## Article

# Application of the Entropy Method and Color Difference Formula to the Evaluation of Round Brilliant Cut Diamond Scintillation 

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#### Abstract

A modeling approach combining the entropy method and color difference formula is proposed in order to quantitatively evaluate diamond scintillation. The images of 66 diamonds were captured from $0^{\circ}$ to $105^{\circ}$ rotation at $15^{\circ}$ intervals. The color difference of corresponding pixels in adjacent rotation angle images was calculated using a MatLab r2014a program, which indicated the diamond's color change due to its scintillation. A threshold (10) was determined to divide the color difference into seven color difference intervals, the percentage of which indicated the color-change area. The color difference and the percentage were comprehensively analyzed using the entropy method to evaluate diamond scintillation objectively and quantitatively. Lightness was the main factor affecting the diamond scintillation while chroma and hue also significantly affected it.


Keywords: entropy method; evaluation; diamond; image processing

## 1. Introduction

As a mineral, the diamond has certain indicators in geology [1,2]. Because of their high hardness, high refractive index, and high dispersion, larger and more pure diamonds are often used as gemstones. Diamonds are evaluated in terms of four aspects: carat weight, color, clarity, and cut in various standards. Faceted diamonds can also be characterized by scintillation, a dynamic and comprehensive display of fire and brightness. Moses et al. [3] defined scintillation as the appearance, or extent, of spots of light seen in a polished diamond when viewed face-up that flash as the diamond, observer, or light source moves (sparkle), as well as the relative size, arrangement, and contrast of bright and dark areas that result from internal and external reflections seen in a polished diamond when viewed face-up while that diamond is either still or moving (pattern). By trial and error, ancient jewelers produced cuts that refracted light and gave illumination life to gemstones. Tolkowsky systematically analyzed the diamond's optical characteristics and estimated the best proportions for round brilliant cut diamonds, which today's standards for "ideal-cut" diamonds are based upon with minor changes [4].

When a polished diamond is viewed face-up, brightness and fire refer to the appearance or extent of internal and external reflections of "white" light and light dispersed into spectral colors, respectively [3]. Thus, gemstone scintillation is related to brightness and fire, as well as the patterns produced by various optical effects [5,6]. In order to observe the optical effects of the gemstone, FireScope, Brilliance Scope, Diamond Integrator, ASET instruments, GemCad, GemRay, DiamCalc, and other tools and computer software have been developed [7-14].

For diamond images, scintillation implies a color change in the visual perception, which can be evaluated by the color difference [9]. The quantitative measurement of object color is extremely complex. Since it involves the observer's visual physiology, visual psychology, lighting conditions, observation conditions, and many other factors. In order to obtain a consistent measurement effect, the Commission Internationale de l'Eclairage (CIE) has stipulated a set of standard color systems. Subsequently, the CIE 1931 standard colorimetric observer, CIE 1964 supplementary standard colorimetric observer, CIE 1976 Lab color space, and so on were proposed [15]. Currently, the most common color difference formula in research are the CIE 1976 Lab color difference formula $[16,17]$ and the CIE DE2000 color difference formula [18-23]. The geometric meaning of the CIE 1976 Lab color difference formula is an ellipsoid centered on the coordinate point of the standard color sample in a uniform color space. The color space is not uniform in practical situations, however. The color tolerance is different at different positions of the chromaticity diagram-it is smallest in the blue portion and largest in the green portion [24-26]. The color difference formula is constantly being improved. McDonald, Witt, RIT-Dupont [27], and others have conducted a series of experiments and collected important data sets for the proposal or prediction of color difference formulas [28-31]. The CIE DE2000 color difference formula is currently the closest color difference formula to human vision. It is both more accurate and more computationally demanding [32].

The entropy method uses the effective information provided by the measurement data to make an objective evaluation of the evaluation object. In the evaluation process, a smaller entropy value of an evaluation index leads to larger amount of information being provided, and a larger corresponding weight; otherwise, the index weight is smaller [33]. The entropy method is widely used in various fields such as computer algorithms [34], evaluation of handling equipment [35], and product quality assessment [36].

## 2. Materials and Methods

### 2.1. Materials

Sixty-six non-fancy-color round brilliant cut diamonds were provided by the School of Gemmology, China University of Geosciences (Beijing). The stones were graded and certified by professionals at the China National Gemstone Training Center (NGTC). Based on the Chinese national standards, these diamonds were evaluated in terms of five aspects: carat weight, color, clarity, cut, and fluorescence.

