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# Supply Chain Coordination with a Loss-Averse Retailer and Combined Contract

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**Abstract:** This paper studies the supply chain coordination where the retailer is loss-averse, and a combined buyback and quantity flexibility contract is introduced. The loss-averse retailer's objective is to maximize the Conditional Value-at-Risk of utility. It is shown the combined contract can coordinate the chain and a unique coordinating wholesale price exists if the confidence level is below a threshold. Moreover, the retailer's optimal order quantity, expected utility and coordinating wholesale price are decreasing in loss aversion and confidence levels, respectively. We also find that when the contract parameters are restricted, the combined contract may coordinate the supply chain even though neither of its component contracts coordinate the chain.

**Keywords:** loss aversion; Conditional Value-at-Risk; supply chain coordination; contract design

## 1. Introduction

Double marginalization prevails in supply chain management and hurts the supply chain performance tremendously. Therefore, contracts play an important role in supply chain management. A variety of different contracts are proposed to stimulate all the members and then coordinate the supply chain over recent decades, such as buyback, quantity flexibility, revenue sharing and quantity discount (e.g., [1–5]). The review of contract problem can be found in Cachon [6] and Tsay et al. [7].

Traditionally, all agents are assumed to be risk-neutral when designing a contract. Each seeks to maximize expected profit or minimize expected cost. However, many experimental studies (e.g., [8–10]) have asserted that most decision makers often defy risk neutrality assumption and their actual behaviors deviate from expected profit maximizing, which is referred to as "decision bias" by Wang and Webster [11]. Thus, some researchers have advocated relaxing the restrain of risk neutrality and adopting other alternative choice models to characterize the decision-making behavior. Among them the model based on loss aversion is popular in recent years. Loss aversion originates from prospect theory and means that people facing equivalent gains and losses are more averse to the latter [12].

The loss-averse newsvendor problem is initially addressed by Schweitzer and Cachon [13]. Since then, loss-averse preferences have drawn a great deal of attention and the contract models based on loss aversion have been studied in various contexts. Nevertheless, the risk management of the loss-averse decision makers yet receives little attention in the existing papers. The recent studies on portfolio management show that the loss-averse investors usually take on more risks (e.g., [14,15]). Therefore, how to hedge against the loss-averse members' risk in the supply chain develops into an interesting issue. However, there are now only a few researchers introduce Conditional Value-at-Risk

(CVaR) measure into the loss-averse newsvendor models. When incorporating risk management, the loss-averse decision makers' ordering policy in a supply chain and supply chain performance are still unclear.

To fill this research gap, loss aversion and risk management are jointly considered in our paper. CVaR measure, which is originally proposed by Rockafellar and Uryasev [16,17], has received considerable attention as a coherent and easily computed risk criterion in the portfolio and operation management literature. In view of this, CVaR measure is introduced into our supply chain coordination problem to hedge against the risks for the loss-averse retailer. We devote to answer the following questions: (1) How does the loss-averse retailer select the order quantity maximizing CVaR of utility in a supply chain? (2) How to devise the contract coordinating the chain? (3) How does the loss aversion and confidence level affect the optimal order quantity and supply chain performance?

To address the above problems, this paper analytically models and characterizes the coordination of a supply chain in which the retailer is loss-averse and CVaR measure is introduced. The retailer's optimal order quantity maximizing the CVaR of utility in the decentralized case is less than the optimal solution in the integrated case. Then we devise a composite contract by combining buyback and quantity flexibility contracts to induce the retailer to order more products and thus coordinate the supply chain. The major contributions of our work is as follows. First, the loss-averse retailer's optimal ordering policy is characterized. Second, we establish the sufficient condition under which there is a unique coordinating wholesale price. Third, the effects of loss aversion, confidence level and contract parameters on optimal order quantity, expected utility and coordinating wholesale price are analyzed, respectively. Fourth, we point out that when the contract parameters are restricted, a coordinating combined contract may exist even though neither of the coordinating component contracts exist.

The rest of this paper is organized as follows. In next section, we summarize the related literature. In Section 3, we present the description of the model and identify the bilateral supply chain with wholesale price contract cannot be coordinated. In Section 4, we devise a composite contract by combining buyback and quantity flexibility contracts and provide the loss-averse retailer's ordering policy. In Section 5, we investigate the coordination of supply chain. In Section 6, we conduct numerical experiments which corroborate the results in Sections 4 and 5. In Section 7, we conclude our paper and identify areas for future research.

## 2. Literature Review

The relevant literature will be reviewed according to three streams: supply chain coordination based on loss aversion, supply chain coordination under CVaR criterion and inventory problem based on loss aversion under CVaR criterion.

The first stream is on the supply chain coordination with loss-averse agents. Wang and Webster [18] devise a gain/loss-sharing-and-buyback contract to investigate the supply chain with a loss-averse retailer, and then compare the performance of three special cases of this contract. Deng et al. [19] further extend their work by considering the asymmetric loss aversion level and quantify the influence of information asymmetry. When both yield and demand are stochastic, Luo et al. [20] study the supply chain coordination under three different contracts. Du et al. [21] also consider the yield risk and demand uncertainty and study a supply chain with loss-averse supplier and retailer. Their optimal policies are obtained, respectively. Hu et al. [22] consider the three-echelon supply chain under revenue sharing contract. They derive each member's optimal policy and find that Pareto improvement can be achieved. Huang et al. [23] introduce a composed option and cost-sharing contract to study a vendor-managed inventory supply chain. They show that supply chain coordination and Pareto improvement can be achieved under this composite contract. When only partial information of demand distribution is known, Zhai and Yu [24] design a combined buyback and loss-sharing contract and employ the robust approach to study the supply chain coordination.

The second stream is on the supply chain coordination under CVaR criterion. Chen et al. [25] consider a supply chain in which a single retailer and multiple suppliers are all risk-averse. They study the performance of the contract and its stability. Chen et al. [26] incorporate default probability into the supply chain. They show quantity discount contract can coordinate the chain if the risk-averse manufacturer's confidence level is small enough. Li et al. [27] study a supply chain where both risk-averse manufacturer and risk-neutral retailer face demand uncertainty. They devise an improved risk-sharing contract to coordinate the chain. Yang et al. [28] use several single contracts to study the supply chain with a risk-averse retailer, but fail to consider composite form. Xie et al. [29] further extend their model by using mean-CVaR as the loss-averse retailer's criterion and investigate the supply chain coordination. Luo et al. [30] use buyback contract to study the supply chain coordination under mean-CVaR criterion, and investigate the effect of allocation rule in the platform selling model.

The third stream is on the inventory problem based on loss aversion under CVaR criterion, and existing literature all focuses on the newsvendor problem and its extensions. Sun and Xu [31] study the newsvendor problem and show that the optimal order quantity maximizing the CVaR of utility is less than that maximizing expected utility. Xu et al. [32] extend their model to the case where excess demand can be partially backlogged, and obtain the similar results. Xu et al. [33] investigate the newsvendor model in the case with and without shortage cost, respectively. They also analyze the relation between the newsvendor's loss aversion level and fill rate targets. Xu et al. [34] propose the legacy loss and investigate the loss-averse newsvendor's optimal ordering policy in three different decision objectives. Chan and Xu [35] suggest a new loss aversion utility function. In the case of maximizing expected utility and CVaR of utility, they obtain the optimal order quantities, respectively. Table 1 compares the contribution of different authors.

**Table 1.** Comparison between contributions of different authors.

| Author(s) | Supply Chain Coordination | Loss Aversion | CVaR |
|---|---|---|---|
| Xiao et al. [1] | √ | | |
| Xiong et al. [2] | √ | | |
| Wan and Chen [3] | √ | | |
| Li et al. [4] | √ | | |
| Sarkar [5] | √ | | |
| Wang and Webster [18] | √ | √ | |
| Deng et al. [19] | √ | √ | |
| Luo et al. [20] | √ | √ | |
| Du et al. [21] | √ | √ | |
| Hu et al. [22] | √ | √ | |
| Huang et al. [23] | √ | √ | |
| Zhai and Yu [24] | √ | √ | |
| Chen et al. [25] | √ | | √ |
| Chen et al. [26] | √ | | √ |
| Li et al. [27] | √ | | √ |
| Yang et al. [28] | √ | | √ |
| Xie et al. [29] | √ | | √ |
| Luo et al. [30] | √ | | √ |
| Sun and Xu [31] | | √ | √ |
| Xu et al. [32] | | √ | √ |
| Xu et al. [33] | | √ | √ |
| Xu et al. [34] | | √ | √ |
| Chan and Xu [35] | | √ | √ |
| This paper | √ | √ | √ |

In summary, previous works in the literature reviewed above do not address the supply chain coordination when jointly considering loss aversion and risk management. On the other hand, Xiong et al. [2] consider the supply chain based on risk neutrality and show the combined buyback and quantity flexibility contract is superior to both component contracts (buyback and quantity flexibility contracts) from the perspective of coordination. When the retailer has loss-averse preferences and incorporating risk management, can this combined contract coordinate the chain yet? These motivate the study in this paper.

