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A Piecewise Linear FGM Approach for Efficient and Accurate FAHP Analysis: Smart Backpack Design as an Example

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Abstract: Most existing fuzzy AHP (FAHP) methods use triangular fuzzy numbers to approximate the fuzzy priorities of criteria, which is inaccurate. To obtain accurate fuzzy priorities, time-consuming alpha-cut operations are usually required. In order to improve the accuracy and efficiency of estimating the fuzzy priorities of criteria, the piecewise linear fuzzy geometric mean (PLFGM) approach is proposed in this study. The PLFGM method estimates the α cuts of fuzzy priorities and then connects these α cuts with straight lines. As a result, the estimated fuzzy priorities will have piecewise linear membership functions that resemble the real shapes. The PLFGM approach has been applied to the identification of critical features for a smart backpack design. According to the experimental results, the PLFGM approach improved the accuracy and efficiency of estimating the fuzzy priorities of these critical features by 33% and 80%, respectively.

Keywords: fuzzy analytic hierarchy process; fuzzy geometric mean; alpha-cut operations; piecewise linear

1. Introduction

The analytic hierarchy process (AHP), proposed by Saaty [1], is a well-known multi-criteria decision-making method. AHP is based on the pairwise comparison of criteria, which is a subjective process. To better consider such subjectivity, fuzzy logic has been incorporated into AHP, which resulted in various fuzzy AHP (FAHP) methods [2]. FAHP have been extensively applied to a number of topics in various fields, e.g., supplier selection [3–6], project selection and risk assessment/management [7,8], personnel selection [9,10], failure mode and effect analysis [11,12], strategy analysis and technology selection [13–16], etc.

In a FAHP problem, deriving the values of fuzzy eigenvalue and eigenvector requires a number of fuzzy multiplication operations, which is a time-consuming task [17]. For this reason, most existing FAHP methods [18–26] estimate, rather than derive, the values of fuzzy eigenvalue and eigenvector. To improve both the efficiency and accuracy of solving a FAHP problem, a piecewise linear fuzzy geometric mean (PLFGM) approach is proposed in this study. The PLFGM approach can be viewed as a hybrid of alpha-cut operations (ACO) [18] and fuzzy geometric mean (FGM) [22]. In the PLFGM approach, some α cuts of fuzzy eigenvalue and eigenvector are estimated using FGM. Then, these α cuts are connected with straight lines. As a result, the membership functions of the estimated fuzzy eigenvalue and eigenvector become piecewise linear functions, rather than triangular functions. In this

way, the estimated fuzzy eigenvalue and eigenvector better approximate their exact values. In addition, the required calculations can be done quickly, even for a large-scale FAHP problem. The novelty of the proposed methodology resides in the following:

- (1) The priority of a criterion is approximated with a polygon fuzzy number, rather than a triangular fuzzy number (TFN).
- (2) The commonly used FGM method is modified, and the PLFGM approach is proposed to improve the accuracy of deriving the priorities of criteria.
- (3) The proposed PLFGM approach is similar in nature to the ACO method, but much more efficient than it.
- (4) The center-of-gravity (COG) [27] of a polygon fuzzy number is derived.

The remainder of this paper is organized as follows. Section 2 is dedicated to the literature review. Section 3 is a preliminary of some existing FAHP methods. Section 4 introduces the proposed PLFGM approach. Section 5 details the application of the PLFGM approach to the identification of critical features of a smart backpack design. Several existing methods were also applied to the same problem for comparison. Section 6 concludes this study and puts forth some topics for future investigation.

2. Related Work

In theory, the fuzzy eigenvalue and eigenvector of a fuzzy judgment matrix can be derived using ACO [18]. To enhance the computational efficiency, some researchers modified the definition of consistency, so as to derive fuzzy eigenvalue and vector in a different way (i.e., not fuzzy eigenanalysis) [19,20]. In addition, many existing FAHP methods approximate, rather than derive, the values of fuzzy eigenvalue and eigenvector using techniques such as fuzzy extent analysis (FEA) [21], FGM [22], and the fuzzy inverse of column sum (FICSM) [23]. However, such approximation may lead to incorrect decisions [24,25]. To address this problem, Chen et al. [26] modified the ACO method and proposed the approximating alpha-cut operations (xACO) method that derived the values of fuzzy eigenvalue and eigenvector without enumerating all possible α cuts of a fuzzy judgment matrix. However, Chen et al.'s method was still time-consuming for a large-scale FAHP problem.

In the recent literature, Sirisawat and Kiatcharoenpol [28] ranked a few solutions for reverse logistics barriers using technique for order preference by similarity to ideal solution (TOPSIS). Factors critical to the ranking process were prioritized by solving a FAHP problem using the FEA method. Chen et al. [29] considered a FAHP problem as a fuzzy collaborative forecasting process [30–33], in which the fuzzy priorities of criteria, rather than experts' fuzzy pairwise comparison results, were aggregated. Lyu et al. [34] compared the effects of various risks on constructing a metro tunnel, for which the FEA method was applied to solve the FAHP problem. Chen and Wu [35] decomposed an inconsistent fuzzy judgment matrix into several consistent fuzzy subjudgment matrixes, so as to assess the suitability of a smart technology application for e-health. Boral et al. [36] combined FAHP and fuzzy multi-attribute ideal deal comparative analysis (fuzzy MAIRCA) for comparing risk factors in conducting a failure mode and effect analysis. For evaluating the sustainability of a smart technology application to mobile health care, Chen [37] applied the FGM method to aggregate the pairwise comparison results by multiple experts, and then derived the fuzzy priorities of criteria using the ACO method. The differences between the proposed methodology and some existing methods are summarized in Table 1.

Table 1. Differences between the proposed methodology and some existing methods.

Method	Type of Eigenvalue and Eigenvector	Shape of Membership Functions	Efficiency	Accuracy
FGM [22]	Fuzzy	Triangular	Very high	Low
FEA [21,28,34]	Crisp	-	Very high	Very low
FICSM [23]	Fuzzy	Triangular	Very high	Low
ACO [18,37]	Fuzzy	Nonlinear	Very low	Very high
xACO [26]	Fuzzy	Logarithmic	Low ~ moderate	High
The proposed methodology	Fuzzy	Piecewise Linear	Very high	Moderate ~ High

3. Preliminary

3.1. FAHP

In a FAHP problem, a decision maker compares the relative priority of a criterion over that of another using linguistic terms such as “as equal as,” “weakly more important than,” “strongly more important than,” “very strongly more important than,” and “absolutely more important than.” These linguistic terms are usually mapped to TFNs within [1,9] (see Table 2) [38,39].

