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Four Distances for Circular Intuitionistic Fuzzy Sets

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Abstract: In the paper, for the first time, four distances for Circular Intuitionistic Fuzzy Sets (C-IFSs) are defined. These sets are extensions of the standard IFS that are extensions of Zadeh's fuzzy sets. As it is shown, the distances for the C-IFS are different than those for the standard IFSs. At the moment, they do not have analogues in fuzzy sets theory. Examples, comparing the proposed distances, are given and some ideas for further research are formulated.

Keywords: distance; intutionistic fuzzy set; circular intutionistic fuzzy set

MSC: 03E72

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1. Introduction

The concept of an Intuitionistic Fuzzy Set (IFS, see [1,2]) (introduced in 1983) was one of the first in time extensions of L (Zadeh's fuzzy set [3]) On the other hand, the IFS is also an object of different extensions. One extensions is the Circular IFS (C-IFS, see [4]). It is defined as follows.

Let us have a fixed universe E and its subset A. The set $A_r^* = \{\langle x, \mu_A(x), \nu_A(x); r\rangle \mid x \in E\}$, where $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $r \in [0, \sqrt{2}]$ is a radius of the circle around each element $x \in E$, is called a C-IFS and functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ represent the degree of membership (validity, etc.) and non-membership (non-validity, etc.) of element $x \in E$ to a fixed set $A \subseteq E$. Now, we can define also function $\pi_A : E \to [0,1]$ by means of $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ and it corresponds to degree of indeterminacy (uncertainty, etc.). Let us remark that in [4], the radius r was defined to take values from the interval $[0,1]$. Here, we extended the region of r values to be $[0, \sqrt{2}]$ because we would like the points with center $\langle 0,1\rangle$ and $\langle 1,0\rangle$ to be able to cover the whole IFS triangle, which can be valid only if $r \geq \sqrt{2}$. In future, it would be appropriate using the herewith presented form of the C-IFS definition.

When $r = 0$, the C-IFS is transformed to an IFS. On the other hand, when $r > 0$, the C-IFS is an object that is different from the ordinary IFS. In reality, in ordinary IFS theory, there is a way to represent the existance of circles around the elements of universe E (see Figures 1 and 2).

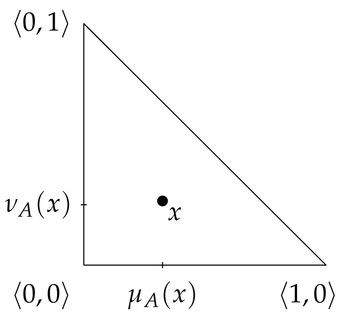

Figure 1. Geometrical interpretation of an element of an IFS.

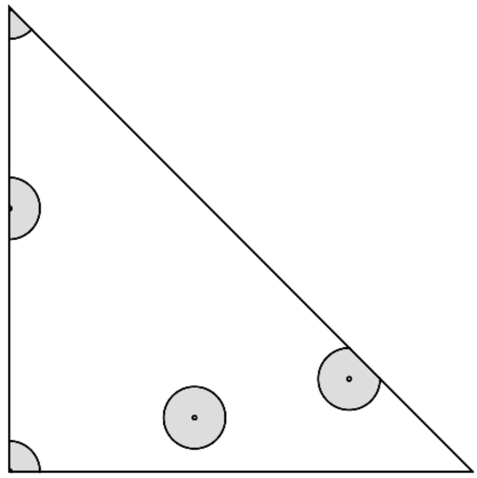

Figure 2. Geometric representation of different circular IFSs onto the IFS interpretation triangle.

A metric (topological) space can be thought of as a very basic space that satisfies a few axioms. The ability to measure and compare distances between elements of a set is often crucial, and it provides more structure than general topological space possesses (see, [5,6]).

When we refer to the elements or "points" of the underlying set, we do not necessarily refer to geometrical points, although this is how most of us usually visualize them. They may be objects of any type, such as sequences, functions, images, sounds, signals, decisions, etc.