## 3. Model Description

Consider a single period supply chain where the retailer has the loss-averse preferences. Before the selling period, the retailer decides on the quantity purchased from the manufacturer. The manufacturer produces the products at a cost $c$ in response to his orders and sells it at a price $w$. Then the customers arrive, and the selling season starts. All unsold products will be salvaged at the end of the period. Let $p$ and $s$ be the selling price and salvage value per unit, respectively. To avoid triviality, it is assumed that $s < c < w < p$. The unsatisfied demand will be lost, and shortage cost is not considered. Notions concerned in this paper are listed in Table 2, and some is introduced when needed.

**Table 2.** Summary of the notations.

| Notation | Description |
|---|---|
| $p$ | Selling price per unit, |
| $w$ | Wholesale price per unit, |
| $c$ | Production cost per unit, |
| $s$ | Salvage value per unit, $p > w > c > s$, |
| $b$ | Buyback price per unit, |
| $Q$ | Order quantity, |
| $D$ | Random demand, |
| $f(x)$ | Probability density function of $D$, |
| $F(x)$ | Cumulative distribution function of $D$, |
| $\overline{F}(x)$ | Tail distribution of $F(x)$, i.e., $\overline{F}(x) = 1 - F(x)$, |
| $\lambda$ | Loss aversion level, $\lambda \geq 1$, |
| $\alpha$ | Confidence level, $0 \leq \alpha < 1$, |
| $\beta$ | minimum purchase rate, $0 < \beta \leq 1$, |
| $Q_0^*$ | Optimal production quantity in the integrated case, |
| $Q^*$ | Optimal order quantity maximizing CVaR of utility, |
| $Q_1^*$ | Optimal order quantity maximizing expected utility, |
| $Q_2^*$ | Optimal order quantity maximizing CVaR of profit, |
| $Q_3^*$ | Optimal order quantity maximizing expected profit, |
| $w^*(b,\beta)$ | Coordinating wholesale price under combined contract, |
| $w_Q^*(\beta)$ | Coordinating wholesale price under quantity flexibility contract, |
| $w_B^*(b)$ | Coordinating wholesale price under buyback contract. |

To provide a benchmark, we first analyze the optimal solution of the integrated supply chain. In the integrated case, the manufacturer and retailer are owned by one risk-neutral firm. When the production quantity is $Q$, the integrated firm's total expected profit is

$$E[\Pi(Q,D)] = \int_0^Q [(p-s)x - (c-s)Q]dF(x) + \int_Q^{+\infty} (p-c)QdF(x). \tag{1}$$

It is easy to prove that $E[\Pi(Q,D)]$ is concave and then we have

$$Q_0^* = F^{-1}\left(\frac{p-c}{p-s}\right). \tag{2}$$

Now consider a decentralized case where both the manufacturer and retailer are independent agents. As in [36], it is reasonable to assume the manufacturer is risk-neutral when it can diversify its risk by serving many independent retailers. The retailer's utility function is (see e.g., [11,18,32–34]):

$$U(\pi(Q,D)) = \begin{cases} \pi(Q,D), & \pi(Q,D) \geq 0, \\ \lambda\pi(Q,D), & \pi(Q,D) < 0, \end{cases} \tag{3}$$

where $\pi(Q, D)$ is the retailer's profit and $\lambda \geq 1$ is his loss aversion level. The retailer is loss-neutral if $\lambda = 1$.

Under CVaR criterion, when the retailer's confidence level is $\alpha \in [0, 1)$, the VaR and CVaR of utility $U(\pi(Q, D))$ are defined as [33]

$$VaR_\alpha[U(\pi(Q, D))] = \sup\{y \in R : P[U(\pi(Q, D)) \geq y] \geq \alpha\} \tag{4}$$

and

$$CVaR_\alpha[U(\pi(Q, D))] = E[U(\pi(Q, D))|U(\pi(Q, D)) \leq VaR_\alpha[U(\pi(Q, D))]], \tag{5}$$

respectively. Please note that the retailer's CVaR of utility reduces to the expected utility when $\alpha = 0$, and reduces to the CVaR of profit when $\lambda = 1$, and further reduces to the expected profit when $\alpha = 0$ and $\lambda = 1$ simultaneously, i.e., the expected utility, CVaR of profit and expected profit are all the special cases of the CVaR of utility. The retailer's objective is to maximize $CVaR_\alpha[U(\pi(Q, D))]$.

It follows from Xu et al. [33] that the optimal order quantity $Q^*$ satisfies

$$(\lambda - 1)(w - s)F\left(\frac{w - s}{p - s}Q^*\right) + \alpha(p - w) + [(p - s)F(Q^*) - (p - w)] = 0. \tag{6}$$

Moreover, they demonstrate that $Q^*$ is decreasing in $\lambda$ and $\alpha$. Then we have

$$Q^* \leq F^{-1}\left(\frac{p - w}{p - s}\right) < F^{-1}\left(\frac{p - c}{p - s}\right) = Q_0^*. \tag{7}$$

Thus, the retailer's optimal order quantity maximizing the CVaR of utility is less than the integrated optimal solution, which is due to loss aversion (the first term of (6)), risk aversion (the second term of (6)) and double marginalization (the third term of (6)). To eliminate these effects, we then devise a composite contract by combining buyback and quantity flexibility contracts to stimulate the retailer order more products and then coordinate the supply chain.

## 4. Loss-Averse Retailer's Ordering Policy under the Combined Contract

The operation sequence and steps of the combined contract are described as follows: (1) Before the selling period starts, the manufacturer offers a contract $(w, b, \beta)$. (2) The retailer decides the order quantity $Q$ of the product at a wholesale price $w$ according to the preliminary demand forecasts. The manufacturer responds to produce the quantity the retailer orders. (3) When the retailer further receives actual information on customer demand, he has a chance to modify his order within $[\beta Q, Q]$, where $\beta \in (0, 1]$ is the minimum purchase rate the retailer commits. The orders are delivered immediately and then demand occurs during the period. (4) The retailer will return the unsold products with the price $b \in [s, w)$ at the end of the period. Please note that both the quantity flexibility contract and buyback contract are the special cases of this combined contract. More specifically, the combined contract reduces to quantity flexibility contract if $b = s$, and reduces to buyback contract if $\beta = 1$.

Under this combined contract, the retailer's profit is

$$\pi(Q, D, w, b, \beta) = \begin{cases} (p-b)D-(w-b)\beta Q, & 0 \leq D < \beta Q, \\ (p - w)D, & \beta Q \leq D < Q, \\ (p - w)Q, & D \geq Q, \end{cases} \tag{8}$$

from which it follows that the retailer will face losses if realized demand is less than $\frac{w-b}{p-b}\beta Q$. Otherwise, he will face gains.

**Theorem 1.** *The retailer's optimal order quantity $Q^*$ under combined contract is unique and satisfies:*

$$(\lambda - 1)(w - b)\beta F\left(\frac{w-b}{p-b}\beta Q^*\right) + \beta(w-b)F(\beta Q^*) + (p-w)[\alpha - \overline{F}(Q^*)] = 0. \tag{9}$$

**Proof.** See Appendix A. □

The following corollary indicates how would the optimal order quantity $Q^*$ change when loss aversion or confidence level increases.

**Corollary 1.** *The impacts of loss aversion, confidence level and contract parameters on the optimal order quantity are as follows:*

*(i)* $Q^*$ *is decreasing in* $\lambda$*;*
*(ii)* $Q^*$ *is decreasing in* $\alpha$*;*
*(iii)* $Q^*$ *is decreasing in* $\beta$ *and* $w$*, while increasing in* $b$*.*

**Proof.** See Appendix A. □

When loss aversion or confidence level increases, the retailer becomes more conservative. Thus, he will order less products to hedge against the potential loss or risk. The higher the loss aversion or confidence lever is, the less he orders. Moreover, the retailer will order less as minimum purchase rate or wholesale price increases, while order more as buyback price increases, which are intuitive.

