Article

Some New Results for Jaggi- $\mathcal{W}$ -Contraction-Type Mappings on b-Metric-like Spaces

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Abstract: In this article, we generalize, improve, unify and enrich some results for Jaggi- $\mathcal{W}$ -contraction-type mappings in the framework of b-metric-like spaces. Our results supplement numerous methods in the existing literature, and we created new approach to prove that a Picard sequence is Cauchy in a b-metric-like space. Among other things, we prove Wardowski's theorem, but now by using only the property ($\mathcal{W}1$). Our proofs in this article are much shorter than ones in recently published papers.

Keywords: Banach principle; Jaggi- $\mathcal{W}$ -contractive mapping; Jaggi- $\mathcal{W}$ -Suzuki-contractive mapping; fixed point; b-metric-like space

MSC: 47H10; 54H25

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1. Introduction and Preliminaries

In 2012, Wardowski [1] introduced a new type of contraction called a $\mathcal{W}$ -contraction and generalized the Banach contraction principle [2]. Motivated by Wardowski's result, many authors generalized his findings in many fields. See [3–12] for more details.

Definition 1. [1] Let $(\mathcal{Q}, d)$ be a metric space. A mapping $\mathcal{A} : \mathcal{Q} \rightarrow \mathcal{Q}$ is called an $\mathcal{W}$ -contraction provided that for some $\omega > 0$ and $\mathcal{W} \in \mathfrak{W}$ one has

$$\omega + \mathcal{W}(d(\mathcal{A}\mu, \mathcal{A}v)) \leq \mathcal{W}(d(\mu, v)) \quad (1)$$

for all $\mu, v \in \mathcal{Q}$ with $d(\mathcal{A}\mu, \mathcal{A}v) > 0$, where $\mathfrak{W}$ is the set of all functions $\mathcal{W} : (0, +\infty) \rightarrow \mathbb{R}$ so that:

- ($\mathcal{W}1$) $\mathcal{W}$ is strictly increasing: $\mu < v$ implies that $\mathcal{W}(\mu) < \mathcal{W}(v)$;
- ($\mathcal{W}2$) $\lim_{n \rightarrow +\infty} \alpha_n = 0$ if and only if $\lim_{n \rightarrow +\infty} \mathcal{W}(\alpha_n) = -\infty$, for each sequence $\{\alpha_n\}_{n \in \mathbb{N}}$ in $(0, +\infty)$;
- ($\mathcal{W}3$) $\lim_{\alpha \rightarrow 0^+} \alpha^k \mathcal{W}(\alpha) = 0$, for some $k \in (0, 1)$.

Remark 1. Obviously, if $\mathcal{A}$ satisfies the condition (1) and $\mathcal{W}$ is an increasing function (not necessarily strictly increasing), then $\mathcal{A}$ is contractive, i.e., $d(\mathcal{A}\mu, \mathcal{A}v) < d(\mu, v)$, for all $\mu, v \in \mathcal{Q}$ with $\mu \neq v$ and so $\mathcal{A}$ is continuous.

Wardowski's first important result is:

Theorem 1. [1] Let (Q, d) be a complete metric space and let $\mathcal{A} : Q \rightarrow Q$ be an $\mathcal{W}$ -contraction. Then, $\mathcal{A}$ has a unique fixed point $\mu^* \in Q$ provided that $\mathcal{W}$ be a continuous mapping. On the other hand, the sequence $\{\mathcal{A}^n \mu\}_{n \in \mathbb{N}}$ converges to μ^* for every $\mu \in Q$.

We can prove Wardowski's result via an easier method using the condition (W1) and the next very well known Lemma.

Lemma 1. [13] Let $\{\mu_n\}$ be a sequence in metric space (Q, d) such that $\lim_{n \rightarrow +\infty} d(\mu_n, \mu_{n+1}) = 0$. If $\{\mu_n\}$ is not a Cauchy sequence in (Q, d) , as a result, there exist two sequences $\{n_k\}$ and $\{m_k\}$ of natural numbers such that $n_k > m_k > k$ and

$$d(\mu_{n_k}, \mu_{m_k}), d(\mu_{n_k+1}, \mu_{m_k}), d(\mu_{n_k}, \mu_{m_k-1}), d(\mu_{n_k+1}, \mu_{m_k-1}), d(\mu_{n_k+1}, \mu_{m_k+1}), \dots \quad (2)$$

tend to ε^+ , as $k \rightarrow +\infty$, for some $\varepsilon > 0$.

Now, the proof of Theorem 1 is:

Proof. It is clear that (1) implies the continuity of the mapping $\mathcal{A}$ as well as the uniqueness of the fixed point of $\mathcal{A}$ if it exists. In order to show that $\mathcal{A}$ has a fixed point, let μ_0 be an arbitrary point in Q . Further, we define a sequence $\{\mu_n\}_{n \in \mathbb{N} \cup \{0\}}$ by $\mu_{n+1} = \mathcal{A}\mu_n$. If there exists $k \in \mathbb{N} \cup \{0\}$ such that $\mu_k = \mu_{k+1}$, then μ_k is a unique fixed point and the proof of Theorem 1 is finished. Therefore, suppose that $\mu_{n+1} \neq \mu_n$, for every $n \in \mathbb{N} \cup \{0\}$.

