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Cubic q-Rung Orthopair Fuzzy Heronian Mean Operators and Their Applications to Multi-Attribute Group Decision Making

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Abstract: The cubic q-rung orthopair fuzzy set (Cq-ROFS) contains much more information to determine the interval valued q-rung orthopair fuzzy sets (IVq-ROFSs) and q-rung orthopair fuzzy sets (q-ROFSs) simultaneously for coping with the vagueness in information. It provides more space for decision makers (DMs) to describe their opinion in the environment of fuzzy set (FS) theory. In this paper, firstly, we introduce the conception of Cq-ROFS and their characteristics. Further, the Heronian mean (HM) operator based on Cq-ROFS, called the weighted HM operator, are explored. To overcome the deficiency of HM operator and keeping in mind the partitioned structure in real decision situations, we offer Cubic q-rung orthopair fuzzy partitioned HM operator and its weighted shape. An algorithm of the proposed operators based on multi-attribute group decision making (MAGDM) problems for the selection of best alternative among the given ones is established. Lastly, we provide an example to depict the authenticity and advantages of the exposed methods by contrasting with other existing drawbacks.

Keywords: cubic q-rung orthopair fuzzy sets; Heronian mean operators; partitioned structure; multi-attribute group decision making

1. Introduction

Decision making is a useful tool to attain the first alternatives among the given alternatives. Many researchers have been given mixture of concept to get the correct results. In the past, decisions were made on bases of crisp numbers. But this approach is less applicable in making the suitable decisions. However, with the passage of time, due to difficulties in method, it was difficult for the decision makers (DMs) to hold the vagueness in facts by using the traditional approaches. Thus, researchers have given the data in terms of fuzzy sets (FSs). The concept of FS was initially proposed by Zadeh [1] in 1965, which was the huge achievement and it has many implementations in different areas. Basically, FS consists of only positive degree belonging to [0, 1]. FS is a better apparatus for understanding the uncertain and ambiguous statements. FS has obtained powerful attention by the researchers everywhere in the world and they had studied it both definite and pure aspects. Considerable developments of FS had been established such as interval valued fuzzy set (IVFS) [2] in which membership degree (MD) is equal to interval value belonging to [0, 1]. During the last decades, researchers are paying more attentions to these approaches and fruitfully practiced it to numerous circumstances in the decision making (DM) process. The idea of FSs was further generalized by intuitionist fuzzy set (IFS) established by Atanassov [3] in which MD 'u' as well as non-membership degree (NMD) 'v' is also including having condition that $0 \leq u^q + v^q \leq 1$. Some aggregation operators based on IFS were defined by Xu [4], which aggregated the given data to single value and make an easy task for decision makers to make their judgments. Atanassov generalized the idea of IFS to interval valued intuitionist fuzzy set (IVIFS) [5] and some IVIF aggregation operators were defined by Wang et al. [6]. In IFS, if NMD become '0' then we have FS. The IFS cannot be described effectively, for handling such type of situation, when decision makers give '0.2' as a MD and '0.9' as NMD. Then the concept of Pythagorean fuzzy set (PFS) was established by Yager [7] for dealing with this kind of problems. It is a broader concept in which MD "u" and NMD "v" must assure the situation $0 \leq u^2 + v^2 \leq 1$. Further, Pythagorean fuzzy power aggregation operators and Pythagorean fuzzy Einstein prioritized aggregation operators for PFSs were discussed in [8,9]. The concept of PFS was further generalized by interval value Pythagorean fuzzy set (IVPFS) and some fundamental properties of IVPF aggregation operators were discussed by Peng and Yang [10]. In addition, some aggregation operators based on IVPFSs were proposed by Rahman et al. [11]. Further, the q-ROFS proposed by Yager [12] can generalize IFS and PFS. In q-ROFS MD "u" and NMD "v" must assure the condition $0 \leq u^q + v^q \leq 1$. The q-ROFS allows the decision makers more space in making their opinion. Some others views on q-ROFS are given in [13]. Some q-ROFBM and q-ROFHM operators are discussed in [14–16]. Joshi et al. [17] established the idea of IVq-ROFS and the IVq-ROF Archimedean Muirhead mean operators are discussed in [18]. The idea of cubic set (CS) was established by Jun et al. [19] using the combination of IVFS and FS, and they defined some basic operations on CSs. Fahmi et al. [20] introduced some cubic fuzzy Einstein aggregation operators and also discussed its implementations to judgment process. In addition, the trapezoidal cubic fuzzy number Einstein hybrid weighted averaging operators are discussed in [21]. Mahmood et al. [22] established the idea of cubic hesitant fuzzy sets (CHFSs) and their aggregation operators in decision making process. Further, Harish et al. [23] generalized the concept of cubic sets to cubic intuitionist fuzzy set (CIFS) and some CIF aggregation operators are discussed in [24]. Abbas et al. [25] introduced the concept of cubic Pythagorean fuzzy set (CPFS) and some cubic pythagorean fuzzy weighted averaging (CPFWA)and cubic pythagorean fuzzy weighted geometric (CPFWG) aggregation operators were defined by them. From the above written work, we can see all actual study mostly target the FS, IVFS and their corresponding functions.

CS considers only the membership intervals and non-membership portion of the data entities is not discussed. Usually, it is hard to demonstrate the assessment of MD by definite value in a FS. In such conditions, it could be simple to interpret ambiguity and hesitancy in the real world using an interval value and an exact value rather than unique interval/exact values. As a result, the hybrid form of an interval value might be exceptionally useful to interpret the hesitancy because of his/her slow judgment in composite decision making problems. Because of this logic, the idea of cubic intuitionist fuzzy set (CIFS) was established by Kaur and Garg [23], which is narrated by two components at the same time, where one represents the MDs by an IVIF numbers (IVIFNs) and the other represents the MD by IF numbers (IFNs). Hence, a CIFS is the hybrid set which is the combination of both IVIFNs and IFNs. Evidently, the edge of the CIFS is that it can carry significantly more data to demonstrate the IVIFNs and IFNs simultaneously. For instance, suppose a manager has to evaluate the work of his teammates. The teammate provides him with his self-analyzed report saying that he has completed 20–30% and simultaneously has not accomplished 50–60% of the work assigned to him. After analyzing his report by the manager, he gives his judgment under IFS environment by saying that he disagrees with the completed work by 20% and agrees to the incomplete work by 10%. Then, in that case, CIFS is formulated as an R-order given by $(([0.20, 0.30], [0.50, 0.60]), (0.20, 0.10))$. On the other hand, if the manager agrees by 40% and disagree to the incomplete work by 50% then P-order CIFS is formed as $(([0.20, 0.30], [0.50, 0.60]), (0.50, 0.60))$. Therefore, this domain increases the level of accuracy by boosting the range of the membership interval by assuming a fuzzy set membership value interrelated to it. Hence, it is a useful apparatus for holding the vague and unclear information during the decision making procedure under the ambiguous domain. For more related work on IFS, PFS, we many refer to Ref. [24,25].

However, all the above approaches are unsuitable to aggregate these cubic q-rung orthopair fuzzy numbers (Cq-ROFNs) on the basis of the Heronian mean (HM) and the geometric Heronian mean. Thus, how to aggregate these Cq-ROFNs on the basis of the Heronian mean and the geometric Heronian mean is an interesting topic. To solve this issue, in this paper, we shall develop some cubic q-rung orthopair fuzzy aggregation operators on the basis of the traditional generalized Heronian mean and geometric Heronian mean [26]. The traditional HM operators are defined on the assumption that each attribute is related to all the other attributes. Obviously, in many practical decision situations, this assumption does not always hold. For example, when selecting a car, we may consider the following five attributes: purchase price (c_1), car brand (c_2), operating performance (c_3), customer excitement (c_4), and safety (c_5). It is easy to observe that not all of the attributes in this example are interrelated. In fact, the attributes can be divided into two distinct partitions: $D_1 = \{c_1, c_3, c_5\}$ and $D_2 = \{c_2, c_4\}$. We can find that the attributes belonging to the same partition are interrelated, but the attributes of distinct partitions are independent. For example, c_1, c_3, and c_5 are interrelated, but there is no relationship between c_1 and c_2. Under such situations, the traditional HM operator cannot solve such decision situations. Motivated by the idea of partitioned Bonferroni mean [27], we may divide these attributes into several distinct partitions according to a specific relationship pattern between attributes. To aggregate input arguments with the partition structure, we develop several q-rung orthopair fuzzy Heronian mean operators.

When a decision maker provides $(([0.8, 0.9], [0.82, 0.92]), (0.82, 0.9))$ for MD and NMD, the existing CIFS and CPFS cannot be described effectively. Keeping the advantages of HM and PHM operators and restriction of the existing methods, in this paper, we present the concept of Cq-ROFS which is described by two portions at the same time, where one part represent the MDs by IVq-ROFSs and the other represent MDs by q-ROFSs. Hence, Cq-ROFS is the combination of both IVq-ROFSs and q-ROFSs, respectively. On the other hand, Cq-ROFS contain much more information in the form of IVq-ROFS and q-ROFS because it is the combination of both of these q-RIVOFS and q-ROFS. By the concept of HM operators and by taking the advantages of Cq-ROFS, we propose new aggregation operator as *Cq-ROFHM* as well as cubic q-rung orthopair fuzzy weighted Heronian mean operator (*Cq-ROFWHM*) and discuss its properties. Further, we will discuss cubic q-rug orthopair fuzzy partitioned Heronian mean operators (*Cq-ROFPHM*), and its weighted expressions and properties. Decision making approach has been proposed based on established operators for the selection of best solution. A numerical example is explored to demonstrate the given approach. This paper ends with the conclusion remarks.

Further, we arrange our paper as follows: In Section 2, different basic ideas related to q-ROFSs, IVq-ROFSs are reviewed. In Section 3, we define basic definition of Cq-ROFS and its basic operations. In Section 4, we propose HM operator and its weighted shape. Moreover, to reduce the deficiency of HM operator, choosing the partitioned structure in real decision situation, we suggest Cq-ROFPHM operator and its weighted form. In Section 5, we propose an algorithm for *Cq-ROFHM* operators based on multi-attribute group decision (MAGDM) problems for the selection of best alternative among the given ones. Finally, we use an example to demonstrate the efficiency and consequences of the established method by comparing with other existing techniques. In Section 6, we give the conclusion remarks of this paper.

2. Preliminaries

In this portion, we will discuss about the basic ideas of q-ROFSs, IVq-ROFSs, their basic operations, score functions (SF), accuracy functions (AF) and their properties. The basic notions of HM operator, weighted HM operators for non-negative real numbers, and its general form with parameters are also discussed.

Definition 1. *Let* $X \neq \phi$ *be a universal collection. A q-ROFS on* X *is an object of the shape:*

$$A = \{< x, f_A(x), g_A(x) > | x \in X\} \tag{1}$$

where $f_A(x)$ *and* $g_A(x)$ *are MD and NMD respectively having extra condition that* $0 \leq (f_A(x))^q + (g_A(x))^q \leq 1$, *where* $q \geq 1$. *In general,* $h_A(x) = \sqrt[q]{1 - (f_A(x))^q - (g_A(x))^q}$ *is the hesitancy degree of* x *to* A. *For simplicity* $A = \langle f_A, g_A \rangle$ *denotes the q-ROFN.*

Definition 2 [12]**.** *Let* $A_i = \{< f_{A_i}, g_{A_i} >\}\ (i = 1, 2)$ *be a family of q-ROFNs,* $Ƕ > 0$. *Then fundamental operations of q-ROFNs are defined by*

1. $A_1 \oplus A_2 = \left\{ < \left(\left(f_{A_1}\right)^q + \left(f_{A_2}\right)^q - \left(f_{A_1}\right)^q \left(f_{A_2}\right)^q \right)^{\frac{1}{q}}, g_{A_1} g_{A_2} > \right\}$;
2. $A_1 \otimes A_2 = \left\{ < f_{A1} f_{A_2}, \left(\left(g_{A_1}\right)^q + \left(g_{A_2}\right)^q - \left(g_{A_1}\right)^q \left(g_{A_2}\right)^q \right)^{\frac{1}{q}} > \right\}$;
3. $ǶA_1 = \left\{ < \left(\left(1 - \left(1 - f_{A_1}\right)^{Ƕ} \right)^{\frac{1}{q}}, \left(g_{A_1}\right)^{Ƕ} \right) > \right\}$;
4. $A_1^{Ƕ} = \left\{ < \left(\left(f_{A_1}\right)^{Ƕ}, \left(1 - \left(1 - g_{A_1}\right)^{Ƕ} \right)^{\frac{1}{q}} \right) > \right\}$;
5. $A_1^c = \{g_A(x), f_A(x)\}$.

Definition 3 [12]**.** *Let* $A_i = \{< f_{A_i}, g_{A_i} >\}\ (i = 1, 2)$ *be a family of q-ROFNs,* $Ƕ > 0$ *then their SFs and AFs are defined, respectively as:*

$$Sc(A_i) = \left(f_{A_i}\right)^q - \left(g_{A_i}\right)^q (i = 1, 2) \tag{2}$$

and

$$H(A_i) = \left(f_{A_i}\right)^q + \left(g_{A_i}\right)^q (i = 1, 2) \tag{3}$$

To compare the two q-ROFNs, we have:

1. *If* $S(A_1) > S(A_2)$, *then* $A_1 > A_2$.
2. *If* $S(A_1) = S(A_2)$, *then*
   i. *If* $H(A_1) > H(A_2)$, *then* $A_1 > A_2$.
   ii. *If* $H(A_1) = H(A_2)$, *then* $A_1 = A_2$.

Definition 4 [17]**.** *Let* $X \neq \phi$ *be a universal collection. An IVq-ROFS on* X *is a set:*

$$A = \{< x, f_A(x), g_A(x) > | x \in X\} \tag{4}$$

where in $0 \leq f_A(x) \leq 1$ *and* $0 \leq g_A(x) \leq 1$ *are MD and NMD respectively,* $f_A(x) = \left[f_A^L(x), f_A^U(x)\right]$, $g_A(x) = \left[g_A^L(x), g_A^U(x)\right]$ *with the condition* $0 \leq \left(f_A^U(x)\right)^q + \left(g_A^U(x)\right)^q \leq 1$, $\forall x \in X$, $q \geq 1$. $h_A(x) = \left[h_A^L(x), h_A^U(x)\right] = \left[\begin{array}{c} \sqrt[q]{1 - \left(\left(f_A^U(x)\right)^q + \left(g_A^U(x)\right)^q \right)}, \\ \sqrt[q]{1 - \left(\left(f_A^L(x)\right)^q + \left(g_A^L(x)\right)^q \right)} \end{array} \right]$ *is called refusal degree of* x *to* A.

Definition 5 [17]**.** *Let* $A_i = \left\{ < f_{A_i}, g_{A_i} > \right\} (i = 1, 2)$ *be a family of IVq-ROFNs,* $Ƕ > 0$. *Fundamental operations for IVq-ROFNs are defined by*

1. $A_1 \oplus A_2 = \left\{ < \left[\sqrt[q]{\left(f^L_{A_1}\right)^q + \left(f^L_{A_2}\right)^q - \left(f^L_{A_1}\right)^q\left(f^L_{A_2}\right)^q}, \sqrt[q]{\left(f^U_{A_1}\right)^q + \left(f^U_{A_2}\right)^q - \left(f^U_{A_1}\right)^q\left(f^U_{A_2}\right)^q} \right], \left[g^L_{A_1} g^L_{A_2}, g^U_{A_1} g^U_{A_2}\right] > \right\};$
2. $A_1 \otimes A_2 = \left\{ \begin{array}{c} \left[f^L_{A_1} f^L_{A_2}, f^U_{A_1} f^U_{A_2}\right], \\ \left[\sqrt[q]{\left(g^L_{A_1}\right)^q + \left(g^L_{A_2}\right)^q - \left(g^L_{A_1}\right)^q\left(g^L_{A_2}\right)^q}, \sqrt[q]{\left(g^U_{A_1}\right)^q + \left(g^U_{A_2}\right)^q - \left(g^U_{A_1}\right)^q\left(g^U_{A_2}\right)^q} \right] > \end{array} \right\};$
3. $\text{Ϧ}A_1 = \left\{ < \left[\sqrt[q]{\left(1 - \left(1 - f^L_{A_1}\right)^{\text{Ϧ}}\right)}, \sqrt[q]{\left(1 - \left(1 - f^U_{A_1}\right)^{\text{Ϧ}}\right)} \right], \left[\left(g^L_{A_1}\right)^{\text{Ϧ}}, \left(g^U_{A_1}\right)^{\text{Ϧ}} \right] > \right\};$
4. $A_1^{\text{Ϧ}} = \left\{ < \left(\left[\left(f^L_{A_1}\right)^{\text{Ϧ}}, \left(f^U_{A_1}\right)^{\text{Ϧ}} \right], \left[\sqrt[q]{\left(1 - \left(1 - g^L_{A_1}\right)^{\text{Ϧ}}\right)}, \sqrt[q]{\left(1 - \left(1 - g^U_{A_1}\right)^{\text{Ϧ}}\right)} \right] \right) > \right\};$
5. $A_1^c = \{g_{A_1}(x), f_{A_1}(x)\}$.

Definition 6 [17]. *Let* $A_i = \{< f_{A_i}, g_{A_i} >\}$ $(i = 1, 2)$ *be a family of IVq-ROFNs,* $\text{Ϧ} > 0$, *then the SFs and AFs are defined by:*

$$Sc(A_i) = \frac{1}{4}\left[\left(1 + \left(f^L_{A_i}\right)^q - \left(g^L_{A_i}\right)^q\right) + \left(1 + \left(f^U_{A_i}\right)^q - \left(g^U_{A_i}\right)^q\right)\right] \tag{5}$$

and

$$H(A_i) = \frac{\left(f^L_{A_i}\right)^q + \left(f^U_{A_i}\right)^q + \left(g^L_{A_i}\right)^q + \left(g^U_{A_i}\right)^q}{2} \tag{6}$$

respectively. Then, comparison method for two q-ROFNs is defined by:

1. *If* $S(A_1) < S(A_2)$, *then* $A_1 < A_2$.
2. *If* $S(A_1) = S(A_2)$, *then*

    (1) *If* $H(A_1) < H(A_2)$, *then* $A_1 < A_2$.
    (2) *If* $H(A_1) = H(A_2)$, *then* $A_1 = A_2$

Definition 7 [16]. *Let* $(r_1, r_2, \ldots.., r_n)$ *be a collection of non-negative real numbers such that* $r_i \in [0, 1], (i = 1, 2, \ldots., n)$, *then HM is defined as follows:*

$$HM(r_1, r_2, \ldots., r_n) = \left(\frac{2}{n(n+1)} \sum_{\text{Ϧ}=1}^{n} \sum_{t=i}^{n} \sqrt{r_{\text{Ϧ}} r_t}\right) \tag{7}$$

Definition 8 [16]. *Let* $a, b \geq 0$ *with* $a + b > 0$, $(r_1, r_2, \ldots., r_n)$ *be a family of positive real numbers and* $r_i \in [0, 1] (i = 1, 2, \ldots., n)$, *if*

$$HM^{a,b}(r_1, r_2, \ldots.., r_n) = \left(\frac{2}{n(n+1)} \sum_{\text{Ϧ}=1}^{n} \sum_{t=i}^{n} {r_{\text{Ϧ}}}^a {r_t}^b\right)^{\frac{1}{a+b}} \tag{8}$$

Then $HM^{a,b}$ Is the generalized HM operator having parameters a, b. Note that, if $a = b = \frac{1}{2}$, then $HM^{a,b}$ degenerate into above-mentioned traditional HM.

3. Cubic q-Rung Orthopair Fuzzy Sets (Cq-ROFS)

In this portion, we establish the fundamental definition of Cq-ROFS, their basic operations, score functions and accuracy functions for family of cubic q-rung orthopair fuzzy numbers (Cq-ROFNs).

