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Adomian Decomposition Method with Orthogonal Polynomials: Laguerre Polynomials and the Second Kind of Chebyshev Polynomials

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Abstract: In this paper, a new efficient and practical modification of the Adomian decomposition method is proposed with Laguerre polynomials and the second kind of Chebyshev polynomials which has not been introduced in other articles to the best of our knowledge. This approach can be utilized to approximately solve linear and nonlinear differential equations. The proposed formulations are examined by a representative example and the numerical results confirm their efficiency and accuracy.

Keywords: Adomian decomposition method; Laguerre polynomials; Chebyshev polynomials

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1. Introduction

The Adomian decomposition method (ADM) was first introduced by the American mathematician Adomian and has been widely used for a class of deterministic and stochastic problems in scientific research fields [1–6]. It is based on looking for a solution in view of a series $u(x,t) = \sum_{k=0}^{\infty} u_k(x,t)$ and the decomposition of the nonlinear operator to a series $\mathcal{N}(u) = \sum_{k=0}^{\infty} \Lambda_k(u_0, u_1, \ldots, u_k)$. Consequently, these terms can be computed recursively through using Adomian polynomials Λ_k [7].

The modification of ADM itself has acquired a lot of remarkable results and it can be flexibly applied to kinds of complex higher order equations, even partial differential equations. However, for some special right hand terms of equations, the definite integrals with parameters in the domain cannot be solved explicitly, not to mention the approximate numerical integrations. Therefore, it is necessary to approximate the right terms by series before using the ADM. This paper aims at modifying ADM using orthogonal polynomials. Hosseini [8] firstly introduced the idea of combining ADM with the first kind of Chebyshev polynomials, and the effectiveness and reliability of this frame was proved to be suitable for linear and nonlinear equations. Subsequently, Liu [9] employed Legendre polynomials to the ADM and compared them to ones using the existing Chebyshev polynomials.

The same as the Chebyshev polynomials and Legendre polynomials, Laguerre polynomials are also categorized as the Jacobi orthogonal polynomials, and are the eigenfunctions of certain singular Sturm-Liouville [10]. By the use of the series method, the researchers [11] have studied the Hyers-Ulam stability of the associated homogeneous Laguerre differential equation in a subclass of analytic functions. For more details, the readers can refer to the book [12]. The most significant difference between them shows that the orthogonal interval of the former two class polynomials is $[-1,1]$ and the later is $[0,+\infty]$. Mathematically speaking, Laguerre polynomials are solutions to Laguerre's differential equation

$$x\frac{\mathrm{d}^2 L_n(x)}{\mathrm{d}x^2} + (1-x)\frac{\mathrm{d}L_n(x)}{\mathrm{d}x} + nL_n(x) = 0. \tag{1}$$

The recurrence relation of Laguerre polynomials is

$$L_{n+1}(x) = (2n+1-x)L_n(x) - n^2 L_{n-1}(x), \quad n \geq 1 \tag{2}$$

and Rodrigues' formula is

$$L_n(x) = \frac{e^{-x}}{n!}\frac{\mathrm{d}}{\mathrm{d}x}\left(x^n e^{-x}\right). \tag{3}$$

In the same way, the second kind of Chebyshev polynomials are solutions to Chebyshev's differential equation

$$(1-x^2)\frac{\mathrm{d}^2 U_n(x)}{\mathrm{d}x^2} - 3x\frac{\mathrm{d}U_n(x)}{\mathrm{d}x} + n(n+2)U_n(x) = 0. \tag{4}$$

The recurrence relation of the second kind of Chebyshev polynomials is

$$U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x), \quad n \geq 1 \tag{5}$$

and the correspond trigonometric identity is

$$U_n(\cos\theta) = \frac{\sin((n+1)\theta)}{\sin\theta}. \tag{6}$$

This paper firstly employs Laguerre polynomials and the second kind of Chebyshev polynomials to modify the ADM, that is, at the beginning of implementation of ADM, Laguerre and the second kind of Chebyshev orthogonal polynomials are used to expand the right hand terms which fail to integrate with parameters. The modified ADM is then demonstrated by applying it to solve a representable numerical problem which involves a nonlinear operator and complicated right hand term. The obtained results are studied to show the superiority of this modified ADM. Moreover, the modified ADM presented in this paper is compared to ones with the Taylor expansion, the first kind of Chebyshev polynomials and Legendre polynomials. The results show that the Chebyshev expansion possesses the absolute advantage in respect of error and approximation order, while the error derived by Laguerre expansion is about five orders of magnitude higher than the former method, because each Laguerre polynomial contains all items from the lowest to the highest, which leads to larger truncation errors. Moreover, the Taylor expansion at zero makes the error further and further away from the origin.

