mathematics

Article

Mathematics Training in Engineering Degrees: An Intervention from Teaching Staff to Students

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Abstract: There has always been a great concern about the teaching of mathematics in engineering degrees. This concern has increased because students have less interest in these studies, which is mainly due to the low motivation of the students towards mathematics, and which is derived in most cases from the lack of awareness of undergraduate students about the importance of mathematics for their career. The main objective of the present work is to achieve a greater motivation for engineering students via an intervention from the teaching staff to undergraduate students. This intervention consists of teaching and learning mathematical concepts through real applications in engineering disciplines. To this end, starting in the 2017/2018 academic year, sessions addressed to the teaching staff from Universitat Politècnica de Catalunya in Spain were held. Then, based on the material extracted from these sessions, from 2019/2020 academic year the sessions "Applications of Mathematics in Engineering I: Linear Algebra" for undergraduate students were offered. With the aim of assessing these sessions, anonymous surveys have been conducted. The results of this intervention show an increase in students' engagement in linear algebra. These results encourage us to extend this experience to other mathematical subjects and basic sciences taught in engineering degrees.

Keywords: mathematics education; engineering degrees; STEM; student motivation

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1. Introduction

To improve the economy of countries, a key factor is to encourage technology. Technological production begins by encouraging and supporting students to develop professional careers in fields related to science, technology, engineering and mathematics (STEM). Therefore, STEM disciplines are considered essential for the economic development of technological societies. The critical role of integrating STEM disciplines into the promotion of students who need to equip themselves with 21st century skills has attracted much attention. Several countries have promoted STEM education as the benefits of this education for quality learning have been recognized. In addition, it has been shown that STEM education could improve the integration of students' skills and the fact that they are better prepared for their professional activity. The 21st century, as the age of information-based technology, brings new job prospects as well as upcoming jobs that demand new skills from workers. Technology is currently used in many jobs in areas such as science, business, engineering, etc.

In addition, high employment in STEM disciplines is expected [1–3]. As technological knowledge becomes increasingly specialized and economically important, more jobs are needed in STEM disciplines and this demand is expected to increase further in the coming years, as noted in [4,5]. However, in most countries, the number of students enrolling in STEM-related disciplines has decreased at secondary and tertiary levels [6]. Currently, this concern has increased because a lower interest in STEM disciplines among European university students has been detected. Therefore, the engineering education community is working to identify the factors that provoke this scenario, as indicated by [1].

One of the issues that has gained a lot of interest in academic research is the worrying levels of dropout in higher education. It was found that approximately one-third of entering college students leave their institution of higher education without obtaining a degree, especially during their first year [7]. The dropout rate increases in STEM careers, as can be observed in [8]. Several studies have focused on the importance of students' motivation and engagement [9,10], in particular for technological degrees [11]. So, to reduce the number of dropouts in the early stages of study, it is necessary to promote student engagement [12,13], which is directly related to motivation [14], student achievement [15] and academic performance [11].

Teachers have always been the most crucial element in educational reform [16,17]. Previous studies on education reform stated that teachers were the main drivers behind students' interest in STEM and their achievements [18]. Most efforts to reform undergraduate STEM education are based on a presumptive reform model related to teacher participation, based primarily on classroom innovation and the teaching–learning process. The self-efficacy and involvement of teachers in classroom teaching play a key role in the realization of integrated education in STEM [19,20]. An important aspect of mathematics education research is to address significant ways for learning and change of mathematics teachers [21–23].

Many university departments offer math courses to their first-year college students and are generally a mandatory part of their departmental programs [24]. In [25], the relationship between basic subjects and applied engineering subjects in higher engineering education curricula is evaluated. Different approaches to teaching mathematics have been considered in several works. There are numerous studies that confirm that active learning has a positive effect on increasing students' motivation and their improvement in learning, which implies enhanced performance, as indicated in [26–29]. For example, the concept of mathematical creativity and the relevance of problem-solving in teaching mathematics have been studied in [30]. In addition, key skills and qualifications expected from employees change from performing routine tasks to solving problems comprised of complex systems through construction, description, explanation, manipulation and prediction. That is, employees are expected to have problem-solving and analytical thinking skills, as well as the conceptual tools to communicate and outsource them.

Rarely do math teacher training programs include a focus on mathematical modeling or the use of models in future teachers' math courses [31,32]. The use of problem-posing in engineering degrees is a profitable tool to increase student involvement and it is known that in engineering education practical and real applications used in basic sciences encourage student engagement and motivation [14], as has been developed in previous studies [33–35]. With this methodology, students are given a problem related to a technological field, which will drive the learning process and allow students to discover what they need to learn to solve the problem. Moreover, it helps students to develop skills and competencies, such as continuous learning, autonomy, teamwork, critical thinking, communication and planning [36], which are considered very important in their profession [37]. Furthermore, theory and practice are integrated, and motivation is enhanced, which results in increased academic performance [38–41]. Another approach to math education based on action learning has been considered in [42]. Various studies have described the benefits of integrating information and communication technologies (ICT) into education [43–45]. It is hoped that the 21st century math teachers will be able to figure out how to integrate technology into all aspects of education [46,47]. Computational thinking is another essential skill to incorporate in math education [48].

In this article, we conjecture the challenge of generating an integrated STEM curriculum. In particular, the aim of this study is to present a contribution to the relationship between mathematical applications and integrated STEM education. The main objective of the present work is to achieve greater motivation for undergraduate engineering students by contextualizing basic sciences, mainly mathematics, through applications to the disciplines of technological degrees. It is expected that the material developed in this work will be introduced for future adaptation of mathematics into core subjects of engineering degrees.

Research Rationale and Research Questions

Engineering students generally do not perceive mathematics in the same way as people who want to pursue this discipline. Engineering students think differently; they want to solve engineering problems and mathematics is just one tool like any other. They need to be told what their knowledge of mathematics is for and the extent to which it is essential to their studies and their future profession. In this sense, the motivation and commitment of the students is considered a key element, making clear the relevance of these basic disciplines to the later technologies and professional exercise.

For an intervention to be more likely to be successful, it must be contextually appropriate in its disciplinary and institutional environment. In this sense, the first part of the work consists of considering the competence specificities related to engineering disciplines that focus on a type of problems of their own, as well as the type of knowledge and learning their own skills. It is very convenient that this task is done in collaboration with the teachers of mathematics and technology departments. Next, it is a question of conducting an analysis looking for a systemic understanding that goes beyond the appreciation of the individual components, to extract the mathematical concepts of the different engineering problems posed previously. This task will be carried out by the teachers of mathematics departments who will then make the extracted material available to undergraduate engineering students.

The aim of this work is to present proposals for the implementation of problems arising from the technology faced by engineering students, which will be complementary to their regular courses. These problems will be multidisciplinary and have in common the idea that mathematics is a necessary skill for solving them. Having realized the need for their knowledge of mathematical methods, students are looking forward to solving the posed problems, thus turning their attention to their mathematical education.

This paper focuses on these research questions:

- How can the mathematical curriculum of an engineering program be adapted to include technological applications?
- How do teachers value this intervention?
- How do students value this intervention?