1. Carat weight: Carat weight refers to the weight of the diamond, and its unit is generally the carat (ct), with $1 \mathrm{ct}=0.2 \mathrm{~g}$.
2. Color: Non-fancy-color diamonds often have a yellow tone. Depending on the amount of yellow tone, the diamond's color is classified as D, E, F, G, H, I, J, K, L, M, N, and < N; D has almost no yellow tone.
3. Clarity: Diamonds often contain a variety of inclusions, including solids, liquids, gases, and cracks. According to the diamond's inclusions seen under a $10 \times$ magnifying glass, the diamond's clarity is classified as LC (Loupe Clean), VVS (Very Very Slight Inclusions), VS (Very Slight Inclusions), SI (Slight Inclusions), and P (Pique), and further subdivided into 11 small levels, i.e., FL (Flawless), IF (Internally Flawless), VVS1, VVS2, VS1, VS2, SI1, SI2, P1, P2, and P3.
4. Cut: After a rough diamond is cut, light beams are reflected, refracted and dispersed, making the diamond very beautiful. Based on proportion and finish, the diamond cut is classified as EX (Excellent), VG (Very Good), G (Good), F (Fair), and P (Poor).
5. Fluorescence: According to the intensity of light emitted by diamonds under long-wavelength ultraviolet light, a diamond's fluorescence is divided into four levels: STR (Strong), MED (Medium), FNT (Faint) and NON (None).

The grading results of the diamonds in this study were as follows:

1. Shape: round brilliant
2. Weight: 0.11-0.40ct
3. Color: D-M
4. Clarity: LC-P
5. Cut: P-EX
6. Fluorescence: NON-STR

### 2.2. Diamond Images

The diamonds were placed face-up on a white rotating table marked with a scale from $0^{\circ}$ to $360^{\circ}$ in a standard light source box. The box has the following advantages [35]:

1. It can provide various illuminants.
2. Its size allows the observer to observe the diamonds at what is an optimal viewing distance of the diamonds from both the light source and the observer.
3. It provides adequate shielding from extraneous light.
4. It is deep enough to eliminate color distractions from the surrounding area.
5. Its interior color is Munsell "N7 neutral gray," which can reduce color contrasts between the diamonds and the background.

The diamonds were exposed to the CIE standard illuminant D65, which simulated daylight with a correlated color temperature of approximately 6500 K . The relative spectral power distribution was extrapolated based on the measured daylight data. This is the best illumination for evaluating a gemstone's color [37,38]. The illumination parameters where the diamonds were positioned were measured by a chroma meter (CL200, Minolta, Japan), with the following results:

1. Illuminance: $\mathrm{Ev}=1011 \mathrm{~lx}$
2. Chromaticity coordinates: $\mathrm{u}^{\prime}=1976 ; \mathrm{v}^{\prime}=4784$
3. Tristimulus value: $\mathrm{X}=978 ; \mathrm{Y}=1052 ; \mathrm{Z}=1012$

The CIE color system is a color mixture system based on the fundamental that each color can be made from a mixture of three primary colors in a certain proportion. The number of the three primary colors required to match the color to be measured is known as the tristimulus value. $\mathrm{X}, \mathrm{Y}$, and Z are the tristimulus values defined in the CIE 1931-XYZ system and are widely used in other color systems; $\mathrm{u}^{\prime}, \mathrm{v}^{\prime}$ are the chromaticity coordinates obtained from $\mathrm{X}, \mathrm{Y}$, and Z [15].

The diamonds were photographed using a Canon EOS 200D at ISO 640, f/6.3, and 1/200 s. The observer (camera) and illumination were held still, and the diamonds were rotated counterclockwise from $0^{\circ}$ to $105^{\circ}$ at $15^{\circ}$ intervals. They were parallel to the stage during the rotation. Eight images were taken at each chosen angle for every diamond (Figure 1).


Figure 1. Schematics illustrating photographic conditions. (A) Side view. (B) Main view.

Each diamond image was set to $500 \times 500$ pixels using Adobe Photoshop CC2018. In order to calculate the color difference between pixels of adjacent rotation angle images, and eight images of each diamond were rotated to the same position as the first image. For example, when Dia 1708 was not rotated (rotation angle $=0^{\circ}$ ), the first image was recorded as Dia 1708-1; the second image (recorded as Dia 1708-2) was rotated counterclockwise by $15^{\circ}$. Dia 1708-2 was then rotated clockwise by $15^{\circ}$ to its original orientation, i.e., that of Dia 1708-1. Dia 1708-3, Dia 1708-4, Dia 1708-5, Dia 1708-6, Dia 1708-7, and Dia 1708-8 were similarly reoriented for evaluation (Figure 2A).


