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Eliminating Rank Reversal Problem Using a New Multi-Attribute Model—The RAFSI Method

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Abstract: Multi-attribute decision-making (MADM) methods represent reliable ways to solve real-world problems for various applications by providing rational and logical solutions. In reaching such a goal, it is expected that MADM methods would eliminate inconsistencies like rank reversal issues in a given solution. In this paper, an endeavor is taken to put forward a new MADM method, called RAFSI (Ranking of Alternatives through Functional mapping of criterion sub-intervals into a Single Interval), which successfully eliminates the rank reversal problem. The developed RAFSI method has three major advantages that recommend it for further use: (i) its simple algorithm helps in solving complex real-world problems, (ii) RAFSI method has a new approach for data normalization, which transfers data from the starting decision-making matrix into any interval, suitable for making rational decisions, (iii) mathematical formulation of RAFSI method eliminates the rank reversal problem, which is one of the most significant shortcomings of existing MADM methods. A real-time case study that shows the advantages of RAFSI method is presented. Additional comprehensive analysis, including a comparison with other three traditional MADM methods that use different ways for data normalization and testing the resistance of RAFSI method and other MADM methods to rank the reversal problem, is also carried out.

Keywords: multi-criteria optimization; RAFSI method; performance comparison; rank reversal

1. Introduction

Multi-criteria optimization (MCO) methods represent powerful tools for making rational decisions while being engaged in various types of activities. Studies in MCO problems have particularly been prevalent in recent decades [1]. The reasons for such developments lie both in theoretical and practical points of view. In a theoretical sense, MCO is attractive as it studies insufficiently structured problems, while, in a practical sense, MCO represents a powerful way for choosing adequate actions. Furthermore, MCO methods are unavoidable for designing appropriate tools to explore diverse systems.

MCO methods can be classified into five groups [2]: (1) methods for determining non-inferior solutions that determine the set of non-inferior solutions, while it depends on the decision-makers (DMs) to adopt the final solution based on their preferences. The following methods belong to this

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group: weighting coefficient methods (the restriction method in the criteria functions environment, as well as the Simplex method), (2) methods with a predetermined preference, which are used to form synthesizing (resultant) criterion function (it includes almost all multi-attribute decision-making (MADM) methods, (3) interactive methods in which DMs express their preferences interactively, (4) stochastic methods where indicators of uncertainty are included in the optimization model, and (5) methods for emphasizing a subset of non-inferior solutions that narrow down the subset of non-inferior results, which are achieved by introducing additional elements for making rational decisions.

MADM methods involve sound mathematical steps for processing information to evaluate alternatives concerning a predetermined set of criteria, which is the main focus of this paper. It is performed to establish a ranking of solutions and the best choice. Some of the most predominant representative methods of this group are

- Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) [3],
- Više Kriterijumska optimizacija i Kompromisno Rešenje (VIKOR) [4,5],
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [6],
- Analytical Hierarchy Process (AHP) [7],
- Elimination Et Choice Translating Reality (ELECTRE) [8],
- Multi-Attributive Border Approximation Area Comparison (MABAC) [9],
- Complex Proportional Assessment (COPRAS) [10],
- Combinative Distance-based Assessment (CODAS) [11],
- lattice MADM methods [12].

MADM methods play a significant role in solving real-world problems in several areas. Let us mention some interesting studies, which show the diversity of applications of MADM methods. Orji and Wei [13] applied a hybrid decision-making trial and evaluation laboratory (DEMATEL)-TOPSIS model for sustainable supplier selection. Rabbani et al. [14] modified traditional MADM methods using fuzzy sets and demonstrated their application in logistics. Mahdi Paydar et al. [15] applied the fuzzy Multi-Objective Optimization Method by Ratio Analysis (MOORA) and Failure Mode and Effects Analysis (FMEA) methods in the Iranian chemical industry application. Zhou and Xu [16] used DEMATEL, Analytic Network Process (ANP), and VIKOR methods in sustainable supplier selection. Lu et al. [17] extended the ELECTRE method using a rough set theory. Si et al. [18] showed the possibilities of applying picture fuzzy numbers in MADM. Noureddine and Ristic [19] combined the Full Consistency Method (FUCOM), TOPSIS, and MABAC with the Dijkstra algorithm for optimizing the transport of dangerous cargo. Badi et al. [20] used a gray-based assessment model to evaluate healthcare waste treatment alternatives in Libya. Krmac and Djordjevic [21] applied the TOPSIS method for evaluating the influence of the Train Control Information System on capacity utilization.