Definition 9. *Let* $X \neq \phi$ *be a universal set. A Cq-ROFS is of the shape:*

$$\mathfrak{C} = \{< x, A(x), \lambda(x) >\} \tag{9}$$

where, $A(x)$ *is an IVq-ROFS and* $\lambda(x)$ *is a q-ROFS. Here* $A(x) = \{[u^L, u^U], [v^L, v^U]\}$ *with* $0 \leq (u^U)^q + (v^U)^q \leq 1$ *and* $\lambda(x) = (u, v)$ *with* $0 \leq u^q + v^q \leq 1$. *For the simplicity, we express this pair as* $\mathfrak{C} = \{< A, \lambda >\}$, *where* $A = \{[u^L, u^U], [v^L, v^U]\}$ *and* $\lambda = (u, v)$ *and is called a Cq-ROFN.*

Definition 10. *A Cq-ROFS "C "defined in (9) is called internal Cq-ROFS, if* $u \in [u^L, u^U]$ *and* $v \in [v^L, v^U]$ *for all* $x \in X$, *otherwise called external Cq-ROFS.*

Definition 11. *For any collection of Cq-ROFNs* $\{\mathfrak{C}_i, i \in I\}$, *we have:*

1. (P-union) $\cup_{P\,I\in I}\, \mathfrak{C}_i = \left(\left(\left[\overset{\sup}{i \in I} u_i^L, \overset{\sup}{i \in I} u_i^U\right], \left[\overset{\inf}{i \in I} v_i^L, \overset{\inf}{i \in I} v_i^U\right]\right), \left(\overset{\sup}{i \in I} u_i, \overset{\inf}{i \in I} v_i\right)\right)$;
2. (P-intersection) $\cap_{P\,I\in I}\, \mathfrak{C}_i = \left(\left(\left[\overset{\inf}{i \in I} u_i^L, \overset{\inf}{i \in I} u_i^U\right], \left[\overset{\sup}{i \in I} v_i^L, \overset{\sup}{i \in I} v_i^U\right]\right), \left(\overset{\inf}{i \in I} u_i, \overset{\sup}{i \in I} v_i\right)\right)$;
3. (R-union) $\cup_{R\,I\in I}\, \mathfrak{C}_i = \left(\left(\left[\overset{\sup}{i \in I} u_i^L, \overset{\sup}{i \in I} u_i^U\right], \left[\overset{\inf}{i \in I} v_i^L, \overset{\inf}{i \in I} v_i^U\right]\right), \left(\overset{\inf}{i \in I} u_i, \overset{\sup}{i \in I} v_i\right)\right)$;
4. (R-intersection) $\cap_{R\,I\in I}\, \mathfrak{C}_i = \left(\left(\left[\overset{\inf}{i \in I} u_i^L, \overset{\inf}{i \in I} u_i^U\right], \left[\overset{\sup}{i \in I} v_i^L, \overset{\sup}{i \in I} v_i^U\right]\right), \left(\overset{\sup}{i \in I} u_i, \overset{\inf}{i \in I} v_i\right)\right)$;

Example 1. *Let* $\mathfrak{C}_1 = \left\{\begin{array}{c}([0.2, 0.3], [0.4, 0.5]), \\ (0.3, 0.4)\end{array}\right\}$, $\mathfrak{C}_2 = \left\{\begin{array}{c}([0.4, 0.6], [0.1, 0.2]), \\ (0.4, 0.5)\end{array}\right\}$, $\mathfrak{C}_3 = \left\{\begin{array}{c}([0.2, 0.5], [0.1, 0.3]), \\ (0.3, 0.6)\end{array}\right\}$ *be the three Cq-ROFNs. Then, for any family of Cq-ROFNs* $\{\mathfrak{C}_i, i \in I\} (i = 1, 2, 3)$, *we have:*

1. (P-union) $\cup_{P\,I\in I}\, \mathfrak{C}_i = \left(\begin{array}{c}\left(\begin{array}{c}\left[\overset{\sup}{i \in I}(0.2, 0.4, 0.2), \overset{\sup}{i \in I}(0.3, 0.6, 0.5)\right], \\ \left[\overset{\inf}{i \in I}(0.4, 0.1, 0.1), \overset{\inf}{i \in I}(0.5, 0.2, 0.3)\right]\end{array}\right). \\ \left(\overset{\sup}{i \in I}(0.3, 0.4, 0.3), \overset{\inf}{i \in I}(0.4, 0.5, 0.6)\right)\end{array}\right) = \left\{\begin{array}{c}([0.4, 0.6], [0.1, 0.2]), \\ (0.4, 0.4)\end{array}\right\}$;
2. (P-intersection) $\cap_{P\,I\in I}\, \mathfrak{C}_i = \left(\begin{array}{c}\left(\begin{array}{c}\left[\overset{\inf}{i \in I}(0.2, 0.4, 0.2), \overset{\inf}{i \in I}(0.3, 0.6, 0.5)\right], \\ \left[\overset{\sup}{i \in I}(0.4, 0.1, 0.1), \overset{\sup}{i \in I}(0.5, 0.2, 0.3)\right]\end{array}\right). \\ \left(\overset{\inf}{i \in I}(0.3, 0.4, 0.3), \overset{\sup}{i \in I}(0.4, 0.5, 0.6)\right)\end{array}\right) = \left\{\begin{array}{c}([0.2, 0.3], [0.4, 0.5]), \\ (0.3, 0.6)\end{array}\right\}$;
3. (R-union) $\cup_{R\,I\in I}\, \mathfrak{C}_i = \left(\begin{array}{c}\left(\begin{array}{c}\left[\overset{\sup}{i \in I}(0.2, 0.4, 0.2), \overset{\sup}{i \in I}(0.3, 0.6, 0.5)\right], \\ \left[\overset{\inf}{i \in I}(0.4, 0.1, 0.1), \overset{\inf}{i \in I}(0.5, 0.2, 0.3)\right]\end{array}\right), \\ \left(\overset{\inf}{i \in I}(0.3, 0.4, 0.3), \overset{\sup}{i \in I}(0.4, 0.5, 0.6)\right)\end{array}\right) = \left\{\begin{array}{c}([0.4, 0.6], [0.1, 0.2]), \\ (0.3, 0.6)\end{array}\right\}$;

4. (R-intersection) $\cap_{R\, I\in I} \complement_i = \left(\begin{array}{c}\left(\begin{array}{c}\left[\overset{\inf}{i\in I}(0.2,\,0.4,\,0.2),\ \overset{\inf}{i\in I}(0.3,\,0.6,\,0.5)\right],\\ \left[\overset{\sup}{i\in I}(0.4,\,0.1,\,0.1),\ \overset{\sup}{i\in I}(0.5,\,0.2,\,0.3)\right]\end{array}\right),\\ \left(\overset{\sup}{i\in I}(0.3,\,0.4,\,0.3),\ \overset{\inf}{i\in I}(0.4,\,0.5,\,0.6)\right)\end{array}\right) = \left\{\begin{array}{c}([0.2,\,0.3],[0.4,\,0.5]),\\ (0.4,\,0.4)\end{array}\right\};$

Definition 12. *Let* $\complement_1 = \{([u_1^L, u_1^U],[v_1^L, v_1^U]),(u_1, v_1)\}$ *and* $\complement_2 = \{([u_2^L, u_2^U],[v_2^L, v_2^U]),(u_2, v_2)\}$ *be the two Cq-RONs. Then:*

1. (Equality) $\complement_1 = \complement_2$ if and only if $[u_1^L, u_1^U] = [u_2^L, u_2^U]$, $[v_1^L, v_1^U] = [v_2^L, v_2^U]$, $u_1 = u_2$, $v_1 = v_2$;
2. (P-order) $\complement_1 \subseteq \complement_2$ if $[u_1^L, u_1^U] \subseteq [u_2^L, u_2^U]$, $[v_1^L, v_1^U] \supseteq [v_2^L, v_2^U]$, $u_1 \leq u_2$, $v_1 \geq v_2$;
3. (R-order) $\complement_1 \subseteq \complement_2$ if $[u_1^L, u_1^U] \subseteq [u_2^L, u_2^U]$, $[v_1^L, v_1^U] \supseteq [v_2^L, v_2^U]$, $u_1 \geq u_2$, $v_1 \leq v_2$;

Definition 13. *For ranking the family of Cq-RONs* $\complement_i = \{\left(\left[u_i^L, u_i^U\right],\left[v_i^L, v_i^U\right]\right),(u_i, v_i)\}$*, we define the score functions under R-order as follows:*

$$Sc(\complement_i) = \frac{1}{2}\left[\frac{1}{4}\left[\left(1+(u_i^L)^q-(v_i^L)^q\right)+\left(1+(u_i^U)^q-(v_i^U)^q\right)\right]+\left(v_i^q-u_i^q\right)\right] \tag{10}$$

While for P-order as

$$Sc(\complement_i) = \frac{1}{2}\left[\frac{1}{4}\left[\left(1+(u_i^L)^q-(v_i^L)^q\right)+\left(1+(u_i^U)^q-(v_i^U)^q\right)\right]+\left(u_i^q-v_i^q\right)\right] \tag{11}$$

Further, an accuracy functions are defined by:

$$H(\complement_i) = \frac{1}{2}\left[\frac{1}{2}\left[\left((u_i^L)^q+(u_i^U)^q\right)+\left((v_i^L)^q+(v_i^U)^q\right)\right]+\left(u_i^q+v_i^q\right)\right] \tag{12}$$

It is evident that $-1 \leq Sc(\complement_i) \leq 1$ and $0 \leq H(\complement_i) \leq 1$.

Definition 14. *For any two Cq-ROFNS* $\complement_1$, $\complement_2$ *the following comparison rules have been defined:*

1. *If* $Sc(\complement_1) > Sc(\complement_2)$ *then* $\complement_1$ *is preferable over* $\complement_2$ *and is expressed by* $\complement_1 \succ \complement_2$;
2. *If* $Sc(\complement_1) = Sc(\complement_2)$

   (1) $H(\complement_1) > H(\complement_2)$ *then* $\complement_1 \succ \complement_2$.
   (2) $H(\complement_1) = H(\complement_2)$ *then* $\complement_1 \sim \complement_2$*, where* $\sim$ *represent "equivalent to".*

Example 2. *Let* $\complement_1 = \{([0.4, 0.5],[0.3, 0.4]),(0.2, 0.7)\}$ *and* $\complement_2 = \{([0.2, 0.7],[0.1, 0.2]),(0.3, 0.5)\}$*, then score function under R-order can be calculated as:*

$$Sc(\complement_1) = \frac{1}{2}\left[\frac{1}{4}\left[\left(1+(0.4)^3-(0.3)^3\right)+\left(1+(0.5)^3-(0.4)^3\right)\right]+\left((0.7)^3-(0.2)^3\right)\right] = 0.42025$$

$$Sc(\complement_2) = \frac{1}{2}\left[\frac{1}{4}\left[\left(1+(0.2)^3-(0.1)^3\right)+\left(1+(0.7)^3-(0.2)^3\right)\right]+\left((0.5)^3-(0.3)^3\right)\right] = 0.34175$$

Since, $Sc(\complement_1) > Sc(\complement_2)$ then $\complement_1$ is preferable over $\complement_2$ and is denoted by $\complement_1 \succ \complement_2$.

Theorem 1. *For Cq-ROFSs* $\complement_1 = <A, \alpha>$, $\complement_2 = <B, \beta>$, $\complement_3 = <\complement, \gamma>$, $\complement_4 = <D, \delta>$*, where* A, B, $\complement$, D *are IVq-ROFSs and* α, β, γ, δ *are q-ROFSs in* X*, we have*

1. *If* $Ȼ_1 \subseteq_P Ȼ_2$ *and* $Ȼ_2 \subseteq_P Ȼ_3$ *then* $Ȼ_1 \subseteq_P Ȼ_3$.
2. *If* $Ȼ_1 \subseteq_P Ȼ_2$ *then* $Ȼ_2^c \subseteq_P Ȼ_1^c$.
3. *If* $Ȼ_1 \subseteq_P Ȼ_2$ *and* $Ȼ_1 \subseteq_P Ȼ_3$ *then* $Ȼ_1 \subseteq_P Ȼ_2 \cap_P Ȼ_3$.
4. *If* $Ȼ_1 \subseteq_P Ȼ_2$ *and* $Ȼ_3 \subseteq_P Ȼ_2$ *then* $Ȼ_1 \cup_P Ȼ_3 \subseteq_P Ȼ_2$.
5. *If* $Ȼ_1 \subseteq_P Ȼ_2$ *and* $Ȼ_3 \subseteq_P Ȼ_4$ *then* $Ȼ_1 \cup_P Ȼ_3 \subseteq_P Ȼ_2 \cup_P Ȼ_4$ *and* $Ȼ_1 \cap_P Ȼ_3 \subseteq_P Ȼ_2 \cap_P Ȼ_4$.
6. *If* $Ȼ_1 \subseteq_R Ȼ_2$ *and* $Ȼ_2 \subseteq_R Ȼ_3$ *then* $Ȼ_1 \subseteq_R Ȼ_3$.
7. *If* $Ȼ_1 \subseteq_R Ȼ_2$ *then* $Ȼ_2^c \subseteq_R Ȼ_1^c$.
8. *If* $Ȼ_1 \subseteq_R Ȼ_2$ *and* $Ȼ_1 \subseteq_R Ȼ_3$ *then* $Ȼ_1 \subseteq_R Ȼ_2 \cap_R Ȼ_3$.
9. *If* $Ȼ_1 \subseteq_R Ȼ_2$ *and* $Ȼ_3 \subseteq_R Ȼ_2$ *then* $Ȼ_1 \cup_R Ȼ_3 \subseteq_R Ȼ_2$.
10. *If* $Ȼ_1 \subseteq_R Ȼ_2$ *and* $Ȼ_3 \subseteq_R Ȼ_4$ *then* $Ȼ_1 \cup_R Ȼ_3 \subseteq_R Ȼ_2 \cup_R Ȼ_4$ *and* $Ȼ_1 \cap_R Ȼ_3 \subseteq_P Ȼ_2 \cap_R Ȼ_4$.

Proof. This is straightforward, so proof is neglected. □

Definition 15. *Let* $Ȼ_1 = \{([u_1^L, u_1^U], [v_1^L, v_1^U]), (u_1, v_1)\}$ *and* $Ȼ_2 = \{([u_2^L, u_2^U], [v_2^L, v_2^U]), (u_2, v_2)\}$ *be the two Cq-ROFS. Then basic operations of these Cq-ROFS are given by:*

1. $$Ȼ_1 \oplus Ȼ_2 = \left\{ \begin{array}{c} \left(\left[\left(u_1^{Lq} + u_2^{Lq} - u_1^{Lq}u_2^{Lq}\right)^{\frac{1}{q}}, \left(u_1^{Uq} + u_2^{Uq} - u_1^{Uq}u_2^{Uq}\right)^{\frac{1}{q}}\right], \left[v_1^{Lq}v_2^{Lq}, v_1^{Uq}v_2^{Uq}\right]\right) \\ , \left(\left(u_1^q + u_2^q - u_1^q u_2^q\right)^{1/q}, v_1 v_2\right) \end{array} \right\}.$$
2. $$Ȼ_1 \otimes Ȼ_2 = \left\{ \begin{array}{c} \left(\left[u_1^L u_2^L, u_1^U u_2^U\right], \left[\left(v_1^{Lq} + v_2^{Lq} - v_1^{Lq}v_2^{Lq}\right)^{1/q}, \left(v_1^{Uq} + v_2^{Uq} - v_1^{Uq}v_2^{Uq}\right)^{1/q}\right]\right) \\ , \left(u_1 u_2, \left(\left(v_1^q + v_2^q - v_1^q v_2^q\right)^{1/q}\right)\right) \end{array} \right\}.$$
3. $$ƘȻ_1 = \left\{ \begin{array}{c} \left(\left[\left(1 - \left(1 - u_1^{Lq}\right)^{Ƙ}\right)^{\frac{1}{q}}, \left(1 - \left(1 - u_1^{Uq}\right)^{Ƙ}\right)^{\frac{1}{q}}\right], \left[v_1^{LƘ}, v_1^{UƘ}\right]\right), \\ \left(\left(1 - \left(\left(1 - u_1^q\right)^{Ƙ}\right)\right)^{\frac{1}{q}}, v_1^{Ƙ}\right) \end{array} \right\}, Ƙ > 0;$$
4. $$Ȼ_1^{Ƙ} = \left\{ \begin{array}{c} \left(\left[u_1^{LƘ}, u_1^{UƘ}\right], \left[\left(1 - \left(1 - v_1^{Lq}\right)^{Ƙ}\right)^{\frac{1}{q}}, \left(1 - \left(1 - v_1^{Uq}\right)^{Ƙ}\right)^{\frac{1}{q}}\right]\right), \\ \left(u_1^{Ƙ}, \left(1 - \left(\left(1 - v_1^q\right)^{Ƙ}\right)\right)^{\frac{1}{q}}\right) \end{array} \right\}$$
5. $$Ȼ_1^c = \left\{\left(\left[v_1^L, v_1^U\right], \left[u_1^L, u_1^U\right]\right), (v_1, u_1)\right\}.$$

4. Cubic q-Rung Orthopair Fuzzy Heronian Mean (Cq-ROFHM) Operators

In this portion, we discuss the notions of *Cq-ROFHM* operators, its properties, its weighted form, cubic q-rung orthopair fuzzy partitioned Heronian mean (Cq-ROFPHM) operators, its properties and its weighted form, respectively.

Definition 16. *Let* $Ȼ_{Ƙ}(Ƙ = 1, 2, \ldots\ldots\ldots, n)$ *be a family of Cq-ROFS,* $a \geq 0$, $b \geq 0$ *and* $a + b \geq 0$, *if*

$$\mathrm{Cq-ROFHM}^{a,b}(Ȼ_1, Ȼ_2, \ldots\ldots\ldots.., Ȼ_n) = \left(\frac{2}{n(n+1)} \sum\nolimits_{Ƙ=1}^{n} \sum\nolimits_{t=i}^{n} Ȼ_{Ƙ}^{a} Ȼ_t^{b}\right)^{\frac{1}{a+b}} \tag{13}$$

Then Cq-ROFHM is called Cq-ROFHM operator.

Using Definition 15, the next theorem can be obtained.