Figure 2. Schematic of the experimental process. (A) Each sample was photographed after being rotated counterclockwise by $0^{\circ}$ to $105^{\circ}$ at $15^{\circ}$ intervals and the color difference between pixels of adjacent rotation angle images was calculated (the size of each image was $500 \times 500$ pixels, and take Dia 1708 as an example). (B) The color difference was divided into seven color difference intervals for subsequent analysis by entropy method. (C) In order to divide the color difference interval, contrasting blocks on the diamond image were selected (seven of which are shown in this figure). The standard deviation (representing the resolution of chromaticity) of the pixels' color difference were calculated to determine the color difference threshold. (D) The box plot showed the percentage of the number of data in each color difference interval to the number of all data for each sample, which decreased with increases in color difference intervals. The entropy method was used to analyze these data to quantitatively evaluate the diamond scintillation. (E) The diamond scintillation was visually evaluated by seven experienced graders with professional knowledge to verify the result by entropy method.

### 2.3. Digital Image Information

### 2.3.1. Calculation of Color Difference

The corresponding pixels and color differences in adjacent rotation angle images were calculated via the CIE DE2000 color difference formula [39-41].

$$
\begin{equation*}
\Delta \mathrm{E}_{00}=\left\{\left[\Delta \mathrm{L}^{\prime} /\left(\mathrm{k}_{\mathrm{L}} \mathrm{~S}_{\mathrm{L}}\right)\right]^{2}+\left[\Delta \mathrm{C}^{\prime} /\left(\mathrm{k}_{\mathrm{C}} \mathrm{~S}_{\mathrm{C}}\right)\right]^{2}+\left[\Delta \mathrm{H}^{\prime} /\left(\mathrm{k}_{\mathrm{H}} \mathrm{~S}_{\mathrm{H}}\right)\right]^{2}+\mathrm{R}_{\mathrm{T}}\left[\Delta \mathrm{C}^{\prime} /\left(\mathrm{k}_{\mathrm{C}} \mathrm{~S}_{\mathrm{C}}\right)\right]\left[\Delta \mathrm{H}^{\prime} /\left(\mathrm{k}_{\mathrm{H}} \mathrm{~S}_{\mathrm{H}}\right)\right]\right\}^{1 / 2} \tag{1}
\end{equation*}
$$

In Equation (1), $\Delta \mathrm{E}_{00}$ is the total color difference calculated with CIE DE2000. The terms $\Delta \mathrm{L}^{\prime}, \Delta \mathrm{C}^{\prime}$, and $\Delta \mathrm{H}^{\prime}$ are the CIE Lab metric lightness, chroma, and hue differences, respectively. While the terms $\mathrm{S}_{\mathrm{L}}, \mathrm{S}_{\mathrm{C}}$, and $\mathrm{S}_{\mathrm{H}}$ are the weighting functions for the lightness, chroma, and hue components, respectively. The calculated values vary according to the positions of the color pair being considered in CIELab color space [40,41].

$$
\begin{gather*}
\mathrm{S}_{\mathrm{L}}=1+0.015\left(\mathrm{~L}_{\mathrm{m}}^{\prime}-50\right)^{2} /\left[20+\left(\mathrm{L}_{\mathrm{m}}{ }^{\prime}-50\right)^{2}\right]^{1 / 2}  \tag{2}\\
\mathrm{~S}_{\mathrm{C}}=1+0.045 \mathrm{C}_{\mathrm{m}}^{\prime}{ }^{\prime}  \tag{3}\\
\mathrm{S}_{\mathrm{H}}=1+0.015 \mathrm{C}_{\mathrm{m}}^{\prime}{ }^{\prime} \mathrm{T} .  \tag{4}\\
\mathrm{T}=1-0.17 \cos \left(\mathrm{~h}_{\mathrm{m}}{ }^{\prime}-30\right)+0.24 \cos \left(2 \mathrm{~h}_{\mathrm{m}}{ }^{\prime}\right)+0.32 \cos \left(3 \mathrm{~h}_{\mathrm{m}}{ }^{\prime}+6\right)-0.20 \cos \left(4 \mathrm{~h}_{\mathrm{m}}{ }^{\prime}-63\right) . \tag{5}
\end{gather*}
$$

where $\mathrm{L}_{\mathrm{m}}{ }^{\prime}, \mathrm{C}_{\mathrm{m}}{ }^{\prime}$, and $\mathrm{h}_{\mathrm{m}}{ }^{\prime}$ respectively represent the arithmetic mean values of the lightness, chroma, and hue of the color sample and the standard color sample calculated by CIE DE2000.

