Controlling Waste and Carbon Emission for a Sustainable Closed-Loop Supply Chain Management under a Cap-and-Trade Strategy

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Abstract: Considering the increasing number of end-of-life goods in the context of improving the ambience and health of a population and their destructive impacts, recycling strategies are important for industries and organizations. In this article, a closed-loop supply chain management containing a single manufacturer, a single retailer, and a third party is introduced in which the manufacturer first propagates newly finished goods and then dispatches some of the finished goods to the retailer considering a single-setup multi-delivery policy. Due to shipping, carbon emission is taken into account as well as a carbon emission trading mechanism to curb the amount of carbon emissions by the retailer. For recycling through collection, inspection, remanufacturing, and landfill, the third party collects the end-of-life goods from its customers and ships perfect products to the manufacturer after a two-stage inspection. In this model, major sources of emissions such as shipping, replenishment orders, and inventory have been taken care of. The minimizing of the total cost relating to the container capacity, shipment numbers, and replenishment cycle length is the main objective of the closed-loop supply chain management for making the system more profitable. Expository numerical explorations, analysis, and graphic representations are conferred to elucidate this model, and it is observed that this model saves some percentage of the cost compared to the existing literature.

Keywords: supply chain management; reverse logistics; remanufacturing; inventory control; production

1. Introduction

A sustainable supply chain (SSC) gives maximum profit and reduces both the expenditure of resources and the environmental impact by using smart technology (Tayyab et al. [1]). Tiwari et al. [2] developed a sustainable ordering policy under trade credit. Due to careless use of smart technology, the life cycle of many products decreases the recyclability of end-of-life (EOL) goods. To lessen the environmental and atmospheric impact, much contemplation has been made of reverse logistics (RL) for waste management. Although reverse operations reduce EOL waste through remanufacturing, helping to contribute to a SSC (Cárdenas-Barrón et al. [3]) there must be reimbursement for the costs of collection, inspection, landfill, buying used products, remanufacturing, and inventory handling.

Thus, shipping costs must be added into closed-loop supply chain management (CLSCM), which is conventionally introduced as a component of the ordering or setup cost in a traditional supply chain (SC) model (Cárdenas-Barrón and Sana [4]). Taleizadeh et al. [5] considered an improved strategy for

replenishment frequency optimization through a vendor-managed-inventory policy (VMIP). Recently, global SC models have applied single-setup multi-delivery (SSMD) policies, because of the increase to the number of shipments. Again, as the distances between players are not the same, a variable shipment cost is introduced on top of fixed shipment cost. However, increasing the number of shipments increases the percentage of carbon emissions. Recently, Hota et al. [6] discussed the effect of variable transportation and carbon emissions in an unreliable SSC.

To reduce global warming, many international organizations such as the United Nations and the European Union have introduced legislative mechanisms to alleviate the volume of carbon emission. Among these mechanisms, trading carbon emission is usually accepted as an important operative market-based strategy that has been embraced by many governments. The strategy of trading carbon has been introduced using the name cap-and-trade (CAPT). A firm/industry is allotted a cap or limit to emissions of carbon. If the volume of carbon emissions surpasses the carbon limit, the company can purchase the right to emit additional carbon (Daryanto et al. [7]). Many industries neglect this policy to protect the environment. According to a survey report by Accenture, only 10% of companies designed their supply chain by actively considering carbon footprint and fulfilled fruitful sustainability initiatives. More than 37% of SC managers have not properly considered SC emissions in their SC network. In fact, 70% of directors have not created an authentic approach to establish sustainability in their organizations. Thus, extensive research is essential to acquire the knowledge about supply chains regarding the dominance of sustainability in decision-making. Mishra et al. [8] discussed a green supply chain management (GSCM) model to control carbon emissions by considering carbon tax and CAPT policy.

This paper represents a CLSCM with objectives of maintaining environmental impact and enhancing resource use. Carbon emissions, shipping, carbon CAPT strategy, landfill, and additional costs related to manufacturing are introduced in this study. Ullah and Sarkar [9] developed a hybrid manufacturing/remanufacturing system considering smart technology. A hybrid system is discussed, where the return rate is high, and the manufacturer fulfills the refuted demand through remanufacturing and then the remaining demand comes from newly finished products. When firms and warehouses consider carbon emissions and shipment costs, the results may vary for relatively high costs and emissions. Taleizadeh et al. [10] suggested that a pure policy is more profitable than a hybrid policy. Ahamed and Sarkar [11] designed a multi-objective SSC model of second-generation biofuel supply under carbon tax and CAPT policy. Sarkar et al. [12] described a multi-attribute CLSCM for self-healing polymer-based returnable transport packaging under budget and storage constraints, but they did not consider the CAPT strategy and landfill concept in a CLSC involving third party logistics (3PL). Therefore, this research completes the research gap. After detecting which items are imperfect used products, they are used as landfill, which is a new purpose of this paper. In this study, the CAPT mechanism is considered for the retailer to allocate a carbon emission limit associated with the shipment and carbon emission costs. The minimizing of the total (expected) cost with respect to contemporary optimization of container capacity, the required shipment numbers, and the cycle time of the players of the CLSCM is the main objective in order to increase profit.

2. Literature Review

An SSC is described as a collaboration between players to communicate planning, environmentally friendly materials, and information with the end customer. SSC is also described as cooperation among players for transferring merchandise to the exact location at a specific time so that the entire expenses of the SSC are optimized (Sarkar and Majumder [13]). The main intention of SSC is to optimize costs. Due to the increasing number of industries, overall waste has gradually increased. Sustainability is defined as the concern of each firm to reach economic goals and manage societal manifestation of waste. This strategy may affect the activity of the other members. Thus, more extensive research to optimize the performance of whole SSC is needed; herein, many researchers are cited regarding

SSC and inventory (Sarkar [14]; Cárdenas-Barrón et al. [15]; Ullah and Sarkar [9]; Sarkar et al. [16]; Sarkar et al. [17]; Cárdenas-Barrón and Sana [18]).

Environmental issues are now becoming significant. Therefore, industries and organizations are researching their own techniques to assure more environmentally friendly products. Many researchers have widely studied GSCM and CLSCM. To protect the ambience and atmosphere from the destructive effects of EOL products, much attention is paid to CLSCM through recycling. CLSCM is characterized as the management and activity of organizations to maximize profit and productive value recovery by considering the life cycle of different returnable products (Guide and Wassenhove [19]). Saxena et al. [20] considered a CLSCM with primary/secondary-market concepts for manufacturing and remanufacturing. A special compensation to curb global warming and landfill, as well as to reduce waste by returning EOL products, was made by (Govindan and Soleimani [21]; Govindan et al. [22]). The performance of the RL method depends on the customer's consent to send back used products to restore their life cycle (Saharudin et al. [23]). Huang et al. [24] discussed coordination based on the quantity discount strategy to reduce a customer's false failure return. Heydari et al. [25] proposed a reverse logistics supply chain (RLSC) and CLSCM by considering government roles with a price discount offered by the retailer. In this paper, a CLSCM is designed, where 3PL gives a price discount to the manufacturer to buy perfect used products from the manufacturer. Govindan and Popiuc [26] developed the idea of RL in supply chains (two or three echelons) by premising a revenue-dividing contract.

Remanufacturing is an important aspect for sustainable improvement, as it extends the product's life cycle and decreases resource waste. Thus, to reduce waste, the effect of inspection plays an important role. Konstantaras et al. [27] elaborated on a large CLSCM model to initiate several inspection and sorting policies. But they did not make zero waste within the CLSCM. Sarkar et al. [16] showed that the returnable number of used products is directly proportional to customer demand. Never are all products returned.

Recently, global SSC models have considered SSMD policies to transport products and reduce holding cost. Therefore, the number of shipments increases. As the distances between players and the numbers of containers are not the same, variable shipment costs are added along with fixed shipment costs. Khouja [28] and Cárdenas-Barrón [29] considered a fixed transportation cost in their SSC. Ertogral et al. [30] developed a vendor–buyer inventory model considering shipment lot size and shipment cost. Sarkar [14] proposed a two-echelon supply chain management (SCM) with a variable shipment cost using a SSMD policy. Sarkar et al. [31] discussed variable shipment strategy depending on the lot size in a three-layer SCM.

As the shipment numbers increase for an SSMD policy, it affects the environment through carbon emissions. Sarkar et al. [31] designed a three-layer SC model with a variable carbon emission cost depending on the demand rate and fixed carbon emission cost. This study is initiated by Hua et al. [32], who extended the basic SCM model using the concept of carbon emissions. Shaw et al. [33] discussed the joint effect of carbon emissions in inventory management. Environmental issues have received little study in CLSC. Sarkar et al. [16] made an economic and environmental assessment of a three-layer CLSCM considering remanufacturing, returnable transport items, and fixed-plus-variable carbon emission costs with the assumption that the variable cost due to carbon emission depends on the shipment cost, which itself varies directly by shipment size. The present study is different from the others, as no one has yet considered a variable carbon emission cost in a CLSCM (three-layer) model assuming dependence on the number of containers and the distance.

