

Article

# Clustering-Based Binarization Methods Applied to the Crow Search Algorithm for 0/1 Combinatorial Problems

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**Abstract:** Metaheuristics are smart problem solvers devoted to tackling particularly large optimization problems. During the last 20 years, they have largely been used to solve different problems from the academic as well as from the real-world. However, most of them have originally been designed for operating over real domain variables, being necessary to tailor its internal core, for instance, to be effective in a binary space of solutions. Various works have demonstrated that this internal modification, known as binarization, is not a simple task, since the several existing binarization ways may lead to very different results. This of course forces the user to implement and analyze a large list of binarization schemas for reaching good results. In this paper, we explore two efficient clustering methods, namely KMeans and DBscan to alter a metaheuristic in order to improve it, and thus do not require on the knowledge of an expert user for identifying which binarization strategy works better during the run. Both techniques have widely been applied to solve clustering problems, allowing us to exploit useful information gathered during the search to efficiently control and improve the binarization process. We integrate those techniques to a recent metaheuristic called Crow Search, and we conduct experiments where KMeans and DBscan are contrasted to 32 different binarization methods. The results show that the proposed approaches outperform most of the binarization strategies for a large list of well-known optimization instances.

**Keywords:** clustering techniques; crow search algorithm; binary domains; metaheuristics; bio-inspired computing

## 1. Introduction

Optimization problems can be seen in different areas of the modern world, such as bridge reinforcement [1], load dispatch [2], location of emergency facilities [3], marketing [4], and social networks [5], among others. To solve them, we must identify and understand to which model the problem belongs. There are three types of models, the first has discrete variables, the second has continuous variables, and the third has both types of variables. Discrete optimization has widely been used to define a large number of problems belonging to the NP-hard class [6]. It is necessary

to emphasize that the time to solve this type of problem increases exponentially according to its size. For this reason, we consider it totally prudent to solve these problems through metaheuristics [7,8], which deliver acceptable results in a limited period of time.

There are many metaheuristics and we can see a complete collection of all nature-inspired algorithms in [9] but most work on continuous space. Some researchers have been working in depth on developing binary versions that make these metaheuristics operate in binary spaces. A study of putting continuous metaheuristics to work in binary search spaces [10] clearly describes the two-step technique. These techniques require a large number of combinations and tuning of parameters to binarize and thus be able to obtain the best of all, which is a great waste of time.

Despite successful published work, binary techniques require offline analysis and an expert user in the field to adjust the metaheuristic to the problem domain in addition to working with the various available combinations of the two-step technique. Using gained expertise, our work is focused on two binarization clustering methods to smartly modify a set of real solutions to binary solutions by grouping decision variables, without the need for combinations as in the two-step technique, saving time and algorithm manipulation. The first one proposes to use KMeans [11,12] clustering algorithm, the most common unsupervised machine learning algorithm, to assign each variable in defined groups by the user. It is necessary to assign values of transition to each cluster in order to binarize each variable. The second one integrates DBscan [13,14] as a powerful clustering method for arraying values in a nonapriori determined number of clusters. Unlike KMeans, this technique creates the clusters by itself just knowing the number of variables required to be included in a cluster and the distance between each point in the data set. Furthermore, the transition value to binarize is calculated online and depends on each cluster. To test our approaches, we include these clustering methods in the Crow Search Algorithm (CSA) [15] which is a population-based metaheuristic inspired by the behavior of crows (easy implementation, it works very fast, and there are few parameter settings). In this method, each individual follows others trying to discover the places where they hide the food in order to steal it.

Based on [16,17], both works were used as gold standards, and we perform an experimental design led by quantitative methodology. We implement different criteria such as quality of solutions, robustness in terms of the instances, solving large-scale of well known binary optimization problems: Set Covering Problem (SCP) [18] and 0/1 Knapsack Problem (KP) [19]. These two problems are very popular benchmarks to address and try new strategies with because they have known results with which to compare.

The remainder of this manuscript is organized as follows: Section 2 presents the related work. Section 3 exposes binarization strategies. CSA is described in Section 4. Section 5 explains how to integrate binarizations on CSA. Next, experimental results including a research methodology are explained in Section 6. Discussions are shown in Section 7. At the end of the manuscript, conclusions are presented in Section 8.

## 2. Related Work

As previously mentioned, binarization is mandatory for continuous metaheuristics when the problem ranges in a binary domain. This task is not simple since experimentation with different binarization schemes is needed to reach good results. The transfer function is the most used binarization method and it was introduced in [20]. The transfer function is a very cheap operator, his range provides probabilities values and tries to model the transition of the particle positions. This function is responsible for the first step of the binarization method which corresponds to map the  $\mathbb{R}^n$  solutions in  $[0, 1]^n$  solutions. Two types of functions have been used in the literature, the S-shaped [21] and V-shaped [22]. The second step is to apply a binarization rule to the result of the transfer function. Examples of binarization rules are a complement, roulette, static probability, and elitist [22]. “Two-step” binarization is employed to refer to the use of transfer functions and binarization rule together. A detailed description of binarization techniques can be seen in [10].

Our goal here is to alleviate the user involvement in binarization testing and to provide a unique alternative that works better on average when contrasted to a large set of “Two-step” binarization techniques. In this context, the use of machine learning techniques appears as a good candidate to support the binarization process and as a consequence to obtain better search processes. Machine learning has already been used to improve search in metaheuristics [23]. For instance, the first trend is to employ machine learning for parameter tuning, as in [24] the authors use machine learning to parameter control in metaheuristics. The techniques reported in this context can be organized in four groups: Parameter control strategies [25], parameter tuning strategies [26], instance-specific parameter tuning strategies [27], and reactive search [28]. The general idea is to exploit search information in order to find good parameter configurations; this may lead to better quality solutions. An example of this is in [29], where the author uses a metaheuristic to determine the optimal number of hidden nodes and their proper initial locations on neural networks. Machine learning has also been used to build approximation models when objective function and constraints are expensive computationally [30]. Population management can also be improved via machine learning algorithms, obtaining information from already visited solutions, and using it to build new ones [31].

Machine learning has also been used to reduce the search space, for instance by using clustering [32] and neural networks [33]. In these works, two main areas of machine learning were used: forecasting and classify. It is important to mention the uses of machine learning with metaheuristics in these areas. It has been used to forecast different kinds of problems, like optimizing the low-carbon flexible job Shop Scheduling problem [34], electric load [35,36] use to forecast economic recessions in Italy, and other industrial problems as in [37]. As we mentioned, machine learning also can be used to classify some different groups of data sets; in this scenario, this technique has also been used in different problems as the image classification [38], electronic noise classification of bacterial foodbones pathogens [39], urban management [40], to mention some recent studies.

Although the participation of machine learning in the metaheuristic field is large, the specific use in binarization is indeed very limited. To the best of our knowledge, only a few experiments with the clustering techniques DBscan and KMeans have been reported [13,41]. In this way, we believe that binarization has a lot of room for exploration beyond the improvement of metaheuristics, the use of just one machine learning technique to binarization, compared to the 32 combinations of two-step technique that let the user gain simplicity, speediness, and less effort to use the binarization on metaheuristics, which is the aim pursued in this work.

### 3. Binarization

A study of putting continuous metaheuristics to work in binary search spaces [10] exposes two main groups of binarization techniques. The first group called two-step binarization allows one to put in work the continuous metaheuristics without operator modifications and the second group called continuous-binary operator transformation redefines the algebra of the search space, thereby reformulating the operators.

In this work, we used the first group to test our experiments. These two steps refer to transfer functions, which are responsible for mapping solutions in the real domain to a binary domain. Then we are going to use a machine learning strategy of clustering called KMeans in order to cluster the column of solutions and another strategy called DBscan that will cluster all the solutions (matrix). Both strategies will be used to map solutions between 0 and 1.

#### 3.1. Two Step Binarization

Two-step binarization consists of applying, as its name says, two stages in order to bring the variables of the real search space to the world of binary variables. The first step uses eight transfer functions and the second step uses four discretization functions, therefore generating 32 possible strategies to apply.

### 3.1.1. First Step—Transfer Functions

These operators are very cheap to implement in the algorithm and their range provides probability values and attempts to model the transition of the particle positions; in other words, these functions transform a real solution that belongs to a real domain, to another real solution but restricted to a domain between 0 and 1. For that, there exist two classic types in the literature [42,43] called S-Shape and V-Shape, which are used.

### 3.1.2. Second Step—Binarization

A set of rules can be applied to transform a solution in the real domain to a solution in the binary domain. We use Standard, Complement, Static probability, and Elitist. In particle swarm optimization, this approach was first used in [20].

## 3.2. KMeans

KMeans is one of the simplest and most popular unsupervised machine learning algorithms for clustering. The main goal of KMeans [44,45] is to group similar data points given certain similarities. To achieve this objective, a number of clusters in a data set must be set. Then, every data point is allocated to each of the clusters, the arithmetic representation is as follows, see Equation (1):

$$J = \sum_{j=1}^k \sum_{i=1}^n \|x_i^j - c_l\|^2 \quad (1)$$

where  $k$  represents the number of clusters,  $n$  depicts the number of cases,  $x_i^j$  defines  $j$ th case of  $i$ th solution, and  $c_l$  exposes the  $l$ th centroid (see pseudo-code in Algorithm 1).

---

#### Algorithm 1: KMeans pseudo-code

---

```

1 Input: data set of P = {p1...pn}
2 Input: a number of cluster k
3 Output: a set of k clusters.
4 while stop criterion do
5   |-(re) assign each point to a cluster where the point is the most similar, based on the mean
      | value of the points of the cluster;
6   | -Update the clusters' mean value;
7 end

```

---

The mechanism initially requires the  $k$  value as the number of the clusters and the best solution vector of dimension  $d$  composed by the decision variables  $\hat{X} = \langle x_1^1, \dots, x_j^1, \dots, x_k^1, \dots, x_l^1, \dots, x_d^1 \rangle$ . This vector is passed as input data for being grouped (see the left side of Figure 1). Iteratively, KMeans assigns each decision variable to one group based on the similarity of their features and these are allocated to the closest centroid, minimizing Euclidean distances. After, KMeans recalculates centroids by taking the average of all decision variables allocated to the cluster of that centroid, thus decreasing the complete intracluster variance over the past phase.

The right side of Figure 1 illustrates the computed groups. Positions are adjusted in each iteration of the process until the algorithm converges.

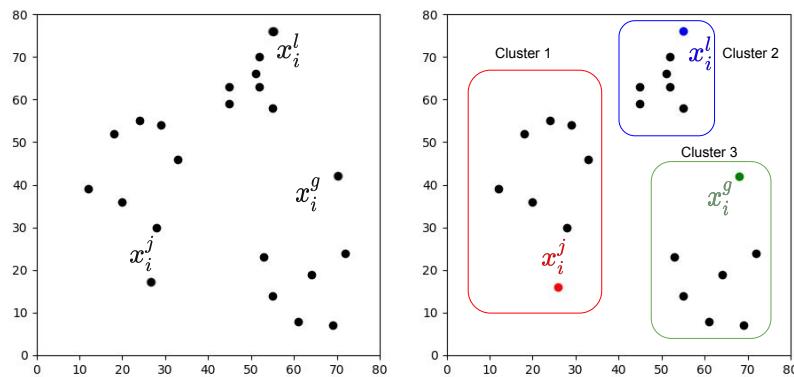
### 3.3. DBscan

Density-Based Spatial Clustering of Applications with Noise (DBscan) [46] is a well-known data clustering algorithm which is commonly used in data mining and machine learning. It can be used to

identify clusters of any shape in a data set containing noise and outliers. Clusters are thick data space areas separated by areas with reduced points density.

The goal is to define thick areas that can be measured by the amount of near-point objects. It is indispensable to know that DBscan has two important parameters that are required for work.

1. Epsilon ( $\epsilon$ ): Sets how close points should be regarded as part of a cluster to each other.
2. Minimum points (MinPts): The minimum number of points to form a dense region.

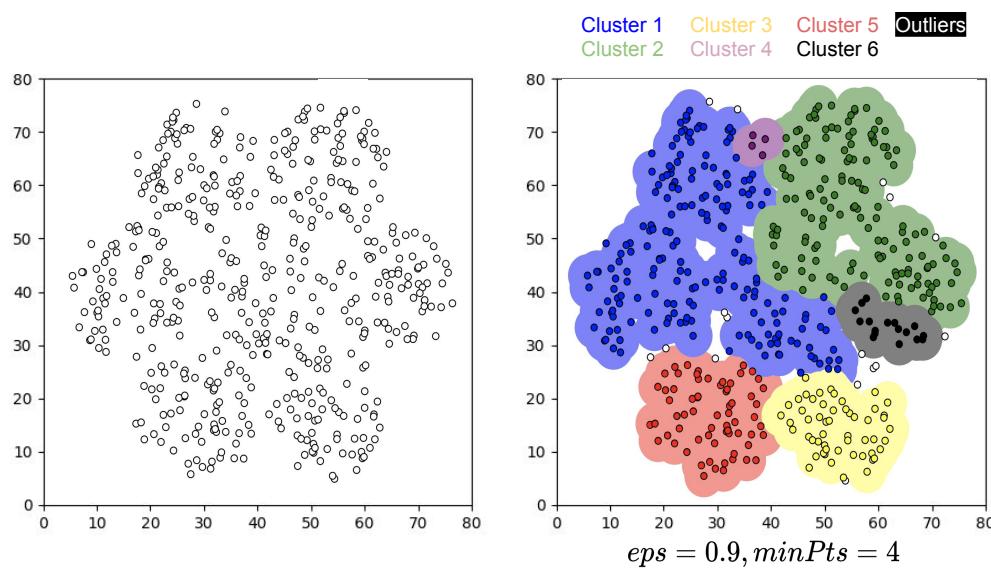


**Figure 1.** Clustering process of the KMeans algorithm applied to the decision variables.

The main idea is that there must be at least a minimum number of points for each group in the neighborhood of a given radius. The  $\epsilon$  parameter defines the neighborhood radius around a point  $x$ , any point  $x$  in the data set with a neighbor above or equal to MinPts is marked as a center point, and  $x$  is a limit point whether the number of its neighbors is less than MinPts. Finally, if a point is not a core or a point of the edge, then it is called a point of noise or an outer point.

As a summary, this technique helps when you do not know much information about data, see pseudo-code in Algorithm 2.

The right side of Figure 2 illustrates the computed groups. Positions are adjusted in each iteration of the process until the algorithm converges.



**Figure 2.** Clustering process of the DBScan algorithm applied to the best solution.

**Algorithm 2:** DBscan pseudo-code

---

```

1 Input: data set of P = { $p_1 \dots p_n$ };
2 Input:  $\text{eps}$ ;
3 Input: MinPts;
4 Output: a set of  $k$  clusters. ;
5 for each unvisited point in P do
6   mark point as visited;
7   neighborPts = regionQuery(point,  $\text{eps}$ );
8   C=0
9   if sizeof(neighborPts)<MinPts then
10    | mark point as noise;
11   else
12    | C= next cluster;
13    | expandCluster(point, neighborPts, C,  $\text{eps}$ , MinPts)
14   end
15 end
16 expandCluster(point, neighborPts, C,  $\text{eps}$ , MinPts);
17 add point to cluster C;
18 for each point' in neighborPts do
19   if point' is not visited then
20    | mark point' as visited;
21    | neighborPts' = regionQuery(point',  $\text{eps}$ );
22    | if sizeof(neighborPts')>=MinPts then
23     | | neighborPts = neighborPts joined neighborPts'
24    | end
25   end
26   if point' is not yet member of any cluster then
27    | | add point' to cluster C
28   end
29 end
30 regionQuery(point,  $\text{eps}$ );
31 return all point within point'  $\text{eps}$ -neighborhood(including point)

```

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#### 4. Crow Search Algorithm

Crows (crow family or corvids) are considered the most intelligent birds. They contain the largest brain relative to their body size. Based on a brain-to-body ratio, their brain is slightly lower than a human brain. Evidence of the cleverness of crows is plentiful. They can remember faces and warn each other when an unfriendly one approaches. Moreover, they can use tools, communicate in sophisticated ways, and recall their food hiding place up to several months later.

CSA tries to simulate the intelligent behavior of the crows to find the solution of optimization problems [15]. Optimization point of view: the crows are searchers, the environment is the search space, each position of the crow corresponds to a feasible solution, the quality of food source is the objective function, and the best food source of the environment is the global solution of the problem. Below, parameters for CSA are shown.

- N: Population.
- AP: Awareness probability.
- $fl$ : Flight length.
- $iter$ : Maximum number of iterations.

When CSA is working, two things can happen. For this, let Crow 1 and Crow 2 be different possible solutions belonging to the search space. Firstly, Crow 1 does not know that Crow 2 is following it. As a result, Crow 2 will approach the hiding place of Crow 1. This action is called State 1. On the other hand, Crow 1 knows that Crow 2 is following it. Finally, to protect its hidden place from being stolen, Crow 1 will fool Crow 2 by going to another position of the search space. This action is called state 2. The representation of these states is represented for Equation (2).

$$x_{i,j}^{iter+1} = \begin{cases} x_{i,j}^{iter} + r_1 fl(m_{i,j}^{iter} - x_{i,j}^{iter}) & r_2 \geq AP \\ \text{a random position} & \text{otherwise} \end{cases} \quad (2)$$

where:

- $r_1$  and  $r_2$  are random numbers with uniform distribution between 0 and 1.
- $fl$  denotes the flight length of crows.
- $AP$  denotes the awareness probability of crows (intensification and diversification).

Below, pseudo-code of CSA is shown in Algorithm 3.

---

**Algorithm 3: CSA pseudo-code**


---

```

1 Randomly initialize the position of a flock of N crows in the search space
2 Evaluate the position of the crows
3 Initialize the memory of each crow
4 while iter < itermax do
5   for i = 1 to N do
6     c = Randomly choose one of the crows to follow
7     for j = 1 to D do
8       if r2 ≥ AP then
9         | xi,jiter+1 = xc,jiter + r1 fl (mc,jiter - xc,jiter)
10        else
11          | xi,jiter+1 = a random position of the search space
12        end
13      end
14    end
15    Check the feasibility of new positions
16    Evaluate the new position of the crows
17    Update the memory of crows
18 end
```

---

## 5. Integration: Binarizations on CSA

As we shown in Section 3, to work in a binary search space CSA [15], one must apply the binarization techniques. Firstly, we will solve the problem using Two-steps technique, then we will apply KMeans technique.

To have a high level explanation, the steps are described below:

When CSA evaluate a  $x_i$ , a real value is generated, then:

1. Two-steps:  $x_i$  in t + 1 is binarized column by column applying the first step getting a continuous solution that enters as an input to the second step making the binarization evaluate the solution in the Objective function.
2. KMeans: In this case,  $x_i$  in t + 1 is clustered. Then each cluster is assigned to a cluster transition value already defined as a parameter. Small centroids get the small cluster transition value and a

high centroid value gets a high value of transition. It is necessary to evaluate every point in the clusters, asking if random values between 0 and 1 are equal or greater than a cluster transition value. If we get the position  $x_{i,j}$  and apply the complement to the  $x_i$  in t, then the objective function is evaluated.

