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Truss Sizing Optimization with a Diversity-Enhanced Cyclic Neighborhood Network Topology Particle Swarm Optimizer

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Abstract: This study presents a reliable particle swarm optimizer for sizing optimization of truss structures. This population-based stochastic optimization approach is based on the principle that each particle communicates its position and function value to a number of successively numbered neighboring particles via a fixed cyclic interaction structure. Therefore, such a neighborhood structure changes the movement pattern of the entire swarm, and allows each particle's movement not to be driven by one global best particle position, which enhances the diversification attitude. Further, by transforming the objective function, it is possible to steer the search towards feasible regions of design space. The efficiency of the proposed approach is demonstrated by solving four classical sizing optimization problems of truss structures.

Keywords: particle swarm optimization; constrained optimization; neighboring particle selection; neighborhood topology; truss sizing optimization

1. Introduction

Sizing optimization of truss structures is one of the most important topics in structural optimization [1–8]. In these problems, the cross-sectional areas of various elements are included as design variables, while the structure layout and topology are fixed. Efficient optimization algorithms are needed to find the global optimum, satisfying optimization constraints. A number of mathematical programming methods, such as sequential unconstrained minimization technique [9], feasible directions method [10,11], moving asymptotes method [12], and sequential quadratic programming [13] exhibit fast convergence and high accuracy. However, they inherently exploit specific problem-dependent properties, such as differentiability and convexity. These may limit their applicability to truss optimization problems [5].

Meta-heuristic optimization methods, including genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO), simulated annealing (SA), differential evolution (DE), harmony search (HS) teaching-learning-based optimization (TLBO), and ray optimization (RO), underwent a tremendous surge in popularity and applicability over the past few decades. However, employing meta-heuristic methods in structural optimization problems may be very cumbersome, as these techniques were originally developed for unconstrained problems. In such cases, various constraint-handling methods are available. For example, there are the modified feasible-based mechanism [1,3], penalty approach [4,14], augmented Lagrange multiplier approach [5,15], violated design points redirection [7,16], fly-back mechanism method [8], problem reformulation as an unconstrained multi-objective formulation [17,18], and serial sequential quadratic programming sub-problems [19]. However, because these methods may not guarantee that strict constraints will be satisfied, their use in real-life problems may be limited [5].

Details of conventional constraint handling schemes can be found in Jansen [5] and Saka [20], and some state-of-the-art meta-heuristic algorithms used in structural optimization problems were recently reviewed by Lamberti [21] and Saka [20].

The PSO algorithm has proven itself capable of finding global optima in a variety of engineering applications, including structural design problems [5,14,22–25]. An important factor facilitating reliable global searches in structural optimization is to keep an adequate balance between intensification and diversification [2,26]. Intensification searches around the current best region serve to refine and select the best candidate solution, whereas diversification allows different regions of the design space to be explored efficiently, thus assisting the global search process to find a good optimum. These two tasks are somewhat conflicting but equally important; hence, a balance between these two objectives must be achieved. It should be noted that most population-based meta-heuristic algorithms contract the search space as they approach the global optimum. However, after performing a certain number of swarm movements, particles may be trapped in a region that does not even contain local optima. A common problem with PSO is the poor diversification of particles, which results in a strong tendency to premature convergence. In standard PSO, poor diversification is mainly caused by the social structure of the swarm, the so called “neighborhood topology.” The movement of each particle is guided by the collective information generated by other particles connected with it. One of the most common topologies is the star topology, in which the neighborhood of each individual is the entire swarm. This type of fully connected neighborhood topology allows relatively good solutions to be found, and exhibits faster convergence than other neighborhood topologies. However, because of poor exploration performance, this may frequently result in premature convergence to local optima. To overcome this limitation, the ring neighborhood topology, in which each particle is only influenced by its two immediately adjacent neighbors, was developed. As the ring topology tends to reduce the convergence rate of the swarm, its exploration ability is improved. However, this topology has a serious drawback in the higher computation time required by the optimization process. Hence, it appears that one of the most important issues in complex structural optimization problems is to determine the best connection between neighboring particles, while ensuring a good balance of intensification and diversification.

The aim of this study is to develop an easy-to-use and reliable PSO formulation for the sizing optimization of truss structures. The key improvement of our novel PSO scheme is that the population diversity results are derived mainly from distributed social learning among each particle’s neighborhood, rather than learning from only one global best particle in the whole swarm. This neighborhood structure makes each particle to exchange some information via a fixed cyclic interaction structure that contains a series of successively numbered particles with itself as the center, which is repeated for all particles in the swarm in a cyclic manner. Thus, each particle belongs to at least two neighborhood groups of nearby particles so the successful global-best search history of a particle has an indirect effect on the social interactions among all particles. In this structure, a particle’s neighbors are not necessarily particles that are close to each other in hyperdimensional search space, but instead this refers to particles that share information related to their individual fitness values. From this viewpoint, the proposed optimizer is called as a cyclic neighborhood network topology particle swarm optimizer, or CNNT-PSO. The constraint-handling strategy utilized in this research relies on a simple transformation of the original cost function of the optimization problem. This is converted into a pseudo-function that ensures all particles move toward a feasible region of the search space from the beginning of the optimization process. Compared with conventional constraint handling methods, this is relatively simple to implement, and does not require problem-dependent or user-defined parameters such as penalty factors or Lagrange multipliers. This PSO scheme has recently been applied by some of the present authors to the synthesis of fixed-structure robust controllers [27]. In this paper, we carry out a comprehensive analysis of its performance in complicated constrained optimization tasks such as structural optimization problems. The numerical efficiency of the proposed

approach is demonstrated by the results obtained in four classical weight-minimization problems of planar and spatial truss structures.

The remainder of this study is organized as follows. Section 2 presents the formulation of the structural optimization problem with various types of constraints; the proposed meta-heuristic search engine is also described. The optimization results are discussed in Section 3, where extensive comparisons with methods in the literature are presented, and the sensitivity of the algorithm to the neighborhood size (i.e., the number of neighboring particles) is analyzed. Finally, Section 4 summarizes the main findings of the study and outlines directions for future research.

2. Description of the Optimization Framework

In meta-heuristic optimization, social diversity is usually defined as the level of dispersion of candidate solutions around the search space. PSO and most other population-based methods contract the search space to increase the probability of obtaining a fitter population. The basic swarm movement pattern usually forces the whole population to move as a single group toward a promising region of the contracting search space. However, this may cause all particles to become trapped in a region far from the optimum [28,29]. In view of this, the main scope of the present study is to develop a simple diversity-enhanced CNNT-PSO scheme for truss-size optimization.