The emission of CO_2 is the main source of the greenhouse effect that causes global warming. Controlling carbon emissions is thus an efficient way to obstruct carbon emission. Carbon CAPT is an important regulatory strategy to lessen carbon emissions throughout the entire world. There are various reputable trading markets promoted in the USA, the EU, and Australia, such as the European, Chicago, and Australia Climate Exchange (Tiwari et al. [34]), respectively. The CAPT mechanism is used to allot carbon emission limits for a company, while the CAPT strategy ascertains a carbon

limit for a definite interval in which companies can sell or purchase carbon credit (Tiwari et al. [34]; Ahmed and Sarkar [35]). Wang et al. [36] considered carbon emission constraints in an optimal production quantity model for new/remanufactured products to gain maximum profit. This paper is distinct from other papers in that no one has yet considered a CAPT mechanism for the retailer in a CLSCM model.

An expansive literature review of sustainable CLSCM shows that much research has been guided by sustainability designs, and furthermore that no researcher has investigated the behavior of a CLSCM model under the existence of such indicators for recycling, such as product collection, inspection, landfill, remanufacturing and reuse of products, considering variable shipment and carbon emission costs and a CAPT strategy for the retailer. It is clear that the SSC arguments may only focus on the minimum cost, ignoring environmental components. Considering this research gap, a three-layer CLSCM diagram is studied to minimize the cost in total while assuming hybrid policy. The economic benefits of considering landfill, transportation, carbon emission, and CAPT costs are discussed here. The proposed model uses the resources through recycling and variable shipment and carbon emission costs for the manufacturer and 3PL, and landfill for the 3PL provider and a CAPT strategy are introduced to provide certified logistics. The comparison between contributions of different authors is shown Table 1.

This paper is structured as follows: Section 3 interprets notation and assumptions including problem definition. Section 4 illustrates a mathematical model along with its corresponding solution methodology. Section 5 provides numerical results. Section 6 discusses a sensitivity analysis for this model with respect to the input parameters. Section 7 ascertains implications along with managerial insights, and the conclusions, major findings, and further extensions are investigated in Section 8.

Table 1. Author(s) comparison Table.

Author(s)	CLSCM and SSMD	Shipment	Environmental Effect	Reuse	Strategy
Cárdenas-Barrón and Sana [18]	—	Const.	—	—	Delay in payment
Daryanto et al. [7]	SSMD	Variable	CE, carbon tax	—	—
Hua et al. [32]	—	—	CE, carbon CAPT	—	—
Huang et al. [24]	SSMD	Const.	—	—	Quantity discount
Heydari et al. [25]	CLSCM	Const.	—	Remanuf-acturing	Price discount
Sarkar et al. [16]	CLSCM and SSMD	Variable	CE	Remanuf-acturing	—
Taleizadeh et al. [10]	—	—	CE	—	Shortage, back-ordering
Khouja et al. [28]	SSMD	Const.	—	—	—
Sarkar et al. [37]	SSMD	Variable	CE	Reworking	—
This model	CLSCM and SSMD	Variable	CE, carbon CAPT	LF	Price discount

"—" indicates the concept does not exist for the research, Const. means constant, CE means Carbon emission, LF means landfill.

3. Problem Definition, Notation, and Assumptions

This section first defines the problem and then assigns assumptions including notation for this model.

3.1. Problem Definition

Due to short-term use of new technology, the life cycle of products decreases gradually. As a result, the amount of EOL waste is increased which results in environmental pollution and damages human health. Therefore, recycling the waste through collection, inspection, remanufacturing, landfill, and reusing besides manufacturing newish products is the ultimate goal of this paper. A CLSCM is apprised here in which manufacturing and remanufacturing occur at the same stage and SSMD strategy is considered by the players to reduce their holding cost. Also, variable and fixed shipments and carbon emission for the manufacturer and the 3PL are envisaged for this model. The retailer uses a CAPT strategy to control carbon emissions. The 3PL is responsible for all activities such as collection, inspection, landfill, and deliveries of product to the manufacturer for remanufacturing. Then, the cycle is repeated. The whole scenario is depicted in Figure 1.
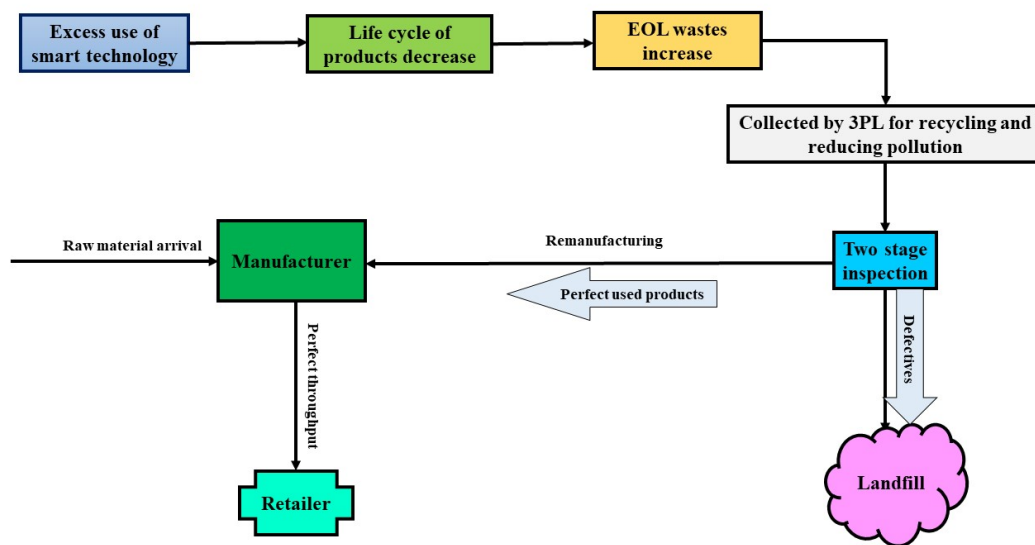

Figure 1. Problem description.

3.2. Notation

The notations are elaborately discussed in Table 2.

Table 2. Symbol description

	Decision variables
T_l	replenishment cycle length (year)
s_m	shipment numbers in a cycle for delivering the finished goods to the retailer via manufacturer (integer number)
s_l	shipment numbers in a cycle for delivering the used goods to the manufacturer via 3PL (integer number)
β	capacity of a single container (units)
	Dependent variable
X	transfer quantity of carbon emissions (units)
	Random variables
ρ	collection rate of 3PL for the used products (units/cycle)
ξ	fraction of collected products that are defective due to inspection and are used for landfill
r_1	random variable presenting Type I error
r_2	random variable presenting Type II error
	Parameters
Q	retailer's ordering quantity (units)
P	production rate per year (units/year)
D	rate of demand (units/year)
C_m^P	manufacturer's unit production expense ($/unit)
h_s	manufacturer's holding cost to hold finished products ($/unit/unit time)

A_s	manufacturer's setup expense per setup (\$/setup)
O_s	manufacturer's per-order ordering expense (\$/order)
B_m^P	buying price of raw material for manufacturing (\$/unit)
B_m^B	manufacturer's remanufacturing cost (\$/unit)
H_s	manufacturer's holding cost to hold used goods (\$/unit/unit time)
B_m^S	buying price of new/remanufactured products of the retailer (\$/unit)
H_m^R	retailer's holding cost to hold finished goods (\$/unit)
O_r	retailer's ordering cost per order (\$/order)
δ	carbon emission quota (units/cycle)
e	unit carbon emission linked to the replenishment order (unit)
w	unit variable carbon emission at the warehouse due to retailer's inventory (unit)
c	carbon price of the retailer (\$/unit)
B_m^R	raw material price for remanufacturing paid by the manufacturer to 3PL (\$/unit)
C_l^C	collection cost of 3PL (\$/unit)
C_s^T	setup cost paid by 3PL per setup (\$/setup)
H^C	holding cost paid by 3PL to hold collected products (\$/unit/unit time)
C_c^I	inspection cost of 3PL (\$/unit)
C_i^V	variable inspection cost of 3PL container to deliver products to the manufacturer (\$/container)
C_l^T	landfill cost of 3PL (\$/unit)
d_c^T	price discount offered to the manufacturer by 3PL (\$/unit)
C_a^T	cost of wrongly accepting faulty items (\$/unit)
C_r^T	cost of wrongly rejecting faultless items (\$/unit)
F	fixed shipment cost of the CLSC (\$/shipment)
C_t	shipment cost conferred by the manufacturer and 3PL per container per unit distance (\$/container/km)
l_{mr}	distance of the retailer from manufacturer (kilometers)
l_{ml}	distance of the manufacturer from 3PL (kilometers)
C_{fcs}	fixed carbon emission cost paid by manufacturer per shipment for the delivery of finished products (\$/shipment)
C_{vcs}	carbon emission cost paid by manufacturer per container at unit distance for the delivery of finished products (\$/container/kilometer)
C_{fcm}	fixed carbon emission cost paid by 3PL per shipment to deliver used products to the manufacturer (\$/shipment)
C_{vcm}	carbon emission cost paid by 3PL per container per unit distance to deliver used products to the manufacturer (\$/container/kilometer)

3.3. Assumptions

The following assumptions are made for designing the model.