2. Materials and Methods

The study was conducted at the Universitat Politècnica de Catalunya-BarcelonaTech (UPC) (www.upc.edu, accessed on 1 May 2021), a public university specializing in STEM. During the 2017/2018 academic year, the seminar "Contextualization of mathematics in engineering degrees" was inaugurated at UPC, supervised by one of the authors of this paper and promoted by the vice rector's Office for Academic Policy. The intervention was done in several stages, each dealing with one science subject (mathematics, physics, chemistry, etc.). In the first stage, the intervention was based on mathematics, which began in the 2017/2018 academic year and continues today.

First, these seminars consisted of lectures (an hour and a half per session) for teachers. This is the fourth academic year of this seminar for teachers, called "Contextualization of mathematics in engineering degrees", which aims to illustrate the applications of mathematics in different technological areas. Then, in accordance with the results obtained in the previous seminars for teachers, teaching is carried out for undergraduate engineering students (weekly sessions, an hour and a half each session). Previous sessions were aimed at teachers starting the 2017/2018 academic year and the 2019/2020 academic year sessions for undergraduate engineering students focused on mathematics began. Students' sessions were called "Applications of mathematics in engineering". This seminar for students aimed to bring to the classroom the material extracted from the previous seminar for teachers, in order to improve academic performance and reduce the dropout rate at the UPC, promoting students' engagement. This intervention began in the 2017/2018 academic year with teachers to enable them to implement the material extracted from these sessions later with students in the 2019/2020 academic year. Currently, the intervention is carried out in parallel with teachers and students to expand the material available.

To evaluate these interventions, anonymous surveys were conducted, both for teachers and students of each of the sessions. These questionnaires analyze the impact of the experience and collect assessments from all project members, which will be used to tailor science content to the needs and expectations of undergraduate students in upcoming academic years.

2.1. Teachers' Intervention

The teacher's intervention based on mathematics consists of sessions focused on different engineering disciplines (automation, electricity, mechanics, electronics, etc.). In each of these areas, engineering cases are presented and explained using the mathematical tools needed to solve them. To study and solve these exercises, it is necessary to apply mathematical concepts and techniques: equations of the linear system, complex numbers, matrix modeling, etc. The sessions of this seminar are taught by both teachers of the departments of basic and applied subjects engineering departments. To date, there have been eighteen sessions of math contextualization. The titles are detailed in Table 1.

Table 1. Seminar of contextualization of mathematics in engineering degrees.

Session	Title	Date
1	"Invitation to the Educative Renewal of Mathematics in Engineering degrees"	10 April 2018
2	"Network Flows"	25 April 2018
3	"Engagement with the First Course Students of Civil Engineering"	15 May 2018
4	"Mathematics of Google"	23 May 2018
5	"Numerical Factory: a Numerical Tasting about the Teaching of Mathematics in Engineering"	5 June 2018
6	"How Mathematical Tools help to Manufacture Mechanical Parts"	3 October 2018
7	"One Proposal for the Teaching of Mathematics in Computer Science"	16 October 2018
8	"Mathematical Applications in Elasticity and Resistance of Materials"	7 November 2018
9	"A Historically Problematic Relationship: Mathematics in Engineering"	27 November 2018
10	"Virtual Reality Applications for Biomedical Engineering"	27 February 2019
11	"Fundamental Mathematical Concepts and Tools in Electronic Engineering"	21 March 2019
12	"Modelling and Linear Ordinary Differential Equations Systems"	10 April 2019
13	"Determined Linear Systems for Consecutive Values of States"	2 May 2019
14	"Mathematical Concepts and Tools in Automatic"	22 May 2019
15	"Animated Mathematics"	16 October 2019
16	"Probabilities and Communication Theory: Random Walks in Graphs and Algorithms"	2 December 2020
17	"Cryptography: the Arithmetic of Large Numbers"	17 March 2021
18	"Mathematics at the Service of Engineering Attitudes"	4 May 2021

To evaluate this experience, anonymous surveys were conducted at the end of each session with the aim of analyzing teachers' opinions about the applications and practical exercises introduced.

2.2. Students' Intervention

The material from these teachers' sessions has been adapted to be useful to students. Thus, since the 2019/2020 academic year, weekly sessions have been given to undergraduate students in the first semester on "Applications of Mathematics in Engineering", based on the subject of linear algebra. This students' intervention is designed to increase the engagement and motivation of students in the early stages of their studies. These are voluntary sessions and the UPC recognizes 1 European Credit Transfer and Accumulation System (ECTS) for each semester of student attendance.

Sessions aimed at undergraduate students are organized according to the different concepts of linear algebra. The sessions "Applications of Mathematics in Engineering I: Linear Algebra" for undergraduate students consists of 10 sessions. The sessions of this seminar (Table 2) were organized following the contents of linear algebra in the first course of an engineering degree in order to show students that the concepts they are learning are useful and necessary for their degree.

Table 2. Applications of mathematics in engineering I: linear algebra.

Session	Title
1	"Complex Numbers on the Study of Price Fluctuations"
2	"Complex Numbers on the Study of Alternating Current"
3	"Indeterminate Systems: Control Variables"
4	"Mesh Flushes: a Basis of Conservative Fluxes Vector Subspace"
5	"Addition and Intersection of Vector Subspaces in Discrete Dynamical Systems"
6	"Linear Applications and Associated Matrix"
7	"Basis Changes"
8	"Eigenvalues, Eigenvectors and Diagonalization in Engineering"
9	"Modal Analysis in Discrete Dynamical Systems"
10	"Difference Equations"

To evaluate this experience, anonymous surveys were conducted at the end of each session, with the aim of analyzing students' appreciation of the applications and practical exercises introduced. In order to extract more information from the students attending the sessions "Applications of Mathematics in Engineering I: Linear Algebra", personal interviews were undertaken at the end of these sessions.

3. Results

3.1. Teachers' Results

3.1.1. Teachers' Mathematical Contents

As an example of the teachers' intervention and in order to show the seminars and the development of a session, the session "Network flows" is summarized below. The applications and linear algebra contents from this session are detailed in Table 3.

Table 3. Session 2 ("Network flows") from the seminar of contextualization of mathematics in engineering degrees.

Applications	Linear Algebra Contents
Roundabout traffic	Matrices and determinants. Equation systems.
Electrical network	Equation systems. Vectorial spaces. Vectorial subspaces. Linear applications.
Bus station	Discrete linear systems: contagious matrix, eigenvectors and eigenvalues.
Google: webs classification	Discrete linear systems: contagious matrix, eigenvectors and eigenvalues, Gould accessibility index.

With the aim of being profitable in the future and being able to be consulted and used by any member of the educational community, the sessions of the "Seminar of contextualization of mathematics in engineering degrees" have been recorded. These recordings are available in a repository of UPC (https://upcommons.upc.edu/handle/2117/118481, Catalan language, accessed on 1 May 2021).

3.1.2. Teachers' Surveys Results

The number of teachers attending the sessions "Seminar of contextualization of mathematics in engineering degrees" undertaken to date (18 sessions) is 612 (among them, around 150 different teachers), which means an average of 34 teachers per session. It is worth noting that not only the teachers from the mathematics department attended these sessions, but also those from engineering departments.