By (1), it follows that

$$\mathcal{W}(d(\mu_n, \mu_{n+1})) < \omega + \mathcal{W}(d(\mu_n, \mu_{n+1})) \leq \mathcal{W}(d(\mu_{n-1}, \mu_n)), \quad (3)$$

for all $n \in \mathbb{N}$, that is, according to (W1) $d(\mu_n, \mu_{n+1}) < d(\mu_{n-1}, \mu_n)$ for all $n \in \mathbb{N}$. This further means that $d(\mu_n, \mu_{n+1}) \rightarrow d^* \geq 0$ as $n \rightarrow +\infty$. If $d^* > 0$, we obtain from the previous relation that

$$\mathcal{W}(d^*) \leq \omega + \mathcal{W}(d^*) \leq \mathcal{W}(d^*), \quad (4)$$

which is a contradiction. Hence, $\lim_{n \rightarrow +\infty} d(\mu_n, \mu_{n+1}) = 0$.

Now, we prove that $\{\mu_n\}_{n \in \mathbb{N} \cup \{0\}}$ is a Cauchy sequence by supposing the contrary. When we put $\mu = \mu_{n_k}, \nu = \mu_{m_k}$ in (1) we obtain:

$$\omega + \mathcal{W}(d(\mu_{n_k+1}, \mu_{m_k+1})) \leq \mathcal{W}(d(\mu_{n_k}, \mu_{m_k})). \quad (5)$$

Since both $d(\mu_{n_k+1}, \mu_{m_k+1})$ and $d(\mu_{n_k}, \mu_{m_k})$ tend to ε^+ as $k \rightarrow +\infty$, we obtain

$$\omega + \mathcal{W}(\varepsilon^+) \leq \mathcal{W}(\varepsilon^+), \quad (6)$$

which is a contradiction. Hence, the sequence $\{\mu_n\}_{n \in \mathbb{N} \cup \{0\}}$ is a Cauchy sequence.

Since (Q, d) is a complete metric space, the sequence $\{\mu_n\}_{n \in \mathbb{N} \cup \{0\}}$ converges to some point $\mu^* \in Q$. Continuity of the mapping $\mathcal{A}$ implies that $\mathcal{A}\mu^* = \mu^*$, i.e., μ^* is a unique fixed point of $\mathcal{A}$. $\square$

The contractive condition in Banach's theorem has been weakened in several ways while a well-known generalization of metric spaces are b-metric-like spaces. Additionally, there are many combinations of these orientations.

Very recently, O. Popescu and G. Stan [14] proved the following two theorems:

Theorem 2. [14] Let $\mathcal{A}$ be a self-mapping from a complete metric space (Q, d) into itself. Suppose that there exists $\omega > 0$ such that $d(\mathcal{A}\mu, \mathcal{A}\nu) > 0$, this implies that

$$\begin{aligned} & \omega + \mathcal{W}(d(\mathcal{A}\mu, \mathcal{A}\nu)) \\ & \leq \mathcal{W}(\alpha \cdot d(\mu, \nu) + \beta \cdot d(\mu, \mathcal{A}\mu) + \gamma \cdot d(\nu, \mathcal{A}\nu) + \delta \cdot d(\mu, \mathcal{A}\nu) + L \cdot d(\nu, \mathcal{A}\mu)), \end{aligned}$$

for all $\mu, v \in \mathcal{Q}$, where $\mathcal{W} : (0, +\infty) \rightarrow \mathbb{R}$ is an increasing mapping, $\alpha, \beta, \gamma, \delta, L$ are non-negative numbers such that $\delta < \frac{1}{2}$, $\gamma < 1$, $\alpha + \beta + \gamma + 2\delta = 1$ and $0 < \alpha + \delta + L \leq 1$. $\mathcal{A}$ has a unique fixed point $\mu^* \in \mathcal{Q}$ and the sequence $\{\mathcal{A}^n \mu\}_{n \in \mathbb{N}}$ converges to μ^* for every $\mu \in \mathcal{Q}$.

Theorem 3. [14] Let $\mathcal{A}$ be a self-mapping from a complete metric space $\mathcal{Q}$ into itself. Suppose that there exists $\omega > 0$ such that

$$d(\mathcal{A}\mu, \mathcal{A}v) > 0 \text{ implies that } \omega + \mathcal{W}(d(\mathcal{A}\mu, \mathcal{A}v)) \leq \mathcal{W}(d(\mu, v)), \quad (7)$$

for all $\mu, v \in \mathcal{Q}$, where $\mathcal{W} : (0, +\infty) \rightarrow \mathbb{R}$ is a mapping satisfying the conditions (W2) and (W3''), where

(W3'') $\mathcal{W}$ is continuous on $(0, \alpha)$, where α is a positive real number.

$\mathcal{A}$ has a unique fixed point $\mu^* \in \mathcal{Q}$ and the sequence $\{\mathcal{A}^n \mu\}_{n \in \mathbb{N}}$ converges to μ^* for every $\mu \in \mathcal{Q}$.

The authors of [15] introduced the following contractive-type mapping and proved the corresponding fixed point result.

Definition 2. [15] Let $\mathcal{A}$ be a self-mapping on a b -metric-like space $(\mathcal{Q}, \sigma)$ with parameter $s \geq 1$. Then, the mapping $\mathcal{A}$ is said to be generalized (s, q) -Jaggi $\mathcal{W}$ -contraction-type if there is $\mathcal{W} \in \mathfrak{W}$ and $\omega > 0$ such that

$\sigma(\mathcal{A}\mu, \mathcal{A}v) > 0$ implies that

$$\omega + \mathcal{W}(s^q \sigma(\mathcal{A}\mu, \mathcal{A}v)) \leq \mathcal{W}\left(\alpha \frac{\sigma(\mu, \mathcal{A}\mu)\sigma(v, \mathcal{A}v)}{\sigma(\mu, v)} + \beta \sigma(\mu, v) + \gamma \sigma(v, \mathcal{A}\mu)\right), \quad (8)$$

for all $\mu, v \in \mathcal{Q}$ and $\alpha, \beta, \gamma \geq 0$ with $\alpha + \beta + 2\gamma s < 1$, for some $q > 1$.