One of the most important problems that occur in most MADM methods with predetermined preferences is the lack of resistance to rank reversal problems. If unexpected changes in the ranking of alternatives occur when any non-optimal alternative is added or deleted from the existing set of alternatives, this indicates serious mathematical issues in the applied MADM method. This problem can be illustrated with the following example in which three candidates are examined (candidates A, B, and C) who applied for the same work position. A MADM method is used to rank the candidate alternatives and the method suggested the following ranking of the candidates: A > B > C. Furthermore, it is assumed that candidate B (with the second rank) is replaced with a poor candidate D, which kept candidates A and C unchanged. If this new set of alternatives (A, D, and C) is now ranked by the same method under the same criteria weights, it is expected that the applied MADM method would again suggest candidate A as the best solution under the new conditions. However, in actual practice, some unwanted changes in the ranking order of the alternatives occur for the majority of the MADM methods [22].

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The rank reversal problem was noticed and presented for the first time by Belton and Gear [22], who analyzed the use of Analytic Hierarchy Process (AHP) for ranking alternatives. In their research, they conducted a simple experiment in which three alternatives and two criteria were analyzed. After the initial ranking of the alternatives, they formed a new set of alternatives by introducing a copy of the non-optimal alternative. After evaluating this new set of alternatives while keeping the same criteria weights, inconsistencies were observed as the ranking order of the best alternative was changed. Thus, they concluded that AHP suffers from rank reversal phenomena. A few years later, Triantaphyllou and Mann [23] noticed the same problem again in AHP when the worst alternative was replaced by a non-optimal alternative. Triantaphyllou and Mann [23] also conducted the same experiment on two other methods, which included the Weighted Sum Model (WSM) and Weighted Product Model (WPM), and concluded that none of these methods were efficient in solving the rank reversal problem. Afterward, Triantaphyllou and Lin [24] further tested five MADM methods, including WSM, WPM, AHP, revised AHP, and TOPSIS in terms of the same two evaluative criteria in the fuzzy environment and came to the same conclusions. Then, many authors pointed out the rank reversal problem in many other MADM methods [25–30].

Furthermore, there is a large number of MADM methods already developed in the past few years, which give successful results for solving practical problems [31]. Nevertheless, most of these methods are not able to successfully eliminate the rank reversal problem. Among such methods, only the lattice MADM method can successfully eliminate the rank reversal problem [12]. However, this method has a complex mathematical algorithm and requires profound knowledge in net theory [32]. The complexity of the lattice algorithm significantly limits its broader use [33]. Moreover, several studies have shown that the rank reversal problem can be solved when traditional methods are substantially modified [34–36]. Keeping in mind that MADM methods are often used in the condition of dynamic changes in the initial decision matrix, authors of this research have paid attention to the development of a new MADM method, called Ranking of Alternatives through Functional mapping of criterion sub-intervals into a Single Interval (RAFSI) method that eliminates rank reversal problems. Besides eliminating the rank reversal problem, RAFSI method is also characterized by simple mathematical formulations that can be easily used for solving complex problems. RAFSI method integrates three starting points for making consistent decisions, which encompass (1) defining referential criteria points including ideal and anti-ideal criteria values, (2) defining relations between the considered alternatives and ideal/anti-ideal values, and (3) using a new technique for data normalization, based on defining criteria functions that map criteria sub-intervals into a unique criteria interval.

According to the results shown in this paper, three main advantages of the RAFSI method distinguish it from the other traditional MADM methods, which include (1) a simple algorithm of RAFSI method that enables DMs to solve complex problems, (2) use a new data normalization technique that converts an initial decision matrix into a unique criterion interval, and (3) resistance of the RAFSI method to rank reversal problems. We are emphasizing this phenomenon since it can be especially seen in dynamic conditions of decision-making where some alternatives often change during the process of making decisions, and MADM methods are often used in such conditions. Based on these advantages of RAFSI method, one of the most important contributions of this paper is to enrich the MADM research domain by developing a new method, which enables the DMs to make stable and coherent decisions in dynamic and uncertain environments.

After the introductory discussion on motivation, goals, and contributions, the content of the paper is presented as follows. In Section 2, the mathematical formulation of the RAFSI method is presented. Section 3 covers the application of RAFSI method for a real-time case study by considering six alternatives and five criteria. Results' validation and performance comparisons are presented in Section 4. Lastly, Section 5 concludes the paper with future research directions.

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2. RAFSI Method

Let us assume that the DMs have to rank m alternatives on the basis of n criteria C_1, C_2, \ldots, C_n . Criteria weights $(w_j, j = 1, 2, ..., n)$ meet the following condition $\sum_{j=1}^{n} w_j = 1$. Criteria $C_1, C_2, ..., C_n$ can be maximizing type (max) or minimizing type (min). Alternatives $A_i (i = 1, 2, ..., m)$ are defined by their respective values (a_{ij}) on each criterion (c_i) . The initial decision matrix is shown as follows

$$N = \begin{bmatrix} C_1 & C_2 & \dots & C_n \\ A_1 & n_{11} & n_{12} & \dots & n_{1n} \\ A_2 & n_{21} & n_{22} & \dots & n_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_m & n_{m1} & n_{m2} & \dots & n_{mn} \end{bmatrix}$$
(1)

The RAFSI method has the following steps.