2. Modification of Adomian Decomposition Method

Before starting our programme, we present a quick review of the ADM. Consider the differential equation

$$\mathcal{L}(u) + \mathcal{R}(u) + \mathcal{N}(u) = f(x), \tag{7}$$

where $\mathcal{L}$ is the highest order derivative, which is supposed to be invertible, $\mathcal{R}$ is a linear operator of less order than $\mathcal{L}$, $\mathcal{N}$ is a nonlinear operator, and $f(x)$ is the right hand term which can be considered as a source term.

Suppose u and $\mathcal{N}(u)$ can be decomposed as

$$u(x,t) = \sum_{k=0}^{\infty} u_k(x,t), \quad \mathcal{N}(u) = \sum_{k=0}^{\infty} \Lambda_k(u_0, u_1, \ldots, u_k),$$

where Λ_k is the Adomian polynomials of $u_0, u_1, \ldots, u_k$ generated by

$$\Lambda_k = \frac{1}{k!}\frac{\mathrm{d}^k}{\mathrm{d}\delta^k}\left(\mathcal{N}\left(\sum_{i=0}^{\infty}\delta^i u_i\right)\right)\Bigg|_{\delta=0}, \quad k = 0, 1, 2, \cdots \tag{8}$$

Here, we enumerate several leading terms of Λ_k,

$$\begin{aligned}
&\Lambda_0 = \mathcal{N}(u_0), \\
&\Lambda_1 = u_1 \frac{\mathrm{d}}{\mathrm{d}u_0}\mathcal{N}(u_0), \\
&\Lambda_2 = u_2 \frac{\mathrm{d}}{\mathrm{d}u_0}\mathcal{N}(u_0) + \frac{u_1^2}{2!}\frac{\mathrm{d}^2}{\mathrm{d}u_0^2}\mathcal{N}(u_0), \\
&\Lambda_3 = u_3 \frac{\mathrm{d}}{\mathrm{d}u_0}\mathcal{N}(u_0) + u_1 u_2 \frac{\mathrm{d}^2}{\mathrm{d}u_0^2}\mathcal{N}(u_0) + \frac{u_1^3}{3!}\frac{\mathrm{d}^3}{\mathrm{d}u_0^3}\mathcal{N}(u_0), \\
&\cdots .
\end{aligned} \tag{9}$$

By applying the inverse operator $\mathcal{L}^{-1}$ to (7) with the given conditions, we have

$$u = \Psi(x) + \mathcal{L}^{-1}(g) - \mathcal{L}^{-1}\mathcal{R}(u) - \mathcal{L}^{-1}\mathcal{N}(u), \tag{10}$$

where $\Psi(x)$ is generated by some corresponding initial conditions from integrating the source term. From the idea of ADM, we can determine all the components of the decomposition solution. The term u_0 is given by

$$u_0 = \Psi(x) + \mathcal{L}^{-1} f(x) \tag{11}$$

and the components of the ADM follow in terms of u_0 and hence u can be completely determined by

$$u_{k+1} = -\mathcal{L}^{-1}(\mathcal{R}u_k) - \mathcal{L}^{-1}(\mathcal{N}u_k) = -\mathcal{L}^{-1}(\mathcal{R}u_k) - \mathcal{L}^{-1}(\Lambda_k), \quad k \geq 0. \tag{12}$$