Table 2. Linguistic terms for expressing relative priorities.

Symbol	Linguistic Term	TFN
L1	As equal as	(1, 1, 3)
L2	As equal as or weakly more important than	(1, 2, 4)
L3	Weakly more important than	(1, 3, 5)
L4	Weakly or strongly more important than	(2, 4, 6)
L5	Strongly more important than	(3, 5, 7)
L6	Strongly or very strongly more important than	(4, 6, 8)
L7	Very strongly more important than	(5, 7, 9)
L8	Very or absolutely strongly more important than	(6, 8, 9)
L9	Absolutely more important than	(7, 9, 9)

Based on pairwise comparison results, the fuzzy judgment matrix $\tilde{\mathbf{A}}_{n \times n} = [\tilde{a}_{ij}]$ is constructed as:

$$\begin{aligned} \tilde{a}_{ji} &= (a_{ji1}, a_{ji2}, a_{ji3}) \\ &= 1/\tilde{a}_{ij} \\ &\cong (1/a_{ij3}, 1/a_{ij2}, 1/a_{ij1}) \end{aligned} \tag{1}$$

$$\tilde{a}_{ii} = 1 \tag{2}$$

The fuzzy eigenvalue and eigenvector of $\tilde{\mathbf{A}}$, indicated with $\tilde{\lambda}$ and $\tilde{\mathbf{x}}$ respectively, satisfy [40]:

$$\det(\tilde{\mathbf{A}}(-)\tilde{\lambda}\mathbf{I}) = 0 \tag{3}$$

and

$$(\tilde{\mathbf{A}}(-)\tilde{\lambda}\mathbf{I})(\times)\tilde{\mathbf{x}} = 0 \tag{4}$$

where $(-)$ and (\times) denote fuzzy subtraction and multiplication, respectively. To derive the values of $\tilde{\lambda}$ and $\tilde{\mathbf{x}}$, a number of fuzzy multiplication operations need to be performed. However, the multiplication of TFNs does not yield a TFN [41]. Therefore, $\tilde{\lambda}$ and $\tilde{\mathbf{x}}$ are not TFNs anymore, as illustrated in Figure 1. Approximating them with TFNs may lead to incorrect decisions.

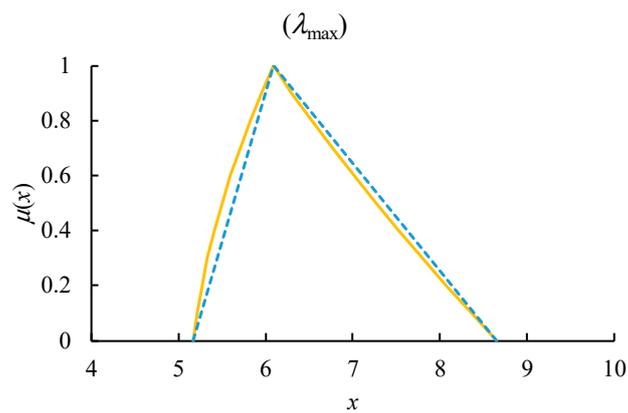


Figure 1. Non-TFN nature of a fuzzy eigenvalue.

3.2. ACO

In the ACO method, fuzzy parameters and variables in Equations (3) and (4) are replaced with their α cuts:

$$\det(\tilde{A}(\alpha) - \tilde{\lambda}(\alpha)\mathbf{I}) = 0 \tag{5}$$

$$(\tilde{A}(\alpha) - \tilde{\lambda}(\alpha)\mathbf{I})\tilde{\mathbf{x}}(\alpha) = 0 \tag{6}$$

Each α cut is an interval:

$$\tilde{a}_{ij}(\alpha) = [a_{ij}^L(\alpha), a_{ij}^R(\alpha)] \tag{7}$$

$$\tilde{\lambda}(\alpha) = [\lambda^L(\alpha), \lambda^R(\alpha)] \tag{8}$$

$$\tilde{\mathbf{x}}(\alpha) = [\mathbf{x}^L(\alpha), \mathbf{x}^R(\alpha)] \tag{9}$$

If α takes 11 possible values (0, 0.1, . . . , 1), Equations (5) and (6) must be solved $11 \cdot 2^{C_2^n}$ times to derive the membership functions of fuzzy eigenvalue and eigenvector as [26]:

$$\lambda^L(\alpha) = \min_{\det([a_{ij}^*(\alpha)] - \lambda_t(\alpha)\mathbf{I})=0} (\lambda_t(\alpha)) \tag{10}$$

$$\lambda^R(\alpha) = \max_{\det([a_{ij}^*(\alpha)] - \lambda_t(\alpha)\mathbf{I})=0} (\lambda_t(\alpha)) \tag{11}$$

$$\mathbf{x}^L(\alpha) = \min_{([a_{ij}^*(\alpha)] - \lambda_t(\alpha)\mathbf{I})\mathbf{x}_t(\alpha)=0} (\mathbf{x}_t(\alpha)) \tag{12}$$

$$\mathbf{x}^R(\alpha) = \max_{([a_{ij}^*(\alpha)] - \lambda_t(\alpha)\mathbf{I})\mathbf{x}_t(\alpha)=0} (\mathbf{x}_t(\alpha)) \tag{13}$$

where $*$ = L or R. $\lambda_t^L(\alpha)$, $\lambda_t^R(\alpha)$, $\mathbf{x}_t^L(\alpha)$, and $\mathbf{x}_t^R(\alpha)$ are the results derived from the t -th combination; $t = 1 \sim 11 \cdot 2^{C_2^n}$. Although the ACO method can derive the membership functions of fuzzy eigenvalue and eigenvector accurately, it is time-consuming.

Based on $\tilde{\mathbf{x}}$, the fuzzy priorities of criteria can be derived as [40]:

$$\begin{aligned} \tilde{w}_i &= \frac{\tilde{x}_i}{\sum_{j=1}^n \tilde{x}_j} \\ &= \frac{1}{1 + \sum_{j \neq i} \frac{\tilde{x}_j}{\tilde{x}_i}} \end{aligned} \tag{14}$$

In addition, based on $\widetilde{\lambda}_{\max}$, fuzzy consistency ratio can be assessed as [40]:

$$\widetilde{CR} = \frac{\widetilde{\lambda}_{\max} - n}{n - 1} \cdot \frac{1}{RI} \tag{15}$$

where RI random consistency index. If $\widetilde{CR} \leq 0.1$, then the decision maker’s pairwise comparison results are consistent. Neither \widetilde{w}_i nor \widetilde{CR} are TFNs [26].

