diagnosis using the revised method in Definition 11 is presented. In a given pathology, suppose that  $S$  is a set of symptoms,  $D$  is a set of diseases and  $P$  is the set of patients. The fuzzy medical knowledge of the fuzzy soft set is constructed as fuzzy soft matrices.

Construct a fuzzy soft set  $(\lambda, D)$  over  $S, \lambda : D \rightarrow \mathcal{S}$ , where  $\mathcal{S}$  is the power set of  $S$ . A relation matrix  $M_1$  is obtained from  $(\lambda, D)$  called symptom-disease fuzzy soft relation matrix. The complement of  $(\lambda, D), (\lambda, D)^c$  denoted by  $M_2$  is called non symptom-disease fuzzy soft relation matrix.

Construct another fuzzy soft set  $(\gamma, S)$  over  $P$ ,  $\lambda : S \rightarrow \mathcal{P}$ , where  $\mathcal{P}$  is the power set of  $P$ . A relation matrix  $N_1$  is obtained from  $(\gamma, S)$  called *patients-symptom fuzzy soft relation matrix*, and its complement  $(\gamma, S)^c$  denoted by  $N_2$  is called *patients-non symptoms fuzzy soft relation matrix*.

Using Definition 10, obtain two new relation matrices  $Y_1 = N_1 \varphi M_1$  (called *patients-symptom disease fuzzy soft relation matrix*) and  $Y_2 = N_1 \varphi M_1^c$  (called *patients-symptom non disease fuzzy soft relation matrix*). Similarly, obtain relation matrices  $Y_3 = N_1^c \varphi M_1$  (called *patients-non symptom disease fuzzy soft relation matrix*) and  $Y_4 = N_1^c \varphi M_1^c$  (called *patients-non symptom non disease fuzzy soft relation matrix*). Thus,

$$Y_1 = N_1 \varphi M_1, \quad Y_2 = N_1 \varphi M_1^c, \quad Y_3 = N_1^c \varphi M_1, \quad Y_4 = N_1^c \varphi M_1^c.$$

Using Definition 6, obtain the corresponding membership value matrices  $MV(Y_1)$ ,  $MV(Y_2)$ ,  $MV(Y_3)$  and  $MV(Y_4)$ .

Calculate the diagnosis scores  $Z_{T_1}$  and  $Z_{T_2}$  for and against the disease, respectively, as

$$Z_{T_1} = [\delta(T_1)]_{ij} = \eta_{(T_1)ij} - \eta_{(T_3)ij},$$

and

$$Z_{T_2} = [\delta(T_2)]_{ij} = \eta_{(T_2)ij} - \eta_{(T_4)ij}.$$

Thus, if  $\max(Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j))$  occurs for exactly  $(p_i, d_k)$  only, then we conclude that the diagnostic hypothesis for patient  $p_i$  is the disease  $d_k$ . These steps are summarized in Algorithm 1.

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**Algorithm 1** Algorithm for Diagnostic Scores of Patients

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1. Choose the parameter set.
  2. Construct the FSS  $(\lambda, D)$  and  $(\lambda, D)^c$ . Compute associated FSMs  $M_1$  and  $M_1^c$ .
  3. Construct the FSS  $(\gamma, S)$  and  $(\gamma, S)^c$ . Compute associated FSMs  $N_1$  and  $N_1^c$ .
  4. Compute  $Y_1 = N_1 \varphi M_1$ ,  $Y_2 = N_1 \varphi M_1^c$  and their corresponding membership value matrices  $MV(Y_1)$ ,  $MV(Y_2)$ .
  5. Compute the  $Y_3 = N_1^c \varphi M_1$ , and  $Y_4 = N_1^c \varphi M_1^c$  and their corresponding membership value matrices  $MV(Y_3)$  and  $MV(Y_4)$ .
  6. Compute the diagnosis scores  $Z_{T_1}$  and  $Z_{T_2}$ .
  7. Find  $Z_k = \max\{Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j)\}$ . We conclude that the patient  $P_i$  is suffering from disease  $D_k$ .
- 

4.1. Case Study

The case studies in [24,37] were used to verify the effectiveness of our new operation.