Theorem 4. [15] Let $(\mathcal{Q}, \sigma)$ be a 0- σ -complete b -metric-like space with a coefficient $s \geq 1$ and let $\mathcal{A}$ be a self-mapping satisfying a generalized (s, q) -Jaggi $\mathcal{W}$ -contraction-type (8). $\mathcal{A}$ has a unique fixed point whenever $\mathcal{W}$ or $\mathcal{A}$ is continuous.

Further, the same authors in [15] introduced the following new type of contractive mapping and proved the corresponding result.

Definition 3. [15] Let $\mathcal{A}$ be a self-mapping on a b -metric-like space $(\mathcal{Q}, \sigma)$ with parameter $s \geq 1$. The mapping $\mathcal{A}$ is said to be a generalized (s, q) -Jaggi $\mathcal{W}$ -Suzuki contraction-type if there exists $\mathcal{W} \in \mathfrak{W}$ and $\omega > 0$ such that

$$\frac{1}{2s} \sigma(\mu, \mathcal{A}\mu) < \sigma(\mu, v) \text{ implies that}$$

$$\omega + \mathcal{W}(s^q \sigma(\mathcal{A}\mu, \mathcal{A}v)) \leq \mathcal{W}\left(\alpha \frac{\sigma(\mu, \mathcal{A}\mu)\sigma(v, \mathcal{A}v)}{\sigma(\mu, v)} + \beta \sigma(\mu, v) + \gamma \sigma(v, \mathcal{A}\mu)\right), \quad (9)$$

for all $\mu, v \in \mathcal{Q}$ and $\alpha, \beta, \gamma \geq 0$ with $\alpha + \beta + 2\gamma s < 1$, for some $q > 1$ and satisfying $\sigma(\mathcal{A}\mu, \mathcal{A}v) > 0$.

Theorem 5. [15] Let $(\mathcal{Q}, \sigma)$ be a 0- σ -complete b -metric-like space with a coefficient $s \geq 1$ and let $\mathcal{A}$ be a self-mapping satisfying a generalized (s, q) -Jaggi $\mathcal{W}$ -Suzuki contraction (9). $\mathcal{A}$ has a unique fixed point whenever $\mathcal{W}$ or $\mathcal{A}$ is continuous.

Note that Panwar and Anita [16] derived Suzuki-type common fixed point theorems for hybrid pairs of mappings in metric-like spaces using a Hausdorff metric-like space. They also investigated the presence of a common solution for a class of functional equations generated in dynamic programming as a consequence of their main result.

Now we present some definitions and basic notions of b -metric-like spaces as a generalization of a usual metric space.

Definition 4. [15] A b -metric-like on a nonempty set $\mathcal{Q}$ is a function $\sigma : \mathcal{Q} \times \mathcal{Q} \rightarrow [0, +\infty)$ such that for all $\mu, \nu, z \in \mathcal{Q}$ and a constant $s \geq 1$, the following three conditions are satisfied:

- ($\sigma 1$) $\sigma(\mu, \nu) = 0$ implies that $\mu = \nu$;
- ($\sigma 2$) $\sigma(\mu, \nu) = \sigma(\nu, \mu)$;
- ($\sigma 3$) $\sigma(\mu, z) \leq s[\sigma(\mu, \nu) + \sigma(\nu, z)]$.

In this case, the triple $(\mathcal{Q}, \sigma, s \geq 1)$ is called a b -metric-like space with constant s or a b -dislocated metric space by some authors. For some examples of metric-like and b -metric-like spaces, see [15,17–23]. Additionally, for various metrics in the framework of the complex domain, see [24,25].

In partial metric, metric-like, partial b -metric, and b -metric-like spaces, the definitions of convergent and Cauchy sequences are the same. As a result, we only discuss the definitions of convergence and Cauchyness in b -metric-like spaces.

Definition 5. [26] Let $\{\mu_n\}$ be a sequence in a b -metric-like space $(\mathcal{Q}, \sigma, s \geq 1)$.

- (i) The sequence $\{\mu_n\}$ is said to be convergent to μ if $\lim_{n \rightarrow +\infty} \sigma(\mu_n, \mu) = \sigma(\mu, \mu)$;
- (ii) The sequence $\{\mu_n\}$ is said to be σ -Cauchy in $(\mathcal{Q}, \sigma, s \geq 1)$ if $\lim_{n,m \rightarrow +\infty} \sigma(\mu_n, \mu_m)$ exists and is finite. If $\lim_{n,m \rightarrow +\infty} \sigma(\mu_n, \mu_m) = 0$, then $\{\mu_n\}$ is called a 0- σ -Cauchy sequence.

- (iii) One says that a b -metric-like space $(\mathcal{Q}, \sigma, s \geq 1)$ is σ -complete (resp. 0- σ -complete) if for every σ -Cauchy (resp. 0- σ -Cauchy) sequence $\{\mu_n\}$ in it there exists an $\mu \in \mathcal{Q}$, such that $\lim_{n,m \rightarrow +\infty} \sigma(\mu_n, \mu_m) = \lim_{n \rightarrow +\infty} \sigma(\mu_n, \mu) = \sigma(\mu, \mu)$.