The COG method can be applied to defuzzify a fuzzy priority as [27]:

$$COG(\widetilde{w}_i) = \frac{\int_0^1 x \mu_{\widetilde{w}_i}(x) dx}{\int_0^1 \mu_{\widetilde{w}_i}(x) dx} \tag{16}$$

However, the ACO method takes samples uniformly along the y axis, while COG requires that samples be taken regularly along the x axis [26]. To resolve this discrepancy, the range of \widetilde{w}_i can be partitioned into Γ equal intervals [42]:

$$\widetilde{w}_i = \left\{ \left[\frac{\Gamma - \eta + 1}{\Gamma} w_i^L(0) + \frac{\eta - 1}{\Gamma} w_i^R(0), \frac{\Gamma - \eta}{\Gamma} w_i^L(0) + \frac{\eta}{\Gamma} w_i^R(0) \right] \mid \eta = 1 \sim \Gamma \right\} \tag{17}$$

The center of the η -th interval is indicated with $C_i(\eta)$:

$$\begin{aligned} C_i(\eta) &= \frac{1}{2} \left(\frac{\Gamma - \eta + 1}{\Gamma} w_i^L(0) + \frac{\eta - 1}{\Gamma} w_i^R(0) + \frac{\Gamma - \eta}{\Gamma} w_i^L(0) + \frac{\eta}{\Gamma} w_i^R(0) \right) \\ &= \frac{2\Gamma - 2\eta + 1}{2\Gamma} w_i^L(0) + \frac{2\eta - 1}{2\Gamma} w_i^R(0) \end{aligned} \tag{18}$$

The membership of $C_i(\eta)$ is determined by interpolating those of the two closest α cuts of \widetilde{w}_i :

$$\begin{aligned} \mu_{\widetilde{w}_i}(C_i(\eta)) &= \frac{C_i(\eta) - \max_{w_i^*(\alpha) \leq C_i(\eta)} w_i^*(\alpha)}{\min_{w_i^*(\alpha) \geq C_i(\eta)} w_i^*(\alpha) - \max_{w_i^*(\alpha) \leq C_i(\eta)} w_i^*(\alpha)} \cdot \min_{w_i^*(\alpha) \geq C_i(\eta)} \alpha \\ &+ \frac{\min_{w_i^*(\alpha) \geq C_i(\eta)} w_i^*(\alpha) - C_i(\eta)}{\min_{w_i^*(\alpha) \geq C_i(\eta)} w_i^*(\alpha) - \max_{w_i^*(\alpha) \leq C_i(\eta)} w_i^*(\alpha)} \cdot \max_{w_i^*(\alpha) \leq C_i(\eta)} \alpha \end{aligned} \tag{19}$$

where * can be R or L . Then, the COG of \widetilde{w}_i is calculated based on the centers of the intervals:

$$COG(\widetilde{w}_i) = \frac{\sum_{\eta=1}^{\Gamma} (\mu_{\widetilde{w}_i}(C_i(\eta)) C_i(\eta))}{\sum_{\eta=1}^{\Gamma} \mu_{\widetilde{w}_i}(C_i(\eta))} \tag{20}$$

3.3. FGM

The FGM method estimates the fuzzy priority of criterion i as [38]:

$$\widetilde{w}_i \cong \frac{\sqrt[n]{\prod_{j=1}^n \widetilde{a}_{ij}}}{\sum_{k=1}^n \frac{\sqrt[n]{\prod_{j=1}^n \widetilde{a}_{kj}}}{n}} \tag{21}$$

When \widetilde{w}_i is approximated with a TFN, i.e., $\widetilde{w}_i = (w_{i1}, w_{i2}, w_{i3})$, the following theorem holds.

Theorem 1 ([39]).

$$w_{i1} \cong \frac{1}{1 + \sum_{k \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{kj3}}}{\sqrt[n]{\prod_{j=1}^n a_{ij1}}}} \tag{22}$$

$$w_{i2} \cong \frac{1}{1 + \sum_{k \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{kj2}}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}}}} \tag{23}$$

$$w_{i3} \cong \frac{1}{1 + \sum_{k \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{kj1}}}{\sqrt[n]{\prod_{j=1}^n a_{ij3}}}} \tag{24}$$

The COG method can be applied to defuzzify a TFN-based fuzzy priority as [27]

$$COG(\tilde{w}_i) = \frac{w_{i1} + w_{i2} + w_{i3}}{3} \tag{25}$$

The fuzzy maximal eigenvalue $\tilde{\lambda}_{max}$ can be estimated as [38]

$$\tilde{\lambda}_{max} \cong \frac{1}{n} \sum_{i=1}^n \frac{\sum_{j=1}^n (\tilde{a}_{ij}(\times) \tilde{w}_j)}{\tilde{w}_i}. \tag{26}$$

The following theorem holds if $\tilde{\lambda}_{max}$ is approximated with a TFN.

Theorem 2 ([39]).

$$\lambda_{max,1} \cong 1 + \frac{1}{n} \sum_{i=1}^n \sum_{j \neq i} \frac{a_{ij1} w_{j1}}{w_{i3}} \tag{27}$$

$$\lambda_{max,2} \cong 1 + \frac{1}{n} \sum_{i=1}^n \sum_{j \neq i} \frac{a_{ij2} w_{j2}}{w_{i2}} \tag{28}$$

$$\lambda_{max,3} \cong 1 + \frac{1}{n} \sum_{i=1}^n \sum_{j \neq i} \frac{a_{ij3} w_{j3}}{w_{i1}}. \tag{29}$$

Based on $\tilde{\lambda}_{max}$, fuzzy consistency ratio, in terms of a TFN, can be evaluated according to Equation (15) as

$$CR_1 = \frac{\lambda_{max,1} - n}{n-1} RI \tag{30}$$

$$CR_2 = \frac{\lambda_{max,2} - n}{n-1} RI \tag{31}$$

$$CR_3 = \frac{\lambda_{max,3} - n}{n-1} RI. \tag{32}$$

4. The PLFGM Approach

4.1. Assumptions and Limitations

The following assumptions are made in this study:

- (1) The decision-maker is able to compare the relative priorities of criteria in pairs.
- (2) Pairwise comparison results are consistent.
- (3) An efficient ACO-based method for solving large-scale FAHP problems is still lacking.