As mentioned above, the retailer's CVaR objective of utility is the expected utility when $\alpha = 0$. In such a case, $Q^*$ reduces to the optimal order quantity that maximizes expected utility that is often studied in the supply chain models with loss-averse decision makers (e.g., [21–23]). Denote it by $Q_1^*$ and then from (9) it satisfies

$$(\lambda - 1)(w - b)\beta F\left(\frac{w-b}{p-b}\beta Q_1^*\right) + \beta(w-b)F(\beta Q_1^*) - (p-w)\overline{F}(Q_1^*) = 0. \tag{10}$$

Similarly, if $\lambda = 1$, then $Q^*$ reduces to the optimal order quantity that maximizing CVaR of profit that is often studied in the supply chain models under CVaR criterion (e.g., [25–27]). Denote it by $Q_2^*$ and then it satisfies

$$\beta(w-b)F(\beta Q_2^*) + (p-w)[\alpha - \overline{F}(Q_2^*)] = 0. \tag{11}$$

Furthermore, if both $\alpha = 0$ and $\lambda = 1$, then $Q^*$ reduces to the optimal order quantity maximizing expected profit that is studied in most traditional supply chain models with risk-neutral decision makers. Please note that in this case, our model is the same as that in Xiong et al. [2]. Denote it by $Q_3^*$ and then it satisfies

$$\beta(w-b)F(\beta Q_3^*) - (p-w)\overline{F}(Q_3^*) = 0. \tag{12}$$

Summarizing the above analysis and combining with Corollary 1, we have

**Corollary 2.** *For any given* $\lambda$ *and* $\alpha$*, then* $Q^* \le Q_1^* \le Q_3^*$ *and* $Q^* \le Q_2^* \le Q_3^*$*.*

This results show that due to the joint effects of loss aversion and risk aversion, the optimal order quantity maximizing CVaR of utility is always less than that maximizing expected utility or CVaR of profit, and further less than that maximizing expected profit.

**Theorem 2.** *The retailer's expected utility* $E[U(\pi(Q^*, D, w, b, \beta))]$ *under* $Q^*$ *is decreasing in* $\alpha$ *and* $\lambda$*.*

**Proof.** See Appendix A. □

This theorem indicates that as the loss aversion or confidence level increases, the retailer will face a lower expected utility if he wants to order less to reduce the potential risk. The higher the loss aversion or confidence lever is, the lower his expected utility is. On the contrary, to obtain a higher expected utility, the retailer must order more products, which enhances the risk of excess order and requires him to have greater risk tolerance. This demonstrates that high risk implies high return, and vice versa.

## 5. Supply Chain Coordination

We now discuss whether there is a wholesale price that can coordinate the supply chain. Please note that the integrated firm's solution is used as the benchmark and the condition of supply chain coordination is $Q^* = Q_0^*$, i.e., the retailer orders the system optimal production quantity (e.g., [2,22,29]).

**Theorem 3.** *If $\alpha < \overline{F}(Q_0^*) = \frac{c-s}{p-s}$, then there is a unique wholesale price $w^*(b,\beta)$ under the combined contract that can coordinate the supply chain and satisfies*

$$(\lambda - 1)(w^* - b)\beta F\left[\frac{\beta(w^* - b)}{p - b}Q_0^*\right] + \beta(w^* - b)F\left(\beta Q_0^*\right) + (p - w^*)[\alpha - \overline{F}(Q_0^*)] = 0. \quad (13)$$

*Otherwise, there is not a wholesale price coordinating the supply chain.*

**Proof.** See Appendix A. □

This theorem shows the sufficient condition for the existence and uniqueness of coordinating wholesale price. There is a threshold of confidence level that is independent of loss aversion and contract parameters, below which the supply chain can be coordinated. In particular, if $\alpha = 0$, that is the objective is to maximize the expected utility, then there is always a wholesale price that can coordinate the chain. Moreover, if both $\alpha = 0$ and $\lambda = 1$, the coordinating contract given by (13) are just that given by Xiong et al. [2] for risk-neutral setting. This result also offers an important managerial implication. When the downstream retailer is loss-averse and operates under CVaR, his loss-averse preferences and risk attitude have a significant impact on supply chain coordinating. In practical situations, the loss aversion and confidence levels can be determined through experiments (e.g., [37]). If his confidence level is small, i.e., $\alpha < \overline{F}(Q_0^*) = \frac{c-s}{p-s}$, then the upstream manufacturer can select the wholesale price according to (13). Otherwise, supply chain cannot be coordinated no matter how the manufacturer sets the price.

The impacts of loss aversion, confidence level and contract parameters on the coordinating wholesale price are as follows.

**Corollary 3.** *If $\alpha < \overline{F}(Q_0^*) = \frac{c-s}{p-s}$, then*

*(i)* $w^*(b,\beta)$ *is decreasing in* $\lambda$;
*(ii)* $w^*(b,\beta)$ *is decreasing in* $\alpha$;
*(iii)* $w^*(b,\beta)$ *is decreasing in* $\beta$*, while increasing in* $b$.

**Proof.** See Appendix A. □

We have shown above that when loss aversion, confidence level or minimum purchase rate increases, the retailer will decrease the order quantity. Therefore, to induce the retailer to order more products, the manufacturer must decrease the price. However, the manufacturer will undertake more risk and must charge a higher price as the buyback price increases.

Then we will compare the coordinating wholesale price under combined contract with that under its two component contracts. On the one hand, the combined contract reduces to quantity flexibility contract if $b = s$. Since $w^*(b, \beta)$ is increasing in $b$, then the coordinating wholesale price $w_Q^*(\beta)$ under quantity flexibility contract satisfies $w_Q^*(\beta) \leq w^*(b, \beta)$. On the other hand, the combined contract reduces to buyback contract if $\beta = 1$. Since $w^*(b, \beta)$ is decreasing in $\beta$, then the coordinating wholesale price $w_B^*(b)$ under buyback contract satisfies $w_B^*(b) \leq w^*(b, \beta)$. Therefore, in light of the above analysis, we can directly obtain the following results.

**Corollary 4.** *If* $\alpha < \overline{F}(Q_0^*) = \frac{c-s}{p-s}$, *then*

*(i)* *there is a unique wholesale price* $w_B^*(b)$ *under the buyback contract that can coordinate the supply chain and satisfies*

$$(\lambda - 1)(w_B^* - b)F\left(\frac{w_B^* - b}{p - b} Q_0^*\right) + (p - b)F\left(Q_0^*\right) + (p - w_B^*)(\alpha - 1) = 0. \tag{14}$$

*Moreover, it is decreasing in* $\lambda$ *and* $\alpha$, *while increasing in* $b$.

*(ii)* *there is a unique wholesale price* $w_Q^*(\beta)$ *under the quantity flexibility contract that can coordinate the supply chain and satisfies*

$$(\lambda - 1)(w_Q^* - s)\beta F\left[\frac{\beta(w_Q^* - s)}{p - s} Q_0^*\right] + \beta(w_Q^* - s)F\left(\beta Q_0^*\right) + (p - w_Q^*)[\alpha - \overline{F}(Q_0^*)] = 0. \tag{15}$$

*Moreover, it is decreasing in* $\lambda$, $\alpha$ *and* $\beta$.

*(iii)* $w^*(b, \beta) \geq \max\{w_B^*(b), w_Q^*(\beta)\}$.

This corollary shows that the coordinating wholesale price under combined contract is higher than that under two component contracts. In contrast with the component contracts, the manufacturer will undertake more risk under combined contract and thus must charge a higher wholesale price. Moreover, note that if $\lambda = 1$, the coordinating contracts given by (14) and (15) are just that given by Yang et al. [28] for risk-averse setting.

Now we will consider the case that the contract parameters are restricted to certain ranges. As pointed out by Xiong et al. [2], this often occurs in practice and for example, the percentage of returning unsold books in publishing industry is usually no more than 35%. In this case, a contract is called feasible when each parameter is within its range.

**Theorem 4.** *Suppose that* $w \in [\underline{w}, \overline{w}]$, $b \in [s, \overline{b}]$ *and* $\beta \in [\underline{\beta}, 1]$. *If* $\max\{w_B^*(\overline{b}), w_Q^*(\underline{\beta})\} < \underline{w} \leq w^*(\overline{b}, \underline{\beta})$, *then there is not feasible coordinating buyback or quantity flexibility contract, while is a feasible coordinating combined contract.*

**Proof.** See Appendix A. □

Since buyback and quantity flexibility contracts are the special cases of combined contract, there is always a feasible coordinating combined contract if either a feasible coordinating buyback or quantity flexibility contract exists, but not vice versa. When contract parameters are constrained, this theorem shows that the combined contract may coordinate the supply chain even though neither of its component contracts coordinate the chain. In this sense, combined contract is superior to its both component contracts.

