

Article

Strategies for Solving Addition Problems Using Modified Schema-Based Instruction in Students with Intellectual Disabilities

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Abstract: A study is presented on the strategies employed to solve additive change problems by three students with intellectual disabilities (two of them with autism spectrum disorder). The students followed a program involving modified schema-based instruction. The results show an improvement in the problem-solving skills of the three students, who achieved successful formal strategies associated with identifying the operation. We analyze the importance of adapting and/or emphasizing certain steps in the instruction process in order to tailor them to the difficulties of each student.



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Keywords: problem solving; strategies; additive change problem; autism spectrum disorder; intellectual disabilities; learning difficulties

1. Introduction

Many hours of mathematics teaching in the initial stages of education are devoted to solving addition word problems, meaning those that are presented in text form (oral or written) and are solved with one or more addition or subtraction operations. In general, solving these problems requires going through several phases: understanding the situation given in the problem, organizing the quantities, deciding on the appropriate mathematical operation, executing it, and checking the result [1,2]. These phases call for different skills: understanding the terminology, creating a mental representation of the situation, numerical reasoning, and algorithmic knowledge of operations. The complexity of this process will depend on factors such as, for example, the semantic structure of the problem, the size of the numbers, the algorithm (with or without carrying/borrowing), and the linguistic characteristics (lexicon, verb tenses, etc.) [3]. Problem solving also requires cognitive processes (attention, memory, processing written or oral information) and executive function processes (planning, decision-making, etc.) [4].

Since students with intellectual disabilities (ID) often exhibit deficits in the aforementioned skills and processes, many tend to have difficulties solving word problems [5]. Specifically, they may have difficulties understanding the wording in order to be able to construct a coherent representation of the situation [6]. Students with ID have more difficulties extracting the information from the problem and deciding which operation is required to solve it, than with the calculation itself [7]. Because of this, research conducted on students with ID focuses on their understanding of the mathematical relationships given in the problems, and on how they connect them with the right operations [2,6,8–10]. Such is the case of those studies that analyze interventions with schema-based instruction [2,6,8] or conceptual model problem solving [9].

Schema-based instruction (SBI) emphasizes the semantic analysis of problems, and proposes using a visual schema for each problem type (e.g., change, combination, and compare, for addition problems). Students learn to associate the problem with a corresponding schema in which they place the numerical information from the problem and thus identify the unknown quantity. This helps them decide which operation solves the problem [9].

SBI has been used successfully to teach addition problems to students with disabilities [2,8,9,11,12]. Some authors resort to a modified schema-based instruction (MSBI) approach that adds visual supports, task analysis (such as a heuristic in place of a mnemonic) and systematic prompting to traditional SBI. For example, ref. [12] demonstrated that MSBI was effective in teaching change problems to three students with autism spectrum disorder (ASD) and moderate ID. Ref. [13] used an MSBI approach that included pictorial task analysis, graphic organizers, and systematic prompting with feedback to teach addition and subtraction word problems to eight students with moderate ID. Along similar lines, ref. [14] demonstrated the efficiency of the MSBI approach to teach mathematical problem-solving flexibility and communication to two middle school students with ASD. Most of the studies referenced measured the effect of instruction on student progress in terms of their problem-solving success rate. However, they did not focus on analyzing the solution strategies followed by students, in the sense of the work by [15]. It is important to observe which strategies students with ID resort to and how they evolve during an instructional process, since this can be used to improve the instruction and adapt it to the characteristics of these students.

Simple addition problems (meaning those that are solved with just one operation) have different semantic structures depending on the problem: combination, change, and compare [16]. This research focuses on change problems, which involve a situation in which there is an initial quantity that is either increased or decreased by an action, resulting in a different final quantity. For the situation “Juan has 10 marbles, he loses 7 marbles and ends up with 3 marbles,” there are three types of change problems depending on the location of the unknown quantity: FAU (final amount unknown), CAU (change amount unknown), and IAU (initial amount unknown). Each of these types can also be of the “get more” or “get less” type, depending on the action of the verb. For typically developing students, it has been shown that FAU problems are the easiest, followed by CAU and IAU problems [17].

Change problems can be further classified as (lexically) consistent and (lexically) inconsistent, depending on whether the arithmetic operation required to solve them corresponds or not to the action of the verb. Inconsistent problems are those in which the required arithmetic operation is contrary to the problem’s relational term. For example, the IAU change problem “Dan had some marbles. He found 9 more marbles. Now he has 15 marbles. How many marbles did he have to start with?” is a “get more” problem that requires subtraction to find the solution, so it is an inconsistent problem [18]. There are three types of inconsistent change problems, namely, of the CAU “get more,” IAU “get more,” and IAU “get less” types. Research indicates that many typically developing students solve these problems by following a strategy of literally translating the sentence, relating the meaning of words to the operation (“get more” to add or “get less” to subtract), which leads to errors in inconsistent problems. For example, typically developing students performed better on consistent than on inconsistent language problems. Moreover, almost all erroneous answers were due to reversal errors (choosing addition instead of subtraction or the reverse) [19].

Less research has been conducted on inconsistent problems with ID students. Ref. [20] compared outcomes when solving arithmetic problems in students with and without disabilities in grades 3 through 8. They concluded that students with disabilities performed considerably worse than general education students, and did not improve their performance across grade levels at the same rate. Specifically, the authors concluded that the IAU “get more” and “get less” word problems were more difficult for students with disabilities because the cue words misguided the students.