1. All players of the CLSCM are in a cooperation and the power of each player is the same. No game strategy is considered for the symmetric power of the SC players. This study elaborates on a coordination policy for the cost minimization. The demand and production rates are deterministic with $D < P$. Thus, shortage is not allowed for this model.
2. Constant and variable shipment carbon emission costs are introduced for the SSMD policy. This study adopts the variable shipment and carbon emission costs depending on the container quantity and distance between players.
3. To help alleviate global warming due to carbon emissions, a CAPT strategy is considered by the retailer for a definite period in which it can sell or buy carbon credit regarding its own carbon emissions quantity to help control carbon emission (Hua et al. [32]).
4. Two kinds of goods (newish from raw materials and remanufactured from collected used goods) are produced. The retailer and end customer bare a similar cost for both. It is further assumed that the manufacturing cost is larger than the remanufacturing cost, which is larger than the raw material price for remanufacturing used products, i.e., $B_m^R < B_m^B < C_m^P$.
5. A fraction ρ of the total demand D is collected by 3PL as recyclable goods, and it is obtained that ξ is the fraction of collected products that are defective after inspection, which are fully used for landfill, where $0 < \xi < 1$. A two-stage inspection policy is performed by 3PL before the delivery of products to the manufacturer. At the time of inspection, inspectors may wrongly reject a faultless product as faulty, which is defined as a Type I error, and may wrongly accept a faulty product as faultless, which is defined as a Type II error (Sarkar and Saren [38]). Hence, two kinds of inspection errors may occur during human inspection.
6. The manufacturer's manufacturing/remanufacturing capacity is unlimited and both remanufactured and newly finished products have the same quality and price. The players use same type container for the delivery of products (Sarkar et al. [16]). The container number is

considered as an integer. Collection, shipping, inspection, and delivery of the used products to the manufacturer are all the responsibility of 3PL. It is assumed that the random variables ρ, ξ, r_1, r_2 are mutually independent and each pursues the same distribution (uniform) in this model.

4. Model Formulation

The retailer's order size is Q while the manufacturer's production volume at a single stage is $s_m Q$. The finished products and used products are transferred between players in same-sized separate batches through a SSMD policy. Thus, a fixed and variable shipment cost is considered here. A carbon cap policy is adopted by the retailer for controlling carbon emissions due to replenishment order and inventory. After collecting used goods from the customers, the 3PL inspects the used goods and separates the defective items for landfill. In this model, the 3PL offers price discounts for each product such that the manufacturer accepts used products from the 3PL, even though the manufacturer bares the raw material price for remanufacturing each item. On this basis, this paper generates a mathematical diagram on recycling to reach the main goal of the CLSCM (see Figure 2).
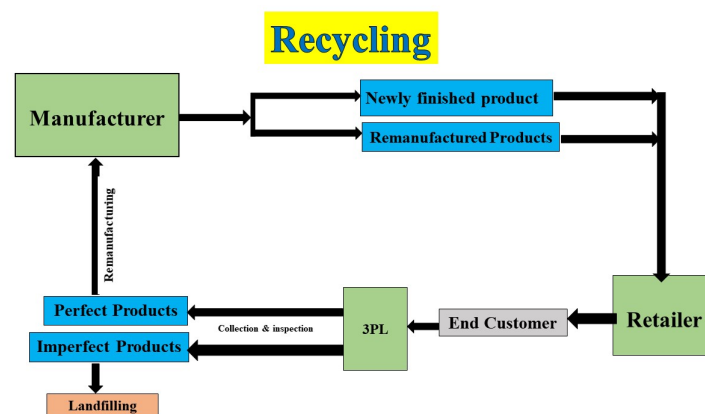

Figure 2. CLSCM diagram for recycling.

4.1. Manufacturer's Model

The manufacturer has an infrastructure for producing $s_m Q$ products, where the retailer's order size is Q. The manufacturer has to pay holding costs for both newly finished products and collected used products. The manufacturer uses the SSMD policy for product delivery to the retailer. Therefore, the number of shipments increases, causing carbon emissions to increase. Besides the fixed shipment, and carbon emission costs, variable shipment, and carbon emission costs are also introduced. Manufacturing, remanufacturing, and buying raw materials for production are all the responsibility of the manufacturer. All cost components relative to manufacturer are discussed below.

4.1.1. Manufacturer's Ordering Cost (OC_m)

Within a certain time period, the manufacturer orders raw materials for manufacturing new products and remanufacturing collected used products, with an ordering cost of

$$OC_m = O_s. \tag{1}$$

4.1.2. Setup Cost of the Manufacturer (SC_m)

The manufacturer produces $s_m Q$ finished goods in each production cycle. Thus, the setup cost of manufacturer is

$$SC_m = A_s. \tag{2}$$

4.1.3. Manufacturer's Holding Cost (HC_m)

The costs to hold finished products and collected used products are not the same and therefore they are evaluated separately.

Manufacturer's holding cost for newly made products (HCN_m)

The manufacturer produces $s_m Q$ quantity and delivers Q quantity per shipment to the retailer through s_m shipments using the SSMD policy during production. Thus, the average inventory of the manufacturer is (Kim and Sarkar [39]).

$$
\begin{aligned}
Inv_{Avg(finished)} &= \left[\left\{ s_m Q \left(\frac{Q}{P} + (s_m - 1) \frac{Q}{D} \right) - \frac{s_m^2 Q^2}{2P} \right\} - \left\{ \frac{Q^2}{D} (1 + 2 + \ldots + (s_m - 1)) \right\} \right] \\
&= \frac{s_m Q^2}{P} - \frac{s_m^2 Q^2}{2P} + \frac{s_m (s_m - 1) Q^2}{2D}.
\end{aligned}
\tag{3}
$$

Therefore, the manufacturer's holding expense for newly manufactured products is

$$
HCN_m = \left[\frac{s_m Q^2}{P} - \frac{s_m^2 Q^2}{2P} + \frac{s_m (s_m - 1) Q^2}{2D} \right] h_s.
\tag{4}
$$

Manufacturer's holding cost for collected used products (HCR_m)

As a large amount of finished goods are manufactured from EOL goods, the manufacturer has to maintain an EOL product inventory, and thus the cost function must include the manufacturer's holding cost for collected used goods. The average stock of the manufacturer for collected used goods (Sarkar et al. [16]) is

$$
\begin{aligned}
Inv_{Avg(used)} &= E[\rho]D(1 - E[\xi]) \left(T_l - \frac{s_m Q}{P} \right) + E[\rho]D(1 - E[\xi]) \frac{s_m Q}{2P} \\
&= E[\rho]D(1 - E[\xi]) \left(T_l - \frac{s_m Q}{2P} \right).
\end{aligned}
\tag{5}
$$

Therefore, the manufacturer's holding expense for collected used goods is

$$
HCR_m = \left[E[\rho]D(1 - E[\xi]) \left(T_l - \frac{s_m Q}{2P} \right) \right] H_s.
\tag{6}
$$

Finally, the manufacturer's holding cost in total is

$$
\begin{aligned}
HC_m &= HCN_m + HCR_m \\
&= \left[\frac{s_m Q^2}{P} - \frac{s_m^2 Q^2}{2P} + \frac{s_m (s_m - 1) Q^2}{2D} \right] h_s + \left[E[\rho]D(1 - E[\xi]) \left(T_l - \frac{s_m Q}{2P} \right) \right] H_s.
\end{aligned}
\tag{7}
$$