The minimum weight problem for a truss structure can be formulated as follows:

$$\min_{A:=\{A_1, A_2, \dots, A_{ng}\} \in \mathbb{F}} W(A) := \sum_{k=1}^{ng} \left(A_k \cdot \sum_{l=1}^{mk} \rho_l L_l \right) \quad (1)$$

subject to

$$\mathbb{F} := \{A \in \mathbb{R}^{ng} \mid h_1(A) \leq \mathbf{0}, h_2(A) \leq \mathbf{0}, h_3(A) \leq \mathbf{0}, h_4(A) \leq \mathbf{0}\}, \quad (2)$$

where A_k is the cross-sectional area of members belonging to the k th group, ρ_l is the density of the l th member in the k th group, L_l is the length of the l th member in the k th group, ng is the total number of member groups, mk is the total number of members in the k th group, and $\mathbf{0}$ is the zero vector of adequate dimensions. In (2), $h_1(A)$ and $h_2(A)$ include stress and buckling constraints for the i th truss member, respectively, and $h_3(A)$ and $h_4(A)$ include nodal displacement and cross-sectional area constraints, respectively. Details of the above constraint conditions are as follows:

$$h_1(A) = \begin{bmatrix} \sigma_{\min,1} - \sigma_1 \\ \sigma_{\min,2} - \sigma_2 \\ \vdots \\ \sigma_{\min,nm} - \sigma_{nm} \\ \sigma_1 - \sigma_{\max,1} \\ \sigma_2 - \sigma_{\max,2} \\ \vdots \\ \sigma_{nm} - \sigma_{\max,nm} \end{bmatrix}, \quad h_2(A) = \begin{bmatrix} \sigma_1^b - \sigma_1 \\ \sigma_2^b - \sigma_2 \\ \vdots \\ \sigma_{ncm}^b - \sigma_{ncm} \\ \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_{ncm} \end{bmatrix}, \quad h_3(A) = \begin{bmatrix} \delta_{\min,1} - \delta_1 \\ \delta_{\min,2} - \delta_2 \\ \vdots \\ \delta_{\min,nm} - \delta_{nm} \\ \delta_1 - \delta_{\max,1} \\ \delta_2 - \delta_{\max,2} \\ \vdots \\ \delta_{nm} - \delta_{\max,nm} \end{bmatrix}, \quad h_4(A) = \begin{bmatrix} A_{\min,1} - A_1 \\ A_{\min,2} - A_2 \\ \vdots \\ A_{\min,ng} - A_{ng} \\ A_1 - A_{\max,1} \\ A_2 - A_{\max,2} \\ \vdots \\ A_{ng} - A_{\max,ng} \end{bmatrix}, \quad (3)$$

where nm is the number of elements, ncm is the number of elements subject to compression, nn is the number of nodes; σ_j is the stress in the j th element, σ_j^b is the allowable buckling stress for the j th member subject to compression, δ_j is the nodal displacement of the j th free node, and $\{\cdot\}_{\max}$, $\{\cdot\}_{\min}$ respectively denote upper and lower bounds for stress, displacement, and cross-sectional area.

The proposed CNNT-PSO algorithm proceeds as follows.

Step 0. Transformation of the minimum weight problem into a pseudo-function-based problem.

The CNNT-PSO algorithm considers the constraint conditions of (2) to calculate the fitness value of each particle and determine the optimal design-variable vector $\mathbf{x}^* := \{A_1^*, A_2^*, \dots, A_{ng}^*\} \in \mathbb{F}$. Our constraint-handling strategy relies on a simple transformation of (1) subject to (2) into an

unconstrained optimization problem using a pseudo-function such as $W_v(x) := \arctan\{W(x)\} - \pi/2$. The modified unconstrained problem is then formulated as

$$\min_{x \in \mathbb{R}^n} \mathcal{W}(x) := \begin{cases} h_{\max}(x), & \text{if } h_{\max}(x) := \max[h_1(x), h_2(x), \dots, h_{2(nm+ncm+nn+ng)}(x)] > 0; \\ W_v(x), & \text{otherwise,} \end{cases} \quad (4)$$

where $h_j(\cdot)$ denotes any of the constraint function vectors $h_1(x)$, $h_2(x)$, $h_3(x)$ and $h_4(x)$ in (2). Because $W_v(x) < 0$ for any x , the condition, $W_v(x) < h_{\max}(x)$, holds for an infeasible solution $x \notin \mathbb{F}$ satisfying $h_{\max}(x) \geq 0$. The present approach ensures that all particles move towards the feasible design space since the very beginning of the optimization process. Because PSO requires neither a derivable cost function nor an explicit relationship between cost function and constraints, the $\mathcal{W}(x)$ functional can be used regardless of the nature of the cost function and the type of constraints.

Remark 1. A pseudo-objective function $W_v(x)$ can be any function for which the following conditions simultaneously hold: (i) $W_v(x) < 0$ for any x yielding $h_j(x) \leq 0$ ($j = 1, 2, \dots, 2(nm + ncm + nn + ng)$); (ii) $W_v(x_i) \leq W_v(x_j)$, whenever $W(x_i) \leq W(x_j)$. The function $W_v(x) := \arctan\{W(x)\} - \pi/2$ can be used for various constrained optimization problems.

Step 1. Initialization of PSO parameters and start of the optimization procedure.

Let $\mathbb{D}$ denote the hyperdimensional search space of the design vector $x_i^\ell := \{A_1^\ell, A_2^\ell, \dots, A_{ng}^\ell\} \in \mathbb{R}^{ng}$, where $i = 1, 2, \dots, n_p$ is the particle index, $\ell (= 0, 1, \dots, \ell_{\max})$ is the iteration number, and $\ell_{\max}$ is the maximum number of searches. Initialize n_p particles with randomly selected positions $x_i^0 := \{A_1^0, A_2^0, \dots, A_{ng}^0\} \in \mathbb{D}$ and velocities $v_i^0 = 0 \in \mathbb{R}^{ng}$. Let $x_{pbest,i}^\ell$ denote each particle's best previous position, that is, that which yields the minimum fitness value of the modified cost function $\mathcal{W}(\cdot)$ in (4). This initial value is $x_{pbest,i}^0 = x_i^0$. Next, we introduce $x_{sbest,i}^\ell$, which denotes the best position in the social neighborhood of the i th particle in the current iteration ℓ . Mathematically, this can be written as

$$x_{sbest,i}^\ell := \arg \min_{x \in \{x_j^\ell \mid j=i-\frac{n_s}{2}, \dots, i+\frac{n_s}{2}\}} \mathcal{W}(x), \quad (5)$$

where “arg min” denotes the set of points at which the cost function $\mathcal{W}(\cdot)$ attains its minimum value, the (even) parameter $n_s (\leq n_p)$ is the number of neighbors of the i th particle, and $x_j^\ell := x_{(j-1 \bmod n_p)+1}^\ell$ for $j < 1$ or $n_p + 1 \leq j$. Therefore, the initial $x_{sbest,i}^0$ is set using a previously determined n_s as $x_{sbest,i}^0 := \arg \min_{x \in \{x_j^0 \mid j=i-\frac{n_s}{2}, \dots, i+\frac{n_s}{2}\}} \mathcal{W}(x)$.

The overall schematic representation of the proposed CNNT-PSO with $n_p = 9$ and $n_s = 4$ is shown in Figure 1. This cyclic network topology-based neighborhood structure forces each particle to share information via a fixed near-neighbor interaction structure containing a series of successively numbered particles, with itself at the center (e.g., $x_{i-\frac{n_s}{2}}, x_{i-\frac{n_s}{2}+1}, \dots, x_{i+\frac{n_s}{2}}$).