In addition, the proposed PLFGM approach is subject to the following limitations:

- (1) The PLFGM approach can only improve the accuracy of α cuts when α is not equal to 0 or 1.
- (2) When pairwise comparison results are inconsistent, the effect of the PLFGM method is limited.
- (3) When the uncertainty of pairwise comparison results is not high, the effect of the PLFGM method is also limited.

A flowchart is provided in Figure 2 to illustrate the procedure of the PLFGM approach.

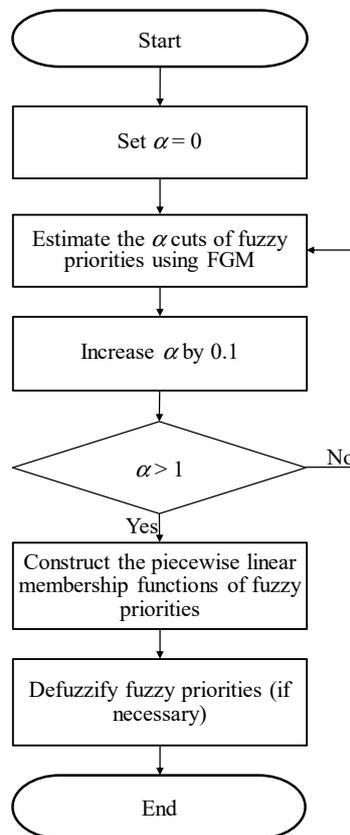


Figure 2. Procedure of the proposed methodology.

4.2. Piecewise Linear Membership Functions

Letting the left and right α cuts of \tilde{w}_i be indicated with $w_i^L(\alpha)$ and $w_i^R(\alpha)$, respectively. According to Theorem 1:

$$w_i^L(\alpha) \cong \frac{1}{1 + \sum_{k \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{kj}^R(\alpha)}}{\sqrt[n]{\prod_{j=1}^n a_{ij}^L(\alpha)}}} \tag{33}$$

$$w_i^R(\alpha) \cong \frac{1}{1 + \sum_{k \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{kj}^L(\alpha)}}{\sqrt[n]{\prod_{j=1}^n a_{ij}^R(\alpha)}}} \tag{34}$$

In PLFGM, a fuzzy priority is estimated by connecting some of its α cuts with straight lines, as illustrated in Figure 3, in which the membership function on either side is approximated by connecting four α cuts with straight lines [43]. FGM is a special case of PLFGM because only the α cuts when $\alpha = 0$ and 1 are connected.

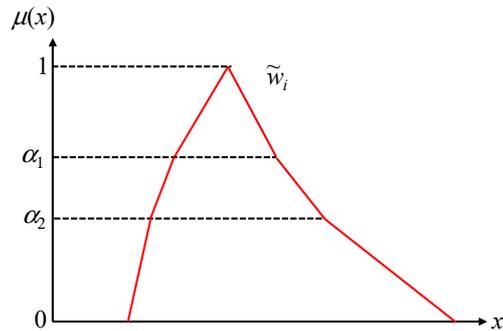
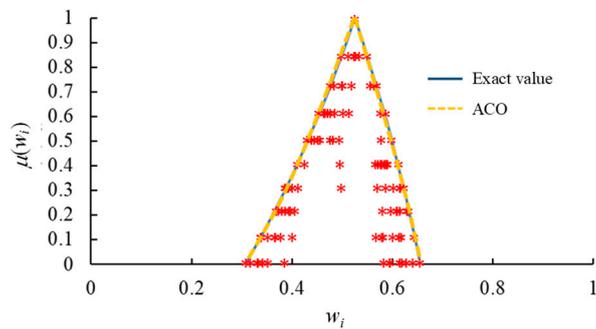


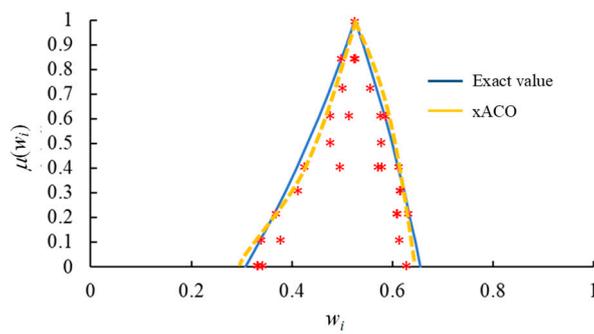
Figure 3. A fuzzy priority estimated using PLFGM.

An example is provided in Figure 4 that illustrates the differences among ACO, xACO, FGM, and PLFGM.

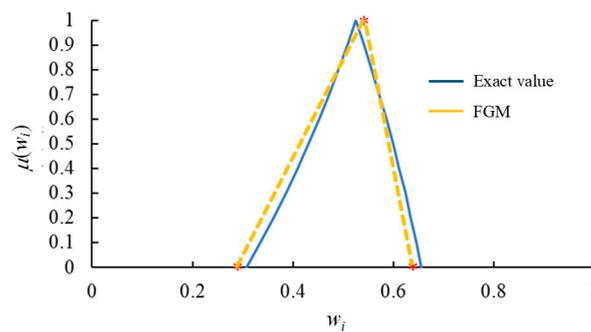


(a) ACO

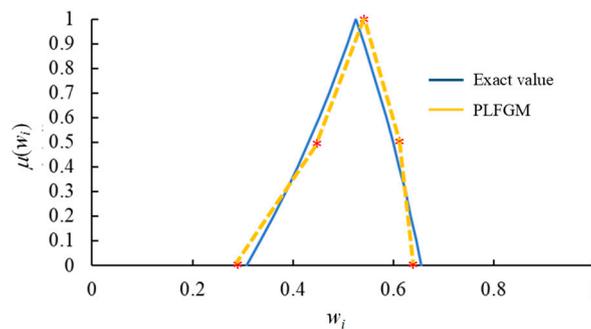
Figure 4. Cont.



(b) xACO



(c) FGM



(d) PLFGM

Figure 4. Differences among ACO, xACO, FGM, and PLFGM (* denotes a data point) (a) ACO; (b) xACO; (c) FGM; (d) PLFGM.