4.1.1. Case 1

The data are as in [24]. Suppose that Lukky, Joe and Paul are three patients  $p = \{p_1, p_2, p_3\}$  taken to a laboratory for a test with symptoms  $s = \{s_1, s_2, s_3, s_4\}$ , where  $s_1, s_2, s_3, s_4$  represent temperature, headache, cough and stomach problems, respectively. Let the possible diseases relating to the above symptoms be  $d = \{d_1, d_2\}$ , where  $d_1, d_2$  represent viral fever and malaria, respectively.

Assume that the FSS  $(\lambda, D)$  over  $S$ , where  $\lambda : D \rightarrow \mathcal{S}$  gives appropriate explanation of fuzzy soft medical knowledge between diseases and symptoms. Let

$$\lambda(d_1) = \{(s_1, 0.85, 0.00), (s_2, 0.25, 0.00), (s_3, 0.55, 0.00), (s_4, 0.30, 0.00)\}$$

$$\lambda(d_2) = \{(s_1, 0.75, 0.00), (s_2, 0.50, 0.00), (s_3, 0.45, 0.00), (s_4, 0.45, 0.00)\}.$$

These FSSs  $\lambda(d_i), i = 1, 2$  are represented by a single FSM  $M_1$ , called *symptom-disease fuzzy soft relation matrix*.

$$M_1 = \begin{matrix} & d_1 & d_2 \\ s_1 & \begin{bmatrix} (0.85, 0.00) & (0.75, 0.00) \end{bmatrix} \\ s_2 & \begin{bmatrix} (0.25, 0.00) & (0.50, 0.00) \end{bmatrix} \\ s_3 & \begin{bmatrix} (0.55, 0.00) & (0.45, 0.00) \end{bmatrix} \\ s_4 & \begin{bmatrix} (0.30, 0.00) & (0.45, 0.00) \end{bmatrix} \end{matrix}.$$

Similarly, the complements of  $\lambda(d_i), i = 1, 2$

$$\lambda(d_1)^c = \{(s_1, 1.00, 0.85), (s_2, 1.00, 0.25), (s_3, 1.00, 0.55), (s_4, 1.00, 0.30)\}$$

$$\lambda(d_2)^c = \{(s_1, 1.00, 0.75), (s_2, 1.00, 0.50), (s_3, 1.00, 0.45), (s_4, 1.00, 0.45)\}$$

are represented by single FSM  $M_1^c$ , called *non symptom-disease fuzzy soft relation matrix*.

$$M_1^c = \begin{matrix} & d_1 & d_2 \\ s_1 & \begin{bmatrix} (1.00, 0.85) & (1.00, 0.75) \end{bmatrix} \\ s_2 & \begin{bmatrix} (1.00, 0.25) & (1.00, 0.50) \end{bmatrix} \\ s_3 & \begin{bmatrix} (1.00, 0.55) & (1.00, 0.45) \end{bmatrix} \\ s_4 & \begin{bmatrix} (1.00, 0.30) & (1.00, 0.45) \end{bmatrix} \end{matrix}.$$

Suppose that the FSS  $(\gamma, S)$  over  $P$ , where  $\gamma : S \rightarrow \mathcal{P}$ , gives an appropriate explanation of patients-symptoms in the laboratory.

$$(\gamma, S) = \left\{ \begin{array}{l} \gamma(s_1) = \{(p_1, 0.75, 0.00), (p_2, 0.40, 0.00), (p_3, 0.70, 0.00)\} \\ \lambda(s_2) = \{(p_1, 0.40, 0.00), (p_2, 0.50, 0.00), (p_3, 0.40, 0.00)\} \\ \lambda(s_3) = \{(p_1, 0.90, 0.00), (p_2, 0.30, 0.00), (p_3, 0.60, 0.00)\} \\ \lambda(s_4) = \{(p_1, 0.75, 0.00), (p_2, 0.40, 0.00), (p_3, 0.30, 0.00)\} \end{array} \right\}.$$