Remark 2. The standard PSO algorithm adopts the following form for the unique global best position found by the entire swarm:

$$x_{gbest}^\ell := \arg \min_{x \in \{x_i^\ell \mid i=1, 2, \dots, n_p, j=1, 2, \dots, \ell\}} \mathcal{W}(x). \quad (6)$$

This means that $x_{gbest}^\ell := \min[x_{pbest,1}^\ell, x_{pbest,2}^\ell, \dots, x_{pbest,n_p}^\ell]$; that is, x_{gbest}^ℓ depends on all of the available $x_{pbest,i}^\ell$ in each iteration. This uniqueness of x_{gbest}^ℓ may be the main cause of the strong tendency of PSO to converge prematurely. To overcome this limitation, the particles' flexibility is increased by introducing an additional diversity-boosting tool that exploits each particle's local social learning based on the concept of the cyclic network topology-based neighborhood structure.

Figure 1. Explanation of the CNNT-PSO search mechanism ($n_p=9$; $n_s=4$).

Step 2. Apply the CNNT-PSO algorithm to update the positions x_i^ℓ and velocities v_i^ℓ of all particles.

After initializing x_i^0 , v_i^0 , $x_{pbest,i}^0$, and $x_{sbest,i}^0$ in Step 1, the following position and velocity update laws are applied to all particles ($i = 1, 2, \dots, n_p$):

$$v_i^{\ell+1} \leftarrow \chi \left[v_i^\ell + c_1 r_{1,i}^\ell (x_{pbest,i}^\ell - x_i^\ell) + c_2 r_{2,i}^\ell (x_{sbest,i}^\ell - x_i^\ell) \right], \quad (7)$$

$$x_i^{\ell+1} \leftarrow x_i^\ell + v_i^{\ell+1}, \quad (8)$$

where $r_{1,i}^\ell$ and $r_{2,i}^\ell$ denote random numbers generated uniformly within $[0, 1]$, c_1 is the cognitive scaling factor, c_2 is the social scaling factor, and χ is the constriction factor, defined as

$$\chi = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \quad (9)$$

for $\varphi := c_1 + c_2 (> 4)$ [30]. In this paper, c_1 and c_2 are set to 2.05. Next, we set $\ell \leftarrow \ell + 1$ and update $x_{pbest,i}^\ell$ and $x_{sbest,i}^\ell$ as follows:

$$x_{pbest,i}^\ell \leftarrow \arg \min_{x \in \{x_j^\ell \mid j=1,2,\dots,\ell\}} \mathcal{W}(x), \quad x_{sbest,i}^\ell \leftarrow \arg \min_{x \in \{x_j^\ell \mid j=i-\frac{n_s}{2}, \dots, i+\frac{n_s}{2}\}} \mathcal{W}(x). \quad (10)$$

We now briefly discuss some distinctive features of our CNNT-PSO scheme. Unlike standard PSO, the third term of (7) implies social cooperation in the neighborhood of the i th particle. Hence, the main novelty introduced by the present approach is that population diversity is derived from a process of distributed social learning in each particle's neighborhood, rather than learning from only the global best particle x_{gbest}^ℓ , as in the canonical star topology. Furthermore, because each particle belongs to at least two neighborhood groups of nearby particles, the successful global-best search history of a particle will indirectly affect its social interactions.

Remark 3. Because of the probabilistic nature and problem-dependent characteristics of PSO, it may be difficult to set absolute criteria that ensure the choice of n_s gives a good balance between the particle's global search capacity and the convergence rate of the swarm. The ratio n_s/n_p is a reliable indicator of the efficiency of the optimization search. If n_s/n_p is small, the successful social-best search history of a particle is slowly conveyed to particles belonging to other neighboring groups. This may lead to slow down convergence of particles toward the true solution, but also to considerably reduce the risk of premature convergence. Conversely, if n_s/n_p is large, particles gather lots of information from many other neighboring particles: the convergence rate will be high, but there will be the risk of premature convergence. The sensitivity of the CNNT-PSO algorithm to the number of neighboring particles will be analyzed in great detail in the next section.

Remark 4. The proposed CNNT-PSO scheme and other modern optimization algorithms require the setting of a large number of parameters to optimize their performance. However, manually exploring the resulting

combinatorial space of parameter settings may be tedious and tends to lead to unsatisfactory outcomes. From this viewpoint, *Irace* packag [31] and *SMAC* (sequential model-based algorithm configuration) [32] can be used as useful tools for finding the best parameter settings of an optimizer.

Step 3. Termination criterion.

If the user-defined termination criterion (e.g., $\ell > \ell_{\max}$) of the optimization process is satisfied, the iteration of swarm movements following CNNT-PSO stops, and the optimum design is determined as follows:

$$\mathbf{x}^* (= \{A_1^*, A_2^*, \dots, A_{ng}^*\}) := \arg \min_{\mathbf{x} \in \{x_i^j \mid i=1,2,\dots, np; j=1,2,\dots, \ell\}} \mathcal{L}(\mathbf{x}). \quad (11)$$

If the termination criterion is not satisfied, return to Step 2.

3. Test Problems and Optimization Results

The CNNT-PSO algorithm described in Section 2 was tested in four classical weight minimization problems of truss structures including sizing variables. A total of 30 independent runs were carried out for each neighborhood size, where all runs used different initial populations to properly account for the stochastic nature of the optimizer. Statistical data were compared with results in the literature, in particular with Degertekin [1,3] that presented an extensive survey of the optimization results obtained using various meta-heuristic methods (see Table 1). In addition, a sensitivity analysis on the effect of n_s was carried out. The proposed PSO algorithm was implemented in MATLAB Version 7.14, and computations were executed on a PC with a 3.4 GHz Intel Core i7 processor and 8 GB RAM.

Table 1. Survey of literature studies that have analyzed the test cases considered in this research.

Problem	Investigated Optimization Techniques
10-bar planar truss (Cases 1 and 2)	Harmony search (HS) algorithm [33]
	Particle swarm optimization (PSO) [7,8]
	Particle swarm optimization with passive congregation (PSOPC) [8]
	Heuristic particle swarm optimization (HPSO) [8]
	Heuristic particle swarm ant colony optimization (HPSACO) [34]
	Improved harmony search (IHS) algorithm [35]
	Artificial bee colony algorithm with an adaptive penalty function approach (ABC-AP) [36]
	Efficient harmony search (EHS) algorithm [3]
	Self-adaptive harmony search (SAHS) algorithm [3]
	Teaching-learning-based optimization (TLBO) [1]
25-bar spatial bar	Harmony search (HS) algorithm [33]
	Particle swarm optimization (PSO) [8]
	Particle swarm optimization with passive congregation (PSOPC) [8]
	Heuristic particle swarm optimization (HPSO) [8]
	Big bang-big crunch (BB-BC) optimization [37]
	Hybrid big bang-big crunch (HBB-BC) optimization [38]
	Improved harmony search (IHS) algorithm [35]
	Corrected multi-level & multi-point simulated annealing (CMLPSA) algorithm [6]
	Efficient harmony search (EHS) algorithm [3]
	Self-adaptive harmony search (SAHS) algorithm [3]
72-bar spatial truss	Teaching-learning-based optimization (TLBO) [1]
	Harmony search (HS) algorithm [33]
	Particle swarm optimization (PSO) [7,8]
	Particle swarm optimization with passive congregation (PSOPC) [8]
	Heuristic particle swarm optimization (HPSO) [8]
	Big bang-big crunch (BB-BC) optimization [37]
	Hybrid big bang-big crunch (HBB-BC) optimization [38]
	Efficient harmony search (EHS) algorithm [3]
	Self-adaptive harmony search (SAHS) algorithm [3]
	Teaching-learning-based optimization (TLBO) [1]
200-bar planar truss	Harmony search (HS) algorithm [33]
	Particle swarm optimization with passive congregation (PSOPC) [34]
	Heuristic particle swarm ant colony optimization (HPSACO) [34]
	Corrected multi-level & multi-point simulated annealing (CMLPSA) algorithm [6]
	Efficient harmony search (EHS) algorithm [3]
	Self-adaptive harmony search (SAHS) algorithm [3]
	Teaching-learning-based optimization (TLBO) [1]