Step 1: Define ideal and anti-ideal values. For each criterion $C_i (j = 1, 2, ..., n)$, the DM defines two values a_{I_i} and a_{N_i} , where a_{I_i} represents the ideal value of criterion C_i , while a_{N_i} represents an anti-ideal value of criterion C_j . It is clear that $a_{I_i} > a_{N_i}$ for max criteria and $a_{I_i} < a_{N_i}$ for min criteria.

Step 2: Mapping of elements of the initial decision matrix into criteria intervals. In the previous part, criteria intervals are defined below.

- (a) $C_j \in [a_{N_j}, a_{I_j}]$, when C_j belongs to max type criteria and (b) $C_j \in [a_{I_j}, a_{N_j}]$, when C_j belongs to min type criteria.

In order to make all criteria of the initial decision matrix equal or transfer them into the criteria interval $[n_1, n_{2k}]$, we are forming a sequence of numbers from the k interval in the way where k-1points are inserted between the highest and the lowest values of the criteria interval.

$$n_1 < n_2 \le n_3 < n_4 \le n_5 < n_6 \dots \le n_{2k-1} < n_{2k}$$
 (2)

The criteria interval is constant for all criteria and it has n_1 and n_{2k} fixed points. Then we can map sub-intervals of the criteria into criteria intervals using functions f_1 , f_2 , f_3 , that is f_s , as shown in Figure 1.

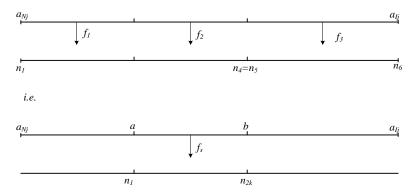


Figure 1. Mapping of sub-intervals into the criteria interval.

The map of minimum value a_{N_i} (for max criteria) and a_{I_i} (for min criteria) is n_1 . Additionally, the map of maximum value a_{I_i} (for max criteria) and a_{N_i} (for min criteria) is n_{2k} . It is suggested that the ideal value is at least six times better than the anti-ideal (barely acceptable value), or $n_1 = 1$ and $n_{2k} = 6$. However, the DM can use other preferred values such as $n_1 = 1$ and $n_{2k} = 9$.

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We define a function $f_s(x)$, which maps sub-intervals into the criteria interval $[n_1, n_{2k}]$ by Formula (3) below. The endpoints of the interval $[n_1, n_{2k}]$ determine the ratio of a barely acceptable alternative to the ideal alternative. This ratio is set up by the DM.

$$f_s(x) = \frac{n_{2k} - n_1}{a_{I_i} - a_{N_i}} x + \frac{a_{I_j} \cdot n_1 - a_{N_j} \cdot n_{2k}}{a_{I_i} - a_{N_i}}$$
(3)

where n_{2k} and n_1 represent the relation that shows the extent to which the ideal value is preferred over the anti-ideal value, and where a_{I_j} and a_{N_j} represent ideal and anti-ideal values of criteria C_j , respectively.

Expression (3), as a function, can be part of the function, which maps a part of the interval $[a_{N_j}, a_{I_j}]$ into interval $[n_1, n_{2k}]$. In this case, all these parts, that is, all functions $f_1(x)$, $f_2(x)$..., $f_n(x)$, represent a function $f_s(x)$ that maps the entire criterion interval into a defined numerical interval. Thus, Expression (3) can represent a function that maps a part of an interval, but can also map a complete criterion interval into the corresponding numerical interval. Therefore, the numbers a_{I_j} and a_{N_j} can represent: (1) values from inside the criterion interval or (2) endpoints of the criterion interval. The second possibility is used in this paper.

In this way, the standardized decision matrix $S = \left[s_{ij}\right]_{m \times n}$ (i = 1, 2, ..., m, j = 1, 2, ..., n) is obtained in which all elements of the matrix are mapped into the interval $[n_1, n_{2k}]$. After functional mapping of the elements of the initial decision matrix into criteria interval N $[n_1, n_{2k}]$, the condition $n_1 \le s_{ij} \le n_{2k}$ is achieved for every I, j.

$$S = \begin{pmatrix} C_1 & C_2 & \dots & C_n \\ A_1 & s_{11} & s_{12} & \dots & s_{1n} \\ A_2 & s_{21} & s_{22} & \dots & s_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & s_{m1} & s_{m2} & \dots & s_{mn} \end{pmatrix}$$
(4)

In the above formula, the elements of the matrix s_{ij} are obtained by using expression (3), that is, $s_{ij} = f_{A_i}(C_j)$.

