Article

Photo-Thermal Interactions in a Semiconducting Media with a Spherical Cavity under Hyperbolic Two-Temperature Model

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Abstract: This article highlights the study of photo-thermoelastic interaction in an unbounded semiconductor medium containing a spherical cavity. This problem is solved using the new hyperbolic two-temperature model. The bounding surface of the cavity is traction free and loaded thermally by exponentially decaying pulse boundary heat flux. In addition, the carrier density is prescribed on the inner surface of the cavity in terms of the recombination speed. The techniques of Laplace transforms are used to get the analytical solutions of the problem in the transformed domain by the eigenvalues method. The inversions of Laplace transformations have been carried out numerically. The outcomes also display that the analytical schemes can overcome the mathematical problem to analyze this problem. Numerical outcomes for a semiconductor material are performed and demonstrated graphically. According to the numerical results, this new hyperbolic two-temperature model of thermoelasticity offers finite speed of the thermal wave and mechanical wave propagation.

Keywords: Laplace transforms; hyperbolic two-temperature; spherical cavity; eigenvalues method; semiconductor medium

1. Introduction

The thermoelasticity model with the classical two-temperature model is presented by Williams and Gurtin [1], Chen and Gurtin [2] and Chen et al. [3]; when using another temperature depending on the classic two temperatures (thermodynamic temperature T^* and conductivity temperature ϕ^*), the classical inequality of Clausius-Duhem has been substituted. The first is due to the thermal processes and the second is due to the inherent mechanical processes between the particles and the elastic material layers. Only in the last decade were these processes observed, developed in many works in the theory of thermoelasticity under two temperatures, and applications found mainly in problems in which the discontinuities of restrictions have no physical interpretations. Ezzat et al. [4] investigated the effects of fractional order and two-temperature theory in magneto-thermoelasticity under dual-phase-lag heat transfer. Ezzat and Youssef [5] studied the two-temperature model as a three-dimensional problem for a thermoelastic plane due to ramp type heating. Deswal and Kalkal [6] have used the state space formulations to study the impacts of initial stresses under two-temperature magneto-thermoelasticity. Abbas et al. [7] have studied the response of a heating source in a transversely isotropic thermoelastic medium under two temperatures without energy dissipation. Youssef and El-Bary [8] have presented new generalized thermoelastic interactions under the hyperbolic two-temperature model.

In integrated circuit fabrication, silicon is normally used as the basis material and produces an N-type or P-type semiconductor by adopting diffusion technology. On applying the thermal effects, the surface of steel parts can be carburized by applying the diffusion principle. In the electronics

domain, the diffusion movement is also utilized in forming a semiconductor PN junction with free electrons and holes. The understanding of transport phenomena in solids through the expansions of spatially resolved in situ investigations has gained major attention. This paper describes investigations that try to measure the transport process based on the principles of optical beams deviation through a photothermal technique that can be considered as an expansion of the photothermal deviation approach. These techniques are characterized as having no contact and directly producing the parameters of electronics and heat transport on the surface of the semiconductor or at the interface and within most of the semiconducting material. Marin et al. [9,10] have investigated some problems for elastic dipolar bodies.

Pure silicon is an intrinsic semiconductor and is used in a wide range of the semiconductor industry, for instance, for monocrystal lines Si is used to produce silicon wafers. Todorovic [11,12] has studied the plasma and thermal-elastic waves in semiconductor material. The change in the propagation of thermal, elastic and plasma waves are due to linear coupling through heat and mass transport (i.e., thermal-diffusive). In materials science, the heating conductivity variables that depend on temperature are very significant and have numerous applications. Song et al. [13] studied vibration using the generalized thermoelastic model subjected to optically excited semiconductors as micro-conductors. Yasein et al. [14] studied the influence of variable thermal conductivity of one-dimensional semiconducting media under photothermal excitations due to ramp type heating. Ailawalia and Kumar [15] applied ramp type heating to study the semiconducting media under the photothermal theory. Abbas and Hobiny [16] studied the photo-thermal-elastic interaction in an unbounded semiconducting medium with spherical cavity, due to pulse heat flux, by using the theory of coupled plasma, thermal, and elastic wave. Yasein, et al. [14] studied the influence of variable thermal conductivity of a semiconductor elastic medium during photothermal excitation subjected to thermal ramp type. Lotfy et al. [17] investigated the response of a semiconductor medium of variable thermal conductivity due to laser pulses with two temperatures through the photothermal process. Many researchers [18–26] have solved various problems under generalized thermoelastic models. In the Laplace domain, the eigenvalues approach gave an analytical solution without any supposed restrictions on the factual physical variables.