The material developed in the sessions "Seminar of contextualization of mathematics in engineering degrees" has been analyzed taking into account the results of the anonymous surveys conducted by the attending professors. Teachers' surveys assess the academic aspects of each lecture, as well as the clarity of the speaker and the general organization of the activity. The participants answered three questions valued on a 5-point scale (1 = strongly disagree, 2 = disagree, 3 = neither agree nor disagree, 4 = agree, 5 = strongly agree). In addition, there is an open field with the possibility to add a comment about the session.

Teachers' questionnaires of the sessions held until now have already been analyzed. The participants in these surveys were 337 teachers (55% of the assistants to the sessions). In Table 4 the questions and the average results are detailed.

Table 4. Teachers' surveys results.

Survey Question	Average
The assessment of academic aspects is positive	4.56
The level of satisfaction regarding the speaker is positive	4.62
General organization of the activity has been appropriate	4.56

The response of the teachers participating in these sessions has been very positive, as can be seen in Table 4. In addition, it is worth mentioning the comments of some teachers expressed in the open field of the surveys, the main themes were:

- Innovative problems.
- Examples with applications in different fields.
- Interesting works linked to social needs.

3.2. Students' Results

3.2.1. Students' Mathematical Contents

Some examples of the applications and problems addressed to students in the sessions "Applications of Mathematics in Engineering I: Linear Algebra" are summarized below. They consist of applications of linear algebra related to engineering which can be understood by undergraduate students in first-year courses.

1. Session 1 (complex numbers on the study of price fluctuations):

In dynamical systems, oscillatory modes with the following form are frequent:

$$p(k) = p_e + c\|\lambda\|^k \cos(k\varphi + \varphi_0),\ k = 0,\ 1,\ 2,\ \ldots \tag{1}$$

determined by:

$$\lambda = \|\lambda\| e^{j\varphi} \in \mathbb{C} \tag{2}$$

In this study, $p(k)$ is the merchandise price in the k-th sales season.

Price expectation for the next season from the previous season is in general:

$$\hat{p}(k) = \beta_1 p(k-1) + \beta_2 p(k-2) + \cdots \tag{3}$$

$$\beta_1 + \beta_2 + \cdots = 1 \tag{4}$$

It is demonstrated that:

$$p(k) = p_e + c\|\lambda\|^k \cos(k\varphi + \varphi_0) \tag{5}$$

where:

$$\lambda = \|\lambda\| e^{j\varphi} \in \mathbb{C} \tag{6}$$

is the "dominant root" of:

$$t^k + \frac{b}{a}\left(\beta_1 t^{k-1} + \beta_2 t^{k-2} + \cdots\right) = 0 \tag{7}$$

called "characteristic polynomial".

- Application session 1: spiderweb model:

In the spiderweb model, producers take as "price expectative" the price from the previous season:

$$\hat{p}(k) = p(k-1) \Rightarrow \beta_1 = 1,\ \beta_2 = \beta_3 = \cdots = 0 \Rightarrow \tag{8}$$

$$\Rightarrow \lambda \text{ is the dominant root of : } t + \frac{b}{a} = 0 \Rightarrow \tag{9}$$

$$\Rightarrow \lambda = -\frac{b}{a} = \frac{b}{a}e^{j\pi} \Rightarrow \begin{cases} \|\lambda\| = \frac{b}{a} \\ \text{Biannual periodicity} \end{cases} \tag{10}$$

- Application session 1: producers' reference to two previous years:

Suppose that $\frac{b}{a} = 1$, but producers refer to the two previous years:

$$\hat{p}(k) = \frac{p(k-1) + p(k-2)}{2} \Rightarrow \beta_1 = \beta_2 = \frac{1}{2}, \beta_3 = \beta_4 = \cdots = 0 \Rightarrow \tag{11}$$

$$\Rightarrow \lambda \text{ is the dominant root of : } t^2 + \frac{1}{2}(t+1) = 0 \Rightarrow \lambda = \frac{-1 \pm j\sqrt{7}}{4} \tag{12}$$

Therefore:

$$\begin{cases} \text{Triennial periodicity} \\ \text{Attenuated oscillations } \left(\text{with } \frac{\mathrm{b}}{\mathrm{a}} = 1\right) \end{cases} \tag{13}$$

Particularly, the condition $\frac{b}{a} < 1$ can be changed to $\frac{b}{a} < 2$:

$$\frac{b}{a} = 2 \Rightarrow \lambda \ \text{ is the dominant root of : } t^2 + t + 1 = 0 \Rightarrow \lambda = \frac{-1 \pm j\sqrt{3}}{2} \Rightarrow \|\lambda\| = 1 \tag{14}$$

- Application session 1: price cycle of pork meat:

In almost a century, four times/year oscillations were observed in the production of pork fat meat in the USA. It is necessary to find a model that fits into it and deduce the $\frac{b}{a}$ value to attenuate it.

It must be considered that there are two seasons of production in each year (spring and autumn) and that the raising period of fat pork is approximately one year. Therefore, k variable corresponds to semester and the "decision/production" is two of these periods (that is, $\beta_1 = 0$).

Supposing:

$$\hat{p}(k) = \frac{1}{5}(p(k-2) + p(k-3) + p(k-4) + p(k-5) + p(k-6)) \tag{15}$$

results:

$$t^6 + \frac{b}{a}\frac{1}{5}\left(t^4 + t^3 + t^2 + t + 1\right) = 0 \tag{16}$$

In fact, four times-year oscillations are obtained:

$$\lambda = e^{j\frac{\pi}{4}} \Longrightarrow \begin{cases} \lambda^6 = -j \\ \lambda^4 + \lambda^3 + \lambda^2 + \lambda + 1 \cong 2.4j \end{cases} \Rightarrow \frac{b}{a} \cong \frac{5}{2.4} \cong 2.08 \tag{17}$$

Thus, it must be forced:

$$\frac{b}{a} < 2.08 \tag{18}$$

2. Session 2 (Complex numbers on the study of alternating current):

In this session, several applications of electricity in alternating current were explained, in which the use of complex numbers was necessary to solve these problems. See [38] for further information. The applications dealt in this session were:

– Analysis of alternating current circuits: an alternating current $i(t)$ must be calculated in a node, knowing the values of three alternating currents in the same node. Kirchhoff's current law is used, and currents are converted into the complex form.
– Triphasic distribution: phase/neutral voltage and phase/phase voltage must be calculated in a triphasic distribution. To solve it, voltages are converted into the complex form, and phasor representation is used in order to explain the relation between phase/neutral voltage and phase/phase voltage.
– RLC circuit: a circuit with resistance, inductance and capacitor is solved using the complex impedance.
– Resonances: the conditions in which resonance is produced in a parallel circuit must be determined.
– Annulation of reactive power: in this exercise, the capacity of a capacitor must be calculated which has to be in parallel with impedance so that the equivalent impedance is real. That means that reactive power disappears, and performance is optimized.

3. Session 3 (indeterminate systems: control variables):

The third session showed applications of indeterminate equations systems.

• Application session 3: the roundabout traffic:

One of the exercises consisted of a roundabout traffic where three double-ways converge (Figure 1). It was explained how it can be described by a linear equations system and the compatibility conditions were found and interpreted [20].

