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Complex Intuitionistic Fuzzy Soft Lattice Ordered Group and Its Weighted Distance Measures

S. Rajareega, J. Vimala *  and D. Preethi

Department of Mathematics, Alagappa University, Karaikudi, Tamil Nadu 630003, India; reega948@gmail.com (S.R.); preethi06061996@gmail.com (D.P.)

* Correspondence: vimaljey@alagappauniversity.ac.in

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Abstract: In recent years, the complex fuzzy set theory has intensified the attention of many researchers. This paper focuses on developing the algebraic structures pertaining to lattice ordered groups and lattice ordered subgroups for complex intuitionistic fuzzy soft set theory. Furthermore, some of their properties and operations are discussed. In addition, the weighted distance measures between two complex intuitionistic fuzzy soft lattice ordered groups such as weighted hamming, weighted normalized hamming, weighted euclidean and weighted normalized euclidean distance measures were introduced and also some of the algebraic properties of the weighted distance measures are verified. Moreover, the application of complex intuitionistic fuzzy soft lattice ordered groups by using the weighted distance measures is analysed.

Keywords: complex fuzzy sets; complex intuitionistic fuzzy sets; complex intuitionistic fuzzy soft sets; complex intuitionistic fuzzy soft lattice ordered group; weighted distance measures

MSC: 06D72

1. Introduction

Vagueness and various uncertainties are characteristics that are pervasive in problems which occurring in engineering, medical science, economics, environments, etc. To exceed these uncertainties and vagueness, some kinds of theories were given like fuzzy set theory [1], intuitionistic fuzzy set theory [2], soft set theory [3] and intuitionistic fuzzy soft theory [4]. Although all of these theories posed a challenge to handle the periodicity or seasonality that to be in many real life problems. This led to Daniel Ramot [5] to present a new innovative concept called complex fuzzy sets.

Complex fuzzy sets are able to handle the problems that are either very difficult or impossible to address with one-dimensional grades of membership. Since then, Kumar T. [6] found an application in multicriteria decision making problem on the basis of the proposed complex intuitionistic fuzzy soft sets and the notion of complex intuitionistic fuzzy soft sets was conferred by Alkouri [7]. Subsequently, Selvachandran G. and Quek S.G. [8] introduced and developed the notion of complex intuitionistic fuzzy soft groups. Distance measure is a numerical measurement between two objects. Moreover distance measure is an important issue on fuzzy sets, soft sets and other hybrid structures. The distance measure and similarity measure for the hybrid structures of fuzzy sets such as fuzzy soft sets [9], intuitionistic fuzzy sets [10], intuitionistic fuzzy soft sets [11], interval-valued complex fuzzy sets [12], complex vague soft sets [13] and complex intuitionistic fuzzy sets [14] were introduced. On the other hand, the lattice

ordered algebraic systems play an important role in algebra. The concept of lattice theory was originated by Birkhoff [15]. Satya Saibaba G.S.V. and Vimala J. [16,17] introduced the study of fuzzy lattice ordered groups in a different manner. The combination of fuzzy sets and lattice ordered structures may provide more new interesting topics [18–20] which intensified the attention of many researchers and decision makers. Here, we introduce and develop the theory of complex intuitionistic fuzzy soft lattice ordered group ($CI FSL\mathcal{G}$). The main contribution of this study includes

1. A concept of lattice ordered algebraic structures of complex intuitionistic fuzzy soft sets is originated.
2. $CI FSL\mathcal{G}$'s pertinent properties are obtained with its operations such as union, intersection, complement, AND.
3. Weighted distance measures between $CI FSL\mathcal{G}$ s, namely the weighted hamming, weighted normalized hamming, weighted euclidean and weighted normalized euclidean distance measures are introduced. It is possible to handle the seasonality and two-dimensional problems with membership and non-membership grades through $CI FSL\mathcal{G}$.
4. The application of weighted distance measures between $CI FSL\mathcal{G}$ s to find the best work stream for employees is analysed.

The remaining sections of this paper are organized as follows. In Section 2, the important concepts are recapitulated and presented. In Section 3, we introduced the notion of $CI FSL\mathcal{G}$ and find the other supporting properties based lattice ordered group structure. Furthermore explore some operations on $CI FSL\mathcal{G}$. In Section 4, the axiomatic definition of the distance function is presented and also the weighted distance measures between $CI FSL\mathcal{G}$ s are introduced. Subsequently, some of the algebraic properties of these weighted distance measures are also utilized. We find an application of $CI FSL\mathcal{G}$ using weighted distance measures on $CI FSL\mathcal{G}$ s.

2. Preliminaries

In this section, we recapitulate some of the important preliminaries pertaining to the development of $CI FSL\mathcal{G}$.

Definition 1 ([1]). The fuzzy set X over a set H is a set $X = \{ \langle h, \mu(h) \rangle \mid h \in H \}$ where $\mu : H \rightarrow [0, 1]$ which is called the membership function of X and $\mu(h)$ is called the membership value of H in X .

Definition 2 ([15]). A Lattice ordered group is a system $G = (G, +, \leq)$ where

- (i) $(G, +)$ is a group
- (ii) $(G, \leq)$ is a lattice
- (iii) $x \leq y \Rightarrow a + x + b \leq a + y + b$, for all $a, b, x, y \in G$.

Definition 3 ([16]). A fuzzy subset λ of a lattice-ordered group G is said to be a fuzzy lattice-ordered subgroup or briefly, fuzzy ℓ -subgroup if

- (i) $\lambda(xy) \geq \lambda(x) \wedge \lambda(y)$
- (ii) $\lambda(x^{-1}) \geq \lambda(x)$
- (iii) $\lambda(x \vee y) \geq \lambda(x) \wedge \lambda(y)$
- (iv) $\lambda(x \wedge y) \geq \lambda(x) \wedge \lambda(y)$, $\forall x, y \in G$.

