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Modeling an Uncertain Productivity Learning Process Using an Interval Fuzzy Methodology

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Abstract: Existing methods for forecasting the productivity of a factory are subject to a major drawback—the lower and upper bounds of productivity are usually determined by a few extreme cases, which unacceptably widens the productivity range. To address this drawback, an interval fuzzy number (IFN)-based mixed binary quadratic programming (MBQP)–ordered weighted average (OWA) approach is proposed in this study for modeling an uncertain productivity learning process. In the proposed methodology, the productivity range is divided into the inner and outer sections, which correspond to the lower and upper membership functions of an IFN-based fuzzy productivity forecast, respectively. In this manner, all actual values are included in the outer section, whereas most of the values are included within the inner section to fulfill different managerial purposes. According to the percentages of outlier cases, a suitable forecasting strategy can be selected. To derive the values of parameters in the IFN-based fuzzy productivity learning model, an MBQP model is proposed and optimized. Subsequently, according to the selected forecasting strategy, the OWA method is applied to defuzzify a fuzzy productivity forecast. The proposed methodology has been applied to the real case of a dynamic random access memory factory to evaluate its effectiveness. The experimental results indicate that the proposed methodology was superior to several existing methods, especially in terms of mean absolute error, mean absolute percentage error, and root mean square error in evaluating the forecasting accuracy. The forecasting precision achieved using the proposed methodology was also satisfactory.

Keywords: productivity; learning; interval fuzzy number; mixed binary quadratic programming; ordered weighted average

1. Introduction

Productivity is a measure of how efficient a system is in converting inputs into outputs and is usually measured by dividing the quantity or value of outputs by that of inputs [1–3]. Productivity can be measured at different levels, such as for a factory (or store), city, or even country [4]. This study focuses on the productivity of a factory. In a factory, productivity increases with time because of operators becoming more familiar with their tasks, equipment engineers becoming skilled in maintaining and repairing machines, product engineers becoming more experienced in solving product quality problems, and other reasons [5].

Factories are adopting an increasing number of information technologies (ITs) that include software, hardware, and artificial intelligence [6,7]. For example, factories rely on transaction processing systems (TPSs) to automate routine operations, which obviously elevates productivity [8–11]. Consequently, human workers are now trained to be familiar with MISs rather than with routine operat ions. The emergence of Industry 4.0 has created opportunities for further enhancing productivity. For example, when wireless sensors are

incorporated in a machine, the sensors can detect abnormal operating conditions before a serious shutdown that results in the loss of productivity, thereby enabling predictive maintenance [12]. Although some researchers have asserted that artificial intelligence will eventually replace human workers for performing many tasks, the applications of artificial intelligence do not necessarily enhance productivity due to reasons such as false hopes, mismeasurement, redistribution, and implementation lags [13]. Nevertheless, productivity improves as users learn to master IT. Although productivity improves by conducting activities involving substantial human intervention, productivity is subject to considerable uncertainty [14,15]. To address this problem, fuzzy logic [16] has been extensively applied to model productivity. For example, in a study by Hougaard [17], the inputs and outputs of a production plan were given in or estimated with fuzzy numbers. After enumerating all possible values of fuzzy inputs and outputs, the αcuts of fuzzy productivity were derived. Finally, a triangular fuzzy number was used to approximate fuzzy productivity. Similarly, Emrouznejad et al. [18] modeled inputs, outputs, and prices through fuzzy numbers. The αcuts of fuzzy parameters were fed as interval data into a data envelopment analysis model to calculate the overall profit Malmquist productivity index. Wang and Chen [19] proposed a fuzzy collaborative forecasting approach for forecasting the productivity of a factory. In the fuzzy collaborative forecasting approach, multiple experts fitted a fuzzy productivity learning process with quadratic or nonlinear programming models to forecast productivity. The fuzzy productivity forecasts by experts were aggregated using fuzzy intersection. Then, the aggregation result was defuzzified using a back propagation network. In a study by Chen and Wang [20], fuzzy productivity forecasts were compared with a competitive region to assess the productivity competitiveness of a factory. Recently, Chen et al. [21] proposed a heterogeneous fuzzy collaborative forecasting approach in which experts constructed either mathematical programming models or artificial neural networks to forecast productivity. The adoption of different fuzzy forecasting methods contributed to the diversification of fuzzy productivity forecasts, which was considered a favorable property for a multiple-expert forecasting problem.

However, a problem associated with existing methods is that the lower and upper bounds on a fuzzy productivity forecast are usually determined by a few extreme cases [20]. Moreover, other cases may lie considerably close to cores (or centers), which unreasonably widens the range of a fuzzy productivity forecast, as illustrated in Figure 1, in which red circles represent extreme cases. There exist two types of extreme cases, namely better-than-anticipated (BTA) and poorer-than-expected (PTE) cases.

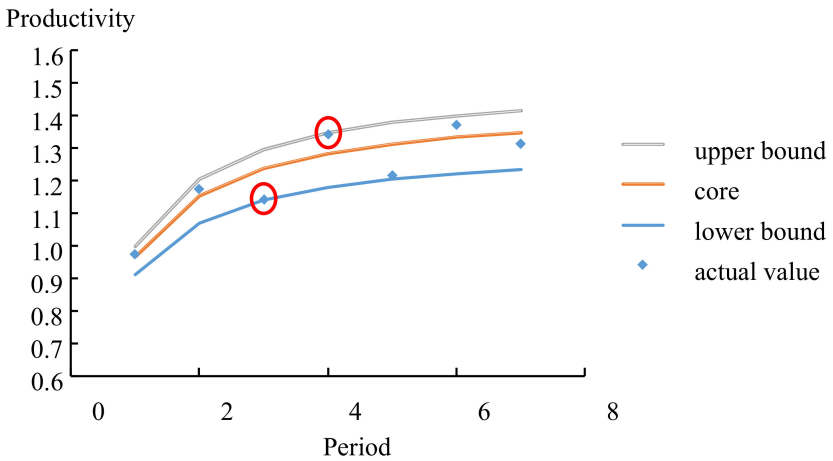


Figure 1. Lower and upper bounds determined by extreme cases.

Therefore, a desirable option is to form a narrow interval that contains most of the collected data by excluding extreme cases, as illustrated in Figure 2. To this end, an interval fuzzy number (IFN) [22–24] is a viable option. There exist two membership functions in an IFN, one of which is suitable for modeling the inner part of a fuzzy productivity forecast, whereas the other is suitable for modeling the outer part.