## 6. Numerical Experiments

We carry out numerical experiments to illustrate our theoretical results and present some managerial insights in this section. Let $p = 6$, $w = 5$, $c = 4$ and $s = 1$. The demand is a truncated normal random variable whose cumulative distribution function is $F(x) = (G(x) - G(0))\,/(1 - G(0))$, where $G(x) = 1/(\sqrt{2\pi}\sigma)\int_{-\infty}^{x} e^{-(t-\mu)^2/2\sigma^2}dt$ and mean $\mu = 200$ as well as standard deviation $\sigma = 50$.

Then we can easily get the optimal production quantity $Q_0^* = 187.34$ and the total profit $E(\Pi) = 303.44$ in the centralized supply chain. To gain some insights on the effects of loss aversion, confidence level and contract parameters on the optimal order quantity, coordinating wholesale price and expected profit under combined contract, we conduct two sets of numerical experiments. In the first set, we fix $\alpha = 0.2$, $\beta = 0.8$ and vary $\lambda$ from 1 to 3 in steps of 0.1. Three different buyback prices are considered: $b = 2$, $b = 3$ and $b = 4$. In the second set, we fix $\lambda = 2$, $b = 3$ and vary $\alpha$ from 0 to $\frac{c-s}{p-s} = 0.6$ in steps of 0.05. Three different minimum purchase rates are considered: $\beta = 0.6$, $\beta = 0.7$ and $\beta = 0.8$.

Figures 1 and 2 illustrate the effects of loss aversion on the order quantity and coordinating wholesale price, respectively. As shown in these two figures, for any given buyback price $b$, the order quantity and coordinating wholesale price decrease with loss aversion level. Figures 3 and 4 illustrate that for any given purchase rate $\beta$, the optimal order quantity and coordinating wholesale price also decrease with confidence level, i.e., the higher the loss aversion or confidence level is, the lower the order quantity and coordinating wholesale price are. These results are in accordance with Corollaries 1 and 3. When the loss aversion or confidence level increases, the retailer becomes more conservative and will select a smaller order quantity to hedge against the potential loss or risk that comes from the possible excess order. Thus, the manufacturer must decrease the wholesale price to induce the retailer to enhance order quantity. Figures 1 and 2 (Figures 3 and 4) also show that the buyback price (purchase rate) affects the sensitivity of optimal order quantity and coordinating wholesale price when loss aversion level (confidence level) changes. For instance, the optimal order quantity when $b = 2$ decreases more rapidly than that when $b = 4$ in Figure 1. This indicates that for lower buyback price, the change to loss aversion level has larger effect on the order quantity. In addition, Figure 1 implies that the order quantity maximizing CVaR of utility (i.e., $\lambda > 1$) is less than that maximizing CVaR of profit (i.e., $\lambda = 1$). Figure 3 implies that the order quantity maximizing CVaR of utility (i.e., $\alpha > 0$) is less than that maximizing expected utility (i.e., $\alpha = 0$). Please note that according to Corollaries 1 and 3, the monotonicity of both the optimal order quantity and coordinating wholesale price are independent on the demand distribution and parameter values. Similarly, according to Corollary 2, the relation between the optimal order quantity maximizing CVaR of utility, maximizing CVaR of profit and maximizing expected utility are independent on them as well.

Figures 5 and 6 show profit allocations for different loss aversion levels and confidence levels in the coordinating supply chain, respectively. When loss aversion or confidence level increases, the retailer's profit increases while the manufacturer's decreases. This means that to induce the retailer to increase order quantity and coordinate the supply chain, the manufacturer must transfer a portion of the profit to the retailer. From Figure 6 we also observe that supply chain coordination only exists within a certain range, e.g., $\beta = 0.8$ and $\alpha \leq 0.5$. Otherwise, the manufacturer's profit is less than zero and channel coordination cannot be achieved. Moreover, these two figures illustrate that the higher the buyback price (purchase rate) is, the larger (less) the manufacturer's profit is. One may intuitively think that the manufacturer should prefer a lower buyback price or higher purchase rate, but in fact the opposite is true. Since Figures 2 and 4 show that a higher $b$ or lower $\beta$ will result in a higher coordinating wholesale price, the manufacturer actually can gain more profit of the entire supply chain.