Theorem 2. *Let* $a \geq 0$, $b \geq 0$ *and* $a + b \geq 0$, $\mathcal{C}_{\kappa} = \left(A_{\mathcal{C}_{\kappa}}, \lambda_{\mathcal{C}_{\kappa}}\right)(\kappa = 1, 2, \ldots\ldots, n)$ *be a family of Cq-ROFNs, then result obtained by applying Equation (13) is also Cq-ROFSs, and*

$$\mathrm{Cq-ROFHM}\left(\mathcal{C}_1,\ \mathcal{C}_2, \ldots\ldots\ldots \mathcal{C}_n\right) = \left(\begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \left(1 - \prod_{\kappa=1}^{n} \prod_{t=\kappa}^{n}\left(1 - \left(\left(u^{L}_{c_{\kappa}}\right)^{a}\left(u^{L}_{c_t}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}, \\ \left(1 - \prod_{\kappa=1}^{n} \prod_{t=\kappa}^{n}\left(1 - \left(\left(u^{U}_{c_{\kappa}}\right)^{a}\left(u^{U}_{c_t}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}} \end{array}\right], \\ \left[\begin{array}{c} \left(1 - \left(1 - \prod_{\kappa=1}^{n} \prod_{t=\kappa}^{n}\left(1 - \left(1 - \left(v^{L}_{c_{\kappa}}\right)^{q}\right)^{a}\left(1 - \left(v^{L}_{c_t}\right)^{q}\right)^{b}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}}, \\ \left(1 - \left(1 - \prod_{\kappa=1}^{n} \prod_{t=\kappa}^{n}\left(1 - \left(1 - \left(v^{U}_{c_{\kappa}}\right)^{q}\right)^{a}\left(1 - \left(v^{U}_{c_t}\right)^{q}\right)^{b}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}} \end{array}\right] \end{array}\right), \\ \left(\begin{array}{c} \left(1 - \prod_{\kappa=1}^{n} \prod_{t=\kappa}^{n}\left(1 - \left(\left(u_{c_{\kappa}}\right)^{a}\left(u_{c_t}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}, \\ \left(1 - \left(1 - \prod_{\kappa=1}^{n} \prod_{t=\kappa}^{n}\left(1 - \left(1 - \left(v_{c_{\kappa}}\right)^{q}\right)^{a}\left(1 - \left(v_{c_t}\right)^{q}\right)^{b}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}} \end{array}\right) \end{array}\right) \tag{14}$$

Proof. By using Definition 15, we can get

$$\mathcal{C}^{a}_{\kappa} = \left(\begin{array}{c} \left(\left[\left(u_{\mathcal{C}_{\kappa}}{}^{L}\right)^{a}, \left(u_{\mathcal{C}_{\kappa}}{}^{U}\right)^{a}\right], \left[\left(1 - \left(1 - \left(v_{\mathcal{C}_{\kappa}}{}^{L}\right)^{q}\right)^{a}\right)^{\frac{1}{q}}, \left(1 - \left(1 - \left(v_{\mathcal{C}_{\kappa}}{}^{U}\right)^{q}\right)^{a}\right)^{\frac{1}{q}}\right]\right), \\ \left(u_{\mathcal{C}_{\kappa}}{}^{a}, \left(1 - \left(1 - \left(v_{\mathcal{C}_{\kappa}}\right)^{q}\right)^{a}\right)^{\frac{1}{q}}\right) \end{array}\right)$$

$$\mathcal{C}^{b}_{t} = \left(\begin{array}{c} \left(\left[\left(u_{\mathcal{C}_t}{}^{L}\right)^{b}, \left(u_{\mathcal{C}_t}{}^{U}\right)^{b}\right], \left[\left(1 - \left(1 - \left(v_{\mathcal{C}_t}{}^{L}\right)^{q}\right)^{b}\right)^{\frac{1}{q}}, \left(1 - \left(1 - \left(v_{\mathcal{C}_t}{}^{U}\right)^{q}\right)^{b}\right)^{\frac{1}{q}}\right]\right), \\ \left(u_{\mathcal{C}_t}{}^{b}, \left(1 - \left(1 - \left(v_{\mathcal{C}_t}\right)^{q}\right)^{b}\right)^{\frac{1}{q}}\right) \end{array}\right)$$

Then

$$\mathcal{C}^{a}_{\kappa}\mathcal{C}^{b}_{t} = \left(\begin{array}{c} \left(\left[\left(u_{\mathcal{C}_{\kappa}}{}^{L}\right)^{a}\left(u_{\mathcal{C}_t}{}^{L}\right)^{b}, \left(u_{\mathcal{C}_{\kappa}}{}^{U}\right)^{a}\left(u_{\mathcal{C}_t}{}^{U}\right)^{b}\right], \left[\begin{array}{c} \left(\left(1 - \left(1 - \left(v_{\mathcal{C}_t}{}^{L}\right)^{q}\right)^{a}\left(1 - \left(v_{\mathcal{C}_t}{}^{L}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}}, \\ \left(\left(1 - \left(1 - \left(v_{\mathcal{C}_t}{}^{U}\right)^{q}\right)^{a}\left(1 - \left(v_{\mathcal{C}_t}{}^{U}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}} \end{array}\right]\right), \\ \left(u_{\mathcal{C}_{\kappa}}{}^{a}u_{\mathcal{C}_t}{}^{b}, \left(\left(1 - \left(1 - \left(v_{\mathcal{C}_{\kappa}}\right)^{q}\right)^{a}\left(\left(1 - \left(v_{\mathcal{C}_t}\right)^{q}\right)^{b}\right)\right)\right)^{\frac{1}{q}}\right) \end{array}\right)$$

Then we can get

$$\sum\nolimits_{t=\kappa}^{n} \mathcal{C}^{a}_{\kappa}\mathcal{C}^{b}_{t} = \left(\begin{array}{c} \left(\left[\begin{array}{c} \left(1 - \prod_{t=\kappa}^{n}\left(1 - \left(\left(u_{\mathcal{C}_{\kappa}}{}^{L}\right)^{a}\left(u_{\mathcal{C}_t}{}^{L}\right)^{b}\right)^{q}\right)\right)^{\frac{1}{q}}, \\ \left(1 - \prod_{t=\kappa}^{n}\left(1 - \left(\left(u_{\mathcal{C}_{\kappa}}{}^{U}\right)^{a}\left(u_{\mathcal{C}_t}{}^{U}\right)^{b}\right)^{q}\right)\right)^{\frac{1}{q}} \end{array}\right], \left[\begin{array}{c} \prod_{t=\kappa}^{n}\left(\left(1 - \left(1 - \left(v_{\mathcal{C}_{\kappa}}{}^{L}\right)^{q}\right)^{a}\left(1 - \left(v_{\mathcal{C}_t}{}^{L}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}}, \\ \prod_{t=\kappa}^{n}\left(\left(1 - \left(1 - \left(v_{\mathcal{C}_{\kappa}}{}^{U}\right)^{q}\right)^{a}\left(1 - \left(v_{\mathcal{C}_t}{}^{U}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}} \end{array}\right]\right), \\ \left(\begin{array}{c} \left(1 - \prod_{t=\kappa}^{n}\left(1 - \left(u_{\mathcal{C}_{\kappa}}{}^{a}u_{\mathcal{C}_t}{}^{b}\right)^{q}\right)\right)^{\frac{1}{q}}, \\ \prod_{t=\kappa}^{n}\left(\left(1 - \left(1 - \left(v_{\mathcal{C}_{\kappa}}\right)^{q}\right)^{a}\left(1 - \left(v_{\mathcal{C}_t}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}} \end{array}\right) \end{array}\right)$$

And

$$\sum_{\varkappa=1}^{n}\sum_{t=\varkappa}^{n}\mathfrak{C}_{\varkappa}^{a}\mathfrak{C}_{t}^{b}=\begin{pmatrix}\left(\begin{bmatrix}\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(\left(u_{\mathfrak{C}_{\varkappa}}{}^{L}\right)^{a}\left(u_{\mathfrak{C}_{t}}{}^{L}\right)^{b}\right)^{q}\right)\right)^{\frac{1}{q}},\\ \left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(\left(u_{\mathfrak{C}_{\varkappa}}{}^{U}\right)^{a}\left(u_{\mathfrak{C}_{t}}{}^{U}\right)^{b}\right)^{q}\right)\right)^{\frac{1}{q}}\end{bmatrix},\begin{bmatrix}\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(\left(1-\left(1-\left(v_{\mathfrak{C}_{\varkappa}}{}^{L}\right)^{q}\right)^{a}\left(1-\left(v_{\mathfrak{C}_{t}}{}^{L}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}},\\ \prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(\left(1-\left(1-\left(v_{\mathfrak{C}_{\varkappa}}{}^{U}\right)^{q}\right)^{a}\left(1-\left(v_{\mathfrak{C}_{t}}{}^{U}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}}\end{bmatrix}\right)\\ \begin{pmatrix}\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(u_{\mathfrak{C}_{\varkappa}}{}^{a}u_{\mathfrak{C}_{t}}{}^{b}\right)^{q}\right)\right)^{\frac{1}{q}},\\ \prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(\left(1-\left(1-\left(v_{\mathfrak{C}_{\varkappa}}\right)^{q}\right)^{a}\left(1-\left(v_{\mathfrak{C}_{t}}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{q}}\end{pmatrix}\end{pmatrix}$$

Further results can be derived as follows

$$\frac{2}{n(n+1)}\sum_{\varkappa=1}^{n}\sum_{t=\varkappa}^{n}\mathfrak{C}_{\varkappa}^{a}\mathfrak{C}_{t}^{b}=\begin{pmatrix}\left(\begin{matrix}\begin{bmatrix}\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(\left(u_{\mathfrak{C}_{\varkappa}}{}^{L}\right)^{a}\left(u_{\mathfrak{C}_{t}}{}^{L}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}},\\ \left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(\left(u_{\mathfrak{C}_{\varkappa}}{}^{U}\right)^{a}\left(u_{\mathfrak{C}_{t}}{}^{U}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\end{bmatrix}\\ \begin{bmatrix}\left(\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(\left(1-\left(1-\left(v_{\mathfrak{C}_{\varkappa}}{}^{L}\right)^{q}\right)^{a}\left(1-\left(v_{\mathfrak{C}_{t}}{}^{L}\right)^{q}\right)^{b}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}},\\ \left(\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(\left(1-\left(1-\left(v_{\mathfrak{C}_{\varkappa}}{}^{U}\right)^{q}\right)^{a}\left(1-\left(v_{\mathfrak{C}_{t}}{}^{U}\right)^{q}\right)^{b}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\end{bmatrix}\end{matrix}\right),\\ \begin{pmatrix}\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(u_{\mathfrak{C}_{\varkappa}}{}^{a}u_{\mathfrak{C}_{t}}{}^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}},\\ \left(\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(\left(1-\left(1-\left(v_{\mathfrak{C}_{\varkappa}}\right)^{q}\right)^{a}\left(1-\left(v_{\mathfrak{C}_{t}}\right)^{q}\right)^{b}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\end{pmatrix}\end{pmatrix}$$

And

$$\left(\frac{2}{n(n+1)}\sum_{\varkappa=1}^{n}\sum_{t=\varkappa}^{n}\mathfrak{C}_{\varkappa}^{a}\mathfrak{C}_{t}^{b}\right)^{\frac{1}{a+b}}=\begin{pmatrix}\left(\begin{matrix}\begin{bmatrix}\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(\left(u_{c_{\varkappa}}^{L}\right)^{a}\left(u_{c_{t}}^{L}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}},\\ \left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(\left(u_{c_{\varkappa}}^{U}\right)^{a}\left(u_{c_{t}}^{U}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}\end{bmatrix},\\ \begin{bmatrix}\left(1-\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(1-\left(v_{c_{\varkappa}}^{L}\right)^{q}\right)^{a}\left(1-\left(v_{c_{t}}^{L}\right)^{q}\right)^{b}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}},\\ \left(1-\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(1-\left(v_{c_{\varkappa}}^{U}\right)^{q}\right)^{a}\left(1-\left(v_{c_{t}}^{U}\right)^{q}\right)^{b}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}}\end{bmatrix}\end{matrix}\right),\\ \begin{pmatrix}\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(\left(u_{c_{\varkappa}}\right)^{a}\left(u_{c_{t}}\right)^{b}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}},\\ \left(1-\left(1-\prod_{\varkappa=1}^{n}\prod_{t=\varkappa}^{n}\left(1-\left(1-\left(v_{c_{\varkappa}}\right)^{q}\right)^{a}\left(1-\left(v_{c_{t}}\right)^{q}\right)^{b}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}}\end{pmatrix}\end{pmatrix}$$

Thus the proof is completed. □

In addition, the characteristics of Cq-ROFHM operator are as follows.

Theorem 3 (Impotency). *Assume* $\mathfrak{C}_{\mathfrak{k}} = \left(\mathrm{A}_{\mathfrak{C}_{\mathfrak{k}}}, \lambda_{\mathfrak{C}_{\mathfrak{k}}}\right)(\mathfrak{k} = 1, 2, \ldots\ldots, n)$ *be a family of Cq-ROFNs, if all* $\mathfrak{C}_{\mathfrak{k}}$ *are same, that is,* $\mathfrak{C}_{\mathfrak{k}} = \mathfrak{C} = \left(\mathrm{A}_{\mathfrak{k}}, \lambda_{\mathfrak{k}}\right)$ *for all k, then*

$$\mathrm{Cq-ROFHM}^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots\ldots\ldots, \mathfrak{C}_n) = \mathfrak{C}. \tag{15}$$

Proof. As, $\mathfrak{C}_{\mathfrak{k}} = \mathfrak{C}, \forall\ \mathfrak{k}$, we have

$$\mathrm{Cq-ROFHM}^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots\ldots\ldots, \mathfrak{C}_n) = \left(\frac{2}{n(n+1)} \sum\nolimits_{\mathfrak{k}=1}^{n} \sum\nolimits_{t=\mathfrak{k}}^{n} \mathfrak{C}_{\mathfrak{k}}^{a} \mathfrak{C}_{t}^{b}\right)^{\frac{1}{a+b}} = \left(\mathfrak{C}^{a+b}\right)^{\frac{1}{a+b}} = \mathfrak{C}$$

□

Theorem 4 (Monotonicity). *Assume* $\alpha_{\mathfrak{k}}, \beta_{\mathfrak{k}}\ (\mathfrak{k} = 1, 2, 3, \ldots, n)$ *be the two families of Cq-ROFNs, if* $\alpha_{\mathfrak{k}} \le \beta_{\mathfrak{k}}, \forall\ \mathfrak{k} = 1, 2, 3, \ldots, n$ *then*

$$\mathrm{Cq-ROFHM}^{a,b}(\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots, \alpha_n) \le \mathrm{Cq-ROFHM}^{a,b}(\beta_1, \beta_2, \ldots\ldots\ldots\ldots, \beta_n) \tag{16}$$

Proof. Since, $\alpha_{\mathfrak{k}} \le \beta_{\mathfrak{k}}$ and $\alpha_j \le \beta_j$ for $\mathfrak{k} = 1, 2, 3 \ldots, n$ and $t = i, i+1, \ldots, n$ we have

$$\alpha_k^a \alpha_t^b \le \beta_k^a \beta_t^b$$

Then

$$\frac{2}{n(n+1)} \sum\nolimits_{\mathfrak{k}=1}^{n} \sum\nolimits_{t=\mathfrak{k}}^{n} \alpha_{\mathfrak{k}}^{a} \alpha_{t}^{b} \le \frac{2}{n(n+1)} \sum\nolimits_{\mathfrak{k}=1}^{n} \sum\nolimits_{t=\mathfrak{k}}^{n} \beta_{\mathfrak{k}}^{a} \beta_{t}^{b}$$

So,

$$\left(\frac{2}{n(n+1)} \sum\nolimits_{\mathfrak{k}=1}^{n} \sum\nolimits_{t=\mathfrak{k}}^{n} \alpha_{\mathfrak{k}}^{a} \alpha_{t}^{b}\right)^{\frac{1}{a+b}} \le \left(\frac{2}{n(n+1)} \sum\nolimits_{\mathfrak{k}=1}^{n} \sum\nolimits_{t=\mathfrak{k}}^{n} \beta_{\mathfrak{k}}^{a} \beta_{t}^{b}\right)^{\frac{1}{a+b}}$$

And,

$$\mathrm{Cq-ROFHM}^{a,b}(\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots, \alpha_n) \le \mathrm{Cq-ROFHM}^{a,b}(\beta_1, \beta_2, \ldots\ldots\ldots\ldots, \beta_n)$$

□

Theorem 5 (Boundedness). *The Cq-ROFHM operator lies between the max and min operators.*

$$\min(\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \ldots\ldots\ldots, \mathfrak{C}_n) \le \mathrm{Cq-ROFHM}^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots\ldots\ldots, \mathfrak{C}_n) \le \max(\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \ldots\ldots\ldots, \mathfrak{C}_n). \tag{17}$$

Proof. Let $c = \min(\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \ldots\ldots\ldots, \mathfrak{C}_n)$, $d = \max(\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \ldots\ldots\ldots, \mathfrak{C}_n)$.

According to Theorem 4, we have

$$\mathrm{Cq-ROFHM}^{a,b}(c, c, \ldots\ldots\ldots\ldots, c) \le \mathrm{Cq-ROFHM}^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots\ldots\ldots, \mathfrak{C}_n) \le \mathrm{Cq-ROFHM}^{a,b}(d, d, \ldots\ldots\ldots\ldots, d)$$

Further,

$$\mathrm{Cq-ROFHM}^{a,b}(c, c, \ldots\ldots\ldots\ldots, c) = c \text{ and } \mathrm{Cq-ROFHM}^{a,b}(d, d, \ldots\ldots\ldots\ldots, d) = d.$$

So,

$$c \le \mathrm{Cq-ROFHM}^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots\ldots\ldots, \mathfrak{C}_n) \le d$$

And,

$$\min(\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \ldots\ldots.., \mathfrak{C}_n) \leq \mathrm{Cq} - \mathrm{ROFHM}^{a,b}(\mathfrak{C}_1,\ \mathfrak{C}_2, \ldots\ldots\ldots.., \mathfrak{C}_n) \leq \max(\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \ldots\ldots.., \mathfrak{C}_n)$$

The objects 'a' and 'b' has very essential part in the accumulated results. Further, we analyze some particular cases of Cq-ROFHM operator using different values of the objects a, b. □

Case 1. If $a = b = \frac{1}{2}$ then proposed Cq-ROFHM renovate into Cq-ROF fundamental HM operator, it follows that

$$\mathrm{Cq} - \mathrm{ROFHM}^{\frac{1}{2},\frac{1}{2}}\left(\mathfrak{C}_1,\ \mathfrak{C}_2, \ldots\ldots\ldots.., \ \mathfrak{C}_n\right) =$$

$$\left(\begin{array}{c} \left(\left[\begin{array}{c} \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \sqrt{\left(u^{L}_{\mathfrak{C}_{\mathfrak{K}}} u_{\mathfrak{C}_t}{}^{L}\right)^{q}}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}, \\ \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \sqrt{\left(u^{U}_{\mathfrak{C}_{\mathfrak{K}}} u_{\mathfrak{C}_t}{}^{U}\right)^{q}}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}} \end{array}\right], \left[\begin{array}{c} \prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \sqrt{\left(\left(1 - \left(v^{L}_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\left(1 - \left(v^{L}_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\right)}\right)^{\frac{2}{n(n+1)} * \frac{1}{q}}, \\ \prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \sqrt{\left(\left(1 - \left(v^{U}_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\left(1 - \left(v^{U}_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\right)}\right)^{\frac{2}{n(n+1)} * \frac{1}{q}} \end{array}\right]\right) \\ \left(\left(\begin{array}{c} \left(1 - \prod_{\mathfrak{C}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \sqrt{\left(u_{\mathfrak{C}_{\mathfrak{K}}} u_{\mathfrak{C}_t}\right)^{q}}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}, \\ \left(\prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \sqrt{\left(\left(1 - \left(v_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\left(1 - \left(v_{\mathfrak{C}_t}\right)^{q}\right)\right)}\right)^{\frac{2}{n(n+1)} * \frac{1}{q}},\right) \end{array}\right)\right) \end{array}\right) \tag{18}$$

Case 2. If $a = b = 1$ Equation (14) renovate into

$$\mathrm{Cq} - \mathrm{ROFHM}^{1,1}(\mathfrak{C}_1,\ \mathfrak{C}_2, \ldots\ldots\ldots.., \ \mathfrak{C}_n) =$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{t=\mathfrak{K}}^{n}\left(1 - \left(u^{L}_{\mathfrak{C}_{\mathfrak{K}}} u^{L}_{[?]_t}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2q}}, \\ \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{t=\mathfrak{K}}^{n}\left(1 - \left(u^{U}_{\mathfrak{K}} u^{U}_{[?]_t}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2q}} \end{array}\right] \\ \left(1 - \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \left(1 - \left(v^{L}_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\left(1 - \left(v^{L}_{\mathfrak{C}_t}\right)^{q}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q}}, \\ \left(1 - \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \left(1 - \left(v^{U}_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\left(1 - \left(v^{U}_{\mathfrak{C}_t}\right)^{q}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q}} \end{array}\right), \\ \left(\begin{array}{c} \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{t=\mathfrak{K}}^{n}\left(1 - \left(u_{\mathfrak{C}_{\mathfrak{K}}} u_{\mathfrak{C}_t}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2q}}, \\ \left(1 - \left(1 - \prod_{\mathfrak{K}=1}^{n} \prod_{\mathfrak{K}=t}^{n}\left(1 - \left(1 - \left(v_{\mathfrak{C}_{\mathfrak{K}}}\right)^{q}\right)\left(1 - \left(v_{\mathfrak{C}_t}\right)^{q}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q}} \end{array}\right) \end{array}\right) \tag{19}$$

Which is a cubic q-rung orthopair fuzzy generalized interrelated square mean.

Case 3. If $a \to 0$, Equation (19) decrease to

$$\lim_{a \to 0} Cq-ROFHM^{a,\,b}(\mathfrak{C}_1,\ \mathfrak{C}_2,\ldots\ldots\ldots.,\ \mathfrak{C}_n) = \left(\frac{1}{n}\sum_{\mathfrak{h}=1}^{n}\mathfrak{C}_{\mathfrak{h}}^{a}\right)^{\frac{1}{a}} = \left(\begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \left(1-\prod_{\mathfrak{h}=1}^{n}\left(\left(1-\left(\left(u_{\mathfrak{C}_{\mathfrak{h}}}^{L}\right)^{b}\right)^{q}\right)^{\frac{1}{n}}\right)\right)^{\frac{1}{(a+b)}}, \\ \left(1-\prod_{\mathfrak{h}=1}^{n}\left(\left(1-\left(\left(u_{\mathfrak{C}_{\mathfrak{h}}}^{U}\right)^{b}\right)^{q}\right)^{\frac{1}{n}}\right)\right)^{\frac{1}{(a+b)}} \end{array}\right], \\ \left[\begin{array}{c} \left(1-\left(1-\prod_{\mathfrak{h}=1}^{n}\left(1-\left(1-\left(\left(v_{\mathfrak{C}_{\mathfrak{h}}}^{L}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{b}}\right)^{\frac{1}{q}}, \\ \left(1-\left(1-\prod_{\mathfrak{h}=1}^{n}\left(1-\left(1-\left(\left(v_{\mathfrak{C}_{\mathfrak{h}}}^{U}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{b}}\right)^{\frac{1}{q}} \end{array}\right] \end{array}\right), \\ \left(\begin{array}{c} \left(1-\prod_{\mathfrak{h}=1}^{n}\left(\left(1-\left(\left(u_{\mathfrak{C}_{\mathfrak{h}}}\right)^{b}\right)^{q}\right)^{\frac{1}{n}}\right)\right)^{\frac{1}{(a+b)}}, \\ \left(1-\left(1-\prod_{\mathfrak{h}=1}^{n}\left(1-\left(1-\left(\left(v_{\mathfrak{C}_{\mathfrak{h}}}\right)^{q}\right)^{b}\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{b}}\right)^{\frac{1}{q}} \end{array}\right) \end{array}\right) \tag{20}$$

Which is a cubic q-rung orthopair fuzzy generalized mean.