Definition 1 ([5,6])**.** *A **metric** on a set X is a function $d : X \times X \to \mathbb{R}$ with the following properties:*

1. *$d(x,y) \geq 0$ for all $x, y \in X$, and equality holds if and only if (iff) $x = y$.*
2. *$d(x,y) = d(y,x)$ for all $x, y \in X$ (symmetry).*
3. *$d(x,z) \leq d(x,y) + d(y,z)$ for all $x, y, z \in X$ (the triangle inequality).*

*We call $d(x,y)$ the **distance** between x and y, and the pair (X,d) a **metric space**.*

It is evident that d has the properties we expect when we measure a distance between points in rigid geometry. Let us now introduce two from the most popular metrics in R^n, for any positive number n.

Definition 2 ([6,7])**.** *Taking any $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n) \in R^n$, let us define:*

1. ***Euclidean metric:***

$$d_2(x,y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

2. ***Manhattan (Hamming) metric:***

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|$$

In the present paper, for the first time, we will introduce distances over two C-IFSs. Ideas for norms, metrics and distances over IFSs were originally introduced in [8] and described in more details in [9], where the first two distances were given. The next two distances, which are extensions of the first two, were introduced in [10] by E. Szmidt and J. Kacprzyk. In [2], these two distances were called after their names. Later, a lot of other distances were introduced over IFSs (see, e.g., [7,11–39]).

The first four distances over IFSs are the following.

Let us have two Intuitionistic Fuzzy Sets (IFSs; see [2,9]) A and B:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$
$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\},$$

where $\mu_A, \nu_A, \mu_B, \nu_B : E \to [0,1]$ and $\mu_A(x) + \nu_A(x) \leq 1, \mu_B(x) + \nu_B(x) \leq 1$ for each $x \in E$.

Let everywhere below $\mathcal{C}_E$ be the cardinality of universe E. In [2,9] the following distances are described:

$$H_2(A,B) = \frac{1}{2\mathcal{C}_E}.\sum_{x\in E}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|), \quad (1)$$

(intuitionistic fuzzy Hamming distance)

$$E_2(A,B) = \sqrt{\frac{1}{2\mathcal{C}_E}.\sum_{x\in E}((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2)}, \quad (2)$$

(intuitionistic fuzzy Euclidean distance)

$$H_3(A,B) = \frac{1}{2\mathcal{C}_E}.\sum_{x\in E}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|), \quad (3)$$

(Szmidt and Kacprzyk's form of intuitionistic fuzzy Hamming distance)

$$E_3(A,B) = \sqrt{\frac{1}{2\mathcal{C}_E}.\sum_{x\in E}((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2) + (\pi_A(x) - \pi_B(x))^2)} \quad (4)$$

(Szmidt and Kacprzyk's form of intuitionistic fuzzy Euclidean distance).

2. Definitions of the First Four Distances over C-IFSs

Here, we introduce the following four distances for C-IFS that are modifications of distances (1)–(4) as follows:

$$H_2(A,B) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \frac{1}{2C_E}\sum_{x\in E}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|)\right) \quad (5)$$

(intuitionistic fuzzy Hamming distance),

$$E_2(A,B) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \sqrt{\frac{1}{2\mathcal{C}_E}.(\sum_{x\in E}(\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2)}\right) \quad (6)$$

(intuitionistic fuzzy Euclidean distance),

$$H_3(A,B) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \frac{1}{2C_E}.\sum_{x\in E}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)|)\right) \quad (7)$$

(Szmidt and Kacprzyk's form of intuitionistic fuzzy Hamming distance),

$$E_3(A,B)$$

$$\frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \sqrt{\frac{1}{2\mathcal{C}_E}.(\sum_{x\in E}((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2) + (\pi_A(x) - \pi_B(x))^2)}\right) \quad (8)$$

(Szmidt and Kacprzyk's form of intuitionistic fuzzy Euclidean distance).

Obviously, if A and B are standard IFS, i.e., $r_A = r_B = 0$, the new distances coincide with (1)–(4).

Let $C_IFS(E)$ and $IFS(E)$ be the sets of all C-IFSs and of all IFSs over the universe E, respectively. As we mentioned above, $A \in IFS(E)$ iff $A \in$ C-IFS(E) and $r_a = 0$.

Theorem 1. *For any $A_{r_A}, B_{r_B} \in$ C-IFS(E), that is $A, B \in IFS(E)$ where $r_A, r_B \in [0, \sqrt{2}]$, the expressions (5)–(8) are well-defined metrics (distances).*

Proof. We need to show that the formulas $H_2(A_{r_A}, B_{r_B})$, $H_3(A_{r_A}, B_{r_B})$, $E_2(A_{r_A}, B_{r_B})$, $E_3(A_{r_A}, B_{r_B})$ stated in expressions (5)–(8) obey the three axioms for a metric from Definition 1.