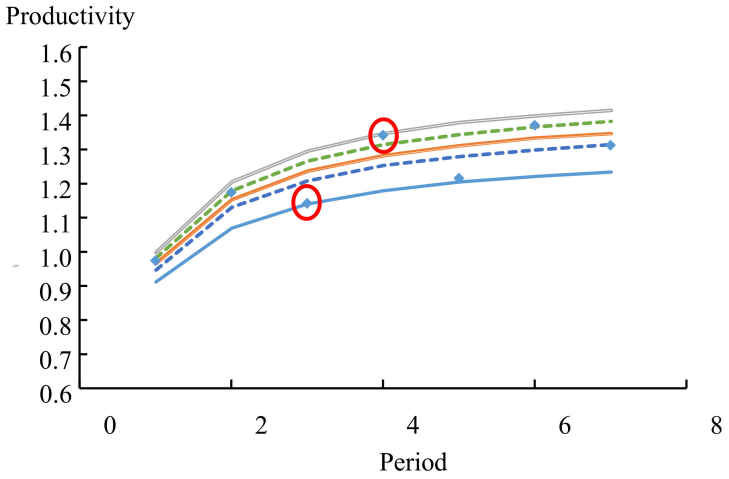


Figure 2. Narrow interval that contains most of the collected data.

Due to the aforementioned reasons, an IFN-based mixed binary quadratic programming (MBQP)–ordered weighted average (OWA) approach is proposed in this study for modeling an uncertain productivity learning process by distinguishing between BTA and PTE cases. The motives for this study are explained as follows:

(1) Owing to the existence of extreme cases, fuzzy productivity forecasts generated using an existing fuzzy forecasting method are not sufficiently precise.
(2) Fuzzy productivity forecasts generated using existing fuzzy forecasting methods are usually type-1 fuzzy numbers [2,15,19]. Compared with type-1 fuzzy numbers, IFNs can better consider uncertainty [25,26]. However, fuzzy forecasting methods that generate IFN-based fuzzy productivity forecasts are not widely used.
(3) A special defuzzifier needs to be proposed for an IFN-based fuzzy productivity forecast that separates extreme cases from normal cases.

To the best of our knowledge, the present study is the first attempt of its kind. The parameters of the IFN-based fuzzy productivity learning model are given in the form of IFNs. Consequently, fuzzy productivity forecasts generated by the IFN-based fuzzy productivity learning model are also in the form of IFNs. In the proposed methodology, the range of productivity is divided into the inner and outer sections that correspond to the lower and upper membership functions of an IFN-based fuzzy productivity forecast, respectively. In this manner, all actual values are included in the outer section, whereas most of the values lie within the inner section. Moreover, the ratio of the number of PTE cases to the number of BTA cases is a useful factor for selecting a suitable forecasting strategy. To derive the values of parameters in the IFN-based fuzzy productivity learning model, an MBQP model is proposed and optimized. Finally, according to the selected forecasting strategy, the OWA method [27] was applied to defuzzify a fuzzy productivity forecast.

The remainder of this paper is organized as follows. First, some arithmetic operations on IFNs are introduced in Section 2. The proposed methodology is detailed in Section 3. To illustrate the applicability of the proposed methodology, a real case is discussed in Section 4. The performance of the proposed methodology is also compared with those of several existing methods. Finally, the conclusions of this study and some directions for future research are provided in Section 5.

2. Preliminary

IFNs have been extensively applied in multiple-criteria decision-making problems. For example, Hu et al. [28] considered a multiple-criteria decision-making problem in which criteria took the values of IFNs. Moreover, some of the weights assigned to criteria were unknown. To address this problem,

an expected value function was optimized through a maximizing deviation method. However, in existing studies on IFN applications, the motives for adopting IFNs are not clear or strong. By contrast, in this study, the motive for adopting an IFN to represent a fuzzy productivity forecast is clear.

This section introduces some arithmetic operations on IFNs. First, the definition of an IFN is given as follows [29]:

Definition 1. *An IFN $\widetilde{A}$ is a subset of real numbers R and is defined as the set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))| x\mu_{\widetilde{A}}(x) \in R\}$, where $\mu_{\widetilde{A}}(x) : R \rightarrow [0, 1]$ is the interval-valued membership function of $\widetilde{A}$.*

If $\widetilde{A}$ is Moore-continuous, then there exist two membership functions for $\widetilde{A}$, namely the lower membership function (LMF) $\mu_{\widetilde{A}_l}(x)$ and the upper membership function (UMF) $\mu_{\widetilde{A}_u}(x)$, such that $\mu_{\widetilde{A}}(x) = [\mu_{\widetilde{A}_l}(x), \mu_{\widetilde{A}_u}(x)]$. An IFN is a special case of type-II fuzzy sets [30].

Some attributes of an IFN are defined as follows:

Definition 2. *The inner support, outer support, and core of an IFN $\widetilde{A}$ of R are defined, respectively, as follows:*

$$isupp_{\widetilde{A}} = \{x \in \mathbf{R} | \mu_{\widetilde{A}_l}(x) > 0\} \tag{1}$$

$$osupp_{\widetilde{A}} = \{x \in \mathbf{R} | \mu_{\widetilde{A}_u}(x) > 0\} \tag{2}$$

$$core_{\widetilde{A}} = \{x \in \mathbf{R} | \mu_{\widetilde{A}_l}(x) = \mu_{\widetilde{A}_u}(x) = 1\} \tag{3}$$

Definition 3. *An IFN $\widetilde{A}$ is an interval triangular fuzzy number (ITFN) if both the LMF and UMF of $\widetilde{A}$ are triangular functions,*

$$\mu_{\widetilde{A}_l}(x) = \begin{cases} \frac{x - A_{l1}}{A_2 - A_{l1}} & if \quad A_{l1} \le x < A_2 \\ \frac{A_{l3} - x}{A_{l3} - A_2} & if \quad A_2 \le x < A_{l3} \\ 0 & otherwise \end{cases} \tag{4}$$

$$\mu_{\widetilde{A}_u}(x) = \begin{cases} \frac{x - A_{u1}}{A_2 - A_{u1}} & if \quad A_{u1} \le x < A_2 \\ \frac{A_{u3} - x}{A_{u3} - A_2} & if \quad A_2 \le x < A_{u3} \\ 0 & otherwise \end{cases} \tag{5}$$

$\widetilde{A}$ can be briefly denoted by $((A_{l1}, A_2, A_{l3}), (A_{u1}, A_2, A_{u3}))$ or $(A_{u1}, A_{l1}, A_2, A_{l3}, A_{u3})$.