In the case of word problems, research with typically developing students has identified three basic solution strategies: (1) direct modeling, when they use concrete objects or drawings to count; (2) counting, when they count based on a number series (there is no direct modeling); and (3) known or derived facts, when the result of the operation is recognized mentally [15,16]. In the case of addition problems, a distinction is made in direct modeling and counting if the student counts all (objects or fingers), counts up from the first addend, counts up from the larger addend (minimum addend strategy), etc.

The aforementioned studies with typically developing children show an evolution in strategies, from informal (modeling and counting) to the more formal strategy of recalling a number fact [21]. Research indicates that students with learning disabilities use less effective strategies [22–24]. Ref. [22] found that children with learning disabilities had more problems than their typically developing peers in making the transition from informal strategies to advanced abstract additive reasoning strategies. In a similar vein, ref. [23] showed that students with learning disabilities in mathematics used less sophisticated strategies and made more mistakes than their typically developing peers when solving simple and complex addition problems. These differences between groups in the use and accuracy of the strategy were related, in part, to differences in working memory and counting management.

Ref. [25] also found that, compared to students without difficulties, children with learning difficulties are less flexible when using strategies in mathematics. In particular, they tend to use the same strategy repeatedly, unlike their typically developing peers, who use a variety of strategies to solve problems. Ref. [25] used the term “strategy rigidity” to describe this. One reason for this rigidity is a lack of conceptual understanding of the quantitative relationships of the problem, which prevents drawing a mental representation of its structure [26]. In the specific case of students with ASD, they have also been observed to frequently use rudimentary strategies focused on counting [27,28]. This has been attributed to cognitive flexibility in students with ASD being significantly weaker than in their typically developing peers [14].

Since the use of these rudimentary strategies can hinder the subsequent learning of arithmetic operations, different researchers propose teaching methodologies so that students with learning difficulties can learn effective solving strategies [29,30].

Ref. [31] also proposed effective experiments for teaching advanced multiplicative strategies to children with learning difficulties. In the case of additive relationships, ref. [30] compared the drill-and-practice approach with that of teaching the minimum addend strategy to students with learning difficulties and their typically developing peers. The results of the study showed how, of the students with learning difficulties, only those who followed the strategy taught improved significantly, whereas the general education students showed an improvement with both methods. In the case of students with ASD, ref. [14] suggested the need to provide instruction that helps students with this disorder to solve mathematical word problems using effective strategies. Specifically, the authors showed how an MSBI approach helped two students with ASD to develop more flexible and appropriate mathematical problem-solving strategies.

For students with ID, the study of informal strategies takes on special relevance, as it provides information that can be used to develop appropriate interventions [22]. Our goal in this paper is to analyze how MSBI makes it possible for students with ID to advance in the use of strategies when solving addition problems, and what aspects of the instruction allow for this advance. Specifically, we seek to answer the following questions:

- (1) What additive change problem-solving strategies do three students with ID exhibit when they follow MSBI?
- (2) What aspects of MSBI have been shown to be relevant to helping students who encounter difficulties during the process of solving additive change problems?

2. Materials and Methods

2.1. Participants

The participants in the study were three male students with ID (we will refer to them as students 1, 2, and 3) enrolled in the same special education center, two of them (students 1 and 2) diagnosed with ASD.

A purposeful selection was employed for participant recruitment of the study. The inclusion criteria for participation were (1) to be identified as having mild intellectual disability (IQ between 50–55 and 70) [32], (2) identified by their tutors as struggling with math and receiving learning support, and (3) having obtained a minimum score of 50 in the Test of Early Mathematics Ability [33] to guarantee a baseline knowledge of addition and subtraction number facts (Table 1).

Table 1. Student demographics.

Variable	Student 1	Student 2	Student 3
Gender	Male	Male	Male
Ethnicity	Caucasian	Caucasian	Caucasian
Age (years:months)	14:10	13:4	17:4
Disability Category	ASD	ASD	GDD
IQ	55 (WISC-IV)	54 (WISC-V)	58 (WISC-IV)
Schooling	Combined	Special	Special
Math Achievement	71 (TEMA-3)	56 (TEMA-3)	50 (TEMA-3)
Number Skills	100%	96%	96%
Number Comparison	100%	50%	50%
Calculation Skills	88%	50%	50%
Concepts	71%	71%	60%
Number Facts	100%	78%	44%

Note. GDD: global developmental delay, ASD: autism spectrum disorder, TEMA-3: Test of Early Mathematics Ability [33].

Two teachers with more than 8 years of experience teaching students with disabilities conducted the instruction. Students 1 and 3 were taught by the same teacher. The sessions took place in a classroom at the special education center where the three participants were enrolled, separated from the rest of the classes and without distractions. One session per week was held for each student, in sequence, as indicated below.

2.2. Modified Schema-Based Instruction (MSBI)

The MSBI approach was adapted from [13,14] and included (1) explicit instruction on using a self-instruction sheet (Figure 1, right), (2) use of schematic diagrams and manipulatives (Figure 1, left), and (3) use of systematic prompting and feedback. The instructional sessions followed a model, lead, and test format, as described below.