Note the following:

- (a) for max type criteria, if there is a_{x_i} where $a_{x_i} > a_{I_i}$, then we have equality $f(a_{x_i}) = f(a_{I_i})$
- (b) for *min* type criteria, if there is a_{x_i} where $a_{x_i} < a_{I_i}$, then we have equality $f(a_{x_i}) = f(a_{I_i})$

Step 3: Calculate arithmetic and harmonic means. Using expressions (5) and (6), arithmetic and harmonic means are calculated for minimum and maximum sequence of the elements n_1 and n_{2k} .

$$A = \frac{n_1 + n_{2k}}{2} \tag{5}$$

$$H = \frac{2}{\frac{1}{n_1} + \frac{1}{n_{2k}}}\tag{6}$$

Step 4: Form normalized decision matrix $\hat{S} = \left[\hat{s}_{ij}\right]_{m \times n}$ (i = 1, 2, ..., m, j = 1, 2, ..., n). Using expressions (7) and (8), elements of the matrix S are normalized, and transferred into the interval [0,1].

(a) for the criteria C_i (j = 1, 2, ..., n) max type:

$$\hat{s}_{ij} = \frac{s_{ij}}{2A} \tag{7}$$

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(b) for the criteria C_j (j = 1, 2, ..., n) min type:

$$\hat{s}_{ij} = \frac{H}{2s_{ij}} \tag{8}$$

In this way, a new normalized decision matrix is created, as shown below.

$$\hat{S} = \begin{pmatrix} C_{1} & C_{2} & \dots & C_{n} \\ A_{1} & \hat{s}_{11} & \hat{s}_{12} & \dots & \hat{s}_{1n} \\ \hat{s}_{21} & \hat{s}_{22} & \dots & \hat{s}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m} & \hat{s}_{m1} & \hat{s}_{m2} & \dots & \hat{s}_{mn} \end{pmatrix}$$

$$(9)$$

where $\hat{s}_{ij} \in [0,1]$ represents normalized elements of \hat{S} .

For the elements of the normalized decision matrix $\hat{S} = [\hat{s}_{ij}]_{m \times n'}$, which are defined using Expressions (7) and (8), the following relations can apply.

(a) For max type criteria C_j (j = 1, 2, ..., n), we have the following condition.

$$0 < \frac{n_1}{2A} \le \hat{s}_{ij} \le \frac{n_{2k}}{2A} < 1 \tag{10}$$

Proof of (10):

$$\frac{n_{2k}}{2A} = \frac{n_{2k}}{2^{\frac{n_1 + n_{2k}}{2}}} = \frac{n_{2k}}{n_1 + n_{2k}} < \frac{n_{2k} + n_1}{n_1 + n_{2k}} = 1$$

(b) for min type criteria C_j (j = 1, 2, ..., n), we have the following condition.

$$0 < \frac{H}{2n_{2k}} \le \hat{s}_{ij} \le \frac{H}{2n_1} < 1 \tag{11}$$

Proof of (11):

$$\frac{H}{2n_1} = \frac{\frac{2}{\frac{1}{n_{2k}} + \frac{1}{n_1}}}{2n_1} = \frac{1}{n_1(\frac{1}{n_{2k}} + \frac{1}{n_1})} = \frac{1}{1 + \frac{n_1}{n_{2k}}} < 1$$

Additionally, for the boundary values of criteria intervals n_1 and n_{2k} , we have the following equality (12) and (13).

$$\frac{n_1}{2A} = \frac{H}{2n_{2k}} \tag{12}$$

Proof of (12):

$$\frac{n_1}{2A} = \frac{H}{2n_{2k}} \Rightarrow \frac{n_1}{A} = \frac{H}{n_{2k}}$$

$$\frac{n_1}{A} = \frac{n_1}{\frac{n_1 + n_{2k}}{2}} = \frac{2}{\frac{n_1 + n_{2k}}{n_1}} = \frac{2}{1 + \frac{n_{2k}}{n_1}}$$

$$= \frac{2}{\frac{n_{2k}}{n_{2k}} + \frac{n_{2k}}{n_1}} = \frac{2}{n_{2k} \left(\frac{1}{n_{2k}} + \frac{1}{n_1}\right)} = \frac{\frac{2}{\frac{1}{n_{2k}} + \frac{1}{n_1}}}{n_{2k}} = \frac{H}{n_{2k}}$$

$$\frac{n_{2k}}{2A} = \frac{H}{2n_1}$$
(13)