An ITFN is shown in Figure 3, in which $\widetilde{A}$ = ((5, 9, 12), (2, 8, 13)) or (2, 5, 9, 12, 13).

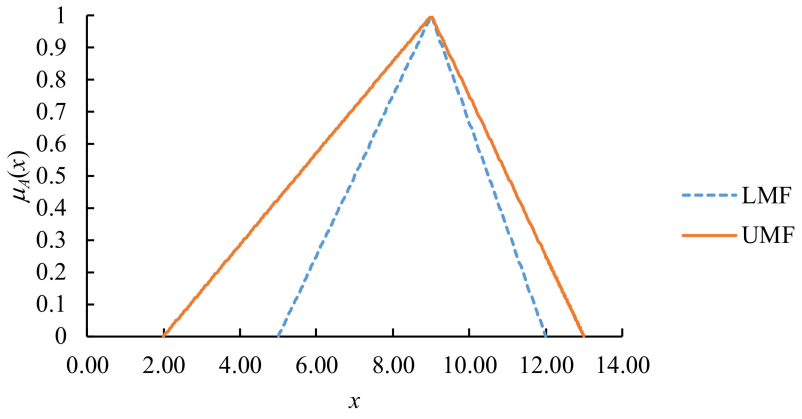


Figure 3. An ITFN.

Property 1. *The inner support, outer support, and core of an ITFN* $\widetilde{A}$ *can be derived as follows*:

$$isupp_{\widetilde{A}} = [A_{l1}, A_{l3}] \tag{6}$$

$$osupp_{\widetilde{A}} = [A_{u1}, A_{u3}] \tag{7}$$

$$core_{\widetilde{A}} = A_2 \tag{8}$$

Some arithmetic operations on ITFNs are summarized in the following theorem [31–33].

Theorem 1. *(Arithmetic Operations on ITFNs)*

(1) *Fuzzy addition:* $\widetilde{A}(+)\widetilde{B} = (A_{u1} + B_{u1}, A_{l1} + B_{l1}, A_2 + B_2, A_{l3} + B_{l3}, A_{u3} + B_{u3})$.

(2) *Fuzzy subtraction:* $\widetilde{A}(-)\widetilde{B} = (A_{u1} - B_{u3}, A_{l1} - B_{l3}, A_2 - B_2, A_{l3} - B_{l1}, A_{u3} - B_{u1})$.

(3) *Fuzzy product (or multiplication):* $\widetilde{A}(\times)\widetilde{B} \cong (A_{u1}B_{u1}, A_{l1}B_{l1}, A_2B_2, A_{l3}B_{l3}, A_{u3}B_{u3})$ *whenever* $0 \notin \widetilde{B}$.

(4) *Fuzzy division:* $\widetilde{A}(/)\widetilde{B} = (A_{u1}/B_{u3}, A_{l1}/B_{l3}, A_2/B_2, A_{l3}/B_{l1}, A_{u3}/B_{u1})$*whenever* $0 \notin \widetilde{B}$.

(5) *Exponential function:* $e^{\widetilde{A}} \cong (e^{A_{u1}}, e^{A_{l1}}, e^{A_2}, e^{A_{l3}}, e^{A_{u3}})$.

(6) *Logarithmic function:* $\ln \widetilde{A} \cong (\ln A_{u1}, \ln A_{l1}, \ln A_2, \ln A_{l3}, \ln A_{u3})$ *whenever* $A_{u1} \geq 0$.

3. Proposed Methodology

The proposed methodology comprises the following steps. First, the collected productivity data are analyzed to make sure that a productivity learning process exists. Subsequently, all parameters in the productivity learning model are fuzzified as IFNs to consider uncertainty. To derive the values of IFN-based fuzzy parameters, an MBQP model is proposed and optimized. Finally, the OWA method is applied to defuzzify an IFN-based fuzzy productivity forecast. IFNs, rather than general type-2 fuzzy numbers, are adopted in the proposed methodology because the mathematics needed for IFNs, primarily interval arithmetic, is much simpler than that needed for general type-2 fuzzy numbers [34].

3.1. Data Preanalysis

In a factory, many performance measures exhibit learning phenomena [35–37]. However, the fuzzy learning model of productivity is different from that of other performance measures, such as yield or unit cost, because the asymptotic or final value of productivity is unbounded, whereas that of yield or unit cost is bounded.

Productivity $\rightarrow \infty$; yield $\rightarrow$ 100%; the unit cost $\rightarrow 0$

Therefore, before applying the proposed methodology, it should be ensured that the collected productivity data follow a learning process:

$$P_t = P_0 e^{-\frac{b}{t} + r(t)} \tag{9}$$

where P_t is the productivity forecast at time period t ($t = 1 - T$); P_0 is the asymptotic or final productivity; $b > 0$ is the learning constant; and $r(t)$ is a homoscedastical and serially uncorrelated error term that is often ignored. Taking the logarithmic values of both sides gives the following Equation:

$$\ln P_t = \ln P_0 - \frac{b}{t} + r(t) \tag{10}$$

A linear regression model is presented in the aforementioned Equation, whose validity can be measured in terms of the coefficient of determination R^2, which is given as follows:

$$R^2 = \frac{S_{xy}^2}{S_{xx}S_{yy}} \tag{11}$$

where

$$S_{xx} = \sum_{t=1}^{T} (-\frac{1}{t})^2 - T(\frac{\sum_{t=1}^{T} (-\frac{1}{t})}{T})^2 \tag{12}$$

$$S_{yy} = \sum_{t=1}^{T} (\ln P_t)^2 - T(\frac{\sum_{t=1}^{T} (\ln P_t)}{T})^2 \tag{13}$$

$$S_{xy} = \sum_{t=1}^{T} (-\frac{\ln P_t}{t}) - T(\frac{\sum_{t=1}^{T} (-\frac{1}{t})}{T})(\frac{\sum_{t=1}^{T} \ln P_t}{T}) \tag{14}$$

R^2 is expected to approach a value of 1 if the collected productivity data follow a learning process.