The modified ADM in this topic is introduced to solve differential equations where source term $f(x)$ is not polynomial in general. To perform the modified ADM, $f(x)$ is expanded in Taylor series for an arbitrary natural number l (at $x = 0$),

$$f_{Tayler}(x) \approx \sum_{k=0}^{l-1} \frac{f^{(k)}(0)}{k!} x^k. \tag{13}$$

Hosseini [8] modified the ADM by expanding $f(x)$ in the first kind of Chebyshev polynomials $T_k(x)$

$$f_T(x) \approx \sum_{k=0}^{l-1} a_k T_k(x) \tag{14}$$

and Liu [9] modified the ADM by expanding $f(x)$ in Legendre polynomials $P_k(x)$

$$f_P(x) \approx \sum_{k=0}^{l-1} b_k P_k(x). \tag{15}$$

Alternatively, in the present paper, the source term is expressed by Laguerre polynomials $L_k(x)$

$$f_L(x) \approx \sum_{k=0}^{l-1} c_k L_k(x), \tag{16}$$

and the second kind of Chebyshev polynomials $U_k(x)$

$$f_U(x) \approx \sum_{k=0}^{l-1} d_k U_k(x), \tag{17}$$

where $L_n(x)$, $U_k(x)$ are the related orthogonal polynomials and we can deduce that

$$\begin{aligned}
&L_0(x) = 1,\\
&L_1(x) = -x + 1,\\
&L_2(x) = x^2 - 4x + 2,\\
&L_3(x) = -x^3 + 9x^2 - 18x + 6,\\
&\cdots .\\
&U_0(x) = 1,\\
&U_1(x) = 2x,\\
&U_2(x) = 4x^2 - 1,\\
&U_3(x) = 8x^3 - 4x,\\
&\cdots .
\end{aligned}$$

Taking the Laguerre expansion as an example and plugging (16) into (11), we have

$$\begin{aligned}
u_0 &= \mathcal{L}^{-1}(c_0 L_0(x) + c_1 L_1(x) + \cdots + c_l L_l(x)) + \Psi(x),\\
u_1 &= -\mathcal{L}^{-1}(\mathcal{R}u_0) - \mathcal{L}^{-1}(\Lambda_0),\\
u_2 &= -\mathcal{L}^{-1}(\mathcal{R}u_1) - \mathcal{L}^{-1}(\Lambda_1),\\
u_3 &= -\mathcal{L}^{-1}(\mathcal{R}u_2) - \mathcal{L}^{-1}(\Lambda_2),\\
&\cdots .
\end{aligned} \tag{18}$$

Alternatively, Wazwaz [13] had rewritten (18) as

$$\begin{aligned}
u_0 &= \mathcal{L}^{-1}(c_0 L_0(x)) + \Psi(x),\\
u_1 &= \mathcal{L}^{-1}(c_1 L_1(x)) - \mathcal{L}^{-1}(\mathcal{R}u_0) - \mathcal{L}^{-1}(\Lambda_0),\\
u_2 &= \mathcal{L}^{-1}(c_2 L_2(x)) - \mathcal{L}^{-1}(\mathcal{R}u_1) - \mathcal{L}^{-1}(\Lambda_1),\\
u_3 &= \mathcal{L}^{-1}(c_3 L_3(x)) - \mathcal{L}^{-1}(\mathcal{R}u_2) - \mathcal{L}^{-1}(\Lambda_2),\\
&\cdots .
\end{aligned} \tag{19}$$

Both (18) and (19) are governing equations of modified ADM using Laguerre polynomials. The n-term approximation of u is obtained from these equations as $u_n = \sum_{k=0}^{n-1} u_k$, which can be very close to the Laguerre expansion of the exact solution u. Convergence is well established in [14,15]. Since the decomposition series is very rapidly convergent, it will be shown by a representative example that the number of terms required to obtain an accurate solution is very small. Next, we will compare the obtained approximate u to the ones obtained from the Taylor expansion, Chebyshev expansion, Legendre expansion and Laguerre expansion to validate the accuracy of the obtained solution as well as the proposed method.

3. Test Problem

In this section, an initial ordinary differential equation is considered and the problem is solved by modified ADM (18) with the Taylor expansion $u_{Taylor}(x)$, Chebyshev expansion $u_T(x)$, $u_U(x)$, Legendre expansion $u_P(x)$ and Laguerre expansion $u_L(x)$. Note that the approximate solution u derived by (18) and (19) has similar behaviour compared to the corresponding orthogonal polynomials expansion of the exact solution, so we only calculate the first modified ADM (18).