For these two techniques we take the same strategy that is illustrated in Algorithm 4.

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**Algorithm 4:** CSA + Two-steps and Kmeans binarization techniques pseudo-code
 

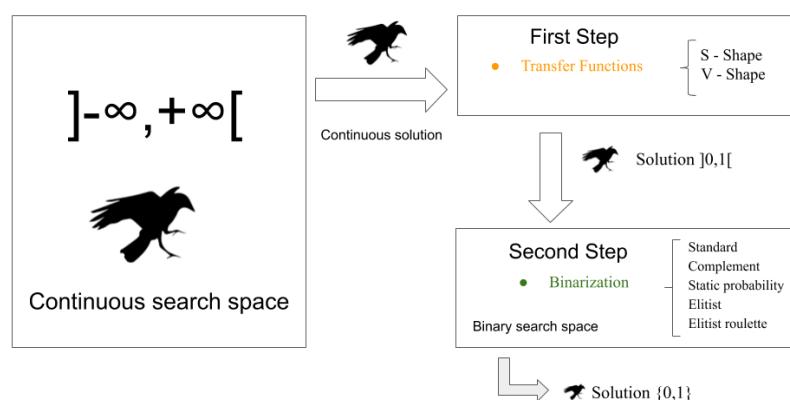
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```

1 Randomly initialize the position of a flock of N crows in the search space
2 Evaluate the position of the crows
3 Initialize the memory of each crow
4 while  $iter < iter_{max}$  do
5   for  $i = 1$  to  $N$  do
6     c=Randomly choose one of the crows to follow
7     for  $j = 1$  to  $D$  do
8       if  $r_2 \geq AP$  then
9          $x_{i,j}^{iter+1} = x_{c,j}^{iter} + r_1 fl(m_{c,j}^{iter} - x_{c,j}^{iter})$ 
10      else
11         $x_{i,j}^{iter+1} = \text{a random position of the search space}$ 
12      end
13      Apply binarization to  $x^{i,iter+1}$ 
14    end
15  end
16  Check the feasibility of new positions
17  Evaluate the new position of the crows
18  Update the memory of crows
19 end
  
```

---

A graphic representation can be seen in Figure 3. A continuous solution from the metaheuristic is transformed into a solution that belongs to real domain to a solution in a real domain but restricted to a domain between 0 and 1, applying the first step. Then, the solution again is transformed into a solution in a binary domain using step 2. Figure 4 shows how the real solution is clusterized and then transformed to a binary solution applying cluster transition value using KMeans.

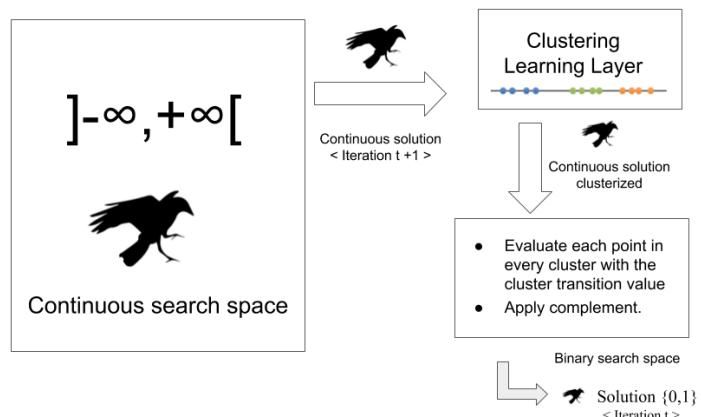


**Figure 3.** Continuous to Binary Two-steps.

Figure 4 shows how the real solution is clusterized and then is transformed to a binary solution applying cluster transition value using KMeans.

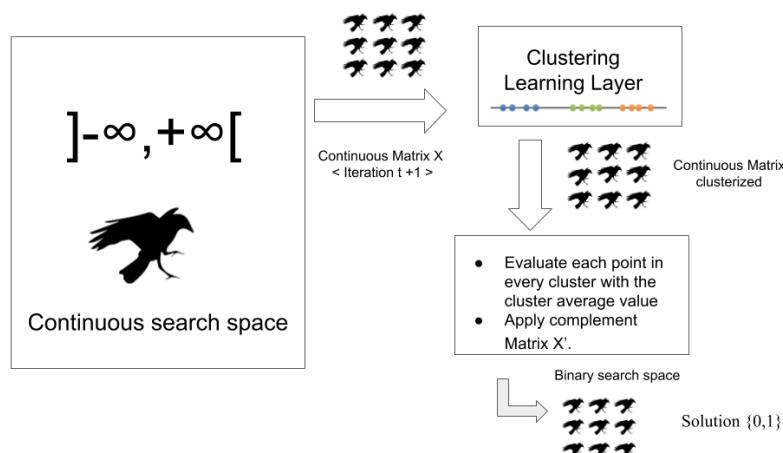
The third technique used is DBscan. This strategy is slightly different from previous ones because what we do is binarize the complete matrix instead of binarizing the solution vector. For that, we will create a matrix replica in its initial state ( $x'$ ), then the metaheuristic will normally work by performing its movements in the real search space. Once each iteration is complete, the entire matrix will be clustered. As we have seen before, DBscan receives a distance ( $\epsilon$ ) and a minimum of points (minPts) as input parameter, with this data the algorithm will only deliver a number of clusters depending on the data behavior. The following steps are performed after the clustering has been completed:

1. Calculate the mean of the points to every cluster.
2. Sort the clusters from least to greatest taking into account the average value of each cluster.
3. Evaluate each point in the clusters by checking if a random value between 0 and 1 is equal to or greater than the average of each cluster; if it is, then we get the position  $x_{i,j}$  and we apply the complement to the replica of the matrix ( $x'$ ). Then the objective function is evaluated.



**Figure 4.** Continuous to Binary KMeans.

For this technique, we take the strategy that is illustrated in Algorithm 5. Finally, Figure 5 shows how the matrix  $x$  is clusterized and then, it is transformed to a binary matrix applying Dbscan.



**Figure 5.** Continuous to Binary DBscan.

**Algorithm 5:** CSA + DBscan binarization technique pseudo-code

---