3.2. IFN-Based Fuzzy Productivity Learning Model

The IFN-based fuzzy productivity learning model is proposed by defining the parameters in (8) with ITFNs.

$$\widetilde{P}_t = \widetilde{P}_0(\times)e^{-\frac{\widetilde{b}}{t}+r(t)} \tag{15}$$

where

$$\widetilde{P}_t = (P_{tu1}, P_{tl1}, P_{t2}, P_{tl3}, P_{tu3}) \tag{16}$$

$$\widetilde{P}_0 = (P_{0u1}, P_{0l1}, P_{02}, P_{0l3}, P_{0u3}) \tag{17}$$

$$\widetilde{b} = (b_{u1}, b_{l1}, b_2, b_{l3}, b_{u3}) \tag{18}$$

An IFN-based fuzzy productivity forecast is meaningful in practice. The interpretation of (16) is that, according to a historical experience, the productivity within the t-th period would be within P_{0u1} and P_{0u3}. If this range is very wide, then a narrower range (from P_{0l1} to P_{0u3}) is very likely to contain actual value.

Because $t \neq 0$, according to the formula of fuzzy division, dividing $\widetilde{b}$ by $-t$ gives the following Equation:

$$-\frac{\widetilde{b}}{t} = (-\frac{b_{u3}}{t}, -\frac{b_{l3}}{t}, -\frac{b_2}{t}, -\frac{b_{l1}}{t}, -\frac{b_{u1}}{t}) \tag{19}$$

By taking the exponential of (19), we obtain the following Equation:

$$e^{-\frac{\widetilde{b}}{t}} \cong (e^{-\frac{b_{u3}}{t}}, e^{-\frac{b_{l3}}{t}}, e^{-\frac{b_2}{t}}, e^{-\frac{b_{l1}}{t}}, e^{-\frac{b_{u1}}{t}}) \tag{20}$$

$\widetilde{P}_t$ can be derived by multiplying $\widetilde{P}_0$ to both sides of (20) by using the formula of fuzzy multiplication:

$$\widetilde{P}_t = \widetilde{P}_0(\times)e^{-\frac{\widetilde{b}}{t}} \cong (P_{0u1}e^{-\frac{b_{u3}}{t}}, P_{0l1}e^{-\frac{b_{l3}}{t}}, P_{02}e^{-\frac{b_2}{t}}, P_{0l3}e^{-\frac{b_{l1}}{t}}, P_{0u3}e^{-\frac{b_{u1}}{t}}) \tag{21}$$

3.3. MBQP Model for Deriving the Values of Fuzzy Parameters

Mathematical programming models involving type-2 or other types of fuzzy numbers have been extensively applied in the literature [38–40]. By taking the logarithm of (15), we obtain the following Equation:

$$\ln \widetilde{P}_t \cong (\ln P_{0u1} - \frac{b_{u3}}{t}, \ln P_{0l1} - \frac{b_{l3}}{t}, \ln P_{02} - \frac{b_2}{t}, \ln P_{0l3} - \frac{b_{l1}}{t}, \ln P_{0u3} - \frac{b_{u1}}{t}) \quad (22)$$

The following MBQP model is optimized to derive the values of fuzzy parameters.

Model MBQP:

$$\text{Min } Z = \sum_{t=1}^{T} (\ln P_{0u3} - \frac{b_{u1}}{t} - \ln P_{0u1} + \frac{b_{u3}}{t} + \ln P_{0l3} - \frac{b_{l1}}{t} - \ln P_{0l1} + \frac{b_{l3}}{t}) \quad (23)$$

subject to

$$\ln P_t \geq (1-s)(\ln P_{0u1} - \frac{b_{u3}}{t}) + s(\ln P_{0u2} - \frac{b_{u2}}{t});\ t = 1 \sim T \quad (24)$$

$$\ln P_t \leq (1-s)(\ln P_{0u3} - \frac{b_{u1}}{t}) + s(\ln P_{0u2} - \frac{b_{u2}}{t});\ t = 1 \sim T \quad (25)$$

$$\frac{\sum_{t=1}^{T} X_{t1}X_{t2}}{T} \geq (1-\alpha) \quad (26)$$

$$\ln P_t \geq X_{t1}(\ln P_{0l1} - \frac{b_{l3}}{t});\ t = 1 \sim T \quad (27)$$

$$\ln P_t \leq X_{t2}(\ln P_{0l3} - \frac{b_{l1}}{t});\ t = 1 \sim T \quad (28)$$

$$X_{t1}, X_{t2} \in \{0, 1\};\ t = 1 \sim T \quad (29)$$

$$\ln P_{0u1} \leq \ln P_{0l1} \leq \ln P_{02} \leq \ln P_{0l3} \leq \ln P_{0u3} \quad (30)$$

$$0 \leq b_{u1} \leq b_{l1} \leq b_2 \leq b_{l3} \leq b_{u3} \quad (31)$$

The objective function minimizes the sum of the widths of fuzzy productivity forecasts by considering both LMF and UMF, thereby narrowing both the ranges of LMF and UMF (Figure 4) to maximize the forecasting precision [41]. Constraints (24) and (25) suggest that the membership of an actual value in the corresponding fuzzy forecast should be higher than the satisfaction level (s) based on UMF. X_{t1} and X_{t2} are binary variables, as defined in (29). When both X_{t1} and X_{t2} are equal to 1, an actual value lies within the range of LMF, as suggested by Constraints (27) and (28). Otherwise, the actual value lies outside the LMF range. In this manner, the inclusion level [42] is higher than $100(1-\alpha)\%$ (Figure 5), as required by Constraint (26). Constraints (26)–(29) are quadratic constraints or can be converted into quadratic constraints. Constraints (30) and (31) define the sequences of endpoints in the ITFNs. The MBQP model has one linear objective function, $2T + 6$ variables, $4T + 9$ linear constraints, and $2T + 1$ quadratic constraints.

By moving variables independent of t out of the summation function, the objective function changes as follows:

$$\text{Min } Z = T \ln P_{0u3} - T \ln P_{0u1} + T \ln P_{0l3} - T \ln P_{0l1} + \sum_{t=1}^{T} (-\frac{b_{u1}}{t} + \frac{b_{u3}}{t} - \frac{b_{l1}}{t} + \frac{b_{l3}}{t}) \quad (32)$$

Let

$$\sum_{t=1}^{T} \frac{1}{t} = K \quad (33)$$

Then,

$$Z = T\ln P_{0u3} - T\ln P_{0u1} + T\ln P_{0l3} - T\ln P_{0l1} - Kb_{u1} + Kb_{u3} - Kb_{l1} + Kb_{l3} \tag{34}$$

Note that (33) is a divergent harmonic series [43].