The main objective of the present contribution is to study the effects of hyperbolic two-temperature in a semiconductor medium with a spherical cavity under pulse heat flux. The basic equations are processed by using the eigenvalues method and Laplace transforms. Employing Laplace transform as tools, the solutions of the thermo-physical quantities have been obtained in the transformed domain, and the numerical inversion of the Laplace transform is carried out incorporating a method proposed by Stehfest. Finally, the outcomes are illustrated graphically to show the differences among the hyperbolic two-temperature model, the classical two-temperature model, and the one temperature model.

2. Formulation of the Problem

The basic governing equations under the hyperbolic two-temperature photothermal theory for a homogeneous, isotropic semiconductor medium in the absence of body forces and heat sources are given by [8,27,28]:

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} - \gamma_n N_{,i} - \gamma_t T_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (1)$$

$$D_e N_{,jj} - \frac{N}{\tau} + \frac{k}{\tau} T = \frac{\partial N}{\partial t}, \quad (2)$$

$$K \phi_{,jj} + \frac{E_g}{\tau} N = \rho c_e \frac{\partial T}{\partial t} + \gamma_t T_o \frac{\partial u_{j,j}}{\partial t}, \quad (3)$$

$$\ddot{\phi} - \ddot{T} = a \phi_{,jj}, \quad (4)$$

where $i, j, k = 1, 2, 3$, $N = n - n_o$, n_o is the concentration of carrier at equilibrium, u_i are the components of displacement, $T = T^* - T_o$, T^* is the thermodynamic temperature increment, $\phi = \phi^* - T_o$, ϕ^* is the

temperature conductivity increment, ρ is the medium density, T_0 is the reference temperature, $a > 0$ is the parameter of two-temperature, λ, μ are the Lamé's constants, σ_{ij} are the components of stresses, c_e is the specific heat at constant strain, $\gamma_n = (3\lambda + 2\mu)d_n$, d_n is the coefficient of electronic deformation, $\gamma_t = (3\lambda + 2\mu)\alpha_t$, α_t is the linear thermal expansion coefficient, K is the thermal conductivity, τ is the lifetime of a photogenerated carrier, D_e is the coefficient of carrier diffusion, $k = \frac{\partial n_0}{\partial T}$ is the coupling parameter of thermal activation [28], r is the position vector, and t is the time. Let us consider an isotropic, homogeneous unbounded semiconductor medium with a spherical cavity, whose state can be expressed in terms of the time t and the space variable r which occupies the region $R \leq r < \infty$. Due to symmetry, the radial displacement $u_r = u(r, t)$ is only non-vanishing, therefore the Equations (1)–(4) can be written as:

$$(\lambda + 2\mu)\left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \gamma_n \frac{\partial N}{\partial r} - \gamma_t \frac{\partial T}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (5)$$

$$D_e\left(\frac{\partial^2 N}{\partial r^2} + \frac{2}{r}\frac{\partial N}{\partial r}\right) = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \frac{k}{\tau}T, \quad (6)$$

$$K\left(\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r}\right) = \rho c_e \frac{\partial T}{\partial t} - \frac{E_g}{\tau}N + \gamma_t T_0 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right), \quad (7)$$

$$\frac{\partial^2 T}{\partial t^2} = \frac{\partial^2 \phi}{\partial t^2} - a\left(\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r}\right), \quad (8)$$

with

$$\sigma_{rr} = (\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda\frac{2u}{r} - \gamma_n N - \gamma_t T \text{ and} \quad (9)$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = (\lambda + 2\mu)\frac{u}{r} + \lambda\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \gamma_n N - \gamma_t T. \quad (10)$$