3.1. Planar 10-Bar Truss Structure

The first test problem regards the 10-bar truss shown in Figure 2. The Young's modulus and material density of the truss members are 10 Msi and 0.1 lb/in³, respectively. The cross-sectional area of each member is included as a design variable. Two variants of this problem were considered: (1) Case 1: $P_1 = 100$ kips and $P_2 = 0$; (2) Case 2: $P_1 = 150$ kips and $P_2 = 50$ kips. The cross-sectional areas of elements were permitted to vary between 0.1 and 35 in². The optimization problem includes 22 nonlinear constraints on element stresses (stress limit in tension and compression is 25,000 psi) and nodal displacements (± 2 in in both coordinate directions X and Y).

A population of 250 individuals was used in this test case. The initial population in each optimization run was generated randomly. The maximum number of iterations $\ell_{\max}$ was set to 500. Tables 2 and 3 compare the optimization results with literature for Cases 1 and 2, respectively. The algorithm's performance was compared on a statistical basis by considering the best, worst, and average weight obtained in the optimization runs. The feasibility of all optimized designs listed in the tables was checked with MATLAB and ANSYS. The present PSO algorithm always found optimized designs consistent with the best designs reported in the literature. However, the proposed algorithm was definitely the most robust optimizer in terms of the dispersion of the optimized weight. Furthermore, some of the algorithms taken as the basis of comparison with CNNT-PSO converged to slightly infeasible designs. Tables 4 and 5 analyze the effect of the neighborhood size on the performance of CNNT-PSO. Remarkably, the proposed algorithm always found a feasible optimized design for each of the 30 optimization runs, regardless of the value of n_s/n_p . It appears that setting the neighborhood size to twice the number of design variables allows the optimal solution to be obtained. The convergence characteristics of the objective function of 30 independent optimization runs are illustrated in Figure 3. Tables 4 and 5 show that the best solution is far less sensitive than the worst solution to the number of neighboring particles. CNNT-PSO always converged to the best solution overall when the n_s/n_p ratio was less than or close to 10%. A similar trend can be observed in the other truss design problems. However, if n_s is very small (e.g., n_s/n_p approximately 4% in this test case), more iterations may be required to converge to the global optimum.

Figure 2. Schematic of the planar 10-bar truss structure.

Figure 3. Convergence curves for the 10-bar truss problem ($n_s = 20$).

Table 2. Comparison of optimization results with literature: Case 1 of 10-bar truss problem.

Optimal Design Variables A_i (in ²)	Lee & Geem: HS [33]	Li et al. [8]		Perez & Behdinan: PSO [7]	Kaveh & Talatahari: HPSACO [34]	Lamberti & Pappalettere: IHS [35]	Degertekin [3]		Sonmez [36] ABC-AP	Degertekin & Hayalioglu: TLBO [1]	This Study ($n_s = 20$) CNNT-PSO
		PSOPC	HPSO				EHS	SAHS			
A_1	30.15	30.569	30.704	33.500	30.307	30.5222	30.208	30.394	30.548	30.4286	30.5171
A_2	0.102	0.100	0.100	0.100	0.100	0.1000	0.100	0.100	0.100	0.1000	0.1000
A_3	22.71	22.974	23.167	22.766	23.434	23.2005	22.698	23.098	23.180	23.2436	23.1999
A_4	15.27	15.148	15.183	14.417	15.505	15.2232	15.275	15.491	15.218	15.3677	15.2198
A_5	0.102	0.100	0.100	0.100	0.100	0.1000	0.100	0.100	0.100	0.1000	0.1000
A_6	0.544	0.547	0.551	0.100	0.5241	0.5513	0.529	0.529	0.551	0.5751	0.5512
A_7	7.541	7.493	7.460	7.534	7.4365	7.4572	7.558	7.488	7.463	7.4404	7.4574
A_8	21.56	21.159	20.978	20.467	21.079	21.0367	21.559	21.189	21.058	20.9665	21.03931
A_9	21.45	21.156	21.508	20.392	21.229	21.5288	21.491	21.342	21.501	21.5330	21.5310
A_{10}	0.100	0.100	0.100	0.100	0.100	0.1000	0.100	0.100	0.100	0.1000	0.1000
Best weight (lb)	5057.88	5061.00	5060.92	5024.21	5056.56	5060.82	5062.39	5061.42	5060.880	5060.96	5060.8540
Worst weight	N/A	N/A	N/A	N/A	N/A	N/A	5065.83	5063.39	N/A	5063.23	5060.8743
Average weight	N/A	N/A	N/A	N/A	N/A	N/A	5063.73	5061.95	N/A	5062.08	5060.8581
Standard deviation	N/A	N/A	N/A	N/A	N/A	N/A	1.98	0.71	N/A	0.79	0.0054
Analysis of the optimization results using ANSYS/MATLAB programs											
Weight (lb)	5058.336	5040.669	5060.906	5024.248	5056.59	5060.93	5062.39	5061.275	5060.888	5060.956	5060.8540
Feasibility	Infeasible	Infeasible	Feasible	Infeasible	Infeasible	Feasible	Feasible	Feasible	Feasible	Feasible	Feasible
Infeasible node displacement (in) ¹	−2.0018 (1st node)	−2.0085 (1st node)	None	−2.0389 (1st node)	−2.002 (1st node)	None	None	None	None	None	None
Infeasible bar stress (ksi) ²	None	25.00118 (5th bar)	None	25.01712 (5th bar)	None	None	None	None	None	None	None

¹ An example infeasible node displacement is shown in this and following tables. ² A typical stress example from the infeasible cases is shown in this and following tables.

Table 3. Comparison of optimization results with literature: Case 2 of 10-bar truss problem.