Definition 4 ([5]). A complex fuzzy set A defined on a universe of discourse U is characterized by a membership function $\mu_A(x)$ that assigns a complex-valued grade of membership in A to any element $x \in U$. By definition,

all values of $\mu_A(x)$ lie within the unit circle in the complex plane and are expressed by $\mu_A(x) = r_A(x)e^{iw_A(x)}$, where $i = \sqrt{-1}$, $r_A(x)$ and $w_A(x)$ are both real-valued, $r_A(x) \in [0, 1]$ and $w_A(x) \in (0, 2\pi]$. A complex fuzzy set A is thus of the form

$$A = \{ \langle x, \mu_A(x) \rangle : x \in U \} = \{ \langle x, r_A(x)e^{iw_A(x)} \rangle : x \in U \}$$

Definition 5 ([7]). A complex intuitionistic fuzzy set S , defined on U , is characterized by membership and non-membership functions $\mu_S(x)$ and $\nu_S(x)$ respectively, that assign any element $x \in U$ a complex-valued grade of both membership and non-membership in S . By definition, the values of $\mu_S(x)$, $\nu_S(x)$, and their sum may receive all lying within the unit circle in the complex plane, and are on the form $\mu_S(x) = r_S(x)e^{iw_{\mu_S}(x)}$ for membership function, $\nu_S(x) = k_S(x)e^{iw_{\nu_S}(x)}$ for non-membership function, where $i = \sqrt{-1}$ each of $r_S(x)$ and $k_S(x)$ are real valued and both belong to the interval $[0, 1]$ such that $0 \leq r_S(x) + k_S(x) \leq 1$ also, $w_{\mu_S}(x)$ and $w_{\nu_S}(x)$ are real valued. We represent the complex intuitionistic fuzzy set S as, $S = \{ \langle x, \mu_S(x) = a, \nu_S(x) = a' \rangle : x \in U \}$, where $\mu_S : U \rightarrow \{a \mid a \in \mathbb{C}, |a| \leq 1\}$, $\nu_S : U \rightarrow \{a' \mid a' \in \mathbb{C}, |a'| \leq 1\}$ and $|\mu_S(x) + \nu_S(x)| \leq 1$.

Definition 6 ([6]). Let E be a set of parameters, $CIFS(U)$ denote the collection of all complex intuitionistic fuzzy sets on U , and $\tilde{F}$ be a function from E to $CIFS(U)$. Then the set of ordered pairs $\{(\varepsilon, \tilde{F}(\varepsilon)) : \varepsilon \in E, \tilde{F}(\varepsilon) \in CIFS(U)\}$, denoted by $(\tilde{F}, E)$, is called a complex intuitionistic fuzzy soft set (CIFSS) on U .

Definition 7 ([6]). Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2)$ be two CIFSSs over the X . Define $R = E_1 \cup E_2$ and $S = E_1 \cap E_2$

$$\tilde{H}(a) = \begin{cases} \tilde{F}_1(a), & a \in E_1 - S \\ \tilde{F}_2(a), & a \in E_2 - S \\ \tilde{F}_1(a) \cup \tilde{F}_2(a), & a \in S \end{cases}, \quad \tilde{K}(a) = \tilde{F}_1(a) \cap \tilde{F}_2(a), \forall a \in S$$

Then

- (i) $(\tilde{H}, R)$ is called the union of $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2)$ and is denoted as $(\tilde{H}, R) = (\tilde{F}_1, E_1) \cup (\tilde{F}_2, E_2)$
- (ii) $(\tilde{K}, S)$ is called the intersection of $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2)$ and is denoted as $(\tilde{K}, S) = (\tilde{F}_1, E_1) \cap (\tilde{F}_2, E_2)$
- (iii) The complement of $(\tilde{F}_1, E_1)$ is denoted as $(\tilde{F}_1, E_1)^c = (\tilde{F}_1^c, \neg E_1)$, where $\tilde{F}_1^c : \neg E \rightarrow CIFS(U)$ is mapping given by

$$\tilde{F}_1^c(\neg a) = \{ \langle x, \nu_{\tilde{F}_1(a)}(x), \mu_{\tilde{F}_1(a)}(x) \rangle \mid x \in X \}, \forall \neg a \in \neg E_1$$

Definition 8 ([6]). Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2)$ be two CIFSSs over the X . Then we say that $(\tilde{F}_1, E_1) \subseteq (\tilde{F}_2, E_2)$, if

- (i) $E_1 \subseteq E_2$
- (ii) $\tilde{F}_1(a_1) \subseteq \tilde{F}_2(a_1)$, that is $\mu_{\tilde{F}_1(a_1)}(x) \leq \mu_{\tilde{F}_2(a_1)}(x)$ and $\nu_{\tilde{F}_2(a_1)}(x) \leq \nu_{\tilde{F}_1(a_1)}(x)$, $\forall x \in X, a_1 \in E_1$

3. Complex Intuitionistic Fuzzy Soft Lattice Ordered Group

In this section, we introduce the notion of Complex Intuitionistic Fuzzy Soft Lattice Ordered Group. Throughout this paper $G = (G, \cdot, \vee, \wedge)$ denotes the lattice ordered group, where $\vee$ and $\wedge$ are the maximum and minimum operator respectively and also

- (i) $\mu \geq \nu$, if both $r \geq \tau$ and $w \geq \psi$ and
- (ii) $\mu \leq \nu$, if both $r \leq \tau$ and $w \leq \psi$, where $\mu = re^{iw}$ and $\nu = \tau e^{i\psi}$, with $r, \tau \in [0, 1]$ and $w, \psi \in (0, 2\pi]$ [8].

Definition 9. Let G be a lattice ordered group and Let $\mathcal{N} = \{ \langle x, \mu_{\mathcal{N}}(x), \nu_{\mathcal{N}}(x) \rangle \mid x \in G \}$ be a complex intuitionistic fuzzy set on G . Then $\mathcal{N}$ is said to be a complex intuitionistic fuzzy lattice ordered subgroup (CIFL-Subgroup) of G , if the following conditions holds for all $x, y \in G$:

- (i) $\mu_{\mathcal{N}}(xy) \geq \mu_{\mathcal{N}}(x) \wedge \mu_{\mathcal{N}}(y)$
- (ii) $\nu_{\mathcal{N}}(xy) \leq \nu_{\mathcal{N}}(x) \vee \nu_{\mathcal{N}}(y)$
- (iii) $\mu_{\mathcal{N}}(x^{-1}) \geq \mu_{\mathcal{N}}(x)$
- (iv) $\nu_{\mathcal{N}}(x^{-1}) \leq \nu_{\mathcal{N}}(x)$
- (v) $\mu_{\mathcal{N}}(x \vee y) \geq \mu_{\mathcal{N}}(x) \wedge \mu_{\mathcal{N}}(y)$
- (vi) $\mu_{\mathcal{N}}(x \wedge y) \geq \mu_{\mathcal{N}}(x) \wedge \mu_{\mathcal{N}}(y)$
- (vii) $\nu_{\mathcal{N}}(x \vee y) \leq \nu_{\mathcal{N}}(x) \vee \nu_{\mathcal{N}}(y)$
- (viii) $\nu_{\mathcal{N}}(x \wedge y) \leq \nu_{\mathcal{N}}(x) \vee \nu_{\mathcal{N}}(y)$.

Definition 10. Let $(\tilde{F}, E) \in \text{CIFSS}(G)$. Then $(\tilde{F}, E)$ is said to be a complex intuitionistic fuzzy soft lattice ordered group (CIFSSL- $\mathcal{G}$) on G if $\tilde{F}(a)$ is a complex intuitionistic fuzzy lattice ordered subgroup of G , for all $a \in \rho(\tilde{F}, E)$, Where $\rho(\tilde{F}, E) = \{a \in E : \tilde{F}(a) \text{ is non null}\}$ is the support set of $(\tilde{F}, E)$.