3. Initial and Boundary Conditions

Take into account the boundary with initial conditions before solving the problem. In this problem, the initial conditions are assumed to be homogeneous and are supplemented by considering the boundary on the internal surface of the cavity. It is stress free, therefore the boundary condition can be given as:

$$\sigma_{rr}(R, t) = 0. \quad (11)$$

The conditions on the inner surface of the cavity are due to heat flux with an exponentially decaying pulse [29]

$$-K \frac{\partial \phi(r, t)}{\partial r} \Big|_{r=R} = q_0 \frac{t^2 e^{-\frac{t}{t_p}}}{16t_p^2}, \quad (12)$$

where t_p is the pulsing heat flux characteristic time and q_0 is a constant. The carrier density boundary condition on the inner surface of the cavity may be expressed by

$$D_e \frac{\partial N(r, t)}{\partial r} \Big|_{r=R} = s_a N(R, t), \quad (13)$$

where s_a is the speed of recombination on the surface. For appropriateness, the non-dimensional parameters can be taken as

$$\begin{aligned} N^\circ &= \frac{N}{n_0}, \phi^\circ = \frac{\phi}{T_0}, T^\circ = \frac{T}{T_0}, (r^\circ, u^\circ) = \eta c(r, u), (\sigma_{rr}^\circ, \sigma_{\theta\theta}^\circ) = \frac{(\sigma_{rr}, \sigma_{\theta\theta})}{\lambda + 2\mu}, \\ (t^\circ, \tau^\circ, t_p^\circ) &= \eta c^2(t, \tau, t_p), q_0^\circ = \frac{q_0}{\eta c T_0 K}, a^\circ = \frac{a}{c^2} \end{aligned} \quad (14)$$

where $\eta = \frac{\rho c_e}{K}$ and $c^2 = \frac{\lambda + 2\mu}{\rho}$.

In these dimensionless terms of the physical quantities in Equation (14), the above Equations (6)–(14) can be expressed as (after neglecting the superscript ° for convenience)

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} - \frac{2u}{r^2} - n_1 \frac{\partial N}{\partial r} - n_2 \frac{\partial T}{\partial r} = \frac{\partial^2 u}{\partial t^2}, \quad (15)$$

$$\frac{\partial^2 N}{\partial r^2} + \frac{2}{r} \frac{\partial N}{\partial r} = n_3 \frac{\partial N}{\partial t} + \frac{n_3}{\tau} N - \frac{n_4}{\tau} T, \quad (16)$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} = \frac{\partial T}{\partial t} - \frac{n_5}{\tau} N + v_6 \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial r} + \frac{2u}{r} \right). \quad (17)$$

$$\frac{\partial^2 T}{\partial t^2} = \frac{\partial^2 \phi}{\partial t^2} - a \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} \right), \quad (18)$$

$$\sigma_{rr} = \frac{\partial u}{\partial r} + n_7 \frac{2u}{r} - n_1 N - n_2 T, \quad (19)$$

$$\sigma_{\theta\theta} = n_7 \frac{\partial u}{\partial r} + (1 + n_7) \frac{u}{r} - n_1 N - n_2 T, \quad (20)$$

$$\left. \frac{\partial \phi(r, t)}{\partial r} \right|_{r=R} = -q_0 \frac{t^2 e^{-\frac{t}{t_p}}}{16t_p^2}, \quad \sigma_{rr}(R, t) = 0, \quad \left. \frac{\partial N(r, t)}{\partial r} \right|_{r=R} = f_8 N(R, t), \quad (21)$$

where $n_1 = \frac{n_0 \gamma n}{\lambda + 2\mu}$, $n_2 = \frac{T_0 \gamma t}{\lambda + 2\mu}$, $n_3 = \frac{1}{\eta D_e}$, $n_4 = \frac{k T_0}{n_0 \eta D_e}$, $n_5 = \frac{n_0 E_g}{\rho c_e T_0}$, $n_6 = \frac{\gamma t}{\rho c_e}$, $n_7 = \frac{\lambda}{\lambda + 2\mu}$, $n_8 = \frac{S_a}{\eta c D_e}$.