Optimal Design Variables A_i (in ²)	Lee & Geem: HS [33]	Li et al. [8]		Kaveh & Talatahari: HPSACO [34]	Degertekin [3]		Sonmez [36] ABC-AP	Degertekin & Hayalioglu: TLBO [1]	This study ($n_s = 20$) CNNT-PSO
		PSOPC	HPSO		EHS	SAHS			
A_1	23.250	23.473	23.353	23.194	23.589	23.525	23.4692	23.524	23.5186
A_2	0.1020	0.101	0.100	0.100	0.101	0.100	0.1005	0.1000	0.1000
A_3	25.730	25.287	25.502	24.585	25.422	25.429	25.2393	25.441	25.2897
A_4	14.510	14.413	14.250	14.221	14.488	14.488	14.354	14.479	14.3685
A_5	0.1000	0.100	0.100	0.100	0.100	0.100	0.1001	0.1000	0.1000
A_6	1.9770	1.969	1.972	1.969	1.975	1.992	1.9701	1.995	1.9697
A_7	12.210	12.362	12.363	12.489	12.362	12.352	12.4128	12.334	12.3894
A_8	12.610	12.694	12.984	12.925	12.682	12.698	12.8925	12.689	12.8299
A_9	20.360	20.323	20.356	20.952	20.322	20.341	20.3343	20.354	20.3371
A_{10}	0.100	0.103	0.1010	0.101	0.100	0.100	0.1000	0.1000	0.1000
Best weight (lb)	4668.81	4677.70	4677.29	4675.78	4679.02	4678.84	4677.077	4678.31	4676.9239
Worst weight	N/A	N/A	N/A	N/A	4684.28	4682.26	N/A	4681.23	4679.2153
Average weight	N/A	N/A	N/A	N/A	4681.61	4680.08	N/A	4680.12	4677.0500
Standard deviation	N/A	N/A	N/A	N/A	2.51	1.89	N/A	1.016	0.4491
Analysis of the optimization results using ANSYS/MATLAB									
Weight (lb)	4669.365	4667.76	4681.93	4675.797	4679.0148	4678.8476	4677.0754	4678.3149	4676.9239
Feasibility	Infeasible	Infeasible	Infeasible	Infeasible	Feasible	Feasible	Feasible	Feasible	Feasible
Infeasible node	−2.0039	−2.005	None	−2.00158	None	None	None	None	None
displacement (in)	(2nd node)	(2nd node)		(2nd node)					
Infeasible bar	25.04062	25.00876	25.07655	25.00189	None	None	None	None	None
stress (ksi)	(5th bar)	(6th bar)	(5th bar)	(6th bar)					

Table 4. Sensitivity of optimized weight to neighborhood size: Case 1 of 10-bar truss problem.

Neighbor Size: n_s		Objective Function Value			
(Ratio of n_s to n_p)		Best	Average	Worst	Standard Deviation
10	(4.0%)	5060.9167908101	5061.2950060723	5062.0421888195	0.2907511199
20	(8.0%)	5060.8540025711	5060.8580488412	5060.8742859840	0.0054030957
40	(16.0%)	5060.8536755940	5061.4278657355	5076.6697042583	2.8820669209
80	(32.0%)	5061.0075312501	5093.2458964858	5173.6500952290	30.7951400369
140	(56.0%)	5061.8921893369	5168.3643472361	5409.6582554952	85.3588142997
200	(80.0%)	5097.2427471776	5282.2050683960	5778.8410590943	189.9139754508
250	(100.0%)	5066.4836471986	5336.3103411734	5895.1836530033	180.8318776546

Table 5. Sensitivity of optimized weight to neighborhood size: Case 2 of 10-bar truss problem.

Neighbor Size: n_s		Objective Function Value			
(Ratio of n_s to n_p)		Best	Average	Worst	Standard Deviation
10	(4.0%)	4677.1658402493	4678.2375183454	4680.5386535189	0.9051300720
20	(8.0%)	4676.9238853154	4677.0500355970	4679.2153171228	0.4490860051
40	(16.0%)	4676.9228600423	4677.2863340068	4680.5057282055	0.8084333711
80	(32.0%)	4679.1222079698	4717.6958257250	4846.8397665392	39.7234590669
140	(56.0%)	4690.6747765927	4876.4393422376	5155.7746654222	144.7862819775
200	(80.0%)	4721.7716442903	5011.4643856148	5512.8166815800	210.2604407706
250	(100.0%)	4825.5787830425	5176.9813415952	5635.2466296128	232.7394293553

3.2. Spatial 25-Bar Truss Structure

The second test problem concerns the spatial 25-bar truss tower shown in Figure 4. Elements were divided into 8 groups, as shown in Table 6, which also lists the allowable compressive/tensile stress limits for each group. The Young's modulus is 10 Msi and the material density is 0.1 lb/in³. The structure was subjected to two independent loading conditions:

- Condition 1:* 20 kips acting in the positive Y-direction and 5 kips acting in the negative Z-direction at node 1, 20 kips acting in the negative Y-direction and 5 kips acting in the negative Z-direction at node 2;
- Condition 2:* 1 kip acting in the positive X-direction at node 1, 0.5 kip acting in the positive X-direction at nodes 3 and 6, 10 kips acting in the positive Y-direction at nodes 1 and 2, 5 kips acting in the negative Z-direction at nodes 1 and 2.

The cross-sectional areas of each group of elements were permitted to vary between 0.01 and 3.4 in². The displacements of all free nodes in the coordinate directions X, Y and Z must be less than ± 0.35 in. Therefore, the optimization problem includes 8 sizing variables and 110 nonlinear constraint conditions.

CNNT-PSO parameters were set as follows: $n_p = 200$ and $\ell_{\max} = 500$. Feasible solutions were obtained for all 30 trials. The optimization results are compared with literature in Table 7. It can be seen that the present algorithm was the most efficient optimizer in terms of optimized weight and robustness. The statistical data given in Table 8 confirm that the optimal size of the neighborhood is about 10% of the population size. Figure 5 shows the convergence characteristics of the objective function for 30 independent optimization runs. However, for very small or large values of n_s , CNNT-PSO could not achieve an adequate trade-off between population diversity enhancement and convergence speed.

Figure 4. Schematic of the spatial 25-bar truss structure.

Table 6. Member grouping and stress limits for the spatial 25-bar truss problem.

Member Group	Compressive Stress Limit, Ksi (MPa)	Tensile Stress Limit, Ksi (MPa)
1 A_1	35.092 (241.96)	40.0 (275.80)
2 $A_2 \sim A_5$	11.590 (79.913)	40.0 (275.80)
3 $A_6 \sim A_9$	17.305 (119.31)	40.0 (275.80)
4 $A_{10} \sim A_{11}$	35.092 (241.96)	40.0 (275.80)
5 $A_{12} \sim A_{13}$	35.092 (241.96)	40.0 (275.80)
6 $A_{14} \sim A_{17}$	6.7590 (46.603)	40.0 (275.80)
7 $A_{18} \sim A_{21}$	6.9590 (47.982)	40.0 (275.80)
8 $A_{22} \sim A_{25}$	11.082 (76.410)	40.0 (275.80)