Case 4. If $a = 1,\ b \to 0$, Equation (14) reduce to a cubic q-rung orthopair fuzzy average mean given by

$$\lim_{b \to 0} Cq-ROFHM^{1,\,b}(\mathfrak{C}_1,\ \mathfrak{C}_2,\ldots\ldots\ldots.,\ \mathfrak{C}_n) = \left(\frac{1}{n}\oplus_{\mathfrak{h}=1}^{n}\mathfrak{C}_{\mathfrak{h}}\right) = \left(\begin{array}{c} \left(\begin{array}{c} \left[\left(1-\otimes_{\mathfrak{h}=1}^{n}\left(1-\left(u_{\mathfrak{C}_{\mathfrak{h}}}^{L}\right)^{q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}, \left(1-\otimes_{\mathfrak{h}=1}^{n}\left(1-\left(u_{\mathfrak{C}_{\mathfrak{h}}}^{U}\right)^{q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}\right], \\ \left[\begin{array}{c} \left(1-\left(1-\otimes_{\mathfrak{h}=1}^{n}\left(1-\left(1-\left(v_{\mathfrak{C}_{\mathfrak{h}}}^{L}\right)^{q}\right)\right)^{\frac{1}{n}}\right)\right)^{\frac{1}{q}}, \\ \left(1-\left(1-\otimes_{\mathfrak{h}=1}^{n}\left(1-\left(1-\left(v_{\mathfrak{C}_{\mathfrak{h}}}^{U}\right)^{q}\right)\right)^{\frac{1}{n}}\right)\right)^{\frac{1}{q}} \end{array}\right] \end{array}\right), \\ \left(\left(\left(1-\otimes_{\mathfrak{h}=1}^{n}\left(1-\left(u_{\mathfrak{C}_{\mathfrak{h}}}\right)^{q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}\right), \left(1-\left(1-\otimes_{\mathfrak{h}=1}^{n}\left(1-\left(1-\left(v_{\mathfrak{C}_{\mathfrak{h}}}\right)^{q}\right)\right)^{\frac{1}{n}}\right)\right)^{\frac{1}{q}}\right) \end{array}\right) \tag{21}$$

Case 5. If $a \to 0,\ b = 0$, then the established Cq-ROFHM renovate into

$$\lim_{a \to 0} Cq-ROFHM^{a,\,0}(\mathfrak{C}_1,\ \mathfrak{C}_2,\ldots\ldots\ldots.,\ \mathfrak{C}_n) = \lim_{a \to 0}\left(\frac{1}{n}\oplus_{\mathfrak{h}=1}^{n}\mathfrak{C}_{\mathfrak{h}}^{a}\right)^{\frac{1}{b}} = \otimes_{\mathfrak{h}=1}^{n}\left(\mathfrak{C}_{\mathfrak{h}}\right)^{\frac{1}{n}} \tag{22}$$

Note that if we assign different values to parameter 'q' we attain different kinds of orthopair fuzzy sets. For example, if a = 1, Cq-ROFS transform into CIFS. If a = 2 then Cq-ROFS transform into CPFS. Therefore using different special cases of Cq-ROFHM operator, if q = 1 0r 2, some other different particular cases of Cq-ROFHM operator can be obtained. For example, if a = 1, Equation (18) further disintegrates into a cubic intuitionistic fuzzy Heronian mean operator, Equation (19) further degenerates into a cubic intuitionistic fuzzy interrelated square Heronian mean, Equation (20) further disintegrates into a cubic intuitionistic fuzzy generalized mean, Equation (21) further reduces to a cubic intuitionistic fuzzy average operator and Equation (22) further reduces to a cubic intuitionistic fuzzy geometric mean.

Normally, in most realistic MADM problems, different attributes have different level of significance. However, the above established Cq-ROFHM operators pay no attention to this feature. Further the weighted form of Cq-ROFHM operator is given by:

Definition 17. *Let* $\mathcal{C}_{\eth} = \left(A_{\mathcal{C}_{\eth}}, \lambda_{\mathcal{C}_{\eth}}\right) (\eth = 1, 2, \ldots\ldots, n)$ *be a family of Cq-ROFNs,* $a \geq 0,\ b \geq 0, a + b \geq 0$, *and* $w = (w_1, w_2, \ldots.., w_n)$ *represent the weight vector of Cq-ROFNs and* $\sum_{\eth=1}^{n} w_{\eth} = 1$*. If*

$$\mathrm{Cq-ROFWHM}_w^{a,b}(\mathcal{C}_1, \mathcal{C}_2, \ldots\ldots, \mathcal{C}_n) = \left(\frac{2}{n(n+1)} \sum_{\eth=1}^{n} \sum_{t=\eth}^{n} \left(w_{\eth}\mathcal{C}_{\eth}\right)^a \left(w_t \mathcal{C}_t\right)^b\right)^{\frac{1}{a+b}} \tag{23}$$

Then, the above Equation (23) represents the Cq-ROFWHM operator.

Theorem 6. *Let* $\mathcal{C}_{\eth} = \left(A_{\mathcal{C}_{\eth}}, \lambda_{\mathcal{C}_{\eth}}\right) (\eth = 1, 2, \ldots\ldots, n)$ *be a set of Cq-ROFNs,* $a \geq 0,\ b \geq 0, a + b \geq 0$*, and* $w = (w_1, w_2, \ldots.., w_n)$ *is the weight vector of Cq-ROFNs,* $w_{\eth} \in [0, 1]$ *and* $\sum_{\eth=1}^{n} w_{\eth} = 1$*, then the resultant equation by using Equation (23) is also Cq-ROFNs as follows:*

$$\mathrm{Cq-ROFWHM}_w^{a,b}(\mathcal{C}_1, \mathcal{C}_2, \ldots\ldots, \mathcal{C}_n) =$$
$$\left(\begin{array}{c}\left(\left[\begin{array}{c}\left(1 - \prod_{\eth=1}^{n} \prod_{t=\eth}^{n} \left(1 - \left(u_{\eth}^{L}\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}, \\ \left(1 - \prod_{\eth=1}^{n} \prod_{t=\eth}^{n} \left(1 - \left(u_{\eth}^{U}\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}\end{array}\right], \left[\begin{array}{c}\left(1 - \left(1 - \prod_{\eth=1}^{n} \prod_{t=\eth}^{n} \left(v_{\eth}^{L}\right)^{\frac{2q}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}}, \\ \left(1 - \left(1 - \prod_{\eth=1}^{n} \prod_{t=\eth}^{n} \left(v_{\eth}^{U}\right)^{\frac{2q}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}}\end{array}\right]\right), \\ \left(\left(\left(1 - \prod_{\eth=1}^{n} \prod_{t=\eth}^{n} \left(1 - \left(u_{\eth}\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}\right), \left(1 - \left(1 - \prod_{\eth=1}^{n} \prod_{t=\eth}^{n} \left(v_{\eth}\right)^{\frac{2q}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}}\right)\end{array}\right) \tag{24}$$

where

$$\left[u_{\eth}^{L}, u_{\eth}^{U}\right] = \left[\begin{array}{c}\left(1 - \left(1 - \left(u_{\eth}^{L}\right)^q\right)^{w_{\eth}}\right)^{\frac{a}{q}} \left(1 - \left(1 - \left(u_t^{L}\right)^q\right)^{w_t}\right)^{\frac{b}{q}}, \\ \left(1 - \left(1 - \left(u_{\eth}^{U}\right)^q\right)^{w_{\eth}}\right)^{\frac{a}{q}} \left(1 - \left(1 - \left(u_t^{U}\right)^q\right)^{w_t}\right)^{\frac{b}{q}}\end{array}\right], \left[v_{\eth}^{L}, v_{\eth}^{U}\right] =$$
$$\left[\begin{array}{c}\left(1 - \left(1 - \left(v_{\eth}^{L}\right)^{w_{\eth} q}\right)^a \left(1 - \left(v_t^{L}\right)^{w_t q}\right)^b\right)^{\frac{1}{q}}, \\ \left(1 - \left(1 - \left(v_{\eth}^{U}\right)^{w_{\eth} q}\right)^a \left(1 - \left(v_t^{U}\right)^{w_t q}\right)^b\right)^{\frac{1}{q}}\end{array}\right],$$

$$u_{\eth} = \left(1 - \left(1 - \left(u_{\eth}\right)^q\right)^{w_{\eth}}\right)^{\frac{a}{q}} \left(1 - \left(1 - \left(u_t\right)^q\right)^{w_t}\right)^{\frac{b}{q}},\ v_{\eth} = \left(1 - \left(1 - \left(v_{\eth}\right)^{w_{\eth} q}\right)^a \left(1 - \left(v_t\right)^{w_t q}\right)^b\right)^{\frac{1}{q}}$$

The HM operator has a well-known property that it can establish the relationship structure between any two attributes. In HM operators, each attribute is presumed to be related with other attributes. But for most realistic DM problems this supposition cannot always hold true. The real DM problems are often that different partitions are used for the separation of attributes, by keeping in mind the association structure between attributes. The attributes which are separated in two different partitions have no connection between them. Each attribute of the same partition has a connection with each other. Clearly, this type of situation cannot be handled by the usual HM operator. Now, we offer the cubic q-rung orthopair fuzzy partitioned Heronian mean (Cq-ROFPHM) operator to explain this situation.

The overhead mention executive situation can be deliberate mathematically by: Let $\text{Ꞓ}_{\text{ƙ}} = (\text{ƙ} = 1, 2, 3, \ldots, n)$ be a set of Cq-ROFNs, separated into "g" different partitions $F_1, F_2, \ldots\ldots\ldots, F_g$ with $F_i \cap F_j = \varnothing$ and $\cup_{i=1}^{g} F_i = \{\text{Ꞓ}_i\} F_i = \left\{\text{Ꞓ}_{i1}, \text{Ꞓ}_{i2}, \ldots . \text{Ꞓ}_{i|F_i|}\right\}$ where $|F_i|$ indicate the cardinality of partitions F_i and $\sum_{i=1}^{g} |F_i| = n$. Based on the above suppositions, the Cq-ROFPHM operator is defined as follows.

Definition 18. *Let* $\text{Ꞓ}_{\text{ƙ}} = \left(A_{\text{Ꞓ}_{\text{ƙ}}}, \lambda_{\text{Ꞓ}_{\text{ƙ}}}\right)(\text{ƙ} = 1, 2, \ldots\ldots, n)$ *be a family of Cq-ROFNs,* $a \geq 0$, $b \geq 0$, $a + b \geq 0$. *The Cq-ROFPHM operator is a function* $Cq - ROFPHM : [0,\ 1]^n \to [0,\ 1]$ *such that*

$$Cq - ROFPHM^{a,\,b}(\text{Ꞓ}_1, \text{Ꞓ}_2, \ldots\ldots, \text{Ꞓ}_n) = \frac{1}{g}\left(\sum_{i=1}^{g}\left(\frac{2}{|F_i|(|F_i|+1)}\sum_{\text{ƙ}=1}^{|F_i|}\sum_{t=\text{ƙ}}^{|F_i|}\left(\text{Ꞓ}_{i\text{ƙ}}\right)^a \otimes \left(\text{Ꞓ}_{it}\right)^b\right)^{\frac{1}{a+b}}\right) \tag{25}$$

Theorem 7. *Let* $\text{Ꞓ}_{\text{ƙ}} = \left(A_{\text{Ꞓ}_{\text{ƙ}}}, \lambda_{\text{Ꞓ}_{\text{Ꞓ}}}\right)(\text{ƙ} = 1, 2, \ldots\ldots, n)$ *be a collection of Cq-ROFNs,* $a \geq 0, b \geq 0$ *and* $a + b \geq 0$, *then the resultant equation by using Equation (25) is also a Cq-ROFN given by*

$$\begin{gathered} Cq - ROFPHM^{a,\,b}(\text{Ꞓ}_1, \text{Ꞓ}_2, \ldots\ldots, \text{Ꞓ}_n) = \\ \left(\begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \left(1 - \prod_{i=1}^{g}\left(1 - \left(\left(1 - \left(1 - \left(u_i^L\right)^q\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}}, \\ \left(1 - \prod_{i=1}^{g}\left(1 - \left(\left(1 - \left(1 - \left(u_i^U\right)^q\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}} \end{array}\right], \\ \left[\begin{array}{c} \prod_{i=1}^{g}\left(1 - \left(1 - \left(v_i^L\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}}, \\ \prod_{i=1}^{g}\left(1 - \left(1 - \left(v_i^U\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}} \end{array}\right] \end{array}\right), \\ \left(\begin{array}{c} \left(1 - \prod_{i=1}^{g}\left(1 - \left(\left(1 - \left(1 - (u_i)^q\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}}, \\ \prod_{i=1}^{g}\left(1 - \left(1 - (v_i)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}} \end{array}\right) \end{array}\right) \end{gathered} \tag{26}$$

where

$$u_i^L = \left(1 - \prod_{\text{ƙ}=1}^{|F_i|}\prod_{t=\text{ƙ}}^{|F_i|}\left(1 - \left(A_{\text{Ꞓ}_{i\text{ƙ}}}{}^a A_{\text{Ꞓ}_{it}}{}^b\right)^q\right)\right)^{\frac{1}{q}}, u_i^U = \left(1 - \prod_{\text{ƙ}=1}^{|F_i|}\prod_{t=\text{ƙ}}^{|F_i|}\left(1 - \left(A_{\text{Ꞓ}_{i\text{ƙ}}}{}^a A_{\text{Ꞓ}_{it}}{}^b\right)^q\right)\right)^{\frac{1}{q}}$$

$$v_i^L = \prod_{\text{ƙ}=1}^{|F_i|}\prod_{t=\text{ƙ}}^{|F_i|}\left(1 - \left(1 - \lambda^q_{\text{Ꞓ}_{i\text{ƙ}}}\right)^a\left(1 - \lambda^q_{\text{Ꞓ}_{it}}\right)^b\right)^{\frac{1}{q}}, v_i^U = \prod_{\text{ƙ}=1}^{|F_i|}\prod_{t=\text{ƙ}}^{|F_i|}\left(1 - \left(1 - \lambda^q_{\text{Ꞓ}_{i\text{ƙ}}}\right)^a\left(1 - \lambda^q_{\text{Ꞓ}_{it}}\right)^b\right)^{\frac{1}{q}}$$

$$u_i = \left(1-\prod_{Ꞩ=1}^{|F_i|}\prod_{t=Ꞩ}^{|F_i|}\left(1-\left(A_{\complement_{iꞨ}}{}^{a}A_{\complement_{it}}{}^{b}\right)^{q}\right)\right)^{\frac{1}{q}}, v_i = \prod_{Ꞩ=1}^{|F_i|}\prod_{t=Ꞩ}^{|F_i|}\left(1-\left(1-\lambda_{\complement_{iꞨ}}^{q}\right)^{a}\left(1-\lambda_{\complement_{it}}^{q}\right)^{b}\right)^{\frac{1}{q}}$$

Theorem 8. *Let* $a \geq 0,\ b \geq 0,\ a+b \geq 0, \complement_Ꞩ = \left(A_{\complement_Ꞩ}, \lambda_{\complement_Ꞩ}\right)(Ꞩ = 1,2,\ldots\ldots,n)$ *be a collection of Cq-ROFNs having "g" distinct subsets* $F_Ꞩ(Ꞩ = 1,2,\ldots\ldots,n)$. *Then Cq-ROFPHM operator has the following characteristics.*

1. **Idempotency:** If all $\complement_Ꞩ$ are same that is, $\complement_Ꞩ = \complement = (A_\complement, \lambda_\complement)\forall\ Ꞩ$, then

$$Cq-ROFPHM^{a,b}(\complement_1, \complement_2, \ldots\ldots, \complement_n) = \complement = (A_\complement, \lambda_\complement) \tag{27}$$

Proof.

$$Cq-ROFPHM^{a,b}(\complement_1, \complement_2, \ldots\ldots, \complement_n) = \frac{1}{g}\left(\sum_{i=1}^{g}\left(\frac{2}{|F_i|(|F_i|+1)}\sum_{Ꞩ=1}^{|F_i|}\sum_{t=Ꞩ}^{|F_i|}\left(\complement_{iꞨ}\right)^{a}\otimes(\complement_{it})^{b}\right)^{\frac{1}{a+b}}\right) =$$
$$\frac{1}{g}\left(\sum_{i=1}^{g}\left(\frac{2}{|F_i|(|F_i|+1)}\sum_{Ꞩ=1}^{|F_i|}\sum_{t=Ꞩ}^{|F_i|}(\complement)^{a}\otimes(\complement)^{b}\right)^{\frac{1}{a+b}}\right) = \frac{1}{g}\left(\sum_{i=1}^{g}\left(\frac{2}{|F_i|(|F_i|+1)}\frac{|F_i|(|F_i|+1)}{2}\complement^{a+b}\right)^{\frac{1}{a+b}}\right) = \frac{1}{g}\left(\sum_{Ꞩ=1}^{g}\complement\right) = \complement$$

2. **Monotonicity:** Let $B_Ꞩ = \left(A_{B_Ꞩ}, \lambda_{B_Ꞩ}\right)(Ꞩ = 1,2,\ldots\ldots,n)$ be a set of Cq-ROFNs having same partition structure as $\complement_Ꞩ = \left(A_{\complement_Ꞩ}, \lambda_{\complement_Ꞩ}\right)(Ꞩ = 1,2,\ldots\ldots,n)$, $A_{B_Ꞩ} \geq A_{\complement_Ꞩ}$ and $\lambda_{B_Ꞩ} \leq \lambda_{\complement_Ꞩ}$ for all k, then

$$(B_1, B_2, \ldots\ldots, B_n) \geq Cq-ROFPHM^{a,b}(\complement_1, \complement_2, \ldots\ldots, \complement_n) \tag{28}$$

□

Proof. Since $A_{B_Ꞩ} \geq A_{\complement_Ꞩ}$ and $\lambda_{B_Ꞩ} \leq \lambda_{\complement_Ꞩ}$ for all k, using Definition 6, we can obtain, $B_Ꞩ \geq \complement_Ꞩ$ for all k, then $A_{B_{iꞨ}}^{a}A_{B_{it}}^{b} \geq A_{\complement_{iꞨ}}^{a}A_{it}^{b}$ and

$$1-\left(1-\lambda_{B_{iꞨ}}^{q}\right)^{a}\left(1-\lambda_{B_{it}}^{q}\right)^{b} \geq 1-\left(1-\lambda_{\complement_{iꞨ}}^{q}\right)^{a}\left(1-\lambda_{\complement_{it}}^{q}\right)^{b}$$

Further

$$u_{B_i} = \left(1-\prod_{Ꞩ=1}^{|F_i|}\prod_{t=Ꞩ}^{|F_i|}\left(1-\left(A_{B_{iꞨ}}^{a}A_{B_{it}}^{b}\right)^{q}\right)\right)^{\frac{1}{q}} \geq \left(1-\prod_{Ꞩ=1}^{|F_i|}\prod_{t=Ꞩ}^{|F_i|}\left(1-\left(A_{\complement_{iꞨ}}^{a}A_{\complement_{it}}^{b}\right)^{q}\right)\right)^{\frac{1}{q}} = u_{\complement_i}$$