The $\mathrm{k}_{\mathrm{L}}, \mathrm{k}_{\mathrm{C}}$, and $\mathrm{k}_{\mathrm{H}}$ values are parametric factors that are adjusted according to different viewing parameters for the lightness, chroma, and hue components, respectively. $\mathrm{R}_{\mathrm{T}}$ is intended to improve the performance of the color difference equation for fitting chromatic differences in the blue region [42]. In gemology, $\mathrm{k}_{\mathrm{L}}=\mathrm{k}_{\mathrm{C}}=\mathrm{k}_{\mathrm{H}}=1$ [38].

$$
\begin{gather*}
\mathrm{R}_{\mathrm{T}}=-\sin (2 \Delta \theta) \mathrm{R}_{\mathrm{C}}  \tag{6}\\
\Delta \theta=30 \exp \left\{-\left[\left(\mathrm{h}_{\mathrm{m}}^{\prime}-275\right) / 25\right]^{2}\right\}  \tag{7}\\
\mathrm{R}_{\mathrm{C}}=2\left[\mathrm{C}_{\mathrm{m}}^{7} /\left(\mathrm{C}_{\mathrm{m}}^{\prime 7}+25^{7}\right)\right]^{1 / 2} \tag{8}
\end{gather*}
$$

$\mathrm{R}_{\mathrm{T}}$ is intended to improve the performance of the color difference equation for fitting chromatic differences in the blue region [42]. $\Delta \theta$ is the rotation angle determined by the hue angle, and $R_{C}$ is the rotation amplitude according to the change in chroma [41].

The color difference was calculated via a program written in MatLab r2014a software (MathWorks, USA). Two images of the same diamond were opened in the software and the color parameters ( $\mathrm{L}, \mathrm{a}, \mathrm{b}$ ) of the images could be read and be converted into L (lightness), C (chroma), h (hue). Then the color difference of the corresponding pixels was calculated by the Equation (1). The results were exported as an Excel spreadsheet and used to draw the color difference images. Each diamond could produce seven Excel spreadsheets and seven color difference images of the $0^{\circ}$ image and $30^{\circ}$ image, $30^{\circ}$ image and $60^{\circ}$ image, $60^{\circ}$ image and $90^{\circ}$ image, $90^{\circ}$ image and $120^{\circ}$ image, and $120^{\circ}$ image and $150^{\circ}$ image (labeled SD1-2, SD2-3, SD3-4, SD4-5, SD5-6, SD6-7, and SD7-8, respectively, in Figure 2A).

### 2.3.2. Determining the Color Difference Interval Threshold

A very small color change would cause a large color difference value change, and the color space was not uniform, so it was necessary to divide the color difference into several intervals for further analysis. We borrowed McAdam's method of determining the boundary of McAdam's ellipse to determine the threshold of the color difference interval [24]. With the NBS as the unit which derived using the CIE 1964 uniform color space color difference formula, when the color difference value is $1.5-3.0 \mathrm{NBS}$, and the human eye can detect the color change. When the color difference value is $3.0-6.0$ NBS, the human eye can recognize the color change. When the color difference value is 6.0-12.0 NBS, the human eye can perceive a large color change. When the color difference value is
$>12$ NBS, the human eye can perceive a very large color change [15]. The color difference results calculated using different color difference formulas were different, thus the color difference formula used must be noted when performing the calculation. Each group of data was divided into several different color difference intervals based on the large color changes that the human eye can perceive using the CIE DE2000 color difference formula (Figure 2B). The color difference could be further analyzed by the standard deviation to determine the threshold of the color difference intervals [24].

1. Contrasting blocks: Eighteen pairs of areas with a very large color change were selected (Figure 2C). These were mainly changed in hue and lightness. Each area contained $3 \times 3$ to $20 \times 20$ pixels. These color areas were referred to as contrasting blocks. For example, C-A1 and C-B1 were a pair of contrasting blocks. Portions of selected contrasting blocks are listed in Table 1 (labeled C).
2. Standard blocks: The color was selected at the center of each contrasting block. The color blocks of the same size as the corresponding contrasting blocks were produced according to the selected color's parameters using Adobe Photoshop CC2018. These blocks were called standard blocks (labeled S in Table 1). Thus, S-A1 was the standard blocks of C-B1.
3. Normal distribution: The color difference between each contrasting block and the standard block (such as C-A1 and S-A1) was calculated via the program written in MatLab r2014a and output as an Excel spreadsheet that obeyed the normal distribution $\mathrm{N}\left(\Delta \mathrm{E}_{\mathrm{A}}, \Delta \mathrm{E}_{\mathrm{SD}}{ }^{2}\right)$ (Figure 3). Here, $\Delta \mathrm{E}_{\mathrm{SD}}$ represents the standard deviation of the color difference, and $\Delta \mathrm{E}_{\mathrm{A}}$ represents the average of the color difference. Thus, there was a total of 36 Excel spreadsheets.
4. Color difference: The color difference between each pair of contrasting blocks (such as C-A1 and C-B1) was calculated and output as an Excel spreadsheet. There was a total of 18 Excel spreadsheets. Each pair of contrasting block color difference was represented by each spreadsheet's average color differences $(\Delta \mathrm{E})$. Each pair of contrasting blocks contained two kinds of standard color, and thus each color difference value corresponded to two standard blocks (such as S-A1 and S-B1).
5. $N \Delta E: N \Delta E$ was calculated, meaning that $\Delta E$ was on the order of a normal distribution $N$ $\left(\Delta \mathrm{E}_{\mathrm{A}}, \Delta \mathrm{E}_{\mathrm{SD}^{2}}{ }^{2}\right)$.