```

1 Randomly initialize the position of a flock of N crows in the search space
2 Evaluate the position of the crows
3 Initialize the memory of each crow
4 while  $iter < iter_{max}$  do
5   for  $i = 1$  to  $N$  do
6     c=Randomly choose one of the crows to follow
7     for  $j = 1$  to  $D$  do
8       if  $r_j \geq AP$  then
9          $x_{i,j}^{iter+1} = x_{c,j}^{iter} + r_i fl(m_{c,j}^{iter} - x_{c,j}^{iter})$ 
10      else
11         $x_{i,j}^{iter+1} = \text{a random position of the search space}$ 
12      end
13    end
14  end
15  Apply binarization to,  $x$ 
16  Check the feasibility of new positions
17  Evaluate the new position of the crows
18  Update the memory of crows
19 end

```

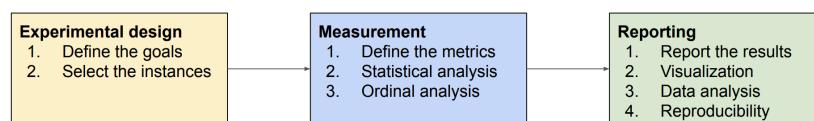
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## 6. Experimental Results

We propose to test our approach by solving the instances of the SCP [47] and the 0/1 Knapsack Problem [48].

### 6.1. Methodology

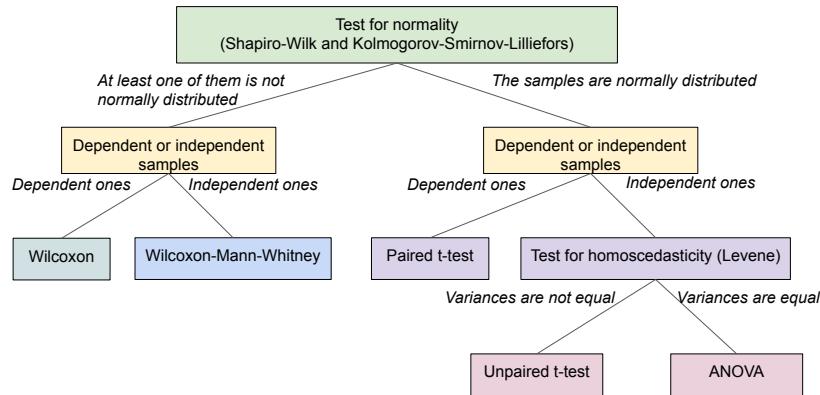
A performance analysis is totally necessary to properly evaluate the performance of metaheuristics [49]. In this study, we use the well known benchmark taken from OR-Library [50] in order to compare the given best solution of the CSA against the best known result of the benchmark. Figure 6 illustrates the steps for making an exhaustive analysis of the improved metaheuristic. For experimental design, we design aims and guidelines to demonstrate the proposed approach is a real alternative to binarization systems. Next, we establish the best value as the critical metric for evaluating future results. In this context, we apply an ordinal analysis and we employ statistical tests to determine which approach is significantly better. Finally, we describe the used hardware and software features to reproduce computational experiments, and then we report all results in tables and distribution charts.



**Figure 6.** Steps in performance analysis of a metaheuristic.

Moreover, it is very important to show how significant of a difference the three strategies have in this type of problem. For this reason we perform a contrast statistical test for each instance through the Kolmogorov-Smirnov-Lilliefors test to determine the independence of samples [51] and the Mann-Whitney-Wilcoxon test [52] to compare the results statistically (see Figure 7).

Kolmogorov-Smirnov-Lilliefors test allows us to analyze the independence of the samples by determining the  $Z_{MIN}$  or  $Z_{MAX}$  (depending if the problem is minimization or maximization) obtained from the 31 executions of each instance.



**Figure 7.** Statistical significance test.

The results are evaluated using the relative percentage deviation (RPD). The RPD value quantifies the deviation of the objective value  $Z_{min}$  from  $Z_{opt}$ , which is the minimal best-known value for each instance in our experiment, and it is calculated as follows:

$$RDP = \left( \frac{Z_{min} - Z_{opt}}{Z_{opt}} \right) \quad (3)$$

## 6.2. Set Covering Problem (SCP)

The Set Covering Problem [47] is a classic combinatorial optimization problem, belonging to the NP-hard class [53], that has been applied in several real-world problems like the location of emergency facilities [3], airline and bus crew scheduling [54], vehicle scheduling [55], steel production [56], among others.

Formally, the SCP is defined as follows: let  $A = (a_{ij})$  be a binary matrix with  $M$ -rows ( $\forall i \in I = \{1, \dots, M\}$ ) and  $N$ -columns ( $\forall j \in J = \{1, \dots, N\}$ ), and let  $C = (c_j)$  be a vector representing the cost of each column  $j$ , assuming that  $c_j > 0$ ,  $\forall j = \{1, \dots, N\}$ . Then, it observes that a column  $j$  covers a row  $i$  if  $a_{ij} = 1$ . Therefore, it has:

$$a_{ij} = \begin{cases} 1, & \text{if row } i \text{ can be covered by column } j \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

The SCP consists of finding a set of elements that cover a range of needs at the lowest cost. In its matrix form, a feasible solution corresponds to a subset of columns and the needs are associated with the rows and treated as constraints. The problem aims at selecting the columns that optimally cover all the rows.

The Set Covering Problem finds a minimum cost subset  $S$  of columns such that each row is covered by at least one column of  $S$ . An integer programming formulation of the SCP is as follows:

$$\begin{aligned} & \text{minimize} \sum_{j=1}^n c_j x_j \\ & \text{subject to:} \\ & \sum_{j=1}^n a_{ij} x_j \geq 1 \quad \forall i \in I \\ & x_j \in \{0, 1\} \quad \forall j \in J \end{aligned} \quad (5)$$

Instances: In order to evaluate the algorithm performance when solving the SCP, we use 65 instances taken from the Beasley's OR-library [50], which are organized in 11 sets.

Table 1 describes instance group, number of rows  $M$ , number of columns  $N$ , range of costs, density (percentage of nonzeroes in the matrix).

**Table 1.** Instances taken from the Beasley's OR-Library.

Instance Group	$M$	$N$	Cost Range	Density (%)	Best Known
4	200	1000	[1,100]	2	Known
5	200	2000	[1,100]	2	Known
6	200	1000	[1,100]	5	Known
A	300	3000	[1,100]	2	Known
B	300	3000	[1,100]	5	Known
C	400	4000	[1,100]	2	Known
D	400	4000	[1,100]	5	Known
NRE	500	5000	[1,100]	10	Unknown (except NRE.1)
NRF	500	5000	[1,100]	20	Unknown (except NRF.1)
NRG	1000	10,000	[1,100]	2	Unknown (except NRG.1)
NRH	1000	10,000	[1,100]	5	Unknown

Reducing the instance size of SCP: In [57] different preprocessing methods have particularly been proposed to reduce the size of the SCP, two of them have been taken as the most effective ones, which are Column Domination and Column Inclusion; these methods are used to accelerate the processing of the algorithm.

Column Domination: It consists of eliminating the redundant columns of the problem in such a way that it does not affect the final solution.

Steps:

- All the columns are ordered according to their cost in ascending order.
- If there are equal cost columns, these are sorted in descending order by the number of rows that the column  $j$  covers.
- Verify if the column  $j$  whose rows can be covered by a set of other columns with a cost less than  $c_j$  (cost of the column  $j$ ).
- It is said that column  $j$  is dominated and can be eliminated from the problem.

Column Inclusion: If a row is covered only by one column after the domination process, it means that there is no better column to cover those rows; consequently, this column must be included in the optimal solution.

This reduction will be used as input data of instances.

### 6.3. Knapsack (KP)

0–1 Knapsack Problem is still an actual issue [58] which is a typical NP-hard problem in operations research. There are many studies that use this problem to prove their algorithm, such as in [59,60]. The problem may be defined as follows: given a set of objects from  $N$ , each  $o$  object has a certain weight and value. Furthermore, the number of items to be included in the collection must be determined to ensure that the full weight is less than or equal to the limit and that the full value is as large as possible.

$$\chi_j = \begin{cases} 1 & \text{if the item is included} \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in J = \{1, 2, \dots, N\}. \quad (6)$$

The problem is to optimize the objective function:

$$\max \sum_{j=1}^n p_j x_j \quad (7)$$

Subject to:

$$\begin{aligned} \sum_{j=1}^n w_j x_j &\leq b \\ x_j &\in \{0, 1\}, \forall j \in J \end{aligned} \quad (8)$$

Binary decision variables  $x_j$  are used to indicate if item  $j$  is included in the knapsack or not. It can be assumed that all benefits and weights are positive and that all weights are smaller than capacity  $b$ .

Instances: In order to evaluate the algorithm performance of solving the KP, we used 10 instances [61].

Table 2 describes instance, dimension, and parameters: weight  $w$ , profit  $p$ , and capacity of the bag  $b$ .

**Table 2.** 0/1 Knapsack Problem instances.

Instance	Dimension	Parameters
f1	10	$w = \{95, 4, 60, 32, 23, 72, 80, 62, 65, 46\}$ $p = \{55, 10, 47, 5, 4, 50, 8, 61, 85, 87\}$ $b = 269$
f2	20	$w = \{92, 4, 43, 83, 84, 68, 92, 82, 6, 44, 32, 18, 56, 83, 25, 96, 70, 48, 14, 58\}$ $p = \{44, 46, 90, 72, 91, 40, 75, 35, 8, 54, 78, 40, 77, 15, 61, 17, 75, 29, 75, 63\}$ $b = 878$
f3	4	$w = \{6, 5, 9, 7\}$ $p = \{9, 11, 13, 15\}$ $b = 20$
f4	4	$w = \{2, 4, 6, 7\}$ $p = \{6, 10, 12, 13\}$ $b = 11$
f5	15	$w = \{56.358531, 80.87405, 47.987304, 89.59624, 74.660482, 85.894345, 51.353496, 1.498459, 36.445204, 16.589862, 44.569231, 0.466933, 37.788018, 57.118442, 60.716575\}$ $p = \{0.125126, 19.330424, 58.500931, 35.029145, 82.284005, 17.41081, 71.050142, 30.399487, 9.140294, 14.731285, 98.852504, 11.908322, 0.89114, 53.166295, 60.176397\}$ $b = 75$
f6	6	$w = \{30, 25, 20, 18, 17, 11, 5, 2, 1, 1\}$ $p = \{20, 18, 17, 15, 15, 10, 5, 3, 1, 1\}$ $b = 60$
f7	7	$w = \{31, 10, 20, 19, 4, 3, 6\}$ $p = \{70, 20, 39, 37, 7, 5, 10\}$ $b = 50$
f8	23	$w = \{983, 982, 981, 980, 979, 978, 488, 976, 972, 486, 486, 972, 485, 485, 969, 966, 483, 964, 963, 961, 958, 959\}$ $p = \{81, 980, 979, 978, 977, 976, 487, 974, 970, 85, 485, 970, 970, 484, 484, 976, 974, 482, 962, 961, 959, 958, 857\}$ $b = 10,000$
f9	5	$w = \{15, 20, 17, 8, 31\}$ $p = \{33, 24, 36, 37, 12\}$ $b = 80$
f10	20	$w = \{84, 83, 43, 4, 44, 6, 82, 92, 25, 83, 56, 18, 58, 14, 48, 70, 96, 32, 68, 92\}$ $p = \{91, 72, 90, 46, 55, 8, 35, 75, 61, 15, 77, 40, 63, 75, 29, 75, 17, 78, 40, 44\}$ $b = 879$

As we can see in the instance Table 2, distinct sorts of problems are solved, each with specific sizes and values to get a wide concept of the algorithm's behavior.

Software and Hardware: CSA was implemented in Java 1.8. The characteristics of the Personal Computer (PC) were: Windows 10 with an Intel Core i7-6700 CPU (3.4Ghz), 16 GB of RAM.

Parameter settings: Below, Table 3 shows the setup for Two-steps strategy, Table 4 shows the setup for KMeans strategy, and finally Table 5 shows setup for DBscan strategy.

**Table 3.** Crow Search Algorithm (CSA) Parameters for Set Covering Problem (SCP) and Knapsack Problem (KP)—Two-steps.

Population	Awareness Probability	Flight Length	Iterations
30	0.1	2	3000

**Table 4.** CSA Parameters for SCP and KP—KMeans.

Population	Awareness Probability	Flight Length	Number of Clusters	Cluster Transition Value	Iterations
30	0.3	2	3	0.04, 0.06, 0.08	3000

**Table 5.** CSA Parameters for SCP and KP—DBscan.

Population	Awareness Probability	Flight Length	Minimum Number of Points	Maximum Distance	Iterations
30	0.1	2	2	0.009	1000

Sixty-five instances of the SCP and ten instances of the 0/1 Knapsack Problem have been considered, which were executed 31 times each.

#### 6.4. Comparison Results and Strategies

In [62] and in Appendix section (Appendices A.1 and A.2), we can see the results obtained; the information is detailed, also indicating each of the transfer functions and the binarization methods that were used. The data is grouped as follows: instance: name of the instance, MIN: the minimum reached value, MAX: the maximum value reached, AVG: the average value, BKS: the best known solution, RPD it is defined by the Equation (3) and finally TIME: time of execution in seconds.

As we mentioned, the first strategy implemented was Two-steps, the second strategy implemented was KMeans, and the last strategy implemented was DBscan.

#### Set Covering Problem Results

Comparison strategy: Table 6 shows a summary of the instances grouped by their complexity; we define three groups: G1 (easy), G2 (medium), and G3 (hard). The first group includes instances 4.1 to 6.5, the second group includes instances a.1 to d.5, and the third group includes instances nre.1 to nrh.5.

To get more information of the comparison between strategies, see [62]. It will easily realize how complex it is to develop all the combinations of two steps in order to get good results vs. clustering techniques.

Summary: Next, we rank the strategies in order of BKS obtained. In addition, the instances that obtained BKS in the 3 strategies are shown.

- Score
  1. Two-steps got 51/65 BKS.
  2. DBscan got 25/65 BKS.
  3. KMeans got 16/65 BKS.
- The three strategies got BKS in the same instances: 4.1-4.3-4.6-4.7-5.1-5.4-5.5-5.6-5.7-5.8-5.9-6.2-6.4-a.5

**Table 6.** CSA/SCP—Two-steps vs. KMeans vs. DBscan resume summary (t.o = time out).

Instance	BKS	Stand.		Comp.		Static		Elitist		KMeans		DBscan	
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG
S1 G1	336.08	344.92	356.75	373.96	382.11	420.88	491.21	346.92	360.91	336.80	339.43	335.84	346.80
S1 G2	151.55	156.85	164.18	174	177.14	544.45	719.47	157.25	163.91	155.70	157.81	156.55	164
S1 G3	67.10	81.25	91.63	83.70	85.27	3673.70	4385.35	73.50	78.78	75.75	75.80	t.o.	t.o.
AVG	184.91	194.34	204.18	210.55	214.84	1546.34	1865.34	192.55	201.20	189.41	191.01	t.o.	t.o.
S2 G1	336.08	341.56	354.92	372.64	381.55	716	842.13	348.24	367.78	336.80	339.43	335.84	338.25
S2 G2	151.55	156.45	262.97	173.70	177.05	1222.70	1383.50	157.70	164.15	155.70	157.81	156.55	164
S2 G3	67.10	366.80	423.83	83.50	85.19	5619.10	5986.45	74.05	79.27	75.45	75.50	t.o.	t.o.
AVG	184.91	288.27	347.24	209.94	214.59	2519.26	2737.36	193.33	203.73	189.31	190.91	t.o.	t.o.
S3 G1	336.08	340.80	353.72	373.20	380.81	854.32	809.68	347.28	362.93	336.80	339.44	335.84	338.25
S3 G2	151.55	126.86	133.19	142.20	144.55	1243	1417.11	127.26	132.79	126.46	128.65	127.73	136.57
S3 G3	67.10	625.10	712.35	83.80	85.31	5291.90	5656.64	84.15	96.71	75.75	75.80	t.o.	t.o.
AVG	184.91	364.25	399.75	199.73	203.55	2463.07	2627.81	186.23	197.47	179.67	181.29	t.o.	t.o.
S4 G1	336.08	341.64	353.17	372.04	380.58	364.56	393.54	336.12	339.52	336.80	339.43	335.84	338.25
S4 G2	151.55	156.55	163.31	173.80	176.94	309.35	390.49	151.70	155.96	155.70	157.81	156.55	164
S4 G3	67.10	973.95	1071.68	84.05	85.49	2168.80	2546.21	69.80	73.84	75.75	75.80	t.o.	t.o.

**Table 6.** Cont.

Instance	BKS	Stand.		Comp.		Static		Elitist		KMeans		DBscan		
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	
AVG	184.91	490.71	529.38	209.96	214.33	947.57	1110.08	185.87	189.77	189.41	191.01	t.o.	t.o.	
V1 G1	336.08	738.40	849.97	385.88	507.63	742.76	883.61	580.48	659.84	336.80	339.44	335.84	338.25	
V1 G2	151.55	1290.05	1435.96	172.75	177.36	1287.55	1467.41	809.90	964.76	155.70	157.82	156.55	164	
V1 G3	67.10	5741.30	6160.88	83.45	85.41	5732.75	6209.07	3679.25	4089.29	75.75	75.80	t.o.	t.o.	
AVG	184.91	2589.91	2815.60	214.02	256.80	2587.68	2853.36	1689.87	1904.63	189.41	191.02	t.o.	t.o.	
V2 G1	336.08	726.88	827.56	376	387.02	710.48	834.56	565.16	653.04	336.80	339.43	335.84	338.25	