4.1.4. Shipment Cost of the Manufacturer (SHC_m)

When the finished products are transported via manufacturer to the retailer, the manufacturer has a shipment cost. As all products (manufactured/remanufactured) are produced through a single stage and are supplied to retailer through s_m lots, the fixed shipment cost is $s_m F$. As both the quantity of containers and the distance between the manufacturer and retailer may vary, a variable shipping cost is added as $C_t l_{mr} \frac{s_m Q}{\beta}$. Thus, the total shipping cost of the manufacture is

$$
SHC_m = s_m F + C_t l_{mr} \frac{s_m Q}{\beta}.
\tag{8}
$$

4.1.5. Carbon Emission Cost (CEC_m)

Due to the increasing number of shipments, the amount of carbon emissions increases. Thus, two kinds of carbon emission costs (fixed plus variable) are adopted in this model. The fixed carbon emission cost is given by $C_{fcs}s_m$. As carbon emissions depend upon the quantity of containers and the distance between the manufacturer and retailer, the variable carbon emission cost is given by $C_{vcs}l_{mr}\frac{s_m Q}{\beta}$. Thus, the total carbon emission cost is

$$CEC_m = C_{fcs}s_m + C_{vcs}l_{mr}\frac{s_m Q}{\beta}. \tag{9}$$

4.1.6. Manufacturing Cost of the Manufacturer (MC_m)

The manufacturer produces $s_m Q - E[\rho]D(1 - E[\xi])$ products from new raw materials at a rate of C_m^P per unit. Therefore, the manufacturer's manufacturing cost is

$$MC_m = C_m^P \left\{ s_m Q - E[\rho]D(1 - E[\xi]) \right\}. \tag{10}$$

4.1.7. Remanufacturing Cost of the Manufacturer (RC_m)

The manufacturer remanufactures $E[\rho]D(1 - E[\xi])$ products at an expense of B_m^B per unit, on that account, the remanufacturing cost of the manufacturer is

$$RC_m = E[\rho]D(1 - E[\xi])B_m^B. \tag{11}$$

4.1.8. Raw Material Price for Remanufacturing (RP_m)

A third party gives a price discount of d_c^T for each used product during delivery to the manufacturer, thus the total recycling fee is given by

$$RP_m = E[\rho]D(1 - E[\xi]) \left(B_m^R - d_c^T \right). \tag{12}$$

4.1.9. Buying Price for Raw Materials at the Manufacturer (BP_m)

The buying price of raw materials for each unit is B_m^P. Therefore, the total buying price of raw material is given by

$$BP_m = \left[s_m Q - E[\rho]D(1 - E[\xi]) \right] B_m^P. \tag{13}$$

4.1.10. Manufacturer's Total Cost per Production Run (TC_m)

The total cost related to manufacturer is achieved by adding all costs relative to the manufacturer per cycle. On that account, the expected cost in total of the manufacturer per cycle is

$$
\begin{aligned}
TC_m\left(s_m, \beta, T_l\right) &= \frac{1}{T_l}\left[OC_m + SC_m + HC_m + SHC_m + CEC_m + M_m^C + RC_m + RP_m + BP_m\right] \\
&= \frac{A_s + O_s}{T_l} + h_s D^2 T\left[\frac{1}{Ps_m} - \frac{1}{2P} + \frac{1}{2D}\left(1 - \frac{1}{s_m}\right)\right] + H_s E[\rho]D(1 - E[\xi])\left(l - \frac{D}{2P}\right) \\
&\quad + \frac{1}{T_l}\left[s_m F + C_{fcs}s_m + E[\rho]D\left(1 - E[\xi]\right)\left(B_m^R - d_c^T - B_m^P + B_m^B - C_m^P\right)\right] \\
&\quad + C_t l_{mr}\frac{D}{\beta} + C_{vcs}l_{mr}\frac{D}{\beta} + D(C_m^P + B_m^P).
\end{aligned}
\tag{14}
$$

4.2. Retailer's Model

The retailer orders Q quantity from the manufacturer with an ordering expense of O_r. The retailer pays a holding cost for finished products, buys finished products for a given price, and bares a carbon emission cost depending on the replenishment order, and a carbon emission cost related to the inventory of the retailer. A CAPT strategy is considered by the retailer to allay the volume of carbon emissions in which they can buy, sell, or transfer extra emission credit. All costs relative to retailer are described.

4.2.1. Retailer's Ordering Cost (OC_r)

During the cycle time T_l, the retailer orders for Q finished products and pays an ordering cost of O_r for each order. Therefore, the ordering cost conferred by the retailer is given by

$$OC_r = O_r. \tag{15}$$

4.2.2. Retailer's Holding Cost (HC_r)

During the period T_l, the retailer receives Q products. The retailer's holding cost for finished products is given by (Kim and Sarkar [39])

$$HC_r = \frac{QT_l H_m^R}{2}. \tag{16}$$

4.2.3. Buying Price of Finished Products of the Retailer (BP_r)

As the retailer pays the same amount for new/remanufactured products therefore, the total buying price paid by retailer for new/remanufactured products is given by

$$BP_r = QB_m^S. \tag{17}$$

4.2.4. Carbon Emission Cost Linked to the Replenishment Order (CER_r)

As e unit is the amount of carbon emissions linked to the replenishment order, the carbon emission costs of the retailer related to the replenishment order is given by

$$CER_r = ec. \tag{18}$$

4.2.5. Carbon Emission Cost due to the Inventory of the Retailer (CEI_r)

As w unit is the amount carbon emission cost due to the inventory at retailer, hence, the average stock of the retailer is $\frac{QT_l}{2}$, and the carbon emission costs relative to the retailer for this stock is given by

$$CEI_r = \frac{T_l Qwc}{2}. \tag{19}$$

4.2.6. Total Cost Related to Retailer (TC_r)

The retailer's aggregate cost is obtained by adding all costs related to the retailer. Consequently, the retailer's aggregate cost is given by (see for reference Hua et al. [32]).

$$
\begin{aligned}
TC_r &= \frac{1}{T_l}\left[OC_r + HC_r + BP_r - cX\right] \\
&= \frac{1}{T_l}\left[O_r + \frac{QT_l H_m^R}{2} + QB_m^S - cX\right],
\end{aligned} \tag{20}
$$

subject to, $CER_r + CEI_r + cX = c\delta$, i.e.,

$$c\left(\frac{T_l Qw}{2} + e + X\right) = c\delta, \tag{21}$$

where Equation (21) is the balance constraint of carbon, δ is the carbon emission quota, c is the carbon price per unit, and X is considered to be the excess carbon credit at retailer which he sells to another company or industry.

Thus, the retailer's per cycle expected total cost is

$$
\begin{aligned}
TC_r\left(T_l\right) &= \frac{O_r + \frac{QT_l H_m^R}{2} + QB_m^S + c\left(\frac{T_l Qw}{2} + e - \delta\right)}{T_l} \\
&= \frac{O_r + ce}{T_l} + \frac{DT_l(H_m^R + cw)}{2} - \frac{c\delta}{T_l} + DB_m^S.
\end{aligned} \tag{22}
$$

4.3. 3PL's Model (TC_l)

After collecting used items from the customers, the 3PL inspects them to detect any defective items. Then, the 3PL delivers the perfect items to the manufacturer for remanufacturing with a discount price and uses the defective products for landfill. The manufacturer pays a raw material price for these perfect items for remanufacturing. All costs relative to 3PL are described as follows:

4.3.1. 3PL's Collection Cost (CT_c)

The 3PL collects EOL goods from the end consumer. The collection cost of 3PL for each unit is C_l^C. Therefore, the total collection cost is given by

$$CT_c = E[\rho]DC_l^C. \tag{23}$$

4.3.2. Setup Cost of 3PL (SC_l)

The setup cost for setting up the infrastructure for collecting used products is given by

$$SC_l = C_s^T. \tag{24}$$

4.3.3. 3PL's Holding Cost (HC_s)

The 3PL collects $E[\rho]D$ used items and stocks them until the manufacturer asks for a delivery. The 3PL's inventory increases gradually during this period. These products are delivered to the manufacturer after inspection. On that account, 3PL's holding cost to hold the used products during this period is given by

$$HC_s = \frac{E[\rho]DH_l^C T_l}{2}. \tag{25}$$

4.3.4. Inspection Cost (IC_l)

The 3PL applies a two-stage inspection policy to detect faulty goods and prevent the shipment of faulty goods. In this model, 3PL first applies a product inspection policy and then applies a container inspection policy for the transportation of perfect goods to the manufacturer for remanufacturing. Thus, the total inspection cost is

$$IC_l = C_c^I E[\rho]D + C_i^V \frac{E[\rho]D(1 - E[\xi])}{\beta}. \tag{26}$$

4.3.5. Landfill Cost of 3PL (LC_l)

Please note that ξ is the fraction of collected products that are defective after inspection and hence, the number of defective items after inspection is $E[\rho]DE[\xi]$, which are used for landfill). Thus, the number of non-defective items after inspection is given by $E[\rho]D(1 - E[\xi])$. As $E[\rho]DE[\xi]$ is the quantity of products used for landfill, the total landfill cost is

$$LC_l = E[\rho]DE[\xi]C_l^T. \tag{27}$$

4.3.6. Price Discounts (PD_l)

3PL offers a price discount d_c^T for each item to motivate the manufacturer into purchasing products. The price discount offered by 3PL to the manufacturer is given by

$$PD_l = E[\rho]D(1 - E[\xi])d_c^T. \tag{28}$$

4.3.7. Shipment Cost During Delivery to the Manufacturer (SHC_l)

The variable shipment cost is evaluated based on the container numbers that are used for transportation and the distance between 3PL and the manufacturer. Since 3PL ships $E[\rho]D(1 - E[\xi])$ perfect products to the manufacturer, the fixed shipment cost for delivery to the manufacturer is Fs_l and the variable shipment cost is $C_t l_{ml} \frac{E[\rho]D(1-E[\xi])}{\beta}$. Therefore, the total shipping cost for the delivery of perfect products is given by

$$SHC_l = Fs_l + C_t l_{ml} \frac{E[\rho]D(1 - E[\xi])}{\beta}. \tag{29}$$

4.3.8. Carbon Emission Cost During Dispatch to the Manufacturer (CEC_l)

Similar to previously, the fixed carbon emission cost is $C_{fcm}s_l$ and the variable carbon emission cost is $C_{vcm}l_{ml} \frac{E[\rho]D(1-E[\xi])}{\beta}$. Therefore, the carbon emission cost in total paid by the retailer during delivery to the manufacturer is given by

$$CEC_l = C_{fcm}s_l + C_{vcm}l_{ml} \frac{E[\rho]D(1 - E[\xi])}{\beta}. \tag{30}$$

4.3.9. Inspection Error (IE_l)

At the time of inspection, inspectors may wrongly reject a faultless product as faulty, which is defined as Type I error, and may wrongly accept a faulty product as faultless, which is called a Type II error. These two kinds of inspection errors may be founded during human inspection.