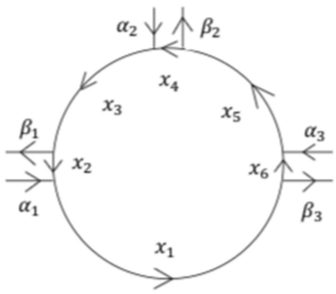

Figure 1. The roundabout traffic.

In this practical exercise it was asked to:

1. Prove that it can be described by the following linear equation system:

$$A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ -\beta_1 \\ \alpha_2 \\ -\beta_2 \\ \alpha_3 \\ -\beta_3 \end{pmatrix}, A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{19}$$

2. Find and interpret the compatibility conditions.
3. In such case, prove that it is a 1-indeterminate system, and a solution basis of the homogeneous system is $x_1 = \cdots = x_6 = 1$.
4. How many traffic measures are needed to know $(x_1, \ldots, x_6)$?
5. Deduce that there exist solutions with $x_i \geq 0$ and that there exists a unique solution with $x_i \geq 0$ and some $x_{i_0} = 0$.
6. Interpret the solutions with $x_i > 0$.

• Application session 3: flow distribution:

Another application dealt in this session was the following flow distribution (Figure 2):

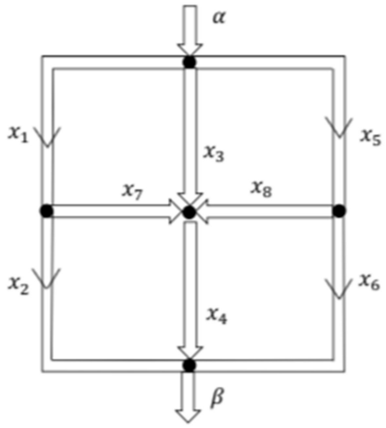

Figure 2. Flow distribution.

In this exercise it was asked:

1. To study the compatibility conditions of the system.
2. To determine how many flows must be measured to know the global circulation of the system.
3. If global circulation can be calculated measuring the flows of the four peripheric points.
4. If global circulation can be calculated measuring the flows of the four intern points.
5. To generalize the study to three branches with more the one interconnexion.

4. Session 4 (mesh fluxes: a basis of vector subspace of conservative fluxes):

In this session a simple electrical network (Figure 3) was solved in order to demonstrate that mesh fluxes are a basis of conservative fluxes. See [38] for further information.

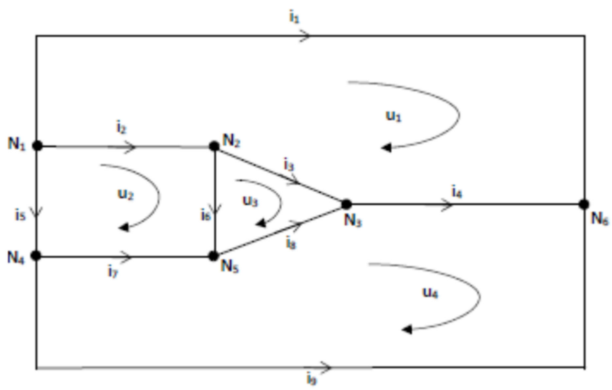

Figure 3. An electrical network.

E being the set of possible current distributions, it was demanded to find the subset $F \subset E$ verifying KCL (Kirchhoff's Current Law); that is, at each of the nodes sum of input currents must be equal to the sum of output currents.

In practice, the used currents are not the above ones indicates in the figure, but the so-called mesh currents (I_1, I_2, I_3, I_4).

To justify this use, it is asked to:

1. Prove that *E* is a vector space of dimension 9 and that *F* is a subspace of *E* of dimension 4.
2. Determinate a basis of *F* so that (I_1, I_2, I_3, I_4) are its coordinates.
3. Prove that one of Kirchhoff's equations is redundant; that is, if it is verified at 5 nodes, it must also be verified at the 6th node.

5. Session 5 (addition and intersection of vector subspaces in discrete dynamical systems):

In this session some examples about control linear systems were explained. See [49] for further information on control linear systems.

The following figure shows the diagram of a general control linear system (Figure 4):

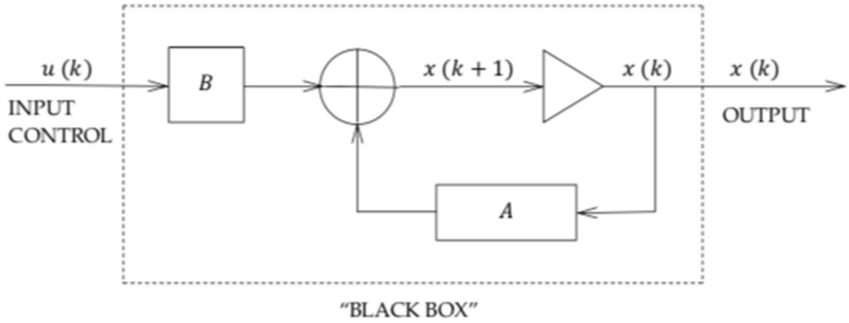

Figure 4. General control linear system.

The state of a general control linear system is:

$$x(k+1) = Ax(k) + Bu(k) \tag{20}$$

Here, different cases of control linear systems are presented.

- Application session 5: one-control case:

In the case of one control and the initial state equal to zero:

$$x(k+1) = Ax(k) + bu(k) \tag{21}$$

$$x(0) = 0 \tag{22}$$

In this case examples were proposed in which states were calculated and it was asked to find the control functions to reach a certain state.

- Application session 5: multi-control case:

In the case of multi-control and the initial state equal to zero:

$$x(k+1) = Ax(k) + Bu(k),\ B = (b_1 \dots b_m) \tag{23}$$

$$x(0) = 0 \tag{24}$$

The examples held in the multi-control case explained how to calculate the states in two conditions:

- With all of the controls, as an addition of subspaces.
- With any of the controls, as an intersection of subspaces.

- Application session 5: Kalman decomposition:

In the case of more general systems:

$$x(k+1) = Ax(k) + Bu(k) \tag{25}$$

$$y(k) = Cx(k) \tag{26}$$

Controllability subspace and observability subspace were defined.

Kalman decomposition was used to solve this case.

6. Session 6 (linear applications and associated matrix):

The applications dealt in this session were examples of linear applications and the associated matrix defined by the images of a basis.

Given a vector space E (with basis $(u_1, \dots, u_n)$), which has as image the vector space F, the following property is defined:

$$E \xrightarrow{f} F \tag{27}$$

$$u_1 \longrightarrow f(u_1) \tag{28}$$

$$u_n \longrightarrow f(u_n) \tag{29}$$

Being:

$$x = x_1 u_1 + \dots + x_n u_n \tag{30}$$

$$f(x) = x_1 f(u_1) + \cdots + x_n f(u_n) \tag{31}$$

If the basis of F is $(v_1, \ldots, v_m)$:

$$x \xrightarrow{f} f(x) \equiv y = y_1 v_1 + \cdots + y_m v_m \tag{32}$$

Therefore:

$$\begin{pmatrix} x_1 \\ \cdots \\ x_n \end{pmatrix} \xrightarrow{f} \begin{pmatrix} y_1 \\ \cdots \\ y_m \end{pmatrix} = \underbrace{\underbrace{\begin{pmatrix} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \end{pmatrix}}_{f(u_1)\ldots f(u_n)}}_{f\ MATRIX\ in\ BASES\ \left(\begin{smallmatrix} u_1, \ldots, u_n \\ v_1, \ldots, v_m \end{smallmatrix}\right)} \begin{pmatrix} x_1 \\ \cdots \\ x_n \end{pmatrix} \tag{33}$$

This property was applied in the examples hereunder.