4.3. Defuzzification

To defuzzify a fuzzy priority estimated using the PLFGM approach, the following theorems are helpful:

Theorem 3 ([6]). The integral of a non-normal trapezoidal fuzzy number (TrFN) \tilde{P} , shown in Figure 5, is:

$$\int_{x_1}^{x_2} \mu_{\tilde{P}(x)}(x)dx = \frac{\mu_2 x_2^2 + \mu_1 x_2^2 - 2\mu_2 x_1 x_2 + \mu_1 x_1^2 - 2\mu_1 x_1 x_2 + \mu_2 x_1^2}{2(x_2 - x_1)} \tag{35}$$

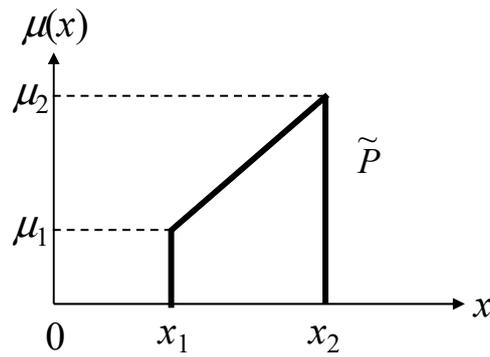


Figure 5. A non-normal TrFN.

Theorem 4 ([6]).

$$\int_{x_1}^{x_2} x \mu_{\tilde{P}(x)}(x) dx = \frac{2\mu_2 x_2^3 + \mu_1 x_2^3 - 3\mu_2 x_1 x_2^2 + \mu_2 x_1^3 + 2\mu_1 x_1^3 - 3\mu_1 x_1^2 x_2}{6(x_2 - x_1)}. \tag{36}$$

A fuzzy priority estimated using the PLFGM approach can be decomposed into several non-normal TrFNs, as illustrated in Figure 6. In this figure, there are four non-normal TrFNs, whose corner data are summarized in Table 3. Then, the defuzzified value of \tilde{w}_i can be derived by applying Theorems 3 and 4 as follows.

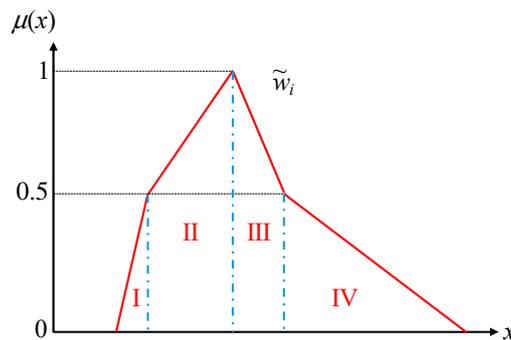


Figure 6. Decomposing a fuzzy priority estimated using PLFGM into several non-normal TrFNs.

Table 3. Corner data of the non-normal TrFNs.

	I	II	III	IV
x_1	$w_i^L(0)$	$w_i^L(0.5)$	$w_i^*(1)$	$w_i^R(0.5)$
x_2	$w_i^L(0.5)$	$w_i^*(1)$	$w_i^R(0.5)$	$w_i^R(0)$
μ_1	0	0.5	1	0.5
μ_2	0.5	1	0.5	0

Theorem 5. Let \tilde{w}_i be a polygonal fuzzy number as shown in Figure 6. Then the COG of \tilde{w}_i is:

$$COG(\tilde{w}_i) = \frac{w_i^L(0.5)^3\gamma_2\gamma_3\gamma_4 - 1.5w_i^L(0)w_i^L(0.5)^2\gamma_2\gamma_3\gamma_4 + 0.5w_i^L(0)^3\gamma_2\gamma_3\gamma_4 + 2.5w_i^*(1)^3\gamma_1\gamma_3\gamma_4 - 3w_i^L(0.5)w_i^*(1)^2\gamma_1\gamma_3\gamma_4 + 2w_i^L(0.5)^3\gamma_1\gamma_3\gamma_4 - 1.5w_i^L(0.5)^2w_i^*(1)\gamma_1\gamma_3\gamma_4 + 2w_i^R(0.5)^3\gamma_1\gamma_2\gamma_4 - 1.5w_i^*(1)w_i^R(0.5)^2\gamma_1\gamma_2\gamma_4 + 2.5w_i^*(1)^3\gamma_1\gamma_2\gamma_4 - 3w_i^*(1)^2w_i^R(0.5)\gamma_1\gamma_2\gamma_4 + 0.5w_i^R(0)^3\gamma_1\gamma_2\gamma_3 + w_i^R(0.5)^3\gamma_1\gamma_2\gamma_3 - 1.5w_i^R(0.5)^2w_i^R(0)\gamma_1\gamma_2\gamma_3}{1.5w_i^L(0.5)^2\gamma_2\gamma_3\gamma_4 - 3w_i^L(0)w_i^L(0.5)\gamma_2\gamma_3\gamma_4 + 1.5w_i^L(0)^2\gamma_2\gamma_3\gamma_4 + 7.5w_i^*(1)^2\gamma_1\gamma_3\gamma_4 - 9w_i^L(0.5)w_i^*(1)\gamma_1\gamma_3\gamma_4 + 4.5w_i^L(0.5)^2\gamma_1\gamma_3\gamma_4 + 7.5w_i^R(0.5)^2\gamma_1\gamma_2\gamma_4 - 9w_i^*(1)w_i^R(0.5)\gamma_1\gamma_2\gamma_4 + 4.5w_i^*(1)^2\gamma_1\gamma_2\gamma_4 + 1.5w_i^R(0)^2\gamma_1\gamma_2\gamma_3 + 1.5w_i^R(0.5)^2\gamma_1\gamma_2\gamma_3 - 3w_i^R(0.5)w_i^R(0)\gamma_1\gamma_2\gamma_3} \tag{37}$$

where $\gamma_1 = w_i^L(0.5) - w_i^L(0)$; $\gamma_2 = w_i^*(1) - w_i^L(0.5)$; $\gamma_3 = w_i^R(0.5) - w_i^*(1)$; $\gamma_4 = w_i^R(0) - w_i^R(0.5)$.