4. Method of Solution

For the function $h(r, t)$, the Laplace transform is defined by

$$\bar{h}(r, s) = L[h(r, t)] = \int_0^\infty h(r, t) e^{-st} dt, \quad s > 0, \quad (22)$$

where s is the parameter of the Laplace transform. Hence, the above equations can be written in the following forms

$$\frac{d^2 \bar{u}}{dr^2} + \frac{2}{r} \frac{d\bar{u}}{dr} - \frac{2\bar{u}}{r^2} - n_1 \frac{d\bar{N}}{dr} - n_2 \frac{d\bar{T}}{dr} = s^2 \bar{u}, \quad (23)$$

$$\frac{d^2 \bar{N}}{dr^2} + \frac{2}{r} \frac{d\bar{N}}{dr} = n_3 \left(s + \frac{1}{\tau} \right) \bar{N} - \frac{n_4}{\tau} \bar{T}, \quad (24)$$

$$\frac{d^2 \bar{\phi}}{dr^2} + \frac{2}{r} \frac{d\bar{\phi}}{dr} = s \bar{T} - \frac{n_5}{\tau} \bar{N} + s n_6 \left(\frac{d\bar{u}}{dr} + \frac{2\bar{u}}{r} \right), \quad (25)$$

$$\bar{T} = \bar{\phi} - \frac{a}{s^2} \left(\frac{d^2 \bar{\phi}}{dr^2} + \frac{2}{r} \frac{d\bar{\phi}}{dr} \right), \quad (26)$$

$$\bar{\sigma}_{rr} = \frac{d\bar{u}}{dr} + n_7 \frac{2\bar{u}}{r} - n_1 \bar{N} - n_2 \bar{T}, \quad (27)$$

$$\bar{\sigma}_{\theta\theta} = n_7 \frac{d\bar{u}}{dr} + (1 + n_7) \frac{\bar{u}}{r} - n_1 \bar{N} - n_2 \bar{T}, \quad (28)$$

$$\left. \frac{d\bar{\phi}(r, t)}{dr} \right|_{r=R} = \frac{-q_0 t_p}{8(st_p + 1)^3}, \quad \bar{\sigma}_{rr}(R, r) = 0, \quad \left. \frac{d\bar{N}(r, t)}{dr} \right|_{r=R} = n_8 \bar{N}(R, t). \quad (29)$$

Differentiating Equations (23)–(25) with respect to r and using Equation (26), which can be written as

$$\frac{d^2 \bar{u}}{dr^2} + \frac{2}{r} \frac{d\bar{u}}{dr} - \frac{2\bar{u}}{r^2} = a_1 \bar{u} + a_2 \frac{d\bar{N}}{dr} + a_3 \frac{d\bar{\phi}}{dr}, \quad (30)$$

$$\frac{d^2}{dr^2} \left(\frac{d\bar{N}}{dr} \right) + \frac{2}{r} \frac{d}{dr} \left(\frac{d\bar{N}}{dr} \right) - \frac{2}{r^2} \left(\frac{d\bar{N}}{dr} \right) = a_4 \bar{u} + a_5 \frac{d\bar{N}}{dr} + a_6 \frac{d\bar{\phi}}{dr}, \quad (31)$$

$$\frac{d^2}{dr^2} \left(\frac{d\bar{\phi}}{dr} \right) + \frac{2}{r} \frac{d}{dr} \left(\frac{d\bar{\phi}}{dr} \right) - \frac{2}{r^2} \left(\frac{d\bar{\phi}}{dr} \right) = a_7 \bar{u} + a_8 \frac{d\bar{N}}{dr} + a_9 \frac{d\bar{\phi}}{dr}, \quad (32)$$

where

$$\begin{aligned} a_1 &= \left(s^2 - n_2 a_7 \frac{a}{s^2} \right), \quad a_2 = \left(n_1 - n_2 a_8 \frac{a}{s^2} \right), \quad a_3 = n_2 \left(1 - \frac{a}{s^2} a_9 \right) \\ a_4 &= \frac{n_4}{\tau} \frac{a}{s^2} a_7, \quad a_5 = \left(n_3 \left(s + \frac{1}{\tau} \right) + \frac{n_4}{\tau} \frac{a}{s^2} a_8 \right), \quad a_6 = \frac{n_4}{\tau} \left(\frac{a}{s^2} a_9 - 1 \right) \\ a_7 &= \frac{n_6 s^3}{\left(1 + \frac{a}{s} (1 + n_6 n_2) \right)}, \quad a_8 = \frac{\left(s n_6 n_1 - \frac{n_5}{\tau} \right)}{\left(1 + \frac{a}{s} (1 + n_6 n_2) \right)}, \quad a_9 = \frac{s(1 + n_6 n_2)}{\left(1 + \frac{a}{s} (1 + n_6 n_2) \right)}. \end{aligned}$$