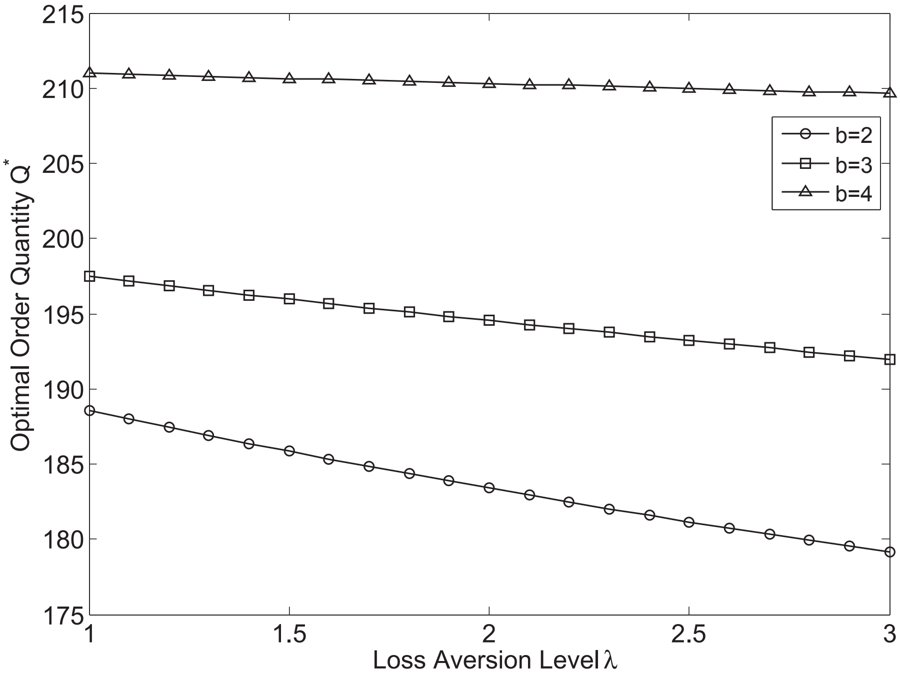


**Figure 1.** Optimal order quantity vs. loss aversion level for different buyback prices.

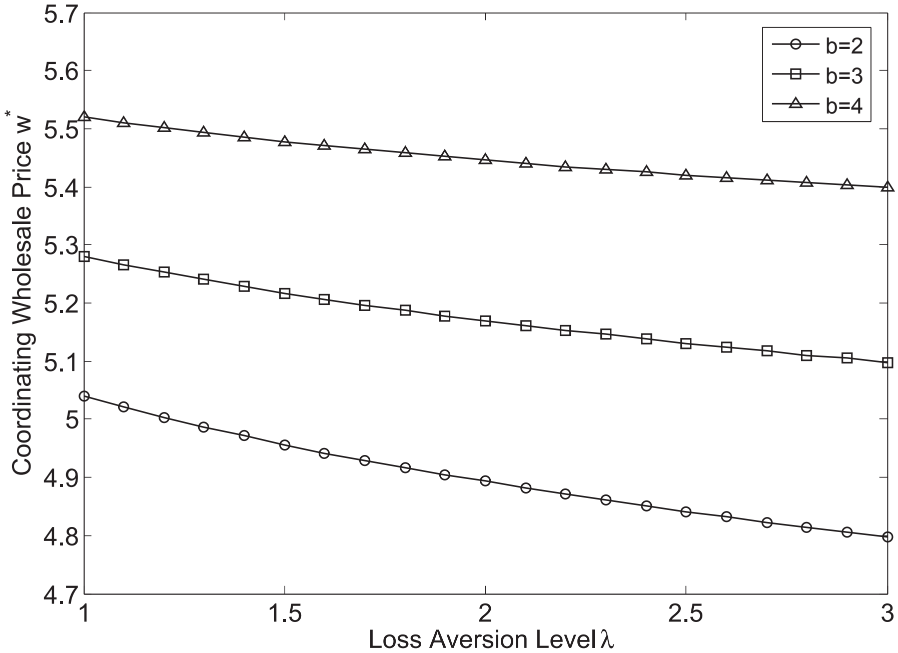


**Figure 2.** Coordinating wholesale price vs. loss aversion level for different buyback prices.

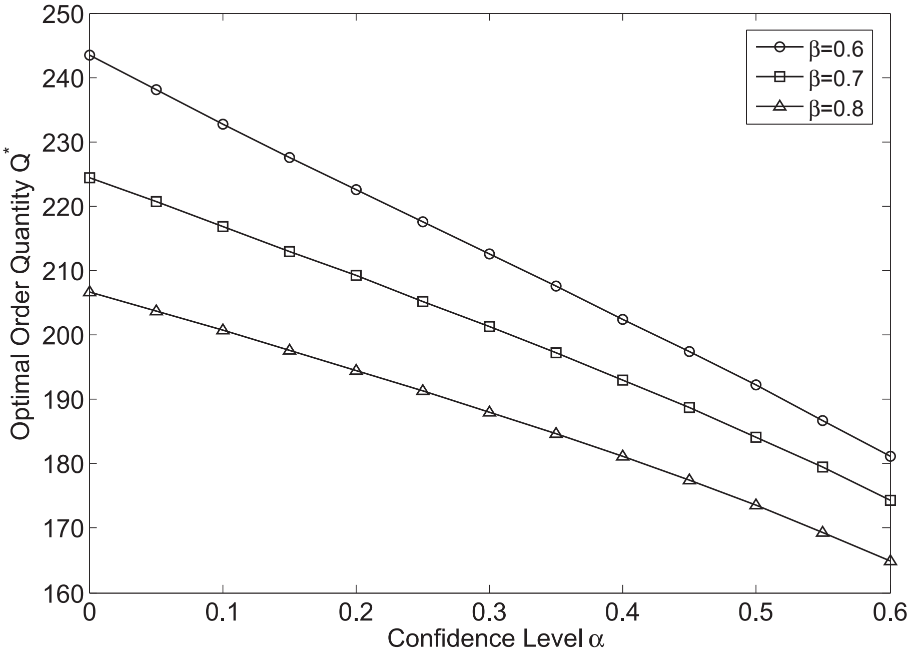


**Figure 3.** Optimal order quantity vs. confidence level for different minimum purchase rates.

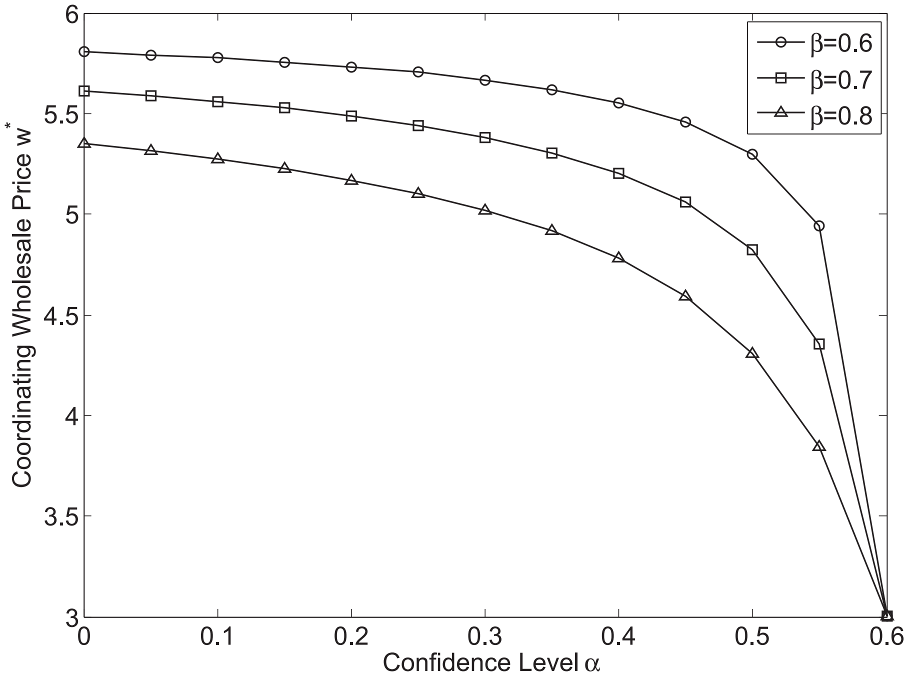


**Figure 4.** Coordinating wholesale price vs. confidence level for different minimum purchase rates.

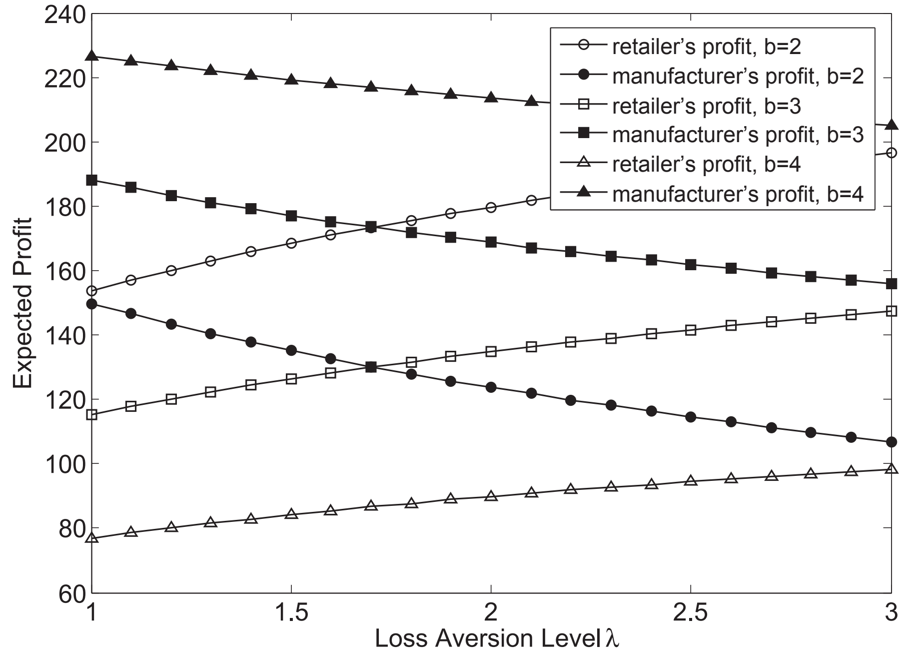


**Figure 5.** Expected profit vs. loss aversion level for different buyback prices.

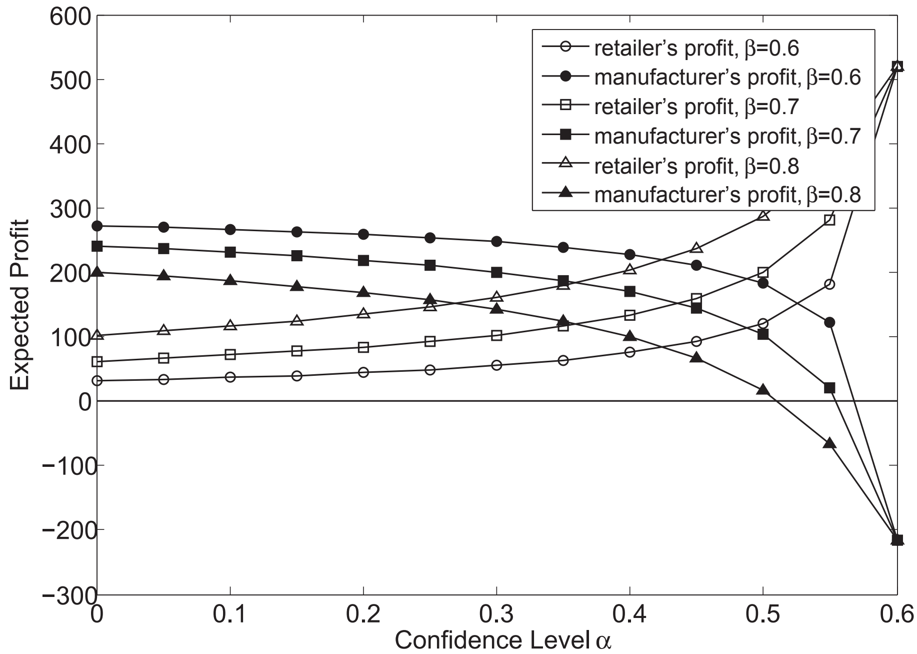


**Figure 6.** Expected profit vs. confidence level for different purchase rates.

## 7. Discussion and Conclusions

This paper studies the coordination of a supply chain where the retailer has loss-averse preferences and his objective is to maximize the CVaR of utility. Due to the jointly effect of loss aversion, risk aversion and double marginalization, the retailer's order quantity in decentralized supply chain is less than the optimal solution in integrated case. Then a composite form by combining buyback and quantity flexibility contracts is devised to coordinate the chain. The loss-averse retailer's optimal order quantity under combined contract is derived. Furthermore, there is a unique wholesale price coordinating supply chain if the confidence level is less than a critical value. When the retailer's loss aversion or confidence level increases, his optimal order quantity, expected utility and the coordinating wholesale price all decrease. Thus, when jointly considering loss aversion and risk management ($\lambda > 1$ and $0 \leq \alpha < 1$), the coordinating wholesale price is always less than that in the risk-neutral case ($\lambda = 1$ and $\alpha = 0$) studied by Xiong et al. [2]. This shows when the retailer is loss-averse and incorporating CVaR measure, his loss-averse preferences and risk attitude have a significant impact on supply chain coordinating. Furthermore, as the special cases of combined contract, the coordinating wholesale prices under buyback and quantity flexibility contracts are directly obtained and less than that under combined contract. We also find situations where a feasible coordinating combined contract may exist even though neither of its feasible coordinating component contracts exist. This implies that the combined contract creates more opportunities to coordinate the chain than buyback and quantity flexibility contracts that studied by Yang et al. [28].

This paper investigates a supply chain with a single retailer. Future research may consider the competition between multiple loss-averse retailers. Moreover, when the agents are loss-averse, supply chain coordination with random yield or capacity under CVaR criterion deserves further study.

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## Appendix A

**Proof of Theorem 1.** Define an auxiliary function

$$
\begin{aligned}
H(Q,v) =& v - \frac{1}{1-\alpha} E[v - U(\pi(Q,D,w,b,\beta))]^+ \\
=& v - \frac{1}{1-\alpha} \left\{ \int_0^{\frac{w-b}{p-b}\beta Q} [v - \lambda(p-b)x + \lambda(w-b)\beta Q]^+ dF(x) \right. \\
&+ \int_{\frac{w-b}{p-b}\beta Q}^{\beta Q} [v - (p-b)x + (w-b)\beta Q]^+ dF(x) \\
&\left. + \int_{\beta Q}^{Q} [v - (p-w)x]^+ dF(x) + \int_Q^{+\infty} [v - (p-w)Q]^+ dF(x) \right\}.
\end{aligned} \tag{A1}
$$