Remark 5. In the sizing optimization of skeletal structures, the computational cost is generally determined by the total number of function evaluations that is the product of the population size (n_p) and the number of optimization iterations ($\ell_{\max}$). This is because each computation of $\mathcal{W}(\mathbf{x})$ in (4) entails a structural analysis to check whether the optimization constraints are satisfied. Hence, although the proposed CNNT-PSO algorithm includes a large number of particles, it does not necessarily have a high computational cost. For example, in the 10-bar planar truss problem, Degertekin [3] set the HMS (harmony memory size, which corresponds to population size in PSO) to 20, and obtained optimum designs from EHS and SAHS after about 12,000 and 8000 optimization iterations, respectively. Therefore, the objective function was evaluated 240,000 times for EHS and 160,000 times for SAHS. However, using $n_s = 250$ and $\ell_{\max} = 500$, the proposed CNNT-PSO method requires the objective function to be evaluated 125,000 times, which is 48% fewer than EHS and 22% fewer than SAHS. Further, the number of evaluations of the objective function in Degertekin [1], which used a population size of 30 and 16,872 optimization iterations, was 506,160, which is about 4 times more than with CNNT-PSO. For the 25-bar spatial truss problem, the number of objective function evaluations performed by the proposed method was between 20.2 and 35.7% of those in Degertekin [3] and Degertekin [1]. On the other hand, $\mathcal{W}(\mathbf{x})$ in (4) must be evaluated $n_s \times \ell_{\max}$ times to account for the influence of all particles included in the neighborhood of a given particle. However, this is simple arithmetic to find $\mathbf{x}_{\text{best},i}^{\ell}$ ($\ell = 1, 2, \dots, \ell_{\max}$) in (10), and thus, the computational burden of this evaluation is not significant. This suggests that the CNNT-PSO scheme does not incur a greater computational cost than other schemes reported in the literature.

Table 7. Comparison of optimization results with literature: 25-bar truss problem.

Optimal Design Variables A_i (in ²)	Lee & Geem: HS [33]	Li et al. [8]		Camp [37] BB-BC	Kaveh & Talatahari: HPSACO [34]	Kaveh & Talatahari: HBB-BC [38]	Lamberti & Pappalettere: IHS [35]	Lamberti [6] CMLPSA	Degertekin [3]		Degertekin & Hayalioglu: TLBO [1]	This study ($n_s = 20$) CNNT-PSO
		PSOPC	HPSO						EHS	SAHS		
A_1	0.047	0.010	0.010	0.010	0.010	2.6622	0.0100	0.0100	0.010	0.010	0.0100	0.0100
A_2 – A_5	2.022	1.979	1.970	2.092	2.054	1.993	1.9871	1.9870	1.995	2.074	2.0712	1.9870
A_6 – A_9	2.950	3.011	3.016	2.964	3.008	3.056	2.9935	2.9935	2.980	2.961	2.9570	2.9935
A_{10} – A_{11}	0.010	0.100	0.010	0.010	0.010	0.010	0.0100	0.0100	0.010	0.010	0.0100	0.0100
A_{12} – A_{13}	0.014	0.100	0.010	0.010	0.010	0.010	0.0100	0.0100	0.010	0.010	0.0100	0.0100
A_{14} – A_{17}	0.688	0.657	0.694	0.689	0.679	0.665	0.6839	0.6894	0.696	0.691	0.6891	0.6839
A_{18} – A_{21}	1.657	1.678	1.681	1.601	1.611	1.642	1.6769	1.6769	1.679	1.617	1.6209	1.6769
A_{22} – A_{25}	2.663	2.693	2.643	2.686	2.678	2.679	2.6622	2.6621	2.652	2.674	2.6768	2.6622
Best weight (lb)	544.38	545.27	545.19	545.38	544.99	545.16	545.15	545.15	545.49	545.12	545.09	545.1627
Worst weight	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	548.04	546.60	546.33	545.1627
Average weight	N/A	N/A	N/A	545.78	545.52	545.66	N/A	N/A	546.52	545.94	545.41	545.1627
Standard deviation	N/A	N/A	N/A	0.491	0.315	0.367	N/A	N/A	1.05	0.91	0.42	6.94×10^{-6}
Analysis of the optimization results using ANSYS/MATLAB												
Weight (lb)	544.36	547.96	545.238	545.52	544.991	565.03	545.1658	545.554	545.486	545.118	545.08	545.1627
Feasibility	Infeasible	Feasible	Feasible	Infeasible	Infeasible	Infeasible	Feasible	Infeasible	Feasible	Infeasible	Infeasible	Feasible
Infeasible node displacement (in)	0.350709 (1st bar)	None	None	None	0.35004015 (1st node)	0.35001849 (1st node)	None	None	None	None	None	None
Infeasible bar stress (ksi) ¹	6.973365 (19th bar)	None	None	7.227284 (19th bar)	7.2042322 (19th bar)	None	None	6.9597203 (19th bar)	None	7.166396 (19th bar)	7.1501796 (19th bar)	None

¹ All of the items shown violated the predefined compressive stress limit specified in Table 6.

Table 8. Sensitivity of optimized weight to neighborhood size: 25-bar truss problem.

Neighbor Size: n_s		Object. Funct. Value			
(Ratio of n_s to n_p)		Best	Average	Worst	Standard Deviation
6	(3.0%)	545.2022914740	545.3321179916	545.5017575323	0.0826258622
10	(5.0%)	545.1629167122	545.1648971493	545.1687823043	0.0014564516
20	(10.0%)	545.1627105408	545.1627143990	545.1627412660	0.0000069387
50	(25.0%)	545.1627102443	545.1628795084	545.1655031932	0.0005456476
100	(50.0%)	545.1627427736	545.8373723082	549.1823298747	1.1695878525
150	(75.0%)	545.2296934557	552.9448115665	583.3440908564	10.1769607107
200	(100.0%)	545.8290039641	569.5171789211	607.8854689086	16.7379238301

Figure 5. Convergence curves for the 25-bar truss problem ($n_s = 20$).

3.3. Spatial 72-Bar Truss Structure

The third test case is the sizing optimization of the spatial 72-bar truss shown in Figure 6. The Young's modulus of the material is 10 Msi and the material density is 0.1 lb/in³. Truss members were divided into 16 groups based on structural symmetry, and the cross-sectional areas of the bars in each group were included as design variables: (1) A_1 – A_4 , (2) A_5 – A_{12} , (3) A_{13} – A_{16} , (4) A_{17} – A_{18} , (5) A_{19} – A_{22} , (6) A_{23} – A_{30} , (7) A_{31} – A_{34} , (8) A_{35} – A_{36} , (9) A_{37} – A_{40} , (10) A_{41} – A_{48} , (11) A_{49} – A_{52} , (12) A_{53} – A_{54} , (13) A_{55} – A_{58} , (14) A_{59} – A_{66} , (15) A_{67} – A_{70} , (16) A_{71} – A_{72} . Hence, this optimization problem includes 16 design variables and 264 nonlinear constraints on the member stresses ($\pm 25,000$ psi) and displacements of top nodes 17–20 (± 0.25 in both the X- and Y-directions). The cross-sectional areas were permitted to vary between 0.1 and 4 in². This truss structure was optimized for two independent loading conditions:

- Condition 1:* 5 kips acting in the positive X- and Y-directions, and in the negative Z-direction at node 17;
- Condition 2:* 5 kips acting in the negative Z-direction at nodes 17 through 20.