Figure 3. Normal distribution of color difference between C-A6 and S-A6. The color difference between C-A6 and S-A6 obeyed the normal distribution $\mathrm{N}\left(5.88,4.53^{2}\right)$. When the color difference a certain color and S-A6 was 19.47, N $\Delta \mathrm{E}$ was 3; when the color difference between a certain color and S-A6 was $37.59, \mathrm{~N} \Delta \mathrm{E}$ was 7 . Because the color difference was a positive number, the negative number had no reference value.

Table 1. Calculation results of contrasting blocks and standard blocks.

|  | C | S | $\Delta \mathrm{E}_{\text {SD }}$ | $\Delta \mathrm{E}_{\mathrm{A}}$ | N $\Delta \mathrm{E}$ |  | C | S | $\Delta \mathrm{E}_{\text {SD }}$ | $\Delta \mathrm{E}_{\mathrm{A}}$ | N $\Delta \mathrm{E}$ | $\Delta \mathrm{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A1 |  |  | 0.76 | 1.23 | 6.95 | B1 |  |  | 0.23 | 0.64 | 25.42 | 6.54 |
| A2 |  |  | 2.00 | 3.17 | 4.50 | B2 |  |  | 1.99 | 3.36 | 4.43 | 12.17 |
| A3 |  |  | 3.65 | 4.70 | 4.72 | B3 |  |  | 1.21 | 2.50 | 15.99 | 21.93 |
| A4 |  |  | 5.90 | 10.49 | 2.97 | B4 |  |  | 4.37 | 6.95 | 4.82 | 28.02 |
| A5 |  |  | 1.08 | 2.71 | 26.87 | B5 |  |  | 4.96 | 8.94 | 4.59 | 31.68 |
| A6 |  |  | 4.53 | 5.88 | 5.09 | B6 |  |  | 1.42 | 2.73 | 18.43 | 28.96 |

### 2.4. Evaluation of Scintillation by Entropy Method

After the threshold of the color difference intervals was determined, the percentage of the number of data in each color difference interval to the number of all data was calculated.

$$
\begin{equation*}
\mathrm{R}_{\mathrm{e}}=\mathrm{m}_{\mathrm{e}} / \mathrm{m}^{*} \tag{9}
\end{equation*}
$$

In Equation (9), $m_{e}$ represents the amount of the data in the specific color difference interval $e$, and $\mathrm{m}^{*}$ represents the amount of all of the data.

During rotation, the largest degree of brightness and fire color represented the diamond's scintillation quality. The $R_{e}$ value of the spreadsheet with the largest standard deviation among the seven Excel spreadsheets produced by a given diamond's eight images was selected as the basis for quantitatively evaluating the diamond scintillation. All of the diamond data that were ultimately used for calculation were plotted in a box map (Figure 2D). The color difference indicated the change of brightness and fire color; the percentage of the number of data in each color difference interval to the number of all data $\left(R_{e}\right)$ was indicated by the area of the color change. $R_{e}$ was found to decrease with increasing color difference intervals.

### 2.5. Evaluation of Scintillation by Naked Eye

The objective evaluation results obtained by the entropy method required verification by visual evaluation. It was difficult to directly evaluate the diamond scintillation when they were rotated. The eight images of each diamond were made into a GIF animation with each image having a dwell time of 1 s . The sixty-six diamonds were divided into ten groups on average based on the cut. The diamond scintillation was determined by the changes to the degree and area of the diamonds' fire color and brightness. The sixty-six diamonds were evaluated by seven experienced graders with professional knowledge, each of whom had received the diploma of HRD Antwerp Certified Diamond Grader. The final results of evaluating diamond scintillation by the naked eye was the dominant evaluation of the seven observers (Figure 2E). Each observer was required to be in good health with normal vision, aged 22-24 years, and to have extensive experience. The criteria of the naked eye observations were the following:

1. Excellent (EX): The diamond was extremely bright with extremely high fire. The changes in the diamond table's brightness and fire color were extremely obvious when the diamond was rotated.
2. Very good (VG): The diamond was very bright with a great deal of fire. The changes in the diamond table's brightness and fire color were quite obvious when the diamond was rotated.
3. Good (G): The diamond was quite bright with some fire. The changes in the diamond table's brightness and fire color were obvious when the diamond was rotated.
4. Fair (F): The diamond was bright with some fire. The changes in the diamond table's brightness and fire color were obvious when the diamond was rotated.
5. Poor (P): The diamond was not bright with little fire. The changes in the diamond table's brightness and fire color were not obvious when the diamond was rotated.