- Application session 6: rotation of 30°:

In this example it was required to rotate a vector 30°, therefore the linear application is defined as:

$$\mathbb{R}^2 \longrightarrow \mathbb{R}^2 \tag{34}$$

It was asked to find:

$$f\begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{35}$$

The matrix in ordinary bases is calculated:

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \tag{36}$$

Therefore:

$$f\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3\frac{\sqrt{3}}{2} + 2\frac{-1}{2} \\ 3\frac{1}{2} + 2\frac{\sqrt{3}}{2} \end{pmatrix} \tag{37}$$

In general:

$$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2}x_1 - \frac{1}{2}x_2 \\ \frac{1}{2}x_1 + \frac{\sqrt{3}}{2}x_2 \end{pmatrix} \tag{38}$$

- Application session 6: change to italics:

This example showed how to change a letter to italics (Figure 5):

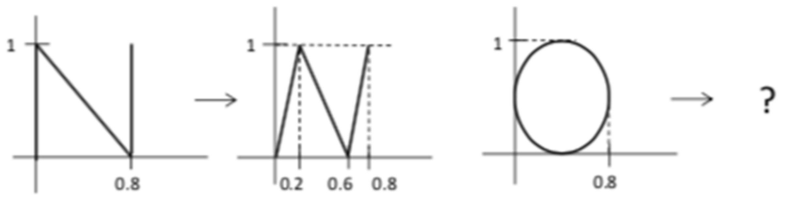

Figure 5. Change a letter to italics.

First, the matrix in ordinary bases is calculated:

$$\begin{pmatrix} 0.75 & 0.2 \\ 0 & 1 \end{pmatrix} \tag{39}$$

Therefore:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \begin{pmatrix} 0.75 & 0.2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0.75x_1 + 0.2x_2 \\ x_2 \end{pmatrix} \tag{40}$$

Indeed:

$$\begin{pmatrix} 0.8 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0.75 & 0.2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0.8 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 1 \end{pmatrix} \tag{41}$$

In general, fixed points are defined by:

$$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} 0.75 & 0.2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Longleftrightarrow \cdots \Longleftrightarrow x_1 = 0.8x_2 \tag{42}$$

7. Session 7 (Basis changes):

This session presented examples of basis changes in vectors and basis changes in linear applications. Finally, some applications in control theory were dealt. Here, one of the examples treated in the session is explained.

- Application session 7: color filters:

This is an example of basis changes in vectors.

Colors form a vector space with dimension 3. For example: yellow, green, red and blue are not linearly independent.

Different bases of three colors are used depending on if the mixed is additive (light) or subtractive (pigments), as it is going to be detailed hereunder.

The three chosen colors are called primary colors and the mixed of only two of them are called secondary colors.

Likewise, in international congress CIE (Commission Internationale de l'Éclairage) of 1931, new coordinates which depend on luminosity were stablished.

The human retina contains 6.5 million cone cells and 120 million rod cells.

The three types of cone cells respond to light of short (S cones), medium (M cones) and long (L cones) wavelengths. L cones more readily absorb red, M cones, green and S cones absorb blue.

Rod cells are sensitive to brightness and produce a black and white response.

For that reason, colors red, green and blue are used for additive mixing as primary colors.

Natural code for screens is RGB code: red (R), green (G) and blue (B). Secondary colors result as (Figure 6):

$$\begin{pmatrix} \mathrm{R} \\ \mathrm{G} \\ \mathrm{B} \end{pmatrix} \tag{43}$$

G + B = CYAN (C)
R + B = MAGENTA (M)
R + G = YELLOW
R + G + B = WHITE

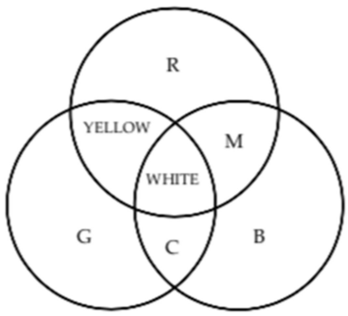

Figure 6. Colors.

But black cannot be obtained.

For subtractive mixing (printers, pigments, etc.), code CMY is used, which has as primary colors cyan, magenta and yellow:

$$\begin{pmatrix} \mathrm{C} \\ \mathrm{M} \\ \mathrm{Y} \end{pmatrix} \tag{44}$$

Secondary colors are the primary colors in the natural code:

MAGENTA + YELLOW = RED
CYAN + YELLOW = GREEN
CYAN + MAGENTA = BLUE
CYAN + MAGENTA + YELLOW = BLACK

Likewise, black is often added as a fourth pigment for saving reasons.

In additive mixing it was verified that human retina is especially sensible to brightness (black and white). For this reason, in CIE congress of 1931, the CIE code was established:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{45}$$

where x ($\equiv \mathrm{C_R}$) $\cong$ RED, y brightness and z ($\equiv \mathrm{C_B}$) $\cong$ BLUE.

A usual transformation is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.61 & 0.29 & 0.15 \\ 0.35 & 0.59 & 0.063 \\ 0.04 & 0.12 & 0.787 \end{pmatrix} \begin{pmatrix} \mathrm{R} \\ \mathrm{G} \\ \mathrm{B} \end{pmatrix} \tag{46}$$

8. Session 8 (eigenvalues, eigenvectors and diagonalization in engineering):

In this session, multiple applications of eigenvalues and eigenvectors in engineering were exposed: materials resistance, mechanics, elasticity, control, dynamics, electricity, population models, etc. Here, two of the examples are developed. More examples can be found in [50].

- Application session 8: prey/predator:

Supposing a prey (p) and predator (d) model, where respective next year populations $d(k+1)$, $p(k+1)$ depend linearly on present year populations $d(k)$, $p(k)$:

$$\begin{pmatrix} d(k+1) \\ p(k+1) \end{pmatrix} = \begin{pmatrix} 0.5 & 0.4 \\ -0.125 & 1.1 \end{pmatrix} \begin{pmatrix} d(k) \\ p(k) \end{pmatrix} \tag{47}$$

It was asked to determine the eigenvalues an eigenvectors of the matrix, which are:

$$\lambda_1 = 1;\ v_1 = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \tag{48}$$

$$\lambda_2 = 0.6; v_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{49}$$

The first one indicates a stationary distribution of 4 predators for each 5 preys, which maintains the total populations constant ($\lambda_1 = 1$).

The second one indicates another stationary distribution (4 predators for each prey), with a yearly decrease of the total population of 40% ($\lambda_2 = 0.6$).