V2 G2	151.55	1229.95	1392.4395	174.70	179.786	1232	1431.86	782.50	914.29	155.70	157.82	156.55	164	
V2 G3	67.10	5590.55	5979.80	83.85	85.37	5595.90	6012.68	3532.40	3891.67	75.75	75.80	t.o.	t.o.	
AVG	184.91	2515.79	3403.68	211.51	217.39	2512.79	2759.70	1626.68	1819.66	189.41	191.01	t.o.	t.o.	
V3 G1	336.08	698.84	787.76	372.80	380.01	697	800.87	530.80	606.17	336.80	339.44	335.84	338.25	
V3 G2	151.55	1144.75	1303.63	174.05	176.59	1174.15	1342.70	665.60	785.03	155.70	157.82	156.55	164	
V3 G3	67.10	5255.20	5610.33	84.10	85.67	5215.45	5633.99	3056.45	3403.96	75.75	75.80	t.o.	t.o.	
AVG	V3	184.91	2366.26	2567.24	210.31	214.09	2362.20	2592.52	1417.61	1598.38	189.41	191.02	t.o.	t.o.
V4 G1	336.08	685.24	776.24	370	394.93	656.96	763.27	501.60	573.14	336.80	430	335.84	338.25	
V4 G2	151.55	1165.20	1347.83	173.50	176.32	1189.12	1198.33	589	697	155.70	157.82	156.55	164	
V4 G3	67.10	5241.35	5615.42	97.90	109.59	4802.40	5148.97	2548.55	2937.04	77.93	75.80	t.o.	t.o.	
AVG	184.91	2363.93	2579.83	213.8	226.94	2216.16	2370.19	1213.05	1402.39	190.14	221.20	t.o.	t.o.	

**Instance distribution:** Now that we have all the results of all the strategies that solved SCP, we compare the distribution of the samples of each instance through a box plot that shows the full distribution of the data. In order to make a summary of all the instances below, we present and describe the hardest instances of each group (4.10, 5.10, 6.5, a.5, b.5, c.5, d.5, nre.5, nrf.5, nrg.5 and nrh.5):

We can see that Figures 8 and 9 are alike. The Two-steps and DBscan strategies obtained BKS but KMeans did not. However, the distribution is quite homogeneous. If we see the instance 4.10 illustrated in Figure 8, DBscan's behavior is remarkable.

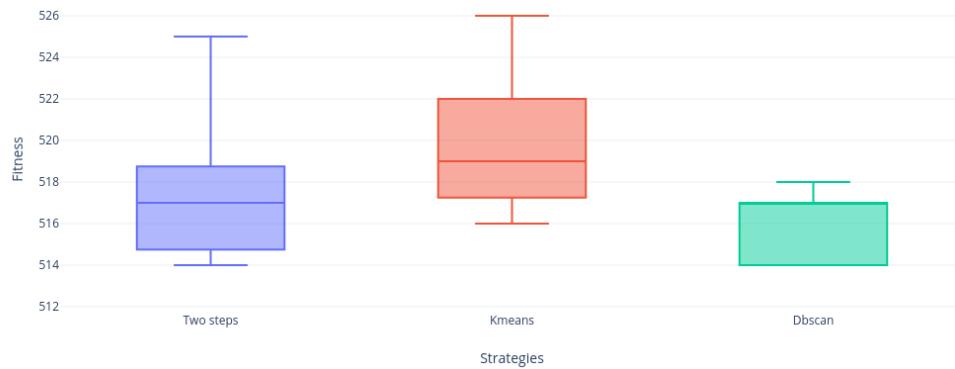
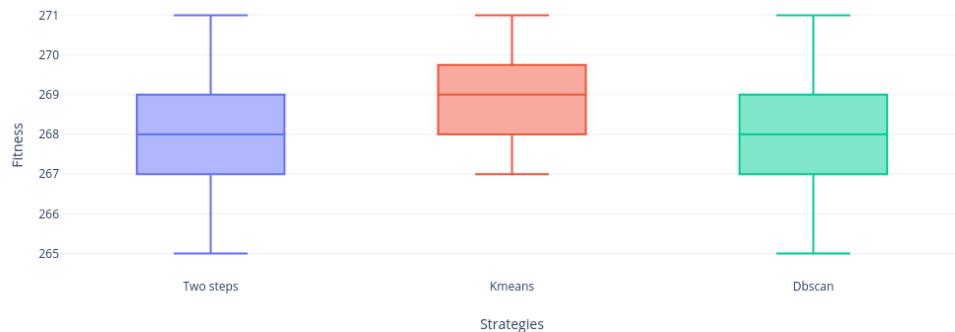
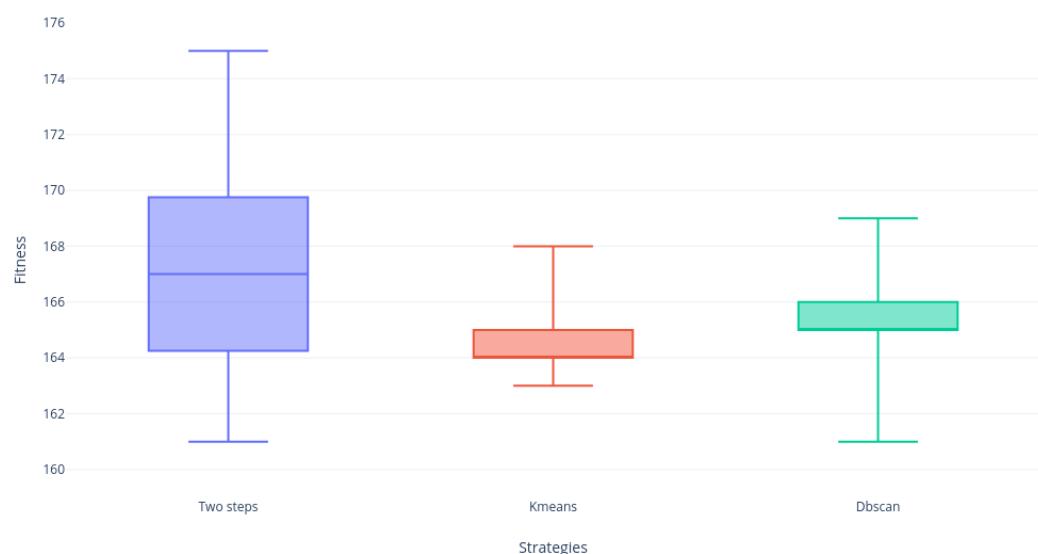
**Figure 8.** Instance 4.10 distribution.**Figure 9.** Instance 5.10 distribution.

Figure 10 shows again that the Two-steps and DBscan strategies obtained BKS and the KMeans strategy did not. However, it can be appreciated again that DBscan is far more compact than Two-steps, which indicates that clustering processing is accurate in this instance. Then, Figure 11 shows us for the first time a very nice and solidly compact distribution. It is necessary to emphasize that the three strategies obtained BKS. Finally, Figure 12 shows how the Two-steps and KMeans strategies obtained BKS while DBscan did not. However, if it was very close, despite this the most compact strategies in its distribution were KMeans and DBscan.

If we continue analyzing the samples, we can realize that Figures 13–17 are very similar, the two-step strategies and KMeans far exceeded DBscan. This gives us an indication that DBscan works better by solving small instances with this metaheuristic, perhaps due to a large number of calculations made by grouping the complete set of solutions versus KMeans that groups the solution and Two-steps that perform binarization instantly on each vector column solution.

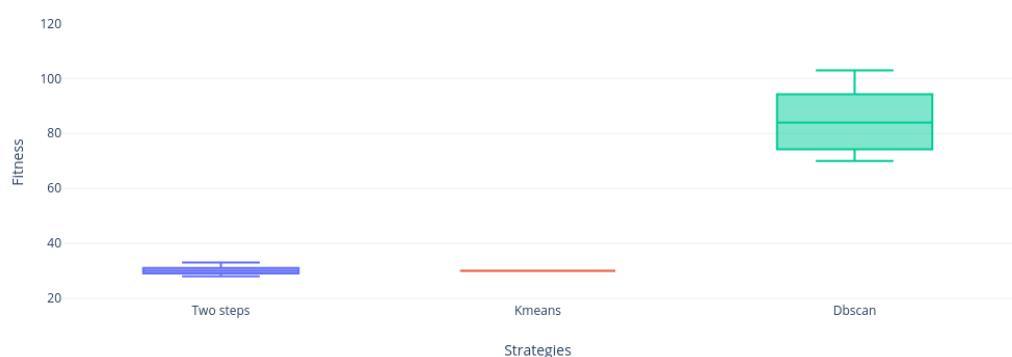
Finally, Figure 18 show us a null contribution of DBscan, who could not complete the executions due to the excessive delay of the strategy when clustering the most difficult instances; later we can observe KMeans absolutely trapped in an optimal local. However, the Two-steps approach continues to show good results, being the most robust at the BKS level as mentioned earlier in the strategy comparison.

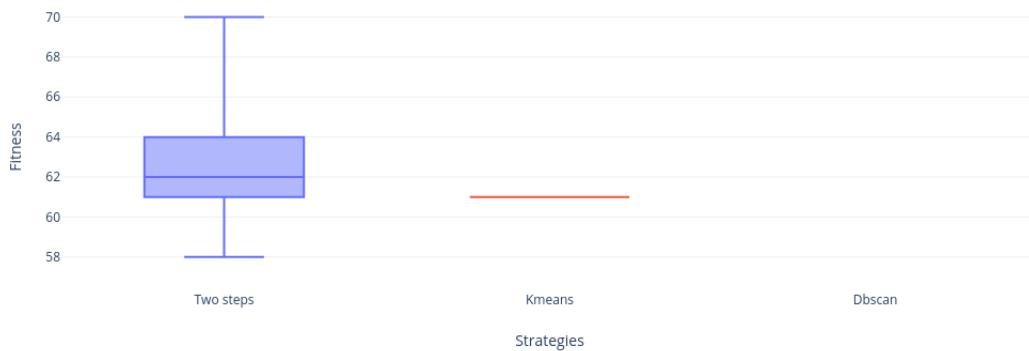


**Figure 10.** Instance 6.5 distribution.



**Figure 11.** Instance a.5 distribution.

**Figure 12.** Instance b.5 distribution.**Figure 13.** Instance c.5 distribution.**Figure 14.** Instance d.5 distribution.**Figure 15.** Instance nre.5 distribution.

**Figure 16.** Instance nrf.5 distribution.**Figure 17.** Instance nrg.5 distribution.**Figure 18.** Instance nrh.5 distribution.

### 6.5. Knapsack Problem Results

Unlike the previous problem (SCP), the Knapsack problem has fewer instances, so it was not necessary to group by complexity.

Comparison strategy: Table 7 shows a summary of the instances.

To get more information of the comparison between strategies, see [62].

Summary: Next, we rank the strategies in order of BKS obtained. In addition, the instances that obtained BKS in the three strategies are shown.

- Score
  - Two-steps got 6/10 BKS.
  - DBscan got 5/10 BKS.
  - KMeans got 4/10 BKS.

- The three strategies got BKS in the same instances: f3-f4-f7-f9

**Table 7.** CSA/KP—Two-steps vs. KMeans vs. DBscan resume summary.

Instance	BKS	Stand.		Comp.		Static		Elitist		KMeans		DBscan	
		MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg
S1	1293.91	1145.52	1049.61	1136.45	958.18	1176.22	1124.34	946	789.78	825.81	832.05	813.88	789.28
S2	1293.91	1076.23	1015.91	853.49	800.82	928.31	896.84	931.6	908.27	825.81	832.05	813.88	789.28
S3	1293.91	1164.51	1128.06	1169.75	1139.02	1153.52	1136.82	1176.37	1158.26	825.81	832.05	813.88	789.28
S4	1293.91	1069.53	1035.32	1071.46	1005.99	1117.15	1063.21	1203.77	1163.96	825.81	832.05	813.88	789.28
V1	1293.91	1070.59	1059.06	1199.57	1165.4	1165.53	1112.61	1235.02	1201.62	825.81	832.05	813.88	789.28
V2	1293.91	940.25	903.22	895.97	854.73	789.72	748.06	320.69	1617.22	825.81	832.05	813.88	789.28
V3	1293.91	1230.63	1220.57	1187.74	1153.86	1235.54	1208.77	1237.23	1203.28	825.81	832.05	813.88	789.28
V4	1293.91	1230.63	1220.57	1187.74	1153.86	1235.54	1208.77	1237.23	1203.28	825.81	832.05	813.88	789.28
AVG	1293.91	1115.98	1079.04	1087.77	1028.98	1100.19	1062.43	1036.012	1155.71	825.81	832.05	813.88	789.28

Instance distribution: Now that we have all the results of all the strategies that KP solved, we compare the distribution of the samples of each instance through a box diagram that shows the complete distribution of the data. To summarize all the instances below, we present and describe instances where the optimum was not reached because instances f3, f4, f7, and f9 always obtain BKS in each execution with the three strategies, so it is not necessary to graph these instances.

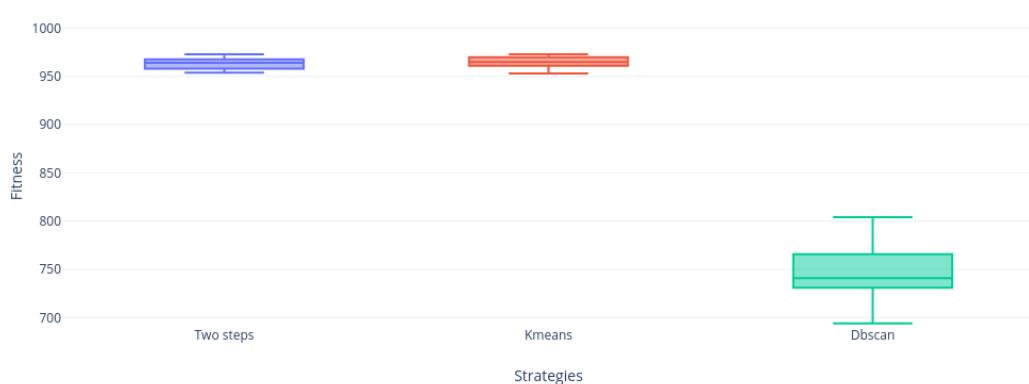
Figures 19–24 shows us that the Two-steps strategy achieved better results but DBscan strategy is far more compact than KMeans strategy. Figure 24 shows the only instance where DBscan did not win to KMeans.



**Figure 19.** Instance f1 distribution.



**Figure 20.** Instance f2 distribution.

**Figure 21.** Instance f5 distribution.**Figure 22.** Instance f6 distribution.**Figure 23.** Instance f8 distribution.**Figure 24.** Instance f10 distribution.

## 7. Statistical Test

As we mentioned, in order to determine independence, we propose the following hypotheses:

- $H_0$ : states that  $Z_{min}/Z_{max}$  follows a normal distribution.
- $H_1$ : states the opposite.

The test performed has yielded  $p\_value$  lower than 0.05; therefore,  $H_0$  cannot be assumed. Now that we know that the samples are independent and it cannot be assumed that they follow a normal distribution, it is not feasible to use the central limit theorem. Therefore, for evaluating the heterogeneity of samples we use a nonparametric evaluation called *Mann–Whitney–Wilcoxon* test. To compare all the results of the hardest instances, we propose the following hypotheses:

- $H_0$ : Two-steps is better than KMeans
- $H_1$ : states the opposite.
- $H_2$ : Two-steps is better than DBscan
- $H_3$ : states the opposite.
- $H_4$ : KMeans is better than DBscan
- $H_5$ : states the opposite.

Finally, the contrast statistical test demonstrates which technique is significantly better.

Tables 8–24 compare SCP and 0/1 KP binarization strategies for the hardest instances tested via the *Wilcoxon's signed rank* test. As the significance level is also established to 0.05, smaller values than 0.05 defines that  $H_0$ ,  $H_2$ , and  $H_4$  cannot be assumed.

We have a method belonging to the PISA system to conduct the test run that supports the study. To this method, we indicate all data distributions (each in a file and each data in a line) and the algorithm will give us a p-value for the hypotheses.

The following tables show the result of the *Mann–Whitney–Wilcoxon* test. To understand them, it is necessary to have knowledge of the following acronyms:

- SWS = Statistically without significance.
- D/A = Does not apply.

SCP— $p$ -value

**Table 8.**  $p$ -values for instance 4.10.

	Two-Steps	KMeans	DBscan
Two-steps	×	$6.47 \times 10^{-4}$	SWS
KMeans	SWS	×	SWS
DBscan	$1.02 \times 10^{-2}$	$2.91 \times 10^{-7}$	×

**Table 9.**  $p$ -values for instance 5.10.

	Two-Steps	KMeans	DBscan
Two-steps	×	$3.34 \times 10^{-2}$	SWS
KMeans	SWS	×	SWS
DBscan	SWS	$3.09 \times 10^{-2}$	×

**Table 10.**  $p$ -values for instance 6.5.

	Two-Steps	KMeans	DBscan
Two-steps	×	SWS	SWS
KMeans	$1.84 \times 10^{-3}$	×	$6.42 \times 10^{-4}$
DBscan	SWS	SWS	×

**Table 11.** *p*-values for instance a.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	SWS	SWS
KMeans	$1.85 \times 10^{-2}$	×	$4.25 \times 10^{-2}$
DBscan	SWS	SWS	×

**Table 12.** *p*-values for instance b.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	SWS	SWS
KMeans	SWS	×	SWS
DBscan	$1.14 \times 10^{-4}$	$9.54 \times 10^{-4}$	×

**Table 13.** *p*-values for instance c.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	SWS	$4.17 \times 10^{-12}$
KMeans	SWS	×	$4.69 \times 10^{-12}$
DBscan	SWS	SWS	×

**Table 14.** *p*-values for instance d.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	$2.29 \times 10^{-9}$	$9.60 \times 10^{-12}$
KMeans	SWS	×	$2.35 \times 10^{-13}$
DBscan	SWS	SWS	×

**Table 15.** *p*-values for instance nre.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	SWS	$5.59 \times 10^{-12}$
KMeans	SWS	×	$2.34 \times 10^{-13}$
DBscan	SWS	SWS	×

**Table 16.** *p*-values for instance nrf.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	$9.16 \times 10^{-12}$	$1.87 \times 10^{-12}$
KMeans	SWS	×	$2.37 \times 10^{-13}$
DBscan	SWS	SWS	×

**Table 17.** *p*-values for instance nrg.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	$3.26 \times 10^{-7}$	$6.49 \times 10^{-12}$
KMeans	SWS	×	$2.37 \times 10^{-13}$
DBscan	SWS	SWS	×

**Table 18.** *p*-values for instance nrh.5.

	<b>Two-Steps</b>	<b>KMeans</b>	<b>DBscan</b>
Two-steps	×	SWS	D/A
KMeans	$5.24 \times 10^{-4}$	×	D/A
DBscan	D/A	D/A	×

KP—*p*-value**Table 19.** *p*-values for instance f.1.

	Two-Steps	KMeans	DBscan
Two-steps	×	SWS	SWS
KMeans	$1.68 \times 10^{-10}$	×	$8.92 \times 10^{-12}$
DBscan	SWS	SWS	×

**Table 20.** *p*-values for instance f.2.

	Two-Steps	KMeans	DBscan
Two-steps	×	SWS	SWS
KMeans	$9.66 \times 10^{-9}$	×	$3.06 \times 10^{-2}$
DBscan	$7.23 \times 10^{-8}$	SWS	×

**Table 21.** *p*-values for instance f.5.

	Two-Steps	KMeans	DBscan
Two-steps	×	SWS	SWS
KMeans	$6.50 \times 10^{-12}$	×	$6.68 \times 10^{-12}$
DBscan	$3.49 \times 10^{-3}$	SWS	×

**Table 22.** *p*-values for instance f.6.

	Two-Steps	KMeans	DBscan
Two-steps	×	SWS	SWS
KMeans	$1.32 \times 10^{-13}$	×	$1.32 \times 10^{-13}$
DBscan	SWS	SWS	×

**Table 23.** *p*-values for instance f.8.

	Two-Steps	KMeans	DBscan
Two-steps	×	SWS	SWS
KMeans	$6.53 \times 10^{-12}$	×	$2.77 \times 10^{-3}$
DBscan	$6.54 \times 10^{-12}$	SWS	×

**Table 24.** *p*-values for instance f.10.

	Two-Steps	KMeans	DBscan
Two-steps	×	SWS	SWS
KMeans	SWS	×	SWS
DBscan	$6.53 \times 10^{-12}$	$6.49 \times 10^{-12}$	×

All reported *p*-values are less than 0.05; as indicated above, SWS indicates that it has no statistical significance, and D/A indicates that the comparison cannot be done because one of the strategies does not contain any data. So, with this information, we can clearly see which strategy was better than the three in each instance reported. As a resume for results for the SCP *p*-values lower than 0.05 were 10 for Two-steps, 10 for KMeans, and 5 for DBscan. On the other hand, results for the 0/1 KP *p*-values lower than 0.05 were 0 for Two steps, 10 for KMeans, and 5 for DBscan.

## 8. Conclusions

After implementing and analyzing binarization strategies through clusters, we can realize that DBscan achieved better results than KMeans at the BKS level. However, the execution times of DBscan were longer than KMeans; we believe that this has a clear explanation. The way in which the experiment was developed with clustering is crucial; KMeans cluster a vector while the BDscan clusters the complete matrix, which makes this technique take longer. Moreover, we do not report the solving time results due to the fact that it is known that this measure basically depends of the characteristics of the used machine (computer). Finally, our work is focused on enhancing the binarization process for metahueristics in order to find better results. In this way, we certainly have sacrificed the solving time but we think that DBscan is very promising because with these experiments we can realize that although we cluster the complete matrix with this technique, which took more time to execute, it still obtained good results.

When solving SCP, we have tested 65 nonunique instances of the Beasley OR Library and 10 well-known instances to solve KP, where classical technique obtained better results than clustering techniques in both problems. We believe that due to the number of combinations made (992 experiments for Two-steps), it allows a better performance on this metahuristic and also that it is necessary to make adjustments, perhaps of autonomous search on the clustering techniques to obtain a better performance.

The best results were obtained with the S-Shape transfer functions (S-Shape 4 predominates over the others) and Elitist binarization method. The worst results were the combination of the V-Shape transfer function with the binarization complement. It is very relevant to say that KMeans did not get better results than Two-steps strategy. On the other hand, it is relevant that DBscan obtained good results, better than KMeans and worse than Two-steps but the quantity of Two-step combinations is a very influential factor since with 32 different different ways of binarization it is far from being an easy way to implement; this requires time and knowledge of an expert user. Which means we reduce to one single experiment (31 executions) with DBscan over the combinatorial strategies of two-step getting good results. Statistically DBscan got better performance.

As future work, we plan to reverse the strategies of KMeans and DBscan by solving the grouping problem as a matrix with KMeans and as a unique solution with DBscan. In addition, we will implement a self-adaptive approach to get better results. Finally, we will apply the online control parameters in the Two-steps strategy to reach BKS in instances that did not reach optimal values.