## 3. Results and Discussion

### 3.1. Division of Color Difference Interval

The larger the $N \Delta E$, the easier it is for the human eye to perceive color change. When $N \Delta E>3$, the human eye exactly perceives the color change. When $\mathrm{N} \Delta \mathrm{E}>7$, the human eye very obviously perceives the color change [43], and the corresponding color difference could be used as the threshold for dividing the color difference interval. Table 1 lists the group when $\mathrm{N} \Delta \mathrm{E} \leq 7$.

The color difference values, $\Delta \mathrm{E}$, were similar for some pairs of contrasting blocks while the $\mathrm{N} \Delta \mathrm{E}$ values were different. For instance, the color difference of the fourth group (28.02) was similar to that of the sixth group (28.96), while the maximum $\mathrm{N} \Delta \mathrm{E}$ of the fourth group (4.82) was significantly different than that of the sixth group (18.43).

For a given pair of color changes, the $\mathrm{N} \Delta \mathrm{E}$ was different if the change direction was different. For example, in the first pair of color changes, $\mathrm{N} \Delta \mathrm{E}$ was 6.95 when changing from A 1 to B 1 ; when changing from B1 to A1, $\mathrm{N} \Delta \mathrm{E}$ was 25.42 . This indicated that the human eye should perceive a higher level of diamond scintillation when B1 changed to A1.

When $\mathrm{N} \Delta \mathrm{E} \leq 7$, the maximum of $\Delta \mathrm{E}_{\mathrm{A}}$ for the diamond was 10.49; hence, the minimum of the color difference that the human eye could perceive significant color change was 10.49. For convenience of calculation, 10 was set as the color difference threshold. Each group color difference of two adjacent images was divided into $0-10,10-20,20-30,30-40,40-50,50-60$, and $>60$, and thus there were seven color difference intervals.

### 3.2. Comparison of Results by Entropy Method and by Naked Eye

The color difference exerted different impacts on the diamond scintillation. The diamond scintillation was quantitatively classified into five levels-EX, VG, G, F, and P—using the entropy method. The seven color difference intervals were identified for each of the 66 diamonds. Here, $\mathrm{x}_{\mathrm{ij}}$ represents the NO. j percentage of the NO. i diamond.

Each variable was normalized. Different $\mathrm{R}_{\mathrm{e}}$ could either inhibit the diamond scintillation or promote it. Variables that had inhibitory effects were described as having a negative index, while those that promoted scintillation were described as positive. A higher positive index implied a better promotion effect.

Positive index:

$$
\begin{equation*}
\mathrm{x}_{\mathrm{ij}}^{\prime}=\left\{\left[\mathrm{x}_{\mathrm{ij}}-\min \left(\mathrm{x}_{1 \mathrm{j}}, \mathrm{x}_{2 \mathrm{j}}, \ldots \mathrm{x}_{\mathrm{nj}}\right)\right] /\left[\max \left(\mathrm{x}_{1 \mathrm{j}}, \mathrm{x}_{2 \mathrm{j}}, \ldots \mathrm{x}_{\mathrm{nj}}\right)-\min \left(\mathrm{x}_{1 \mathrm{j}}, \mathrm{x}_{2 \mathrm{j}}, \ldots \mathrm{x}_{\mathrm{nj}}\right)\right]\right\} \times 100 \tag{10}
\end{equation*}
$$

Negative index:

$$
\begin{equation*}
\mathrm{x}_{\mathrm{ij}}^{\prime}=\left\{\left[\max \left(\mathrm{x}_{1 \mathrm{j}}, \mathrm{x}_{2 \mathrm{j}}, \ldots \mathrm{x}_{\mathrm{nj}}\right)-\mathrm{x}_{\mathrm{ij}} \mathrm{j} /\left[\max \left(\mathrm{x}_{1 \mathrm{j}}, \mathrm{x}_{2 \mathrm{j}}, \ldots \mathrm{x}_{\mathrm{nj}}\right)-\min \left(\mathrm{x}_{1 \mathrm{j}}, \mathrm{x}_{2 \mathrm{j}}, \ldots \mathrm{x}_{\mathrm{nj}}\right)\right]\right\} \times 100\right. \tag{11}
\end{equation*}
$$

To facilitate the calculation, let $\mathrm{x}_{\mathrm{ij}}{ }^{\prime}=\mathrm{x}_{\mathrm{ij}}$.
Calculate the proportion of the NO. i diamond's NO. j percentage using the following:

$$
\begin{equation*}
\mathrm{p}_{\mathrm{ij}}=\frac{\mathrm{x}_{\mathrm{ij}}}{\sum_{\mathrm{i}=1}^{66} \mathrm{x}_{\mathrm{ij}}},(\mathrm{i}=1,2, \cdots, 66 ; \mathrm{j}=1,2, \cdots, 7) \tag{12}
\end{equation*}
$$