Now, it is possible to get the solutions of the coupled system of differential Equations (30)–(32) by the eigenvalues method proposed in [30–34]. From Equations (30)–(32), the matrix-vectors can be expressed in the form

$$LV = AV, \quad (33)$$

where $L = \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{2}{r^2}$, $V = \begin{bmatrix} \bar{u} & \frac{d\bar{N}}{dr} & \frac{d\bar{\phi}}{dr} \end{bmatrix}^T$ and $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$.

The characteristic equation of matrix A can be expressed as

$$\begin{aligned} \omega^3 - \omega^2(a_5 + a_9 + a_1) + \omega(a_5 a_9 - a_9 a_5 + a_5 a_1 + a_9 a_1 - a_4 a_2 - a_7 a_3) + a_6 a_8 a_1 - \\ a_5 a_9 a_1 - a_6 a_7 a_2 + a_4 a_9 a_2 + a_5 a_7 a_3 - a_4 a_8 a_3 = 0. \end{aligned} \quad (34)$$

The eigenvalues of matrix A are the three roots of Equation (34) which are defined by the form $s, \omega_1, \omega_2, \omega_3$. Thus, the corresponding eigenvectors Y can be computed by:

$$Y = \begin{pmatrix} -a_6 a_2 + (-\omega + a_5) a_3 \\ a_6 (-\omega + a_1) - a_4 a_3 \\ -\omega^2 + a_5 (\omega - a_1) + \omega a_1 + a_4 a_2 \end{pmatrix}. \quad (35)$$

The solutions of Equation (33) which are bounded as $r \rightarrow \infty$ can be given by

$$V(r, s) = \sum_{i=1}^3 A_i Y_i X_i r^{-1/2} K_{3/2}(m_i r), \quad (36)$$

where $m_i = \sqrt{\omega_i}$, R_1, R_2 and R_3 are constants that can be determined by using the boundary conditions of the problem and $K_{3/2}$ is the modification of Bessel's function of order $\frac{3}{2}$. Thus, the solutions of variables with respect to r and s can take the forms:

$$\bar{u}(r, s) = \sum_{i=1}^3 A_i U_i r^{-1/2} K_{3/2}(m_i r), \quad (37)$$

$$\bar{N}(r, s) = - \sum_{i=1}^3 A_i N_i r^{-1/2} K_{1/2}(m_i r), \quad (38)$$

$$\bar{\phi}(r, s) = - \sum_{i=1}^3 A_i T_i r^{-1/2} K_{1/2}(m_i r), \quad (39)$$

$$\bar{T}(r, s) = - \sum_{i=1}^3 A_i \frac{T_i(1 - \frac{a}{s^2} m_i^2)}{r^{1/2} m_i} K_{1/2}(m_i r), \quad (40)$$

$$\bar{\sigma}_{rr}(r, s) = \sum_{i=1}^3 A_i \left(\frac{n_1 N_i + (1 - \frac{a}{s^2} m_i^2) n_2 T_i - m_i^2 U_i}{r^{1/2} m_i} K_{1/2}(m_i r) - \frac{2(1 - n_7) U_i}{r^{3/2}} K_{3/2}(m_i r) \right), \quad (41)$$

$$\bar{\sigma}_{\theta\theta}(r, s) = \bar{\sigma}_{\phi\phi}(r, s) = \sum_{i=1}^3 A_i \left(\frac{n_1 N_i + (1 - \frac{a}{s^2} m_i^2) n_2 T_i - m_i^2 n_7 U_i}{r^{1/2} m_i} K_{1/2}(m_i r) + \frac{(1 - n_7) U_i}{r^{3/2}} K_{3/2}(m_i r) \right). \quad (42)$$