A mapping $\mathcal{A} : (\mathcal{Q}, \sigma, s \geq 1) \rightarrow (\mathcal{Q}, \sigma, s \geq 1)$ is called continuous if the sequence $\{\mathcal{A}\mu_n\}$ tends to $\mathcal{A}\mu$, whenever the sequence $\{\mu_n\} \subseteq \mathcal{Q}$ tends to μ in both case as $n \rightarrow +\infty$, that is, $\lim_{n \rightarrow +\infty} \sigma(\mu_n, \mu) = \sigma(\mu, \mu)$ implies that $\lim_{n \rightarrow +\infty} \sigma(\mathcal{A}\mu_n, \mathcal{A}\mu) = \sigma(\mathcal{A}\mu, \mathcal{A}\mu)$.

Remark 2. In a b -metric-like space the limit of a sequence need not be unique and a convergent sequence need not be a σ -Cauchy sequence. However, if the sequence $\{\mu_n\}$ is 0- σ -Cauchy sequence in the σ -complete b -metric-like space $(\mathcal{Q}, \sigma, s \geq 1)$, then the limit of such sequences is unique. Indeed, in such a case if $\mu_n \rightarrow \mu$ as $n \rightarrow +\infty$ we find that $\sigma(\mu, \mu) = 0$. Now, if $\mu_n \rightarrow \mu$ and $\mu_n \rightarrow \nu$ where $\mu \neq \nu$, we obtain:

$$\frac{1}{s} \sigma(\mu, \nu) \leq \sigma(\mu, \mu_n) + \sigma(\mu_n, \nu) \rightarrow \sigma(\mu, \mu) + \sigma(\nu, \nu) = 0 + 0 = 0. \quad (10)$$

From ($\sigma 1$), it follows that $\mu = \nu$, which is a contradiction.

We will prove that certain Picard sequences are σ -Cauchy in this paper using the following result. The proof is exactly the same as the one in [27] (see also [28]).

Lemma 2. Let $\{\mu_n\}$ be a sequence in a b -metric-like space $(\mathcal{Q}, \sigma, s \geq 1)$ such that

$$\sigma(\mu_n, \mu_{n+1}) \leq \lambda \sigma(\mu_{n-1}, \mu_n) \quad (11)$$

for some $\lambda \in [0, \frac{1}{s})$ and each $n \in \mathbb{N}$. $\{\mu_n\}$ is a σ -Cauchy sequence in $(\mathcal{Q}, \sigma, s \geq 1)$ such that $\lim_{n,m \rightarrow +\infty} \sigma(\mu_n, \mu_m) = 0$, that is, it is an 0- σ -Cauchy sequence.

Remark 3. Note that the previous Lemma holds in the framework of b -metric-like spaces for each $\lambda \in [0, 1)$. For more details, see [13,29].

2. Main Result

Our goal in this paper is to improve some fixed point theorems in the setting of generalized metric spaces using (s, q) -Jaggi- $\mathcal{W}$ -contraction and (s, q) -Jaggi- $\mathcal{W}$ -Suzuki contraction.

First, we introduce the next new notion:

Definition 6. Let $\mathcal{A}$ be a self-mapping on a b -metric-like space $(\mathcal{Q}, \sigma, s \geq 1)$. The mapping $\mathcal{A}$ is said to be a generalized (s, q) -Jaggi $\mathcal{W}$ -contractive-type mapping if there is a strictly increasing mapping $\mathcal{W} : (0, +\infty) \rightarrow (-\infty, +\infty)$ and a real positive number ω such that

$\sigma(\mathcal{A}\mu, \mathcal{A}v) > 0$ and $\sigma(\mu, v) > 0$ implies that

$$\omega + \mathcal{W}(s^q \sigma(\mathcal{A}\mu, \mathcal{A}v)) \leq \mathcal{W}\left(\alpha \frac{\sigma(\mu, \mathcal{A}\mu)\sigma(v, \mathcal{A}v)}{\sigma(\mu, v)} + \beta \sigma(\mu, v) + \gamma \sigma(v, \mathcal{A}\mu)\right), \quad (12)$$

for all $\mu, v \in \mathcal{Q}$, where $\alpha, \beta, \gamma \geq 0$ with $\alpha + \beta + 2\gamma s < 1$ and $q > 1$.

Remark 4. In the above definition, we improved Definition 2 from [15].

Remark 5. In the proof of the following result, we will use the fact that $\sigma(\bar{\mu}, \bar{\mu}) = 0$ if the point $\bar{\mu}$ is a fixed point of the mapping $\mathcal{A} : \mathcal{Q} \rightarrow \mathcal{Q}$ satisfying (12). Indeed, if $\sigma(\bar{\mu}, \bar{\mu}) = \sigma(\mathcal{A}\bar{\mu}, \mathcal{A}\bar{\mu}) > 0$, then by (12) we have

$$\omega + \mathcal{W}(s^q \sigma(\bar{\mu}, \bar{\mu})) \leq \mathcal{W}\left(\alpha \frac{\sigma(\bar{\mu}, \bar{\mu})\sigma(\bar{\mu}, \bar{\mu})}{\sigma(\bar{\mu}, \bar{\mu})} + \beta \sigma(\bar{\mu}, \bar{\mu}) + \gamma \sigma(\bar{\mu}, \bar{\mu})\right), \quad (13)$$

that is,

$$\mathcal{W}(s^q \sigma(\bar{\mu}, \bar{\mu})) < \mathcal{W}(\alpha \sigma(\bar{\mu}, \bar{\mu}) + \beta \sigma(\bar{\mu}, \bar{\mu}) + \gamma \sigma(\bar{\mu}, \bar{\mu})), \quad (14)$$

or equivalently,

$$\sigma(\bar{\mu}, \bar{\mu}) < (\alpha + \beta + \gamma) \sigma(\bar{\mu}, \bar{\mu}), \quad (15)$$

because $s^q \geq 1$, which is a contradiction!