Consider for $0 \leq x \leq 1$

$$u'' + xu' + x^2u^3 = (-2 + 2x^2)e^{-x^2} + x^2e^{-3x^2}, \tag{20a}$$

$$u(0) = 1, \quad u'(0) = 0, \tag{20b}$$

with the exact solution $u(x) = e^{-x^2}$, which tends to zero at infinity. Compared to (7), we derive that $\mathcal{L} = \frac{\mathrm{d}^2}{\mathrm{d}x^2}$, $\mathcal{L}^{-1} = \int_0^x \int_0^x (\cdot)\mathrm{d}x\mathrm{d}x$ $\mathcal{R} = x\frac{\mathrm{d}}{\mathrm{d}x}$, $\mathcal{N}(u) = x^2u^3$ and the source term $f(x) = (-2+2x^2)e^{-x^2} + x^2e^{-3x^2}$. Thus, the Adomian polynomials are given by

$$\begin{aligned}
&\Lambda_0 = x^2u_0^3,\\
&\Lambda_1 = x^2(3u_0^2u_1),\\
&\Lambda_2 = x^2(3u_0^2u_2 + 3u_0u_1^2),\\
&\Lambda_3 = x^2(3u_0^2u_3 + 6u_0u_1u_2 + u_1^3),\\
&\cdots.
\end{aligned}$$

Setting $l = n = 7$, we obtain the approximation of u by different orthogonal polynomials.

Case 1. The Taylor expansion of $f(x)$ is written by

$$f_{Taylor}(x) \approx -2 + 5x^2 - 6x^4 + \frac{35x^6}{6}.$$

Then, we have

$$\begin{aligned}
&u_0(x) = \mathcal{L}^{-1}\left(-2 + 5x^2 - 6x^4 + \frac{35x^6}{6}\right) + u(0) + u'(0)(x) = 1 - x^2 + \frac{5x^4}{12} - \frac{x^6}{5} + \frac{5x^8}{48},\\
&u_1(x) = -\mathcal{L}^{-1}\big(xu_0'(x)\big) - \mathcal{L}^{-1}(\Lambda_0) = -\frac{x^4}{12} + \frac{x^6}{10} - \frac{17x^8}{224} + \cdots,\\
&u_2(x) = -\mathcal{L}^{-1}\big(xu_1'(x)\big) - \mathcal{L}^{-1}(\Lambda_1) = -\frac{x^6}{90} - \frac{31x^8}{3360} + \frac{337x^{10}}{37{,}800} + \cdots,\\
&u_3(x) = -\mathcal{L}^{-1}\big(xu_2'(x)\big) - \mathcal{L}^{-1}(\Lambda_2) = \frac{x^8}{840} + \frac{x^{10}}{840} + \cdots,\\
&u_4(x) = -\mathcal{L}^{-1}\big(xu_3'(x)\big) - \mathcal{L}^{-1}(\Lambda_3) = -\frac{x^{10}}{9450} + \cdots,\\
&\cdots.
\end{aligned}$$

Consequently, we obtain

$$u_{Taylor}(x) = 1 - x^2 + \frac{x^4}{3} - \frac{x^6}{9} + \frac{17x^8}{840} + \frac{x^{10}}{18} + \cdots. \tag{21}$$

Case 2. In terms of the first kind Chebyshev expansion for $f(x)$, we have

$$f_T(x) \approx \sum_{k=0}^{6} a_kT_k(2x-1), \quad 0 \le x \le 1,$$

where the coefficients a_k are computed by

$$\begin{aligned}
a_0 &= \frac{1}{\pi}\int_{-1}^{1} \frac{f_T(0.5x+0.5)C_0(x)}{\sqrt{1-x^2}}\mathrm{d}x,\\
a_k &= \frac{2}{\pi}\int_{-1}^{1} \frac{f_T(0.5x+0.5)C_k(x)}{\sqrt{1-x^2}}\mathrm{d}x, \quad k = 1,2,\cdots,6,
\end{aligned}$$

which implies

$$f_T(x) \approx -2 + 0.0019x + 4.9259x^2 + 0.8070x^3 - 9.7599x^4 + 8.4226x^5 - 2.3478x^6.$$