It follows from Rockafellar and Uryasev [16] that the optimal solution to $\max\limits_{Q \geq 0} CVaR_\alpha[U(\pi(Q,D,w,b,\beta))]$ is equal to that to $\max\limits_{Q \geq 0} \left[ \max\limits_{v \in R} H(Q,v) \right]$. Thus, for any given $Q$, we solve the problem $\max\limits_{v \in R} H(Q,v)$ at first and consider five distinct cases:

(i) $v \leq -\lambda(w-b)\beta Q$.

In such a case,

$$H(Q,v) = v. \tag{A2}$$

Then $\frac{\partial H(Q,v)}{\partial v} = 1$ and thus $H(Q,v)$ is increasing in $v$.

(ii) $-\lambda(w-b)\beta Q < v \leq 0$.

Here,

$$H(Q,v) = v - \frac{1}{1-\alpha} \int_0^{\frac{v+\lambda(w-b)\beta Q}{\lambda(p-b)}} [v - \lambda(p-b)x + \lambda(w-b)\beta Q] dF(x). \tag{A3}$$

Then

$$\frac{\partial H(Q,v)}{\partial v} = 1 - \frac{1}{1-\alpha} F\left[ \frac{v + \lambda(w-b)\beta Q}{\lambda(p-b)} \right], \tag{A4}$$

and it is easy to calculate that $\frac{\partial^2 H(Q,v)}{\partial v^2} < 0$, which indicates $H(Q,v)$ is concave. From (A4) we have $\left.\frac{\partial H(Q,v)}{\partial v}\right|_{v=-\lambda(w-b)\beta Q} = 1 > 0$. If $\left.\frac{\partial H(Q,v)}{\partial v}\right|_{v=0} = 1 - \frac{1}{1-\alpha} F\left[\frac{(w-b)\beta Q}{p-b}\right] \leq 0$, that is $Q \geq \frac{(p-b)F^{-1}(1-\alpha)}{(w-b)\beta}$, then there is a unique $v^*$ that satisfies $\left.\frac{\partial H(Q,v)}{\partial v}\right|_{v=v^*} = 0$ and we have $v^* = \lambda(p-b)F^{-1}(1-\alpha) - \lambda(w-b)\beta Q$.

(iii) $0 < v \leq (p-w)\beta Q$.

In such a case,

$$\begin{aligned} H(Q,v) =& v - \frac{1}{1-\alpha}\left\{\int_0^{\frac{w-b}{p-b}\beta Q}[v-\lambda(p-b)x+\lambda(w-b)\beta Q]dF(x)\right. \\ &\left.+\int_{\frac{w-b}{p-b}\beta Q}^{\frac{v+(w-b)\beta Q}{p-b}}[v-(p-b)x+(w-b)\beta Q]dF(x)\right\}. \end{aligned} \tag{A5}$$

Then

$$\frac{\partial H(Q,v)}{\partial v} = 1 - \frac{1}{1-\alpha}F\left[\frac{v+(w-b)\beta Q}{p-b}\right], \tag{A6}$$

and $\frac{\partial^2 H(Q,v)}{\partial v^2} < 0$, which implies $H(Q,v)$ is concave. If $\left.\frac{\partial H(Q,v)}{\partial v}\right|_{v=0} = 1 - \frac{1}{1-\alpha} F\left[\frac{(w-b)\beta Q}{p-b}\right] > 0$ and $\left.\frac{\partial H(Q,v)}{\partial v}\right|_{v=(p-w)\beta Q} = 1 - \frac{1}{1-\alpha}F(\beta Q) \leq 0$, that is $\frac{1}{\beta}F^{-1}(1-\alpha) \leq Q < \frac{(p-b)F^{-1}(1-\alpha)}{(w-b)\beta}$, then the optimal solution is $v^* = (p-b)F^{-1}(1-\alpha) - (w-b)\beta Q$.

(iv) $(p-w)\beta Q < v \leq (p-w)Q$.

Here,

$$\begin{aligned} H(Q,v) =& v - \frac{1}{1-\alpha}\left\{\int_0^{\frac{w-b}{p-b}\beta Q}[v-\lambda(p-b)x+\lambda(w-b)\beta Q]dF(x)\right. \\ &\left.+\int_{\frac{w-b}{p-b}\beta Q}^{\beta Q}[v-(p-b)x+(w-b)\beta Q]dF(x)+\int_{\beta Q}^{\frac{v}{p-w}}[v-(p-w)x]dF(x)\right\}. \end{aligned} \tag{A7}$$

Then

$$\frac{\partial H(Q,v)}{\partial v} = 1 - \frac{1}{1-\alpha}F\left(\frac{v}{p-w}\right), \tag{A8}$$

and $\frac{\partial^2 H(Q,v)}{\partial v^2} < 0$, which indicates $H(Q,v)$ is concave. If $\left.\frac{\partial H(Q,v)}{\partial v}\right|_{v=(p-w)\beta Q} = 1 - \frac{1}{1-\alpha}F(\beta Q) > 0$ and $\left.\frac{\partial H(Q,v)}{\partial v}\right|_{v=(p-w)Q} = 1 - \frac{1}{1-\alpha}F(Q) \leq 0$, that is $F^{-1}(1-\alpha) \leq Q < \frac{1}{\beta}F^{-1}(1-\alpha)$, then the optimal solution is $v^* = (p-w)F^{-1}(1-\alpha)$.

(v) $v \geq (p-w)Q$.

In such a case,

$$\begin{aligned} H(Q,v) =& v - \frac{1}{1-\alpha}\left\{\int_0^{\frac{w-b}{p-b}\beta Q}[v-\lambda(p-b)x+\lambda(w-b)\beta Q]dF(x)\right. \\ &+\int_{\frac{w-b}{p-b}\beta Q}^{\beta Q}[v-(p-b)x+(w-b)\beta Q]dF(x) \\ &\left.+\int_{\beta Q}^{Q}[v-(p-w)x]dF(x)+\int_{Q}^{+\infty}[v-(p-w)Q]dF(x)\right\}. \end{aligned} \tag{A9}$$

Then

$$\frac{\partial H(Q,v)}{\partial v} = 1 - \frac{1}{1-\alpha} \leq 0, \tag{A10}$$

which implies $H(Q,v)$ is decreasing in $v$ and the optimal solution is $v^* = (p-w)Q$.