Type I error

The number of falsely rejected faultless items is $E[\rho]D(1 - E[\xi])E[r_1]$. The total cost due to Type I errors is

$$E[\rho]D(1 - E[\xi])E[r_1]C_r^T.$$

Type II error

The number of falsely accepted defective items is $E[\rho]DE[\xi]E[r_2]$. The total cost due to type II errors is

$$E[\rho]DE[\xi](1 - E[r_2])C_a^T.$$

Hence, the total cost due to inspection errors is given by

$$IE_l = E[\rho]D(1 - E[\xi])E[r_1]C_r^T + E[\rho]DE[\xi](1 - E[r_2])C_a^T. \tag{31}$$

4.3.10. Total Cost of 3PL per Cycle (TC_l)

Thus, the 3PL's expected cost in total per cycle is

$$
\begin{aligned}
TC_l\,&(T_l, s_l, \beta) \\
&= \tfrac{1}{T_l}\left[CT_c + SC_l + HC_s + IC_l + LC_l + PD_l + SHC_l + CEC_l + IE_l\right] \\
&= \tfrac{E[\rho]D}{T_l}\left[C_l^C + C_c^I + E[\xi]C_l^T + (1 - E[\xi])d_c^T + (1 - E[\xi])E[r_1]C_r^T + E[\xi](1 - E[r_2])C_a^T\right] \\
&\quad + \tfrac{1}{T_l}(C_i^V + C_t l_{ml} + C_{vcm} l_{ml})\tfrac{E[\rho]D(1 - E[\xi])}{\beta} + \tfrac{E[\rho]DH_l^C}{2} + \tfrac{C_s^T + s_l(F + C_{fcm})}{T_l}.
\end{aligned}
\tag{32}
$$

4.4. Total Cost of CLSC per Cycle (JTC)

The expected cost in total of the CLSCM is the summation of all costs corresponding to the manufacturer, retailer, and 3PL. Thus, the expected cost in total of the CLSCM per cycle is

$$
\begin{aligned}
JTC\,&(T_l, s_m, s_l, \beta) = TC_m + TC_r + TC_l \\
&= \tfrac{A_s + O_s}{T_l} + h_s D^2 T_l\left[\tfrac{1}{Ps_m} - \tfrac{1}{2P} + \tfrac{1}{2D}\left(1 - \tfrac{1}{s_m}\right)\right] \\
&\quad + H_s E[\rho]D(1 - E[\xi])\left(1 - \tfrac{D}{2P}\right) + \tfrac{1}{T_l}\left\{s_m F + C_{fcs}s_m + E[\rho]D(1 - E[\xi])\left(B_m^R - d_c^T\right.\right. \\
&\quad \left.\left. - B_m^P + B_m^B - C_m^P\right)\right\} + C_t l_{mr}\tfrac{D}{\beta} + C_{vcs}l_{mr}\tfrac{D}{\beta} + D(C_m^P + B_m^P) + \tfrac{O_r + ce}{T_l} \\
&\quad + \tfrac{DT_l(H_m^R + cw)}{2} - \tfrac{c\delta}{T_l} + DB_m^S + \tfrac{E[\rho]D}{T_l}\left[C_l^C + C_c^I + E[\xi]C_l^T + (1 - E[\xi])d_c^T + (1 - E[\xi])E[r_1]C_r^T\right. \\
&\quad \left. + E[\xi](1 - E[r_2])C_a^T\right] + \tfrac{1}{T_l}(C_i^V + C_t l_{ml} + C_{vcm} l_{ml})\tfrac{E[\rho]D(1 - E[\xi])}{\beta} + \tfrac{E[\rho]DH_l^C}{2} + \tfrac{C_s^T + s_l(F + C_{fcm})}{T_l}.
\end{aligned}
\tag{33}
$$

4.5. Solution Methodology

The most important decisions in this model are the optimum shipment numbers for delivery of finished goods to the retailer and used products to the manufacturer to control the waste reduction by the minimum cost. The manufacturer's inventory relies on the quantity of finished goods that have been transported to the retailer. Generally, the retailer's order quantity should be replenished quickly to alleviate any residual stock by the manufacturer. The retailer's inventory plays a significant role in this model by reducing the manufacturer's inventory holding cost. For shortened lead time, the retailer should receive the delivery quickly. Therefore, the manufacturer and the 3PL designate shipment numbers before assigning the value of the other variables. Thus, the optimal value of the replenishment cycle length T_l is acquired by considering the partial derivatives of Equation (33) in regard to T_l.

Rearranging the terms of Equation (33), it is found as

$$
\begin{aligned}
JTC\,(T_l, s_m, s_l, \beta) = \\
\tfrac{1}{T_l}\Bigg[& A_s + O_s + s_m F + C_{fcs}s_m + E[\rho]D(1 - E[\xi])(B_m^R - d_c^T - B_m^P + B_m^B - C_m^P) + O_r + c\,(e - \delta) \\
& + \left\{ E[\rho]D\left(C_l^C + H_l^R + C_c^I + E[\xi]C_l^T + (1 - E[\xi])d_c^T + (1 - E[\xi])E[r_1]C_r^T + E[\xi](1 - E[r_2])C_a^T \right) \right\} \\
& + (C_i^V + C_t l_{ml} + C_{vcm}l_{ml})\tfrac{E[\rho]D(1 - E[\xi])}{\beta} + C_s^T + s_l(F + C_{fcm}) \Bigg] \\
& + T_l\left[h_s D^2\left(\tfrac{1}{Ps_m} + \tfrac{1}{2D}\left(1 - \tfrac{1}{s_m}\right) - \tfrac{1}{2P} \right) + \tfrac{D(H_m^R + cw)}{2} \right] \\
& + H_s E[\rho]D(1 - E[\xi])(1 - \tfrac{D}{2P}) + C_t l_{mr}\tfrac{D}{\beta} + C_{vcs}l_{mr}\tfrac{D}{\beta} + D\,(C_m^P + B_m^P) + DB_m^S + \tfrac{E[\rho]DH_l^C}{2} \\
= & \ \tfrac{1}{T_l}f_1\,(s_m, s_l, \beta) + T_l f_2\,(s_m) + f_3\,(\beta)\,.
\end{aligned}
\tag{34}
$$

[See Appendix A for the values of f_1, f_2, f_3.]

Differentiating Equation (34) with respect to T_l, one can write

$$
\frac{\partial JTC\,(z_r, z_l, T_l, s_m, s_l, \beta)}{\partial T_l} = -\frac{1}{T_l{}^2}f_1\,(s_m, s_l, \beta) + f_2\,(s_m)\,.
\tag{35}
$$

Equating (35) to zero, it is found

$$
T_l{}^* = \sqrt{\frac{f_1\,(s_m, s_l, \beta)}{f_2\,(s_m)}}\,.
\tag{36}
$$

The shipment numbers s_m, s_l, and capacity of single container β can be obtained from the following relations:

$$
JTC\,(s_m{}^* - 1) \geq JTC\,(s_m{}^*) \leq JTC\,(s_m{}^* + 1)\,,
\tag{37}
$$

$$
JTC\,(s_l{}^* - 1) \geq JTC\,(s_l{}^*) \leq JTC\,(s_l{}^* + 1)\,,
\tag{38}
$$

$$
JTC\,(\beta^* - 1) \geq JTC\,(\beta^*) \leq JTC\,(\beta^* + 1)\,.
\tag{39}
$$

Lemma 1. *If s_m, s_l, and β are constants then the aggregate cost of the CLSCM per cycle (JTC) has the global minimum at $T_l{}^* = \sqrt{\dfrac{f_1(s_m, s_l, \beta)}{f_2(s_m)}}$.*

Proof. $JTC\,(T_l, s_m, s_l, \beta)$ will have a least value at $(T_l{}^*)$ (continuous variable) if we can show clearly that the partial derivative second order of JTC (the complete cost function) respecting to $T_l{}^*$ is positive. The partial derivative second order of JTC (the cost function) is

$$
\frac{\partial^2 JTC\,(T_l, s_m, s_l, \beta)}{\partial T_l{}^2} = \frac{2}{T_l{}^3}f_1\,(s_m, s_l, \beta)\,,
\tag{40}
$$

which is greater than zero at $T_l{}^*$. Thus, $JTC\,(T_l, s_m, s_l, \beta)$ has the global minimum if the other three decision variables s_m, s_l, and β are given. $\square$

4.5.1. MHGS Algorithm

The problem in this paper is an MINLP problem, which contains both continuous and discrete variables. A MHGS algorithm is proposed for solving this model.