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## Appendix A

### Appendix A.1. Set Covering Problem Results

#### Appendix A.1.1. SCP—Crow Search Algorithm

**Table A1.** CSA/SCP—SShape 1—Two-steps vs. KMeans vs. DBscan (t.o = time out).

Instance	BKS	S1+Stand.		S1+Comp.		S1+Static		S1+Elitist		KMeans		DBscan	
		MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg
4.1	429	438	444.81	459	463.32	465	494.87	432	443.74	429	430	429	430.38
4.2	512	522	550.35	579	586.39	612	698.06	528	551.10	513	515.84	512	514.77
4.3	516	520	532.58	552	564.19	587	620.47	564	578.36	516	520.61	516	518.87
4.4	494	513	524.58	544	555.26	566	620.13	496	515.84	495	498.87	494	496.10
4.5	512	518	529.23	554	570.45	601	680.39	518	529.55	514	514.90	512	513.23
4.6	560	577	590.23	614	626.23	661	798.55	566	590	560	562.35	560	560.19
4.7	430	436	445.90	485	492.81	498	558.90	438	451.68	430	432.26	430	430.81
4.8	492	501	513.74	532	540.29	585	680.61	499	508.52	493	496.13	492	494.10
4.9	641	675	690.42	722	745.06	807	911.65	657	687.94	645	657.90	641	647.50
4.10	514	534	554.74	569	578.90	595	678.27	528	550.23	516	519.9	514	515.81
5.1	253	265	275.29	291	296.26	309	352.61	258	271.32	253	255.74	253	255.52
5.2	302	310	327.55	352	363.68	421	471.19	317	326.74	308	309.81	302	306.68
5.3	226	230	240.19	249	252.65	278	310.35	229	239.74	228	228.68	226	227.32
5.4	242	242	249.43	263	264.90	301	354.12	254	269.84	242	242.23	242	243.29
5.5	211	211	223.13	242	245.97	247	301.87	245	268.21	211	212.03	211	213.90
5.6	213	222	235.71	251	254.39	267	309.87	226	237.87	213	215.10	213	213.04
5.7	293	305	314.61	327	335.29	360	408.94	305	315.10	293	297.68	293	295.35
5.8	288	302	310.06	342	348.48	370	424.65	299	310.84	288	290.35	288	290.39
5.9	279	283	303.23	321	331.29	357	381.28	287	305.68	279	280.52	279	279.90
5.10	265	271	277.45	294	298.45	335	376.16	273	277.10	267	268.77	265	267.97
6.1	138	146	152.03	159	166	210	287.13	145	151.71	140	142.90	140	143.42
6.2	146	152	157.29	163	169	308	401.10	150	156.10	146	148.87	146	149.71
6.3	154	148	156.52	162	167.23	268	379.39	148	156.65	147	148.16	145	149.65
6.4	131	131	138.26	137	141.94	247	359.87	138	147.64	131	131.68	131	133.03
6.5	161	171	181.35	186	194.50	267	419.94	173	181.42	163	164.68	162	165.39
AVG	336.08	344.92	356.75	373.96	382.11	420.88	491.21	346.92	360.91	336.80	339.43	335.84	346.80
a.1	253	257	263.74	269	272.48	474	584.81	256	261.81	254	255.55	254	256
a.2	252	262	274.42	300	305.23	439	565.55	267	274.74	257	259.48	256	260.65
a.3	232	242	251	256	258.16	410	525.81	242	250.35	235	237.10	233	237.35
a.4	234	239	250.71	264	268.35	409	523.32	236	250.97	235	237.39	236	239.97
a.5	236	239	243.32	258	263.03	421	540.65	238	241.84	236	237.06	236	237.52
b.1	69	75	81.61	82	86.10	347	518.13	72	81.58	74	79.61	69	76.48
b.2	76	84	90.26	92	94.19	411	549.61	81	88.35	83	90.03	81	86.81
b.3	80	80	85.94	91	91.90	507	535.9	85	87.65	84	88.55	82	88.81
b.4	79	83	89.10	96	100.26	517	660.52	84	88.55	84	87.90	83	88.16
b.5	72	72	78.92	82	84.55	498	587.58	76	80.35	72	77.68	78	76.03
c.1	227	232	237.65	263	264.65	512	718.52	233	236.71	228	230.29	233	238.77
c.2	219	228	236.61	259	263.13	617	782.81	226	235.58	226	222.71	226	232.74
c.3	243	257	270.55	297	302.32	738	939.94	261	271	254	256.68	253	264
c.4	219	231	243.45	256	260.17	618	783.23	233	243.58	225	227.97	224	233.10
c.5	215	219	226.81	245	250.29	529	713.77	218	226.06	215	216.35	222	230.42
d.1	60	61	68.29	70	70.87	600	889.42	63	67.23	66	66	68	79.65
d.2	66	70	73.52	77	78.68	686	1001.55	68	74.19	71	71	73	89.71
d.3	72	77	81.68	83	86.74	881	1114.29	78	82.13	82	82	82	100.84
d.4	66	65	68.29	71	71.81	644	945	65	68.23	67	67	70	81.74
d.5	61	64	67.74	69	69.97	631	909.13	63	67.39	66	66	72	81.29
AVG	151.55	156.85	164.18	174	177.14	544.45	719.47	157.25	163.91	155.70	157.81	156.55	164
nre.1	29	30	33.23	32	33	1354	1643.54	30	32.68	30	30	70	80.03
nre.2	30	33	37.45	36	37.61	1405	1732.55	33	36.19	34	34	83	128.32
nre.3	27	28	33.61	32	34.61	1137	1416.77	30	33.61	34	34	74	112.35
nre.4	28	31	34	34	34	1210	1712.48	30	33.90	33	34	83	177.45
nre.5	28	30	32.94	33	33.87	1292	1860.23	30	32.77	30	30	92	150.52
nrf.1	14	17	19	17	17.03	667	903.48	15	18.42	17	17	372	444.19
nrf.2	15	16	18.71	18	18	607	836.29	16	18.61	18	18	74	404.58
nrf.3	14	17	19.39	19	19.90	858	1034.90	17	19.29	19	19	335	487.25
nrf.4	14	15	18.45	17	18.61	667	902.97	16	18.29	18	18	52	420.52
nrf.5	13	15	18.29	16	16.32	653	844.23	15	17.61	16	16	65	371.03
nrg.1	176	201	208.26	230	233.48	4473	5588.65	192	203.39	197	197	287	385.13
nrg.2	154	172	181.06	187	190.48	3848	4648.48	166	174.16	168	168	208	284.29
nrg.3	166	180	190.55	196	197.84	4319	5026.55	180	185.19	183	183	211	347.23
nrg.4	168	187	200.03	216	219.26	4142	4935.29	180	192.10	186	186	250	345.77
nrg.5	168	191	201.10	213	217.32	4198	5155	181	193.84	186	186	230	353.97
nrh.1	63	102	124.68	82	83.65	9080	10,028.42	72	77.42	77	77	t.o.	t.o.
nrh.2	63	92	120.10	81	81.81	9085	10,332.71	71	76.48	71	71	t.o.	t.o.
nrh.3	59	89	120.23	74	75	8101	9886.68	67	73.61	69	69	t.o.	t.o.
nrh.4	58	91	116.52	73	74.97	8592	9899.26	67	70.94	68	68	t.o.	t.o.
nrh.5	55	88	105.13	68	68.74	7786	9318.55	62	67.16	61	61	t.o.	t.o.
AVG	67.10	81.25	91.63	83.70	85.27	3673.70	4385.35	73.50	78.78	75.75	75.80	t.o.	t.o.

**Table A2.** CSA/SCP—SShape 2—Two-steps vs. KMeans vs. DBscan (t.o = time out).

Instance	BKS	S2+Stand.		S2+Comp.		S2+Static		S2+Elitist		KMeans		DBscan	
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG
4.1	429	432	443.48	457	462.81	622	686.58	431	439.73	429	430	429	430.38
4.2	512	517	549.26	578	587.26	929	1084.16	522	549.50	513	515.84	512	514.77
4.3	516	519	532	555	564.23	789	987.25	578	687.47	516	520.61	516	518.87
4.4	494	500	520.55	542	555.81	861	963.39	501	517.03	495	498.87	494	496.10
4.5	512	518	531.84	557	573.48	901	1101.97	514	529.40	514	514.90	512	513.23
4.6	560	566	588.48	610	624.81	1194	1320.35	568	588.70	560	562.35	560	560.19
4.7	430	436	447.13	479	491.35	821	892.06	437	448.60	430	432.26	430	430.81
4.8	492	499	514.74	525	539.52	946	1095.10	493	506.97	493	496.13	492	494.10
4.9	641	641	652.16	722	743.39	1368	1499.48	662	682.20	645	657.90	641	647.50
4.10	514	523	550.52	560	575.35	873	1064.16	518	546.53	516	519.9	514	515.81
5.1	253	257	273.52	287	295.26	490	556.81	260	270.27	253	255.74	253	255.52
5.2	302	312	325.23	352	361.77	717	800.55	308	321.53	308	309.81	302	306.68
5.3	226	231	240.77	250	253.23	466	522.19	229	237.13	228	228.68	226	227.32
5.4	242	244	249.13	261	264.68	357	487.25	247	278.64	242	242.23	242	243.29
5.5	211	215	223.77	242	245.87	287	359.25	287	301.07	211	212.03	211	213.90
5.6	213	217	235.61	245	253.68	422	498.10	224	234	213	215.10	213	213.04
5.7	293	307	312.90	331	334.16	563	650.16	299	311.40	293	297.68	293	295.35
5.8	288	291	310.71	339	346.74	575	654.52	298	308.57	288	290.35	288	290.39
5.9	279	288	306.16	320	331.23	487	587.37	297	357.26	279	280.52	279	279.90
5.10	265	269	276.58	290	297.55	553	609.87	272	278	267	268.77	265	267.97
6.1	138	148	151.67	161	165.26	569	732.77	143	150.77	140	142.90	140	143.42
6.2	146	150	158.58	166	169	774	995.52	151	157.90	146	148.87	146	149.71
6.3	154	150	157.65	162	167.77	766	949.35	150	155.27	147	148.16	145	149.65
6.4	131	134	138.65	139	142.06	778	954.21	147	157.28	131	131.68	131	133.03
6.5	161	175	181.94	186	192.58	792	1000.87	170	179.47	163	164.68	162	165.39
AVG	336.08	341.56	354.92	372.64	381.55	716	842.13	348.24	367.78	336.80	339.43	335.84	338.25
a.1	253	258	264.39	270	273.16	981	1126.26	257	262.60	254	255.55	254	256
a.2	252	263	275.06	301	305.32	943	1066.48	266	274.33	257	259.48	256	260.65
a.3	232	247	250.35	256	257.87	812	998.84	241	250.03	235	237.10	233	237.35
a.4	234	234	240.06	265	268.90	848	977.29	241	251.03	235	237.39	236	239.97
a.5	236	238	244.26	259	264.29	834	971.19	239	242.90	236	237.06	236	237.52
b.1	69	72	80.58	82	85.26	1068	1196.61	76	80.83	74	79.61	69	76.48
b.2	76	76	81.45	93	94	1039	1179.84	80	88.30	83	90.03	81	86.81
b.3	80	82	86.55	91	91.84	1054	1178.89	86	91.04	84	88.55	82	88.81
b.4	79	84	88.68	95	100.23	1153	1353.97	83	87.57	84	87.90	83	88.16
b.5	72	73	79.55	82	84.52	1127	1201.31	78	84.89	72	77.68	78	76.03
c.1	227	233	240.23	262	264.68	1128	1308.87	230	236.47	228	230.29	233	238.77
c.2	219	219	2225.10	254	262.68	1359	1470.74	230	237.47	226	222.71	226	232.74
c.3	243	260	272.03	295	301.52	1447	1734.35	259	269.80	254	256.68	253	264
c.4	219	235	243.74	254	259.42	1365	1463.19	233	243.40	225	227.97	224	233.10
c.5	215	219	225.65	246	249.77	1253	1364.74	219	225.50	215	216.35	222	230.42
d.1	60	63	68.77	70	70.87	1483	1682.68	62	67.67	66	66	68	79.65
d.2	66	69	74.45	77	78.23	1608	1893.90	69	73.57	71	71	73	89.71
d.3	72	78	82.06	83	86.68	1900	2102.94	77	80.40	82	82	82	100.84
d.4	66	64	68.65	70	71.81	1531	1698.81	64	68.27	67	67	70	81.74
d.5	61	62	67.94	69	69.90	1521	1699.13	64	66.87	66	66	72	81.29
AVG	151.55	156.45	262.97	173.70	177.05	1222.70	1383.50	157.70	164.15	155.70	157.81	156.55	164
nre.1	29	29	33.14	32	33	2147	2147.65	30	33.85	30	30	70	80.03
nre.2	30	32	37.29	36	37.61	2652	2897.61	33	37	34	34	83	128.32
nre.3	27	31	34.23	33	34.52	2242	2510.84	30	33.20	34	34	74	112.35
nre.4	28	29	34.32	33	33.97	2519	2804.13	31	33.97	33	34	83	177.45
nre.5	28	30	33.58	33	33.94	2697	2965.90	29	32.67	30	30	92	150.52
nrf.1	14	16	18.16	17	17	1438	1688.13	16	18.53	17	17	372	444.19
nrf.2	15	16	19.23	17	17.97	1335	1516.13	17	18.47	18	18	74	404.58
nrf.3	14	17	19.77	19	19.87	1540	1825.87	17	19.50	19	19	335	487.25
nrf.4	14	16	18.68	17	18.71	1524	1664.94	16	18.37	18	18	52	420.52
nrf.5	13	16	18.10	16	16.32	1313	1445.74	15	17.83	16	16	65	371.03
nrg.1	176	327	371.45	230	233.13	7127	7506.23	192	203.50	197	197	287	385.13
nrg.2	154	233	279.13	185	189.77	5994	6340.61	167	175.87	168	168	208	284.29
nrg.3	166	281	315.81	195	197.71	6580	6997.29	178	186.17	183	183	211	347.23
nrg.4	168	272	303.65	215	218.90	6601	7007.48	183	192.27	186	186	250	345.77
nrg.5	168	276	329	216	217.55	6778	7198.52	184	194.33	186	186	230	353.97
nrh.1	63	1165	1405.97	83	83.77	12,353	12,926.58	72	77.97	71	71	t.o.	t.o.
nrh.2	63	1153	1326.81	79	81.77	11,778	12,818.74	73	78.57	71	71	t.o.	t.o.
nrh.3	59	1199	1361.74	74	74.87	11,992	12,604.52	69	73.53	69	69	t.o.	t.o.
nrh.4	58	1187	1334.77	73	74.84	12,295	12,797.39	66	71.03	68	68	t.o.	t.o.
nrh.5	55	1011	1181.94	67	68.71	11,477	12,064.84	63	68.87	61	61	t.o.	t.o.
AVG	67.10	366.80	423.83	83.50	85.19	5619.10	5986.45	74.05	79.27	75.45	75.50	t.o.	t.o.

**Table A3.** CSA/SCP—SShape 3—Two-steps vs. KMeans vs. DBscan (t.o = time out).

Instance	BKS	S3+Stand.		S3+Comp.		S3+Static		S3+Elitist		KMeans		DBscan	
		MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg
4.1	429	431	442.52	452	461.10	587	667.32	432	439.45	429	430	429	430.38
4.2	512	515	542.26	576	585.35	906	1028.52	515	545.26	513	515.84	512	514.77
4.3	516	516	529.13	554	564.97	798	869.54	579	602.32	516	520.61	516	518.87
4.4	494	495	518.71	544	553.81	828	914.39	501	513.42	495	498.87	494	496.10
4.5	512	517	527.61	559	568.74	863	1036	514	525.35	514	514.90	512	513.23
4.6	560	565	585.53	618	624.13	1082	1239.10	566	587.68	560	562.35	560	560.19
4.7	430	434	445.52	484	491.16	728	830.16	438	447	430	432.26	430	430.81
4.8	492	498	508.68	529	539.26	942	1075.97	497	506.26	493	496.13	492	494.10
4.9	641	654	684.03	726	741.90	1145	1399.77	656	677.03	645	657.90	641	647.50
4.10	514	518	540.97	568	578.26	852	1017.61	517	537.58	516	519.9	514	515.81
5.1	253	260	270.84	289	294.16	471	519.52	257	268.13	253	255.74	253	255.52
5.2	302	313	325.77	349	360.81	694	761.77	314	323.06	308	309.81	302	306.68
5.3	226	229	237	249	252.81	419	476.16	228	234.52	228	228.68	226	227.32
5.4	242	243	248.53	261	264.45	368	589.12	302	317.22	242	242.23	242	243.29
5.5	211	214	221.10	241	244.97	358	475.74	235	247.54	211	212.03	211	213.90
5.6	213	221	233.32	250	253.84	383	469.61	222	234	213	215.10	213	213.04
5.7	293	301	311.84	327	332.84	558	622.35	302	311.52	293	297.68	293	295.35
5.8	288	298	307.87	338	345.74	545	631.48	294	307.29	288	290.35	288	290.39
5.9	279	281	303.13	315	329.26	4412	506.06	301	387.31	279	280.52	279	279.90
5.10	265	271	276.39	294	297.97	497	573.84	268	275.19	267	268.77	265	267.97
6.1	138	145	151.55	159	164.97	701	874.98	144	148.90	140	142.90	140	143.42
6.2	146	151	157.61	166	169.19	803	941.94	148	156.29	146	148.87	146	149.71
6.3	154	148	155.68	160	166.71	788	892.10	148	152.84	147	148.16	145	149.65
6.4	131	132	137	139	141.81	814	912.32	139	150.01	131	131.68	131	133.03
6.5	161	170	180.42	183	192	816	916.68	165	177.97	163	164.68	162	165.39
AVG	336.08	340.80	353.72	373.20	380.81	854.32	809.68	347.28	362.93	336.80	339.44	335.84	338.25
a.1	253	257	262.13	270	273.10	903	1039.42	256	262.06	254	255.55	254	256
a.2	252	261	274.94	301	305.19	861	980.68	264	271.61	257	259.48	256	260.65
a.3	232	241	249.94	255	257.55	812	924.23	242	249.35	235	237.10	233	237.35
a.4	234	243	251.97	246	268.48	828	921.84	240	249.87	235	237.39	236	239.97
a.5	236	238	242.10	260	264.74	846	926.26	237	241.97	236	237.06	236	237.52
b.1	69	74	80.68	82	84.90	962	1110.10	73	80.58	74	79.61	69	76.48
b.2	76	83	88.68	90	93.71	961	1083.71	81	88.68	83	90.03	81	86.81
b.3	80	84	87.13	90	91.71	987	1087.36	86	89.78	84	88.55	82	88.81
b.4	79	84	89.32	95	99	1105	1265.16	81	88.03	84	87.90	83	88.16
b.5	72	74	79.61	80	84.13	1187	1398.65	80	81.27	72	77.68	78	76.03
c.1	227	232	237.48	262	264.52	1098	1230.23	233	236.48	228	230.29	233	238.77
c.2	219	225	236.58	256	262.45	1193	1351.26	225	232.90	226	222.71	226	232.74
c.3	243	257	270.03	298	301.10	1466	1621.74	262	270.23	254	256.68	253	264
c.4	219	230	242.74	256	259.45	1220	1346.16	233	243.19	225	227.97	224	233.10
c.5	215	218	225.13	243	249.65	1118	1289.65	219	225.03	215	216.35	222	230.42
d.1	60	63	68.61	79	70.90	1367	1563.39	61	66.58	66	66	68	79.65
d.2	66	70	74.26	77	78.35	1415	1757.35	69	73.39	71	71	73	89.71
d.3	72	78	81	85	86.71	1665	1942.03	78	81.13	82	82	82	100.84
d.4	66	67	68.68	71	71.90	1458	1610	65	67.84	67	67	70	81.74
d.5	61	64	67.94	69	69.84	1443	1599.94	63	66.87	66	66	72	81.29
AVG	151.55	126.86	133.19	142.20	144.55	1243	1417.11	127.26	132.79	126.46	128.65	127.73	136.57
nre.1	29	30	33.45	32	33	2224	2478.47	35	36.21	30	30	70	80.03
nre.2	30	33	37.26	35	37.10	2535	2724.68	32	36	34	34	83	128.32
nre.3	27	30	33.65	33	34.29	2016	2332.48	30	32.87	34	34	74	112.35
nre.4	28	32	34.58	34	34	2386	2632.77	31	34.29	33	34	83	177.45
nre.5	28	30	33.16	33	33.94	2573	2762.35	28	32.03	30	30	92	150.52
nrf.1	14	15	19.35	16	17	1348	1565.77	15	17.87	17	17	372	444.19
nrf.2	15	16	18.77	18	18	1265	1442.90	16	18.10	18	18	74	404.58
nrf.3	14	17	20.29	19	19.68	1578	1739.68	16	19.52	19	19	335	487.25
nrf.4	14	17	19.06	17	18.68	1386	1546.10	15	17.97	18	18	52	420.52
nrf.5	13	16	18.74	16	16.26	1229	1367.84	15	17.61	16	16	65	371.03
nrg.1	176	585	702	231	233.42	6483	7083.65	200	210.65	197	197	287	385.13