Proof.

$$COG(\tilde{w}_i) = \frac{\int x\mu_{\tilde{w}_i}(x)dx}{\int \mu_{\tilde{w}_i}(x)dx} = \frac{\int_{x \in I} x\mu_{\tilde{w}_i}(x)dx + \int_{x \in II} x\mu_{\tilde{w}_i}(x)dx + \int_{x \in III} x\mu_{\tilde{w}_i}(x)dx + \int_{x \in IV} x\mu_{\tilde{w}_i}(x)dx}{\int_{x \in I} \mu_{\tilde{w}_i}(x)dx + \int_{x \in II} \mu_{\tilde{w}_i}(x)dx + \int_{x \in III} \mu_{\tilde{w}_i}(x)dx + \int_{x \in IV} \mu_{\tilde{w}_i}(x)dx}$$

$$= \frac{\frac{w_i^L(0.5)^3 - 1.5w_i^L(0)w_i^L(0.5)^2 + 0.5w_i^L(0)^3}{6w_i^L(0.5) - 6w_i^L(0)} + \frac{2.5w_i^*(1)^3 - 3w_i^L(0.5)w_i^*(1)^2 + 2w_i^L(0.5)^3 - 1.5w_i^L(0.5)^2w_i^*(1)}{6w_i^*(1) - 6w_i^L(0.5)} + \frac{2w_i^R(0.5)^3 - 1.5w_i^*(1)w_i^R(0.5)^2 + 2.5w_i^*(1)^3 - 3w_i^*(1)^2w_i^R(0.5)}{6w_i^R(0.5) - 6w_i^*(1)} + \frac{0.5w_i^R(0)^3 + w_i^R(0.5)^3 - 1.5w_i^R(0.5)^2w_i^R(0)}{6w_i^R(0) - 6w_i^R(0.5)}}{\frac{0.5w_i^L(0.5)^2 - w_i^L(0)w_i^L(0.5) + 0.5w_i^L(0)^2}{2w_i^L(0.5) - 2w_i^L(0)} + \frac{2.5w_i^*(1)^2 - 3w_i^L(0.5)w_i^*(1) + 1.5w_i^L(0.5)^2}{2w_i^*(1) - 2w_i^L(0.5)} + \frac{2.5w_i^R(0.5)^2 - 3w_i^*(1)w_i^R(0.5) + 1.5w_i^*(1)^2}{2w_i^R(0.5) - 2w_i^*(1)} + \frac{0.5w_i^R(0)^2 + 0.5w_i^R(0.5)^2 - w_i^R(0.5)w_i^R(0)}{2w_i^R(0) - 2w_i^R(0.5)}} \tag{38}$$

$$= \frac{w_i^L(0.5)^3\gamma_2\gamma_3\gamma_4 - 1.5w_i^L(0)w_i^L(0.5)^2\gamma_2\gamma_3\gamma_4 + 0.5w_i^L(0)^3\gamma_2\gamma_3\gamma_4 + 2.5w_i^*(1)^3\gamma_1\gamma_3\gamma_4 - 3w_i^L(0.5)w_i^*(1)^2\gamma_1\gamma_3\gamma_4 + 2w_i^L(0.5)^3\gamma_1\gamma_3\gamma_4 - 1.5w_i^L(0.5)^2w_i^*(1)\gamma_1\gamma_3\gamma_4 + 2w_i^R(0.5)^3\gamma_1\gamma_2\gamma_4 - 1.5w_i^*(1)w_i^R(0.5)^2\gamma_1\gamma_2\gamma_4 + 2.5w_i^*(1)^3\gamma_1\gamma_2\gamma_4 - 3w_i^*(1)^2w_i^R(0.5)\gamma_1\gamma_2\gamma_4 + 0.5w_i^R(0)^3\gamma_1\gamma_2\gamma_3 + w_i^R(0.5)^3\gamma_1\gamma_2\gamma_3 - 1.5w_i^R(0.5)^2w_i^R(0)\gamma_1\gamma_2\gamma_3}{1.5w_i^L(0.5)^2\gamma_2\gamma_3\gamma_4 - 3w_i^L(0)w_i^L(0.5)\gamma_2\gamma_3\gamma_4 + 1.5w_i^L(0)^2\gamma_2\gamma_3\gamma_4 + 7.5w_i^*(1)^2\gamma_1\gamma_3\gamma_4 - 9w_i^L(0.5)w_i^*(1)\gamma_1\gamma_3\gamma_4 + 4.5w_i^L(0.5)^2\gamma_1\gamma_3\gamma_4 + 7.5w_i^R(0.5)^2\gamma_1\gamma_2\gamma_4 - 9w_i^*(1)w_i^R(0.5)\gamma_1\gamma_2\gamma_4 + 4.5w_i^*(1)^2\gamma_1\gamma_2\gamma_4 + 1.5w_i^R(0)^2\gamma_1\gamma_2\gamma_3 + 1.5w_i^R(0.5)^2\gamma_1\gamma_2\gamma_3 - 3w_i^R(0.5)w_i^R(0)\gamma_1\gamma_2\gamma_3}$$

where $\gamma_1 = w_i^L(0.5) - w_i^L(0)$; $\gamma_2 = w_i^*(1) - w_i^L(0.5)$; $\gamma_3 = w_i^R(0.5) - w_i^*(1)$; $\gamma_4 = w_i^R(0) - w_i^R(0.5)$. This completes the proof. \square

Based on the derived (or estimated) fuzzy priorities of criteria, fuzzy weighted average (FWA) [16], multi-attribute utility theory (MAUT) [44], fuzzy technique for order preference by similarity to ideal

solution (fuzzy TOPSIS) [42], or fuzzy VišeKriterijumska Optimizacija I Kompromisno Resenje (fuzzy VIKOR) [45] can be applied to evaluate the overall performances of alternatives.

5. Smart Backpack Design Case

5.1. Application of the Proposed Methodology

A smart backpack, also known as an enhanced backpack, is an innovative application of smart technologies, with functions such as motion detection, navigation, and power generation [46]. However, most of the research and development focus is on rechargeable backpacks with a variety of compartments, that is, placing a mobile power supply in a backpack and connecting the power to the USB plug of each compartment [47]. Although it is very convenient to record activities and navigation using a smart phone, there are still occasions when a smart backpack with functions such as motion detection, navigation, and power generation is required. For example, sometimes it is inconvenient to hold a smart phone, a smart phone is out of power, a mobile power supply is out of power, there is no base station signal, or there is no offline map [48].