Step 1. Obtain shipment numbers s_m^* and s_l^* is found by using a linear search method. Put all parametric data and obtain the optimal value of shipment numbersand assume the manufacturer and 3PL have a clear-cut shipment numbers.

Step 2. Capacity of single container β^*, cycle length T_l^* are obtained.

Step 2.1. Put $\beta = 1$.

Step 2.2. Evaluate T_l^* from Equation (36).

Step 3. Calculate the total cost $JTC\left(T_l^*, s_m^*, s_l^*, \beta\right)$ from Equation (33). $JTC\left(T_l^*, s_m^*, s_l^*, \beta\right)$ is an optimum solution for fixed β.

Step 4. Put $\beta = \beta + 1$ and perform Step 2.2 and 3. If $JTC\left(T_l^*, s_m^*, s_l^*, \beta\right) \leq JTC\left(T_l^*, s_m^*, s_l^*\beta - 1\right)$, reiterate Step 2 and then go to Step 3, if not go to Step 5.

Step 5. Put $JTC\left(T_l^*, s_m^*, s_l^*, \beta^*\right) = JTC\left(T_l^*, s_m^*, s_l^*, \beta - 1\right)$. $JTC\left(z_r^*, z_l^*, T_l^*, s_m^*, s_l^*, \beta^*\right)$ is the global optimal solution.

5. Numerical Experiment

Managing waste and global warming prevention has become a great challenge for industries throughout the whole world. This study proposes a CLSC in which EOL products are collected at a random rate and perfect used products are remanufactured besides the production of newly finished products to recycle and reuse wastes.

Input parameters are provided in Table 3, taken from (Sarkar et al. [31]; Sarkar et al. [16]; Hua et al. [32]; Sarkar and Saren [38]) to validate the model. Expected value of random variables are described in Appendix B. Table 4 presents the output data set relative to the input data set.

Table 3. Input data set.

Parameters for the Manufacturer		
$A_s = \$200/\text{setup}$	$O_s = \$120/\text{order}$	$h_s = \$0.28/\text{unit}/\text{unit time}$
$H_s = \$0.22/\text{unit}/\text{ unit time}$	$P = \$452 \text{ units}/\text{year}$	$F = \$300/\text{shipment}$
$C_{fcs} = \$2/\text{shipment}$	$C_{vcs} = \$0.2/\text{container}/\text{unit distance}$	$l_{mr} = 60 \text{ km}$
$B_m^P = \$60/\text{unit}$	$B_m^R = \$45/\text{unit}$	$B_m^B = \$144/\text{unit}$
$C_t = \$0.01/\text{container}/$ unit distance		$C_m^P = \$158/\text{unit}$
Parameters for the retailer		
$D = \$140 \text{ units}/\text{year}$	$O_r = \$52/\text{order}$	$C = \$0.2/\text{unit}$
$e = 720 \text{ units}$	$H_m^R = \$0.32/\text{unit}/\text{unit time}$	$w = 1.5 \text{ unit}$
$B_m^s = \$300/\text{unit}$	$\delta = 5{,}816 \text{ units}$	
Parameters for the 3PL		
$\rho_1 = 0.8$	$\rho_2 = 0.88$	$\xi_1 = 0.06$
$\xi_2 = 0.14$	$C_s^T = \$35/\text{setup}$	$C_l^C = \$11.6/\text{unit}$
$H_l^C = \$0.22/\text{unit}/\text{unit time}$	$C_c^I = \$.12/\text{unit}$	$C_i^V = \$2/\text{unit}$
$C_l^T = \$.05/\text{unit}$	$d_c^T = \$1/\text{unit}$	$C_{fcm} = \$2/\text{shipment}$
$C_{vcm} = \$0.2/\text{container}/$ unit distance	$r_{11} = 0.006$	$l_{ml} = 25 \text{ km}$
$r_{12} = 0.014$	$r_{21} = 0.02$	$r_{22} = 0.06$
$C_r^T = \$ 0.01/\text{unit}$	$C_a^T = \$0.023/\text{unit}$	

Table 4. Optimal outcomes of the corresponding decision variables.

Decision Variables	s_m^*	s_l^*	T_l^*	β^*	JTC^*
Optimum Results	4	3	0.25	4	73,021

From Table 4, the optimal values for T_l, s_m, s_l, and β are obtained. Subsequently, the corresponding values of JTC and X are determined by putting the optimal results of the corresponding decision variables in Equations (33) and (21), respectively.

The replenishment cycle length $T_l^* = 0.25$ years, the shipment number for transporting newly made products to the retailer by manufacturer is $s_m^* = 4$, the shipment number for transporting the collected used products by 3PL to the manufacturer is $s_l^* = 3$, and the container capacity β^* is 4 units. From Equations (33) and (21), the optimal value are in Table 4.

The expected cost in total is $JTC(T_l^*, s_m^*, s_l^*, \beta^*) = \$73,021$ per cycle and the transfer quantum of carbon emission $X^* = 5,089.44$ unit. The most important scenario overlooked in this section is that the maximum recycling is done for these optimum values of decision variables.

5.1. Special Case 1: Model without a CAPT Strategy for the Retailer

The effect of the CAPT strategy is studied in a special case. Without a CAPT strategy, the carbon emission quota δ becomes zero and the optimal outcomes of the corresponding decision variables are shown in Table 5, given as $T_l^* = 0.96$, $s_m^* = 2$, $s_l^* = 2$, $\beta^* = 4$. The cost of the CLSCM then becomes $\$73,375.7$ per cycle. In this scenario, the outcome of the dependent variable X^* equals 0 unit. Thus, without a CAPT strategy, the management cost increases and hence the players of the CLSCM model are advised to consider various important costs to lessen the system's total cost. In that case, the capacity of each container remains unaltered, which is realistic and thus validates the model. The most important scenario observed in this model is that without CAPT strategy this model converges to Sarkar et al. [16] in which the total cost per cycle was $\$495,874$ and used products were collected at a rate of 0.423. But in this model used products are collected at a rate of 0.84 (i.e., recycling/reusing occurred at maximum level). Thus, one can easily accept this proposed approach to manage waste and reduce the total cost.

5.2. Special Case 2: Model without Reverse Logistics

This case clarifies the status of the model after neglecting the RL operation and in this situation, the model involves the costs related to the forward supply chain. The model is transformed to a function of three variables (s_m, T_l, β). Table 5 displays the optimal outcomes of the corresponding decision variables as $(s_m^* = 2, T_l^* = 0.84, \beta^* = 4$, and the total cost for the CLSCM = $\$73,328.9$ per cycle). The result indicates that after ignoring the RL operation, the cycle time of the manufacturer T_l changes to 0.84 years and the total cost changes to $\$73,328.9$ per cycle. Thus, without RL operation, the system's total cost increases, which can inspire industry to recycle products instead of manufacturing of newly finished products as the industry can produce the same amount of products at minimum cost and manage the waste. The capacity of each container remains unaltered, which is realistic and signifies validation of the model. Again, it is notable that without reverse logistics, this paper is similar to Sarkar et al. [31].

5.3. Special Case 3: Model without Variable Transportation and Carbon Emission

This case demonstrates the status of the model after neglecting variable transportation and carbon emission cost. In this circumstance, the optimal outcomes of the corresponding decision variables are shown in Table 5, given as $T_l^* = 0.69$, $s_m^* = 4$, $s_l^* = 3$, and $\beta^* = 4$. The cost of the CLSCM then becomes $\$72,629$ per cycle. Thus, without variable transportation and carbon emission, the system's cost decreases but as the distances between players are not always same, the advice to the management is to consider these important costs for reality.

Table 5. Optimal outcomes of the corresponding decision variables under Special Cases.