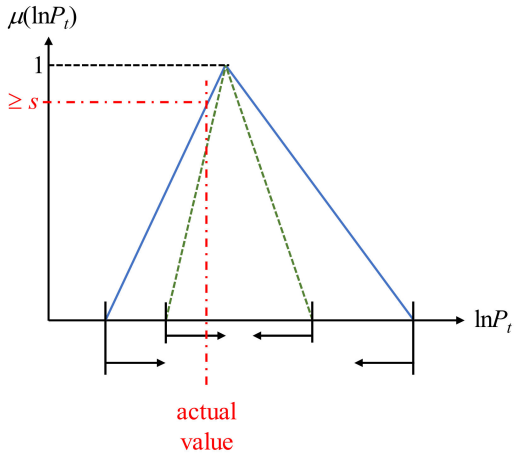


Figure 4. Effects of the objective function.

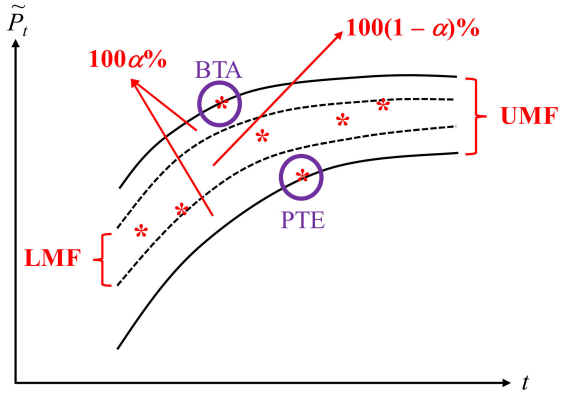


Figure 5. Inclusion interval constructed by solving the MBQP problem.

3.4. OWA for Defuzzifying a Fuzzy Productivity Forecast

In the literature, various formulas have been proposed to defuzzify an ITFN. For example, according to Dahooie et al. [44], an ITFN $\widetilde{A}$ can be defuzzified as follows:

$$D_1(\widetilde{A}) = \frac{A_{u1} + A_{l1} + A_2 + A_{l3} + A_{u3}}{5} \tag{35}$$

which is an extension of the center-of-gravity (COG) formula or

$$D_2(\widetilde{A}) = \frac{(1-\lambda)A_{u1} + \lambda A_{l1} + A_2 + \lambda A_{l3} + (1-\lambda)A_{u3}}{3};\ \lambda \in [0,1] \tag{36}$$

Lee et al. [31] proposed the following formula:

$$D_3(\widetilde{A}) = \frac{A_{u1} + A_{l1} + 4A_2 + A_{l3} + A_{u3}}{8} \tag{37}$$

However, existing defuzzification formulas consider PTE and BTA cases likely, which is questionable because they have distinct meanings in practice.

Definition 4. *A PTE case is a case that lies outside the LMF on the left-hand side, that is,* $P_t \leq P_{tl1}$.

Definition 5. *A BTA case is a case that lies outside the LMF on the right-hand side, that is,* $P_t \geq P_{tl3}$.

To address the aforementioned problem, the concept of OWA is applied in the proposed methodology. The rationale for applying OWA to defuzzify an IFN-based fuzzy productivity forecast is explained as follows:

(1) Using existing defuzzification methods, the defuzzification result of an IFN-based fuzzy productivity forecast is usually the weighted sum of its endpoints. OWA also calculates the weighted sum of data.
(2) OWA aggregates data that have been sorted. The endpoints of an IFN-based fuzzy productivity forecast, from the leftmost to the rightmost, also form a sorted series.

There exist five decision strategies in OWA that assign unequal weights to different attributes according to their performances. The five strategies are optimistic, moderately optimistic, neutral, moderately pessimistic, and pessimistic strategies [44,45]. Most formulas for defuzzifying an ITFN also assign weights to its endpoints. Therefore, assigning weights to the endpoints of $\widetilde{P}_t$ according to their possibilities is reasonable. In the training data, if the number of PTE cases is considerably higher than that of BTA cases, then the "pessimistic" strategy appears to be suitable. By contrast, if the number of BTA cases is considerably higher than that of PTE cases, then the "pessimistic" strategy can be selected. On the basis of these beliefs, a fuzzy productivity forecast is defuzzified according to the selected forecasting strategy, as presented in Table 1. These strategies are subjective selections based on objective historical statistics [46].

Table 1. Defuzzification method based on the forecasting strategy.

Strategy	$D_4(\widetilde{P}_t)$
Optimistic	$0P_{tu1} + 0P_{tl1} + 0P_{t2} + 0P_{tl3} + 1P_{tu3}$
Moderately Optimistic	$0.06P_{tu1} + 0.08P_{tl1} + 0.10P_{t2} + 0.14P_{tl3} + 0.62P_{tu3}$
Neutral	$0.2P_{tu1} + 0.2P_{tl1} + 0.2P_{t2} + 0.2P_{tl3} + 0.2P_{tu3}$
Moderately Pessimistic	$0.49P_{tu1} + 0.30P_{tl1} + 0.15P_{t2} + 0.06P_{tl3} + 0.01P_{tu3}$
Pessimistic	$0.89P_{tu1} + 0.10P_{tl1} + 0.01P_{t2} + 0P_{tl3} + 0P_{tu3}$

Property 2. *The "neutral" forecasting strategy is equivalent to the COG defuzzification method.*

4. Application of the Proposed Methodology to a Real Case

The effectiveness of the proposed methodology was evaluated by applying it for forecasting the productivity of a real dynamic random access memory (DRAM) factory. This case was first investigated by Wang and Chen [19]. In this case, the multi-item productivity of the DRAM factory, which was derived by dividing the monetary value of outputs by that of inputs, was recorded for 14 periods. The recorded data are displayed in Figure 6. Wang and Chen [19] proposed a fuzzy collaborative forecasting approach to forecast the future productivity. For the same purpose, Chen et al. [21] proposed a fuzzy polynomial fitting and mathematical programming approach. The differences between the two approaches and the proposed methodology are summarized in Table 2. The most obvious difference is that only the proposed methodology forecasts productivity with an IFN, thereby differentiating between extreme cases and normal cases to construct a narrow interval of productivity.