And

$$v_{B_i} = \prod_{Ꞩ=1}^{|F_i|}\prod_{t=Ꞩ}^{|F_i|}\left(1-\left(1-\lambda_{B_{iꞨ}}^{q}\right)^{a}\left(1-\lambda_{B_{it}}^{q}\right)^{b}\right)^{\frac{1}{q}} \geq \prod_{Ꞩ=1}^{|F_i|}\prod_{t=Ꞩ}^{|F_i|}\left(1-\left(1-\lambda_{\complement_{iꞨ}}^{q}\right)^{a}\left(1-\lambda_{\complement_{it}}^{q}\right)^{b}\right)^{\frac{1}{q}} = v_{\complement_i}.$$

Thus,

$$\left(\left[\begin{array}{l}\left(1-\prod_{i=1}^{g}\left(1-\left(\left(1-\left(1-\left(u_{B_i}^{L}\right)^{q}\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}},\\ \left(1-\prod_{i=1}^{g}\left(1-\left(\left(1-\left(1-\left(u_{B_i}^{U}\right)^{q}\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}}\end{array}\right]\right) \geq \left(\left[\begin{array}{l}\left(1-\prod_{i=1}^{g}\left(1-\left(\left(1-\left(1-\left(u_{\mathfrak{C}_i}^{L}\right)^{q}\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}},\\ \left(1-\prod_{i=1}^{g}\left(1-\left(\left(1-\left(1-\left(u_{\mathfrak{C}_i}^{U}\right)^{q}\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}},\end{array}\right]\right)$$

And

$$\left(\left[\begin{array}{l}\prod_{i=1}^{g}\left(1-\left(1-\left(v_{B_i}^{L}\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}},\\ \prod_{i=1}^{g}\left(1-\left(1-\left(v_{B_i}^{U}\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}}\end{array}\right]\right) \leq \left(\left[\begin{array}{l}\prod_{i=1}^{g}\left(1-\left(1-\left(v_{\mathfrak{C}_i}^{L}\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}},\\ \prod_{i=1}^{g}\left(1-\left(1-\left(v_{\mathfrak{C}_i}^{U}\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}}\end{array}\right]\right)$$

Finally using Equation (18), we can obtain

$$Cq-ROFPHM^{a,b}(B_1, B_2, \ldots\ldots, B_n) \geq Cq-ROFPHM^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots, \mathfrak{C}_n).$$

3. **Boundedness:** Let $c =< \max_{\mathfrak{h}}(A_{\mathfrak{C}}), \min_{\mathfrak{h}}(\lambda_{\mathfrak{C}}) >$, $d =< \min_{\mathfrak{h}}(A_{\mathfrak{C}}), \max_{\mathfrak{h}}(\lambda_{\mathfrak{C}}) >$, having specific partition structure $F_{\mathfrak{h}}(\mathfrak{h} = 1,\ 2,\ 3,\ \ldots..,\ n)$. Then,

$$c \leq Cq-ROFPHM^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2,\ \mathfrak{C}_3, \ldots.., \mathfrak{C}_n) \leq d \tag{29}$$

□

Proof. Since $c =< \max_{\mathfrak{h}}(A_{\mathfrak{C}}),\ \min_{\mathfrak{h}}(\lambda_{\mathfrak{C}}) >$ and $d =< \min_{\mathfrak{h}}(A_{\mathfrak{C}}), \max_{\mathfrak{h}}(\lambda_{\mathfrak{C}}) >$, then according to monotonicity, we have

$$\begin{aligned}Cq-ROFPHM^{a,b}(c, c,\ c, \ldots.., c) \leq Cq-ROFPHM^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2,\ \mathfrak{C}_3, \ldots.., \mathfrak{C}_n)\\ \leq Cq-ROFPHM^{a,b}(d, d,\ d, \ldots.., d)\end{aligned}$$

Further,
$Cq-ROFPHM^{a,b}(c, c,\ c, \ldots.., c) = c$ and $Cq-ROFPHM^{a,b}(d, d,\ d, \ldots.., d) = d$,
This implies that $c \leq Cq-ROFPHM^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2,\ \mathfrak{C}_3, \ldots.., \mathfrak{C}_n) \leq d$. This completes the proof. □

Different particular cases can be obtained by using the number of partitions and the values of parameters ′a, b ′ of the Cq-ROFPHM operator. If g = 1, then the Cq-ROFPHM operator disintegrate into usual Cq-ROFPHM as follows:

$$Cq-ROFPHM^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2,\ \mathfrak{C}_3, \ldots.., \mathfrak{C}_n) = \left(\frac{2}{|F_1|(|F_1|+1)} \oplus_{\mathfrak{h}=1}^{|F_i|} \oplus_{t=\mathfrak{h}}^{|F_i|} \mathfrak{C}_{1\mathfrak{h}}^{a} \mathfrak{C}_{1t}^{b}\right)^{\frac{1}{a+b}} = \left(\frac{2}{n(n+1)} \oplus_{\mathfrak{h}=1}^{n} \oplus_{t=\mathfrak{h}}^{n} \mathfrak{C}_{\mathfrak{h}}^{a} \mathfrak{C}_{t}^{b}\right)^{\frac{1}{a+b}}. \tag{30}$$

Clearly, if g = 1, and different values are assigned to parameters a, b, we can get the particular cases define in Equations (18)–(22).

Despite the fact that, the established Cq-ROFPHM operator models the associated structure between the input arguments, it does not take the significance of input arguments into account. To solve this disadvantage, we further offer the cubic q-rung orthopair fuzzy weighted partitioned Heronian mean (Cq-ROFWPHM) operator.

Definition 19. *Let* $\mathfrak{C}_{\mathfrak{K}} = \left(A_{\mathfrak{C}_{\mathfrak{K}}}, \lambda_{\mathfrak{C}_{\mathfrak{K}}}\right)(\mathfrak{K} = 1, 2, \ldots\ldots, n)$ *be a family of Cq-ROFNs,* $a \geq 0,\ b \geq 0$ *and* $a + b \geq 0$*, and* $w = (w_1, w_2, \ldots.., w_n)$ *represent the weight vector of Cq-ROFNs,* $w_{\mathfrak{K}} \in [0,\ 1]$ *and* $\sum_{\mathfrak{K}=1}^{n} w_{\mathfrak{K}} = 1$*. Then Cq-ROFWPHM operator is a map* $\mathrm{Cq-ROFWPHM} : [0,\ 1]^n \to [0,\ 1]$ *such that:*

$$\mathrm{Cq-ROFWPHM}_{w}^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots, \mathfrak{C}_n) = \frac{1}{g}\left(\oplus_{i=1}^{g}\left(\frac{2}{|F_i|(|F_i|+1)} \oplus_{\mathfrak{K}=1}^{|F_i|} \oplus_{t=\mathfrak{K}}^{|F_i|}\left(w_{i\mathfrak{K}}\mathfrak{C}_{i\mathfrak{K}}\right)^{a} \otimes (w_{it}\mathfrak{C}_{it})^{b}\right)^{\frac{1}{a+b}}\right) \tag{31}$$

Theorem 9. *Let* $\mathfrak{C}_{\mathfrak{K}} = \left(A_{\mathfrak{C}_{\mathfrak{K}}}, \lambda_{\mathfrak{K}}\right)(\mathfrak{K} = 1, 2, \ldots\ldots, n)$ *be a family of Cq-ROFNs,* $a \geq 0,\ b \geq 0$ *and* $a + b \geq 0$*, and* $w = (w_1, w_2, \ldots.., w_n)$ *represent the weight vector of Cq-ROFNs,* $w_{\mathfrak{K}} \in [0,\ 1]$ *and* $\sum_{\mathfrak{K}=1}^{n} w_{\mathfrak{K}} = 1$*. Then the resultant equation by using Equation (31) is also a Cq-ROFN given by:*

$$\mathrm{Cq-ROFWPHM}_{w}^{a,b}(\mathfrak{C}_1, \mathfrak{C}_2, \ldots\ldots, \mathfrak{C}_n) = \left(\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\left(1 - \prod_{i=1}^{g}\left(1 - \left(\left(1 - \left(1 - \left(u_i^L\right)^q\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}}, \\ \left(1 - \prod_{i=1}^{g}\left(1 - \left(\left(1 - \left(1 - \left(u_i^U\right)^q\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}}\end{array}\right], \\ \left[\prod_{i=1}^{g}\left(1 - \left(1 - \left(v_i^L\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}}, \prod_{i=1}^{g}\left(1 - \left(1 - \left(v_i^U\right)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}}\right]\end{array}\right), \\ \left(\begin{array}{c}\left(1 - \prod_{i=1}^{g}\left(1 - \left(\left(1 - \left(1 - (u_i)^q\right)^{\frac{2}{|F_i|(|F_i|+1)}}\right)\right)^{\frac{1}{a+b}}\right)^{\frac{1}{g}}\right)^{\frac{1}{q}}, \\ \prod_{i=1}^{g}\left(1 - \left(1 - (v_i)^{\frac{2q}{|F_i|(|F_i|+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q*g}}\end{array}\right)\end{array}\right) \tag{32}$$

where

$$u_i^L = \left(1 - \prod_{\mathfrak{K}=1}^{|F_i|}\prod_{t=\mathfrak{K}}^{|F_i|}\left(1 - \left(1 - \left(1 - \mathfrak{C}_{i\mathfrak{K}}^{q}\right)^{w_{i\mathfrak{K}}}\right)^{a}\right)\left(1 - \left(1 - \left(1 - \mathfrak{C}_{it}^{q}\right)^{w_{it}}\right)^{b}\right)\right)^{\frac{1}{q}},$$

$$u_i^U = \left(1 - \prod_{\mathfrak{K}=1}^{|F_i|}\prod_{t=\mathfrak{K}}^{|F_i|}\left(1 - \left(1 - \left(1 - \mathfrak{C}_{i\mathfrak{K}}^{q}\right)^{w_{i\mathfrak{K}}}\right)^{a}\right)\left(1 - \left(1 - \left(1 - \mathfrak{C}_{it}^{q}\right)^{w_{it}}\right)^{b}\right)\right)^{\frac{1}{q}},$$

$$v_i^L = \prod_{\mathfrak{K}=1}^{|F_i|}\prod_{t=\mathfrak{K}}^{|F_i|}\left(1 - \left(1 - v_{i\mathfrak{K}}^{w_{i\mathfrak{K}}q}\right)^{a}\left(1 - v_{it}^{w_{it}q}\right)^{b}\right)^{\frac{1}{q}},$$

$$v_i^U = \prod_{\mathfrak{K}=1}^{|F_i|}\prod_{t=\mathfrak{K}}^{|F_i|}\left(1 - \left(1 - v_{i\mathfrak{K}}^{w_{i\mathfrak{K}}q}\right)^{a}\left(1 - v_{it}^{w_{it}q}\right)^{b}\right)^{\frac{1}{q}},$$

$$u_i = \left(1 - \prod_{\mathfrak{K}=1}^{|F_i|}\prod_{t=\mathfrak{K}}^{|F_i|}\left(1 - \left(1 - \left(1 - \mathfrak{C}_{i\mathfrak{K}}^{q}\right)^{w_{i\mathfrak{K}}}\right)^{a}\right)\left(1 - \left(1 - \left(1 - \mathfrak{C}_{it}^{q}\right)^{w_{it}}\right)^{b}\right)\right)^{\frac{1}{q}},$$

$$v_i = \prod_{\mathfrak{K}=1}^{|F_i|}\prod_{t=\mathfrak{K}}^{|F_i|}\left(1 - \left(1 - v_{i\mathfrak{K}}^{w_{i\mathfrak{K}}q}\right)^{a}\left(1 - v_{it}^{w_{it}q}\right)^{b}\right)^{\frac{1}{q}}.$$

5. A Multi Attribute Group Decision Making (MAGDM) Method Based on Cubic q-Ring Orthopair Fuzzy Weighted Heronian Mean Operator

In this section, we will discuss a new MAGDM method based on Cq-ROFWHM and Cq-ROFWPHM operators to solve MAGDM problems in the environment of Cq-ROFNs.

Let $A = \left\{\widetilde{A_1}, \widetilde{A_2}, \widetilde{A_3}, \ldots, \widetilde{A_m}\right\}$ be a family of alternatives, $B = \left\{\widetilde{B_1}, \widetilde{B_2}, \widetilde{B_3}, \ldots, \widetilde{B_m}\right\}$ be a family of attribute with weighted vector $w = \{w_1, w_2, w_3, \ldots\ldots, w_n\}$ where $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Let $E = \{E_1, E_2, E_3, \ldots.., E_d\}$ be a family of experts with weight vector $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_d\}$ satisfying $\gamma_{\text{ƙ}} \in [0, 1], \text{ƙ} = 1, 2, 3, \ldots\ldots, d$ and $\sum_{\text{ƙ}=1}^{d} \gamma_{\text{ƙ}} = 1$. Assume that the ƙth expert offers his opinion about an alternative $\widetilde{A_i}$ $(i = 1, 2, 3, \ldots\ldots, m)$ with respect to the attribute $\widetilde{B_j}$ $(j = 1, 2, 3, \ldots, n)$ as a Cq-ROFN ${\text{Ͼ}_{ij}}^{\text{ƙ}} =< A^{\text{ƙ}}_{\text{Ͼ}_{ij}}, \lambda^{\text{ƙ}}_{\text{Ͼ}_{ij}} >$. The preference of ƙth expert construct a Cq-ROF decision matrix $Q_{\text{ƙ}} = \left(\text{Ͼ}^{\text{ƙ}}_{ij}\right)_{m\times n}$.

Assume that there exists the particular relationship structure between the attributes and there exists 'g' partitions of the attribute set $F_1, F_2, F_3, \ldots, F_g$ by keeping in view, the inherent relationship structure. Attributes of the same partition contain relationship between each others, and attributes of different partitions have no connection.

After that, we apply the establish operators to solve such DM problems. Steps about algorithm are given by:

Step 1: Normalize the DM information because there are two types of attributes: *benefit-type attributes* and *cost-type attributes*. Therefore, construct the normalized decision matrix as $\widetilde{Q_{\text{ƙ}}} = \left(\widetilde{\text{Ͼ}}^{\text{ƙ}}_{ij}\right)_{m\times n} = \left(\langle A^{\text{ƙ}}_{\widetilde{\text{Ͼ}}_{ij}}, \lambda^{\text{ƙ}}_{\widetilde{\text{Ͼ}}_{ij}} \rangle\right)_{m\times n}$ by converting the cost-type attribute values into *benefit-type attribute* values, and:

$$\widetilde{\text{Ͼ}}^{\text{ƙ}}_{ij} = \begin{cases} \text{Ͼ}^{\text{ƙ}}_{ij} & \textit{for benifit} - \textit{type attribute of } \widetilde{B_j} \\ \left(\text{Ͼ}^{\text{ƙ}}_{ij}\right)^{c} & \textit{for cost} - \textit{type attribute of } \widetilde{B_j} \end{cases} \tag{33}$$

where $\left(\text{Ͼ}^{\text{ƙ}}_{ij}\right)^{c} =< \lambda^{\text{ƙ}}_{\text{Ͼ}_{ij}}, A^{\text{ƙ}}_{\text{Ͼ}_{ij}} >$.

Step 2: the established Cq-ROFWHM operator shown in Equation (34) to aggregate all the individual decision matrices $\widetilde{Q_{k'}}$ $(\text{ƙ} = 1, 2, 3, \ldots\ldots, d)$ into overall decision matrix $M = \left\{\widetilde{\rho_{ij}}\right\}_{m\times n} = \left(< A^{\text{ƙ}}_{\widetilde{\text{Ͼ}}_{ij}}, \lambda^{\text{ƙ}}_{\widetilde{\text{Ͼ}}_{ij}} >\right)_{m\times n}$.

$$\widetilde{\rho_{ij}} = Cq - ROFWHM^{a,\,b}_{\gamma}\left({\widetilde{\text{Ͼ}}_{ij}}^{1}, {\widetilde{\text{Ͼ}}_{ij}}^{2}, {\widetilde{\text{Ͼ}}_{ij}}^{3}, \ldots\ldots., {\widetilde{\text{Ͼ}}_{ij}}^{d}\right) =$$
$$\left(\begin{array}{c} \left(\left[\begin{array}{c} \left(1 - \prod_{\text{ƙ}=1}^{n}\prod_{t=\text{ƙ}}^{n}\left(1 - \left(u^{L}_{\text{ƙ}}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}, \\ \left(1 - \prod_{\text{ƙ}=1}^{n}\prod_{t=\text{ƙ}}^{n}\left(1 - \left(u^{U}_{\text{ƙ}}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}} \end{array}\right], \left[\begin{array}{c} \left(1 - \left(1 - \prod_{\text{ƙ}=1}^{n}\prod_{t=\text{ƙ}}^{n}\left(v^{L}_{\text{ƙ}}\right)^{\frac{2q}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}}, \\ \left(1 - \left(1 - \prod_{\text{ƙ}=1}^{n}\prod_{t=\text{ƙ}}^{n}\left(v^{U}_{\text{ƙ}}\right)^{\frac{2q}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}} \end{array}\right]\right), \\ \left(\begin{array}{c} \left(\left(1 - \prod_{\text{ƙ}=1}^{n}\prod_{t=\text{ƙ}}^{n}\left(1 - \left(u_{\text{ƙ}}\right)^{q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(a+b)}}\right), \\ \left(1 - \left(1 - \prod_{\text{ƙ}=1}^{n}\prod_{t=\text{ƙ}}^{n}\left(v_{\text{ƙ}}\right)^{\frac{2q}{n(n+1)}}\right)^{\frac{1}{a+b}}\right)^{\frac{1}{q}} \end{array}\right) \end{array}\right) \tag{34}$$

where

$$\left[u^{L}_{\mathfrak{K}}, u^{U}_{\mathfrak{K}}\right] = \begin{bmatrix} \left(1-\left(1-\left(u^{L}_{\mathfrak{K}}\right)^{q}\right)^{\lambda_{\mathfrak{K}}}\right)^{\frac{a}{q}}\left(1-\left(1-\left(u^{L}_{t}\right)^{q}\right)^{\lambda_{t}}\right)^{\frac{b}{q}}, \\ \left(1-\left(1-\left(u^{U}_{\mathfrak{K}}\right)^{q}\right)^{\lambda_{\mathfrak{K}}}\right)^{\frac{a}{q}}\left(1-\left(1-\left(u^{U}_{t}\right)^{q}\right)^{\lambda_{t}}\right)^{\frac{b}{q}} \end{bmatrix}, \left[v^{L}_{\mathfrak{K}}, v^{U}_{\mathfrak{K}}\right] = \begin{bmatrix} \left(1-\left(1-\left(v^{L}_{\mathfrak{K}}\right)^{\lambda_{\mathfrak{K}} q}\right)^{a}\left(1-\left(v^{L}_{t}\right)^{\lambda_{t} q}\right)^{b}\right)^{\frac{1}{q}}, \\ \left(1-\left(1-\left(v^{U}_{\mathfrak{K}}\right)^{\lambda_{\mathfrak{K}} q}\right)^{a}\left(1-\left(v^{U}_{t}\right)^{\lambda_{t} q}\right)^{b}\right)^{\frac{1}{q}} \end{bmatrix}$$

$$u_{\mathfrak{K}} = \left(1-\left(1-\left(u_{\mathfrak{K}}\right)^{q}\right)^{\lambda_{\mathfrak{K}}}\right)^{\frac{a}{q}}\left(1-\left(1-\left(u_{t}\right)^{q}\right)^{\lambda_{t}}\right)^{\frac{b}{q}},$$

$$v_{\mathfrak{K}} = \left(1-\left(1-\left(v_{\mathfrak{K}}\right)^{\lambda_{\mathfrak{K}} q}\right)^{a}\left(1-\left(v_{t}\right)^{\lambda_{t} q}\right)^{b}\right)^{\frac{1}{q}}$$

Step 3: ssuming the partition form among the attributes, use Equation (35) to obtain the collective evaluation values $\widetilde{\rho_i} = \left(< A^{\mathfrak{K}}_{\widetilde{\mathfrak{C}}_{ij}}, \lambda^{\mathfrak{K}}_{\widetilde{\mathfrak{C}}_{ij}} >\right) (i = 1, 2, 3, \ldots., m; j = 1, 2, 3, \ldots., n)$ of alternatives $\widetilde{A_i}$.

$$\widetilde{\rho_1} = Cq - ROFWPHM^{a,\, b}_{w}\left(\widetilde{\rho_{i1}}, \widetilde{\rho_{i2}}, \ldots.., \widetilde{\rho_{in}}\right) \tag{35}$$

Step 4: Using Definition 13, calculate the score values $S(\widetilde{\rho_1})$ of each alternative $\widetilde{A_1}(i = 1, 2, 3, \ldots.., m)$ respectively. Further, Definition 14 and values of $S(\widetilde{\rho_1})$, rank $\widetilde{\rho_1}(i = 1, 2, 3, \ldots., m)$ and selects the best solution.