nrg.2	154	441	506.13	186	190.19	5619	5953.74	166	177	168	168	208	284.29
nrg.3	166	484	570.19	196	198.16	5920	6475.87	180	188.23	183	183	211	347.23
nrg.4	168	506	571.71	217	219.74	5971	6549.42	185	196.81	186	186	250	345.77
nrg.5	168	515	620.45	213	218.13	6572	6799.97	186	198.19	186	186	230	353.97
nrh.1	63	2043	2305.06	83	83.97	11,391	12,133.23	119	156.13	77	77	t.o.	t.o.
nrh.2	63	1942	2188.55	81	81.77	11,644	12,162.52	112	141.13	71	71	t.o.	t.o.
nrh.3	59	1870	2244.55	75	75	11,169	11,969.55	100	141.81	69	69	t.o.	t.o.
nrh.4	58	2102	2259.94	74	75.10	11,577	12,029.13	109	143.13	68	68	t.o.	t.o.
nrh.5	55	1778	2010.26	67	68.90	10,952	11,382.81	93	118.71	61	61	t.o.	t.o.
AVG	67.10	625.10	712.35	83.80	85.31	5291.90	5656.64	84.15	96.71	75.75	75.80	t.o.	t.o.

**Table A4.** CSA/SCP—SShape 4—Two-steps vs. KMeans vs. DBscan (t.o = time out).

Instance	BKS	S4+Stand.		S4+Comp.		S4+Static		S4+Elitist		KMeans		DBscan	
		MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg	MIN	Avg
4.1	429	432	440.45	456	461.06	440	455.10	429	430.61	429	430	429	430.38
4.2	512	515	542.77	571	583.74	561	598.77	512	515.74	513	515.84	512	514.77
4.3	516	516	530.25	554	564.32	523	587.21	516	518.87	516	520.61	516	518.87
4.4	494	507	517.45	541	553.48	518	551.81	494	500.10	495	498.87	494	496.10
4.5	512	516	527.71	551	568.74	547	578.55	512	514.61	514	514.90	512	513.23
4.6	560	569	586.65	612	624.77	607	643.74	560	563.61	560	562.35	560	560.19
4.7	430	434	444.84	483	491.55	451	487.13	430	432.26	430	432.26	430	430.81
4.8	492	493	508.06	529	539	516	558.61	492	494.19	493	496.13	492	494.10
4.9	641	668	682.74	728	743.71	705	766.61	647	654.68	645	657.90	641	647.50
4.10	514	522	541.81	564	576.52	558	589.06	514	517.19	516	519.9	514	515.81
5.1	253	254	268.74	288	294.55	279	298.19	253	256.74	253	255.74	253	255.52
5.2	302	315	326.68	349	360.74	335	368.16	302	308.39	308	309.81	302	306.68
5.3	226	228	235.97	246	252.32	244	261.71	226	227.32	228	228.68	226	227.32
5.4	242	242	247.74	262	264.87	268	278.36	243	246.23	242	242.23	242	243.29
5.5	211	212	220.29	241	245.32	250	238.07	212	214.32	211	212.03	211	213.90
5.6	213	223	232.84	247	253.65	236	260.35	213	213.90	213	215.10	213	213.04
5.7	293	301	311.48	327	331.61	322	344.13	293	295.03	293	297.68	293	295.35
5.8	288	296	309.32	336	344.90	323	348.68	288	289.90	288	290.35	288	290.39
5.9	279	279	300.32	314	329.32	280	301.24	280	281.34	279	280.52	279	279.90
5.10	265	272	276.23	292	297.48	288	306.77	265	267.87	267	268.77	265	267.97
6.1	138	145	150.90	158	164.29	157	183.84	138	143.13	140	142.90	140	143.42
6.2	146	150	155.58	165	168.71	180	225.68	146	150.61	146	148.87	146	149.71
6.3	154	148	154.19	162	166.77	170	213.39	145	150.32	147	148.16	145	149.65
6.4	131	133	136.55	138	141.32	148	154.24	132	134.23	131	131.68	131	133.03
6.5	161	171	179.71	187	191.94	208	239.29	161	167	163	164.68	162	165.39
AVG	336.08	341.64	353.17	372.04	380.58	364.56	393.54	336.12	339.52	336.80	339.43	335.84	338.25
a.1	253	253	257.65	270	273.13	322	367.97	254	257.26	254	255.55	254	256
a.2	252	262	272.58	299	304.87	326	381.03	252	260.13	257	259.48	256	260.65
a.3	232	242	249.81	255	257.81	310	348.26	232	236.52	235	237.10	233	237.35
a.4	234	241	250.10	264	267.94	299	348.39	235	240.65	235	237.39	236	239.97
a.5	236	239	243.87	261	266.13	295	350.74	236	237.74	236	237.06	236	237.52
b.1	69	74	80.16	82	85	176	246.52	69	72.97	74	79.61	69	76.48
b.2	76	79	87.81	92	93.74	183	261.26	77	81.45	83	90.03	81	86.81
b.3	80	81	86.06	90	91.58	174	257.03	81	82.34	84	88.55	82	88.81
b.4	79	85	89.68	95	99.13	161	297.45	79	82.55	84	87.90	83	88.16
b.5	72	72	78.58	82	83.90	189	247.98	73	74.32	72	77.68	78	76.03
c.1	227	231	237.90	262	264.55	371	424.48	227	232.77	228	230.29	233	238.77
c.2	219	224	234.55	258	262.90	372	461.16	220	225.10	226	222.71	226	232.74
c.3	243	258	270.42	294	300.68	434	560.23	243	252.81	254	256.68	253	264
c.4	219	233	241.48	256	259.55	353	445.23	219	225.74	225	227.97	224	233.10
c.5	215	219	225.55	245	250.42	374	444.32	216	220.55	215	216.35	222	230.42
d.1	60	63	67.97	69	70.74	376	440.77	60	63	66	66	68	79.65
d.2	66	70	74.32	77	78.45	397	493.06	66	68.39	71	71	73	89.71
d.3	72	76	81.32	85	86.58	439	560.29	72	75.52	82	82	82	100.84
d.4	66	67	68.42	71	71.81	319	450.90	62	65.58	67	67	70	81.74
d.5	61	62	68	69	69.94	317	422.81	61	63.84	66	66	72	81.29
AVG	151.55	156.55	163.31	173.80	176.94	309.35	390.49	151.70	155.96	155.70	157.81	156.55	164
nre.1	29	30	34.23	32	32.84	687	748.32	31	31.65	30	30	70	80.03
nre.2	30	36	39.94	35	37.26	741	904.42	31	33.55	34	34	83	128.32
nre.3	27	31	34.45	33	34.58	622	751.81	27	29.77	34	34	74	112.35
nre.4	28	33	35.52	34	34	688	860.13	28	30.81	33	34	83	177.45
nre.5	28	31	36.13	33	33.94	781	936.06	28	29.74	30	30	92	150.52
nrf.1	14	15	18.58	17	17.03	362	444	14	15.65	17	17	372	444.19
nrf.2	15	16	19.26	17	17.97	268	394.61	15	16.19	18	18	74	404.58
nrf.3	14	17	19.61	19	19.68	355	500	15	16.61	19	19	335	487.25
nrf.4	14	15	18.19	18	18.61	334	429.77	15	15.71	18	18	52	420.52
nrf.5	13	16	18.35	16	16.10	272	377.81	14	15	16	16	65	371.03
nrg.1	176	1071	1186.32	229	234.16	2757	3176.94	179	189.55	197	197	287	385.13
nrg.2	154	758	861.84	189	190.58	2226	2669.29	160	166.23	168	168	208	284.29
nrg.3	166	850	1008.58	197	198.39	2574	2898.26	168	177.65	183	183	211	347.23
nrg.4	168	860	1008.48	217	219.29	2569	2951.26	175	180.74	186	186	250	345.77
nrg.5	168	974	1078.77	216	219.39	2732	3081.35	174	181.13	186	186	230	353.97
nrh.1	63	3065	3318.32	82	84.10	5294	6065.26	71	74.19	77	77	t.o.	t.o.
nrh.2	63	2962	3254.03	81	81.97	5486	5997.68	67	73.48	71	71	t.o.	t.o.
nrh.3	59	3034	3253.35	75	75.39	4853	5984.23	64	69.03	69	69	t.o.	t.o.
nrh.4	58	3010	3255.65	73	75.55	5002	6163.90	62	67.71	68	68	t.o.	t.o.
nrh.5	55	2655	2934.16	68	69.10	4773	5589.06	58	62.45	61	61	t.o.	t.o.
AVG	67.10	973.95	1071.68	84.05	85.49	2168.80	2546.21	69.80	73.84	75.75	75.80	t.o.	t.o.

**Table A5.** CSA/SCP—VShape 1—Two-steps vs. KMeans vs. DBscan (t.o = time out).

Instance	BKS	V1+Stand.		V1+Comp.		V1+Static		V1+Elitist		KMeans		DBscan	
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG
4.1	429	639	699.94	454	463.13	647	723.19	540	600.03	429	430	429	430.38
4.2	512	959	1086.65	573	584.39	953	1127.10	742	881.45	513	515.84	512	514.77
4.3	516	1040	1172.58	665	754.21	754	845.32	701	732.21	516	520.61	516	518.87
4.4	494	869	985.39	539	553.61	817	986.74	723	792.35	495	498.87	494	496.10
4.5	512	925	1102.23	554	561.52	964	1143.55	824	904.32	514	514.90	512	513.23
4.6	560	1259	1357.42	614	624.45	1119	1371.90	939	1038.55	560	562.35	560	560.19
4.7	430	751	875.52	481	493.45	800	913.26	590	710.81	430	432.26	430	430.81
4.8	492	1010	1144.26	530	539.45	1028	1175.71	796	902.58	493	496.13	492	494.10
4.9	641	1315	1513.10	720	740.29	1192	1616.90	1051	1193.26	645	657.90	641	647.50
4.10	514	975	1108.10	566	572.26	987	1131.74	748	863.19	516	519.9	514	515.81
5.1	253	506	566.03	288	296.23	477	557.65	385	446.97	253	255.74	253	255.52
5.2	302	730	821.42	350	359.68	726	849.23	545	622.29	308	309.81	302	306.68
5.3	226	454	502.19	246	251.19	450	537.45	366	407.32	228	228.68	226	227.32
5.4	242	438	519.26	321	354.78	451	534.21	369	409.99	242	242.23	242	243.29
5.5	211	363	402.84	254	2987.54	458	587.32	370	411.21	211	212.03	211	213.90
5.6	213	435	490.16	248	254.16	454	516.84	354	397.32	213	215.10	213	213.04
5.7	293	565	658.52	329	336.03	518	666.68	449	526.74	293	297.68	293	295.35
5.8	288	598	668.61	337	345.42	585	676.74	483	527.29	288	290.35	288	290.39
5.9	279	580	673.10	398	401.05	547	654.21	489	531.07	279	280.52	279	279.90
5.10	265	539	608.13	291	296.48	555	615.87	398	472.52	267	268.77	265	267.97
6.1	138	625	702.68	161	165	628	744.58	413	504.45	140	142.90	140	143.42
6.2	146	708	1013.94	161	167.13	923	1047.35	576	694.87	146	148.87	146	149.71
6.3	154	783	946.39	198	209.19	849	1021.68	529	641.48	147	148.16	145	149.65
6.4	131	534	618.55	180	185.98	847	1035.65	584	601.25	131	131.68	131	133.03
6.5	161	860	1012.23	189	194.06	840	1009.48	548	682.58	163	164.68	162	165.39
AVG	336.08	738.40	849.97	385.88	507.63	742.76	883.61	580.48	659.84	336.80	339.44	335.84	338.25
a.1	253	1030	1148.10	269	271.06	992	1181.42	703	810.94	254	255.55	254	256
a.2	252	903	1068.74	294	303.16	950	1085.06	612	766.45	257	259.48	256	260.65
a.3	232	838	998.48	254	258.10	951	1019.65	598	714.74	235	237.10	233	237.35
a.4	234	908	995.32	263	266.90	925	1026.77	635	706.06	235	237.39	236	239.97
a.5	236	930	1017.94	252	259.16	848	1043.42	653	732.81	236	237.06	236	237.52
b.1	69	1092	1205.55	82	85.42	1091	1235.58	656	781.48	74	79.61	69	76.48
b.2	76	1093	1215.06	92	93.65	1108	1225.10	639	798.13	83	90.03	81	86.81
b.3	80	1375	1546.65	96	100.28	1153	1482.72	689	799.25	84	88.55	82	88.81
b.4	79	1250	1380.39	95	100.19	1194	1402.32	713	914.65	84	87.90	83	88.16
b.5	72	1108	1209.35	87	98.21	1201	1403.21	721	915.41	72	77.68	78	76.03
c.1	227	1163	1349.45	258	262.32	1209	1358.77	804	925.77	228	230.29	233	238.77
c.2	219	1356	1502.10	258	261.97	1311	1534.29	837	1018.81	226	222.71	226	232.74
c.3	243	1660	1769.61	295	301.03	1675	1832.58	1069	1218.94	254	256.68	253	264
c.4	219	1354	1476.10	255	258.23	1331	1500.71	854	1014	225	227.97	224	233.10
c.5	215	1283	1403.81	240	246.81	1265	1448.26	757	980.29	215	216.35	222	230.42
d.1	60	1494	1743.97	69	70.58	1602	1790.90	942	1139.16	66	66	68	79.65
d.2	66	1832	1982.32	76	77.90	1784	2016.32	1121	1287.32	71	71	73	89.71
d.3	72	1955	2154.87	83	86.45	1985	2192.74	1187	1421.10	82	82	82	100.84
d.4	66	1549	1779.29	68	76.26	1634	1786.94	1045	1184.48	67	67	70	81.74
d.5	61	1628	1772.13	69	69.55	1542	1781.55	963	1165.39	66	66	72	81.29
AVG	151.55	1290.05	1435.96	172.75	177.36	1287.55	1467.41	809.90	964.76	155.70	157.82	156.55	164
nre.1	29	2335	2506.26	31	39.36	2501	2975.21	1625	1825.21	30	30	70	80.03
nre.2	30	2782	3022.13	35	37.77	2687	3002.68	1716	1929.81	34	34	83	128.32
nre.3	27	2244	2547.81	32	34.61	2263	2628.39	1507	1700.26	34	34	74	112.35
nre.4	28	2763	2912	34	34	2681	2944.87	1663	1917.81	33	34	83	177.45
nre.5	28	2850	3078.26	33	33.97	2728	3036.26	1674	2017.84	30	30	92	150.52
nrf.1	14	1570	1722.10	17	17	1566	1782.81	986	1139.03	17	17	372	444.19
nrf.2	15	1376	1566	18	18	1450	1578.03	931	1063.00	18	18	74	404.58
nrf.3	14	1733	1933.13	19	19.84	1716	1903.77	1126	1275.97	19	19	335	487.25
nrf.4	14	1543	1729.35	18	18.90	1489	1718.84	1019	1134.26	18	18	52	420.52
nrf.5	13	1334	1488.71	16	16.29	1332	1500.13	820	988.39	16	16	65	371.03
nrg.1	176	7156	7696.32	230	232.68	7405	7914.90	4673	5041.81	197	197	287	385.13
nrg.2	154	6109	6442.68	185	189.29	6155	6470.32	3935	4351.13	168	168	208	284.29
nrg.3	166	6507	7098.19	195	197.42	6542	7166.68	4375	4740.77	183	183	211	347.23
nrg.4	168	6460	7072.45	215	218.19	6792	7302.13	4086	4713.61	186	186	250	345.77
nrg.5	168	6837	7458.77	214	216.81	6712	7387.65	4416	4919.71	186	186	230	353.97
nrh.1	63	12,515	13,137.26	82	83.71	12,574	13,310.87	8001	8869	77	77	t.o.	t.o.
nrh.2	63	12,381	13,125.45	81	81.94	12,227	13,093.87	7802	8797.23	71	71	t.o.	t.o.
nrh.3	59	12,241	13,122.81	74	74.97	12,180	13,132.84	7957	8662.29	69	69	t.o.	t.o.
nrh.4	58	12,464	13,209.45	73	74.97	12,097	12,974.87	7934	8595.19	68	68	t.o.	t.o.
nrh.5	55	11,626	12,348.52	67	68.55	11,558	12,356.23	7339	8103.61	61	61	t.o.	t.o.
AVG	67.10	5741.30	6160.88	83.45	85.41	5732.75	6209.07	3679.25	4089.29	75.75	75.80	t.o.	t.o.

**Table A6.** CSA/SCP—VShape 2—Two-steps vs. KMeans vs. DBscan (t.o = time out).

Instance	BKS	V2+Stand.		V2+Comp.		V2+Static		V2+Elitist		KMeans		DBscan	
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG
4.1	429	616	687.77	453	462.45	637	706.10	498	582.52	429	430	429	430.38
4.2	512	925	1074.84	577	585.19	968	1084.70	769	851.81	513	515.84	512	514.77
4.3	516	999	1129.90	512	548.09	812	987.63	725	974.15	516	520.61	516	518.87
4.4	494	841	946.16	541	553.32	834	946.37	681	774.26	495	498.87	494	496.10
4.5	512	997	1082.71	557	568.26	959	1089.29	748	875.52	514	514.90	512	513.23
4.6	560	1161	1305.23	614	625.03	1148	1310.87	849	990.03	560	562.35	560	560.19
4.7	430	762	847.39	478	495.52	751	869.77	665	704.06	430	432.26	430	430.81
4.8	492	997	1105.65	532	540.29	900	1128.16	749	863.55	493	496.13	492	494.10
4.9	641	1265	1501.42	736	745.16	1345	1512.90	1026	1141.94	645	657.90	641	647.50
4.10	514	889	1045.52	566	574.10	973	1103.13	756	844.03	516	519.9	514	515.81
5.1	253	504	552.81	289	295.29	488	556.55	390	430.32	253	255.74	253	255.52
5.2	302	708	790.77	351	360.06	732	816.40	520	602.45	308	309.81	302	306.68
5.3	226	427	499.35	245	251.35	426	506.33	356	401.32	228	228.68	226	227.32
5.4	242	462	507.42	297	301.02	398	547.14	398	409.94	242	242.23	242	243.29
5.5	211	347	395.90	287	304.21	421	568.96	371	419.36	211	212.03	211	213.90
5.6	213	438	483.77	249	254.45	424	499.13	346	382.42	213	215.10	213	213.04
5.7	293	588	644.39	329	334.81	538	654.47	426	504.13	293	297.68	293	295.35
5.8	288	580	665.32	337	345.74	602	675.07	470	523.06	288	290.35	288	290.39
5.9	279	567	652.19	338	354.32	413	501.06	487	524.98	279	280.52	279	279.90
5.10	265	544	598.87	294	298.23	508	598.17	419	468.97	267	268.77	265	267.97
6.1	138	588	701.55	159	164.06	613	711.50	388	474.45	140	142.90	140	143.42
6.2	146	777	962.42	160	167.03	885	1034.29	502	654.45	146	148.87	146	149.71
6.3	154	802	931.32	157	166.39	807	956.61	475	603.71	147	148.16	145	149.65
6.4	131	518	602.61	154	187.17	420	508.36	576	672.19	131	131.68	131	133.03
6.5	161	870	973.77	188	193.90	760	991.13	539	652.29	163	164.68	162	165.39
AVG	336.08	726.88	827.56	376	387.02	710.48	834.56	565.16	653.04	336.80	339.43	335.84	338.25
a.1	253	1009	1141.32	268	271.23	999	1159.06	706	781.26	254	255.55	254	256
a.2	252	911	1045.35	297	303.90	900	1039.52	589	744.06	257	259.48	256	260.65
a.3	232	899	995.65	256	257.90	876	967.77	593	688.20	235	237.10	233	237.35
a.4	234	875	972.06	266	268.42	805	990.94	585	694.43	235	237.39	236	239.97
a.5	236	862	977.81	254	260.26	836	985.77	533	690.83	236	237.06	236	237.52
b.1	69	1072	1178.35	82	85.58	1089	1186.58	633	732.90	74	79.61	69	76.48
b.2	76	1037	1157.16	92	93.77	984	1174.29	636	753.30	83	90.03	81	86.81
b.3	80	1263	1475.23	100	132.14	1258	1748.19	716	845.14	84	88.55	82	88.81
b.4	79	1170	1346.77	95	100.16	1191	1348.10	730	853.63	84	87.90	83	88.16
b.5	72	1017	1200.29	98	111.12	1098	1569.02	730	784.96	72	77.68	78	76.03
c.1	227	1106	1313.23	260	263.29	1165	1315.68	761	906.87	228	230.29	233	238.77
c.2	219	1295	1461.61	258	262.35	1299	1479.65	811	980.50	226	222.71	226	232.74
c.3	243	1513	1710.23	296	301.29	1552	1728.39	1024	1130.37	254	256.68	253	264
c.4	219	1270	1418.29	256	258.97	1284	1447.32	784	937.70	225	227.97	224	233.10
c.5	215	1230	1372.42	244	247.81	1229	1385.45	828	933.03	215	216.35	222	230.42
d.1	60	1504	1693.84	70	70.81	1509	1685.35	935	1068.37	66	66	68	79.65
d.2	66	1640	1890.45	77	78.52	1750	1912.71	1058	1226.83	71	71	73	89.71
d.3	72	1799	2083.48	85	86.84	1807	2091.03	1153	1321.50	82	82	82	100.84