Our first new result in this paper is the following:

Theorem 6. Let $(\mathcal{Q}, \sigma, s \geq 1)$ be a 0-complete b -metric-like space and let $\mathcal{A}$ be a self-mapping satisfying the generalized (s, q) -Jaggi $\mathcal{W}$ -contractive-type (12). $\mathcal{A}$ has a unique fixed point $\mu^* \in \mathcal{Q}$, whenever $\mathcal{A}$ is continuous. Moreover, the sequence $\{\mathcal{A}^n \mu\}_{n \in \mathbb{N}}$ converges to μ^* , for every $\mu \in \mathcal{Q}$.

Proof. First, we will consider the uniqueness of a possible fixed point. Suppose that $\mathcal{A}$ has two distinct fixed points, $\bar{\mu}$ and $\bar{v}$, in $\mathcal{Q}$. Since $\sigma(\mathcal{A}\bar{\mu}, \mathcal{A}\bar{v}) > 0$ and $\sigma(\bar{\mu}, \bar{v}) > 0$ according to (12), we obtain:

$$\begin{aligned} \mathcal{W}(\sigma(\mathcal{A}\bar{\mu}, \mathcal{A}\bar{v})) &< \omega + \mathcal{W}(s^q \sigma(\mathcal{A}\bar{\mu}, \mathcal{A}\bar{v})) \\ &\leq \mathcal{W}\left(\alpha \frac{\sigma(\bar{\mu}, \mathcal{A}\bar{\mu})\sigma(\bar{v}, \mathcal{A}\bar{v})}{\sigma(\bar{\mu}, \bar{v})} + \beta \sigma(\bar{\mu}, \bar{v}) + \gamma \sigma(\bar{v}, \mathcal{A}\bar{\mu})\right), \end{aligned} \quad (16)$$

that is,

$$\begin{aligned} \mathcal{W}(\sigma(\bar{\mu}, \bar{v})) &< \mathcal{W}\left(\alpha \frac{\sigma(\bar{\mu}, \bar{\mu})\sigma(\bar{v}, \bar{v})}{\sigma(\bar{\mu}, \bar{v})} + \beta \sigma(\bar{\mu}, \bar{v}) + \gamma \sigma(\bar{v}, \bar{\mu})\right) \\ &= \mathcal{W}(\alpha \cdot 0 + \beta \sigma(\bar{\mu}, \bar{v}) + \gamma \sigma(\bar{v}, \bar{\mu})), \end{aligned} \quad (17)$$

or, equivalently,

$$\sigma(\bar{\mu}, \bar{\nu}) < (\beta + \gamma)\sigma(\bar{\mu}, \bar{\nu}).$$

The last obtained condition is in the fact a contradiction. Indeed, $\beta + \gamma \leq \alpha + \beta + 2\gamma s < 1$. Further, (12) implies that

$$\sigma(\mathcal{A}\mu, \mathcal{A}\nu) < \alpha \frac{\sigma(\mu, \mathcal{A}\mu)\sigma(\nu, \mathcal{A}\nu)}{\sigma(\mu, \nu)} + \beta\sigma(\mu, \nu) + \gamma\sigma(\nu, \mathcal{A}\mu), \quad (18)$$

for all $\mu, \nu \in \mathcal{Q}$, whenever $\sigma(\mathcal{A}\mu, \mathcal{A}\nu) > 0$ and $\sigma(\mu, \nu) > 0$.

Now, let μ_0 be an arbitrary point in $\mathcal{Q}$. Consider a sequence $\{\mu_n\}_{n \in \mathbb{N}}$ defined by $\mu_n = \mathcal{A}\mu_{n-1}$. If $\mu_{n-1} = \mu_n$ for some $n \in \mathbb{N}$, then μ_{n-1} is a unique fixed point of $\mathcal{A}$. Suppose further that $\mu_{n-1} \neq \mu_n$ for all $n \in \mathbb{N}$. In this case, $\sigma(\mu_{n-1}, \mu_n) > 0$ for all $n \in \mathbb{N}$. Since $\sigma(\mathcal{A}\mu_{n-1}, \mathcal{A}\mu_n) > 0$ and $\sigma(\mu_{n-1}, \mu_n) > 0$ then according to (12) (putting $\mu = \mu_{n-1}$ and $\nu = \mu_n$), we obtain

$$\begin{aligned} \sigma(\mu_n, \mu_{n+1}) &< \alpha \frac{\sigma(\mu_{n-1}, \mu_n)\sigma(\mu_n, \mu_{n+1})}{\sigma(\mu_{n-1}, \mu_n)} + \beta\sigma(\mu_{n-1}, \mu_n) + \gamma\sigma(\mu_n, \mu_n) \\ &= \alpha\sigma(\mu_n, \mu_{n+1}) + \beta\sigma(\mu_{n-1}, \mu_n) + \gamma\sigma(\mu_n, \mu_n) \\ &\leq \alpha\sigma(\mu_n, \mu_{n+1}) + \beta\sigma(\mu_{n-1}, \mu_n) + 2s\gamma\sigma(\mu_{n-1}, \mu_n). \end{aligned} \quad (19)$$

Obviously, (19) implies that

$$\sigma(\mu_n, \mu_{n+1}) < \frac{\beta + 2s\gamma}{1 - \alpha}\sigma(\mu_{n-1}, \mu_n) < \sigma(\mu_{n-1}, \mu_n). \quad (20)$$