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Proof of equality (13):

$$\frac{n_{2k}}{2A} = \frac{H}{2n_1} \Rightarrow \frac{n_{2k}}{A} = \frac{H}{n_1}$$

$$\frac{n_{2k}}{A} = \frac{n_{2k}}{\frac{n_1 + n_{2k}}{2}} = \frac{2}{\frac{n_1 + n_{2k}}{n_{2k}}} = \frac{2}{\frac{n_1}{n_{2k}} + 1}$$

$$= \frac{2}{\frac{n_1}{n_{2k}} + \frac{n_1}{n_1}} = \frac{2}{n_1(\frac{1}{n_{2k}} + \frac{1}{n_1})} = \frac{1}{n_1} \frac{2}{\frac{1}{n_{2k}} + \frac{1}{n_1}} = \frac{H}{n_1}$$

Step 5: Calculate criteria functions of the alternatives $V(A_i)$. Criteria functions of the alternatives $(V(A_i))$ are calculated according to Equation (14) below. Alternatives are then ranked according to the descending order of the calculated $(V(A_i))$ values.

$$V(A_i) = w_1 \hat{s}_{i1} + w_2 \hat{s}_{i2} + \ldots + w_n \hat{s}_{in}$$
(14)

3. Case Study and Results

In this section, the application of the newly developed RFIS method is presented by giving an example that considers the evaluation of six alternatives A_i ($i=1,2,\ldots,6$) in relation to five criteria C_j ($j=1,2,\ldots,5$). Suppose that the alternatives represent researchers who applied for a job at a scientific research center. Evaluation of the researchers is performed using five criteria. The criteria are arranged in two groups: 1) criteria of maximizing type (max): C1, C2, and C5, and 2) criteria of minimizing type (min): C3 and C4. Criteria weights are estimated by the Level-Based Weight Assessment (LBWA) model [26] as $w_j = (0.35, 0.25, 0.15, 0.15, 0.1)$. The initial decision matrix $(N = \begin{bmatrix} n_{ij} \end{bmatrix}_{m \times n}, i=1,2,\ldots,m, j=1,2,\ldots,n)$ is given below.

Application of RAFSI method is illustrated by following the steps described in Section 2.

Step 1: In the first step, DM defines the set of ideal (a_{I_j}) and anti-ideal values (a_{N_j}) for the considered criteria. In this example, the following ideal and anti-deal points are defined by consensus.

$$a_{I_j} = \{200, 12, 10, 100, 8\}$$

 $a_{N_i} = \{120, 6, 20, 200, 2\}$

Step 2: Based on the defined ideal and anti-ideal points, criteria intervals are formed.

- (a) for max type criteria: $C_1 \in [120, 200]$; $C_2 \in [6, 12]$ i $C_5 \in [2, 8]$,
- (b) for min type criteria: $C_3 \in [10, 20]$ i $C_4 \in [100, 200]$.

To transfer the values of all criteria into a unique interval, a sequence of numbers is chosen where $n_1 < n_2 \le n_3 < n_4 \le n_5 < n_6 \ldots \le n_{2k-1} < n_{2k}$. The final points of the sequence n_1 and n_{2k} define the values determining the number of times the ideal value is better than the anti-ideal value. In other words, points n_1 and n_{2k} determine the boundary values of the interval in which all values of the initial decision matrix are transferred. In this paper, it is assumed that the ideal value is six times better than the barely acceptable value (anti-ideal value). Now, the functions for criteria standardization

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are defined using expression (3). It helps to transfer the values of the initial decision matrix into the interval [1,6]. Therefore, we consider the following functions.

$$f_{A_i}(C_1) = \frac{6-1}{200-120}C_1 + \frac{200\cdot 1-120\cdot 6}{200-120} = 0.06 \cdot C_1 - 6.50$$

$$f_{A_i}(C_2) = \frac{6-1}{12-6}C_2 + \frac{12\cdot 1-6\cdot 6}{12-6} = 0.83 \cdot C_2 - 4.00$$

$$f_{A_i}(C_3) = \frac{6-1}{20-10}C_3 + \frac{20\cdot 1-10\cdot 6}{20-10} = 0.50 \cdot C_3 - 4.00$$

$$f_{A_i}(C_4) = \frac{6-1}{200-10}C_4 + \frac{200\cdot 1-100\cdot 6}{200-100} = 0.05 \cdot C_4 - 4.00$$

$$f_{A_i}(C_5) = \frac{6-1}{8-2}C_5 + \frac{8\cdot 1-2\cdot 6}{8-2} = 0.83 \cdot C_5 - 0.67$$

Based on the defined functions, the elements of the initial decision matrix are mapped into the interval [1,6] and the standardized decision matrix ($S = \left[s_{ij}\right]_{6\times5}$, $i = 1,2,\ldots,6$, $j = 1,2,\ldots,5$) is obtained in which all elements are transferred into the interval [1,6].