Example 1. Consider the ℓ -group $G = (\mathbb{Z}, +, \wedge, \vee)$ and the parameters $E = \{a, b\}$. Next Consider the two CIFSS of G , which are defined below.

- (i) $(\tilde{F}, E) = \{ \langle x, \mu_{\tilde{F}(a)}(x), \nu_{\tilde{F}(a)}(x) \rangle, \langle x, \mu_{\tilde{F}(b)}(x), \nu_{\tilde{F}(b)}(x) \rangle : x \in G \}$, where

$$\mu_{\tilde{F}(a)}(x) = \begin{cases} 0.4e^{i\frac{\pi}{2}}, & \text{if } x = 2k; k \in \mathbb{Z}/\{0\} \\ 0.7e^{i\frac{4\pi}{3}}, & x = 0 \\ 0.2e^{i\frac{\pi}{3}}, & \text{otherwise} \end{cases}, \quad \nu_{\tilde{F}(a)}(x) = \begin{cases} 0.6e^{i\pi}, & \text{if } x = 2k; k \in \mathbb{Z}/\{0\} \\ 0.3e^{i\frac{\pi}{2}}, & x = 0 \\ 0.8e^{i\frac{3\pi}{2}}, & \text{otherwise} \end{cases} \quad \text{and}$$

$$\mu_{\tilde{F}(b)}(x) = \begin{cases} 0.5e^{i\pi}, & \text{if } x = 2k; k \in \mathbb{Z}/\{0\} \\ 0.8e^{i\frac{3\pi}{2}}, & \text{if } x = 0 \\ 0e^{i\frac{\pi}{2}}, & \text{otherwise} \end{cases}, \quad \nu_{\tilde{F}(b)}(x) = \begin{cases} 0.5e^{i\pi}, & \text{if } x = 2k; k \in \mathbb{Z}/\{0\} \\ 0.2e^{i\frac{\pi}{2}}, & \text{if } x = 0 \\ 1e^{i\pi}, & \text{otherwise} \end{cases}$$

- (ii) $(\tilde{G}, E) = \{ \langle x, \mu_{\tilde{G}(a)}(x), \nu_{\tilde{G}(a)}(x) \rangle, \langle x, \mu_{\tilde{G}(b)}(x), \nu_{\tilde{G}(b)}(x) \rangle : x \in G \}$,
where

$$\mu_{\tilde{G}(a)}(x) = \begin{cases} 0.5e^{i\pi}, & \text{if } x = 0 \\ 1e^{i\frac{\pi}{2}}, & \text{otherwise} \end{cases}, \quad \nu_{\tilde{G}(a)}(x) = \begin{cases} 0.5e^{i\frac{\pi}{2}}, & \text{if } x = 0 \\ 0e^{i\pi}, & \text{otherwise} \end{cases}$$

and $\tilde{G}(b) = \tilde{F}(b)$.

From the above $(\tilde{F}, E) \in \text{CIFSSL-}\mathcal{G}(G)$, Whereas $(\tilde{G}, E) \notin \text{CIFSSL-}\mathcal{G}(G)$.

Proposition 1. Let $(\tilde{F}, E) \in \text{CIFSS}(G)$ and e_G be the identity element of G . Then $(\tilde{F}, E) \in \text{CIFSSL-}\mathcal{G}(G)$ if and only if the following conditions are satisfied, for all $a \in E$ and $x, y, e_G \in G$,

- (i) $\mu_{\tilde{F}(a)}(xy^{-1}) \geq \mu_{\tilde{F}(a)}(x) \wedge \mu_{\tilde{F}(a)}(y)$
- (ii) $\nu_{\tilde{F}(a)}(xy^{-1}) \leq \nu_{\tilde{F}(a)}(x) \vee \nu_{\tilde{F}(a)}(y)$
- (iii) $\mu_{\tilde{F}(a)}(x \vee e_G) \geq \mu_{\tilde{F}(a)}(x)$
- (iv) $\nu_{\tilde{F}(a)}(x \vee e_G) \leq \nu_{\tilde{F}(a)}(x)$.

Proposition 2. Let $(\tilde{F}, E) \in CIFSS(G)$. Then for all $a \in E$ and $x, y \in G$ the following are equivalent,

- (i) $\mu_{\tilde{F}(a)}(x) \geq \mu_{\tilde{F}(a)}(y)$ and $\nu_{\tilde{F}(a)}(x) \leq \nu_{\tilde{F}(a)}(y)$ whenever $x \leq y$
- (ii) $\mu_{\tilde{F}(a)}(x \wedge y) \geq \mu_{\tilde{F}(a)}(x) \vee \mu_{\tilde{F}(a)}(y)$ and $\nu_{\tilde{F}(a)}(x \wedge y) \leq \nu_{\tilde{F}(a)}(x) \wedge \nu_{\tilde{F}(a)}(y)$
- (iii) $\mu_{\tilde{F}(a)}(x \vee y) \leq \mu_{\tilde{F}(a)}(x) \wedge \mu_{\tilde{F}(a)}(y)$ and $\nu_{\tilde{F}(a)}(x \vee y) \geq \nu_{\tilde{F}(a)}(x) \vee \nu_{\tilde{F}(a)}(y)$

Proof. (i) $\Leftrightarrow$ (ii) For $x, y \in G$,

$$x \wedge y \leq x \Rightarrow \mu_{\tilde{F}(a)}(x \wedge y) \geq \mu_{\tilde{F}(a)}(x) \text{ and } \nu_{\tilde{F}(a)}(x \wedge y) \leq \nu_{\tilde{F}(a)}(x)$$

$$x \wedge y \leq y \Rightarrow \mu_{\tilde{F}(a)}(x \wedge y) \geq \mu_{\tilde{F}(a)}(y) \text{ and } \nu_{\tilde{F}(a)}(x \wedge y) \leq \nu_{\tilde{F}(a)}(y)$$

$$\text{Hence } \mu_{\tilde{F}(a)}(x \wedge y) \geq \mu_{\tilde{F}(a)}(x) \vee \mu_{\tilde{F}(a)}(y) \text{ and } \nu_{\tilde{F}(a)}(x \wedge y) \leq \nu_{\tilde{F}(a)}(x) \wedge \nu_{\tilde{F}(a)}(y)$$

On the other hand if $x \leq y$, then $x \wedge y = x$

$$\mu_{\tilde{F}(a)}(x) = \mu_{\tilde{F}(a)}(x \wedge y) \geq \mu_{\tilde{F}(a)}(x) \vee \mu_{\tilde{F}(a)}(y) \geq \mu_{\tilde{F}(a)}(y)$$

$$\nu_{\tilde{F}(a)}(x) = \nu_{\tilde{F}(a)}(x \wedge y) \leq \nu_{\tilde{F}(a)}(x) \wedge \nu_{\tilde{F}(a)}(y) \leq \nu_{\tilde{F}(a)}(y)$$

$$\text{Hence } \mu_{\tilde{F}(a)}(x) \geq \mu_{\tilde{F}(a)}(y) \text{ and } \nu_{\tilde{F}(a)}(x) \leq \nu_{\tilde{F}(a)}(y)$$