Since $\frac{\beta + 2s\gamma}{1 - \alpha} < 1$, then by using Lemma 1 we infer that the sequence $\{\mu_n\}_{n \in \mathbb{N}}$ is a 0- σ -Cauchy sequence in 0-complete b -metric-like space $(\mathcal{Q}, \sigma, s \geq 1)$. This means that there exists a unique point $\mu^* \in \mathcal{Q}$ such that

$$\lim_{n, m \rightarrow +\infty} \sigma(\mu_n, \mu_m) = \lim_{n \rightarrow +\infty} \sigma(\mu_n, \mu^*) = \sigma(\mu^*, \mu^*) = 0. \quad (21)$$

Now, we shall prove that μ^* is a fixed point of $\mathcal{A}$. Since the mapping $\mathcal{A}$ is continuous, then we obtain

$$\sigma(\mathcal{A}\mu_n, \mathcal{A}\mu^*) \rightarrow \sigma(\mathcal{A}\mu^*, \mathcal{A}\mu^*), \text{ i.e., } \sigma(\mu_{n+1}, \mathcal{A}\mu^*) \rightarrow \sigma(\mathcal{A}\mu^*, \mathcal{A}\mu^*), \quad (22)$$

as $n \rightarrow +\infty$. Conditions (21) and (22) show that $\mathcal{A}\mu^* = \mu^*$, i.e., μ^* is a fixed point of $\mathcal{A}$. This completes the proof. $\square$

In the sequel, we introduce the notion of a generalized (s, q) -Jaggi- $\mathcal{W}$ -Suzuki contraction-type mapping in the framework of b -metric-like spaces. Our approach generalize and improve the corresponding one in [15].

Definition 7. Let $\mathcal{A}$ be a self-mapping on a b -metric-like space $(\mathcal{Q}, \sigma, s \geq 1)$. Then the mapping $\mathcal{A}$ is said to be a generalized (s, q) -Jaggi- $\mathcal{W}$ -Suzuki contraction if there is an increasing mapping $\mathcal{W} : (0, +\infty) \rightarrow (-\infty, +\infty)$ and a real number $\omega > 0$ such that

$$\frac{1}{2s}\sigma(\mu, \mathcal{A}\mu) < \sigma(\mu, \nu) \text{ and } \sigma(\mathcal{A}\mu, \mathcal{A}\nu) > 0$$

which implies that

$$\omega + \mathcal{W}(s^q\sigma(\mathcal{A}\mu, \mathcal{A}\nu)) \leq \mathcal{W}\left(\alpha \frac{\sigma(\mu, \mathcal{A}\mu)\sigma(\nu, \mathcal{A}\nu)}{\sigma(\mu, \nu)} + \beta\sigma(\mu, \nu) + \gamma\sigma(\nu, \mathcal{A}\mu)\right), \quad (23)$$

for all $\mu, \nu \in \mathcal{Q}$, where $\alpha, \beta, \gamma \geq 0$ with $\alpha + \beta + 2\gamma s < 1$ and $q > 1$.

The following result is an immediately corollary of Theorem 6.

Theorem 7. Let $(\mathcal{Q}, \sigma, s \geq 1)$ be a 0- σ -complete b -metric-like space and let $\mathcal{A}$ be a self-mapping satisfying a generalized (s, q) -Jaggi- $\mathcal{W}$ -Suzuki contraction (23). $\mathcal{A}$ has a unique fixed point $\mu^* \in \mathcal{Q}$ whenever $\mathcal{A}$ is a continuous mapping. Moreover, the sequence $\{\mathcal{A}^n \mu\}_{n \in \mathbb{N}}$ converges to μ^* , for all $\mu \in \mathcal{Q}$.

Proof. From $\sigma(\mu, \mathcal{A}\mu) < 2s\sigma(\mu, \nu)$, we have $\sigma(\mu, \nu) > 0$, for all μ and ν in $\mathcal{Q}$, or, $\sigma(\mu, \nu) = 0$, for some $\mu, \nu \in \mathcal{Q}$. From the second case, it is clear that $\sigma(\mu, \mathcal{A}\mu) = 0$, for some μ , and so, μ is a fixed point of $\mathcal{A}$. Hence, without any loss of generality, we may suppose that $\sigma(\mu, \nu) > 0$ instead of the first Suzuki condition, for all μ and ν in $\mathcal{Q}$. $\sigma(\mu, \nu) > 0$ and $\sigma(\mathcal{A}\mu, \mathcal{A}\nu) > 0$, are the conditions in (12). So, Theorem 7 is a corollary of Theorem 6. $\square$

The following example shows that both conditions $\sigma(\mathcal{A}\mu, \mathcal{A}\nu) > 0$ and $\sigma(\mu, \nu) > 0$ in Definition 6 are necessary. The first is due to the area of definition for the mapping $\mathcal{W}$ and the second is due to the division.

Example 1. Let $\mathcal{Q} = (-\infty, +\infty)$, $\sigma(\mu, \nu) = |\mu| + |\nu|$, and $\mathcal{A} : \mathcal{Q} \rightarrow \mathcal{Q}$ be defined by $\mathcal{A}\mu = \mu + 1$. Then, $(\mathcal{Q}, \sigma, 1)$ is a σ -complete b -metric-like space as well as $\sigma(\mathcal{A}\mu, \mathcal{A}\nu) = |\mathcal{A}\mu| + |\mathcal{A}\nu| = |\mu + 1| + |\nu + 1|$. Further, we see that $\sigma(\mathcal{A}0, \mathcal{A}0) = 2 > 0$, while $\sigma(0, 0) = 0$ and $\sigma(\mathcal{A}(-1), \mathcal{A}(-1)) = |\mathcal{A}(-1)| + |\mathcal{A}(-1)| = 0$ while $\sigma(-1, -1) = |-1| + |-1| = 2 > 0$.