The research and development of smart backpacks is still in a nascent stage. As a result, it is a challenging task to identify factors that are critical to a smart backpack design. After reviewing the relevant literature and current practices, the following five factors were considered critical to a smart backpack design:

- (1) C1: sleek design;
- (2) C2: low price;
- (3) C3: many smart technologies;
- (4) C4: high practicability;
- (5) C5: lightweight.

A designer first compared the relative priorities of these critical factors with linguistic terms. The results are summarized in Table 4.

Table 4. Results of pairwise comparisons.

Critical Factor #1	Critical Factor #2	Relative Priority of Critical Factor #1 Over Critical Factor #2
Low price	Sleek design	Weakly more important than
Many smart technologies	Sleek design	Strongly more important than
Sleek design	High practicability	Weakly more important than
Lightweight	Sleek design	Weakly more important than
Many smart technologies	Low price	Weakly more important than
Low price	High practicability	Weakly more important than
Lightweight	Low price	As equal as
Many smart technologies	High practicability	Strongly more important than
Many smart technologies	Lightweight	Weakly or strongly more important than
High practicability	Lightweight	As equal as

The following fuzzy judgment matrix was constructed:

$$\tilde{A} = \begin{bmatrix} 1 & 1/(1, 3, 5) & 1/(3, 5, 7) & (1, 3, 5) & 1/(1, 3, 5) \\ (1, 3, 5) & 1 & 1/(1, 3, 5) & (1, 3, 5) & 1/(1, 1, 3) \\ (3, 5, 7) & (1, 3, 5) & 1 & (3, 5, 7) & (2, 4, 6) \\ 1/(1, 3, 5) & 1/(1, 3, 5) & 1/(3, 5, 7) & 1 & (1, 1, 3) \\ (1, 3, 5) & (1, 1, 3) & 1/(2, 4, 6) & 1/(1, 1, 3) & 1 \end{bmatrix}$$

At first, the ACO method was applied to derive the exact values of fuzzy maximal eigenvalue and fuzzy priorities from this fuzzy judgment matrix. The results are shown in Figures 7 and 8, respectively. The fuzzy consistency ratio was around 0.096 with a minimum of 0 and a maximum of 0.611. After applying COG to defuzzify fuzzy priorities, the results were 0.121, 0.196, 0.443, 0.11, and 0.174.

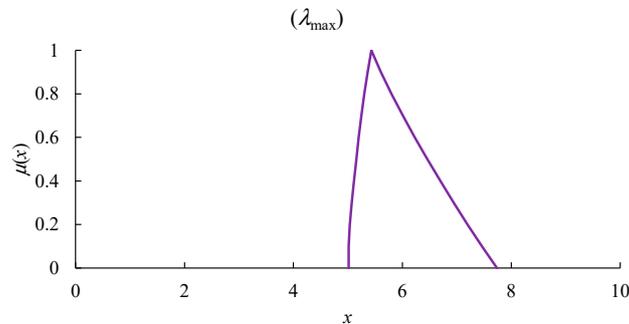


Figure 7. Values of fuzzy maximal eigenvalue derived using ACO.

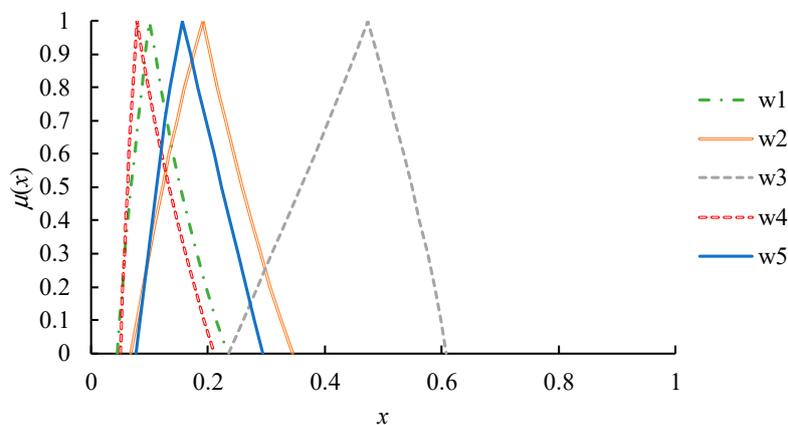
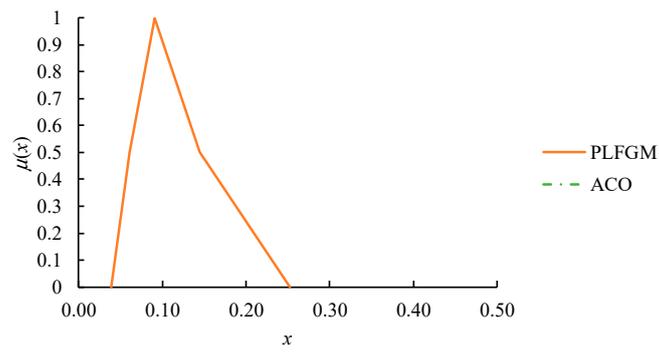


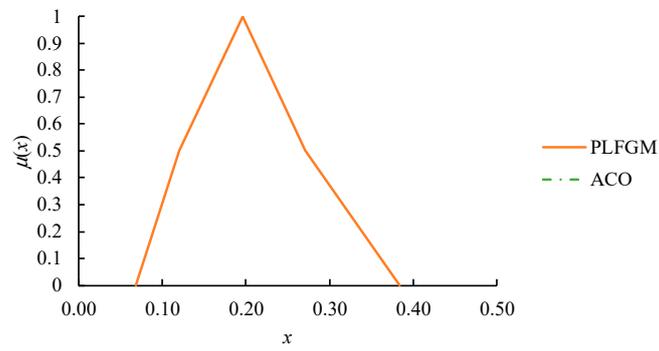
Figure 8. Values of fuzzy priorities derived using ACO.

The ACO method was implemented using MATLAB on a PC with an i7-7700 CPU 3.6 GHz and 8 GB RAM. The execution time was up to 20 s. To enhance computational efficiency, the PLFGM approach was applied.