Model: The instructor provided each student a demonstration and model for a story or change problem. In this case, the instructor followed the six steps of the self-instruction sheet to show how to solve the problem (adapted from [12] and taught the student to relate and place the quantities in the schema in order to help him decide on the appropriate operation to solve the problem (Figure 1).

Lead: After that, the student was guided to solve a maximum of eight problems. These problems were solved by constantly interacting with the student, encouraging him to follow the six steps, and explaining the meaning of the words he did not understand or correcting his mistakes. The situation in the problem was modeled using manipulatives when the student exhibited difficulty understanding it.

Test: At the end of each instructional session, the students solved a probe with six change problems independently, without interacting with the instructor. If the student

scored over 75% on the probe, the next problem type was introduced in the following instruction session.

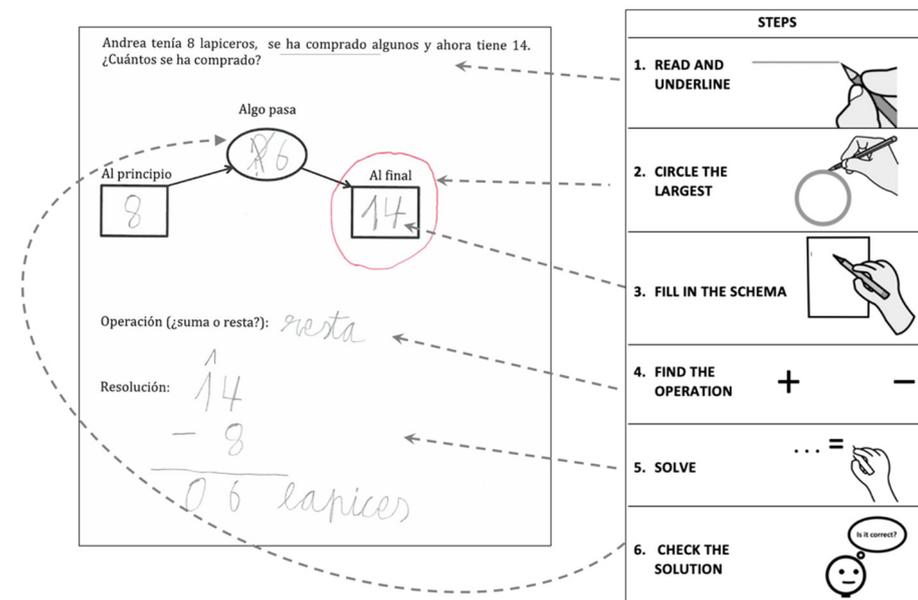


Figure 1. Worksheet (left) and self-instruction sheet (right) for solving one-step change problems. Example with the problem, “Andrea had 8 pens, she bought more, and now she has 14. How many did she buy?”

The instruction began by teaching the student to fill in the schemas in change stories, that is, through change situations without an unknown, in which all three quantities of the problem are provided. The main objective of the session was to teach them how to use the schema by placing the quantities in the boxes and establishing the relationship between them.

In the following instructional sessions, the change problems were presented sequentially—FAU, CAU, IAU—and schematic diagrams were used to solve each type of word problem in terms of the location of the unknown. As in similar studies [34], emphasis was placed on identifying the largest quantity in the schema itself, underscoring the information provided by the action of the verb. Thus, if the verb indicated “get less,” the student was asked, “When did he have more, at the start or at the end?” For example, in the problem in Figure 1, “Andrea had 8 pens, she bought more, and now she has 14. How many did she buy?” Since Andrea “bought pens,” she had fewer at first, so the larger amount at the end is circled.

During the solution process, the instructor and the student interacted in order to clarify the vocabulary, confirm a decision, solve errors, or encourage the student.

After each instructional session, one of the researchers interviewed the instructor to go over the session and plan the next one.

2.3. Design and Data Collection

The study is exploratory in nature [35], as it seeks to analyze the effect of MSBI on students. This is done by analyzing the answers written and through detailed descriptions of the interactions between the students and the teacher during the instruction sessions, which were videotaped.

A multiple baseline across students design [36] was used to establish a functional relationship between MSBI and the students’ performance. Data were collected through written tests at three times during the process: (1) baseline, (2) at the end of each instruction session for the three types of problems (FAU, CAU, IAU), and (3) at the conclusion of the MSBI process (Table 2).

Table 2. Evaluation tests in the study phases.

Session	Baseline			Instruction					Post-Instruction		
	S1, S2, S3	S2, S3	S3	S1, S2, S3					S1, S2, S3		
Probe	B1 B2 B3	B4 B5 B6	B7 B8	I1	I2	I3	I4	I5	PI1 PI2 PI3		
Problems		FAU					FAU	FAU	FAU		
Unknown		CAU		FAU	FAU	FAU	CAU	CAU	CAU		
		IAU			CAU	CAU	IAU	IAU	IAU		

This study used a single-case multiple baseline across students design. In keeping with the methodology of the multiple baseline design, the intervention began with each student at different times to demonstrate the dependence of the instruction and the improvement of the students. All three students took three stable baseline tests (B1, B2, and B3), and the intervention was started with student 1. The instruction problems were sequenced based on the position of the unknown, following the order FAU, CAU, and IAU. At the end of each session, the student was given a probe (I1, I2, I3, I4, and I5). If the goals for the session were completed successfully, the student advanced to the next problem type. Once he achieved mastery (100% correct responses) in the first problem type, students 2 and 3 took new baseline tests to confirm that their initial level was stable (B4, B5, and B6). The intervention was then started with student 2, repeating the same instructional process. Once he achieved mastery in the first problem type, student 3 took new baseline tests (B7 and B8) before beginning his instructional phase. Once the instruction was finished, the three students took three tests: one with the instructor (PI1), another in the classroom with their regular teacher (PI2), and one eight weeks after the MSBI ended (PI3). Table 2 summarizes the distribution of problem types solved by each student in each of the different phases.