Decision Variables	$s_m{}^*$	$s_l{}^*$	$T_l{}^*$	β^*	JTC^*
Optimal results under Special Case 1	2	2	0.96	4	73,375.7
Optimal results under Special Case 2	2	0	0.84	4	73,328.9
Optimal results under Special Case 3	4	3	0.69	4	72,629

From Figures 3–5 one can easily find that the graphs are convex which proves that the total cost is globally minimized.
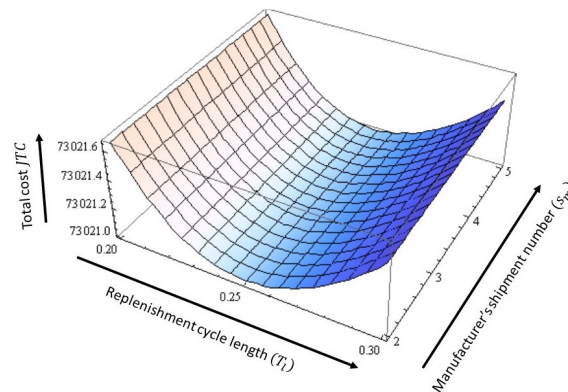
Figure 3. Total cost (JTC) for the CLSC versus manufacturer's shipment number (s_m) and replenishment cycle length (T_l) when s_l and β are fixed, s_m and T_l are variable.
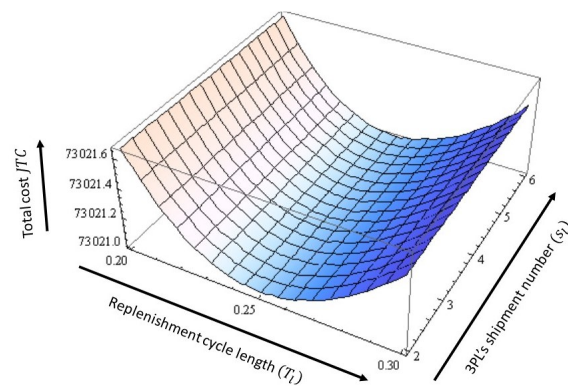
Figure 4. Total cost (JTC) for the CLSC versus 3PL's shipment number (s_l) and replenishment cycle length (T_l) when s_m and β are fixed, s_l and T_l are variable.
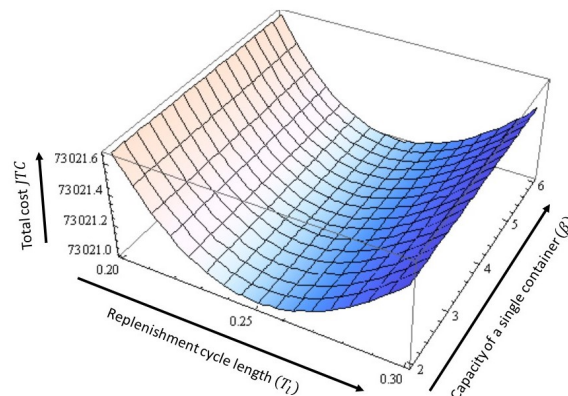
Figure 5. Total cost (JTC) for the CLSC versus capacity of a single container (β) and replenishment cycle length (T_l) when s_m and s_l are fixed, β and T_l are variable.

6. Sensitivity Analysis

The sensitivity analysis in total cost for the CLSCM model with reference to each input data is conferred in Table 6.

1. If the setup cost A_s of the manufacturer for setting up the production infrastructure, ordering costs O_s, O_r relating to manufacturer and retailer, setup cost C_s^T per setup, and the collection cost C_l^C of 3PL are all increased, the total cost increases in that situation. Due to the reductions in the setup cost, ordering cost, and collection cost by 25% or 50%, infeasible results are obtained. These parameters moderately affect the models.

2. The manufacturer's holding costs h_s, H_s for newly finished products and collected used products, retailer's and 3PL's holding costs H_m^R and H_l^C are not sensitive in this study. For negative as well as positive percent changes of the holding cost, the results are similar in magnitude to the total cost. Thus, for both changes, an equilibrium situation is observed in both directions.

3. If the fixed shipment cost of the CLSCM model per shipment F and fixed carbon emission costs C_{fcs} and C_{fcm} per shipment for manufacturer and 3PL are increased by 50% and 25%, respectively, then the expected total cost increases. For decreases of C_{fcs} and C_{fcm} by 50% and F by 25% or more, infeasible results are obtained. Therefore, it can be concluded that these parameters moderately affect the system. Again, The expected total cost increases or decreases according as the shipment cost C_t per container per unit distance increases or decreases.

4. If the carbon emission cost C_{vcs} and C_{vcm} per container per unit distance are increased/decreased, then the expected total cost for the CLSCM model increases/decreases, respectively. The total cost maintains a position of equilibrium and these parameters moderately affect the system.

5. It is observed that for percentage increases in the manufacturer's production cost C_m^P and buying price of raw materials of the manufacturer B_m^P, the changes in total cost is infeasible. If the production cost and buying price of raw materials decreases by 25% or 50%, the total cost decreases. Thus, these parameters are more sensitive.

6. If the raw material's price B_m^R for remanufacturing and the remanufacturing cost of the manufacturer B_m^B increase, then the total cost increases. Decreasing these parameters by 25% or 50% leads to infeasible results. Thus, these are sensitive parameters. Again, if the carbon price c of the retailer decreases, then the total cost increases which is a fully different scenario compared to previous parameters. But due to the increment of this parameter by 25% or 50%, an infeasible result is obtained. If less carbon cost is needed, it is very simple for any CLSC to offset total cost.

7. The buying price B_m^S is the most significant and sensitive parameter for this model. This model considers newly finished and finished products both. Therefore, this cost is very effective and it is clear from sensitivity analysis that if the buying price of finished product B_m^S of retailer decreases by 50%, then the optimal cost decreases by 28.76%.

8. The total cost of CLSCM is increased if per-unit inspection cost C_c^I is increased. Here, it is noticed that if the parameter C_c^I is decreased by 25% or more, an infeasible result is obtained. Again, if the variable inspection cost C_i^V per container increases, then the expected total cost increases. If the parameters C_i^V decreases by 25% or more, an infeasible result is obtained. Therefore, it is not possible to decrease the inspection cost by 25% or more.

9. The landfill cost C_l^T, price discounts d_c^T, cost of wrongly rejecting good items C_r^T, and cost of wrongly accepting faulty items C_a^T are the least sensitive parameters in this model. For positive/negative change of each parameter, an equilibrium situation is arrived.

Table 6. Sensitivity analysis of known parameters.

Parameters	Changes of Inputs (%)	Change of JTC (%)	Parameters	Changes of Inputs (%)	Changes of JTC (%)
A_s	−50	N.F.	C_m^P	−50	−13.3
	−25	N.F.		−25	−6.3
	+25	+ 0.11		+25	N.F.
	+50	+0.17		+50	N.F.
O_s	−50	N.F.	B_m^P	−50	−4.6
	−25	N.F.		−25	−2.1
	+25	+ 0.08		+25	N.F.
	+50	+0.12		+50	N.F.
h_s	−50	−0.002	B_m^R	−50	N.F.
	−25	−0.001		−25	N.F.
	+25	+0.001		+25	+0.66
	+50	+0.002		+50	+.96
H_s	−50	−0.013	B_m^B	−50	N.F.
	−25	−0.007		−25	N.F.
	+25	+0.007		+25	+1.22
	+50	+0.013		+50	+1.74
F	−50	N.F.	O_r	−50	N.F.
	−25	N.F.		−25	N.F.
	+25	+0.43		+25	+0.04
	+50	+0.62		+50	+0.07
C_t	−50	−0.046	c	−50	+0.04
	−25	−0.018		−25	+0.03
	+25	+0.02		+25	N.F.
	+50	+0.03		+50	N.F.
C_{fcs}	−50	N.F.	H_m^R	−50	−0.004
	−25	−0.01		−25	−0.002
	+25	+0.001		+25	+0.002
	+50	+0.018		+50	+0.004
C_{vcs}	−50	−0.29	B_m^S	−50	−28.76
	−25	−0.14		−25	−14.38
	+25	+0.14		+25	+14.38
	+50	+0.29		+50	+28.76
C_s^T	−50	N.F.	d_c^T	−50	0.00
	−25	N.F.		−25	0.00
	+25	+0.033		+25	0.00
	+50	+0.055		+50	0.00
C_l^C	−50	N.F.	C_{fcm}	−50	N.F.
	−25	N.F.		−25	−0.01
	+25	+ 0.334		+25	+0.01
	+50	+ 0.495		+50	+0.02
H_l^C	−50	−0.008	C_{vcm}	−50	−0.29
	−25	−0.004		−25	−0.14
	+25	+0.004		+25	+0.14
	+50	+0.008		+50	+0.29
C_c^I	−50	N.F.	C_r^T	−50	−0.00002
	−25	N.F.		−25	−0.00001
	+25	+0.02		+25	+0.00001
	+50	+0.03		+50	+0.00002
C_l^T	−50	−0.002	C_a^T	−50	−0.0007
	−25	−0.0008		−25	−0.0004
	+25	+0.0008		+25	+0.0004
	+50	+0.002		+50	+0.0007
C_i^V	−50	N.F			
	−25	N.F.			
	+25	+0.05			
	+50	+0.07			

N.F. means not feasible.