- Application session 8: American owl:

In the study of Lamberson [51] about survival of the American owl, he experimentally obtained:

$$\begin{pmatrix} Y(k+1) \\ S(k+1) \\ A(k+1) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0.33 \\ 0.18 & 0 & 0 \\ 0 & 0.71 & 0.94 \end{pmatrix} \begin{pmatrix} Y(k) \\ S(k) \\ A(k) \end{pmatrix} \tag{50}$$

where $Y(k)$, $S(k)$ and $A(k)$ indicate the "young" population (until 1 year old), "subadult" population (between 1 and 2 years old) and "adult" population, respectively, in the year k.

The first row of the matrix is formed by birth rate. So, the young and subadult populations do not procreate, while each adult couple has on average 2 children, each 3 years old. The coefficients 0.18 and 0.71 are the survival indices of the transition young/subadult and subadult/adult, respectively. It is clearly confirmed that the first one is critical: when the young phase finishes, they have to leave the nest, find a hunting domain, find a couple, construct a nest, etc. The coefficient 0.94 indicates that the adult population has a yearly death rate of 6%.

It was asked to find the eigenvalues of the matrix, which are:

$$\lambda_1 = 0.98;\ \lambda_2 = -0.02 \pm 0.21j \tag{51}$$

which means an annual decrease of 2%. In these conditions, the American owl converges to extinction in less than 50 years.

The extinction is avoided if and only if the dominant eigenvalue is greater than 1.

The problem is the low survival index. It was requested to verify that extinction would be avoided if the young survival index is 30% instead of 18%. In this case, the system is:

$$\begin{pmatrix} Y(k+1) \\ S(k+1) \\ A(k+1) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0.33 \\ 0.30 & 0 & 0 \\ 0 & 0.71 & 0.94 \end{pmatrix} \begin{pmatrix} Y(k) \\ S(k) \\ A(k) \end{pmatrix} \tag{52}$$

And the eigenvalues are:

$$\lambda_1 = 1.01;\ \lambda_2 = -0.03 \pm 0.26j \tag{53}$$

In these conditions, there is an annual increase of 1%. The asymptotic population distribution is given by the coordinates of the eigenvector corresponding to the dominant eigenvalue:

$$v_{DOM} \cong \begin{pmatrix} 10 \\ 3 \\ 31 \end{pmatrix} \tag{54}$$

That is, for each 10 young owls, there will be 3 subadult owls and 31 adult owls, with a growth rate of 1%.

9. Session 9 (modal analysis in discrete dynamical systems):

This session showed several exercises about dynamical discrete linear systems: bus station, Gould accessibility index and Google. See [50] for further information related to dynamical discrete systems. The application of a bus station is presented hereunder.

- Application session 9: bus station:

In this exercise four stations (A, B, C and D) were considered. The traffic is determined by the following rules:

- Stations A, B: 1/3 of buses goes to C; 1/3 of buses goes to D; 1/3 of buses remains for maintenance.
- Station C (and respectively D): 1/4 of buses goes to A; 1/4 of buses goes to B; 1/2 of buses goes to D (and respectively C).

It was asked to prove that there is asymptotic stationary distribution of the buses, and to compute it.

10. Session 10 (difference equations):

Some applications of difference equations were held in this session: Shannon information theory, queues theory and "Biking". This last application is developed here.

- Application session 10: "Biking":

It was required to organize, in 4 years, a "biking" with 400 bicycles in permanent regime, buying b bicycles each month.

It is known that 70% of bicycles keep in service, 25% are in the garage and reincorporate the next month, and 5% are irrecoverable.

It was asked the value of b and how many bicycles there would be in 4 years.

3.2.2. Students' Surveys and Interviews Results

So far, two editions of the sessions of "Applications of Linear Algebra in Engineering I: Linear Algebra" have been held, corresponding to the first semesters of the 2019/2020 and 2020/2021 academic years. The number of attending students to the sessions undertaken each semester was 20.

The material developed in these sessions has been analyzed considering the results of the anonymous surveys and interviews conducted to students. Students' surveys evaluate the mathematical and engineering contents and applications of each session, as well as the impact on the motivation of linear algebra. In addition, there is the possibility to add a comment, where students could express their opinion and their impression about the sessions.

Students' surveys of the sessions held until now were analyzed. The surveys were taken in the 2019/2020 and 2020/2021 academic years, when these sessions were held. The results obtained in these two academic years did not present significant differences. Thus, the answers are shown as an average of both years. The participants in these surveys have been all the attending students to the sessions. The participants have answered five questions which must be valued on a 5-point scale (1 = strongly disagree, 2 = disagree, 3 = neither agree nor disagree, 4 = agree, 5 = strongly agree). In the following figures the average of the answers to each question for all the sessions are presented.

The answers to the first question (Figure 7) show that most students, almost 85%, agree with the mathematical contents developed in the sessions.

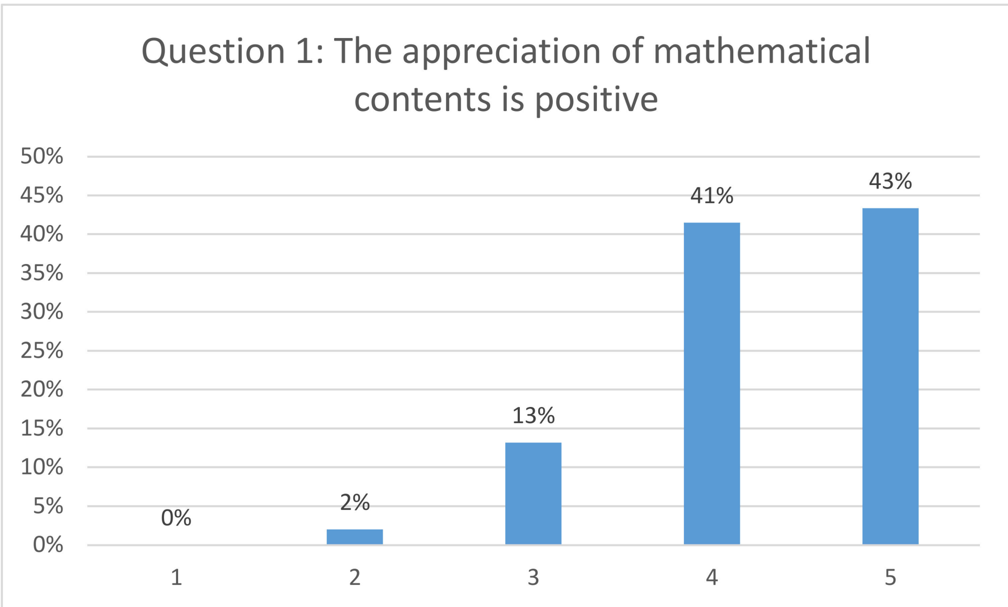

Figure 7. Answers to question 1: the appreciation of mathematical contents is positive.

In the answers to the second question (Figure 8), it can be observed that more than 80% of students agreed with the engineering contents explained in the sessions.

More than 90% of students think that the sessions "Applications of Linear Algebra in Engineering" let them know technological applications of different mathematical concepts (Figure 9).