Every probe (in the baseline, instruction, and post-instruction phases) contained six problems. The baseline and post-instruction tests contained two problems of each of the three types depending on the position of the unknown (two FAU, two CAU, and two IAU), whereas the instruction tests only contained problems of the type that had already been presented.

Information was gathered by videotaping the instructional sessions and compiling all the students’ written work. Written tests and the videos made during the instructional process were used in the data analysis.

Written tests: The written tests were used to analyze the success and the strategy followed by the students. Success was determined by obtaining the correct answer, regardless of the strategy used. The strategies described in [21] were adapted, and the following categories established:

(DA) Direct answer, when the student provides the answer without showing the steps or procedure used to obtain it.

(MC) Modeling with counting, if the student resorts to manipulatives or drawings to represent the action described in the problem and obtains the solution using various counting techniques.

(C) Counting, when the result is obtained by reciting a number series and identifying the action described.

(O) Operations, if an operation is identified and/or written.

A mixed strategy may be employed, in which two or more of the above are used when solving the same problem.

Videos of the sessions: The videos of the instructional sessions were qualitatively analyzed by three researchers, who focused on the student–teacher interactions and on the solutions formulated by the student independently, without the teacher’s help.

Regarding the students, some aspects were observed: their performance (the solution steps they followed with respect to the self-instruction sheet) when they had difficulties or were successful, and their attitude (concentration and motivation). As for the instructor,

the researchers observed the explanations provided to the student and the feedback given when progress or difficulties were experienced.

The researchers analyzed the video sessions separately and agreed on the aspects described above.

2.4. Reliability

Interobserver reliability data were collected during the baseline, instruction generalization, and maintenance phases. A graduate student in education, who was blind to the hypotheses of the study, recoded 30% of the students’ strategies and performance. Interobserver agreement was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100.

Interobserver reliability for strategy categorization was 100% during baseline, 89% during instruction, and 98% in the generalization phase. The mean interobserver reliability agreement for strategy categorization for each student across all phases was 100% for student 1, 88% for student 2, and 100% for student 3. The mean interobserver reliability agreement for solution accuracy was 100% for the three students.

Procedural reliability measured the instructor’s performance regarding the planned behaviors related to (1) number and type of problem, (2) material for each session, (3) following the steps in the self-instruction sheet, and (4) emphasizing the key aspects of each problem type. Procedural agreement was 100% for the three students.

Ethical approval: The study received ethical approval by the Clinical Research Ethics Committee of Cantabria, code 2020.252.

3. Results

This section presents the results of each student sequentially to their participation in the instruction. Tables 3–5 show the test results of the three students at the different phases in the study: baseline, instructional sessions, and post-instruction. They show the success rate and solution strategies. Relevant elements of the instructional sessions during the teacher–student interactions are also described for each student. It is important to remember that the time and progress in the instruction was conditioned by achieving success in the tests (I1, I2, I3, I4, and I5).

Table 3. Number of problems for student 1, with strategies and success.

Session	Baseline			Instruction					Post-Instruction		
	B1	B2	B3	I1	I2	I3	I4	I5	PI1	PI2	PI3
Probe		FAU CAU IAU		FAU	FAU CAU	FAU CAU	FAU CAU IAU	FAU CAU IAU		FAU CAU IAU	
Success	4	4	4	6	5	6	6	6	6	6	6
	Strategies										
Operation	6	6	6	6	6	6	6	6	6	6	6

3.1. Student 1

3.1.1. Baseline

In the initial tests, student 1 showed partial knowledge of change problems, and used the strategy of identifying and writing the operation. He answered all the FAU problems correctly; however, he made mistakes in the CAU and IAU problems that had inconsistent language, choosing the opposite operation to the one that solved the problem (see Figure 2).

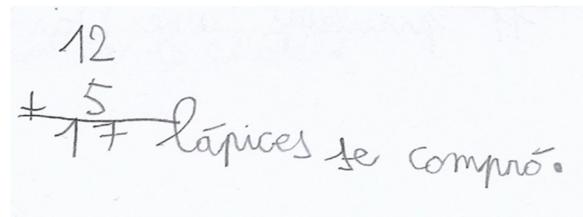


Figure 2. Opposite operation strategy of Student 1 in a baseline problem: “Pedro had 5 pencils, he bought some more, and now he has 12. How many pencils did he buy?”

The instructional objective with this student focused on having him solve problems with inconsistent language with the appropriate operation without being guided solely by the action of the verb.

Table 4. Number problems for student 2, with strategies and success.

Probe	Baseline						Instruction					Post-Instruction		
	B1	B2	B3	B4	B5	B6	I1	I2	I3	I4	I5	PI1	PI2	PI3
			FAU CAU IAU				FAU	FAU CAU	FAU CAU	FAU CAU IAU	FAU CAU IAU		FAU CAU IAU	
Success	6	4	5	4	5	4	6	4	6	3	6	4	4	6
Strategies														
DA		2	3	4	6	5			4					
M	6	4	3	2		1								
OP							6	6	2		6	4	4	5
DA-OP												2	2	
M-OP														1

Table 5. Number of problems for student 3, with strategies and success.