Therefore, we can combine above five cases and express the optimal solution to $\max_{v\in R} H(Q,v)$ as

$$v^* = \begin{cases} \lambda(p-b)F^{-1}(1-\alpha)-\lambda(w-b)\beta Q, & if \quad Q \geq \frac{(p-b)F^{-1}(1-\alpha)}{(w-b)\beta}, \\ (p-b)F^{-1}(1-\alpha)-(w-b)\beta Q, & if \quad \frac{1}{\beta}F^{-1}(1-\alpha) \leq Q < \frac{(p-b)F^{-1}(1-\alpha)}{(w-b)\beta}, \\ (p-w)F^{-1}(1-\alpha), \quad if \quad F^{-1}(1-\alpha) \leq Q < \frac{1}{\beta}F^{-1}(1-\alpha), \\ (p-w)Q, \quad if \quad Q < F^{-1}(1-\alpha). \end{cases} \tag{A11}$$

Next, we will solve the problem $\max_{Q\geq 0} H(Q,v^*)$ and consider the following four cases:

(i) $Q \geq \frac{(p-b)F^{-1}(1-\alpha)}{(w-b)\beta}$.

In such a case, plugging $v^*$ into (A3) we have

$$H(Q,v^*) = \lambda(p-b)F^{-1}(1-\alpha) - \lambda(w-b)\beta Q - \frac{1}{1-\alpha}\int_0^{F^{-1}(1-\alpha)} \lambda(p-b)[F^{-1}(1-\alpha)-x]dF(x). \tag{A12}$$

Then

$$\frac{\partial H(Q,v^*)}{\partial Q} = -\lambda(w-b)\beta < 0, \tag{A13}$$

which implies $H(Q,v^*)$ is decreasing in $Q$.

(ii) $\frac{1}{\beta}F^{-1}(1-\alpha) \leq Q < \frac{(p-b)F^{-1}(1-\alpha)}{(w-b)\beta}$.

Here, plugging $v^*$ into (A5) we have

$$\begin{aligned} H(Q,v^*) =& (p-b)F^{-1}(1-\alpha) - (w-b)\beta Q - \frac{1}{1-\alpha}\left\{\int_0^{\frac{w-b}{p-b}\beta Q}[(p-b)(F^{-1}(1-\alpha)-\lambda x)\right. \\ & \left. +(\lambda-1)(w-b)\beta Q]dF(x) + \int_{\frac{w-b}{p-b}\beta Q}^{F^{-1}(1-\alpha)}(p-b)[F^{-1}(1-\alpha)-x]dF(x)\right\}. \end{aligned} \tag{A14}$$

Then

$$\frac{\partial H(Q,v^*)}{\partial Q} = -(w-b)\beta - \frac{1}{1-\alpha}(\lambda-1)(w-b)\beta F\left(\frac{w-b}{p-b}\beta Q\right) < 0, \tag{A15}$$

and thus $H(Q,v^*)$ is decreasing in $Q$.

(iii) $F^{-1}(1-\alpha) \leq Q < \frac{1}{\beta}F^{-1}(1-\alpha)$.

In such a case, plugging $v^*$ into (A7) we have

$$\begin{aligned} H(Q,v^*) =& (p-w)F^{-1}(1-\alpha) - \frac{1}{1-\alpha}\left\{\int_0^{\frac{w-b}{p-b}\beta Q}[(p-w)F^{-1}(1-\alpha) - \lambda(p-b)x\right. \\ & + \lambda(w-b)\beta Q]dF(x) + \int_{\frac{w-b}{p-b}\beta Q}^{\beta Q}[(p-w)F^{-1}(1-\alpha)-(p-b)x+(w-b)\beta Q]dF(x) \\ & \left. + \int_{\beta Q}^{F^{-1}(1-\alpha)}(p-w)[F^{-1}(1-\alpha)-x]dF(x)\right\}. \end{aligned} \tag{A16}$$

Then

$$\frac{\partial H(Q,v^*)}{\partial Q} = -\frac{1}{1-\alpha}\left[(\lambda-1)(w-b)\beta F\left(\frac{w-b}{p-b}\beta Q\right) + (w-b)\beta F(\beta Q)\right] < 0, \tag{A17}$$

which indicates $H(Q,v^*)$ is decreasing in $Q$.

(iv) $Q < F^{-1}(1-\alpha)$.

Here, plugging $v^*$ into (A9) we have

$$\begin{aligned} H(Q,v^*) =&(p-w)Q - \frac{1}{1-\alpha}\left\{\int_0^{\frac{w-b}{p-b}\beta Q}[(p-w)Q-\lambda(p-b)x+\lambda(w-b)\beta Q]dF(x)\right. \\ &\left.+\int_{\frac{w-b}{p-b}\beta Q}^{\beta Q}[(p-w)Q-(p-b)x+(w-b)\beta Q]dF(x)+\int_{\beta Q}^{Q}(p-w)(Q-x)dF(x)\right\}. \end{aligned} \tag{A18}$$

Then

$$\frac{\partial H(Q,v^*)}{\partial Q} = -\frac{1}{1-\alpha}\left\{(\lambda-1)(w-b)\beta F\left(\frac{w-b}{p-b}\beta Q\right)+\beta(w-b)F(\beta Q)+(p-w)[\alpha-\overline{F}(Q)]\right\}, \tag{A19}$$

and

$$\frac{\partial^2 H(Q,v^*)}{\partial Q^2} = -\frac{1}{1-\alpha}\left[\frac{(w-b)^2}{p-b}(\lambda-1)\beta^2 f\left(\frac{w-b}{p-b}\beta Q\right)+\beta^2(w-b)f(\beta Q)+(p-w)f(Q)\right] < 0, \tag{A20}$$

which implies $H(Q,v^*)$ is concave. Since $\left.\frac{\partial H(Q,v^*)}{\partial Q}\right|_{Q=0} = (p-w) > 0$ and $\left.\frac{\partial H(Q,v^*)}{\partial Q}\right|_{Q=F^{-1}(1-\alpha)} = -\frac{1}{1-\alpha}\left[(\lambda-1)(w-b)\beta F\left(\frac{w-b}{p-b}\beta Q\right)+\beta(w-b)F(\beta Q)\right] < 0$, then there is a unique optimal order quantity $Q^*$ that satisfies $\left.\frac{\partial H(Q,v^*)}{\partial Q}\right|_{Q=Q^*} = 0$, i.e., expression (9).

Combining the above four cases, $Q^*$ is the optimal solution of $\max_{Q\geq 0} H(Q,v^*)$. □

**Proof of Corollary 1.** (i) Let

$$M(Q^*) = (\lambda-1)(w-b)\beta F\left(\frac{w-b}{p-b}\beta Q^*\right)+\beta(w-b)F(\beta Q^*)+(p-w)[\alpha-\overline{F}(Q^*)]. \tag{A21}$$

Then $M(Q^*) = 0$ and

$$\frac{\partial M(Q^*)}{\partial Q^*} = \frac{(\lambda-1)(w-b)^2\beta^2}{p-b}f\left(\frac{w-b}{p-b}\beta Q^*\right)+\beta^2(w-b)f(\beta Q^*)+(p-w)f(Q^*) > 0. \tag{A22}$$

Using the implicit function theorem yields $\frac{dQ^*}{d\lambda} = -\frac{\partial M(Q^*)}{\partial \lambda}\Big/\frac{\partial M(Q^*)}{\partial Q^*}$, where

$$\frac{\partial M(Q^*)}{\partial \lambda} = (w-b)\beta F\left(\frac{w-b}{p-b}\beta Q^*\right) > 0. \tag{A23}$$

Thus, $\frac{dQ^*}{d\lambda} < 0$, which indicates $Q^*$ is decreasing in $\lambda$.