Next, the direct consequences of Theorem 6 are new contraction conditions that generalize and complement the previous results.

Corollary 1. Let $(\mathcal{Q}, \sigma, s \geq 1)$ be a 0- σ -complete b -metric-like space and let $\mathcal{A}$ be a self-mapping satisfying generalized (s, q) -Jaggi- $\mathcal{W}$ -contraction (12) where $\omega_j > 0$, $j = \overline{1, 9}$ such that for all $\mu, \nu \in \mathcal{Q}$ with $\sigma(\mathcal{A}\mu, \mathcal{A}\nu) > 0$ and $\sigma(\mu, \nu) > 0$ any of the following inequalities hold true:

$$\omega_1 + s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu) \leq \mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu), \quad (24)$$

$$\omega_2 + \exp(s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu)) \leq \exp(\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu)), \quad (25)$$

$$\omega_3 - \frac{1}{s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu)} \leq -\frac{1}{\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu)}, \quad (26)$$

$$\omega_4 - \frac{1}{s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu)} + s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu) \leq -\frac{1}{\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu)} + \mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu), \quad (27)$$

$$\omega_5 + \frac{1}{1 - \exp(s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu))} \leq \frac{1}{1 - \exp(\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu))} \quad (28)$$

$$\omega_6 + \frac{1}{\exp(-(s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu))) - \exp(s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu))} \leq \frac{1}{\exp(-\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu)) - \exp(\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu))}, \quad (29)$$

$$\omega_7 + (s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu))^p \leq (\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu))^p, p > 0, \quad (30)$$

$$\omega_8 + s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu) \cdot \exp(s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu)) \leq \mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu) \cdot \exp(\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu)), \quad (31)$$

$$\omega_9 + \exp(s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu)) \cdot \ln(s^q \sigma(\mathcal{A}\mu, \mathcal{A}\nu)) \leq \exp(\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu)) \cdot \ln(\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu)), \quad (32)$$

where

$$\mathcal{N}_{\sigma}^{\alpha, \beta, \gamma}(\mu, \nu) = \alpha \frac{\sigma(\mu, \mathcal{A}\mu)\sigma(\nu, \mathcal{A}\nu)}{\sigma(\mu, \nu)} + \beta \sigma(\mu, \nu) + \gamma \sigma(\nu, \mathcal{A}\mu),$$

$\alpha, \beta, \gamma \geq 0$, $\alpha + \beta + 2\gamma s < 1$ and $q > 1$.

$\mathcal{A}$ has a unique fixed point $\mu^* \in \mathcal{Q}$, if it is continuous and then for every $\mu \in \mathcal{Q}$ the sequence $\{\mathcal{A}^n \mu\}_{n \in \mathbb{N}}$ converges to μ^* .

Proof. First of all, put in Theorem 6: $\mathcal{W}(\phi) = \phi$, $\mathcal{W}(\phi) = \exp(\phi)$, $\mathcal{W}(\phi) = -\frac{1}{\phi}$, $\mathcal{W}(\phi) = \phi - \frac{1}{\phi}$, $\mathcal{W}(\phi) = \frac{1}{1-\exp(\phi)}$, $\mathcal{W}(\phi) = \frac{1}{\exp(-\phi)-\exp(\phi)}$, $\mathcal{W}(\phi) = \phi^p$, $p > 0$, $\mathcal{W}(\phi) = \phi \exp(\phi)$, $\mathcal{W}(\phi) = \exp(\phi) \cdot \ln(\phi)$, respectively. Since every function $\phi \rightarrow \mathcal{W}(\phi)$ is strictly increasing on $(0, +\infty)$, the result follows from Theorems 6. $\square$

3. Application

Now, we consider the following boundary value problem:

$$\begin{cases} v''(\mu) = f(\mu, v(\mu)), & \mu \in [0, 1] \\ v(0) = v(1) = 0, \end{cases}$$

where $f : [0, 1] \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function.

The following Fredholm integral equation can be used to solve the aforementioned equation:

$$v(\mu) = - \int_0^1 K(\mu, t) f(t, v(t)) dt, \quad (33)$$

where the kernel is given by

$$K(\mu, t) = \begin{cases} t(1 - \mu), & \text{if } t \in [0, \mu] \\ \mu(1 - t), & \text{if } t \in [\mu, 1]. \end{cases}$$

See [30] for details.

Now, we provide the existence of solution for (33) that belongs to $\mathcal{Q} = C(I, \mathbb{R})$. Note that we consider the space $\mathcal{Q}$ with the b -metric given by

$$\sigma(\mu, \nu) = \max_{t \in I} (|\mu(t)| + |\nu(t)|)^p$$

for all $\mu, \nu \in \mathcal{Q}$, which is a b -complete b -metric-like space ($s = 2^{p-1}$).

Define $Y : \mathcal{Q} \rightarrow \mathcal{Q}$ by

$$Y\rho(\mu) = - \int_0^1 K(\mu, t) f(t, \rho(t)) dt, \quad \rho \in \mathcal{Q}, \quad \mu \in I.$$

Clearly, a function $u \in \mathcal{Q}$ is a solution of (33) if and only if it is a fixed point of Y .