(i) $\Leftrightarrow$ (iii) For $x, y \in G$,

$$x \vee y \geq x \Rightarrow \mu_{\tilde{F}(a)}(x) \geq \mu_{\tilde{F}(a)}(x \vee y) \text{ and } \nu_{\tilde{F}(a)}(x \vee y) \geq \nu_{\tilde{F}(a)}(x)$$

$$x \vee y \geq y \Rightarrow \mu_{\tilde{F}(a)}(y) \geq \mu_{\tilde{F}(a)}(x \vee y) \text{ and } \nu_{\tilde{F}(a)}(x \vee y) \geq \nu_{\tilde{F}(a)}(y) \text{ and}$$

$$\text{Hence } \mu_{\tilde{F}(a)}(x) \wedge \mu_{\tilde{F}(a)}(y) \geq \mu_{\tilde{F}(a)}(x \vee y) \text{ and } \nu_{\tilde{F}(a)}(x) \vee \nu_{\tilde{F}(a)}(y) \leq \nu_{\tilde{F}(a)}(x \vee y)$$

On the other hand if $x \leq y$, then $x \vee y = y$

$$\mu_{\tilde{F}(a)}(y) = \mu_{\tilde{F}(a)}(x \vee y) \leq \mu_{\tilde{F}(a)}(x) \wedge \mu_{\tilde{F}(a)}(y) \leq \mu_{\tilde{F}(a)}(x)$$

$$\nu_{\tilde{F}(a)}(y) = \nu_{\tilde{F}(a)}(x \vee y) \geq \nu_{\tilde{F}(a)}(x) \vee \nu_{\tilde{F}(a)}(y) \geq \nu_{\tilde{F}(a)}(x)$$

$$\text{Hence } \mu_{\tilde{F}(a)}(y) \leq \mu_{\tilde{F}(a)}(x) \text{ and } \nu_{\tilde{F}(a)}(y) \geq \nu_{\tilde{F}(a)}(x)$$

□

Proposition 3. Let $(\tilde{F}, E) \in CIFSS(G)$. Then for all $a \in E$ and $x, y \in G$ the following are equivalent,

- (i) $\mu_{\tilde{F}(a)}(x) \leq \mu_{\tilde{F}(a)}(y)$ and $\nu_{\tilde{F}(a)}(x) \geq \nu_{\tilde{F}(a)}(y)$ whenever $x \leq y$
- (ii) $\mu_{\tilde{F}(a)}(x \wedge y) \leq \mu_{\tilde{F}(a)}(x) \wedge \mu_{\tilde{F}(a)}(y)$ and $\nu_{\tilde{F}(a)}(x \wedge y) \geq \nu_{\tilde{F}(a)}(x) \vee \nu_{\tilde{F}(a)}(y)$
- (iii) $\mu_{\tilde{F}(a)}(x \vee y) \geq \mu_{\tilde{F}(a)}(x) \vee \mu_{\tilde{F}(a)}(y)$ and $\nu_{\tilde{F}(a)}(x \vee y) \leq \nu_{\tilde{F}(a)}(x) \wedge \nu_{\tilde{F}(a)}(y)$

Proof. The proof is similar to Proposition 2. □

Proposition 4. Let $(\tilde{F}, E) \in CILFSLG(G)$. Then for all $x \in G, a \in E$

- (i) $\mu_{\tilde{F}(a)}(x^+) \geq \mu_{\tilde{F}(a)}(x)$ and $\nu_{\tilde{F}(a)}(x^+) \leq \nu_{\tilde{F}(a)}(x)$
- (ii) $\mu_{\tilde{F}(a)}(x^-) \leq \mu_{\tilde{F}(a)}(x)$ and $\nu_{\tilde{F}(a)}(x^-) \geq \nu_{\tilde{F}(a)}(x)$
- (iii) $\mu_{\tilde{F}(a)}(|x|) \geq \mu_{\tilde{F}(a)}(x)$ and $\nu_{\tilde{F}(a)}(|x|) \leq \nu_{\tilde{F}(a)}(x)$
- (iv) $\mu_{\tilde{F}(a)}((x^{-1})^+) \geq \mu_{\tilde{F}(a)}(x)$ and $\nu_{\tilde{F}(a)}((x^{-1})^+) \leq \nu_{\tilde{F}(a)}(x)$

Proof. Since G is lattice ordered group. For all $x, e_G \in G$, $x^+ = x \vee e_G$, $x^- = x \wedge e_G$, $|x| = x \vee x^{-1}$ and $(x^{-1})^+ = (x^-)^{-1}$.

From the above, result is obvious. $\square$

Proposition 5. Let $(\tilde{F}, E) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$ and e_G be the identity element of G . Then the following results holds for all $x \in G$, $a \in E$

- (i) $\mu_{\tilde{F}(a)}(x^{-1}) = \mu_{\tilde{F}(a)}(x)$ and $\nu_{\tilde{F}(a)}(x^{-1}) = \nu_{\tilde{F}(a)}(x)$
- (ii) $\mu_{\tilde{F}(a)}(e_G) \geq \mu_{\tilde{F}(a)}(x)$ and $\nu_{\tilde{F}(a)}(e_G) \leq \nu_{\tilde{F}(a)}(x)$.

Theorem 1. Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$. If $E_1 \cap E_2 = \emptyset$, then $(\tilde{F}_1, E_1) \tilde{\cup} (\tilde{F}_2, E_2) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$.

Proof. By Definition 7 $(\tilde{F}_1, E_1) \tilde{\cup} (\tilde{F}_2, E_2) = (\tilde{H}, R)$, where $R = E_1 \cup E_2$.

Since $E_1 \cap E_2 = \emptyset$, then $a \in E_1 - S$ or $a \in E_2 - S$ where $S = E_1 \cap E_2$ and for all $a \in R$.

Case: 1 If $a \in E_1 - S$ then $\tilde{H}(a) = \tilde{F}_1(a)$ is $\mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$

Case: 2 If $a \in E_2 - S$ then $\tilde{H}(a) = \tilde{F}_2(a)$ is $\mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$.

Thus $(\tilde{F}_1, E_1) \tilde{\cup} (\tilde{F}_2, E_2) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$. $\square$

Theorem 2. Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$. Then $(\tilde{F}_1, E_1) \tilde{\cap} (\tilde{F}_2, E_2) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$.

Proof. By Definition 7, $(\tilde{F}_1, E_1) \tilde{\cap} (\tilde{F}_2, E_2) = (\tilde{K}, S)$ where $S = E_1 \cap E_2$ and $\tilde{K}(a) = \tilde{F}_1(a) \cap \tilde{F}_2(a)$, for all $a \in E_1 \cap E_2$.