CNNT-PSO was run with $n_p = 800$ and $\ell_{\max} = 1000$. All optimization runs were successful and converged to a feasible design. Table 9 shows that the proposed algorithm was again the most efficient optimizer, as it designed the lightest structure. Furthermore, the optimization results were consistent for all trial runs. The optimal number of neighbors of each particle was again twice the number

of design variables. The convergence characteristics of the objective function for 30 independent optimization runs are shown in Figure 7. Table 10 confirms that a relatively accurate solution could also be obtained when n_s/n_p was less than about 10%.

Figure 6. Schematic of the spatial 72-bar truss structure.

Figure 7. Convergence curves for the 72-bar truss problem ($n_s = 30$).

Table 9. Comparison of optimization results with literature: 72-bar truss problem.

Optimal Design Variables A_i (in ²)	Lee & Geem: HS [33]	Prez & Behdinan: PSO [7]	Li et al. [8]		Camp [37] BB-BC	Kaveh & Talatahari: HBB-BC [38]	Degertekin [3]		Degertekin & Hayalioglu: TLBO [1]	This Study ($n_s = 30$) CNNT-PSO
			PSOPC	HPSO			EHS	SAHS		
A_1-A_4	1.790	1.7427	1.855	1.857	1.8577	1.9042	1.967	1.860	1.9064	1.8861
A_5-A_{12}	0.521	0.5185	0.504	0.505	0.5059	0.5162	0.510	0.521	0.50612	0.5123
$A_{13}-A_{16}$	0.100	0.1000	0.100	0.100	0.1000	0.1000	0.100	0.100	0.100	0.1000
$A_{17}-A_{18}$	0.100	0.1000	0.100	0.100	0.1000	0.1000	0.100	0.100	0.100	0.1000
$A_{19}-A_{22}$	1.229	1.3079	1.253	1.255	1.2476	1.2582	1.293	1.271	1.2617	1.2685
$A_{23}-A_{30}$	0.522	0.5193	0.505	0.503	0.5269	0.5035	0.511	0.509	0.5111	0.5117
$A_{31}-A_{34}$	0.100	0.1000	0.100	0.100	0.1000	0.1000	0.100	0.100	0.100	0.1000
$A_{35}-A_{36}$	0.100	0.1000	0.100	0.100	0.1012	0.1000	0.100	0.100	0.100	0.1000
$A_{37}-A_{40}$	0.517	0.5142	0.497	0.496	0.5209	0.5178	0.499	0.485	0.5317	0.5236
$A_{41}-A_{48}$	0.504	0.5464	0.508	0.506	0.5172	0.5214	0.501	0.501	0.51591	0.5171
$A_{49}-A_{52}$	0.100	0.1000	0.100	0.100	0.1004	0.1000	0.100	0.100	0.100	0.1000
$A_{53}-A_{54}$	0.101	0.1095	0.100	0.100	0.1005	0.1007	0.100	0.100	0.100	0.1000
$A_{55}-A_{58}$	0.156	0.1615	0.100	0.100	0.1565	0.1566	0.160	0.168	0.1562	0.1565
$A_{59}-A_{66}$	0.547	0.5092	0.525	0.524	0.5507	0.5421	0.522	0.584	0.54927	0.5456
$A_{67}-A_{70}$	0.442	0.4967	0.394	0.400	0.3922	0.4132	0.478	0.433	0.40966	0.4103
$A_{71}-A_{72}$	0.590	0.5619	0.535	0.534	0.5922	0.5756	0.591	0.520	0.56976	0.5697
- Best weight (lb)	379.27	381.91	369.65	369.65	379.85	379.66	381.03	380.62	379.63	379.6148
Worst weight	N/A	N/A	N/A	N/A	N/A	N/A	385.50	383.89	380.83	379.6155
Average weight	N/A	N/A	N/A	N/A	382.08	381.85	383.51	382.42	380.20	379.6149
Standard deviation	N/A	N/A	N/A	N/A	1.912	1.201	1.92	1.38	0.41	0.0002
Analysis of the optimization results using ANSYS/MATLAB										
Weight (lb)	379.217	381.936	369.65	369.64	379.84	379.65	381.026	380.837	379.63494	379.6148
Feasibility	Infeasible	Feasible	Infeasible	Infeasible	Feasible	Feasible	Feasible	Feasible	Feasible	Feasible
Infeasible node displacement (in)	0.2505456 (17th node)	None	0.3114043 (17th node)	0.3115011 (17th node)	None	None	None	None	None	None
Infeasible bar stress (ksi)	25.01883 (55th bar)	None	34.762759 (55th bar)	34.768775 (55th bar)	None	None	None	None	None	None

Table 10. Sensitivity of optimized weight to neighborhood size: 72-bar truss problem.

Neighbor Size: n_s		Objective Function Value			
(Ratio of n_s to n_p)		Best	Average	Worst	Standard Deviation
10	(1.250%)	379.6318052789	379.6653403456	379.7761760177	0.0330033631
30	(3.750%)	379.6148044971	379.6149401277	379.6155036251	0.0001752131
60	(7.500%)	379.6148633151	379.6495513185	379.8339175611	0.0568846942
100	(12.50%)	379.7268230162	387.4123385006	407.4333925023	7.6311956102
300	(37.50%)	390.9034206475	471.0343138592	570.4458234840	43.2075800681
400	(50.00%)	426.3544544514	513.5732153123	606.2598561574	46.7613059395
600	(75.00%)	459.8827786937	532.3907876752	637.1533325741	49.0785590185
800	(100.0%)	461.0964821247	569.2890375109	688.3641284045	54.5771993620

3.4. Planar 200-Bar Truss Structure

The final test case concerns the sizing optimization of the planar 200-bar truss with 77 nodes shown in Figure 8. The Young's modulus of the material is 30 Msi and the material density is 0.283 lb/in³. Truss members were divided into 29 groups based on structural symmetry (see Table 11), and the cross-sectional areas of the bars in each group were included as design variables. This optimization problem includes 29 design variables and 1200 nonlinear constraints on member stresses ($\pm 10,000$ psi). Cross-sectional areas must be greater than 0.1 in². This truss structure was designed for three independent loading conditions:

- Condition 1:* 1 kip acting in the positive X-direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, and 71;
- Condition 2:* 10 kips acting in the negative Y-direction at nodes 1–6, 8, 10, 12, 14–20, 22, 24, 26, 28–34, 36, 38, 40, 42–48, 50, 52, 54, 56–62, 64, 66, 68, and 70–75;
- Condition 3:* loading conditions 1 and 2 acting together.

Table 11. Variable linking adopted in the 200-bar truss problem.