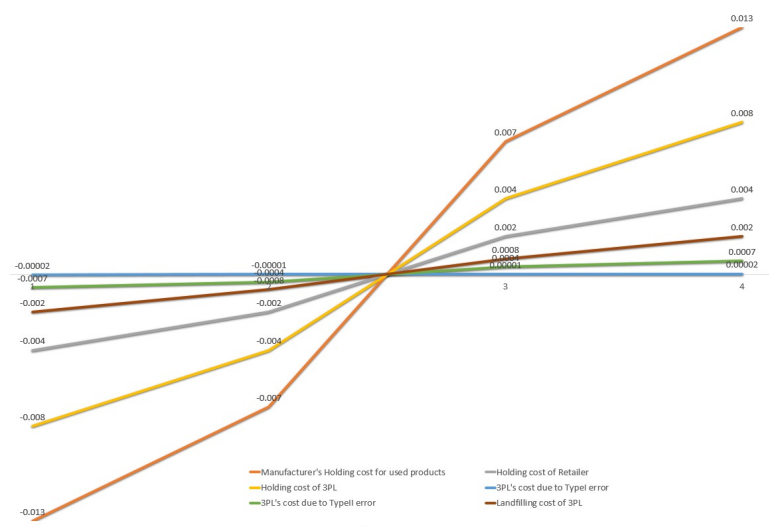Graphically, the effect of changes in total cost for changes of different parameters are illustrated in Figures 6–8.

Figure 6. Effect of changes of total cost for changes of various parameters.
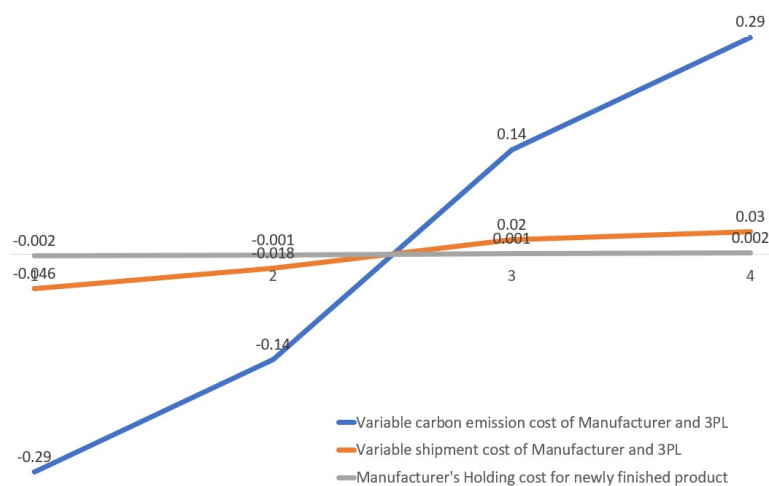

Figure 7. Effect of changes of total cost for changes of various parameters.

Figure 8. Effect of change of total cost for change of buying price of retailer.

7. Implications and Managerial Insights

To recycle the waste and minimize the cost of this CLSCM under a CAPT mechanism, it is very important to verify the total system from all view points.

To increase the lifecycle of each product, the management begins remanufacturing used products after inspecting them. The SSMD policy is introduced here, for which the amount of carbon emission increases. the management has to decide how the carbon emission cost can be reduced, which can be a very difficult situation. This problem is solved by the proposed model.

1. From sensitive analysis, it is evident that if the carbon price of the retailer decreases, the total cost is increased. This is because the retailer uses a CAPT mechanism (in which they can sell the extra carbon credit).
2. It is observed that if the buying cost of newish/remanufactured products is at a minimum, the total cost is at a minimum. Therefore, industry managers can easily make decisions considering these findings. Here, each of the random variables (like return rate of used goods, defective rate of collected used products and also the random variables presenting Type I and Type II errors) follow the uniform distributions, from which the global minimum expense can be calculate by the industry easily.

8. Conclusions

This paper proposed a three-layer CLSCM where the manufacturer was reliable. The ultimate goal of this model was to optimize the aggregate cost through recycling and reusing of waste. Besides fixed shipment and carbon emission costs, this model considered variable shipment and carbon emission costs relying on the number of containers and distance. When using the SSMD policy, the carbon emission cost increased. For this, a carbon CAPT strategy was considered for the retailer in this model. A model (MINLP) and a corresponding algorithm was designed to solve, and the total system cost was minimized along with the replenishment cycle length, shipment numbers, and capacity of each container. Finally, the total cost was minimized. However, what will be the situation if any player in the SCM faces damage due to the coordination policy? What will be the procedure of the compensate policy? This situation can be studied in a future extension of this study. Apart from the symmetric power of coordination, the situation for the asymmetric power for the SC players can be considered for the future extension of the study. Since the used products are collected at a random rate, uncertainty appears in the SC because of uncertain returns, finally leading to uncertain production. Thus, this work can be extended by proposing a variable production rate. A multi-stage production system can also be considered to avoid waste. In this situation, the expected cost in total can be reduced by introducing a discrete setup cost, which can reduce the total cost. For collecting EOL goods, the 3PL can make an elementary investment into the collection centrum. To diminish the environmental impact and enhance resource use, one can also consider a multi-retailer setup for future work. For future investigations, it can be assumed that product delivery is performed offline, whereas orders can be done online. In this case, the demand will become advertisement-dependent. The random variables could also follow Beta, Triangular, or Double Triangular distributions. Finally, this model can be elongated by considering product delivery via drones (i.e., not using human beings).

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

SSC	sustainable supply chain
EOL	end-of-life
CLSCM	closed-loop supply chain management
SSMD	single-setup multi-delivery
SC	supply chain
SCM	supply chain management
CAPT	cap-and-trade
GSCM	green supply chain management
RL	reverse logistics
RLSC	reverse logistics supply chain
VMIP	vendor-managed-inventory policy
3PL	third party logistics

Appendix A

$$f_1\left(s_m, s_l, \beta\right) =$$
$$\left[A_s + O_s + s_m F + C_{fcs}s_m + E[\rho]D(1 - E[\xi])(B_m^R - d_c^T - B_m^P + B_m^B - C_m^P) + O_r + c\left(e - \delta\right) \right.$$
$$+ \left\{ E[\rho]D\left(C_l^C + H_l^R + C_c^I + E[\xi]C_l^T + (1 - E[\xi])d_c^T + (1 - E[\xi])E[r_1]C_r^T + E[\xi](1 - E[r_2])C_a^T\right)\right\}$$
$$+ (C_i^V + C_t l_{ml} + C_{vcm}l_{ml})\tfrac{E[\rho]D(1-E[\xi])}{\beta} + C_s^T + s_l(F + C_{fcm}) \Bigg],$$
$$f_2\left(s_m\right) = \left[h_s D^2\left(\tfrac{1}{Ps_m} + \tfrac{1}{2D}\left(1 - \tfrac{1}{s_m}\right) - \tfrac{1}{2P}\right) + \tfrac{D\left(H_m^R + cw\right)}{2} \right],$$
$$f_3\left(\beta\right) = H_s E[\rho]D(1 - E[\xi])\left(1 - \tfrac{D}{2P}\right) + C_t l_{mr}\tfrac{D}{\beta} + C_{vcs}l_{mr}\tfrac{D}{\beta} + D\left(C_m^P + B_m^P\right)$$
$$+ DB_m^S + \tfrac{E[\rho]DH_l^C}{2}.$$

Appendix B

It is assumed that each of ρ, ξ, r_1, r_2 follows uniform distribution and the probability density functions corresponding to these uniform distributions are

$$f(x) = \begin{cases} \dfrac{1}{\rho_2 - \rho_1}; & \rho_1 \le x \le \rho_2 \\ 0; & \text{Otherwise} \end{cases}$$

$$g(x) = \begin{cases} \dfrac{1}{\xi_2 - \xi_1}; & \xi_1 \le x \le \xi_2 \\ 0; & \text{Otherwise} \end{cases}$$

$$h(x) = \begin{cases} \dfrac{1}{r_{12} - r_{11}}; & r_{11} \le x \le r_{12} \\ 0; & \text{Otherwise} \end{cases}$$

$$u(x) = \begin{cases} \dfrac{1}{r_{22} - r_{21}}; & r_{21} \le x \le r_{22} \\ 0; & \text{Otherwise} \end{cases}$$

The expected value of ρ is

$$E(\rho) = \int_{\rho_1}^{\rho_2} x f(x) dx = \int_{\rho_1}^{\rho_2} \frac{x}{\rho_2 - \rho_1} dx = \frac{\rho_2^2 - \rho_1^2}{2(\rho_2 - \rho_1)} = \frac{\rho_1 + \rho_2}{2}.$$

Similarly, the expected values of ξ, r_1 and r_2 are

$$E(\xi) = \frac{\xi_1 + \xi_2}{2}, E(r_1) = \frac{r_{11} + r_{12}}{2}, E(r_2) = \frac{r_{21} + r_{22}}{2}.$$

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