To prove: $\tilde{F}_1(a) \cap \tilde{F}_2(a) = \{x \mid \mu_{\tilde{F}_1(a)}(x) \wedge \mu_{\tilde{F}_2(a)}(x), \nu_{\tilde{F}_1(a)}(x) \vee \nu_{\tilde{F}_2(a)}(x) \mid x \in G\}$, is $\mathcal{CIFS\mathcal{L}\mathcal{G}}$ -Subgroup.

For $x, y, e_G \in G$ and $a \in E_1 \cap E_2$

$$\begin{aligned}
 (i) \quad \mu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(x \vee e_G) &= [r_{\tilde{F}_1(a)}(x \vee e_G) \wedge r_{\tilde{F}_2(a)}(x \vee e_G)] e^{i[w_{\tilde{F}_1(a)}(x \vee e_G) \wedge w_{\tilde{F}_2(a)}(x \vee e_G)]} \\
 &\geq [(r_{\tilde{F}_1(a)}(x) \wedge r_{\tilde{F}_1(a)}(e_G)) \wedge (r_{\tilde{F}_2(a)}(x) \wedge r_{\tilde{F}_2(a)}(e_G))] \\
 &\quad e^{i[(w_{\tilde{F}_1(a)}(x) \wedge w_{\tilde{F}_1(a)}(e_G)) \wedge (w_{\tilde{F}_2(a)}(x) \wedge w_{\tilde{F}_2(a)}(e_G))]} \\
 &\geq [r_{\tilde{F}_1(a)}(x) \wedge r_{\tilde{F}_2(a)}(x)] e^{i[w_{\tilde{F}_1(a)}(x) \wedge w_{\tilde{F}_2(a)}(x)]} \text{ [By Proposition 5 (ii)]} \\
 &= \mu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(x). \\
 (ii) \quad \nu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(x \vee e_G) &= [\tau_{\tilde{F}_1(a)}(x \vee e_G) \vee \tau_{\tilde{F}_2(a)}(x \vee e_G)] e^{i[\psi_{\tilde{F}_1(a)}(x \vee e_G) \vee \psi_{\tilde{F}_2(a)}(x \vee e_G)]} \\
 &\geq [(\tau_{\tilde{F}_1(a)}(x) \vee \tau_{\tilde{F}_1(a)}(e_G)) \vee (\tau_{\tilde{F}_2(a)}(x) \vee \tau_{\tilde{F}_2(a)}(e_G))] \\
 &\quad e^{i[(\psi_{\tilde{F}_1(a)}(x) \vee \psi_{\tilde{F}_1(a)}(e_G)) \vee (\psi_{\tilde{F}_2(a)}(x) \vee \psi_{\tilde{F}_2(a)}(e_G))]} \\
 &\geq [\tau_{\tilde{F}_1(a)}(x) \vee \tau_{\tilde{F}_2(a)}(x)] e^{i[\psi_{\tilde{F}_1(a)}(x) \vee \psi_{\tilde{F}_2(a)}(x)]} \text{ [By Proposition 5 (ii)]} \\
 &= \nu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(x).
 \end{aligned}$$

Similarly, we can prove

$$(iii) \quad \mu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(xy^{-1}) \geq \mu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(x) \wedge \mu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(y) \text{ and}$$

$$(iv) \quad \nu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(xy^{-1}) \leq \nu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(x) \vee \nu_{\tilde{F}_1(a) \cap \tilde{F}_2(a)}(y)$$

Thus, by Proposition 1 $(\tilde{F}_1, E_1) \tilde{\cap} (\tilde{F}_2, E_2) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$. $\square$

Theorem 3. Let $(\tilde{F}, E) \in \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$. Then $(\tilde{F}, E)^c \notin \mathcal{CIFS\mathcal{L}\mathcal{G}}(G)$.

Proof. By Definition 7

$(\tilde{F}, E)^c = (\tilde{F}^c, \neg E)$, where $\tilde{F}^c : \neg E \rightarrow CIFS(U)$ is mapping given by $\tilde{F}^c(\neg a) = \{ < x, v_{\tilde{F}(a)}(x), \mu_{\tilde{F}(a)}(x) > | x \in X \}$, $\forall \neg a \in \neg E$

$$\begin{aligned} v_{\tilde{F}(a)}(x \wedge y) &= \tau_{\tilde{F}(a)}(x \wedge y) e^{i\psi_{\tilde{F}(a)}(x \wedge y)} \\ &= (1 - r_{\tilde{F}(a)}(x \wedge y)) e^{i(2\pi - w_{\tilde{F}(a)}(x \wedge y))} \\ &\geq \tau_{\tilde{F}(a)}(x) \vee \tau_{\tilde{F}(a)}(y) e^{i(\psi_{\tilde{F}(a)}(x) \vee \psi_{\tilde{F}(a)}(y))} = v_{\tilde{F}(a)}(x) \vee v_{\tilde{F}(a)}(y) \end{aligned}$$

$\therefore (\tilde{F}, E)^c \notin CIFS\mathcal{L}\mathcal{G}(G)$ $\square$

Theorem 4. Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2) \in CIFS\mathcal{L}\mathcal{G}(G)$. Then $(\tilde{F}_1, E_1) \wedge (\tilde{F}_2, E_2) \in CIFS\mathcal{L}\mathcal{G}(G)$.

Proof. Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2) \in CIFS\mathcal{L}\mathcal{G}$ and let $(\tilde{F}_1, E_1) \wedge (\tilde{F}_2, E_2) = (\tilde{G}, C)$ where $C = E_1 \times E_2 \vee (a, b) \in E_1 \times E_2$ and $\tilde{G}(a, b) = \tilde{F}_1(a) \cap \tilde{F}_2(b)$.

We know that $\tilde{F}_1(a)$, $\forall a \in E_1$ and $\tilde{F}_2(b)$, $\forall b \in E_2$ are $CIF\mathcal{L}$ -Subgroup(G) and then $\tilde{G}(a, b) \in CIFS\mathcal{L}\mathcal{G}(G)$, $\forall (a, b) \in E_1 \times E_2$, since by Theorem 2.