Calculate the entropy value of the NO. $j$ percentage, that was, the dispersion of $R_{0-10}, R_{10-20}, \ldots$ $\mathrm{R}_{>60}$.

$$
\begin{equation*}
\mathrm{e}_{\mathrm{j}}=-\frac{\mathrm{k}}{\sum_{\mathrm{i}=1}^{66}\left[\mathrm{p}_{\mathrm{ij}} \ln \left(\mathrm{p}_{\mathrm{ij}}\right)\right]}, \mathrm{k}>0, \mathrm{k}=\frac{1}{[\ln (66)]}, \mathrm{e}_{\mathrm{j}} \geq 0 \tag{13}
\end{equation*}
$$

The difference coefficient of the NO. j percentage defined in Equation (14) was calculated. For the NO. j percentage, a larger difference between the color difference intervals implied a smaller entropy value, and a correspondingly greater impact on the evaluation of the diamond scintillation.

$$
\begin{equation*}
g_{j}=\frac{1-e_{j}}{10-E_{e}}, 0 \leq g_{j} \leq 1, \sum_{j=1}^{7} g_{j}=1, E_{e}=\sum_{j=1}^{7} e_{j} \tag{14}
\end{equation*}
$$

Calculate $\mathrm{g}_{\mathrm{j}}$ of the NO. j color difference interval to account for all the difference in coefficient weights:

$$
\begin{equation*}
\mathrm{w}_{\mathrm{j}}=\frac{\mathrm{g}_{\mathrm{j}}}{\sum_{\mathrm{j}=1}^{7} \mathrm{~g}_{\mathrm{j}}}(1 \leq \mathrm{j} \leq 7) \tag{15}
\end{equation*}
$$

The composite score for each diamond was the sum of the proportional difference of the color difference percentages between adjacent images multiplied by its weighted entropy. The larger the composite score, the stronger the diamond scintillation:

$$
\begin{gather*}
S_{i}=\sum_{j=1}^{7}\left(w_{j} \times p_{i j}\right),(i=1,2, \cdots, 66),  \tag{16}\\
F=1000 \times S_{i} . \tag{17}
\end{gather*}
$$

The F value was divided into different intervals according to its magnitude; each interval corresponds to a different diamond scintillation quality. The evaluation criteria are listed below:

1. $\mathrm{F}<0.8$ : The diamond scintillation under the D65 illumination was P .
2. $0.8 \leq \mathrm{F}<2.0$ : The diamond scintillation under the D65 illumination was F .
3. $2.0 \leq \mathrm{F}<3.5$ : The diamond scintillation under the D65 illumination was G.
4. $\quad 3.5 \leq \mathrm{F}<12$ : The diamond scintillation under the D65 illumination was VG.
5. $\mathrm{F} \geq 12$ : The diamond scintillation under the D65 illumination was EX.

The results are shown in Figure 4. Under these conditions, the consistency and similarity were $51.51 \%$ and $92.42 \%$, respectively, for the results evaluated by the entropy method versus those by the naked eye.

1. Consistency: Implied that the results achieved via the two methods were comparable.
2. Similarity: Meant that they were at the same grade or differed by one level.


Figure 4. Diamond scintillation evaluation results. The larger the F, the stronger the diamond scintillation. This figure shows that the diamond scintillation is generally positively correlated with cut quality.

### 3.3. Factors Affecting Diamond Scintillation

The $\mathrm{R}_{\mathrm{e}}$ value with the largest standard deviation among the seven groups of color differences produced by each diamond's images was selected as the basis to quantitatively evaluate the diamond scintillation. The percentage of the lightness difference $(\Delta \mathrm{L})$, chroma difference $(\Delta \mathrm{C})$, and hue difference $(\Delta h)$ for each group was calculated and recorded as $R_{1}, R_{c}$, and $R_{h}$, respectively (the calculation method was similar to the method used for calculation of $R_{e}$ and the interval threshold was also 10). The Pearson correlation coefficients of $R_{e}$ with $R_{1}, R_{c}$, and $R_{h}$ were analyzed separately in order to determine the factors that affected the non-fancy-colored round brilliant diamond scintillation.