Design Variables	Member Number	Design Variables	Member Number
1	1, 2, 3, 4	16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110,
2	5, 8, 11, 14, 17	17	112, 113, 115, 116, 117, 118
3	19, 20, 21, 22, 23, 24	18	119, 122, 125, 128, 131
4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177	19	133, 134, 135, 136, 137, 138
5	26, 29, 32, 35, 38	20	140, 143, 146, 149, 152
6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37	21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151
7	39, 40, 41, 42	22	153, 154, 155, 156
8	43, 46, 49, 52, 55	23	157, 160, 163, 166, 169
9	57, 58, 59, 60, 61, 62	24	171, 172, 173, 174, 175, 176
10	64, 67, 70, 73, 76	25	178, 181, 184, 187, 190
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75	26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189
12	77, 78, 79, 80	27	191, 192, 193, 194
13	81, 84, 87, 90, 93	28	195, 197, 198, 200
14	95, 96, 97, 98, 99, 100	29	196, 199
15	102, 105, 108, 111, 114

Figure 8. Schematic of the planar 200-bar truss structure.

CNNT-PSO was run with $n_p = 1000$ and $\ell_{\max} = 1500$. All optimization runs were successful and converged to a feasible design. The convergence curve relative to the best optimization run is plotted in Figure 9. Table 12 shows that the present algorithm was once again the most efficient optimizer, as it designed the lightest structure and converged to a feasible design. Furthermore, the optimization results were marginally sensitive to trial runs. The optimal number of neighbors of each particle was once again about twice the number of design variables. It can be seen from Table 13 that a relatively accurate solution could be obtained by setting n_s/n_p to between 6% and 10%.

Figure 9. Convergence curve of the best trial run carried out for 200-bar truss problem ($n_s = 60$).

Table 12. Comparison of optimization results with literature: 200-bar truss problem.

Optimal Design Variables	Lee & Geem [33]	Kaveh & Talatahari [34]		Lamberti [6]	Degertekin [3]		Degertekin & Hayalioglu [1]	This Study ($n_s = 60$)
A_i (in ²)	HS	PSOPC	HPSACO	CMLPSA	EHS	SAHS	TLBO	CNNT-PSO
1	0.1253	0.7590	0.1033	0.1468	0.150	0.154	0.146	0.1482
2	1.0157	0.9032	0.9184	0.9400	0.946	0.941	0.941	0.9405
3	0.1069	1.1000	0.1202	0.1000	0.101	0.100	0.100	0.1000
4	0.1096	0.9952	0.1009	0.1000	0.100	0.100	0.101	0.1000
5	1.9369	2.1350	1.8664	1.9400	1.945	1.942	1.941	1.9408
6	0.2686	0.4193	0.2826	0.2962	0.296	0.301	0.296	0.2975
7	0.1042	1.0041	0.1000	0.1000	0.100	0.100	0.100	0.1000
8	2.9731	2.8052	2.9683	3.1042	3.161	3.108	3.121	3.1067
9	0.1309	1.0344	0.1000	0.1000	0.102	0.100	0.100	0.1000
10	4.1831	3.7842	3.9456	4.1042	4.199	4.106	4.173	4.1067
11	0.3967	0.5269	0.3742	0.4034	0.401	0.409	0.401	0.4057
12	0.4416	0.4302	0.4501	0.1912	0.181	0.191	0.181	0.1897
13	5.1873	5.2683	4.96029	5.4284	5.431	5.428	5.423	5.4343
14	0.1912	0.9685	1.0738	0.1000	0.100	0.100	0.100	0.1000
15	6.2410	6.0473	5.9785	6.4284	6.428	6.427	6.422	6.4340
16	0.6994	0.7825	0.78629	0.5734	0.571	0.581	0.571	0.5745
17	0.1158	0.5920	0.73743	0.1327	0.156	0.151	0.156	0.1366
18	7.7643	8.1858	7.3809	7.9717	7.961	7.973	7.958	7.9803
19	0.1000	1.0362	0.66740	0.1000	0.100	0.100	0.100	0.1000
20	8.8279	9.2062	8.3000	8.9717	8.959	8.974	8.958	8.9802
21	0.6986	1.4774	1.19672	0.7049	0.722	0.719	0.720	0.71089
22	1.5563	1.8336	1.0000	0.4196	0.491	0.422	0.478	0.4659
23	10.9806	10.6110	10.8262	10.8636	10.909	10.892	10.897	10.9110
24	0.1317	0.9851	0.1000	0.1000	0.101	0.100	0.100	0.1000
25	12.1492	12.5090	11.6976	11.8606	11.985	11.887	11.897	11.9112
26	1.6373	1.9755	1.3880	1.0339	1.084	1.040	1.080	1.0712
27	5.0032	4.5149	4.9523	6.6818	6.464	6.646	6.462	6.5030
28	9.3545	9.8000	8.8000	10.8113	10.802	10.804	10.799	10.7210
29	15.0919	14.5310	14.6645	13.8404	13.936	13.870	13.922	13.9310
Best weight (lb)	25,447.1	28,537.8	25,156.5	25,445.63	25,542.5	25,491.9	25,488.15	25,453.0957
Worst weight	N/A	N/A	N/A	N/A	25,838.2	25,799.3	25,563.05	25,466.0958
Average weight	N/A	N/A	N/A	N/A	25,659.71	25,610.2	25,533.14	25,459.1089
Standard deviation	N/A	N/A	N/A	N/A	164.17	141.85	27.44	3.1544
Analysis of the optimization results using ANSYS/MATLAB programs								
Weight (lb)	25,447.5276	28,571.4343	25,155.6741	25,445.9597	25,537.0548	25,491.9226	25,488.1788	25,453.0957
Feasibility	Infeasible	Infeasible	Infeasible	Infeasible	Feasible	Feasible	Feasible	Feasible
Infeasible bar stress (ksi)	−10,369.318165 (122th bar)	−10,744.977441 (76th bar)	−10,996.947443 (184th bar)	1007.084660 (120th bar)	None	None	None	None

Table 13. Sensitivity of optimized weight to neighborhood size: 200-bar truss problem.

Neighbor Size: n_s		Objective Function Value			
(Ratio of n_s to n_p)		Best	Average	Worst	Standard Deviation
30	(3.0%)	25457.3759759502	25476.9767631906	25514.0491597601	17.0656154436
60	(6.0%)	25453.0957113131	25459.1089220335	25466.0958098744	3.1543853566
100	(10.0%)	25453.8020671454	25463.0508675004	25474.0057600994	5.2856406250
250	(25.0%)	25465.3248022958	25495.9601387794	25519.0074637713	14.5041309227
500	(50.0%)	25489.1260391797	25534.2209378792	25574.0068587098	23.1527113502
750	(75.0%)	25513.2968909123	25574.0489259354	25607.9003950622	24.8296616471
1000	(100.0%)	25551.3127913481	25613.7785004692	25651.3237014500	28.1973072143

4. Conclusions

This study presented an easy-to-use and reliable meta-heuristic algorithm for structural optimization. The algorithm implemented a cyclic neighborhood learning-based diversity-enhanced PSO scheme. Diversification was enhanced by the distributed social learning among each particle's neighborhood which replaced the learning from only one global-best particle in the entire swarm as it happens in standard PSO. The proposed CNNT-PSO algorithm was very efficient and reliable in weight-minimization problems of truss structures including up to 29 sizing variables. A comparative study of four classical sizing optimization problems demonstrated that the proposed approach is much better than various meta-heuristic methods in terms of solution quality, robustness, and computational cost.

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