As it has already been shown, the expressions stated in (1)–(4) are well-defined distances in $IFS(E)$. Let us take D to be any one of H_2, H_3, E_2 or E_3. Therefore, D is a metric in $IFS(E)$.

Since it is obvious from the definition of C-IFSs, $A_{r_A} = B_{r_B}$ in C-IFS(E) iff $A = B$ in $IFS(E)$ and $r_A = r_B$. But $A = B$ in $IFS(E)$ iff $D(A, B) = 0$ and the sum of two non-negative numbers is 0 iff both numbers are equal to 0, therefore the validity of the first axiom for a distance is proved.

The validity of the second axiom is obvious since D is symmetric.

In order to show the validity of the third axiom, let us take a third C-IFS C_{r_C} and show that the triangle property

$$D(A_{r_A}, C_{r_C}) \leq D(A_{r_A}, B_{r_B}) + D(B_{r_B}, C_{r_C}) \tag{9}$$

holds.

We know that

$$D(A, C) \leq D(A, B) + D(B, C) \tag{10}$$

holds for the IFSs A, B and C. From well-known inequality $|x| + |y| \geq |x + y|$ for three real numbers it follows that

$$|r_C - r_A| \leq |r_B - r_A| + |r_C - r_B| \tag{11}$$

for all choices of $r_A, r_B, r_C \in [0, \sqrt{2}]$. Therefore, summing up both sides of the last two inequality expressions (10) and (11), the validity of (9), i.e., the third axiom for distance holds. □

Remark 1. *From the definition of H_2, H_3 and E_2, E_3 and since for any two IFS, A, B and $x \in E : |\pi_B(x) - \pi_A(x)| \geq 0$, the following inequalities are valid.*

- $H_2(A, B) \leq H_3(A, B)$
- $E_2(A, B) \leq E_3(A, B)$

3. Numerical Example

A numerical example of an IFS with $E = \{0, 1, 2\}, \mathcal{C}_E = 3$ and $A_{r_A}, B_{r_B} \in C - IFS(E)$ is depicted on Figure 3. For these IFSs, A and B, we have that $r_A = 0.1$ and $r_B = 0.06$ and the degrees of the corresponding elements x from the universe E are given in Table 1.

Table 1. Degrees of the element x.

$x \in E$	$\mu_A(x)$	$\nu_A(x)$	$\pi_A(x)$	$\mu_B(x)$	$\nu_B(x)$	$\pi_B(x)$
0	0.321	0.070	0.609	0.241	0.049	0.710
1	0.099	0.102	0.799	0.075	0.071	0.854
2	0.200	0.699	0.101	0.150	0.489	0.361

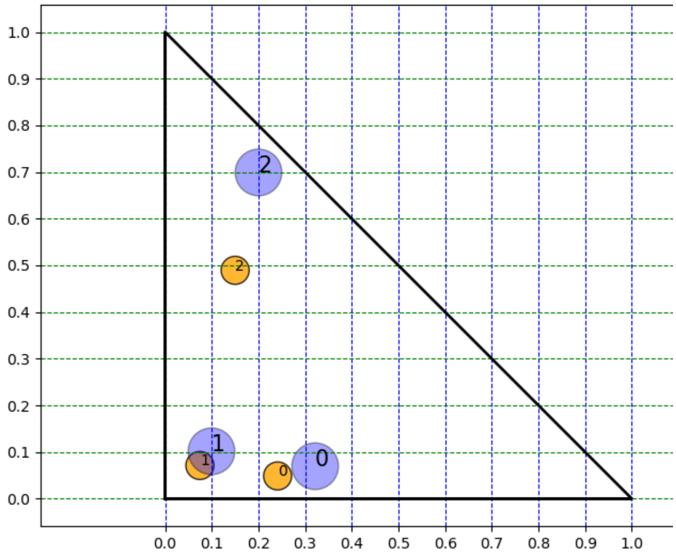

Figure 3. Triangular representation of $A_{r_A}, B_{r_B} \in$ C-IFS(E) with $E = \{0, 1, 2\}$, $C_E = 3$ and $r_A = 0.1$, $r_B = 0.06$.