5.1. Application Steps for an Established Method

Example 3. *In this section, we present a descriptive example to study the results of a new established method and show the efficiency of a new method by comparing it with some existing methods. Using the full benefit of perfect capital, a company administrator determined to put money in one sector.*

In early steps, four sectors have been identified as alternatives. These four sectors include the computer industry $\widetilde{A_1}$, car industry $\widetilde{A_2}$, food industry $\widetilde{A_3}$ and steel industry $\widetilde{A_4}$. There are four experts $\{E_1, E_2, E_3, E_4\}$ in the judgment board with different knowledge backgrounds. Suppose λ represent the different weights of experts, i.e., $\lambda = \{0.24, 0.26, 0.3, 0.2\}$. Five interconnected attribute are presented by assessment committee for assessment: the amount of the capital profit B_1, market potential B_2, the risk of the capital loss B_3, the growth potential B_4 and the stability of the policy B_5. Suppose "w" stands for different weights of attributes, i.e., $w = \{0.22, 0.24, 0.04, 0.4, 0.1\}$. Keeping in mind the inherent relationship structure, these five attributes are divided into two subset, $F_1 = \{B_1, B_3, B_5\}$, $F_2 = \{B_2, B_4\}$. Experts are asked to provide their assessment data in the shape of Cq-ROFNs. Tables 1–4 represent the assessment data given by experts $E_i(i = 1, 2, 3, 4)$. Next, ranking process is illustrated in detail as follows:

Table 1. Cubic q-rung orthopair fuzzy set (Cq-ROF) decision matrix $\mathbf{Q_1}$.

	$\widetilde{\mathbf{B_1}}$	$\widetilde{\mathbf{B_2}}$	$\widetilde{\mathbf{B_3}}$	$\widetilde{\mathbf{B_4}}$	$\widetilde{\mathbf{B_5}}$
$\widetilde{\mathbf{A_1}}$	$\left(\begin{pmatrix}[0.5,0.9],\\ [0.5,0.6]\end{pmatrix}, (0.5,0.7)\right)$	$\left(\begin{pmatrix}[0.6,0.9],\\ [0.4,0.5]\end{pmatrix}, (0.6,0.7)\right)$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.6,0.8]\end{pmatrix}, (0.8,0.5)\right)$	$\left(\begin{pmatrix}[0.6,0.9],\\ [0.6,0.6]\end{pmatrix}, (0.5,0.8)\right)$	$\left(\begin{pmatrix}[0.6,0.6],\\ [0.7,0.9]\end{pmatrix}, (0.6,0.8)\right)$
$\widetilde{\mathbf{A_2}}$	$\left(\begin{pmatrix}[0.5,0.8],\\ [0.6,0.7]\end{pmatrix}, (0.5,0.6)\right)$	$\left(\begin{pmatrix}[0.7,0.9],\\ [0.5,0.6]\end{pmatrix}, (0.7,0.8)\right)$	$\left(\begin{pmatrix}[0.5,0.8],\\ [0.3,0.6]\end{pmatrix}, (0.5,0.7)\right)$	$\left(\begin{pmatrix}[0.7,0.7],\\ [0.5,0.8]\end{pmatrix}, (0.7,0.6)\right)$	$\left(\begin{pmatrix}[0.8,0.9],\\ [0.5,0.6]\end{pmatrix}, (0.7,0.8)\right)$
$\widetilde{\mathbf{A_3}}$	$\left(\begin{pmatrix}[0.8,0.9],\\ [0.5,0.6]\end{pmatrix}, (0.7,0.9)\right)$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.3,0.6]\end{pmatrix}, (0.5,0.7)\right)$	$\left(\begin{pmatrix}[0.5,0.6],\\ [0.4,0.7]\end{pmatrix}, (0.8,0.7)\right)$	$\left(\begin{pmatrix}[0.5,0.6],\\ [0.8,0.9]\end{pmatrix}, (0.6,0.7)\right)$	$\left(\begin{pmatrix}[0.4,0.6],\\ [0.8,0.8]\end{pmatrix}, (0.5,0.7)\right)$
$\widetilde{\mathbf{A_4}}$	$\left(\begin{pmatrix}[0.5,0.7],\\ [0.5,0.6]\end{pmatrix}, (0.7,0.6)\right)$	$\left(\begin{pmatrix}[0.5,0.6],\\ [0.7,0.8]\end{pmatrix}, (0.7,0.6)\right)$	$\left(\begin{pmatrix}[0.7,0.8],\\ [0.5,0.6]\end{pmatrix}, (0.4,0.7)\right)$	$\left(\begin{pmatrix}[0.6,0.8],\\ [0.5,0.7]\end{pmatrix}, (0.6,0.7)\right)$	$\left(\begin{pmatrix}[0.7,0.8],\\ [0.5,0.6]\end{pmatrix}, (0.3,0.9)\right)$

Table 2. Cq-ROF decision matrix $\mathbf{Q_2}$.

	$\widetilde{\mathbf{B}_1}$	$\widetilde{\mathbf{B}_2}$	$\widetilde{\mathbf{B}_3}$	$\widetilde{\mathbf{B}_4}$	$\widetilde{\mathbf{B}_5}$
$\widetilde{\mathbf{A}_1}$	$\left(\left([0.4,0.5],[0.5,0.7]\right),(0.7,0.2)\right)$	$\left(\left([0.6,0.7],[0.2,0.3]\right),(0.8,0.3)\right)$	$\left(\left([0.3,0.5],[0.4,0.6]\right),(0.8,0.6)\right)$	$\left(\left([0.3,0.4],[0.2,0.3]\right),(0.7,0.6)\right)$	$\left(\left([0.3,0.6],[0.2,0.4]\right),(0.8,0.1)\right)$
$\widetilde{\mathbf{A}_2}$	$\left(\left([0.2,0.3],[0.4,0.5]\right),(0.8,0.2)\right)$	$\left(\left([0.1,0.2],[0.6,0.7]\right),(0.9,0.4)\right)$	$\left(\left([0.5,0.7],[0.3,0.4]\right),(0.8,0.5)\right)$	$\left(\left([0.6,0.7],[0.1,0.2]\right),(0.6,0.2)\right)$	$\left(\left([0.1,0.2],[0.4,0.5]\right),(0.7,0.4)\right)$
$\widetilde{\mathbf{A}_3}$	$\left(\left([0.7,0.9],[0.1,0.2]\right),(0.6,0.2)\right)$	$\left(\left([0.4,0.5],[0.2,0.3]\right),(0.7,0.4)\right)$	$\left(\left([0.6,0.8],[0.3,0.7]\right),(0.7,0.7)\right)$	$\left(\left([0.5,0.6],[0.5,0.7]\right),(0.9,0.2)\right)$	$\left(\left([0.4,0.6],[0.2,0.3]\right),(0.6,0.5)\right)$
$\widetilde{\mathbf{A}_4}$	$\left(\left([0.3,0.5],[0.4,0.6]\right),(0.7,0.4)\right)$	$\left(\left([0.2,0.3],[0.1,0.2]\right),(0.6,0.2)\right)$	$\left(\left([0.5,0.6],[0.5,0.5]\right),(0.3,0.5)\right)$	$\left(\left([0.7,0.8],[0.2,0.4]\right),(0.9,0.4)\right)$	$\left(\left([0.2,0.4],[0.1,0.3]\right),(0.5,0.4)\right)$

Table 3. Cq-ROF decision matrix $\mathbf{Q_3}$.

	$\widetilde{\mathbf{B}_1}$	$\widetilde{\mathbf{B}_2}$	$\widetilde{\mathbf{B}_3}$	$\widetilde{\mathbf{B}_4}$	$\widetilde{\mathbf{B}_5}$
$\widetilde{\mathbf{A}_1}$	$\left(\left([0.2,0.4],[0.3,0.5]\right),(0.7,0.1)\right)$	$\left(\left([0.7,0.8],[0.3,0.4]\right),(0.8,0.3)\right)$	$\left(\left([0.4,0.5],[0.1,0.4]\right),(0.6,0.2)\right)$	$\left(\left([0.3,0.5],[0.2,0.4]\right),(0.7,0.1)\right)$	$\left(\left([0.4,0.6],[0.2,0.3]\right),(0.6,0.3)\right)$
$\widetilde{\mathbf{A}_2}$	$\left(\left([0.6,0.8],[0.1,0.2]\right),(0.5,0.3)\right)$	$\left(\left([0.5,0.6],[0.3,0.4]\right),(0.9,0.4)\right)$	$\left(\left([0.4,0.6],[0.2,0.3]\right),(0.7,0.7)\right)$	$\left(\left([0.8,0.9],[0.3,0.5]\right),(0.8,0.2)\right)$	$\left(\left([0.1,0.2],[0.3,0.4]\right),(0.7,0.2)\right)$
$\widetilde{\mathbf{A}_3}$	$\left(\left([0.1,0.2],[0.3,0.4]\right),(0.9,0.6)\right)$	$\left(\left([0.3,0.4],[0.2,0.3]\right),(0.7,0.1)\right)$	$\left(\left([0.5,0.7],[0.1,0.2]\right),(0.8,0.3)\right)$	$\left(\left([0.7,0.8],[0.2,0.3]\right),(0.9,0.2)\right)$	$\left(\left([0.6,0.7],[0.1,0.2]\right),(0.6,0.5)\right)$
$\widetilde{\mathbf{A}_4}$	$\left(\left([0.3,0.4],[0.4,0.6]\right),(0.5,0.5)\right)$	$\left(\left([0.2,0.3],[0.7,0.8]\right),(0.6,0.4)\right)$	$\left(\left([0.2,0.4],[0.3,0.5]\right),(0.4,0.5)\right)$	$\left(\left([0.6,0.7],[0.6,0.8]\right),(0.5,0.3)\right)$	$\left(\left([0.2,0.5],[0.1,0.3]\right),(0.5,0.6)\right)$

Table 4. Cq-ROF decision matrix $\mathbf{Q_4}$.

	$\widetilde{\mathbf{B}_1}$	$\widetilde{\mathbf{B}_2}$	$\widetilde{\mathbf{B}_3}$	$\widetilde{\mathbf{B}_4}$	$\widetilde{\mathbf{B}_5}$
$\widetilde{\mathbf{A}_1}$	$\left(\left([0.4,0.7],[0.3,0.5]\right),(0.4,0.7)\right)$	$\left(\left([0.3,0.5],[0.4,0.6]\right),(0.6,0.4)\right)$	$\left(\left([0.6,0.7],[0.2,0.6]\right),(0.5,0.3)\right)$	$\left(\left([0.6,0.7],[0.1,0.3]\right),(0.5,0.2)\right)$	$\left(\left([0.5,0.6],[0.6,0.8]\right),(0.9,0.4)\right)$
$\widetilde{\mathbf{A}_2}$	$\left(\left([0.4,0.6],[0.5,0.7]\right),(0.7,0.4)\right)$	$\left(\left([0.1,0.2],[0.5,0.6]\right),(0.9,0.6)\right)$	$\left(\left([0.6,0.8],[0.4,0.5]\right),(0.6,0.2)\right)$	$\left(\left([0.4,0.5],[0.4,0.6]\right),(0.7,0.4)\right)$	$\left(\left([0.3,0.5],[0.1,0.2]\right),(0.8,0.3)\right)$
$\widetilde{\mathbf{A}_3}$	$\left(\left([0.8,0.9],[0.2,0.3]\right),(0.5,0.2)\right)$	$\left(\left([0.3,0.4],[0.1,0.2]\right),(0.8,0.2)\right)$	$\left(\left([0.7,0.9],[0.3,0.4]\right),(0.8,0.3)\right)$	$\left(\left([0.5,0.6],[0.1,0.2]\right),(0.7,0.6)\right)$	$\left(\left([0.2,0.3],[0.5,0.6]\right),(0.6,0.4)\right)$
$\widetilde{\mathbf{A}_4}$	$\left(\left([0.3,0.4],[0.5,0.6]\right),(0.5,0.5)\right)$	$\left(\left([0.4,0.6],[0.2,0.3]\right),(0.8,0.3)\right)$	$\left(\left([0.4,0.6],[0.5,0.7]\right),(0.9,0.6)\right)$	$\left(\left([0.2,0.3],[0.1,0.2]\right),(0.6,0.1)\right)$	$\left(\left([0.1,0.3],[0.1,0.2]\right),(0.6,0.1)\right)$

Step 1: Since $\widetilde{B_3}$ represent a cost-type attribute, we have to normalize the decision making information by using Equation (33). Normalized information is shown in Tables 5–8.

Step 2: Use Equation (34) to get the overall decision matrix $M = \left\{\widetilde{\rho_{ij}}\right\}_{4\times 5} = \left\{\left(A_{\rho_{ij}}, \lambda_{\rho_{ij}}\right)\right\}_{4\times 5}$. In addition, we let the parameters $a = 1$, $b = 1$ and $q = 3$. The aim of this MAGDM problem is to select the best alternative. Table 9 show the collective Cq-ROF decision matrix M.

Table 5. Normalized Cq-ROF decision matrix $\widetilde{Q_1}$.

	$\widetilde{B_1}$	$\widetilde{B_2}$	$\widetilde{B_3}$	$\widetilde{B_4}$	$\widetilde{B_5}$
$\widetilde{A_1}$	$\left(\left([0.5,0.9],[0.5,0.6]\right),(0.5,0.7)\right)$	$\left(\left([0.6,0.9],[0.4,0.5]\right),(0.6,0.7)\right)$	$\left(\left([0.6,0.8],[0.6,0.7]\right),(0.5,0.8)\right)$	$\left(\left([0.6,0.9],[0.6,0.6]\right),(0.5,0.8)\right)$	$\left(\left([0.6,0.6],[0.7,0.9]\right),(0.6,0.8)\right)$
$\widetilde{A_2}$	$\left(\left([0.5,0.8],[0.6,0.7]\right),(0.5,0.6)\right)$	$\left(\left([0.7,0.9],[0.5,0.6]\right),(0.7,0.8)\right)$	$\left(\left([0.3,0.6],[0.5,0.8]\right),(0.7,0.5)\right)$	$\left(\left([0.7,0.7],[0.5,0.8]\right),(0.7,0.6)\right)$	$\left(\left([0.8,0.9],[0.5,0.6]\right),(0.7,0.8)\right)$
$\widetilde{A_3}$	$\left(\left([0.8,0.9],[0.5,0.6]\right),(0.6,0.9)\right)$	$\left(\left([0.6,0.7],[0.3,0.6]\right),(0.5,0.7)\right)$	$\left(\left([0.4,0.7],[0.5,0.6]\right),(0.7,0.8)\right)$	$\left(\left([0.5,0.6],[0.8,0.9]\right),(0.6,0.7)\right)$	$\left(\left([0.4,0.6],[0.8,0.8]\right),(0.5,0.7)\right)$
$\widetilde{A_4}$	$\left(\left([0.5,0.7],[0.5,0.6]\right),(0.7,0.6)\right)$	$\left(\left([0.5,0.6],[0.7,0.8]\right),(0.7,0.6)\right)$	$\left(\left([0.5,0.6],[0.7,0.8]\right),(0.7,0.4)\right)$	$\left(\left([0.6,0.8],[0.5,0.7]\right),(0.6,0.7)\right)$	$\left(\left([0.7,0.8],[0.5,0.6]\right),(0.3,0.9)\right)$

Table 6. Normalized Cq-ROF decision matrix $\widetilde{Q_2}$.

	$\widetilde{B_1}$	$\widetilde{B_2}$	$\widetilde{B_3}$	$\widetilde{B_4}$	$\widetilde{B_5}$
$\widetilde{A_1}$	$\left(\left([0.4,0.5],[0.5,0.7]\right),(0.7,0.2)\right)$	$\left(\left([0.6,0.7],[0.2,0.3]\right),(0.8,0.3)\right)$	$\left(\left([0.4,0.6],[0.3,0.5]\right),(0.6,0.8)\right)$	$\left(\left([0.3,0.4],[0.2,0.3]\right),(0.7,0.6)\right)$	$\left(\left([0.3,0.6],[0.2,0.4]\right),(0.8,0.1)\right)$
$\widetilde{A_2}$	$\left(\left([0.2,0.3],[0.4,0.5]\right),(0.8,0.2)\right)$	$\left(\left([0.1,0.2],[0.6,0.7]\right),(0.9,0.4)\right)$	$\left(\left([0.3,0.4],[0.5,0.7]\right),(0.5,0.8)\right)$	$\left(\left([0.6,0.7],[0.1,0.2]\right),(0.6,0.2)\right)$	$\left(\left([0.1,0.2],[0.4,0.5]\right),(0.7,0.4)\right)$
$\widetilde{A_3}$	$\left(\left([0.7,0.9],[0.1,0.2]\right),(0.6,0.2)\right)$	$\left(\left([0.4,0.5],[0.2,0.3]\right),(0.7,0.4)\right)$	$\left(\left([0.3,0.7],[0.6,0.8]\right),(0.7,0.7)\right)$	$\left(\left([0.5,0.6],[0.5,0.7]\right),(0.9,0.2)\right)$	$\left(\left([0.4,0.6],[0.2,0.3]\right),(0.6,0.5)\right)$
$\widetilde{A_4}$	$\left(\left([0.3,0.5],[0.4,0.6]\right),(0.7,0.4)\right)$	$\left(\left([0.2,0.3],[0.1,0.2]\right),(0.6,0.2)\right)$	$\left(\left([0.5,0.5],[0.5,0.6]\right),(0.5,0.3)\right)$	$\left(\left([0.7,0.8],[0.2,0.4]\right),(0.9,0.4)\right)$	$\left(\left([0.2,0.4],[0.1,0.3]\right),(0.5,0.4)\right)$

Table 7. Normalized Cq-ROF decision matrix $\widetilde{Q_3}$.

	$\widetilde{B_1}$	$\widetilde{B_2}$	$\widetilde{B_3}$	$\widetilde{B_4}$	$\widetilde{B_5}$
$\widetilde{A_1}$	$\left(\left([0.2,0.4],[0.3,0.5]\right),(0.7,0.1)\right)$	$\left(\left([0.7,0.8],[0.3,0.4]\right),(0.8,0.3)\right)$	$\left(\left([0.1,0.4],[0.4,0.5]\right),(0.2,0.6)\right)$	$\left(\left([0.3,0.5],[0.2,0.4]\right),(0.7,0.1)\right)$	$\left(\left([0.4,0.6],[0.2,0.3]\right),(0.6,0.3)\right)$
$\widetilde{A_2}$	$\left(\left([0.6,0.8],[0.1,0.2]\right),(0.5,0.3)\right)$	$\left(\left([0.5,0.6],[0.3,0.4]\right),(0.9,0.4)\right)$	$\left(\left([0.2,0.3],[0.4,0.6]\right),(0.7,0.7)\right)$	$\left(\left([0.8,0.9],[0.3,0.5]\right),(0.8,0.2)\right)$	$\left(\left([0.1,0.2],[0.3,0.4]\right),(0.7,0.2)\right)$
$\widetilde{A_3}$	$\left(\left([0.1,0.2],[0.3,0.4]\right),(0.9,0.6)\right)$	$\left(\left([0.3,0.4],[0.2,0.3]\right),(0.7,0.1)\right)$	$\left(\left([0.1,0.2],[0.5,0.7]\right),(0.3,0.8)\right)$	$\left(\left([0.7,0.8],[0.2,0.3]\right),(0.9,0.2)\right)$	$\left(\left([0.6,0.7],[0.1,0.2]\right),(0.6,0.5)\right)$
$\widetilde{A_4}$	$\left(\left([0.3,0.4],[0.4,0.6]\right),(0.5,0.5)\right)$	$\left(\left([0.2,0.3],[0.7,0.8]\right),(0.6,0.4)\right)$	$\left(\left([0.3,0.5],[0.2,0.4]\right),(0.5,0.4)\right)$	$\left(\left([0.6,0.7],[0.6,0.8]\right),(0.5,0.3)\right)$	$\left(\left([0.2,0.5],[0.1,0.3]\right),(0.5,0.6)\right)$

Table 8. Normalized Cq-ROF decision matrix $\widetilde{Q_4}$.