These FSS  $\gamma(s_i), i = 1, 2, 3, 4$  are represented by a FSM  $N_1$ , called *patients-symptoms fuzzy soft relation matrix*

$$N_1 = \begin{matrix} & s_1 & s_2 & s_3 & s_4 \\ p_1 & \begin{bmatrix} (0.75, 0.00) & (0.40, 0.00) & (0.90, 0.00) & (0.75, 0.00) \end{bmatrix} \\ p_2 & \begin{bmatrix} (0.40, 0.00) & (0.50, 0.00) & (0.30, 0.00) & (0.40, 0.00) \end{bmatrix} \\ p_3 & \begin{bmatrix} (0.70, 0.00) & (0.40, 0.00) & (0.60, 0.00) & (0.30, 0.00) \end{bmatrix} \end{matrix}.$$

Similarly, the complements of  $\gamma(s_i), i = 1, 2, 3, 4$

$$(\gamma, S)^c = \left\{ \begin{array}{l} \gamma(s_1)^c = \{(p_1, 1.00, 0.75), (p_2, 1.00, 0.40), (p_3, 1.00, 0.70), \} \\ \gamma(s_2)^c = \{(p_1, 1.00, 0.40), (p_2, 1.00, 0.50), (p_3, 1.00, 0.40)\} \\ \gamma(s_3)^c = \{(p_1, 1.00, 0.90), (p_2, 1.00, 0.30), (p_3, 1.00, 0.60)\} \\ \gamma(s_4)^c = \{(p_1, 1.00, 0.75), (p_2, 1.00, 0.40), (p_3, 1.00, 0.30)\} \end{array} \right\}.$$

are represented by the FSM  $N_2$ , called *patients-non symptoms fuzzy soft relation matrix*.

$$N_2 = N_1^c = \begin{matrix} & s_1 & s_2 & s_3 & s_4 \\ p_1 & \begin{bmatrix} (1.00, 0.75) & (1.00, 0.40) & (1.00, 0.90) & (1.00, 0.75) \end{bmatrix} \\ p_2 & \begin{bmatrix} (1.00, 0.40) & (1.00, 0.50) & (1.00, 0.30) & (1.00, 0.40) \end{bmatrix} \\ p_3 & \begin{bmatrix} (1.00, 0.70) & (1.00, 0.40) & (1.00, 0.60) & (1.00, 0.30) \end{bmatrix} \end{matrix}.$$

Now, the two new fuzzy soft relation matrices  $Y_1 = N_1 \varphi M_1$ , and  $Y_2 = N_1 \varphi M_1^c$  called *patients-symptoms disease* and *patients-symptoms non disease fuzzy soft relation matrices*, respectively, are computed using Definition 9.

$$Y_1 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.80, 0.00) & (0.75, 0.00) \\ p_2 & (0.63, 0.00) & (0.58, 0.00) \\ p_3 & (0.78, 0.00) & (0.73, 0.00) \end{matrix} \quad \text{and} \quad Y_2 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.95, 0.13) & (0.95, 0.23) \\ p_2 & (0.75, 0.13) & (0.75, 0.23) \\ p_3 & (0.85, 0.13) & (0.85, 0.23) \end{matrix}.$$

Then, the membership value matrices  $MV(Y_1)$  and  $MV(Y_2)$  for  $Y_1$  and  $Y_2$  respectively are calculated.

$$MV(Y_1) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.80 & 0.75 \\ p_2 & 0.63 & 0.58 \\ p_3 & 0.78 & 0.73 \end{matrix} \quad \text{and} \quad MV(Y_2) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.83 & 0.73 \\ p_2 & 0.63 & 0.53 \\ p_3 & 0.73 & 0.63 \end{matrix}.$$