Then $(\tilde{F}_1, E_1) \wedge (\tilde{F}_2, E_2) \in CIFS\mathcal{L}\mathcal{G}(G)$. $\square$

Definition 11. Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2) \in CIFS\mathcal{L}\mathcal{G}(G)$. Then we say that

(i) $(\tilde{F}_1, E_1)$ is a complex intuitionistic fuzzy soft l -subgroup of $(\tilde{F}_2, E_2)$ and write $(\tilde{F}_1, E_1) \subseteq (\tilde{F}_2, E_2)$ if the following conditions are satisfied:

- (a) $E_1 \subseteq E_2$
- (b) For all $a \in E_1$, $\tilde{F}_1(a)$ is a complex intuitionistic fuzzy l -subgroup of $\tilde{F}_2(a)$

(ii) $(\tilde{F}_1, E_1) = (\tilde{F}_2, E_2)$ if and only if $(\tilde{F}_1, E_1) \subseteq (\tilde{F}_2, E_2)$ and $(\tilde{F}_1, E_1) \supseteq (\tilde{F}_2, E_2)$

Theorem 5. Let $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2) \in CIFS\mathcal{L}\mathcal{G}(G)$. Then the following holds,

- (i) If $\tilde{F}_2(a) \subset \tilde{F}_1(a)$, for all $a \in E_2 \subseteq E_1$, then $(\tilde{F}_2, E_2) \subseteq (\tilde{F}_1, E_1)$
- (ii) $(\tilde{F}_1, E_1) \cap (\tilde{F}_2, E_2)$ is a complex intuitionistic fuzzy soft l -subgroup of both $(\tilde{F}_1, E_1)$ and $(\tilde{F}_2, E_2)$ if it is non-null

Proof.

- (i) Straight forward from Definition 11.
- (ii) Let $(\tilde{F}_1, E_1) \cap (\tilde{F}_2, E_2) = (\tilde{K}, C)$, where $C = E_1 \cap E_2$.

Since $E_1 \cap E_2 \subseteq E_1$ and by Theorem 2 $\tilde{K}(a) = \tilde{F}_1(a) \cap \tilde{F}_2(a) \in CIFS\mathcal{L}\mathcal{G}(G)$, for every $a \in E_1 \cap E_2$. Therefore $(\tilde{K}, C) \subseteq (\tilde{F}_1, E_1)$.

Similarly, We see that $(\tilde{K}, C) \subseteq (\tilde{F}_2, E_2)$ $\square$

4. Weighted Distance Measures between $CIFS\mathcal{L}\mathcal{G}$ s

In this section, we introduce the distance function and several weighted distance measures on $CIFS\mathcal{L}\mathcal{G}$. And also we present an application of weighted distance measures on $CIFS\mathcal{L}\mathcal{G}$ s.

Definition 12. Let $(\tilde{F}, A), (\tilde{G}, B), (\tilde{H}, C) \in CIFS\mathcal{L}\mathcal{G}(G)$. A real valued non-negative function $d : CIFS\mathcal{L}\mathcal{G}(G) \times CIFS\mathcal{L}\mathcal{G}(G) \rightarrow [0, 1]$ is the distance function between $CIFS\mathcal{L}\mathcal{G}(G)$ s if

- (i) $0 \leq d((\tilde{F}, A), (\tilde{G}, B)) \leq 1$
- (ii) $d((\tilde{F}, A), (\tilde{G}, B)) = d((\tilde{G}, B), (\tilde{F}, A))$
- (iii) $d((\tilde{F}, E), (\tilde{G}, B)) = 0$ iff $(\tilde{F}, A) = (\tilde{G}, B)$
- (iv) If $(\tilde{F}, A) \subseteq (\tilde{G}, B) \subseteq (\tilde{H}, C)$, then $d((\tilde{F}, A), (\tilde{H}, C)) \geq \max\{d((\tilde{F}, A), (\tilde{G}, B)), d((\tilde{G}, B), (\tilde{H}, C))\}$

Definition 13. Let $G = \{x_1, \dots, x_n\}$ be a ℓ -group and let $(\tilde{F}, A), (\tilde{G}, B) \in \mathcal{CIFSLG}$ over G and d a distance measure between $\mathcal{CIFSLG}(G)$. We define some weighted distance measures between $\mathcal{CIFSLG}$ s which are as given below,

(i) The Weighted Hamming Distance:

$$d_w^H((\tilde{F}, A), (\tilde{G}, B)) = \frac{1}{2 \sum_{i=1}^n w_i} \sum_{i=1}^n \sum_{j=1}^m w_i \left[\alpha_1 |r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{G}(b_j)}(x_i)| + \beta_1 |\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{G}(b_j)}(x_i)| \right. \\ \left. + \sigma_1 \max\{|r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{G}(b_j)}(x_i)|, |\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{G}(b_j)}(x_i)|\} \right. \\ \left. + \frac{1}{2\pi} (\alpha_2 |w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{G}(b_j)}(x_i)| + \beta_2 |\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{G}(b_j)}(x_i)| \right. \\ \left. + \sigma_2 \max\{|w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{G}(b_j)}(x_i)|, |\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{G}(b_j)}(x_i)|\}) \right]$$

(ii) The Weighted Normalized Hamming Distance:

$$d_w^{NH}((\tilde{F}, A), (\tilde{G}, B)) = \frac{d_w^H((\tilde{F}, A), (\tilde{G}, B))}{mn}$$

(iii) The Weighted Euclidean Distance Measure:

$$d_w^E((\tilde{F}, A), (\tilde{G}, B)) = \left[\frac{1}{2 \sum_{i=1}^n w_i} \sum_{i=1}^n \sum_{j=1}^m w_i \left[\alpha_1 (r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{G}(b_j)}(x_i))^2 + \beta_1 (\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{G}(b_j)}(x_i))^2 \right. \right. \\ \left. + \sigma_1 \max\{(r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{G}(b_j)}(x_i))^2, (\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{G}(b_j)}(x_i))^2\} \right. \\ \left. + \frac{1}{2\pi} (\alpha_2 (w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{G}(b_j)}(x_i))^2 + \beta_2 (\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{G}(b_j)}(x_i))^2 \right. \\ \left. + \sigma_2 \max\{(w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{G}(b_j)}(x_i))^2, (\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{G}(b_j)}(x_i))^2\}) \right]^{1/2}$$

(iv) The Weighted Normalized Euclidean Distance:

$$d_w^{NE}((\tilde{F}, A), (\tilde{G}, B)) = \frac{d_w^E((\tilde{F}, A), (\tilde{G}, B))}{\sqrt{mn}}$$

where $\alpha_1, \beta_1, \alpha_2, \beta_2, \sigma_1, \sigma_2 \in [0, 1]$, $\alpha_1 + \beta_1 + \sigma_1 = 1$ and $\alpha_2 + \beta_2 + \sigma_2 = 1$, $w_i \in [0, 1], \forall i = 1, 2, \dots, n$ and $a_j \in A, b_j \in B, \forall j = 1, 2, \dots, m$

Proposition 6. All the weighted distance measures defined in Definition 13 are distance function between $\mathcal{CIFSLG}$ s.

Proof. It is obvious that all the weighted distance measures given in Definition 13 satisfy conditions (i)–(iii) of Definition 12. We have to prove the condition (iv) for the weighted Hamming distance.