(ii–iii) We have $\frac{dQ^*}{d\alpha} = -\frac{\partial M(Q^*)}{\partial \alpha}\Big/\frac{\partial M(Q^*)}{\partial Q^*}$, $\frac{dQ^*}{d\beta} = -\frac{\partial M(Q^*)}{\partial \beta}\Big/\frac{\partial M(Q^*)}{\partial Q^*}$, $\frac{dQ^*}{dw} = -\frac{\partial M(Q^*)}{\partial w}\Big/\frac{\partial M(Q^*)}{\partial Q^*}$ and $\frac{dQ^*}{db} = -\frac{\partial M(Q^*)}{\partial b}\Big/\frac{\partial M(Q^*)}{\partial Q^*}$, respectively. Since

$$\frac{\partial M(Q^*)}{\partial \alpha} = (p-w) > 0, \tag{A24}$$

$$\begin{aligned} \frac{\partial M(Q^*)}{\partial \beta} =&(\lambda-1)(w-b)F\left(\frac{w-b}{p-b}\beta Q^*\right)+\frac{(\lambda-1)(w-b)^2\beta Q^*}{p-b}f\left(\frac{w-b}{p-b}\beta Q^*\right) \\ &+(w-b)F(\beta Q^*)+\beta(w-b)Q^*f(\beta Q^*) > 0, \end{aligned} \tag{A25}$$

$$\frac{\partial M(Q^*)}{\partial w} =(\lambda-1)\beta F\left(\frac{w-b}{p-b}\beta Q^*\right)+\frac{(\lambda-1)(w-b)\beta^2 Q^*}{p-b}f\left(\frac{w-b}{p-b}\beta Q^*\right)+\beta F(\beta Q^*)-[\alpha-\overline{F}(Q^*) > 0, \tag{A26}$$

and

$$\frac{\partial M(Q^*)}{\partial b} = -\frac{(\lambda-1)(w-b)\beta^2(p-w)Q^*}{(p-b)^2} f\left(\frac{w-b}{p-b}\beta Q^*\right) - (\lambda-1)\beta F\left(\frac{w-b}{p-b}\beta Q^*\right) - \beta F(\beta Q^*) < 0, \tag{A27}$$

then these results can be proved in a similar way. □

**Proof of Theorem 2.** When the retailer's order quantity is $Q^*$, it follows from (8) that

$$\begin{aligned} E[U(\pi(Q^*, D, w, b, \beta))] = & \int_0^{\frac{w-b}{p-b}\beta Q^*} \lambda[(p-b)x - (w-b)\beta Q^*] dF(x) \\ & + \int_{\frac{w-b}{p-b}\beta Q^*}^{\beta Q^*} [(p-b)x - (w-b)\beta Q^*] dF(x) \\ & + \int_{\beta Q^*}^{Q^*} (p-w)x dF(x) + \int_{Q^*}^{+\infty} (p-w)Q^* dF(x). \end{aligned} \tag{A28}$$

From Corollary 1 we have $\frac{\partial Q^*}{\partial \alpha} < 0$ and then $Q^* \leq Q_1^*$. Since

$$\begin{aligned} & \frac{\partial E[U(\pi(Q^*, D, w, b, \beta))]}{\partial Q^*} \\ = & -(\lambda-1)(w-b)\beta F\left(\frac{w-b}{p-b}\beta Q^*\right) - \beta(w-b)F(\beta Q^*) + (p-w)\overline{F}(Q^*) \\ \geq & -(\lambda-1)(w-b)\beta F\left(\frac{w-b}{p-b}\beta Q_1^*\right) - \beta(w-b)F(\beta Q_1^*) + (p-w)\overline{F}(Q_1^*) \\ = & 0, \end{aligned} \tag{A29}$$

then $\frac{dE[U(\pi(Q^*,D,w,b,\beta))]}{d\alpha} = \frac{\partial E[U(\pi(Q^*,D,w,b,\beta))]}{\partial Q^*} \cdot \frac{\partial Q^*}{\partial \alpha} \leq 0$, which implies that $E[U(\pi(Q^*, D, w, b, \beta))]$ is decreasing in $\alpha$.

Similarly, since $\frac{\partial Q^*}{\partial \lambda} < 0$, then $\frac{dE[U(\pi(Q^*,D,w,b,\beta))]}{d\lambda} = \int_0^{\frac{w-b}{p-b}\beta Q^*} [(p-b)x - (w-b)\beta Q^*] dF(x) + \frac{\partial E[U(\pi(Q^*,D,w,b,\beta))]}{\partial Q^*} \cdot \frac{\partial Q^*}{\partial \lambda} \leq 0$, which shows $E[U(\pi(Q^*, D, w, b, \beta))]$ is decreasing in $\lambda$. □

**Proof of Theorem 3.** Let

$$N(w) = (\lambda-1)(w-b)\beta F\left[\frac{\beta(w-b)}{(p-b)} Q_0^*\right] + \beta(w-b)F\left(\beta Q_0^*\right) + (p-w)[\alpha - \overline{F}(Q_0^*)]. \tag{A30}$$

If $\alpha < \overline{F}(Q_0^*)$, then

$$\begin{aligned} \frac{dN(w)}{dw} = & (\lambda-1)\beta F\left[\frac{\beta(w-b)}{p-b} Q_0^*\right] + \frac{(\lambda-1)\beta^2(w-b)Q_0^*}{p-b} f\left[\frac{\beta(w-b)}{p-b} Q_0^*\right] \\ & + \beta F\left(\beta Q_0^*\right) - [\alpha - \overline{F}(Q_0^*)] > 0, \end{aligned} \tag{A31}$$

which shows $N(w)$ is increasing in $w$. Moreover, we have

$$N(b) = (p-b)[\alpha - \overline{F}(Q_0^*)] < 0, \tag{A32}$$

and

$$N(p) = \lambda(p-b)\beta F\left(\beta Q_0^*\right) > 0. \tag{A33}$$

Thus, there is a unique wholesale price $w^*(b, \beta) \in (b, p)$ that satisfies $N(w^*) = 0$, i.e., expression (13).

If $\alpha \geq \overline{F}(Q_0^*)$, then $N(w)$ is always positive and there is not a coordinating wholesale price. □

**Proof of Corollary 3.** (i) Since $N(w^*) = 0$, by using the implicit function theorem we can get $\frac{dw^*}{d\lambda} = -\frac{\partial N(w^*)}{\partial \lambda} \Big/ \frac{\partial N(w^*)}{\partial w^*}$, where

$$\frac{\partial N(w^*)}{\partial \lambda} = (w^* - b)\beta F\left[\frac{\beta(w^* - b)}{p - b} Q_0^*\right] > 0. \tag{A34}$$

It follows from (A31) that $\frac{\partial N(w^*)}{\partial w^*} > 0$, then $\frac{dw^*}{d\lambda} < 0$, which implies $w^*$ is decreasing in $\lambda$.

(ii–iii) We have $\frac{dw^*}{d\alpha} = -\frac{\partial N(w^*)}{\partial \alpha} \Big/ \frac{\partial N(w^*)}{\partial w^*}$, $\frac{dw^*}{d\beta} = -\frac{\partial N(w^*)}{\partial \beta} \Big/ \frac{\partial N(w^*)}{\partial w^*}$, and $\frac{dw^*}{db} = -\frac{\partial N(w^*)}{\partial b} \Big/ \frac{\partial N(w^*)}{\partial w^*}$, respectively. Since

$$\frac{\partial N(w^*)}{\partial \alpha} = p - w^* > 0, \tag{A35}$$

$$\frac{\partial N(w^*)}{\partial \beta} = (\lambda - 1)(w^* - b)F\left[\frac{\beta(w^* - b)}{p - b} Q_0^*\right] + \frac{(\lambda - 1)(w^* - b)^2 \beta Q_0^*}{p - b} f\left[\frac{\beta(w^* - b)}{p - b} Q_0^*\right] > 0, \tag{A36}$$

and

$$\begin{aligned}\frac{\partial N(w^*)}{\partial b} =& -(\lambda - 1)\beta F\left[\frac{\beta(w^* - b)}{p - b} Q_0^*\right] - \beta F\left(\beta Q_0^*\right) \\ & - \frac{(\lambda - 1)(w^* - b)\beta^2 (p - w^*) Q_0^*}{(p - b)^2} f\left[\frac{\beta(w^* - b)}{p - b} Q_0^*\right] < 0,\end{aligned} \tag{A37}$$

then these results can be obtained in a similar way. □

**Proof of Theorem 4.** It follows from Corollary 4 that $w_B^*(b) \le w_B^*(\bar{b})$ and $w_Q^*(\beta) \le w_Q^*(\underline{\beta})$. Then the first inequality implies that feasible coordinating buyback or quantity flexibility contract does not exist. Similarly, from Corollary 3 we have $w^*(b, \beta) \le w^*(\bar{b}, \underline{\beta})$, then the second inequality implies that a feasible coordinating combined contract exists. □

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