Pr.	Baseline								Instruction					Post-Instruction		
	B1	B2	B3	B4	B5	B6	B7	B8	I1	I2	I3	I4	I5	PI1	PI2	PI3
				FAU CAU IAU					FAU	FAU CAU	FAU CAU	FAU CAU IAU	FAU CAU IAU		FAU CAU IAU	
Success	0	2	2	3	1	3	3	1	6	2	6	6	6	5	6	6
Strategies																
DA	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
OP	0	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

3.1.2. Instruction

The goal of the initial instruction was to familiarize the student with the use of schemas so he could apply them in the remaining problems in the instruction. The student was interested in their use, and even enthusiastic, which he manifested with positive verbal expressions. He showed from the beginning that he understood the meaning of each part of the schema in relation to the story (beginning, change, and end). He also correctly indicated the action of the verb and the numerical relationship, circling the largest number in red, as follows:

[The student reads the problem]

S1: It says “more,” not “how many.” So, I have to find out how many she got, right? I’m getting the hang of it...

Fills in the numbers 9 and 15 in the schema.

S1: We have to figure out what happens in the middle (writes question mark in “Something happens”).

The teacher tells him that this is indeed where he has to put the question mark.

T: Did it go up or down?

S1 correctly circles the larger amount in “At the end.”

S1: The answer is 6 (crosses out the question mark and writes 6).

T: What operation did you do?

S1: Addition.

The teacher explains that even if it “goes up,” this does not always imply addition, and guides him by telling him that if the largest number is 15 (the final amount), the other two numbers have to add up to 15.

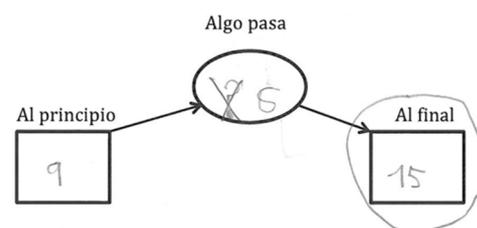
S1: [Thinking] . . . So then it’s... 15–9.

T: That’s right!

S1: I thought it was an addition but no, it’s a subtraction [. . .] It’s not always addition just because it goes up... Since I knew the end and I didn’t know the middle part, it’s a subtraction [he explains enthusiastically].

In the next training phase (I2), the FAU problems were introduced. The student followed all the steps on the self-instruction sheet. He successfully solved these problems with the strategy of identifying and solving the operation.

In the instructional sessions that followed on CAU and IAU problems, he showed a tendency to add when the verb in the problem implied an increase, and to subtract when it implied a decrease. This led him to give an incorrect answer in the problems with inconsistent language, as he had done in the baseline tests (reversal errors). Sometimes, he wrote the opposite operation even though he had mentally found the correct solution. This showed that the student had difficulties connecting the problem with the operation (step 4 in the self-instruction sheet). The instruction focused on making him understand that he could not rely solely on the action of the verb. To underscore this, it was useful to ask him to check the answer, recalling where the largest number was located and checking whether his answer satisfied that condition. He was then asked to think about which operation allowed him to obtain the number sought in the space with the question mark. An example is shown in the transcript of interactions with the student in CAU problem instruction session I2 (Table 4). At times, he did not want to write the operation he had done mentally, so the instructor asked him to write it down, or at least verbalize it (Figure 3).



Operación (¿suma o resta?): *resta*

Resolución: *5 flores*

Figure 3. Student 1’s answer to the CAU problem, “Monica had 9 flowers, somebody gave her more, and now she has 15. How many did she get?”

In general, the student remained motivated in all the instructional sessions. He was receptive to the teacher’s instructions, with whom he communicated and interacted openly. The student was observed seeking the approval (or positive feedback) of the teacher when he found the right answer. This was evidenced by his expressions, “I’m

doing well” and “I’m getting it, right?” This attitude favored the adoption of the steps in the self-instruction sheet.

The student did not exhibit problems following the first three steps in this sheet (read the problem, circle the largest quantity, organize the quantities in the schema), so he did not need to use the manipulatives. His difficulties involved associating the numerical relationships with the right operation, especially the problems with inconsistent language (step 4). At no point did he make mistakes in the arithmetic operation once he identified it (step 5), since his arithmetic procedures were firmly grounded, and in some problems, he even obtained the solution mentally. As noted, checking the answer proved effective, as he again deduced the location of the largest quantity (step 6).

3.1.3. Probes

The student required one instruction session to achieve success in the FAU problems (I1), two instruction sessions for the CAU problems (I2, I3), and another two sessions for the IAU problems (I4, I5). In tests carried out individually without the teacher’s help at the end of each instructional and post-instructional session, the student successfully solved all the problems, following the strategy of identifying the operation, which he had employed from the start of the study.

3.2. *Student 2*

Once student 1’s instruction on FAU problems was completed with mastery, and using the multiple baseline across students design methodology, students 2 and 3 took three new baseline tests, which meant that student 2 began instruction after completing six baseline tests.