### 3.3.1. The Main Factors Affecting Diamond Scintillation

$R_{1}, R_{c}$, and $R_{h}$ were significantly correlated with $R_{e}$. The $100 \%$ diamond correlation of $R_{1}$ and $R_{e}$ were the highest of the three; the $56.06 \%$ diamond correlation of $R_{c}$ and $R_{e}$ were second only to that of $R_{1}$ and $R_{e}$; the $19.70 \%$ diamond correlation of $R_{h}$ and $R_{e}$ were second only to the that of $R_{1}$ and $R_{e}$; the $24.24 \%$ diamond correlation of $R_{e}$ with $R_{c}$, and $R_{e}$ with $R_{h}$ were equal. Lightness, chroma, and hue all significantly affected the diamond scintillation, with lightness being the main factor. Diamond hue was primarily influenced by dispersion, which could cause the diamond to produce colored light resembling a rainbow. The lightness was mainly influenced by its refractive index, and the chroma was influenced by a variety of factors, mainly the refractive index and dispersion. The diamond maximizes its reflective ability after being cut and polished, so most of its surface could reflect white light. Therefore, the area of brightness changed greatly during the rotation process. Although the dispersion of the diamonds was also strong, the area of hue change was much smaller than that of lightness change because of the more stringent criterion of hue change compared with lightness change.

### 3.3.2. Discussion on the Higher Evaluation Level by the Entropy Method

The evaluation results by the naked eye were subjectively affected by the observer. Specifically, $21.21 \%$ diamonds' evaluation results by the entropy method were one level higher than the results obtained by the naked eye. Among these diamonds, the Pearson correlation coefficients of $R_{e}$ with $R_{1}$, $R_{c}$, and $R_{h}$ were not the same (Figure 5A), and were all in the middle range of the correlation of all the diamonds (Figure 5C). For example, the correlation of $R_{e}$ and $R_{c}$ for all the diamonds ranged from 0.942-0.992, while that of the diamonds, whose evaluation results by the naked eye were one level lower than the results by the entropy method, ranged from $0.957-0.986$. Since the influence of $R_{1}$ on $R_{e}$ was significantly higher than the influence of either $R_{c}$ or $R_{h}$, only the influence of chroma and hue on the evaluation results was discussed here. When the degrees of influence of $R_{c}$ and $R_{h}$ on $R_{e}$ were in the moderate range and different, the diamond scintillation perceived by the naked eye could be weaker than that by the entropy method.


Figure 5. Pearson correlation coefficients of $R_{e}$ with $R_{l}, R_{c}$, and $R_{h}$. (A) Pearson correlation coefficients of $R_{e}$ with $R_{c}$, and $R_{h}$ when the results by the entropy method were one level higher than the results by the naked eye and the color difference image, lightness difference image, chroma difference image, and hue difference image of Dia 0035 (within the gray frame). (B) Pearson correlation coefficients of $R_{e}$ with $R_{c}$ and $R_{h}$ when the results by the entropy method were one level lower than the results by the naked eye and the color difference image, lightness difference image, chroma difference image, and hue difference image of Dia 1711 (inside the gray frame). (C) Pearson correlation coefficients ranges of $R_{e}$ with $R_{1}, R_{c}$, and $R_{h}$ when the results by the entropy method were one level higher than the results by the naked eye.

### 3.3.3. Discussion on the Lower Evaluation Level by the Entropy Method

$18.18 \%$ diamonds' evaluation results by the entropy method were one level lower than that by the naked eye. In these diamonds, the Pearson correlation coefficients of $R_{e}$ and $R_{h}$ were the lowest (Figure 5B). Different color changes were perceived by the human eye during observing different areas in the color space with same color difference value. The human eye was more sensitive to the perception of lightness. Therefore, the human eye could perceive a stronger scintillation than the objective evaluation by the entropy method due to the hue had little effect on the color change.

## 4. Conclusions

This study proposed a method combining the entropy method and color difference formula to quantitatively evaluate non-fancy-colored round brilliant diamond scintillation. The color difference of corresponding pixels in two adjacent rotation angle images was calculated using a MatLab r2014a program during the in situ rotation of non-fancy-colored round brilliant diamonds (color changes of fire and brightness during diamond rotation). Each group color difference of two adjacent images was divided into seven color difference intervals with a threshold of 10. The percentage of the number of data in each color difference interval to the number of all data (the area of the color change) was comprehensively analyzed using the entropy method in order to evaluate diamond scintillation quantitatively and objectively.

The Pearson correlation coefficients of color difference percentage with lightness difference percentage, chroma difference percentage, and hue difference percentage in each interval were analyzed separately. The results revealed that the lightness difference percentage, chroma difference percentage, and hue difference percentage were significantly correlated with color difference percentage, with correlation coefficients ranging from 0.979-1.000, $0.942-0.992$, and $0.937-0.992$, respectively. The correlation coefficient of lightness difference percentage and color difference percentage was the highest among the three coefficients in each diamond sample. Therefore, lightness was the main factor affecting the diamond scintillation while chroma and hue also significantly affected it.

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