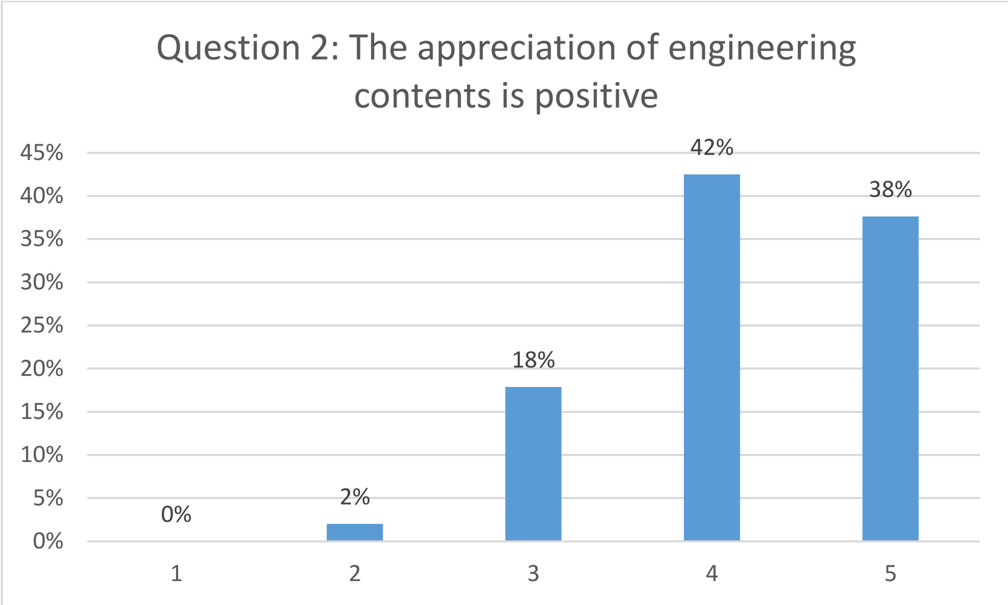

Figure 8. Answers to question 2: the appreciation of engineering contents of this session is positive.

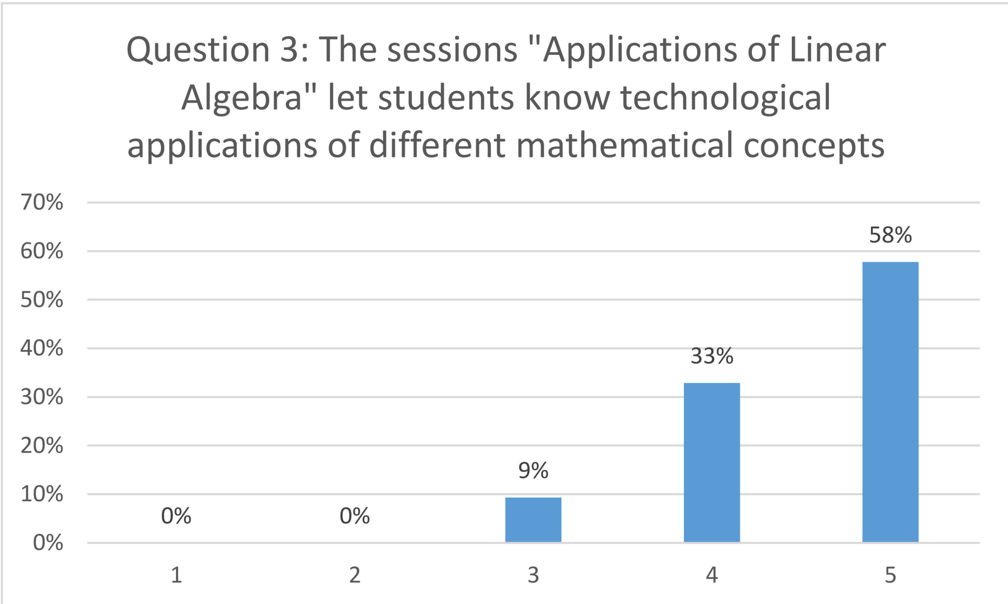

Figure 9. Answers to question 3: the sessions "Applications of Linear Algebra" let students know technological applications of different mathematical concepts.

It can be seen that 90% of students agreed that applications of mathematical concepts succeeded in increasing their motivation to study linear algebra (Figure 10).

Almost 70% of students state that the execution of practical exercises with technological applications improve the learning of mathematical concepts (Figure 11).

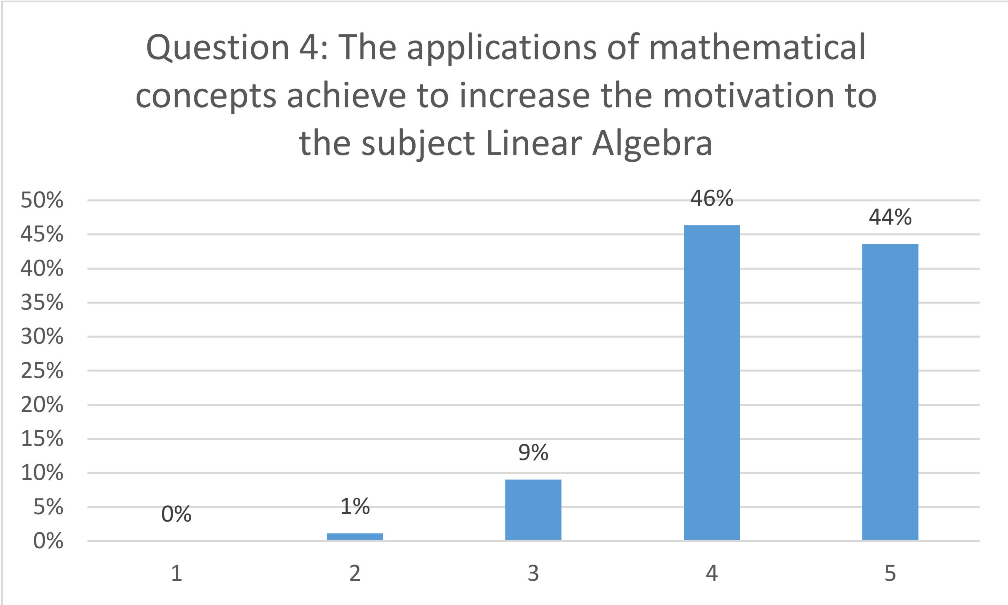

Figure 10. Answers to question 4: the applications of mathematical concepts achieve to increase the motivation to the subject linear algebra.