Take into account the following assumptions:

(C1) For all $u, v \in \mathcal{Q}$ and for all $t \in I$,

$$\left(|f(t, u)| + |f(t, v)| \right) \leq \beta(|u(t)| + |v(t)|).$$

(C2) $f(t, \cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is continuous for all $t \in [0, 1]$.

Theorem 8. Assume that the above assumptions (C1) and (C2) hold. Then, Equation (33) has a solution in $\mathcal{Q}$.

Proof. To show that all of Theorem 6's assumptions are met, it is still necessary to show that Y is a generalized Jaggi- $\mathcal{W}$ -contractive-type mapping.

To show that all assumptions of Theorem 6 are satisfied, it remains to be proved that Y is a generalized $Jaggi\text{-}\mathcal{W}$ -contractive-type mapping. Let $\rho, \varrho \in \mathcal{Q}$. For each $\mu \in I$, we have

$$\begin{aligned} \left(|Y\rho(\mu)| + |Y\varrho(\mu)| \right)^p &= \left(\left| \int_0^1 K(\mu, t) f(t, \rho(t)) dt \right| + \left| \int_0^1 K(\mu, t) f(t, \varrho(t)) dt \right| \right)^p \\ &\leq \left(\left(\int_0^1 |K(\mu, t)|^q dt \right)^{\frac{1}{q}} \left(\int_0^1 (|f(t, \rho(t))| + |f(t, \varrho(t))|)^p dt \right)^{\frac{1}{p}} \right)^p \\ &\leq \left(\int_0^1 |K(\mu, t)|^q dt \right)^{\frac{p}{q}} \int_0^1 (|f(t, \rho(t))| + |f(t, \varrho(t))|)^p dt \\ &\leq \left(\int_0^1 |K(\mu, t)|^q dt \right)^{\frac{p}{q}} \int_0^1 \beta (|\rho(t)| + |\varrho(t)|)^p dt \\ &\leq \left(\int_0^1 |K(\mu, t)|^q dt \right)^{\frac{p}{q}} \int_0^1 \beta \sigma(\rho, \varrho) dt \end{aligned}$$

Via a careful calculation, we obtain

$$\int_0^1 |K(\mu, t)|^q dt = \frac{(1-\mu)^q \mu^{q+1} + \mu^q (1-\mu)^{q+1}}{q+1}, \quad \mu \in [0, 1].$$

So, we obtain that

$$\left(|Y\rho(\mu)| + |Y\varrho(\mu)| \right)^p \leq \beta \frac{(1-\mu)^q \mu^{q+1} + \mu^q (1-\mu)^{q+1}}{q+1} \sigma(\rho, \varrho).$$

Taking the supremum on $\mu \in [0, 1]$, we deduce that

$$\sigma(Y\rho, Y\varrho) \leq \frac{\beta \sigma(\mu, \nu)}{(q+1)^{\frac{p}{q}}}.$$

Now, we infer that

$$\begin{aligned} \ln(\sigma(Y\rho, Y\varrho)) &\leq \ln\left(\frac{\beta \sigma(\mu, \nu)}{(q+1)^{\frac{p}{q}}}\right) \\ &= \ln(\beta \sigma(\mu, \nu)) - \ln((q+1)^{\frac{p}{q}}). \end{aligned}$$

That is,

$$\omega + \mathcal{W}(\sigma(Y\rho, Y\varrho)) \leq \mathcal{W}(\beta \sigma(\rho, \varrho)), \quad (34)$$

where $\mathcal{W}(t) = \ln t$ and $\omega = \ln((q+1)^{\frac{p}{q}})$.

As a result, all of Theorem 6's hypotheses are satisfied, and we derive the existence of an element $u \in \mathcal{Q}$ with the property $u = Yu$. $\square$

4. Conclusions

- Using Lemma 1, we obtained a much shorter proof than the corresponding one in [15]. The cost-effectiveness of our approach is also due to the nonuse of properties (W2) and (W3) of the growing mapping $\mathcal{W} : (0, +\infty) \rightarrow (-\infty, +\infty)$.
- Taking $s = 1, \alpha = \gamma = 0, \beta \in (0, 1)$ in (12), we obtain

$$\omega + \mathcal{W}(\sigma(\mathcal{A}\mu, \mathcal{A}\nu)) \leq \mathcal{W}(\sigma(\mu, \nu)). \quad (35)$$

We therefore obtained the contractive condition of Wardowski and his Theorem 1, but $\mathcal{W}$ satisfies only the condition (W1).

- Taking $s > 1, \alpha = \gamma = 0, \beta \in (0, 1)$ in (12), we obtain

$$\omega + \mathcal{W}(\sigma(\mathcal{A}\mu, \mathcal{A}v)) \leq \mathcal{W}(\sigma(\mu, v)), \quad (36)$$

or

$$\omega + \mathcal{W}(s\sigma(\mathcal{A}\mu, \mathcal{A}v)) \leq \mathcal{W}(\sigma(\mu, v)), \quad (37)$$

i.e., the contractive condition of Lukacs and Kajanto from [18]. Our approach with Lemma 1 gives a much shorter proof of the main result of [18] without adding new assumptions to function $\mathcal{W}$.

- Since the class of b-metric-like spaces contains the other five classes of generalized metric spaces, then our results in this paper generalize and improve many known results in the existing literature.
- Restricting the conditions of control function $\mathcal{W}$, we could remove the term s^q in our new results which are weaker than the previous contractive conditions.

We believe that the idea of further elaborating our method, which was presented in the main result section, is very useful and can be applied to integral and nonlinear fractional differential equations.

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