	$\widetilde{B_1}$	$\widetilde{B_2}$	$\widetilde{B_3}$	$\widetilde{B_4}$	$\widetilde{B_5}$
$\widetilde{A_1}$	$\left(\left([0.4,0.7],[0.3,0.5]\right),(0.4,0.7)\right)$	$\left(\left([0.3,0.5],[0.4,0.6]\right),(0.6,0.4)\right)$	$\left(\left([0.6,0.7],[0.2,0.6]\right),(0.5,0.3)\right)$	$\left(\left([0.6,0.7],[0.1,0.3]\right),(0.5,0.2)\right)$	$\left(\left([0.5,0.6],[0.6,0.8]\right),(0.9,0.4)\right)$
$\widetilde{A_2}$	$\left(\left([0.4,0.6],[0.5,0.7]\right),(0.7,0.4)\right)$	$\left(\left([0.1,0.2],[0.5,0.6]\right),(0.9,0.6)\right)$	$\left(\left([0.6,0.8],[0.4,0.5]\right),(0.6,0.2)\right)$	$\left(\left([0.4,0.5],[0.4,0.6]\right),(0.7,0.4)\right)$	$\left(\left([0.3,0.5],[0.1,0.2]\right),(0.8,0.3)\right)$
$\widetilde{A_3}$	$\left(\left([0.8,0.9],[0.2,0.3]\right),(0.5,0.2)\right)$	$\left(\left([0.3,0.4],[0.1,0.2]\right),(0.8,0.2)\right)$	$\left(\left([0.7,0.9],[0.3,0.4]\right),(0.8,0.3)\right)$	$\left(\left([0.5,0.6],[0.1,0.2]\right),(0.7,0.6)\right)$	$\left(\left([0.2,0.3],[0.5,0.6]\right),(0.6,0.4)\right)$
$\widetilde{A_4}$	$\left(\left([0.3,0.4],[0.5,0.6]\right),(0.5,0.5)\right)$	$\left(\left([0.4,0.6],[0.2,0.3]\right),(0.8,0.3)\right)$	$\left(\left([0.4,0.6],[0.5,0.7]\right),(0.9,0.6)\right)$	$\left(\left([0.2,0.3],[0.1,0.2]\right),(0.6,0.1)\right)$	$\left(\left([0.1,0.3],[0.1,0.2]\right),(0.6,0.1)\right)$

Table 9. Collective Cq-ROF decision matrix M.

	$\widetilde{\mathbf{B_1}}$	$\widetilde{\mathbf{B_2}}$	$\widetilde{\mathbf{B_3}}$	$\widetilde{\mathbf{B_4}}$	$\widetilde{\mathbf{B_5}}$
$\widetilde{\mathbf{A_1}}$	$(([0.02219743, 0.213175], [0.8462167, 0.90304]), (0.144792, 0.842854))$	$(([0.128224, 0.40051], [0.81481, 0.861849]), (0.306328, 0.853026))$	$(([0.019724, 0.15607], [0.870208, 0.908359]), (0.035453, 0.930729))$	$(([0.0448896, 0.216094], [0.781617, 0.845337]), (0.0158308, 0.849051))$	$(([0.045164, 0.136835], [0.847819, 0.90733]), (0.35506, 0.837489))$
$\widetilde{\mathbf{A_2}}$	$(([0.0447384, 0.242076], [0.844096, 0.886662]), (0.183858, 0.9474258337))$	$(([0.044988, 0.135117], [0.872203, 0.90265]), (0.676928, 0.894071))$	$(([0.008071, 0.044183], [0.879227, 0.940818]), (0.129727, 0.924376))$	$(([0.212381, 0.342527], [0.81141, 0.886402]), (0.282355, 0.822124))$	$(([0.017857, 0.075864], [0.811152, 0.854093]), (0.311763, 0.851694))$
$\widetilde{\mathbf{A_3}}$	$(([0.245609297, 0.556753], [0.787562, 0.834762]), (0.251132, 0.867905))$	$(([0.029372, 0.071991], [0.748907, 0.823753]), (0.250083, 0.820084))$	$(([0.007208, 0.10128], [0.901409, 0.94705]), (0.099402, 0.953063))$	$(([0.104385, 0.212804], [0.835258, 0.885618]), (0.519245, 0.847819))$	$(([0.036026, 0.128224], [0.835139, 0.867548]), (0.116593, 0.888141))$
$\widetilde{\mathbf{A_4}}$	$(([0.015799753, 0.071991], [0.863428, 0.90943]), (0.150696, 0.880056))$	$(([0.013391, 0.048896], [0.850735, 0.889705]), (0.232567, 0.834762))$	$(([0.022165, 0.115479], [0.863271, 0.909558]), (0.100525, 0.882359))$	$(([0.122689, 0.292772], [0.820761, 0.889828]), (0.235051, 0.828911))$	$(([0.010909, 0.089044], [0.73512, 0.823753]), (0.057967, 0.8761))$

Step 3: Use Equation (35) to calculate all assessment values of each alternative $\widetilde{A_1}$ and attain the collective assessment values '$\widetilde{\rho_i}$' of each alternative $\widetilde{A_1}$ $(i = 1, 2, 3, 4)$.

$$\widetilde{\rho}_1 = ([0.0000179033, 0.004369], [0.963777, 0.975467], (0.002792115, 0.970406)),$$
$$\widetilde{\rho}_2 = ([0.0000548325, 0.002932], [0.967788, 0.978244], (0.020841102, 0.970889)),$$
$$\widetilde{\rho}_3 = ([0.000244832, 0.005721], [0.960768, 0.972091], (0.0108355387, 0.971571)),$$
$$\widetilde{\rho}_4 = ([0.00000239931, 0.000337], [0.965787, 0.977769], (0.001471525, 0.970592)).$$

Step 4: Based on Equation (10), we calculate score functions $S(\widetilde{\rho_i})$ of $\widetilde{\rho_i}$ as follows:

$$S(\widetilde{\rho_1}) = 0.47898376,\ S(\widetilde{\rho_2}) = 0.477264678$$

$$S(\widetilde{\rho_3}) = 0.482878293,\ S(\widetilde{\rho_4}) = 0.477720557$$

As $S(\widetilde{\rho_3}) > S(\widetilde{\rho_1}) > S(\widetilde{\rho_4}) > S(\widetilde{\rho_2})$. Therefore, $\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$ and $\widetilde{A_3}$ is the best alternative.

5.2. The Effects of the Parameters Values on the Ranking Results

In the following, we will discuss the influence of parameters q, a and b on the ranking results of the alternatives. In above calculation procedure, for our comfort and without loss of generality, we set $a = 1$, $b = 1$ and $q = 3$.

From Table 10, we can notice that the ranking result are same for $q = 4, 5, 7, 8$ and ranking results for these cases are same as $\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$. Finally, we can conclude that results for best alternative do not changes as value of parameter q changes.

Table 10. Ranking result for different values of parameter q.

Q	Score Values	Ranking Results
q = 4	$S_1 = 0.4724, S_2 = 0.4704, S_3 = 0.4775, S_4 = 0.4710$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$
q = 5	$S_1 = 0.4661, S_2 = 0.4638, S_3 = 0.4724, S_4 = 0.4645$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$
q = 7	$S_1 = 0.4540, S_2 = 0.4515, S_3 = 0.4626, S_4 = 0.4524$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$
q = 8	$S_1 = 0.4483, S_2 = 0.44571, S_3 = 0.45798, S_4 = 0.4467$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$

In the following Table 11, when $a = 0$ and $b = 1$, the ranking results are $\widetilde{A_4} > \widetilde{A_1} > \widetilde{A_2} > \widetilde{A_3}$, which are different from the results obtain for $a = 1$ and $b = 1$ having ranking results as $\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$. This means that, when we change the values of parameters a and b, different ranking results may be possible. Different score values and ranking results are possible for fixing any one of the parameter and changing the other. Finally, we can observe that the value of parameters a, b can effect the ranking results and these effects of parameters on the ranking results are shown in Figure 1.

Table 11. Ranking result for different values of parameters **a**, **b**.

a and b	Score Values	Ranking Results
$\mathbf{a = 0,\ b = 1}$	$S_1 = 0.4553, S_2 = 0.3550, S_3 = 0.2983, S_4 = 0.4632$	$\widetilde{A_4} > \widetilde{A_1} > \widetilde{A_2} > \widetilde{A_3}$
$\mathbf{a = 1,\ b = 1}$	$S_1 = 0.4790, S_2 = 0.4773, S_3 = 0.4829, S_4 = 0.4777$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$
$\mathbf{a = 0,\ b = 3}$	$S_1 = 0.4757, S_2 = 0.4596, S_3 = 0.4674, S_4 = 0.4607$	$\widetilde{A_1} > \widetilde{A_3} > \widetilde{A_4} > \widetilde{A_2}$
$\mathbf{a = 1,\ b = 3}$	$S_1 = 0.4764, S_2 = 0.4670, S_3 = 0.4736, S_4 = 0.4667$	$\widetilde{A_1} > \widetilde{A_3} > \widetilde{A_2} > \widetilde{A_4}$
$\mathbf{a = 3,\ b = 1}$	$S_1 = 0.4754, S_2 = 0.4721, S_3 = 0.4797, S_4 = 0.4734$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$
$\mathbf{a = 3,\ b = 3}$	$S_1 = 0.47488, S_2 = 0.4687, S_3 = 0.4751, S_4 = 0.4682$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_2} > \widetilde{A_4}$
$\mathbf{a = 0,\ b = 5}$	$S_1 = 0.4727, S_2 = 0.4527, S_3 = 0.4624, S_4 = 0.4542$	$\widetilde{A_1} > \widetilde{A_3} > \widetilde{A_4} > \widetilde{A_2}$
$\mathbf{a = 1,\ b = 5}$	$S_1 = 0.4739,\ S_2 = 0.4598, S_3 = 0.4681, S_4 = 0.4598$	$\widetilde{A_1} > \widetilde{A_3} > \widetilde{A_4} > \widetilde{A_2}$
$\mathbf{a = 5,\ b = 1}$	$S_1 = 0.4747, S_2 = 0.4674, S_3 = 0.4767, S_4 = 0.4697$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$
$\mathbf{a = 5,\ b = 5}$	$S_1 = 0.4735, S_2 = 0.4646, S_3 = 0.4715, S_4 = 0.4635$	$\widetilde{A_1} > \widetilde{A_3} > \widetilde{A_2} > \widetilde{A_4}$

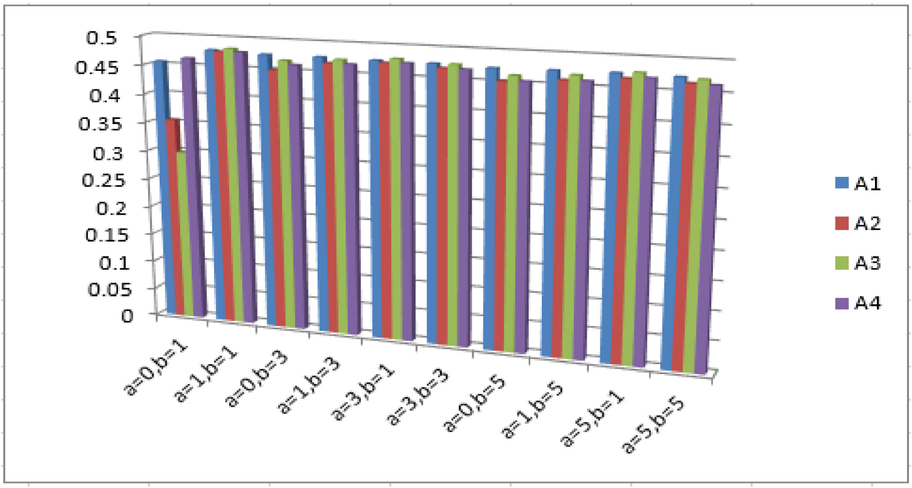

Figure 1. Graphical representations of data discussed in Table 11.

5.3. Comparison with Existing Methods

Here we verify the advantages or efficiency of proposed work with existing methods. We use the cubic intuitionistic Bonferroni mean operators (CIFBM) operator [23], cubic intuitionistic fuzzy weighted averaging and cubic intuitionistic weighted geometric CIFWA operators [24], and (CIFWG) operators [24], cubic Pythagorean fuzzy weighted averaging (CPFWA) operators [25], and cubic Pythagorean fuzzy weighted geometric (CPFWG) operators [25] with proposed operators.

Example 4. *An investment company plans to invest in one area out of four areas as a set of alternative denoted by* $A = \left\{\widetilde{A_1},\ \widetilde{A_2},\ \widetilde{A_3},\ \widetilde{A_4}\right\}$ *where* $\widetilde{A_1},\ \widetilde{A_2},\ \widetilde{A_3}$, and $\widetilde{A_4}$ *respectively, represents the computer industry, car industry, food industry and steel industry. A group of four experts* $\{E_1,\ E_2,\ E_3, E_4\}$ *is invited to evaluate these areas. Suppose ʻλʼ stands for the different weights of experts, i.e.,* $\lambda = \{0.24,\ 0.26,\ 0.3,\ 0.2\}$. *Five interconnected attributes are considered for evaluation denoted by* $\widetilde{B_1},\ \widetilde{B_2},\ \widetilde{B_3},\ \widetilde{B_4},\ \widetilde{B_5}$ *where* $\widetilde{B_1},\ \widetilde{B_2},\ \widetilde{B_3},\ \widetilde{B_4}$ and $\widetilde{B_5}$ *respectively, represent the amount of the capital profit, market potential, the risk of the capital loss, the growth potential and the stability of the policy. Suppose "w" stands for weight vector of attributes, i.e.,* $w = \{0.22,\ 0.24,\ 0.04,\ 0.4,\ 0.1\}$. *According to the inherent relationship structure, we divide these five attributes into two subsets,* $F_1 = \left\{\widetilde{B_1},\ \widetilde{B_3},\ \widetilde{B_5}\right\}$, $F_2 = \left\{\widetilde{B_2},\ \widetilde{B_4}\right\}$. *The information, which is to be evaluated in this example, consists of cubic intuitionistic fuzzy numbers (CIFNs) as shown in Table 12.*

Table 12. Information based on cubic intuitionistic fuzzy sets (CIFS).

	$\widetilde{B_1}$	$\widetilde{B_2}$	$\widetilde{B_3}$	$\widetilde{B_4}$	$\widetilde{B_5}$
$\widetilde{A_1}$	$\left(\left(\begin{array}{c}[0.15,0.20],\\ [0.10,0.20]\end{array}\right),(0.30,0.40)\right)$	$\left(\left(\begin{array}{c}[0.30,0.35],\\ [0.40,0.50]\end{array}\right),(0.10,0.20)\right)$	$\left(\left(\begin{array}{c}[0.30,0.40],\\ [0.20,0.50]\end{array}\right),(0.30,0.20)\right)$	$\left(\left(\begin{array}{c}[0.30,0.35],\\ [0.40,0.50]\end{array}\right),(0.10,0.30)\right)$	$\left(\left(\begin{array}{c}[0.40,0.45],\\ [0.30,0.39]\end{array}\right),(0.40,0.22)\right)$
$\widetilde{A_2}$	$\left(\left(\begin{array}{c}[0.30,0.35],\\ [0.15,0.22]\end{array}\right),(0.20,0.40)\right)$	$\left(\left(\begin{array}{c}[0.10,0.15],\\ [0.18,0.25]\end{array}\right),(0.30,0.10)\right)$	$\left(\left(\begin{array}{c}[0.40,0.45],\\ [0.30,0.29]\end{array}\right),(0.40,0.22)\right)$	$\left(\left(\begin{array}{c}[0.80,0.85],\\ [0.10,0.15]\end{array}\right),(0.12,0.85)\right)$	$\left(\left(\begin{array}{c}[0.15,0.22],\\ [0.12,0.20]\end{array}\right),(0.10,0.60)\right)$
$\widetilde{A_3}$	$\left(\left(\begin{array}{c}[0.70,0.75],\\ [0.10,0.15]\end{array}\right),(0.10,0.80)\right)$	$\left(\left(\begin{array}{c}[0.80,0.82],\\ [0.10,0.15]\end{array}\right),(0.12,0.85)\right)$	$\left(\left(\begin{array}{c}[0.82,0.86],\\ [0.10,0.12]\end{array}\right),(0.10,0.70)\right)$	$\left(\left(\begin{array}{c}[0.10,0.12],\\ [0.82,0.88]\end{array}\right),(0.70,0.10)\right)$	$\left(\left(\begin{array}{c}[0.10,0.20],\\ [0.30,0.35]\end{array}\right),(0.40,0.20)\right)$
$\widetilde{A_4}$	$\left(\left(\begin{array}{c}[0.40,0.48],\\ [0.20,0.28]\end{array}\right),(0.30,0.20)\right)$	$\left(\left(\begin{array}{c}[0.10,0.20],\\ [0.30,0.35]\end{array}\right),(0.40,0.20)\right)$	$\left(\left(\begin{array}{c}[0.15,0.22],\\ [0.12,0.20]\end{array}\right),(0.10,0.60)\right)$	$\left(\left(\begin{array}{c}[0.20,0.28],\\ [0.40,0.48]\end{array}\right),(0.20,0.30)\right)$	$\left(\left(\begin{array}{c}[0.30,0.35],\\ [0.15,0.22]\end{array}\right),(0.20,0.30)\right)$

We use the CIFBWM operators [23], CIFWA operators [24], CIFWG operators [24], CPFWA operators [25] and CPFWG operators [25]. Wang et al. [26] explored the power Maclaurin symmetric mean operators based on Cq-ROFSs to compare with proposed method and the results are shown in Table 13. From Table 13, we can find that we can use the different methods to get the different sorting results under the same evaluation data. Also, the geometrical representation of the information given in Table 13 can be seen from Figure 2.

Table 13. Comparison of established work with existing methods.

Method	Score Values	Ranking Results
Kaur and Garg Method [23]	$S(\widetilde{A_1}) = 0.4253,\ S(\widetilde{A_2}) = 0.4681,\ S(\widetilde{A_3}) = 0.4645,\ S(\widetilde{A_4}) = 0.4616$	$\widetilde{A_2} > \widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4}$
Kaur and Garg Method [24]	$S(\widetilde{A_1}) = 0.4488,\ S(\widetilde{A_2}) = 0.5076,\ S(\widetilde{A_3}) = 0.5065,\ S(\widetilde{A_4}) = 0.4395$	$\widetilde{A_2} > \widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4}$
Abbas et al. Method [25]	$S(\widetilde{A_1}) = 0.4061,\ S(\widetilde{A_2}) = 0.4897,\ S(\widetilde{A_3}) = 0.4778,\ S(\widetilde{A_4}) = 0.3493$	$\widetilde{A_2} > \widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4}$
Wang et al. [26]	$S(\widetilde{A_1}) = 0.3846,\ S(\widetilde{A_2}) = 0.4635,\ S(\widetilde{A_3}) = 0.4612,\ S(\widetilde{A_4}) = 0.3738$	$\widetilde{A_2} > \widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4}$
Proposed Method in this paper	$S(\widetilde{A_1}) = 0.3748,\ S(\widetilde{A_2}) = 0.4774,\ S(\widetilde{A_3}) = 0.4681,\ S(\widetilde{A_4}) = 0.3691$	$\widetilde{A_2} > \widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4}$

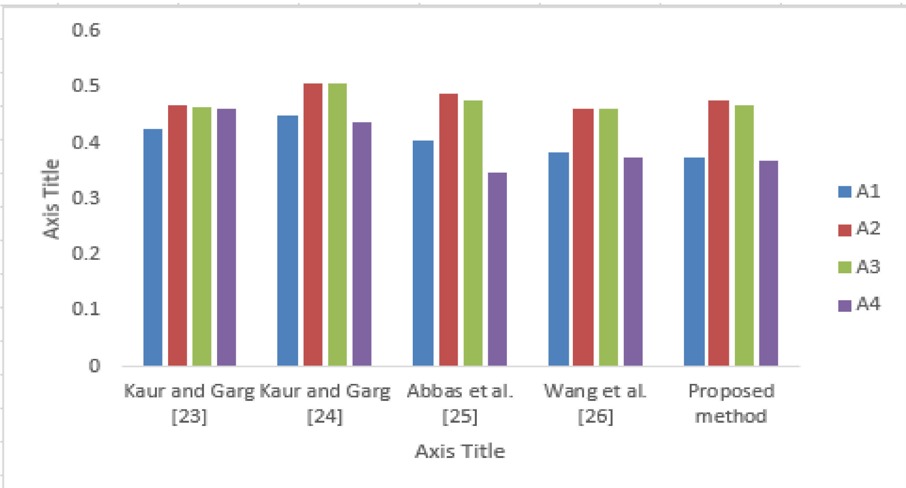

Figure 2. Graphical representations of comparison discussed in Table 13.