Similarly, another two new fuzzy soft relation matrices  $Y_3 = N_1^c \varphi M_1$ , and  $Y_4 = N_1^c \varphi M_1^c$  called *patients - non symptom disease and patients-non symptom non disease fuzzy soft relation matrices*, respectively, are computed using Definition 9.

$$Y_3 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.93, 0.20) & (0.88, 0.20) \\ p_2 & (0.93, 0.15) & (0.88, 0.15) \\ p_3 & (0.93, 0.15) & (0.88, 0.15) \end{matrix} \quad \text{and} \quad Y_4 = \begin{matrix} & d_1 & d_2 \\ p_1 & (1.00, 0.33) & (1.00, 0.45) \\ p_2 & (1.00, 0.35) & (1.00, 0.38) \\ p_3 & (1.00, 0.30) & (1.00, 0.38) \end{matrix}.$$

Then, the membership value matrices  $MV(Y_3)$  and  $MV(Y_4)$  for  $Y_3$  and  $Y_4$  are obtained, respectively.

$$MV(Y_3) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.73 & 0.68 \\ p_2 & 0.78 & 0.73 \\ p_3 & 0.78 & 0.73 \end{matrix} \quad \text{and} \quad MV(Y_4) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.67 & 0.55 \\ p_2 & 0.65 & 0.62 \\ p_3 & 0.70 & 0.62 \end{matrix}.$$

$$Z_{T_1}(p_i, d_j) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.07 & 0.07 \\ p_2 & -0.15 & -0.15 \\ p_3 & 0.00 & 0.00 \end{matrix} \quad \text{and} \quad Z_{T_2}(p_i, d_j) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.15 & 0.17 \\ p_2 & -0.16 & -0.21 \\ p_3 & -0.06 & -0.11 \end{matrix}.$$

**Table 1.** Diagnostic Scores of Patients in [24] Using The New Method.

		$d_1$	$d_2$
$Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j) =$	$p_1$	-0.08	-0.10
	$p_2$	-0.01	0.06
	$p_3$	0.06	0.11

It can be concluded from Table 1 that patients  $p_2$  and  $p_3$  are suffering from the disease  $d_2$ .

4.1.2. Case 2

The data are as in [37]. Suppose that Lukky, Joe and Paul are three patients  $p = \{p_1, p_2, p_3\}$  taken to a laboratory for a test with symptoms  $s = \{s_1, s_2, s_3\}$ , where  $s_1, s_2, s_3$  represent nausea, fever and bleeding from the nose and gums. Let the possible diseases relating to the above symptoms be  $d = \{d_1, d_2\}$ , where  $d_1, d_2$  represent Swine Flu and Dengue, respectively.

Assume that the FSS  $(\lambda, D)$  over  $S$ , where  $\lambda : D \rightarrow S$  gives appropriate explanation of fuzzy soft medical knowledge between diseases and symptoms.

Let

$$\lambda(d_1) = \{(s_1, 0.3, 0.0), (s_2, 0.6, 0.0), (s_3, 0.5, 0.0)\}$$

$$\lambda(d_2) = \{(s_1, 0.9, 0.0), (s_2, 0.7, 0.0), (s_3, 0.8, 0.0)\}.$$

These FSSs  $\lambda(d_i), i = 1, 2$  are represented by FSM  $M_1$ , called *symptom-disease fuzzy soft relation matrix*.

$$M_1 = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} & \begin{bmatrix} (0.3, 0.0) & (0.9, 0.0) \\ (0.6, 0.0) & (0.7, 0.0) \\ (0.5, 0.0) & (0.8, 0.0) \end{bmatrix} \end{matrix}.$$

Similarly, the complements of  $\lambda(d_i), i = 1, 2$

$$\lambda(d_1)^c = \{(s_1, 1.0, 0.3), (s_2, 1.0, 0.6), (s_3, 1.0, 0.5)\}$$

$$\lambda(d_2)^c = \{(s_1, 1.0, 0.9), (s_2, 1.0, 0.7), (s_3, 1.0, 0.8)\}$$

are represented by FSM  $M_1^c$ , called *non symptom-disease fuzzy soft relation matrix*.