3.2.1. Baseline

In the baseline tests, student 2 incorrectly solved eight problems out of the 24 given, of which six had inconsistent language with an IAU structure. He solved the problems with two types of strategies: direct answer (DA) and modeling (M). In none of the 24 problems did he provide a written operation. At times he was thoughtful and gave a direct answer, writing the number of the result, without expressing any other verbal information. In those cases, he could have done a mental calculation of the operation or a mental count by imagining the objects. In other problems, he used the modeling strategy by making drawings that reflected the actions of the verbs in the sentence (adding or crossing out). In Figure 4, for the CAU problem, “Raul had 9 pencils, he lost some, and now he has 3. How many pencils did he lose?,” the student drew nine pencils and crossed them out until he had 3 left, and his answer was the number of crossed out pencils.

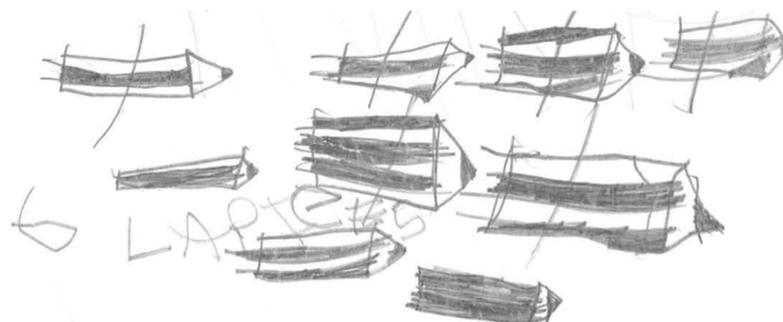


Figure 4. Correct modeling strategy by student 2 in a baseline test problem.

In the problem in Figure 5, with the same structure, “Silvia had 15 grapes, she ate some, and now she has 6. How many grapes did she eat?,” the student crossed out the final number of grapes (6) and gave the remaining grapes as an answer, although he counted incorrectly and gave the answer “10 grapes.”



Figure 5. Incorrect modeling strategy by student 2 in a baseline test problem.

This approach to solving problems showed that the student had an adequate understanding of the problem that allowed him to follow informal modeling strategies involving counting objects or mental calculation (the latter was indicated by the time he spent thinking before giving a numerical result). The instruction had to focus on moving away from these informal strategies, with which he felt safe, towards a formal strategy based on an operational approach. This would be particularly important in problems with inconsistent language.

3.2.2. Instruction

Student 2 understood the use of the schemas without difficulty from reading the stories. The FAU problems were not challenging for him either, and he integrated the strategy of identifying the operation in a single training session, after putting the numbers in the schema, leaving the modeling and direct answer strategy. This was helped by following the steps on the self-instruction sheet (Figure 6). This rejection of the informal strategy indicates that he used it because it made him feel safe, or simply because he liked to draw (as indicated by his tutor), and so he integrated it into his usual procedure.

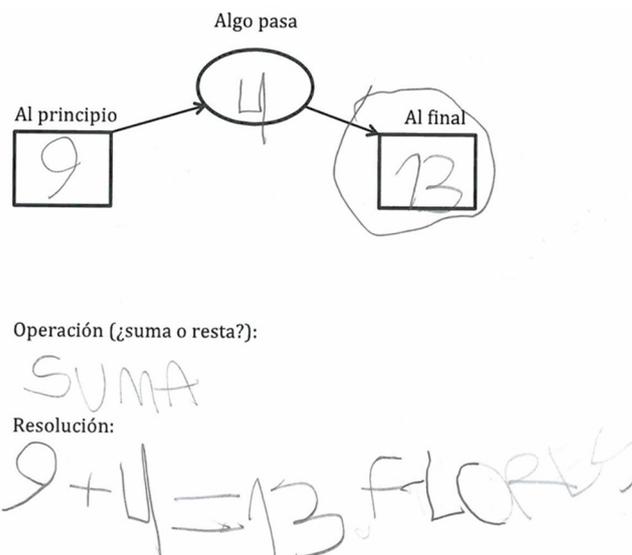


Figure 6. Correct operation strategy by student 2 in the FAU problem, “Silvia has 9 flowers. She buys 4 more flowers. How many does she have now?”

Student 2 found it more difficult to identify the correct operation for CAU and IAU problems, and tended to give direct answers. Two instruction sessions were necessary for the problems with each type of unknown. One advantage this student had was an adequate understanding of the problem and good mental math (or imaginary counting of objects). Sometimes, he only filled in the three numbers in the schema correctly, but

did not write the operation. That is, he did steps 1, 2, and 3, but skipped steps 4, 5, and 6 on the self-instruction sheet. When asked to write the operation, he made the mistake of writing in the number obtained mentally as a result, in one of the two addends of the operation. This is shown in Figure 7, in the IAU problem, “There were some apples on the tree, 7 fell, and now there are 4 apples. How many apples did the tree have to begin with?” The student filled in the schema correctly in his mind and when writing the operation later, he included the number 11 of the solution in the operation ($11 - 7 = 4$). This is an example of a mixed strategy, listed as DA-OP in Table 4.