d.4	66	1638	1721.70	71	71.68	1424	1718.71	899	1108.30	67	67	70	81.74
d.5	61	1489	1693.55	69	69.68	1585	1703.68	946	1103.57	66	66	72	81.29
AVG	151.55	1229.95	1392.4395	174.70	179.786	1232	1431.86	782.50	914.29	155.70	157.82	156.55	164
nre.1	29	2176	2403.26	36	36.87	2548	2857.31	1578	1987.47	30	30	70	80.03
nre.2	30	2695	2918.32	37	37.84	2619	2923.84	1672	1889.33	34	34	83	128.32
nre.3	27	2334	2521.45	33	34.74	2296	2500.48	1435	1614.57	34	34	74	112.35
nre.4	28	2553	2828.48	34	34	2526	2793.87	1614	1813.67	33	34	83	177.45
nre.5	28	2711	2938.65	33	33.87	2654	2970.90	1614	1909.20	30	30	92	150.52
nrf.1	14	1468	1682.45	17	17	1514	1659.13	925	1069.30	17	17	372	444.19
nrf.2	15	1402	1536.06	18	18	1398	1531.61	855	978	18	18	74	404.58
nrf.3	14	1701	1855.81	19	19.97	1628	1858.19	1029	1200.63	19	19	335	487.25
nrf.4	14	1488	1644.90	18	18.77	1542	1674.55	915	1058.73	18	18	52	420.52
nrf.5	13	1296	1442.48	16	16.35	1287	1475.16	782	919.70	16	16	65	371.03
nrg.1	176	7041	7478.61	231	233.19	6869	7443.55	4523	4843.27	197	197	287	385.13
nrg.2	154	5984	6380.45	185	189.77	5805	6303.19	3737	4111.17	168	168	208	284.29
nrg.3	166	6452	6925.32	193	196.97	6559	6929.65	3983	4468.67	183	183	211	347.23
nrg.4	168	6442	6891.16	214	218.90	6441	6973.35	4130	4511.93	186	186	250	345.77
nrg.5	168	6906	7285.71	215	217.35	6755	7169.55	4273	4642.87	186	186	230	353.97
nrh.1	63	12,063	12,811.13	83	83.90	11,907	12,923.58	7492	8301.20	77	77	t.o.	t.o.
nrh.2	63	11,862	12,769.65	81	81.61	12,413	12,925.84	7687	8235.53	71	71	t.o.	t.o.
nrh.3	59	11,860	12,599.94	75	75	12,040	12,671.87	7463	8225.97	69	69	t.o.	t.o.
nrh.4	58	12,029	12,719.58	73	74.81	11,741	12,699.65	7657	8212.10	68	68	t.o.	t.o.
nrh.5	55	11,348	11,962.68	66	68.52	11,376	11,968.42	7284	7840.06	61	61	t.o.	t.o.
AVG	67.10	5590.55	5979.80	83.85	85.37	5595.90	6012.68	3532.40	3891.67	75.75	75.80	t.o.	t.o.

**Table A7.** CSA/SCP—VShape 3—Two-steps vs. KMeans vs. DBscan (t.o = time out).

Instance	BKS	V3+Stand.		V3+Comp.		V3+Static		V3+Elitist		KMeans		DBscan	
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG
4.1	429	598	655.43	457	462.13	602	664.40	513	566.07	429	430	429	430.38
4.2	512	867	1018.73	574	583.63	929	1028.10	746	805.20	513	515.84	512	514.77
4.3	516	982	1085.40	553	560.43	784	995.35	684	789.33	516	520.61	516	518.87
4.4	494	831	930.40	536	552.20	796	909.37	669	732.60	495	498.87	494	496.10
4.5	512	886	1036.67	557	568.50	858	1053.90	749	813.83	514	514.90	512	513.23
4.6	560	1108	1241.53	608	622.70	1095	1250.77	848	949.10	560	562.35	560	560.19
4.7	430	717	827.30	487	494.83	767	833.60	609	662.93	430	432.26	430	430.81
4.8	492	891	1045.40	533	538.27	949	1073.60	694	827.87	493	496.13	492	494.10
4.9	641	1243	1391.57	728	741.90	1279	1403.10	975	1075.33	645	657.90	641	647.50
4.10	514	917	1003.93	561	570.33	929	1026.50	709	804.67	516	519.9	514	515.81
5.1	253	463	515.40	285	294.80	462	512.10	363	407.90	253	255.74	253	255.52
5.2	302	714	762.50	352	361.23	620	759.07	520	570.77	308	309.81	302	306.68
5.3	226	436	471.43	248	251.13	403	481.77	338	373.47	228	228.68	226	227.32
5.4	242	433	493.33	261	263.67	485	501.02	336	489.30	242	242.23	242	243.29
5.5	211	359	388.10	243	246.50	478	503.31	333	401.01	211	212.03	211	213.90
5.6	213	423	458.60	249	253.17	415	459.23	334	362.60	213	215.10	213	213.04
5.7	293	541	603.30	326	332.40	541	613	431	476.60	293	297.68	293	295.35
5.8	288	552	608.03	339	346.10	576	635.13	412	492.63	288	290.35	288	290.39
5.9	279	549	630.70	319	328.03	541	587.01	444	498.21	279	280.52	279	279.90
5.10	265	519	566.27	294	298	506	558.87	402	442.03	267	268.77	265	267.97
6.1	138	545	650.13	160	164.10	496	638.97	355	417.13	140	142.90	140	143.42
6.2	146	824	937.90	161	166.10	682	917.63	466	593.47	146	148.87	146	149.71
6.3	154	778	879.17	162	166.07	737	869.20	430	539.07	147	148.16	145	149.65
6.4	131	488	554.37	141	142.23	714	842.36	441	478.85	131	131.68	131	133.03
6.5	161	807	938.37	186	191.83	781	904.33	469	584.17	163	164.68	162	165.39
AVG	336.08	698.84	787.76	372.80	380.01	697	800.87	530.80	606.17	336.80	339.44	335.84	338.25
a.1	253	951	1070.40	269	271.50	952	1080	563	702.97	254	255.55	254	256
a.2	252	919	1003.17	300	304.80	897	995.60	570	667.50	257	259.48	256	260.65
a.3	232	791	913.70	256	257.70	856	936.30	525	627.07	235	237.10	233	237.35
a.4	234	805	901	265	268.23	773	907.73	497	619.87	235	237.39	236	239.97
a.5	236	828	925.97	257	262.07	826	933.07	523	630.43	236	237.06	236	237.52
b.1	69	979	1084.37	83	85.43	988	1091.90	520	653.60	74	79.61	69	76.48
b.2	76	870	1094.33	92	93.73	975	1107.30	563	671.77	83	90.03	81	86.81
b.3	80	1193	1379.63	90	91.73	1172	1854.32	687	761.87	84	88.55	82	88.81
b.4	79	1076	1238.70	97	99.87	1053	1226	646	769.07	84	87.90	83	88.16
b.5	72	1065	1149.37	81	84.03	1254	1425.65	666	701.35	72	77.68	78	76.03
c.1	227	1089	1238.87	261	263.87	1088	1228.87	663	784.23	228	230.29	233	238.77
c.2	219	1205	1382.37	259	262.17	1245	1386.47	759	873.50	226	222.71	226	232.74
c.3	243	1432	1609.63	296	300.67	1477	1613.73	875	1029.33	254	256.68	253	264
c.4	219	1207	1342.40	258	259.40	1197	1333.23	656	842.50	225	227.97	224	233.10
c.5	215	1036	1276.17	247	248.67	1148	1284.57	215	216.50	215	216.35	222	230.42
d.1	60	1355	1578.83	70	70.90	1434	1562.33	773	967.23	66	66	68	79.65
d.2	66	1597	1747.63	77	78.53	1558	1770.80	957	1081.63	71	71	73	89.71
d.3	72	1728	1964.13	83	86.67	1648	1923.90	1010	1174.70	82	82	82	100.84
d.4	66	1477	1592.73	71	71.77	1472	1603.60	835	966.50	67	67	70	81.74
d.5	61	1292	1579.23	69	70	1470	1588.57	809	959.03	66	66	72	81.29
AVG	151.55	1144.75	1303.63	174.05	176.59	1174.15	1342.70	665.60	785.03	155.70	157.82	156.55	164
nre.1	29	2053	2254.20	33	33	2147	2457.32	1111	1365.21	30	30	70	80.03
nre.2	30	2527	2711.20	36	37.37	2386	2741.63	1378	1644.57	34	34	83	128.32
nre.3	27	2074	2322.83	33	34.60	2185	2346.80	1211	1414.67	34	34	74	112.35
nre.4	28	2433	2608.93	33	33.97	2461	2614	1423	1610.63	33	34	83	177.45
nre.5	28	2609	2775.47	33	33.97	2516	2794.60	1528	1664.90	30	30	92	150.52
nrf.1	14	1416	1569.43	17	17.03	1340	1587.47	829	955.90	17	17	372	444.19
nrf.2	15	1321	1443.47	17	17.97	1271	1446.23	762	868.80	18	18	74	404.58
nrf.3	14	1567	1738.87	19	19.53	1475	1712.23	905	1045.07	19	19	335	487.25
nrf.4	14	1399	1552.83	17	18.47	1274	1547.93	792	935.03	18	18	52	420.52
nrf.5	13	1231	1354.60	16	16.13	1247	1361.70	689	824.87	16	16	65	371.03
nrg.1	176	6430	7032.93	229	234.50	6671	7135.37	3734	4242.67	197	197	287	385.13
nrg.2	154	5334	5918.47	189	191.03	5595	5990.23	3194	3623.40	168	168	208	284.29
nrg.3	166	6059	6399.60	197	198.53	5854	6377.47	3451	3881.77	183	183	211	347.23
nrg.4	168	6162	6558.77	215	219.80	6050	6507.13	3620	3956.83	186	186	250	345.77
nrg.5	168	6343	6718.60	216	219.80	6296	6798.60	3516	4139.43	186	186	230	353.97
nrh.1	63	11,013	11,981	82	84.37	11,191	12,000.60	6901	7398.43	77	77	t.o.	t.o.
nrh.2	63	11,607	12,096.40	81	82.40	11,396	12,110.83	6578	7364.83	71	71	t.o.	t.o.
nrh.3	59	11,473	11,903.43	75	75.83	11,155	11,836.20	6625	7112.07	69	69	t.o.	t.o.
nrh.4	58	11,435	12,043.30	75	75.77	11,313	11,985.40	6649	7158.20	68	68	t.o.	t.o.
nrh.5	55	10,618	11,222.19	69	69.26	10,486	11,328.03	6233	6871.97	61	61	t.o.	t.o.
AVG	67.10	5255.20	5610.33	84.10	85.67	5215.45	5633.99	3056.45	3403.96	75.75	75.80	t.o.	t.o.

**Table A8.** CSA/SCP—VShape 4—Two-steps vs. KMeans vs. DBscan: Groups NRE, NRF, NRG, and NRH (t.o = time out).

Instance	BKS	V4+Stand.		V4+Comp.		V4+Static		V4+Elitist		KMeans		DBscan	
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG
4.1	429	605	664.50	453	459.93	588	636.33	514	535.30	429	430	429	430.38
4.2	512	903	1026.30	571	581.20	873	956.63	672	743.70	513	515.84	512	514.77
4.3	516	952	1078.60	552	558.87	802	879.87	655	748.21	516	520.61	516	518.87
4.4	494	820	908.53	530	546.47	801	858.87	657	702.67	495	498.87	494	496.10
4.5	512	870	1024.23	553	564.67	892	970.57	676	757.03	514	514.90	512	513.23
4.6	560	1087	1236.17	613	621.43	1048	1163	786	882.90	560	562.35	560	560.19
4.7	430	728	808.67	485	490.90	663	781.77	547	629.87	430	432.26	430	430.81
4.8	492	960	1045.10	525	536.23	884	986.07	706	761.53	493	496.13	492	494.10
4.9	641	1229	1415.40	726	737.37	1132	1305.30	909	995.10	645	657.90	641	647.50
4.10	514	905	1011.53	552	566.40	859	943.57	711	757.73	516	519.9	514	515.81
5.1	253	474	524.30	288	292.70	415	486.23	335	380.70	253	255.74	253	255.52
5.2	302	642	755.50	350	358.47	654	712.43	468	523.30	308	309.81	302	306.68
5.3	226	399	472	246	249.97	411	446.87	314	344.33	228	228.68	226	227.32
5.4	242	393	490.40	258	262.10	389	419.68	355	398.21	242	242.23	242	243.29
5.5	211	354	386.63	239	243.93	401	420.12	364	378.21	211	212.03	211	213.90
5.6	213	402	458.30	242	251.33	399	429.60	314	341.67	213	215.10	213	213.04
5.7	293	534	613.47	326	330.13	506	566.87	414	447.57	293	297.68	293	295.35
5.8	288	521	622.43	335	342.77	533	588.17	429	461.73	288	290.35	288	290.39
5.9	279	556	619.40	315	324.60	555	578.98	422	463.14	279	280.52	279	279.90
5.10	265	512	568.17	291	296.20	479	531.17	368	406.97	267	268.77	265	267.97
6.1	138	555	636.10	156	162.13	465	566.53	251	358.43	140	142.90	140	143.42
6.2	146	771	920.67	161	165	683	835.03	400	488.83	146	148.87	146	149.71
6.3	154	710	862.73	160	164.97	700	815.03	379	471.63	147	148.16	145	149.65
6.4	131	447	571.50	138	140.67	647	852.38	477	542.45	131	131.68	131	133.03
6.5	161	802	907.20	185	190.20	645	823.30	417	502.50	163	164.68	162	165.39
AVG	336.08	685.24	776.24	370	394.93	656.96	763.27	501.60	573.14	336.80	430	335.84	338.25
a.1	253	861	1067.30	268	271.07	830	958.87	539	622.40	254	255.55	254	256
a.2	252	843	971	299	303.40	829	902.43	507	568.20	257	259.48	256	260.65
a.3	232	833	924.73	253	256.90	761	860.47	482	546.47	235	237.10	233	237.35
a.4	234	807	917.10	264	267.53	732	850.13	499	553.30	235	237.39	236	239.97
a.5	236	790	917.73	259	262.80	751	846.83	455	550.77	236	237.06	236	237.52
b.1	69	962	1099.53	83	84.73	896	1002.83	440	556.20	74	79.61	69	76.48
b.2	76	977	1078.37	92	93.87	912	1001.17	452	545.80	83	90.03	81	86.81
b.3	80	1217	1404	90	91.63	901	1000.25	547	682.44	84	88.55	82	88.81
b.4	79	1159	1273.37	94	98.63	992	1124.93	559	632.40	84	87.90	83	88.16
b.5	72	974	1119.90	82	84.23	999	1212.21	547	636.96	72	77.68	78	76.03
c.1	227	1143	1229	263	264.57	1011	1134.37	598	689.30	228	230.29	233	238.77
c.2	219	1221	1356.13	258	261.90	1176	1268.40	635	754.17	226	222.71	226	232.74
c.3	243	1317	1619.40	294	299.77	1343	1476.83	704	862.70	254	256.68	253	264
c.4	219	1186	1342.43	257	259.03	1084	1209.93	659	735.50	225	227.97	224	233.10
c.5	215	1196	1292.50	245	248.77	1024	1164.20	607	697.50	215	216.35	222	230.42
d.1	60	1455	1584.57	71	71	1306	1441.97	651	776.13	66	66	68	79.65
d.2	66	1657	1785.97	76	78.23	1444	1626.53	753	894.77	71	71	73	89.71
d.3	72	1791	1922.27	83	86.60	1574	1787.67	801	980.60	82	82	82	100.84
d.4	66	1456	1590.87	70	71.83	1307	1449.17	663	793.40	67	67	70	81.74
d.5	61	1459	1596.77	69	69.87	1296	1463.17	682	801.03	66	66	72	81.29
AVG	151.55	1165.20	1347.83	173.50	176.32	1189.12	1198.33	589	697	155.70	157.82	156.55	164
nre.1	29	2025	2226.93	33	33.33	2168	2541.84	1147	1665.25	30	30	70	80.03
nre.2	30	2472	2763.80	37	38.10	2262	2471.93	1145	1360.57	34	34	83	128.32
nre.3	27	2140	2323.47	34	35.33	1951	2141.83	1025	1160.67	34	34	74	112.35
nre.4	28	2381	2647.10	34	34.60	2194	2398.90	1142	1344.43	33	34	83	177.45
nre.5	28	2465	2776.80	34	34.73	2231	2516.80	1188	1413.73	30	30	92	150.52
nrf.1	14	1467	1572.30	16	17.17	1264	1411.83	683	769.90	17	17	372	444.19
nrf.2	15	1218	1433.20	17	17.97	1123	1323.43	625	712.90	18	18	74	404.58
nrf.3	14	1591	1739.23	19	20.13	1463	1572.83	729	872.40	19	19	335	487.25
nrf.4	14	1446	1550.40	18	18.83	1287	1419.83	641	781.23	18	18	52	420.52
nrf.5	13	1169	1363.90	16	16.47	1125	1244.10	569	677.17	16	16	65	371.03
nrg.1	176	6422	7063.57	245	255.80	5962	6396.57	3202	3556.80	197	197	287	385.13
nrg.2	154	5598	5951.20	194	207.33	4949	5405.80	2708	3021.37	168	168	208	284.29
nrg.3	166	5967	6446.33	206	217.57	5561	5965.03	2886	3267.30	183	183	211	347.23
nrg.4	168	6148	6522.20	231	240.77	5393	5925.83	3007	3284.60	186	186	250	345.77
nrg.5	168	6460	6827.30	229	242.33	5786	6213.17	2984	3381.37	186	186	230	353.97
nrh.1	63	11,451	12,041.17	120	157.23	10,475	11,056.33	5678	6236.73	77	77	t.o.	t.o.
nrh.2	63	11,284	12,032.90	119	162.70	10,574	11,033.20	5389	6096.30	71	71	t.o.	t.o.
nrh.3	59	11,281	11,932.43	117	151.30	10,301	10,855.47	5542	6041.67	69	69	t.o.	t.o.
nrh.4	58	11,166	11,880.47	129	152.83	10,441	10,867.60	5682	6047.33	68	68	t.o.	t.o.
nrh.5	55	10,676	11,213.71	110	137.35	9538	10,217.13	4999	5777.45	61	61	t.o.	t.o.
AVG	67.10	5241.35	5615.42	97.90	109.59	4802.40	5148.97	2548.55	2937.04	77.93	75.80	t.o.	t.o.

## Appendix A.1.2. SCP—Two-Steps

**Table A9.** Two-steps results set.

Instance	Step 1	Step 2	BKS	MIN	MAX	AVG	RPD	TIME
4.1	S-Shape 4	Elitist	429	429	433	438	0	22
4.2	S-Shape 4	Elitist	512	512	564	515.74	0	25
4.3	S-Shape 4	Standard	516	516	548	530.25	0	5.0
4.4	S-Shape 4	Elitist	494	494	512	500.10	0	23
4.5	S-Shape 4	Elitist	512	512	522	514.61	0	24
4.6	S-Shape 4	Elitist	560	560	577	563.61	0	24
4.7	S-Shape 4	Elitist	430	430	444	432.26	0	21
4.8	S-Shape 4	Elitist	492	492	499	494.19	0	24
4.9	S-Shape 2	Standard	641	641	664	652.16	0	95
4.10	S-Shape 4	Elitist	514	514	525	517.19	0	22
5.1	S-Shape 4	Elitist	253	253	262	256.74	0	24
5.2	S-Shape 4	Elitist	302	302	324	308.39	0	27
5.3	S-Shape 4	Elitist	226	226	231	227.32	0	24
5.4	S-Shape 1	Standard	242	242	258	249.43	0	2.9
5.5	S-Shape 1	Standard	211	211	239	223.13	0	2.5
5.6	S-Shape 4	Elitist	213	213	223	213.90	0	23
5.7	S-Shape 4	Elitist	293	293	308	295.03	0	24
5.8	S-Shape 4	Elitist	288	288	295	289.90	0	26
5.9	S-Shape 4	Standard	279	279	328	301.24	0	3.2
5.10	S-Shape 4	Elitist	265	265	271	267.87	0	24
6.1	S-Shape 4	Elitist	138	138	148	143.13	0	22
6.2	S-Shape 4	Elitist	146	146	164	150.61	0	20
6.3	S-Shape 4	Elitist	145	145	158	150.32	0	23
6.4	S-Shape 1	Standard	131	131	144	138.34	0	2.4
6.5	S-Shape 4	Elitist	161	161	175	167	0	23
a.1	S-Shape 4	Standard	253	253	264	257.65	0	180
a.2	S-Shape 4	Elitist	252	252	273	260.13	0	53
a.3	S-Shape 4	Elitist	232	232	246	237.1	0	63
a.4	S-Shape 2	Standard	234	234	249	240.06	0	170
a.5	S-Shape 4	Elitist	236	236	242	273.74	0	55
b.1	S-Shape 4	Elitist	69	69	77	72.97	0	49
b.2	S-Shape 2	Standard	76	76	89	81.45	0	120
b.3	S-Shape 1	Standard	80	80	98	86.99	0	8.9
b.4	S-Shape 4	Elitist	79	79	87	8255	0	57
b.5	S-Shape 1	Standard	72	72	87	78.92	0	5.8
c.1	S-Shape 4	Elitist	227	227	241	232.77	0	96
c.2	S-Shape 2	Standard	219	219	231	225.10	0	330
c.3	S-Shape 4	Elitist	243	243	265	252.81	0	110
c.4	S-Shape 4	Elitist	219	219	237	225.74	0	110
c.5	V-Shape 3	Elitist	215	215	218	216.55	0	130
d.1	S-Shape 4	Elitist	60	60	68	63	0	88
d.2	S-Shape 4	Elitist	66	66	71	68.39	0	100
d.3	S-Shape 4	Elitist	72	72	83	75.52	0	120
d.4	S-Shape 4	Elitist	66	66	72	65.58	0	94
d.5	S-Shape 4	Elitist	61	61	76	63.84	0	93