From Table 13, we observe the same ranking order of alternative, which prove the validity of proposed work. The common property of the proposed Cq-ROFPHM operator and CIFWBM operator proposed by the Harish [23] is that they can consider the interrelationship structure. But CIFWBM [23] operators only consider that each attribute is linked with all other attribute. In all real decision making situation it is not possible because in the above situation the attributes $\widetilde{B_1}$, $\widetilde{B_3}$, $\widetilde{B_5}$ in partition F_1 are related to each other and there is no connection between the attributes $\widetilde{B_2}$, $\widetilde{B_4}$ in partition F_2. The proposed Cq-ROFPHM, Cq-ROFWPHM operators can only deal with such situation. Therefore, the proposed Cq-ROFPHM, Cq-ROFWPHM operators are more suitable.

The CIFWA, CIFWG operators proposed by Harish [24] and CPFWA, CPFWG operators proposed by Abbas et al. [25] can consider only the simple weighted averaging operations. Wang et al. [26] explored the power Maclaurin symmetric mean operators based on Cq-ROFSs and the interrelationship structure between attributes cannot be considered in these operators, while the proposed Cq-ROFPHM operators can handle that situations, which proves the superiority of the proposed work.

Example 5. *An investment company plans to invest in one area out of four areas as a set of alternative denoted by* $A = \left\{\widetilde{A}_1, \widetilde{A}_2, \widetilde{A}_3, \widetilde{A}_4\right\}$ *where* $\widetilde{A}_1, \widetilde{A}_2, \widetilde{A}_3,$ and $\widetilde{A}_4$ *respectively represents a computer industry, car industry, food industry and steel industry. A group of four experts* $\{E_1, E_2, E_3, E_4\}$ *is invited to evaluate these areas. Suppose ′λ′ stands for the weight vector of experts, i.e.,* $\lambda = \{0.24, 0.26, 0.3, 0.2\}$. *Five interconnected attribute are considered for evaluation denoted by* $\widetilde{B}_1, \widetilde{B}_2, \widetilde{B}_3, \widetilde{B}_4, \widetilde{B}_5$ *where* $\widetilde{B}_1, \widetilde{B}_2, \widetilde{B}_3, \widetilde{B}_4$ and $\widetilde{B}_5$ *respectively, represent the amount of the capital profit, market potential, the risk of the capital loss, the growth potential and the stability of the policy. Suppose "w" stands for different weights of attributes, i.e.,* $w = \{0.22, 0.24, 0.04, 0.4, 0.1\}$. *Keeping in view the inherent interrelationship structure, these five attributes are divided into two subsets* $F_1 = \left\{\widetilde{B}_1, \widetilde{B}_3, \widetilde{B}_5\right\}$, $F_2 = \left\{\widetilde{B}_2, \widetilde{B}_4\right\}$. *The information, which is to be evaluated in this example, consists of CPFNs as shown in Table 14.*

Table 14. Information based on cubic Pythagorean fuzzy sets (CPFS).

	$\widetilde{B}_1$	$\widetilde{B}_2$	$\widetilde{B}_3$	$\widetilde{B}_4$	$\widetilde{B}_5$
$\widetilde{A}_1$	$\left(\begin{pmatrix}[0.3,0.4],\\ [0.7,0.8]\end{pmatrix}, (0.4,0.9)\right)$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.5,0.7]\end{pmatrix}, (0.6,0.7)\right)$	$\left(\begin{pmatrix}[0.8,0.9],\\ [0.3,0.4]\end{pmatrix}, (0.7,0.5)\right)$	$\left(\begin{pmatrix}[0.3,0.4],\\ [0.7,0.8]\end{pmatrix}, (0.5,0.8)\right)$	$\left(\begin{pmatrix}[0.8,0.9],\\ [0.3,0.4]\end{pmatrix}, (0.8,0.3)\right)$
$\widetilde{A}_2$	$\left(\begin{pmatrix}[0.4,0.6],\\ [0.6,0.7]\end{pmatrix}, (0.5,0.8)\right)$	$\left(\begin{pmatrix}[0.5,0.6],\\ [0.6,0.7]\end{pmatrix}, (0.6,0.5)\right)$	$\left(\begin{pmatrix}[0.4,0.5],\\ [0.7,0.8]\end{pmatrix}, (0.4,0.7)\right)$	$\left(\begin{pmatrix}[0.8,0.9],\\ [0.3,0.4]\end{pmatrix}, (0.8,0.6)\right)$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.5,0.6]\end{pmatrix}, (0.17,0.6)\right)$
$\widetilde{A}_3$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.5,0.7]\end{pmatrix}, (0.8,0.6)\right)$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.5,0.6]\end{pmatrix}, (0.7,0.6)\right)$	$\left(\begin{pmatrix}[0.7,0.8],\\ [0.4,0.5]\end{pmatrix}, (0.6,0.7)\right)$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.6,0.8]\end{pmatrix}, (0.6,0.7)\right)$	$\left(\begin{pmatrix}[0.8,0.9],\\ [0.3,0.4]\end{pmatrix}, (0.6,0.7)\right)$
$\widetilde{A}_4$	$\left(\begin{pmatrix}[0.6,0.7],\\ [0.5,0.7]\end{pmatrix}, (0.8,0.4)\right)$	$\left(\begin{pmatrix}[0.5,0.8],\\ [0.4,0.6]\end{pmatrix}, (0.4,0.8)\right)$	$\left(\begin{pmatrix}[0.3,0.4],\\ [0.8,0.9]\end{pmatrix}, (0.5,0.8)\right)$	$\left(\begin{pmatrix}[0.4,0.5],\\ [0.5,0.7]\end{pmatrix}, (0.5,0.8)\right)$	$\left(\begin{pmatrix}[0.5,0.8],\\ [0.4,0.6]\end{pmatrix}, (0.4,0.8)\right)$

We still use the CIFBWM operators [23], CIFWA operators [24], CIFWG operators [24], CPFWA operators [25] and CPFWG operators [25], Wang et al. [26] explored the power Maclaurin symmetric mean operators based on Cq-ROFSs to compare with proposed method and the results are shown in Table 15. From Table 15, we can observe that the same evaluation data can be used for the different methods to get the different sorting results. In addition, the graphical representation of the information given in Table 15 can be seen from Figure 3.

Table 15. Comparison of established work with existing methods.

Methods	Score Values	Ranking Results
Kaur and Garg method [23]	Cannot be calculated	Cannot be calculated
Kaur and Garg method [24]	Cannot be calculated	Cannot be calculated
Abbas et al. Method [25]	$S(\widetilde{A}_1) = 0.476688587$, $S(\widetilde{A}_2) = 0.461869231$, $S(\widetilde{A}_3) = 0.485134865$, $S(\widetilde{A}_4) = 0.490034018$	$\widetilde{A}_4 > \widetilde{A}_3 > \widetilde{A}_1 > \widetilde{A}_2$
Wang et al. [26]	$S(\widetilde{A}_1) = 0.45222$, $S(\widetilde{A}_2) = 0.4443$, $S(\widetilde{A}_3) = 0.461213$, $S(\widetilde{A}_4) = 0.471$	$\widetilde{A}_4 > \widetilde{A}_3 > \widetilde{A}_1 > \widetilde{A}_2$
Proposed Method in this paper	$S(\widetilde{A}_1) = 0.469289377$, $S(\widetilde{A}_2) = 0.454639667$, $S(\widetilde{A}_3) = 0.473057851$, $S(\widetilde{A}_4) = 0.481443389$	$\widetilde{A}_4 > \widetilde{A}_3 > \widetilde{A}_1 > \widetilde{A}_2$

From Table 15, we can observe the same ranking order of alternative, which prove the validity of proposed work. The CIFWBM operators proposed by Kaur and Garg [23], CIFWA, CIFWG operators proposed by Kaur and Garg [24], Abbas et al. [25], Wang et al. [26] explored the power Maclaurin symmetric mean operators based on Cq-ROFSs can only evaluate the cubic intuitionistic fuzzy information. According to the definition of CIFS, we have the condition that CIFS must satisfy

$0 \leq \left(u^U\right) + \left(v^U\right) \leq 1$ for IVIFSs and $0 \leq u + v \leq 1$ for IFs. But the values given in the above information do not satisfy such condition, that is $(([0.6,\ 0.7], [0.5,\ 0.6]),\ (0.17,\ 0.6))$. Therefore, these values cannot be aggregated by CIFWBM, CIFWA and CIFWG operators, while the proposed Cq-ROFHM operators can aggregate this type of information. Therefore, the proposed method is superior to the existing methods. In addition, the CPFWA, and CPFWG operators proposed by Abbas et al. [25] cannot consider the interrelationship structure, while the Cq-ROFPHM operators proposed in this paper can deal with such type of situation. Therefore, the proposed work is more superior to existing methods.

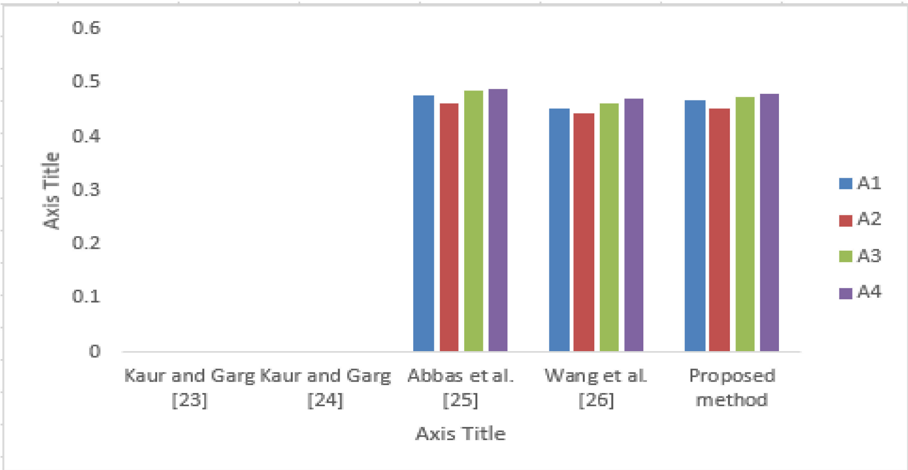

Figure 3. Graphical representation of comparison discussed in Table 15.

Example 6. *An investment company plans to invest in one area out of four areas as a set of alternative denoted by* $A = \left\{\widetilde{A}_1,\ \widetilde{A}_2,\ \widetilde{A}_3,\ \widetilde{A}_4\right\}$ *where* $\widetilde{A}_1,\ \widetilde{A}_2,\ \widetilde{A}_3,$ and $\widetilde{A}_4$ *respectively, represents the computer industry, car industry, food industry and steel industry. A group of four experts* $\{E_1,\ E_2,\ E_3, E_4\}$ *is invited to evaluate these areas. Suppose* ′λ′ *stands for the weight vector of experts, i.e.,* $\lambda = \{0.24,\ 0.26,\ 0.3,\ 0.2\}$. *Five interconnected attribute are considered for evaluation denoted by* $\widetilde{B}_1,\ \widetilde{B}_2,\ \widetilde{B}_3,\ \widetilde{B}_4,\ \widetilde{B}_5$ *where* $\widetilde{B}_1,\ \widetilde{B}_2,\ \widetilde{B}_3,\ \widetilde{B}_4$ and $\widetilde{B}_5$ *respectively, represent the amount of the capital profit, market potential, the risk of the capital loss, the growth potential and the stability of the policy. Suppose "w" stands for weight vector of attributes, i.e.,* $w = \{0.22,\ 0.24,\ 0.04,\ 0.4,\ 0.1\}$. *Keeping in view the inherent relationship structure, these five attributes are divided into two subsets* $F_1 = \left\{\widetilde{B}_1,\ \widetilde{B}_3,\ \widetilde{B}_5\right\}$, $F_2 = \left\{\widetilde{B}_2,\ \widetilde{B}_4\right\}$. *The information, which is to be evaluated in this example, consists of Cq-ROFNs as shown in Table 16.*

Table 16. Information based on Cq-ROFNs.

	$\widetilde{\mathbf{B}}_1$	$\widetilde{\mathbf{B}}_2$	$\widetilde{\mathbf{B}}_3$	$\widetilde{\mathbf{B}}_4$	$\widetilde{\mathbf{B}}_5$
$\widetilde{\mathbf{A}}_1$	$\left(\begin{pmatrix}[0.4, 0.7],\\ [0.3, 0.5]\end{pmatrix}, (0.4, 0.7)\right)$	$\left(\begin{pmatrix}[0.3, 0.5],\\ [0.4, 0.6]\end{pmatrix}, (0.6, 0.4)\right)$	$\left(\begin{pmatrix}[0.2, 0.6],\\ [0.6, 0.7]\end{pmatrix}, (0.3, 0.5)\right)$	$\left(\begin{pmatrix}[0.6, 0.7],\\ [0.1, 0.3]\end{pmatrix}, (0.5, 0.2)\right)$	$\left(\begin{pmatrix}[0.5, 0.6],\\ [0.6, 0.8]\end{pmatrix}, (0.9, 0.4)\right)$
$\widetilde{\mathbf{A}}_2$	$\left(\begin{pmatrix}[0.4, 0.6],\\ [0.5, 0.7]\end{pmatrix}, (0.7, 0.4)\right)$	$\left(\begin{pmatrix}[0.1, 0.2],\\ [0.5, 0.6]\end{pmatrix}, (0.9, 0.6)\right)$	$\left(\begin{pmatrix}[0.4, 0.5],\\ [0.6, 0.8]\end{pmatrix}, (0.2, 0.6)\right)$	$\left(\begin{pmatrix}[0.4, 0.5],\\ [0.4, 0.6]\end{pmatrix}, (0.7, 0.4)\right)$	$\left(\begin{pmatrix}[0.3, 0.5],\\ [0.1, 0.2]\end{pmatrix}, (0.8, 0.3)\right)$
$\widetilde{\mathbf{A}}_3$	$\left(\begin{pmatrix}[0.8, 0.9],\\ [0.2, 0.3]\end{pmatrix}, (0.5, 0.2)\right)$	$\left(\begin{pmatrix}[0.3, 0.4],\\ [0.1, 0.2]\end{pmatrix}, (0.8, 0.2)\right)$	$\left(\begin{pmatrix}[0.3, 0.4],\\ [0.7, 0.9]\end{pmatrix}, (0.3, 0.8)\right)$	$\left(\begin{pmatrix}[0.5, 0.6],\\ [0.1, 0.2]\end{pmatrix}, (0.7, 0.6)\right)$	$\left(\begin{pmatrix}[0.2, 0.3],\\ [0.5, 0.6]\end{pmatrix}, (0.6, 0.4)\right)$
$\widetilde{\mathbf{A}}_4$	$\left(\begin{pmatrix}[0.3, 0.4],\\ [0.5, 0.6]\end{pmatrix}, (0.5, 0.5)\right)$	$\left(\begin{pmatrix}[0.4, 0.6],\\ [0.2, 0.3]\end{pmatrix}, (0.8, 0.3)\right)$	$\left(\begin{pmatrix}[0.5, 0.7],\\ [0.4, 0.9]\end{pmatrix}, (0.6, 0.9)\right)$	$\left(\begin{pmatrix}[0.2, 0.3],\\ [0.1, 0.2]\end{pmatrix}, (0.6, 0.1)\right)$	$\left(\begin{pmatrix}[0.1, 0.3],\\ [0.1, 0.2]\end{pmatrix}, (0.6, 0.1)\right)$

We still use the CIFBWM operators [23], CIFWA operators [24], CIFWG operators [24], CPFWA operators [25] and CPFWG operators [25], Wang et al. [26] explored the power Maclaurin symmetric mean operators based on Cq-ROFSs to compare with proposed method and the results are shown in Table 17. From Table 17, we can observe that the same assessment information can be used for different methods to obtain the different desirable results. In addition, the geometrical picture of the information given in Table 17 can be seen from Figure 4.

Table 17. Comparison of established work with existing methods.

Method	Score Values	Ranking Results
Kaur and Garg Method [23]	Cannot be calculated	Cannot be calculated
Kaur and Garg Method [24]	Cannot be calculated	Cannot be calculated
Abbas et al. Method [25]	Cannot be calculated	Cannot be calculated
Wang et al. [26]	$S(\widetilde{A_1}) = 0.4788$, $S(\widetilde{A_2}) = 0.4712$, $S(\widetilde{A_3}) = 0.4799$, $S(\widetilde{A_4}) = 0.4789$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$
Proposed Method in this paper	$S(\widetilde{A_1}) = 0.47898376$, $S(\widetilde{A_2}) = 0.477264628$, $S(\widetilde{A_3}) = 0.482878293$, $S(\widetilde{A_4}) = 0.477720557$	$\widetilde{A_3} > \widetilde{A_1} > \widetilde{A_4} > \widetilde{A_2}$

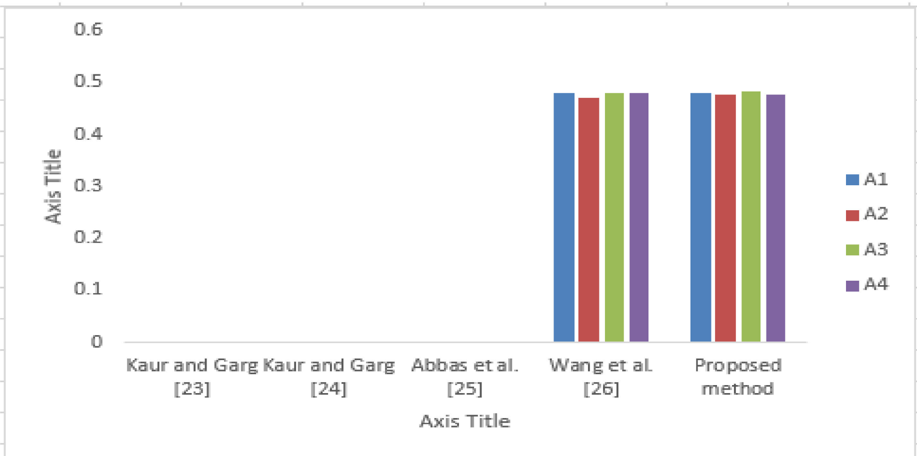

Figure 4. Graphical representations of comparison discussed in Table 17.

The CIFWBM operators proposed by Harish [23], CIFWA, CIFWG operators proposed by Harish [24] can only evaluate the cubic intuitionistic fuzzy information, while operators proposed by Abbas et al. [25] can only evaluate the cubic Pythagorean fuzzy information According to the definition of CIFS, we have the condition that CIFS must satisfy $0 \le (u^U) + (v^U) \le 1$ for IVIFSs and $0 \le u + v \le 1$ for IFSs while for CPFSs, we have the necessary condition $0 \le (u^U)^2 + (v^U)^2 \le 1$ for IVPFSs and $0 \le u^2 + v^2 \le 1$ for PFSs. However, the values which are given in the above information do not satisfy such condition, that is, $(([0.5,\ 0.7], [0.4,\ 0.9]),\ (0.6,\ 0.9))$. Therefore, these values cannot be aggregated by CIFWBM, CIFWA, CIFWGCPFWA and CPFWG operators, while the proposed Cq-ROFHM operators can aggregate this type of information. Therefore, the proposed Cq-ROFHM operators provide more space to decision makers and have a wider range of application.

6. Conclusions

Cq-ROFSs are the strong apparatus for the description of fuzzy data. CIFS and CPFSs are the particular cases of Cq-ROFSs. Cq-ROFSs provide more space to decision makers in MAGD problems because we have combined both q-ROFSs and IVq-ROFSs in one structure. In this paper, we have offered Cq-ROFHM and Cq-ROFWHM operators to analyze the cubic q-rung orthopair fuzzy information.

These operators cannot only aggregate cubic q-rung orthopair fuzzy information but also assume the interrelationship shape among attributes. Moreover, keeping in view the advantages of Cq-ROFHM and Cq-ROFWHM operators, Cq-ROFPHM and Cq-ROFWPHM can models the relationship among the attributes in more logical modes and can remove the effects of relationship between independent attributes on the assessment results. Further, we have discussed some properties of these aggregated operators and argued some special cases and developed algorithm for MAGDM problems using Cq-ROFNs based on these operators. Finally, we illustrated some examples to show the validity and advantages of established work by comparing with existing methods.

In further research, decision making problems [26–39] can be solved through this approach. In addition, some other operators can be developed on the base of this new proposed Cubic q-rung orthopair fuzzy set.

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