The productivity data were divided into two parts, the training data (including the data of the first 10 periods) and test data (including the remaining data). First, to ensure that the collected data followed a learning process, the coefficient of determination (R^2) was calculated. R^2 was found to be 0.87, which was sufficiently high to ensure that the collected data followed a learning process. Subsequently, the training data were used to build the MBQP model, which was solved using a branch-and-bound

algorithm [47–50] on a personal computer with Intel core i7-7700 CPU @ 3.60 GHz and 8 GB RAM in 10 s. Moreover, α was set to 0.2 so that an 80% inclusion interval was constructed. The satisfaction level s was set to 0.3. The optimal solution was

$$\widetilde{P}_0^* = (1.267, 1.343, 1.343, 1.569, 1.683)$$
$$\widetilde{b}^* = (0.990, 0.990, 0.990, 1.260, 1.260)$$

The optimal objective function value Z^* was 5.972. The forecasting results are displayed in Figure 7. The average width of the ranges of LMFs was 0.234. As expected, the ranges of LMFs were too narrow to include all actual values. Nevertheless, most actual values could be contained in such narrow ranges, which is very advantageous for practical applications. The productivity at the eighth period was a PTE case (the purple circle in Figure 7) because the actual value was below the LMF curve, as illustrated in Figure 8. By contrast, no BTA case was observed, which implied that the pessimistic or moderately pessimistic strategy may be suitable.

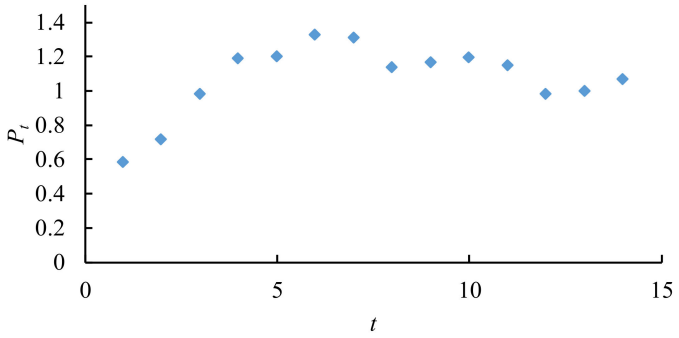


Figure 6. Real case.

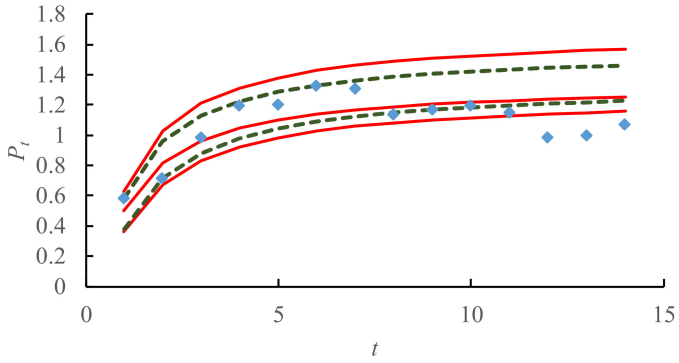


Figure 7. Forecasting results using the proposed methodology.

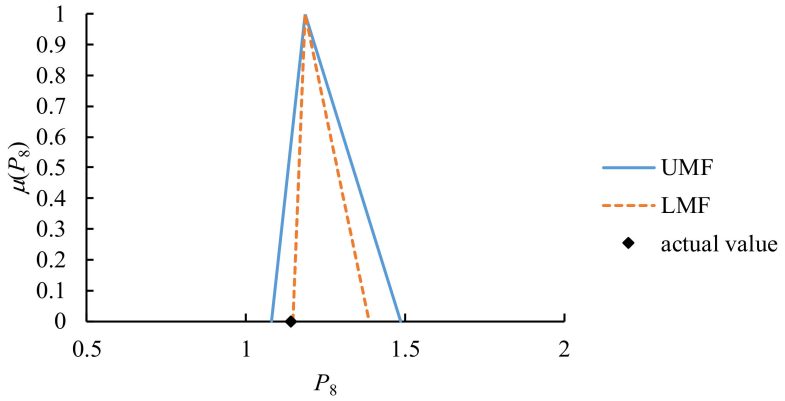


Figure 8. IFN-based fuzzy productivity forecast for t = 8.

Table 2. Differences between the two approaches and the proposed methodology.

Method	Type of Productivity Forecast	Optimization Models	Discriminating Extreme Cases	Number of Experts Required
Wang and Chen [19]	Fuzzy number	NLP, QP	No	Multiple
Chen et al. [21]	Fuzzy number	PP	No	One
The proposed methodology	IFN	MBQP	Yes	One

After applying the proposed methodology to test data, the hit rate was 25%. Subsequently, various formulas were applied to defuzzify interval-valued fuzzy productivity forecasts for test data to evaluate the forecasting accuracy of the proposed methodology in terms of mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean square error (RMSE). The results are summarized in Table 3. The defuzzification formula D4 (the moderately pessimistic strategy) exhibited the best performance.

Table 3. Forecasting accuracy achieved using the proposed methodology (for test data).

Defuzzification Formula	MAE	MAPE	RMSE
D_1	0.270	26.2%	0.279
D_2 ($\lambda = 0.4$)	0.255	24.8%	0.265
D_3	0.240	23.3%	0.250
D_4 (Moderately Optimistic)	0.402	38.9%	0.409
D_4 (Moderately Pessimistic)	0.150	14.7%	0.166

The linear programming (LP) method of Tanaka and Watada [41], quadratic programing (QP) method of Peters [51], QP method of Donoso et al. [52], two NLP models of Chen and Lin [53], artificial neural network (ANN) method of Chen [54], and the PP method of Chen et al. [21] were applied to the real case for comparison. Similar to the proposed methodology, all the aforementioned methods are based on a single expert's forecast.