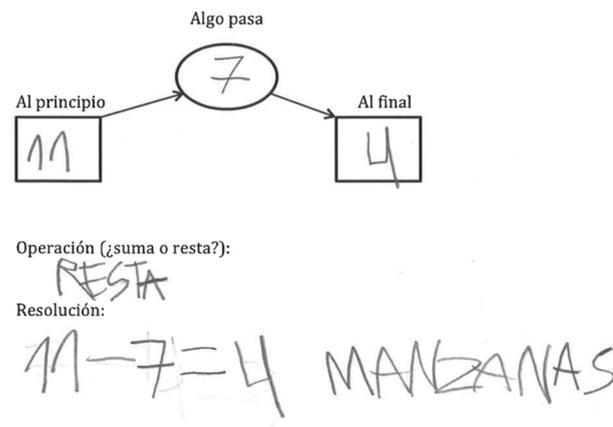


Figure 7. Student 2 incorrect operation strategy when solving the FAU problem, “There were some apples on the tree, 7 fell, and now there are 4 apples. How many apples did the tree have to begin with?”.

To help him overcome this difficulty, the instructor opted to show him the two operations (one correct and another incorrect) in writing, so that he could reason which one was correct based on the schema, and thus check the answer (step 6). The goal was to have him learn the importance of writing the operation and checking the result. These steps on the checklist helped the student overcome these difficulties.

In some problems, the student skipped steps 2 and 3 of the self-instruction sheet and identified the operation directly, following the strategy of using the keywords in the problem and, later, filling in the schema. Such was the case with the incorrect answer to the CAU problem in Figure 8: “There were 9 people in a store, some more went in, and now there are 15. How many people went into the store?” In it, he added 9 and 15, interpreting the action of “going in” as a sum. Because of this, the instructor asked him to read the problem aloud, using the numbers in the diagram, and check whether it made sense, in order to correct it if necessary.

Student 2 did not always follow the sequential order of the steps on the self-instruction sheet, or skipped some. For example, sometimes he did not circle the position of the largest quantity. It was decided to simplify the guidelines and reinforce the importance of steps 3 and 4, with the schema being a way of evaluating the solution in some problems (step 6).

The student also exhibited different degrees of concentration during the sessions. Sometimes it was necessary to teach the material in a session over several days. The student communicated with the instructor through short sentences, which he normally used to ask about the meaning of some words. In return, the instructor provided clear guidelines, also with short sentences that helped the student focus on the actions at hand. The positive feedback that the instructor gave him on the correct actions helped keep the student focused on the task.

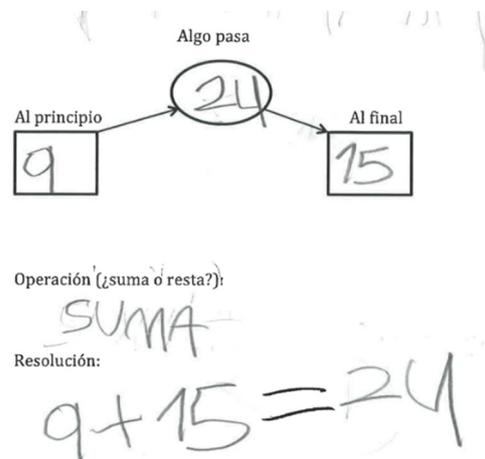


Figure 8. Student 2 incorrect operation strategy in the CAU problem, “There were 9 people in a store, some more went in, and now there are 15. How many people went into the store?”

3.2.3. Probes

The test results at the end of each stage of instruction and post-instruction showed a change in student 2’s strategies. He stopped modeling through drawings in favor of identifying the operations, although for some problems, he continued to use the direct answer strategy, which he gave either as the result of a calculation or a mental count. He achieved a high, but not perfect, rate of success. He made clear progress with the formal strategy through operations.

3.3. Student 3

Once student 2’s instruction on FAU problems was concluded with mastery, student 3 took two new baseline tests, so when the instruction began, the results were analyzed in eight probes.

3.3.1. Baseline

As Table 5 shows, student 3 maintained a stable baseline, exhibiting more difficulties than the other two students in the initial tests. He successfully solved only 15 of the 48 problems given, and he made mistakes in most of the CAU and IAU problems, and even some of the FAUs. He followed two strategies: giving a direct answer (specifically in baseline 1) and identifying the operation. In general, his mistakes in these initial tests were identifying the opposite operation to the one that solved the problem, and giving an incorrect direct answer (providing one of the numbers given in the problem). The goal with Student 3 was to give him an understanding of the problems by relating them to the operation in all the problem types.

3.3.2. Instruction

At the start of the instruction, it was observed that student 3 was hesitant with the problems, exhibited uncertainty when writing the answers, and sought the instructor’s approval in each step. The first difficulty observed was his poor understanding of the problems (step 1). To help him overcome this, he was given manipulatives (cubes) to model the problem and thus arrive at an answer. He was a very disciplined student, so he easily adapted to the steps in the self-instruction sheet. Of note was his ability to work out where the largest quantity was, since this made him understand the problem correctly. A remarkable aspect of student 3 is that he executed the algorithm of the operation correctly (steps 4 and 5), writing it vertically and always counting on his fingers. He was very successful with the operations, but struggled with them, since he needed to write even simple operations as if they involved carries (Figure 9). As with the other subjects, in those problems where he used the opposite operation, verification step 6 was key to achieving

success. He was urged to, once he obtained the result, substitute it for the question mark in the schema, and also in the problem. Such was the case with the CAU problem, “There were 5 people in the classroom, some more came in, and now there are 13. How many people came into the classroom?” in Figure 8. After finding the answer, the student replaced the word “some” with the number 8. He was then asked to read the problem to see whether the answer made sense. On some occasions, when he did the opposite operation, this verification step helped him realize that the problem did not make sense, and that he had to change the operation. This process, used consecutively in several problems, led him to find the right answers. This was important from an attitudinal point of view, since he was more motivated to learn.