The elements of the position A_i - C_1 are obtained using the functions $f_{A_i}(C_1) = 0.06 \cdot C_1 - 6.50$:

$$f_{A_1}(180) = 0.06 \cdot 180 - 6.50 = 4.75, f_{A_2}(165) = 0.06 \cdot 165 - 6.50 = 3.81$$

 $f_{A_3}(160) = 0.06 \cdot 160 - 6.50 = 3.50, f_{A_4}(170) = 0.06 \cdot 170 - 6.50 = 4.13$
 $f_{A_5}(185) = 0.06 \cdot 185 - 6.50 = 5.06, f_{A_6}(167) = 0.06 \cdot 167 - 6.50 = 3.94$

Replacing the values from the initial matrix into functions $f_{A_i}(C_2)$, $f_{A_i}(C_3)$, $f_{A_i}(C_4)$, and $f_{A_i}(C_5)$, we get the remaining values of elements of s_{ij} .

Step 3: Calculating the arithmetic and harmonic means of minimum and maximum elements $n_1 = 1$ and $n_{2k} = 6$.

$$A = (n_1 + n_{2k})/2 = (1+6)/2 = 3.5$$

$$H = \frac{2}{\frac{1}{n_1} + \frac{1}{n_{2k}}} = \frac{2}{\frac{1}{6} + \frac{1}{1}} = 1.71$$

The arithmetic mean for $n_1 = 1$ and $n_{2k} = 6$ is 3.5, while the harmonic mean is 1.71.

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Step 4: Using expressions (7) and (8) elements of matrix S are normalized and transformed, depending on whether they belong to min or max type criteria. In this way, we get a new matrix $\hat{S} = \left[\hat{s}_{ij}\right]_{6\times5}$ (i = 1, 2, ..., 6, j = 1, 2, ..., 5).

For example, the element of the matrix \hat{S} in position A1–C1 is $\hat{s}_{11} = \frac{4.75}{2.3.5} = 0.68$. Moreover, for the min type criteria, A1–C3 is $\hat{s}_{13} = \frac{1.71}{2.3.75} = 0.23$.

Step 5: Using expression (14), criteria functions $V(A_i)$ of the alternatives are calculated, as exhibited in Table 1. Ranking pre-order of the alternatives is derived as per the descending order of $V(A_i)$ values, where the alternative with higher $V(A_i)$ values are always preferred.

Table 1. The function criteria and the final ranking of the researchers/alternatives.
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Alternative	$V(A_i)$	Rank		
A1	0.5081	2		
A2	0.4522	4		
A3	0.4381	5		
A4	0.4560	3		
A5	0.5299	1		
A6	0.4373	6		

Based on the above findings, the researcher A5 is selected as the best alternative candidate for the considered case study.

4. Validation of the Results

4.1. Comparing the Results with Other MADM Methods

For validation, the results of RFIS method are now compared with other traditional MADM methods like TOPSIS [6], VIKOR [4,5], and COPRAS [10]. The same decision matrix and criteria weights are used for this performance comparison. The results of this comparison are shown in Figure 2.

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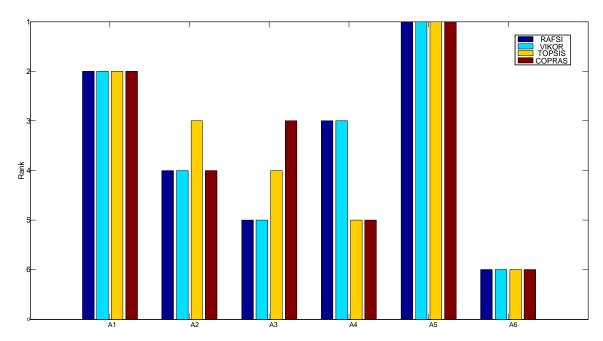


Figure 2. Comparing RAFSI method with other MADM methods.

The ranking orders as obtained by VIKOR and RAFSI methods are in complete agreement, whereas, in COPRAS and TOPSIS methods, the rank similarity is observed only for the first two alternatives {A5, A1}, and the last ranked alternative (A6). For the remaining three alternatives (A2, A3, and A4), COPRAS and TOPSIS methods suggested different rankings. Such a result is a consequence of using different data normalization techniques, such as vector normalization (in TOPSIS method) and additive normalization (in the COPRAS method). To confirm this fact, an experiment is further conducted, which was comprised of the following two stages.

- (1) In the first stage, the COPRAS and TOPSIS methods were slightly modified through the use of additive data normalization techniques in both methods. It was observed that both methods gave the same ranking order (Figure 2) for the considered alternatives under additive data normalization.
- (2) In the second stage, data normalization, as suggested in RAFSI method, was also used for TOPSIS, VIKOR, and COPRAS methods. After using the new normalization technique, identical rankings were obtained by all the methods. Based on these results, it can be concluded that the RAFSI method gives credible and reliable results.