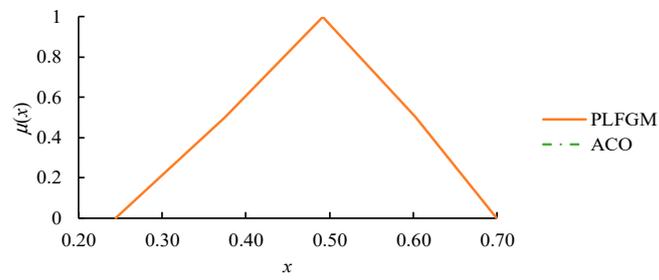
In the PLFGM approach, the α -cuts of fuzzy priorities when α is in $\{0, 0.5, 1\}$ were estimated according to Equations (33) and (34) and then connected, which resulted in their piecewise-linear membership functions, as shown in Figure 9. Obviously, most of the fuzzy priorities estimated using the PLFGM approach resembled their exact values. Subsequently, COG is applied to defuzzify these fuzzy priorities. The results were 0.121, 0.209, 0.482, 0.11, and 0.174. Three of the estimated priorities were equal to the corresponding exact values, showing the effectiveness of the PLFGM approach.



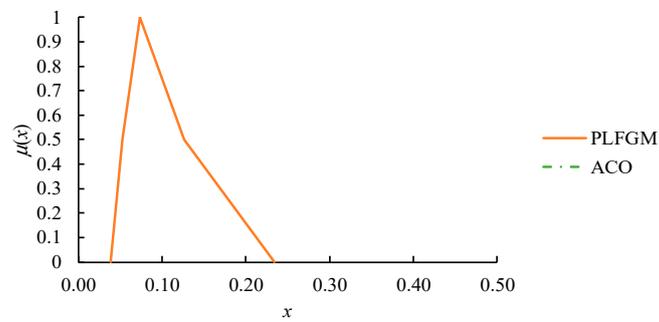
(a) w_1 .



(b) w_2 .

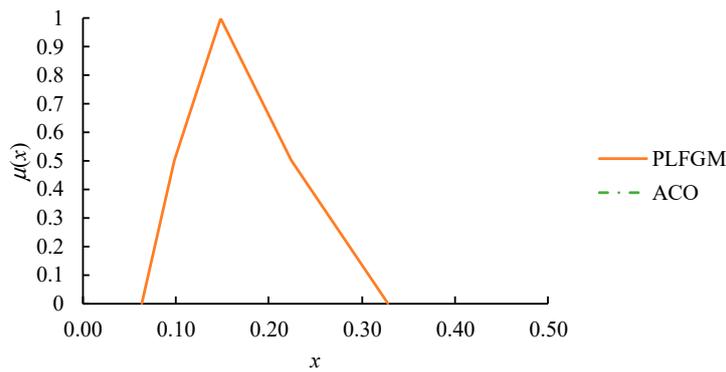


(c) w_3 .



(d) w_4 .

Figure 9. Cont.



(e) w_5 .

Figure 9. Fuzzy priorities estimated using PLFGM (a) w_1 ; (b) w_2 ; (c) w_3 ; (d) w_4 ; (e) w_5 .

5.2. Comparison with Existing Methods

For comparison, three existing methods, FGM, FEA, and xACO were also applied to this case. In the FGM method, fuzzy priorities were approximated with TFNs. In the FEA method, priorities were given in crisp values. In the xACO method, about 20% of the α -cut combinations required by the ACO method were enumerated, which shortened the execution time to about 5 s. Subsequently, the COG method was applied to defuzzify fuzzy priorities. To compare the accuracy achieved using various methods, the average deviation (AD) from exact values was measured:

$$AD = \frac{\sum_{i=1}^n |COG_{method}(w_i) - COG_{ACO}(w_i)|}{n} \tag{39}$$

The results are summarized in Table 5. The execution time for each method was also shown in this table.

Table 5. Performances of various methods.

Method	AD	Execution Time (seconds)
FGM	0.015	1
FEA	0.031	1
xACO	0.01	5
PLFGM	0.01	1

5.3. Discussion

According to the experimental results,

- (1) Both xACO and PLFGM achieved the highest estimation accuracy, followed by FGM. The prevalent FEA method was the least accurate method. Compared to FEA, PLFGM improved the estimation accuracy, in terms of AD, by 33%.
- (2) On the other hand, the execution time of xACO was considerably longer than that of PLFGM, FEA, or FGM. If the size of a FAHP problem becomes larger, xACO will take much more time, while other methods can still be completed instantaneously. Compared to xACO, PLFGM improved the estimation efficiency, in terms of the execution time, by 80%.
- (3) In this case, the PLFGM approach was considered to be superior to the three existing methods, since it achieved the highest estimation accuracy within the shortest execution time.
- (4) The most obvious advantage of the proposed methodology is that it improves the estimation accuracy and efficiency at the same time.

- (5) One disadvantage of the PLFGM approach is the complexity of the formula for calculating the defuzzification value.

6. Conclusions

In a FAHP problem, deriving the fuzzy priorities of criteria is a time-consuming task. As a result, most existing FAHP methods estimate, rather than derive, the values of fuzzy priorities of criteria. In this way, fuzzy priorities are approximated with TFNs. However, the edges of fuzzy priorities are actually curved. Such inaccuracy may lead to incorrect decisions. To address this problem, the PLFGM approach is proposed in this study. The PLFGM approach is a hybrid of ACO and FGM, so it is expected to have the advantages of these two methods. In the PLFGM approach, some α cuts of fuzzy priorities are estimated using the FGM method and connected with straight lines. As a result, the estimated fuzzy priorities have piecewise linear membership functions that resemble the real shapes. In addition, since FGM is much faster than ACO and xACO, the PLFGM approach can greatly improve the efficiency of estimating fuzzy priorities.

The PLFGM approach has been applied to identify the critical features of a smart backpack design. The following conclusions were drawn from the experimental results:

- (1) “Many smart technologies” and “low price” were the two most important features of a smart backpack design. In contrast, “high practicability” was the least important feature.
- (2) Compared to the FGM method, the PLFGM approach improved the estimation accuracy, in terms of AD, by 33%.
- (3) In addition, the efficiency of the PLFGM approach, in terms of the execution time, was 80% higher than that of the xACO method.
- (4) The efficiency of the xACO method deteriorates rapidly as the size of the FAHP problem increases. Therefore, the advantage of the PLFGM approach over the xACO method will be more significant for a larger-scale FAHP problem.

The PLFGM approach needs to be applied to more real cases to further elaborate its effectiveness. In addition, a simpler formula for defuzzifying a polygon fuzzy number must be proposed to enhance the practicability of the PLFGM approach. These constitute some directions for future research.

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