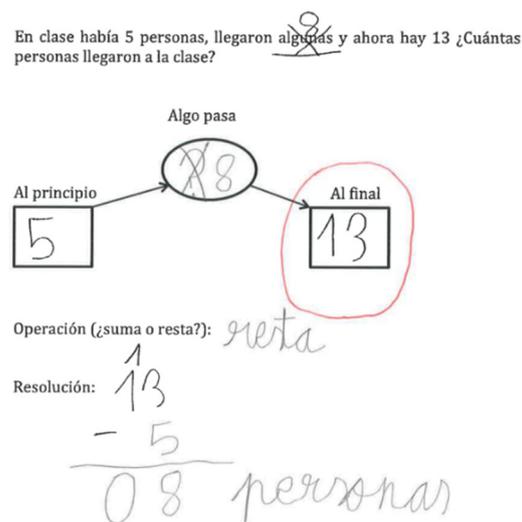


Figure 9. Incorrect operation strategy by student 3 in the CAU problem, “There were 5 people in the classroom, some more came in, and now there are 13. How many people came into the classroom?”

The student concluded the instruction by answering the problems correctly, following all the steps indicated in the self-instruction sheet.

3.3.3. Post-Test

The results of the tests during and after the instruction indicate that the student was successful with all the problem types. His strategies throughout the process relied on identifying the operations, although he required more practice and enhanced knowledge of number facts, the latter of which was beyond the scope of this study.

4. Discussion and Conclusions

This study has shown how three students with ID, two of them diagnosed with ASD, with a different initial profile in terms of their approach to additive change problems, responded appropriately to MSBI.

As in previous studies involving students with ID, the problems in the students’ baseline were related to the conceptual phase, associated with the representation and choice of the appropriate operation, rather than with doing the calculation [7]. Thus, the strategy in the baseline for student 1 was to identify the addition or subtraction operation, sometimes incorrectly, followed by a correct calculation, since he applied known number facts in most cases. Student 3 showed the same strategy, although in his case, the execution was very basic, as he relied on writing the operation vertically and then counting on his fingers. Ref. [23] noted that in mathematics, children with learning difficulties often use strategies associated with younger children and resort to counting with their fingers because it places fewer demands on working memory. Both students continued with the strategy of identifying the operation during the instructional process and in the final tests, although by then their choice of operation was correct, unlike in the baseline.

For his part, student 2 had a very different initial profile, since he used the informal modeling strategy with drawings in the baseline tests, or gave direct answers. The instruction yielded a shift towards the strategy of identifying the operation, which was useful when he had to solve problems with larger numbers where drawings were impractical. However, in the final tests, in some cases he continued to use direct answers (based on a mental count or calculation that he did not verbalize). Because he had a good understanding of the problem, he did not write the operation, which he avoided if he could obtain a mental answer.

The FAU problems were the simplest for the three students, with most of the difficulties being evident in the CAU and IAU problems, especially those with language inconsistencies. Both findings correlate with studies with typically developing students [16] and with students with disabilities [20]. The MSBI that combined the use of schemas with guidelines for solving a problem proved effective. Specifically, all three subjects considerably reduced their use of inappropriate strategies (like wrong choice of operation) throughout the instruction. Moreover, MSBI helped them transition from less sophisticated or inappropriate strategies to more advanced ones based on identifying the correct operation, in agreement with other similar studies [30] and specifically with studies that have used MSBI [14]. The use of the schemas helped the students focus on the story and on carefully reading the problem. By identifying the temporal progression, they were able to avoid giving impulsive answers based on writing the operation associated with the action of the verb. Another aspect of the instruction that proved fundamental was asking the students to indicate in the schema which of the three quantities had to be the largest (step 2 in the self-instruction sheet). This step allowed them to establish numerical relationships, and consequently determine the appropriate operation. One final crucial step in the sequence was to check the answer, substitute it into the problem, and reread the story or problem to see whether it made sense. Student 3 in particular was comfortable with this method, which gave him reassurance.

Ref. [2] noted that students with ID benefit from mathematical instructions that feature visual aids and repetition, and that promote flexible strategies focused on a conceptual understanding. In our research, although the three students followed an MSBI approach, each of them required greater emphasis to be placed on some of their steps, as well as some flexibility. From the start, student 1 was more effective at solving the problems and easily assimilated the use of schemas and the steps in the self-instruction sheet. In the case of student 2, it was decided to make the steps of the self-instruction sheet more flexible, given his greater ease understanding the problem and doing mental counts or calculations. Student 3 was reassured by the self-instruction sheet: Modeling with objects to understand the problem (step 1, read the problem), identifying the position of the largest number, and checking the answer were key aspects in his instruction. All of this indicates the need for flexibility in the instruction to accommodate the needs of each student with ID.

The results are promising, as they provide information on possible effective instruction methods for students with ID. Specifically, the results suggest that the characteristics of an MSBI methodology, such as visual aids and solution guidelines, helped the students with ID and ASD to acquire advanced strategies for solving change problems. Future research should focus on how MSBI might help students with similar characteristics solve problems with different semantic structures and/or operations.

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