Consequently, we derive $u_T(x)$ by a process similar to Taylor expansion

$$\begin{aligned} u_T(x) &= 1 - x^2 + 0.0003x^3 + 0.4938x^4 + 0.0403x^5 - 0.2912x^6 \\ &\quad + 0.1957x^7 - 0.0908x^8 - 0.0207x^9 + 0.0618x^{10} + \cdots . \end{aligned} \tag{22}$$

Case 3. In terms of Legendre expansion for $f(x)$, we have

$$f_P(x) \approx \sum_{k=0}^{6} b_k P_k(2x-1), \quad 0 \le x \le 1,$$

where the coefficients b_k are computed by

$$b_k = \frac{2k+1}{2} \int_{-1}^{1} f_P(0.5x+0.5) P_k(x) \mathrm{d}x, \quad k = 0, 1, \cdots, 6,$$

which implies

$$f_P(x) \approx -1.9996 - 0.0141x + 5.0807x^2 + 0.2047x^3 - 8.6563x^4 + 7.4707x^5 - 2.0361x^6.$$

Consequently, we derive $u_P(x)$ by a process similar to Taylor expansion

$$\begin{aligned} u_P(x) &= 1 - x^2 - 0.0024x^3 + 0.5067x^4 + 0.0106x^5 - 0.2561x^6 \\ &\quad + 0.1768x^7 - 0.0896x^8 - 0.0178x^9 + 0.0614x^{10} + \cdots . \end{aligned} \tag{23}$$

The above three kinds of polynomial expansions are already proposed and confirmed. Next, we introduce two classes of orthogonal polynomials to assist with the application of ADM.

Case 4. In terms of the Laguerre expansion for $f(x)$ proposed in the paper, we have

$$f_L(x) \approx \sum_{k=0}^{6} c_k L_k(x),$$

where the coefficients c_k are computed by

$$c_k = \frac{1}{(k!)^2} \int_{0}^{\infty} f_L(x) L_k(x) e^{-x} \mathrm{d}x, \quad k = 0, 1, \cdots, 6,$$

which implies

$$f_L(x) \approx -2.3931 + 3.8375x - 1.9648x^2 + 0.4388x^3 - 0.0466x^4 + 0.0023x^5 - 0.0001x^6.$$

Consequently, we derive $u_L(x)$ by a process similar to Taylor expansion

$$\begin{aligned} u_L(x) &= 1 - 1.1966x^2 + 0.6396x^3 - 0.0476x^4 - 0.0740x^5 + 0.1245x^6 \\ &\quad - 0.0368x^7 - 0.0875x^8 + 0.0704x^9 + 0.0052x^{10} + \cdots . \end{aligned} \tag{24}$$

Case 5. In terms of the second kind Chebyshev expansion for $f(x)$ proposed in the paper, we have

$$f_U(x) \approx \sum_{k=0}^{6} d_k U_k(x),$$

where the coefficients d_k are computed by

$$d_k = \frac{2}{\pi} \int_{0}^{\infty} f_U(x) U_k(x) \sqrt{1-x^2} \mathrm{d}x, \quad k = 0, 1, \cdots, 6,$$

which implies

$$f_U(x) \approx -2 + 0.0035x + 4.8999x^2 + 0.9414x^3 - 10.0588x^4 + 8.7217x^5 - 2.4583x^6.$$

Consequently, we derive $u_U(x)$ by a process similar to Taylor expansion

$$\begin{aligned} u_U(x) = 1 - x^2 + 0.0006x^3 + 0.4917x^4 + 0.0470x^5 - 0.3008x^6 \\ + 0.2020x^7 - 0.0916x^8 - 0.0215x^9 + 0.0621x^{10} + \cdots . \end{aligned} \tag{25}$$

Finally, in order to demonstrate the accuracy and efficiency of the modified ADM with orthogonal polynomial expansions, we draw the approximate solutions and error curves of various expansions. For comparing the errors of the same order of magnitude accurately, we plotted the absolute errors $|u - u_{Taylor}|$, $|u - u_L|$ in the same coordinate system (see Figure 1) and $|u - u_T|$, $|u - u_P|$, $|u - u_U|$ in another coordinate system (see Figure 2).