By Using Definition 8

$$\begin{aligned} 0 &\leq r_{\tilde{F}(a_j)}(x_i) \leq r_{\tilde{G}(b_j)}(x_i) \leq r_{\tilde{H}(c_j)}(x_i) \leq 1 \text{ and} \\ 0 &\leq \tau_{\tilde{H}(c_j)}(x_i) \leq \tau_{\tilde{G}(b_j)}(x_i) \leq \tau_{\tilde{F}(a_j)}(x_i) \leq 1 \\ 0 &\leq w_{\tilde{F}(a_j)}(x_i) \leq w_{\tilde{G}(b_j)}(x_i) \leq w_{\tilde{H}(c_j)}(x_i) \leq 2\pi \text{ and} \\ 0 &\leq \psi_{\tilde{H}(c_j)}(x_i) \leq \psi_{\tilde{G}(b_j)}(x_i) \leq \psi_{\tilde{F}(a_j)}(x_i) \leq 2\pi \end{aligned}$$

$$\begin{aligned} d_w^H((\tilde{F}, A), (\tilde{G}, B)) &= \frac{1}{2 \sum_{i=1}^n w_i} \sum_{i=1}^n \sum_{j=1}^m w_i \left[\alpha_1 |r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{G}(b_j)}(x_i)| + \beta_1 |\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{G}(b_j)}(x_i)| \right. \\ &\quad \left. + \sigma_1 \max\{|r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{G}(b_j)}(x_i)|, |\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{G}(b_j)}(x_i)|\} \right. \\ &\quad \left. + \frac{1}{2\pi} (\alpha_2 |w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{G}(b_j)}(x_i)| + \beta_2 |\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{G}(b_j)}(x_i)| \right. \\ &\quad \left. + \sigma_2 \max\{|w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{G}(b_j)}(x_i)|, |\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{G}(b_j)}(x_i)|\} \right] \\ &\leq \frac{1}{2 \sum_{i=1}^n w_i} \sum_{i=1}^n \sum_{j=1}^m w_i \left[\alpha_1 |r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{H}(c_j)}(x_i)| + \beta_1 |\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{H}(c_j)}(x_i)| \right. \\ &\quad \left. + \sigma_1 \max\{|r_{\tilde{F}(a_j)}(x_i) - r_{\tilde{H}(c_j)}(x_i)|, |\tau_{\tilde{F}(a_j)}(x_i) - \tau_{\tilde{H}(c_j)}(x_i)|\} \right. \\ &\quad \left. + \frac{1}{2\pi} (\alpha_2 |w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{H}(c_j)}(x_i)| + \beta_2 |\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{H}(c_j)}(x_i)| \right. \\ &\quad \left. + \sigma_2 \max\{|w_{\tilde{F}(a_j)}(x_i) - w_{\tilde{H}(c_j)}(x_i)|, |\psi_{\tilde{F}(a_j)}(x_i) - \psi_{\tilde{H}(c_j)}(x_i)|\} \right] \\ &= d_w^H((\tilde{F}, A), (\tilde{H}, C)) \end{aligned}$$

$$\therefore d_w^H((\tilde{F}, A), (\tilde{G}, B)) \leq d_w^H((\tilde{F}, A), (\tilde{H}, C))$$

$$\text{Similarly, we can get } d_w^H((\tilde{G}, B), (\tilde{H}, C)) \leq d_w^H((\tilde{F}, A), (\tilde{H}, C))$$

$$\text{Then } d_w^H((\tilde{F}, A), (\tilde{H}, C)) \geq \max\{d_w^H((\tilde{F}, A), (\tilde{G}, B)), d_w^H((\tilde{G}, B), (\tilde{H}, C))\} \quad \square$$

Proposition 7. Let d be a weighted distance measure between $\mathcal{CIFSLG}$ s. Then for any $(\tilde{F}, E), (\tilde{G}, B), (\tilde{H}, C) \in \mathcal{CIFSLG}$

1. $d((\tilde{F}, A), (\tilde{G}, B)) = d((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B), (\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))$
2. $d((\tilde{F}, A), (\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)) = d((\tilde{G}, B), (\tilde{F}, A) \tilde{\cup} (\tilde{G}, B))$
3. $d((\tilde{F}, A), (\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) = d((\tilde{G}, B), (\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))$
4. $d((\tilde{F}, A), (\tilde{G}, B) \tilde{\cup} (\tilde{H}, C)) \leq d((\tilde{F}, A), (\tilde{G}, B)) + d((\tilde{F}, A), (\tilde{H}, C))$
5. $d((\tilde{F}, A), (\tilde{G}, B) \tilde{\cap} (\tilde{H}, C)) \leq d((\tilde{F}, A), (\tilde{G}, B)) + d((\tilde{F}, A), (\tilde{H}, C))$

4.1. Application

Suppose the team of analysts decides to analyse the best work stream for employees based on the employees own experience. The following work streams are taken into consideration

Let us define the binary operation ‘ $*$ ’ by $x_1 * x_2$ which means the ‘need’ of x_1 and x_2 . In Table 1, x_2 is the identity element and all the attributes have their own inverse.

Table 1. Binary operation output. x_1 = Information Technology, x_2 = Professional Consulting, x_3 = Financial Services, x_4 = Health Care.

*	x_1	x_2	x_3	x_4
x_1	x_2	x_1	x_4	x_3
x_2	x_1	x_2	x_3	x_4
x_3	x_4	x_3	x_2	x_1
x_4	x_3	x_4	x_1	x_2

Let us take $G = \{x_1, x_2, x_3, x_4\}$. Here, the lattice structure comes under the team's personal preference to the work stream in the decision as well as indicate their equal degree of importance defined by $x_i \succ x_j \Rightarrow x_i = x_j$ for $i, j \in \{1, \dots, 4\}$. Hence $(G, *)$ is a ℓ -group. The parameters are taken as

$$A = \{a_1 = \text{Special and unique benefits, } a_2 = \text{Friendly place to work, } a_3 = \text{Contribution to the community, } a_4 = \text{Accomplish and pride, } a_5 = \text{Honest management}\}$$

For instance, suppose the team of analysts surveys that 70% of the employees of x_1 believe that x_1 is suitable for the first attribute; and 10% of the them believe that x_1 is poor for the first attribute, in which this process is utilized to calculate the amplitude terms for both membership and non membership functions respectively. The phase terms that the range of attribute for the present date for first attribute of x_1 can be given as follows: if 40% employees of x_1 believe that the present date of x_1 is suitable for the first attribute; and 30% of them believe that it is poor. So the first attribute for x_1 can be represented as $(0.7e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.3)})$. The weight vector of the each attribute is $w_j = \{0.2, 0.4, 0.2, 0.2, 0.5\}$ which is based on the employees priorities to the attribute in the decision making.

Next, we list the detailed steps involved in the weighted distance measures based decision making process.