Let us consider the example of the two IFS, A_{r_A} and B_{r_B} from the previous section. A simple computation applying the corresponding formulas and the concrete values for the arbitrary chosen A_{r_A}, B_{r_B} shows that

$$\frac{|r_A - r_B|}{\sqrt{2}} = \frac{|0.1 - 0.06|}{\sqrt{2}} = 0.0283,$$

$$H_2(A, B) = 0.069 \text{ and } E_2(A, B) = 0.096,$$
$$H_3(A, B) = 0.092 \text{ and } E_3(A, B) = 0.123.$$

Hence we conclude that,

$$H_2(A_{r_A}, B_{r_B}) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + H_2(A, B)\right) = 0.049,$$

$$E_2(A_{r_A}, B_{r_B}) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + E_2(A, B)\right) = 0.062,$$

and

$$H_3(A_{r_A}, B_{r_B}) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + H_3(A, B)\right) = 0.083,$$

$$E_3(A_{r_A}, B_{r_B}) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + E_3(A, B)\right) = 0.089.$$

The results from the comparison of the four different distances are shown in Table 2. the following tabular form.

Table 2. Comparison of the four different distances.

D	$H_2(A_{r_A}, B_{r_B})$	$E_2(A_{r_A}, B_{r_B})$	$H_3(A_{r_A}, B_{r_B})$	$E_3(A_{r_A}, B_{r_B})$
$D(A_{r_A}, B_{r_B})$	0.049	0.062	0.083	0.089

The reader may compare the values of the different distances from A and B with Remark 1. The plot from Figure 3 is taken from a software implementation for computation and visualization in an interactive mode of C-IFS and different distances for them. In a future research, the authors will go into more detail introducing multiple distances of C-IFSs and presenting a software implementation about them. Figure 3 was created with the Python's library Matplotlib and it shows what happens if we fix A, B and r_A and change the length of r_B. In future, new distances for C-IFS will be introduced. Each distance over ordinary IFSs can be transformed for the case of C-IFS by the method discussed in Section 2.

4. Result and Discussion

Below we show that each one of the distances satisfies the triangular inequality for three arbitrary C-IFS. Let us consider $A_{r_A}, B_{r_B} \in$ C-IFS(E) from Section 3 and take another $C_{r_C} \in$ C-IFS(E) with degrees of the corresponding elements x from the universe E given in Table 3.

Table 3. Degrees of the element x in C_{r_C}.

$x \in E$	$\mu_C(x)$	$\nu_C(x)$	$\pi_C(x)$
0	0.2411	0.2523	0.5069
1	0.0746	0.3413	0.584
2	0.1501	0.7293	0.120

Applying the formulas from Section 2 as in the previous Section we obtain the values for the different distances between all combinations of pairs from $\{A_{r_A}, B_{r_B}, C_{r_C}\} \in$ C-IFS(E) given in Table 4.

Table 4. Values of the distances H_2, E_2, H_3, E_3.

D	H_2	E_2	H_3	E_3
$D(A_{r_A}, C_{r_C})$	0.058	0.072	0.086	0.088
$D(A_{r_A}, B_{r_B})$	0.049	0.062	0.083	0.089
$D(B_{r_B}, C_{r_C})$	0.067	0.092	0.126	0.127

For the above table if for any of the columns if we pick up an arbitrary permutation of the row indices let the corresponding values be a, b, c. Then it can be easily checked that $a \leq b + c$ which exactly shows the validity of the triangular inequality of any of the proposed distances for the C-IFSs $A_{r_A}, B_{r_B}, C_{r_C}$. As an example, let us take the column H_3 and the permutation of the row indices $3, 1, 2$, then $a = 0.126, b = 0.086, c = 0.083$ and $0.126 < 0.086 + 0.083 = 0.169$.

5. Conclusions

The present paper is the second one devoted to C-IFS. Here, for the first time, distances for C-IFS were introduced. The introduced distances could be applied in diverse areas where objects and processes can be evaluated in more detail compared to an ordinary IFS. In future, new distances over C-IFSs will be introduced and some of their properties will be studied. In the meantime, the concept of an C-IFS was extended to Elliptic IFS (E-IFS) [40]. The present and other distances will be re-formulated for the E-IFSs.

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Writing—original draft, K.A. and E.M. All authors have read and agreed to the published version of the manuscript.

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