$$M_1^c = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} & \begin{bmatrix} (1.0, 0.3) & (1.0, 0.9) \\ (1.0, 0.6) & (1.0, 0.7) \\ (1.0, 0.5) & (1.0, 0.8) \end{bmatrix} \end{matrix}.$$

Suppose that the FSS  $(\gamma, S)$  over  $P$ ,  $\gamma : S \rightarrow \mathcal{P}$ , gives an appropriate explanation of patients-symptoms in the laboratory.

$$(\gamma, S) = \left\{ \begin{array}{l} \gamma(s_1) = \{(p_1, 0.7, 0.0), (p_2, 0.8, 0.0), (p_3, 0.3, 0.0)\} \\ \lambda(s_2) = \{(p_1, 0.8, 0.0), (p_2, 0.5, 0.0), (p_3, 0.6, 0.0)\} \\ \lambda(s_3) = \{(p_1, 0.2, 0.0), (p_2, 0.6, 0.0), (p_3, 0.7, 0.0)\} \end{array} \right\}.$$

These FSSs  $\gamma(s_i), i = 1, 2, 3, 4$  are represented by FSM  $N_1$ , called *patients-symptoms fuzzy soft relation matrix*

$$N_1 = \begin{matrix} & s_1 & s_2 & s_3 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} & \begin{bmatrix} (0.7, 0.0) & (0.8, 0.0) & (0.3, 0.0) \\ (0.8, 0.0) & (0.5, 0.0) & (0.6, 0.0) \\ (0.2, 0.0) & (0.6, 0.0) & (0.7, 0.0) \end{bmatrix} \end{matrix}.$$

Similarly, their complements

$$(\gamma, S)^c = \left\{ \begin{array}{l} \gamma(s_1)^c = \{(p_1, 1.0, 0.7), (p_2, 1.0, 0.8), (p_3, 1.0, 0.3)\} \\ \gamma(s_2)^c = \{(p_1, 1.0, 0.8), (p_2, 1.0, 0.5), (p_3, 1.0, 0.6)\} \\ \gamma(s_3)^c = \{(p_1, 1.0, 0.2), (p_2, 1.0, 0.6), (p_3, 1.0, 0.7)\} \end{array} \right\}.$$

are represented by the FSM  $N_2$ , called *patients-non symptoms fuzzy soft relation matrix*.

$$N_2 = N_1^c = \begin{matrix} & s_1 & s_2 & s_3 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} & \begin{bmatrix} (1.0, 0.7) & (1.0, 0.8) & (1.0, 0.3) \\ (1.0, 0.8) & (1.0, 0.5) & (1.0, 0.6) \\ (1.0, 0.2) & (1.0, 0.6) & (1.0, 0.7) \end{bmatrix} \end{matrix}.$$

Now, the two new fuzzy soft relation matrices  $Y_1 = N_1 \phi M_1$ , and  $Y_2 = N_1 \phi M_1^c$  called patients-symptoms disease and patients-symptoms non disease fuzzy soft relation matrices, respectively, can now be computed using Definition 9.

$$Y_1 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.70, 0.00) & (0.80, 0.00) \\ p_2 & (0.55, 0.00) & (0.85, 0.00) \\ p_3 & (0.60, 0.00) & (0.75, 0.00) \end{matrix} \quad \text{and} \quad Y_2 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.90, 0.15) & (0.90, 0.35) \\ p_2 & (0.80, 0.15) & (0.80, 0.35) \\ p_3 & (0.85, 0.15) & (0.85, 0.35) \end{matrix}.$$

Then, the membership value matrices  $MV(Y_1)$  and  $MV(Y_2)$  for  $Y_1$  and  $Y_2$ , respectively, can be calculated.