4.2. Rank Reversal Problem

One of the ways to check the stability of MADM methods is by introducing new alternatives in the original set or by eliminating poor alternatives from the set. In such conditions, it is expected that the MADM method will not show any drastic change in the ranking of the alternatives. This phenomenon is called the well popular rank reversal problem [13], and considerable attention has already been paid to it in the literature [21,25]. The resistance of the developed RAFSI method to the rank reversal problem is now tested through two experiments. In the first experiment, five scenarios are considered. In each scenario, the worst alternative is eliminated from the set of alternatives, and the impact of this change on ranking and criteria functions of the alternatives are analyzed. In the second experiment, the set of alternatives is further expanded by introducing a new alternative, and the impact of such inclusion on alternatives' rank is analysed.

The first experiment: After applying RAFSI method, the researchers are ranked according to the results shown in scenario S0 (the original rank). In the next scenario (S1), the researcher who achieved the least rank is eliminated. After that, the remaining five candidates are again ranked. Thus,

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a total of five scenarios (S1–S5) are formed, whereby, in each subsequent scenario, the worst-ranked researcher from the set is eliminated. At the same time, we also analyzed the possibility of any changes in criteria function values and rankings of the remaining alternatives for each of the newly formed scenarios. The rankings of the alternatives in all five scenarios are shown in Table 2.

	,	,				
Alternative	S0	S 1	S2	S 3	S4	S 5
A5	1	1	1	1	1	1
A1	2	2	2	2	2	
A4	3	3	3	3		
A2	4	4	4			
A3	5	5				
A6	6					

Table 2. The ranking of the alternatives in scenarios.

From Table 2, it is easy to observe that RAFSI method gives valid results in a dynamic environment. This is also confirmed by criteria function values of the alternatives (f(Ai)). In all these scenarios, the criteria functions of the alternatives remained unchanged. TOPSIS, VIKOR, and COPRAS methods are used in the same condition. All these methods also showed stability and resistance to rank reversal. However, changes in criteria function values are observed in these methods.

The second experiment: In the second experiment, among the six existing candidates, another candidate (A7) is added who achieved the same test results as compared to candidate A6. The new decision matrix is shown below.

After evaluating the new set of candidates by RAFSI, TOPSIS, VIKOR, and COPRAS methods with the same criteria weights, it was observed that the rankings and criteria functions of certain alternatives are changed, as shown in Table 3. To compare the results more comprehensively, a parallel presentation of the results is given using RAFSI, TOPSIS, VIKOR, and COPRAS methods on the new and old set of alternatives.

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Table 3. Ranking pre-orders for the old and new set of the alternatives.

	f(Ai)	A1	A2	A3	A4	A 5	A6	A7
RAFSI	Original $f(Ai)$	f(A1) = 0.508	f(A2) = 0.452	f(A3) = 0.438	f(A4) = 0.456	f(A5) = 0.530	f(A6) = 0.437	
	Original rank	2	4	5	3	1	6	
	New $f(Ai)$	f(A1) = 0.508	f(A2) = 0.452	f(A3) = 0.438	f(A4) = 0.456	f(A5) = 0.530	f(A6) = 0.437	f(A7) = 0.495
	New rank	2	5	6	4	1	7	3
VIKOR	Original $f(Ai)$	f(A1) = 0.350	f(A2) = 0.901	f(A3) = 0.924	f(A4) = 0.801	f(A5) = 0.00	f(A6) = 0.928	
	Original rank	2	4	5	3	1	6	
	New $f(Ai)$	f(A1) = 0.274	f(A2) = 0.817	f(A3) = 1.000	f(A4) = 0.738	f(A5) = 0.00	f(A6) = 0.920	f(A7) = 0.718
	New rank	2	5	7	4	1	6	3
TOPSIS	Original $f(Ai)$	f(A1) = 0.542	f(A2) = 0.464	f(A3) = 0.431	f(A4) = 0.396	f(A5) = 0.704	f(A6) = 0.351	
	Original rank	2	3	4	5	1	6	
	New $f(Ai)$	f(A1) = 0.468	f(A2) = 0.400	f(A3) = 0.410	f(A4) = 0.340	f(A5) = 0.593	f(A6) = 0.311	f(A7) = 0.507
	New rank	3	5	4	6	1	7	2
COPRAS	Original $f(Ai)$	f(A1) = 0.964	f(A2) = 0.950	f(A3) = 0.951	f(A4) = 0.932	f(A5) = 1.00	f(A6) = 0.930	
	Original rank	2	4	3	5	1	6	
	New $f(Ai)$	f(A1) = 0.962	f(A2) = 0.952	f(A3) = 0.957	f(A4) = 0.933	f(A5) = 1.00	f(A6) = 0.933	f(A7) = 0.998
	New rank	3	5	4	6	1	7	2

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After analyzing the results of Table 3, we can conclude the following

(1) COPRAS method: The new candidate A7 was ranked second in the ranking order, so it is clear that all the candidates (except the first ranked) moved one place down in the ranking order. Furthermore, it is expected that the values of criteria functions f(Ai) for an old set of the alternatives f(A1), f(A2), ..., f(A6) would not change, which signifies the function of an f(A7) of the new alternative would be ranked based on the old values of f(Ai). However, from Table 3, after introducing the new alternative, a change in f(Ai) values are observed for the COPRAS method. This fact can cause inconsistencies in ranking order of the alternatives.