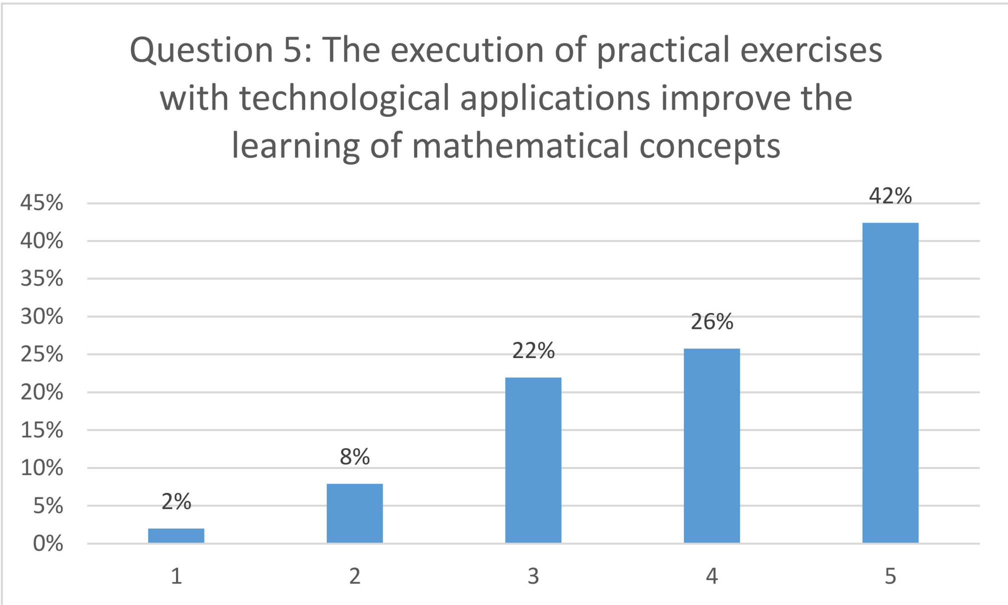

Figure 11. Answers to question 5: the execution of practical exercises with technological applications improve the learning of mathematical concepts.

The response of the attending students to these sessions in 2019/2020 and 2020/2021 academic years was very positive. It is also worth mentioning that some students' comments, expressed in the open field of anonymous surveys in both years, were along the following main themes:

- Applications let students know that mathematics is necessary.
- These sessions achieve the goal to motivate students and let them realize that linear algebra has real applications.
- Context in mathematics increases the interest and the attention of students, both in university and in secondary school.
- Applications helped students learn better linear algebra concepts.

In order to extract more information from students attending the sessions "Applications of Linear Algebra in Engineering", personal interviews were undertaken at the end of all sessions in 2019/2020 and 2020/2021 academic years. These interviews consisted of several open questions, which let students explain in detail their opinion and assessment of the sessions. The main questions set to students were:

1. What aspects do you asses more positively of these sessions?
2. What applications have been more interesting? Why?
3. How have these sessions influenced on your motivation and on your interest toward linear algebra?
4. Have these sessions helped you understand mathematical concepts of the subject linear algebra? What applications? What concepts?
5. After these sessions, do you consider that mathematics are more important and essential to the development of engineering degrees? How? Why?

The information extracted from these answers in both academic years is presented here:

- The sessions "Applications of Linear Algebra in Engineering" let students know real applications in different disciplines of engineering.
- Seeing all these applications let students know what they will be able to do in the following courses and it is very motivating.
- These applications let students realize of how important linear algebra is for engineering degrees and for their future profession.
- Interesting applications: complex numbers (electricity, economy), indeterminate compatible systems (roundabout traffic), vector subspaces (linear control systems), linear applications (change to italics, population fluxes), eigenvectors and eigenvalues (demographic control).

4. Discussion

In this work we contribute to generating an integrated STEM curriculum, presenting an intervention from the teaching staff to students about the relationship between mathematical applications and integrated STEM education. This work contributes to the connection of mathematics with technological disciplines and with technological professions, with the aim of improving the motivation and engagement of undergraduate engineering students.

In the current engineering curriculum, the first two courses consist mainly of math, science, communication and electives courses. Students take very few engineering courses in the first two years. With the intervention presented in this study basic science subjects, as mathematics, can play another role in engineering degrees and offer a wider view in STEM education [52,53]. Here, it has been presented that mathematics courses should cover examples and problems related to the main field of students enrolled degree to improve the understanding and application of these concepts, as [22] states. Teachers have always been the most crucial element in educational reform [16–18]. Teachers' intervention has been done in conjunction with the mathematics department faculty and the engineering department faculty [24,31,32]. As in [25], it is shown that the relationship between basic subjects and applied engineering subjects in higher engineering education curricula is evaluated.

Regarding the teachers' intervention, the seminar based on mathematics consists of sessions focused on electricity, automatics, mechanics, electronics, etc. For example, in the electricity area, different exercises in electrical engineering have been developed by the authors, as [38] shows. Following this structure, other engineering disciplines have been organized, as [49,50] show. Some preliminary results about the teachers' intervention were presented in [54].

Regarding the students' intervention, the main issue has been the relevance of problem-solving in the teaching of mathematics, as it is known that mathematical problem-posing provide better student's critical thinking skills effect than conventional learning [30,36]. Moreover, applications used in basic sciences subjects encourage student engagement and motivation in STEM degrees [14,33–35]. Considering the results shown in Figure 10, it can be seen that 90% of students agreed that applications of mathematical concepts managed to increase their motivation to study linear algebra. Thus, this will lead to reduced dropout, as it is related to motivation [14], student achievement [15] and academic performance [11]. It has been shown that applications let students learn mathematical concepts through practical examples increase students' motivation to study mathematics, as it was confirmed in previous studies like [29]. Moreover, students discover multiple real applications of linear algebra in engineering and other areas, what achieve to motivate them to the study of this subject, as it was developed in previous studies [14,33,34]. From the answers to the questions set to the students in the interviews made after the sessions "Applications of Linear Algebra in Engineering", it can be interpreted that most of the examples have impressed students because they did not know that linear algebra could have applications in so many different disciplines. In addition, knowing what they would be able to do in the next courses using the concepts of linear algebra was really motivating for students, as they realized how essential linear algebra was for their career and they increased their interest towards this subject.

These results confirmed that this experience allows students to get a better understanding of mathematical concepts, as it is concluded in [39], which increases students' performance in mathematical subjects of engineering degrees, as it was analyzed in previous studies like [26]. The work provide evidence that it is possible to structure teacher support so that they can make lasting pedagogical changes, rather than temporary or one-off changes as part of a specific initiative.

5. Conclusions and Future Work

The study presented here has been conducted at the Universitat Politècnica de Catalunya-BarcelonaTech (UPC), a university specialized in STEM disciplines. The work is based on an intervention starting from the teaching staff of basic science departments and engineering departments and finishing with an intervention for undergraduate students. The aim of this study is to present a contribution about the relationship between mathematical applications and integrated STEM education.

Following the successful teachers' intervention with mathematics, in the 2019/20 academic year, the seminar of contextualization for the teaching staff has been expanded to physics creating the seminar "Contextualization of basic sciences in engineering degrees at UPC". Up to now, five sessions have been held regarding physics. It is planned to continue with the other basic sciences. The work of "Contextualization of basic sciences in engineering degrees" consists of seminars which deal with the different basic sciences (mathematics, physics, chemistry, computing, statistics, etc.) in the first courses of engineering degrees.

Similarly, following the successful experience with students, new sessions to expand the applications in engineering to other mathematical subjects are planned. In particular, "Applications of Mathematics in Engineering II" based on multivariable calculus are being to be held the second semester, to complement the ones based on linear algebra held in the first semester.

It is expected that the material developed in this work will be introduced for future adaptation of basic sciences subjects in engineering degrees. This fact will lead to an increase of student engagement and a decrease in dropouts out of engineering degrees. Moreover, secondary teachers have also suggested to expand this experience to secondary education to increase the STEM vocations of the students.

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