$$MV(Y_1) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.70 & 0.80 \\ p_2 & 0.55 & 0.85 \\ p_3 & 0.60 & 0.75 \end{matrix} \quad \text{and} \quad MV(Y_2) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.75 & 0.55 \\ p_2 & 0.65 & 0.45 \\ p_3 & 0.70 & 0.50 \end{matrix}.$$

Similarly, another two new fuzzy soft relation matrices  $Y_3 = N_1^c \varphi M_1$ , and  $Y_4 = N_1^c \varphi M_1^c$  called *patients-non symptom disease and patients-non symptom non disease fuzzy soft relation matrices*, respectively, can also be calculated using Definition 11.

$$Y_3 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.80, 0.15) & (0.95, 0.15) \\ p_2 & (0.80, 0.25) & (0.95, 0.25) \\ p_3 & (0.80, 0.10) & (0.95, 0.10) \end{matrix} \quad \text{and} \quad Y_4 = \begin{matrix} & d_1 & d_2 \\ p_1 & (1.00, 0.40) & (1.00, 0.55) \\ p_2 & (1.00, 0.55) & (1.00, 0.60) \\ p_3 & (1.00, 0.25) & (1.00, 0.55) \end{matrix}.$$

Then, the membership value matrices  $MV(Y_3)$  and  $MV(Y_4)$  for  $Y_3$  and  $Y_4$ , respectively, can be calculated.

$$MV(Y_3) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.65 & 0.80 \\ p_2 & 0.55 & 0.70 \\ p_3 & 0.70 & 0.85 \end{matrix} \quad \text{and} \quad MV(Y_4) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.60 & 0.45 \\ p_2 & 0.45 & 0.40 \\ p_3 & 0.75 & 0.45 \end{matrix}.$$

Next, the diagnosis scores  $Z_{T_1}$  and  $Z_{T_2}$  for and against the disease, respectively, can be calculated.

$$Z_{T_1}(p_i, d_j) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.05 & 0.00 \\ p_2 & 0.00 & 0.15 \\ p_3 & -0.10 & -0.10 \end{matrix} \quad \text{and} \quad Z_{T_2}(p_i, d_j) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.15 & 0.10 \\ p_2 & 0.20 & 0.05 \\ p_3 & -0.05 & 0.05 \end{matrix}.$$

**Table 2.** Diagnostic Scores of Patients in [37] Using The New Method.

		$d_1$	$d_2$
$Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j) =$	$p_1$	-0.10	-0.10
	$p_2$	-0.20	0.10
	$p_3$	-0.05	-0.15

It can be concluded from Table 2 that patient  $p_2$  is suffering from the disease  $d_2$ .

### 5. Discussion

The score  $Z_{T_1}(p_i, d_j)$ , the diagnosis score for the disease, can represent the potency of a disease attacking someone, and  $Z_{T_2}(p_i, d_j)$ , the diagnosis score against the disease, can represent the potency of the immune system of the person to fight against the disease. In addition, note that

$$-1 \leq |Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j)| \leq 1.$$

Hence, if

$$Z_{T_1}(p_i, d_j) = Z_{T_2}(p_i, d_j),$$

then

$$Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j) = 0$$

and the person  $p_i$  is attacked by the disease  $d_j$ , but the person is healthy enough to suppress the effect of the disease.

Additionally, if

$$Z_{T_1}(p_i, d_j) < Z_{T_2}(p_i, d_j),$$

then

$$Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j) < 0$$

and the person  $p_i$  is not attacked by the disease  $d_j$ . In fact, the less the value  $Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j)$ , the less the possibility of the person  $p_i$  to suffer disease  $d_i$ .

However, if

$$Z_{T_1}(p_i, d_j) > Z_{T_2}(p_i, d_j),$$

then

$$Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j) > 0$$

and the person  $p_i$  is suffering from the disease  $d_j$ . In fact, the more the value  $Z_{T_1}(p_i, d_j) - Z_{T_2}(p_i, d_j)$ , the more the possibility of severity of the person  $p_i$  to suffer disease  $d_i$ .