Tanaka and Watada's LP method minimized the sum of the ranges of fuzzy productivity forecasts. The satisfaction level (s) was set to 0.3 for a fair comparison. By contrast, Peters' QP method maximized the forecasting accuracy in terms of the average satisfaction level by requesting the average range of fuzzy productivity forecasts to be less than $d = 1$. To simultaneously optimize the forecasting accuracy and precision, the QP method of Donoso et al. minimized the weighted sum of the squared deviations from the core as well as the squared deviations from the estimated spreads. In this case, the two weights w_1 and w_2 were set to 0.45 and 0.55, respectively. Chen and Lin's two NLP models were extensions of Tanaka and Watada's LP model and Peters's QP model, respectively. The two NLP methods adopted the following high-order objectives and/or constraints: $o = 2$, $s = 0.15$, $m = 2$, and $d = 1.2$, where o and m are the orders of the two objective functions, respectively. In Chen's ANN method, the initial values of the network parameters were set as follows: the connection weight ($\widetilde{w}$) = (0.10, 0.77, 1.15); the threshold ($\widetilde{\theta}$) = (−0.18, −0.12, 0.26); and the learning rate (η) = 0.25. The training of the ANN was completed in 10 epochs. The PP method of Chen et al. overcame the global optimality problem of Chen and Lin's NLP method by converting the NLP models into PP models, for which the Karush–Kuhn–Tucker conditions were easy to solve. The performance of existing methods is summarized in Table 4. A comparison of the performances of existing methods and the proposed methodology is displayed in Figure 9. The "moderately pessimistic" strategy was adopted in the proposed methodology.

Table 4. Forecasting performances of existing methods for test data.

Method	MAE	MAPE	RMSE	Hit Rate	Average Range
Tanaka and Watada's LP method	0.283	27.4%	0.292	25%	0.346
Peters's QP method	0.487	47.0%	0.492	25%	1.233
Donoso et al.'s QP method	0.269	26.1%	0.278	0%	0.273
Chen and Lin's NLP I model	0.276	26.8%	0.285	0%	0.288
Chen and Lin's NLP II model	0.282	27.4%	0.290	100%	1.006
Chen's ANN method	0.185	18.1%	0.198	100%	0.803
Chen et al.'s PP method	0.168	16.4%	0.181	0%	0.249

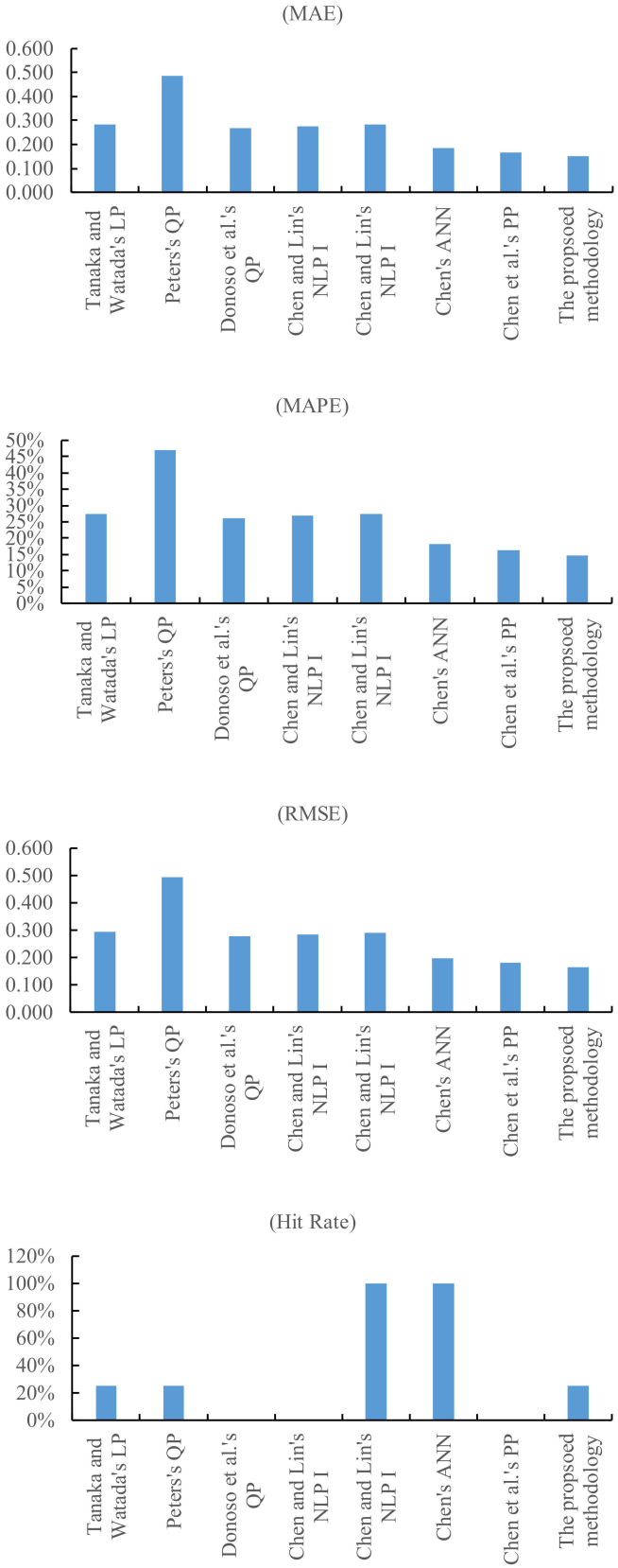


Figure 9. *Cont.*

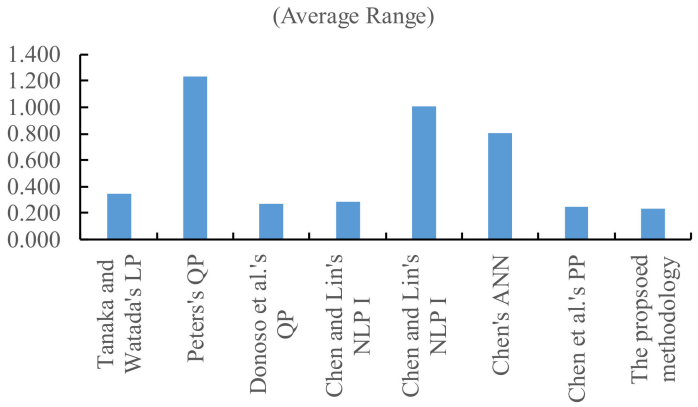


Figure 9. Comparison between the performances of various methods.

According to the experimental results, the following inferences are obtained:

(1) By excluding extreme (PTE and BTA) cases, the average range of fuzzy productivity forecasts was narrowed by 35%. In other words, the average range was widened by 35% when including a single extreme case.