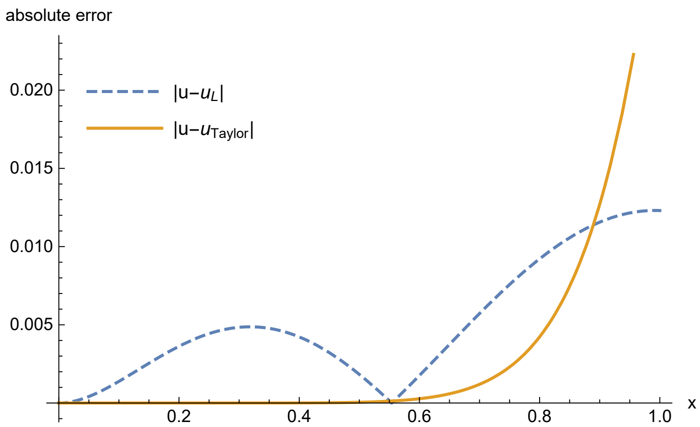

Figure 1. The absolute error between exact solution and u_L and u_{Taylor}.

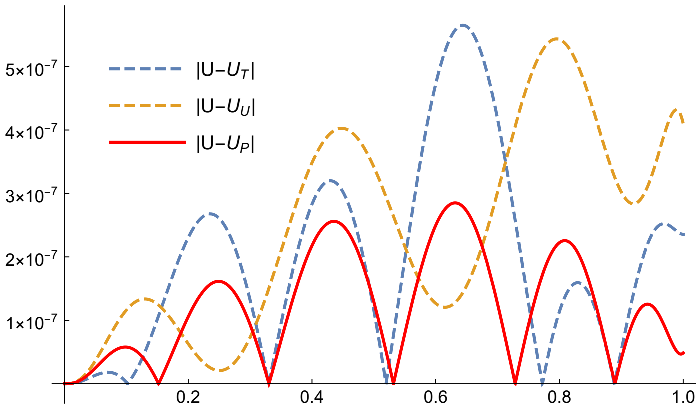

Figure 2. The absolute error between exact solution and u_T, u_P and u_U.

The results show that the Chebyshev and Legendre expansions possess the absolute advantage in respect of error (about 10^{-7}), while the errors derived by Laguerre and Taylor expansions are about five orders of magnitude higher than the former methods (about 10^{-2}), because each Laguerre polynomial contains all items, from the lowest to highest, which leads to larger truncation error. Moreover, the Taylor expansion at zero makes the error further and further away from the origin. From the point of view of errors, we recommend finite interval approximations which include the Chebyshev expansion and Legendre expansion.

4. Concluding Remarks

In this paper, a new efficient and practical modification of the Adomian decomposition method is proposed with Laguerre polynomials and the second kind of Chebyshev polynomials. These orthogonal polynomials can be applied to further improve the ADM and the approximation solution is more accurate than the general ADM. By comparison, Legendre polynomials provide estimations that are a little better than those of Chebyshev polynomials, which is a contradiction with the research of [9]. On the other hand, the orthogonal interval of the Laguerre polynomials is $[0, +\infty]$ so they can be applied to approximate the functions which define a semi-infinite interval. Take the example in this article, the exact solution is $u(x) = e^{-x^2}$, which tends to zero at infinity, and the right hand term is $f(x) = (-2 + 2x^2)e^{-x^2} + x^2e^{-3x^2}$. In fact, the definition interval of $u(x)$ and $f(x)$ is $[-\infty, +\infty]$, so we can use other Hermite polynomials. However, for comparison with other polynomials in one example, such as Chebyshev polynomials, Legendre polynomials and Lagurre polynomials, we chose the common definition interval $[0, 1]$. Therefore, we can choose the most suitable approximation method according to different definition intervals.

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