### 6. Comparison of Results

It should be pointed out that many previous works, to be precise, the works of [24,37], obtained that  $Z_{T_1}(p_i, d_j) > Z_{T_2}(p_i, d_j)$  and that  $Z_{T_k}(p_i, d_j)$  is positive always. This is not true in real life. In such case, it only reveals the degree to which a patient can be affected by a kind of disease. However, the new method in this paper leads to the matrices  $Z_{T_1}(p_i, d_j)$  and  $Z_{T_2}(p_i, d_j)$ , which more realistically reflect medical situations than is represented by [24,37], in that it gives both positive and negative possibilities. This not only reveals the degree to which a patient can be affected by a disease, but it also shows the degree to which a person is not going to be affected by a kind of disease.

To be more precise, taking into consideration some minor computational errors in [24], the following table shows at a glance the diagnoses scores obtained for the patients by the method in [24] and those obtained for the same set of patients by the new method in this article.

**Table 3.** Comparing Diagnostic Scores of Patients Using Method in [24] and the New Method.

Methods Diseases	New Method		[24]	
	$d_1$	$d_2$	$d_1$	$d_2$
$p_1$	-0.08	-0.10	0.25	0.45
$p_2$	-0.01	0.06	0.20	0.55
$p_3$	0.06	0.11	0.40	0.55

It can be seen from Table 3 that, by the method in [24], patient 2(0.55) and patient 3(0.55) suffer more seriously from disease 2 and that patient 1 has some resistance to both disease 1 and disease 2, in which case the possibility level is less than 0.5. These values, however, do not give information about the resistance tendency. The new method proposed in this paper also points out that patient 2(0.06) and patient 3(0.11) suffer more seriously from disease 2 and that patient 1 has some resistance to both disease 1 and disease 2.

Furthermore, the following table will show at a glance the diagnoses scores obtained for the patients by the method in [37] and those obtained for the same set of patients by the new method in this article.

**Table 4.** Comparing Diagnostic Scores of Patients Using Method in [37] and The New Method.

Methods Diseases	New Method		[37]	
	$d_1$	$d_2$	$d_1$	$d_2$
$p_1$	−0.10	−0.10	0.30	0.20
$p_2$	−0.20	0.10	0.30	0.60
$p_3$	−0.05	−0.15	0.50	0.30

From Table 4, in the work of [37], *patient 2*(0.6) was more seriously suffering *disease 2*; *patient 3*(0.5) was observed to suffer more from *disease 1* and *patient 1* suffers *disease 1*(0.3) as nearly as he suffers *disease 2*(0.2), but we cannot say much about *patient 1* exhibiting resistance tendency. As for the new method in this paper, at a glance, it further gives information about the resistance of the patience to the attack of the disease.

Hence, while the new method points out the same persons suffering a particular disease in [24,37], it can also give us some information about the medical status of the resistance level of some persons in relating to each disease. The method did not only inform us whether a patient suffers a disease or not, but it also tells us whether a patient can resist a disease (or not), and to what extent, viz-a-viz being susceptible to the disease.

## 7. Conclusions

In this paper, the Lavanya and Akila's technique for medical diagnosis using fuzzy soft complement matrix initiated by Neog and Sut [23] was studied. In addition, an application of FSM using the max-min average method to extend Sanchez's technique for decision making problems in medical diagnosis was carried out. We established the new method proposed based on the fuzzy reference function is more efficient, in the sense that it gives more information about the health status of a particular patient  $p_i$  in relation to suffering a disease  $d_j$ , when compared with Lavanya and Akila [37].

It should be pointed out that since the end result of this model is some sort of bipolar fuzzy sets (see [38] for more information), this model can be improved in the future to study Bipolar Soft Sets and their use in decision making. It also exhibits some properties of T-spherical fuzzy soft sets (see [39] for more details) in that between negative and positive there is some measure of neutrality. Hence, this model may also be extended to spherical fuzzy sets or fuzzy soft sets. In light of these, it can then be said that this new approach has not extensively used these two concepts and can constitute parts of areas of future extension of this method.

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