nre.1	S-Shape 2	Standard	29	29	41	33.14	0	10.0
nre.2	S-Shape 4	Elitist	30	31	39	33.55	0.033	160
nre.3	S-Shape 4	Elitist	27	27	33	29.77	0	220
nre.4	S-Shape 4	Elitist	28	28	35	30.81	0	150
nre.5	S-Shape 4	Elitist	28	28	33	29.74	0	200
nrf.1	S-Shape 4	Elitist	14	14	18	15.62	0	210
nrf.2	S-Shape 4	Elitist	15	15	18	16.19	0	100
nrf.3	S-Shape 4	Elitist	14	15	19	16.61	0.071	180
nrf.4	S-Shape 4	Elitist	14	15	17	15.71	0.071	91
nrf.5	S-Shape 4	Elitist	13	14	16	15	0.077	160
nrg.1	S-Shape 4	Elitist	176	179	206	189.55	0.017	1100
nrg.2	S-Shape 4	Elitist	154	160	187	166.23	0.039	1000
nrg.3	S-Shape 4	Elitist	166	168	190	177.65	0.012	1200
nrg.4	S-Shape 4	Elitist	168	175	189	180.74	0.042	1000
nrg.5	S-Shape 4	Elitist	168	174	196	181.13	0.036	1000
nrh.1	S-Shape 4	Elitist	63	71	83	74.19	0.127	560
nrh.2	S-Shape 4	Elitist	63	67	86	73.48	0.063	1100
nrh.3	S-Shape 4	Elitist	59	64	80	69.03	0.085	680
nrh.4	S-Shape 4	Elitist	58	62	75	66.71	0.069	950
nrh.5	S-Shape 4	Elitist	55	58	70	62.45	0.055	850

## Appendix A.1.3. SCP—KMeans and DBscan

**Table A10.** KMeans results set.

Ins.	BKS	MIN	MAX	Avg	RPD	TIME
4.1	429	429	430	430	0	47
4.2	512	513	522	515.84	0.002	30
4.3	516	516	526	520.61	0	32
4.4	494	495	505	498.87	0.002	30
4.5	512	514	522	514.90	0.003	30
4.6	560	560	565	562.35	0	33
4.7	430	430	434	432.26	0	27
4.8	492	493	499	496.13	0.002	30
4.9	641	645	666	657.90	0.006	33
4.10	514	516	526	519.9	0.003	28
5.1	253	253	259	255.74	0	30
5.2	302	308	312	309.81	0.019	36
5.3	226	228	230	228.68	0.008	29
5.4	242	242	244	242.23	0	31
5.5	211	211	214	212.03	0	26
5.6	213	213	219	215.10	0	28
5.7	293	293	301	297.68	0	30
5.8	288	288	295	290.35	0	31
5.9	279	279	281	280.52	0	31
5.10	265	267	271	268.77	0.007	31
6.1	138	140	145	142.90	0.014	14
6.2	146	146	151	148.87	0	14
6.3	145	147	151	148.16	0.013	15
6.4	131	131	135	131.68	0	18
6.5	161	163	168	164.68	0.012	29
a.1	253	254	257	255.55	0.004	71
a.2	252	257	262	259.48	0.019	68
a.3	232	235	240	237.10	0.012	71
a.4	234	235	244	237.39	0.004	66
a.5	236	236	238	237.06	0	71
b.1	69	74	80	79.61	0.072	13
b.2	76	83	93	90.03	0.092	14
b.3	80	84	89	88.55	0.050	14
b.4	79	84	89	87.90	0.063	15
b.5	72	72	79	77.68	0	13
c.1	227	228	233	230.29	0.004	130
c.2	219	220	226	222.71	0.004	94
c.3	243	254	260	256.68	0.045	110
c.4	219	225	232	227.97	0.027	90
c.5	215	215	218	216.35	0	140
d.1	60	66	66	66	0.100	24
d.2	66	71	71	71	0.075	26
d.3	72	82	82	82	0.138	27
d.4	62	67	67	67	0.080	24
d.5	61	66	66	66	0.082	24
nre.1	29	30	30	30	0.034	41
nre.2	30	34	34	34	0.133	41
nre.3	27	34	34	34	0.259	38
nre.4	28	33	33	33	0.179	47
nre.5	28	30	30	30	0.071	43
nrf.1	14	17	17	17	0.214	43
nrf.2	15	18	18	18	0.200	40
nrf.3	14	19	19	19	0.357	49
nrf.4	14	18	18	18	0.286	37
nrf.5	13	16	16	16	0.231	37
nrg.1	176	197	197	197	0.119	170
nrg.2	154	168	168	168	0.091	150
nrg.3	166	183	183	183	0.102	160
nrg.4	168	186	186	186	0.107	160
nrg.5	168	186	186	186	0.107	170
nrh.1	63	77	77	77	0.222	340
nrh.2	63	71	71	71	0.127	330
nrh.3	59	69	69	69	0.169	330
nrh.4	58	68	68	68	0.172	330
nrh.5	55	61	61	61	0.109	320

**Table A11.** DBscan results set (t.o = time out).

Ins.	BKS	MIN	MAX	AVG	RPD	TIME
4.1	429	429	432	430.38	0	3700
4.2	512	512	526	514.77	0	2400
4.3	516	516	533	518.87	0	2600
4.4	494	494	500	496.10	0	2400
4.5	512	512	514	513.23	0	2300
4.6	560	560	564	560.19	0	2500
4.7	430	430	435	430.81	0	7300
4.8	492	492	503	494.10	0	3900
4.9	641	641	658	647.50	0	4600
4.10	514	514	518	515.81	0	3900
5.1	253	253	258	255.52	0	4100
5.2	302	302	314	306.68	0	4500
5.3	226	226	229	227.32	0	3600
5.4	242	242	245	243.29	0	4100
5.5	211	211	218	213.90	0	3400
5.6	213	213	214	213.04	0	3700
5.7	293	293	300	295.35	0	4000
5.8	288	288	300	290.39	0	4300
5.9	279	279	280	279.90	0	3800
5.10	265	265	271	267.97	0	4300
6.1	138	140	147	143.42	0.014	2500
6.2	146	146	155	149.71	0	2900
6.3	145	145	154	149.65	0	2700
6.4	131	131	135	133.03	0	2400
6.5	161	162	169	165.39	0.006	2900
a.1	253	254	259	256	0.004	17,000
a.2	252	256	265	260.65	0.015	18,000
a.3	232	233	245	237.35	0.004	17,000
a.4	234	236	245	239.97	0.008	16,000
a.5	236	236	240	237.52	0	17,000
b.1	69	69	85	76.48	0	15,000
b.2	76	81	94	86.81	0.065	16,000
b.3	80	82	99	88.81	0.025	20,000
b.4	79	83	99	88.16	0.050	24,000
b.5	72	78	77	76.03	0.083	16,000
c.1	227	233	247	238.77	0.026	27,000
c.2	219	226	244	232.74	0.032	28,000
c.3	243	253	279	264	0.041	28,000
c.4	219	224	244	233.10	0.022	26,000
c.5	215	222	241	230.42	0.032	26,000
d.1	60	68	90	79.65	0.133	130,000
d.2	66	73	105	89.71	0.106	150,000
d.3	72	82	125	100.84	0.138	160,000
d.4	62	70	96	81.74	0.129	130,000
d.5	61	72	91	81.29	0.180	120,000
nre.1	29	70	103	80.03	1.413	260,000
nre.2	30	83	205	128.32	1.766	420,000
nre.3	27	74	152	112.35	1.740	350,000
nre.4	28	83	799	177.45	1.964	390,000
nre.5	28	92	813	150.52	2.285	490,000
nrf.1	14	372	508	444.19	25.571	250,000
nrf.2	15	74	468	404.58	3.933	210,000
nrf.3	14	335	551	487.25	22.928	240,000
nrf.4	14	52	493	420.52	2.714	190,000
nrf.5	13	65	416	371.03	4	170,000
nrg.1	176	287	436	385.13	0.630	230,000
nrg.2	154	208	327	284.29	0.350	220,000
nrg.3	166	211	417	347.23	0.271	240,000
nrg.4	168	250	402	345.77	0.488	220,000
nrg.5	168	230	398	353.97	0.369	240,000
nrh.1	63	t.o.	t.o.	t.o.	t.o.	t.o.
nrh.2	63	t.o.	t.o.	t.o.	t.o.	t.o.
nrh.3	59	t.o.	t.o.	t.o.	t.o.	t.o.
nrh.4	58	t.o.	t.o.	t.o.	t.o.	t.o.
nrh.5	55	t.o.	t.o.	t.o.	t.o.	t.o.

## Appendix A.2. 0/1 Knapsack Problem Results

### Appendix A.2.1. KP—Crow Search Algorithm

**Table A12.** CSA/KP—SShape 1—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	S1+Stand.		S1+Comp.		S1+Static		S1+Elitist		KMeans		DBscan	
		MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG
f1	295	287	276.12	279	267.21	290	287.32	256	222.81	290	286.32	294	292
f2	1024	789	771.23	729	652.87	782.01	719.08	756	701.33	895	987.23	880	829.87
f3	35	34	34	35	35	34	34	35	35	35	35	35	35
f4	23	23	23	20	20	22	21.50	23	22.50	23	23	23	23
f5	481.06	327.15	310.14	300.47	276.34	312.23	309.93	299.76	280.80	377.07	354.65	436.82	412.19
f6	52	50	50	49	48.45	51	50.50	43	43	51	50.23	52	52
f7	107	100	94.23	90	85.32	99	97.43	105	90.32	107	107	107	107
f8	9767	8743	7843.32	8943	7332.05	9321	8883.45	6983	5574.45	5377	5377.54	5377	5262.35
f9	130	129	129	130	130	128	127.50	129	128.50	130	130	130	130
f10	1025	973	965.04	789	734.55	723	712.66	832	799.12	973	969.52	804	749.39
AVG	1293.91	1145.52	1049.61	1136.45	958.18	1176.22	1124.34	946.185	789.78	825.81	832.05	813.88	789.28

**Table A13.** CSA/KP—SShape 2—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	S2+Stand.		S2+Comp.		S2+Static		S2+Elitist		KMeans		DBscan	
		MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG
f1	295	234	222.03	189	176.29	162	150.32	178	168.20	290	286.32	294	292
f2	1024	900	850	732	699.81	823	800.10	809	771.25	895	987.23	880	829.87
f3	35	33	31.75	19	17.33	23	20.67	24	22.24	35	35	35	35
f4	23	21	19.34	18	13.38	18	16.25	17	16.50	23	23	23	23
f5	481.06	223.32	201.10	199.97	189.71	254.09	250.12	301.01	276.87	377.07	354.65	436.82	412.19
f6	52	51	49.23	50	47.66	50	49.50	42	40.01	51	50.23	52	52
f7	107	102	100.45	99	98.45	83	78.33	97	93.45	107	107	107	107
f8	9767	8231	7822.78	6432	5993.33	6982	6777.89	7014	6897.21	5377	5377.54	5377	5262.35
f9	130	102	98.76	95	93.34	89	85.25	102	97.33	130	130	130	130
f10	1025	865	763.67	701	678.88	799	740.01	732	699.65	973	969.52	804	749.39
AVG	1293.91	1076.23	1015.91	853.49	800.82	928.31	896.84	931.60	908.27	825.81	832.05	813.88	789.28

**Table A14.** CSA/KP—SShape 3—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	S3+Stand.		S3+Comp.		S3+Static		S3+Elitist		KMeans		DBscan	
		MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG
f1	295	289	245.33	282	254.33	276	232.76	281	276.34	290	286.32	294	292
f2	1024	991	937.35	934	900.45	876	835.25	766	750.66	895	987.23	880	829.87
f3	35	35	33.45	33	33	34	34.50	35	35	35	35	35	35
f4	23	22	20.75	21	20.33	23	22.50	23	23	23	23	23	23
f5	481.06	423.05	402.23	333.45	330.45	299.22	272.55	317.65	309.25	377.07	354.65	436.82	412.19
f6	52	49	48.23	50	47.33	47	44.35	51	48.66	51	50.23	52	52
f7	107	102	100.11	98	89.34	103	97.23	88	77.21	107	107	107	107
f8	9767	8653	8432.20	8933	8739.66	9001	8990.78	9112	9004.89	5377	5377.54	5377	5262.35
f9	130	111	101.10	116	109.47	127	119.39	121	118.33	130	130	130	130
f10	1025	970	960.05	897	865.87	749	718.88	969	939.24	973	969.52	804	749.39
AVG	1293.91	1164.51	1128.06	1169.75	1139.02	1153.52	1136.82	1176.37	1158.26	825.81	832.05	813.88	789.28

**Table A15.** CSA/KP—SShape 4—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	S4+Stand.		S4+Comp.		S4+Static		S4+Elitist		KMeans		DBscan	
		MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG
f1	295	276	256.22	273	242.31	222	218.23	250	245.66	290	286.32	294	292
f2	1024	990	934.75	943	913.13	893	833.90	881	843.29	895	987.23	880	829.87
f3	35	29	28.48	27	22.98	30	28.45	29	26.70	35	35	35	35
f4	23	21	19.20	22	20.70	13	11.18	19	17.26	23	23	23	23
f5	481.06	413.31	400.23	388.69	377.09	410.54	399.78	354.76	326.87	377.07	354.65	436.82	412.19
f6	52	42	38.48	49	45.09	51	50.17	52	52	51	50.23	52	52
f7	107	106	105.45	98	79	93	92.50	88	73.19	107	107	107	107
f8	9767	7823	7698.12	7901	7406.21	8432	8023.36	9301	9000.98	5377	5377.54	5377	5262.35
f9	130	123	109.33	114	110.20	126	112.33	101	110.34	130	130	130	130
f10	1025	872	762.98	899	843.21	901	862.23	962	943.32	973	969.52	804	749.39
AVG	1293.91	1069.53	1035.32	1071.46	1005.99	1117.15	1063.21	1203.77	1163.96	825.81	832.05	813.88	789.28

**Table A16.** CSA/KP—VShape 1—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	V1+Stand.		V1+Comp.		V1+Static		V1+Elitist		KMeans		DBscan	
		MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG
f1	295	222	210.12	212	199.33	203	193.45	208	193.66	290	286.32	294	292
f2	1024	777	732.23	789	745.10	798	755.33	801	790.65	895	987.23	880	829.87
f3	35	35	33.33	32	29.67	31	29.22	34	33.50	35	35	35	35
f4	23	23	23	23	22.50	22	20.50	19	17.66	23	23	23	23
f5	481.06	399.98	369.09	382.67	342.65	376.33	323.66	381.23	379.48	377.07	354.65	436.82	412.19
f6	52	49	48.23	50	49.32	51	50.50	47	46.33	51	50.23	52	52
f7	107	99	98.23	90	80.33	95	93.33	100	99.55	107	107	107	107
f8	9767	8010	7988.77	9343	9123.32	8990	8600.45	9734	9456.25	5377	5377.54	5377	5262.35
f9	130	121	120.33	119	118.50	128	125.77	130	130	130	130	130	130
f10	1025	970	967.23	955	943.32	961	933.89	896	869.08	973	969.52	804	749.39
AVG	1293.91	1070.59	1059.06	1199.57	1165.40	1165.53	1112.61	1235.02	1201.62	825.81	832.05	813.88	789.28

**Table A17.** CSA/KP—VShape 2—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	V2+Stand.		V2+Comp.		V2+Static		V2+Elitist		KMeans		DBscan	
		MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG
f1	295	295	294.14	294	293.41	229	219.47	286	278.88	290	286.32	294	292
f2	1024	990	985.66	987	866.62	798	774.44	888	8500.99	895	987.23	880	829.87
f3	35	33	32.45	35	35	25	22.66	29	28.33	35	35	35	35
f4	23	23	23	21	20.66	23	23	20	18.05	23	23	23	23
f5	481.06	288.54	240.11	301.78	252.56	297.23	283.11	334.98	319.56	377.07	354.65	436.82	412.19
f6	52	47	45.87	50	48.59	52	51.50	37	33.87	51	50.23	52	52
f7	107	87	86.23	100	100.76	99	87.78	96	96.06	107	107	107	107
f8	9767	6538	6234.90	6234	6002.76	5372	5008.66	500.986	4990.87	5377	5377.54	5377	5262.35
f9	130	129	127.77	126	126.76	101	109.87	101	99.7651	130	130	130	130
f10	1025	972	962.01	811	800.17	901	900.12	914	907.98	973	969.52	804	749.39
AVG	1293.91	940.25	903.22	895.97	854.73	789.72	748.06	320.69	1617.22	825.81	832.05	813.88	789.28

**Table A18.** CSA/KP—VShape 3—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	V3+Stand.		V3+Comp.		V3+Static		V3+Elitist		KMeans		DBscan	
		MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG
f1	295	287	264.78	279	255.55	190	181.21	257	231.84	290	286.32	294	292
f2	1024	787	754.33	888	854.25	901	890.50	954	927.99	895	987.23	880	829.87
f3	35	35	34.50	35	35	35	33.25	34	33.50	35	35	35	35
f4	23	23	23	23	20.33	20	17.45	23	24.25	23	23	23	23
f5	481.06	214.32	200.21	398.35	345.32	401.39	395.75	436.29	426.35	377.07	354.65	436.82	412.19
f6	52	51	48.25	51	43.33	37	36.87	48	45.50	51	50.23	52	52
f7	107	107	100.55	107	99.10	106	103.98	104	100.66	107	107	107	107
f8	9767	9701	9688.40	8997	8787.23	9565	9333.88	9487	9221.91	5377	5377.54	5377	5262.35
f9	130	130	126.67	129	129.33	129	128.50	130	130	130	130	130	130
f10	1025	971	965.10	970	969.22	971	966.32	899	890.78	973	969.52	804	749.39
AVG	1293.91	1230.63	1220.57	1187.74	1153.86	1235.54	1208.77	1237.23	1203.28	825.81	832.05	813.88	789.28

**Table A19.** CSA/KP—VShape 4—Two-steps vs. KMeans vs. DBscan.

Instance	BKS	V4+Stand.		V4+Comp.		V4+Static		V4+Elitist		KMeans		DBscan	
		MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG	MAX	AVG
f1	295	222	210.58	284	264.58	290	289.98	278	266.66	290	286.32	294	292
f2	1024	990	987.21	988	980.21	850	876.58	884	821.12	895	987.23	880	829.87
f3	35	35	35	35	34.50	34	33.25	35	34.58	35	35	35	35
f4	23	22	20.33	24	23.09	23	23.56	23	24.50	23	23	23	23
f5	481.06	410.25	408.84	378.99	368.29	413.39	410.14	399.36	387.41	377.07	354.65	436.82	412.19
f6	52	52	50.21	52	50.65	50	47.99	52	52	51	50.23	52	52
f7	107	107	106.50	107	105.82	100	98.47	107	107	107	107	107	107
f8	9767	7888	7784.57	8745	8654.25	8932	8774.77	6544	6533.63	5377	5377.54	5377	5262.35
f9	130	129	121.69	130	127.87	119	110.77	128	126.33	130	130	130	130
f10	1025	874	865.22	855	850.26	847	835.66	701	699.50	973	969.52	804	749.39
AVG	1293.91	1072.92	1059.02	1159.89	1145.95	1165.83	1150.12	915.13	905.27	825.81	832.05	813.88	789.28

### Appendix A.2.2. KP—Two-Steps

**Table A20.** KP—Two-steps results.

Instance	Step 1	Step 2	BKS	MIN	MAX	AVG	RPD	TIME
f1	vShape 2	Standard	295	287	295	294.14	0	4.35
f2	sShape3	Standard	1024	821	991	937.35	0.032	4.35
f3	sShape3	elitist	35	35	35	35	0	4.35
f4	sShape3	elitist	23	23	23	23	0	0.2
f5	vShape 3	elitist	481.06	413.36	436.29	426.35	0.093	0.19
f6	vShape4	elitist	52	52	52	52	0	0.07
f7	vShape4	elitist	107	105	107	106	0	0.48
f8	vShape1	elitist	9767	8754	9734	9456.25	0.003	2.6
f9	vShape1	elitist	130	130	130	130	0	0.1
f10	sShape1	Standard	1025	954	973	965.04	0.051	1.2

### Appendix A.2.3. KP—KMeans and DBscan

**Table A21.** KP—KMeans results.

Instance	BKS	MIN	MAX	AVG	RPD	TIME
f1	295	274	290	286.32	0.017	0.27
f2	1024	745	895	987.23	0.126	0.39
f3	35	35	35	35	0	0.25
f4	23	23	23	23	0	0.28
f5	481.06	347	377.07	354.65	0.216	0.32
f6	52	49	51	50.23	0.019	0.29
f7	107	107	107	107	0	0.27
f8	9767	4398	5377	5073.54	0.449	0.25
f9	130	130	130	130	0	0.26
f10	1025	953	973	969.52	0.051	0.40

**Table A22.** KP—DBscan results.

Instance	BKS	MIN	MAX	AVG	RPD	TIME
f1	295	290	294	292	0.003	580
f2	1024	778	880	829.87	0.141	170
f3	35	35	35	35	0	0.22
f4	23	23	23	23	0	0.23
f5	481.06	387.55	436.82	412.19	0.092	140
f6	52	52	52	52	0	0.97
f7	107	107	107	107	0	0.23
f8	9767	5147	5377	5262.35	0.449	0.23
f9	130	130	130	130	0	0.26
f10	1025	694	804	749.39	0.216	1.6

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