- (2) TOPSIS method: The introduction of the new alternative resulted in a significant change in the ranking order as well as changes in f(Ai) values that are also observed. Alternative A7 is placed in the second position. Therefore, it is clear that the ranks of the other alternatives moved one place down. However, the same did not happen for alternative A3 as it remained in the fourth position in both new and old sets of alternatives. Additionally, alternative A2 was third in the old set, while, in the new set, it is in the fifth position instead of the fourth. These kinds of changes in alternatives' ranking are observed with changes in f(Ai) values.
- (3) VIKOR method: In this method, similar changes happened as in the previous two methods. The new alternative A7 is placed in the third position. It is expected that, in the new set of alternatives, all the alternatives below the third rank would move one place down. However, some more drastic changes are noticed in the VIKOR method. For example, alternative A3 was in the fifth rank in the old set, but, in the new set, it is ranked last. Moreover, alternative A6 was last in the old set of alternatives, while it is in the second to last in the new set of alternatives. These changes in the ranking order also followed with the changes in *f*(*Ai*) values.
- (4) RAFSI method: This method showed stability in both sets of alternatives. All the alternatives kept the same f(Ai) values in both sets. Thus, it can be concluded that the RAFSI method has shown logical results following the new set of alternatives.

Based on these analyses, we can conclude that rank reversal problems exist in COPRAS, TOPSIS, and VIKOR methods can lead to irrational results in conditions where we have changeable initial parameters in the decision matrix. At the same time, we can conclude that the developed RAFSI method is resistant to rank reversal problems, which contributes to achieving stable and reliable evaluation results while solving complex real-world problems.

5. Discussion and Conclusions

In this paper, a new MADM method, called RAFSI, is suggested, which shows a high level of reliability in results. This makes this method suitable for solving real-time MADM problems in different areas. The mathematical formulation of the RAFSI method does not use traditional data normalization expression. Instead, a new technique for standardization is suggested that enables data transformation from the initial decision matrix into any interval, which makes this method suitable for rational decision making. The mapping of criteria sub-intervals from the initial decision matrix into a unique criteria interval is done by using criteria functions. After forming a unique criteria interval, using arithmetic and harmonic means, the criteria interval is transformed into a normalized criteria interval. This mapping is done depending on the criteria type. Therefore, we can highlight the following contributions of this paper: (1) the development of a new MADM method for solving real problems in the business world, (2) presentation of the new method that is based on coherent defining relations between ideal and anti-ideal criteria values, (3) it eliminates the rank reversal problem and offers reliable results for making rational decisions, (4) development of a new method for the data normalization, which can be used in various areas, from MADM to heuristic algorithms and artificial intelligence-based methods.

The RAFSI method is validated through a comparison of the results with traditional MADM methods and by checking resistance to rank reversal problems. The performance comparison of

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the results of the RAFSI method is done with TOPSIS, COPRAS, and VIKOR methods. These methods are chosen because they use different ways of data normalization like vector, linear normalization, and additive normalization. The goal of comparison with different methods is to confirm the validity of the new method by concerning traditional MADM methods that have already shown high efficiency in solving real-world problems. The performance comparison results showed a very high level of a positive correlation between the results of the RAFSI method and other widely used MCO methods.

After comparing the ranks in the second phase, the validity of resistance of RAFSI, TOPSIS, COPRAS, and VIKOR methods to rank reversal problem is executed. In these experiments, the change in the number of alternatives is simulated. In the first experiment, the number of alternatives is reduced in five scenarios, while, in the second experiment, a set of alternatives is expanded by introducing one non-optimal alternative. The results showed that the RAFSI method is resistant to the rank reversal problem. On the other hand, the conventional TOPSIS, COPRAS, and VIKOR methods did not show satisfying results. The achieved results confirm the validity of RAFSI methods and can be recommended for using in future research for solving different multi-criteria problems.

The goals of future research should be aimed into the direction of using the RAFSI method for other real problems as well as combining with objective and subjective criteria weighting techniques. Furthermore, one of the goals of future research also lies in expanding RAFSI method by using different uncertainty theories. Using uncertainty theories, it would enable the use of linguistic variables for rational expression of human preferences. In addition, the use of new data normalization techniques in heuristic algorithms and other MADM methods can be a future research scope.

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