(2) The proposed methodology outperformed existing methods in terms of MAE, MAPE, and RMSE in evaluating the forecasting accuracy. The detection of PTE and BTA cases enabled the selection of a suitable forecasting strategy, which contributed to the superiority of the proposed methodology over existing methods. The most significant advantage was over the QP method of Peters. The proposed method was up to 69% more effective than the QP method in minimizing MAPE.

(3) Conversely, the proposed methodology optimized the forecasting precision measured in terms of the average range. Despite such a narrow average range, the hit rate achieved using the proposed methodology was also satisfactory.

(4) To ascertain whether the differences between the performances of various methods were statistically significant, the sums of ranks of all methods were compared [55–57]. The results are presented in Table 5. For example, the proposed methodology ranked the first among the compared methods in reducing MAE, MAPE, RMSE, and the average range, and ranked the fifth in elevating the hit rate. As a result, the sum of ranks was 9 for the proposed methodology. The ranks of methods that performed equally well were averaged. For example, Donoso et al.'s QP method and Chen and Li's NLP I method performed equally well in elevating the hit rate and outperformed the other methods. Therefore, both of their ranks were $(1 + 2)/2 = 1.5$. According to the sums of ranks achieved by these methods, the proposed methodology ranked first, followed by the PP method of Chen et al., the QP method of Donoso et al., and the ANN method of Chen.

(5) To further elaborate the effectiveness of the proposed methodology, it has been applied to another case of forecasting the productivity of a factory. This case was first investigated by Akano and Asaolu [58], in which four factors (preventive maintenance time, off-duty time, machine downtime, and power failure time) were considered to be influential to the productivity of a factory. To forecast the productivity, Akano and Asaolu constructed an adaptive network-based fuzzy inference system (ANFIS), which resulted in a MAPE of up to 34%. In this study, an expert applied the IFN-based MBQP–OWA approach to forecast productivity, for which the neutral strategy was adopted. The forecasting results are shown in Figure 10. The forecasting accuracy, in terms of MAPE, was elevated by 19%.

Table 5. Comparing the sums of ranks of various methods.

Method	Rank (MAE)	Rank (MAPE)	Rank (RMSE)	Rank (Hit Rate)	Rank (Average Range)	Sum of Ranks
Tanaka and Watada's LP	7	7	7	5	5	31
Peters's QP	8	8	8	5	8	37
Donoso et al.'s QP	4	4	4	1.5	3	16.5
Chen and Lin's NLP I	5	5	5	1.5	4	20.5
Chen and Lin's NLP I	6	7	6	7.5	7	33.5
Chen's ANN	3	3	3	7.5	6	22.5
Chen et al.'s PP	2	2	2	3	2	11
The proposed methodology	1	1	1	5	1	9

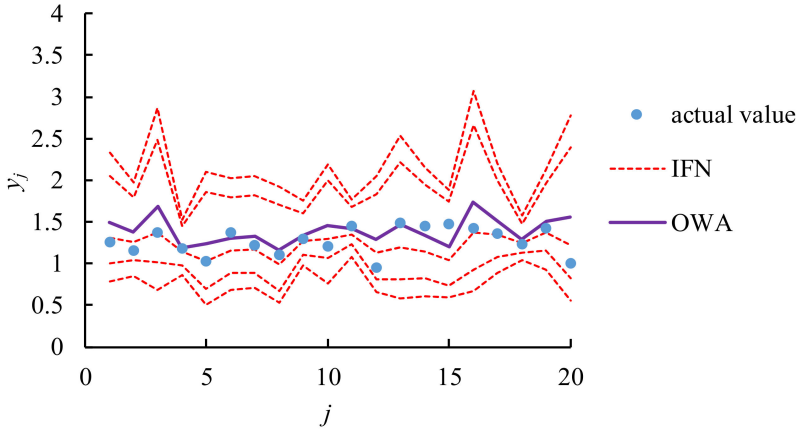


Figure 10. Forecasting results using the IFN-based MBQP–OWA approach.

5. Conclusions

An IFN-based MBQP–OWA approach is proposed in this study to model an uncertain productivity learning process. This study aims to resolve a problem of existing methods, that is, a few extreme (PTE and BTA) cases determine the lower and upper bounds on productivity. This problem causes the range of productivity to be unacceptably wide. To solve this problem, the range of productivity is divided into inner and outer sections that correspond to the LMF and UMF of an IFN-based fuzzy productivity forecast, respectively. In this manner, all actual values are included in the outer section, whereas most of the values lie within the inner section. Moreover, a suitable forecasting strategy can be determined according to the percentages of PTE and BTA cases. To derive the values of parameters in the IFN-based fuzzy productivity learning model, an MBQP model is proposed and optimized. Subsequently, the OWA method based on the selected forecasting strategy is applied to defuzzify the fuzzy productivity forecast. The contribution of this study resides in the following:

(1) Using the characteristics of IFNs, a systematic mechanism was established to avoid extreme cases from widening the ranges of fuzzy productivity forecasts.
(2) An innovative idea was proposed to defuzzify an IFN-based fuzzy productivity forecast using OWA.

The IFN-based MBQP–OWA approach has been applied to a real case of a DRAM factory to evaluate its effectiveness. According to the experiment results, the following findings are obtained:

(1) In terms of MAE, MAPE, and RMSE, the accuracy of the forecasted productivity obtained using the proposed methodology was superior to those obtained using several existing methods.
(2) The forecasting precision achieved using the proposed methodology was also satisfactory, especially for minimizing the average range of fuzzy productivity forecasts.

(3) By identifying PTE and BTA cases, an expert was able to select a suitable forecasting strategy, which further enhanced the forecasting precision and accuracy.

The proposed methodology has several advantages, but there are also some drawbacks. For example, extreme cases may affect the range of productivity in different ways in the future. Nevertheless, in future studies, other types of fuzzy numbers, such as interval-valued intuitionistic fuzzy numbers [59], hesitant IFNs [60,61], Pythagorean fuzzy numbers [62], and interval-valued Pythagorean fuzzy numbers [63,64] can be adopted to model uncertain productivity instead. The proposed methodology can also be applied to other learning processes in various fields that are subject to uncertainty, such as unit cost learning [65] and energy efficiency learning [66]. Another interesting topic is how to build the IFN-based fuzzy productivity learning model if the collected productivity data are incomplete [67]. The proposed methodology can also be extended to fulfill a multiple-expert collaborative forecasting task [68–72].

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