5. Numerical Results and Discussions

Now, we consider a numerical example; for computational purposes, a silicon (Si) like material has been considered. Assuming that an isotropic semiconductor, silicon-like material with the physical constants that are listed below [35]:

$$\begin{aligned} \rho &= 2330 (kg) (m^{-3}), \quad c_e = 695 (J) (kg^{-1}) (K^{-1}), \quad E_g = 1.11 (eV), \quad t_p = 1.2, \\ \lambda &= 3.64 \times 10^{10} (N) (m^{-2}), \quad d_n = -9 \times 10^{-31} (m^3), \quad \alpha_t = 3 \times 10^{-6} (K^{-1}), \\ \mu &= 5.46 \times 10^{10} (N) (m^{-2}), \quad \tau = 5 \times 10^{-5} (s), \quad s_o = 2 (m) (s^{-1}), \quad n_o = 10^{20} (m^{-3}), \\ D_e &= 2.510^{-3} (m^2) (s^{-1}), \quad T_o = 300 (K), \quad t = 2, \quad a = 0.3. \end{aligned}$$

A numerical inversion method adopted the general solutions of the thermodynamic and conductive temperatures, the carrier density, displacement, and the stress distributions; Stehfest's [36] numerical inversion method has been chosen. In this method, the Laplace transforms inverse for $\bar{h}(x, y, s)$ are approximated as

$$h(x, y, t) = \frac{\ln(2)}{t} \sum_{n=1}^N V_n \bar{h}\left(x, y, n \frac{\ln(2)}{t}\right), \quad (43)$$

where

$$V_n = (-1)^{(\frac{N}{2}+1)} \sum_{p=\frac{n+1}{2}}^{\min(n, \frac{N}{2})} \frac{(2p)! p^{(\frac{N}{2}+1)}}{p!(n-p)! (\frac{N}{2}-p)! (2n-1)!}, \quad (44)$$

where N is the term numbers. Based on the Laplace transform's inverse and the above set of constants, the calculations of physical variables with respect to the radial distance r under one temperature, classical two-temperature and hyperbolic two-temperature are presented in Figures 1–6. The variation of conductive temperature, the variation of thermodynamic temperature, the variation of radial displacement, the variation of carrier density, and the distributions of radial and hoop stresses along the radial distance r under the coupled photothermal theories are calculated numerically. Figure 1 displays the variations of conductive temperature versus the radial distance r . It is clear that the conductive temperature has maximum value at the boundary $r = 1$, after that it progressively decreases with the increasing of the radial distance r , to eventually come to zero. Figure 2 shows the thermodynamic temperature variations versus the radial distance r . It is observed that it starts from the highest values at the boundary $r = 1$ and decreases with the increasing of the radial distance r until it reaches zero for sufficiently large values of r . Figure 3 explains the carrier density variations with respect to the radial distance r . It is clear that the density of the carrier begins with its highest values at the boundary $r = 1$, then it progressively decreases with the increasing of the radial distance r until it comes to zero. Figure 4 depicts the variations of radial displacement versus the radial distance r . It can be observed that the displacement attains a maximum negative value then it increases progressively up to peak values in a particular location proximately close to the surface, and then decreases to zero. Figure 5 shows the variations of radial stress with respect to the radial distance r . It is noticed that the radial stress, permanently begins from zero and drops to zero to obey the boundary conditions.

The hoop stress variations versus the radial distance r is shown in Figure 6. It can be noticed that the hoop stress attains a maximum negative value and then continuously increases to zero. Finally, in compression between the solutions, one can conclude that considering the coupled photothermal theory with thermal and conductive temperatures is an important phenomenon and has a great effect on the distribution of field quantities.

Figure 1. The variation of conductive temperature with distance for different models.

Figure 2. The variation of thermodynamic temperature with distance for different models.

Figure 3. The variation of carrier density with distance for different models.

Figure 4. The variation of displacement with distance for different models.

Figure 5. The variation of radial stress with distance for different models.

Figure 6. The variation of hoop stress with distance for different models.

6. Conclusions

Based on the new hyperbolic two-temperature model of thermoelasticity, the variations of the conductive and the thermodynamic temperatures, the variation of radial displacement, and the variation of carrier density, the distributions of radial and hoop stresses in a semiconductor medium are studied. The non-dimensional result has been solved, employing the Laplace transform and the eigenvalues approach. In comparing the solutions, one can conclude that the coupled photo-thermoelastic theory with hyperbolic thermal and conductive temperatures is an important phenomenon and has important effects on the distribution of field quantities. We believe that the analysis of the present study will be useful to understand the basic features of this new model for heat conduction.

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