- Step 1:** The employees reports about their own experience are given in Table 2. From Table 2 $(\tilde{F}, A)$ forms $\mathcal{CIFSLG}$.
- Step 2:** In Table 3, we obtain the weighted distance measures between $(\tilde{F}, A)$ and I , i.e., ideal choice (each rating value is $(1e^{i2\pi}, 0e^{i0\pi})$) by using the Definition 13 with the weight for each attribute $w_j = \{0.2, 0.4, 0.2, 0.2, 0.5\}$ and the weights $\alpha_1 = 0.3, \beta_1 = 0.4, \sigma_1 = 0.3$ for the amplitude term, while $\alpha_2 = 0.1, \beta_2 = 0.5, \sigma_2 = 0.4$ for the phase term.
- Step 3:** The Table 4 involves the ranking of the weighted distance measures obtained.

Table 2. $\mathcal{CIFSLG}$ information about the employees experience.

$(\tilde{F}, A)$	x_1	x_2	x_3	x_4
a_1	$(0.7e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.3)})$	$(0.9e^{i2\pi(0.8)}, 0.1e^{i2\pi(0.1)})$	$(0.8e^{i2\pi(0.4)}, 0.26e^{i2\pi(0.5)})$	$(0.3e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.25)})$
a_2	$(0.6e^{i2\pi(0.7)}, 0.32e^{i2\pi(0.3)})$	$(0.7e^{i2\pi(0.8)}, 0.23e^{i2\pi(0.01)})$	$(0.4e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.2)})$	$(0.7e^{i2\pi(0.7)}, 0.2e^{i2\pi(0.1)})$
a_3	$(0.4e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.39)})$	$(0.8e^{i2\pi(0.9)}, 0.01e^{i2\pi(0.02)})$	$(0.6e^{i2\pi(0.5)}, 0.35e^{i2\pi(0.4)})$	$(0.7e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.1)})$
a_4	$(0.8e^{i2\pi(0.5)}, 0.19e^{i2\pi(0.4)})$	$(0.95e^{i2\pi(0.9)}, 0e^{i2\pi(0.1)})$	$(0.8e^{i2\pi(0.2)}, 0.15e^{i2\pi(0.7)})$	$(0.9e^{i2\pi(0.8)}, 0e^{i2\pi(0.05)})$
a_5	$(0.5e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.6)})$	$(0.8e^{i2\pi(0.8)}, 0.1e^{i2\pi(0.2)})$	$(0.75e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.6)})$	$(0.75e^{i2\pi(0.8)}, 0.05e^{i2\pi(0.2)})$

Table 3. Weighted distance measure of $(\tilde{F}, A)$ and I using Definition 13.

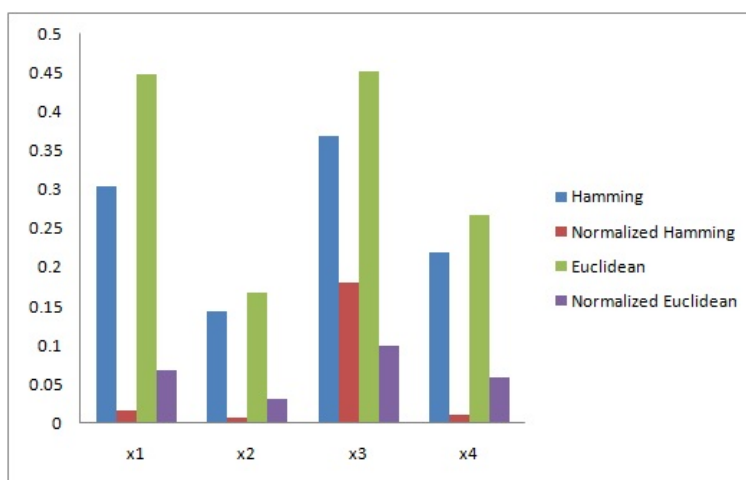
	x_1	x_2	x_3	x_4
$d_w^H((\tilde{F}, A), I)$	0.305	0.143	0.369	0.22
$d_w^{NH}((\tilde{F}, A), I)$	0.016	0.007	0.018	0.011
$d_w^E((\tilde{F}, A), I)$	0.449	0.168	0.452	0.268
$d_w^{NE}((\tilde{F}, A), I)$	0.068	0.031	0.1	0.059

Table 4. Ordering the alternatives.

	Ordering
$d_w^H((\tilde{F}, A), I)$	$x_2 \succ x_4 \succ x_1 \succ x_3$
$d_w^{NH}((\tilde{F}, A), I)$	$x_2 \succ x_4 \succ x_1 \succ x_3$
$d_w^E((\tilde{F}, A), I)$	$x_2 \succ x_4 \succ x_1 \succ x_3$
$d_w^{NE}((\tilde{F}, A), I)$	$x_2 \succ x_4 \succ x_1 \succ x_3$

4.2. Discussion of the Results and Comparative Study

We obtained the weighted distance measures from Table 4. Clearly, the best alternative is x_2 , which is the one with the lowest distance to the ideal choice. It is determined that x_2 is graded as first, x_4 is graded as second, x_1 is graded as third and x_3 is graded as fourth. The result is shown in Figure 1.

**Figure 1.** Weighted distance measure results on $CI FSL-G$.

In Table 2, the phase term of the membership and non-membership values gives the ability to consider values in more accurate compared to fuzzy sets and soft sets and other hybrid structures. The amplitude term gives the ability to deal with periodicity information of membership and non-membership function. On the other hand, different weights lead to different results. In this application, employee indicate their degree of importance to the attributes through the weights. Hence the results obtained by using Definition 13 taken into account the preference to the parameters with the $CI FSL-G$ structure. The results of this application help to identify the gaps and improve their work place culture.

Among the existing methods in literature that is closed to the weighted distance measure on $CI FSL-G$ is the distance measure on complex intuitionistic fuzzy soft sets (CIFSS) [6]. However, using the distance measure on CIFSSs is unfeasible which is discussed above. For instance, in our scenario, intuitionistic fuzzy soft sets (IFSS) [4] and generalized intuitionistic fuzzy soft sets (GIFSS) [9] can

not represent the phase term, which is range at the present date. From this analysis, the weighted distance measure and $CIFSL\mathcal{G}$ structure enables to overcome the problems afflicting in CIFSS, IFSS and GIFSS.

5. Conclusions

The complex intuitionistic fuzzy soft lattice ordered groups ($CIFSL\mathcal{G}$) is a hybrid structure of CIFSS and Lattice ordered group. We found the notion of $CIFSL\mathcal{G}$ and its supporting properties. Furthermore, examined the operations on $CIFSL\mathcal{G}$. Furthermore, we introduced the weighted distance measures and verified some properties. In addition, we demonstrated an application of $CIFSL\mathcal{G}$ to find the best work stream for employees which is accomplished by weighted distance measure between $CIFSL\mathcal{G}$ s. As a future work, we plan to extend the theory by introducing morphisms and some more operations on $CIFSL\mathcal{G}$ and conjointly planned to contribute some real life applications.

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