



A Practical Traffic Assignment Model for Multimodal Transport System Considering Low-Mobility Groups

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Abstract: In this study, we created a practical traffic assignment model for a multimodal transport system considering low-mobility groups with the aim of providing the foundation of transportation network design for low-mobility individuals. First, the route choice equilibrium for walking, non-vehicle, and private car modes is described using the logit function, which is formulated as a variational inequality problem considering different low-mobility groups. Then, the practicalities related to travel times at intersections, traffic barricades between different lanes, and fuel fees of private cars are integrated to design a generalized travel cost function. Last, the method of successive weight averages is used to solve the proposed model. The model and its solution are verified based on a real case study of the city of Wenling in China. The sensitivity of adjustment parameters related to travel costs are analyzed, the practicality of the proposed model is explored, and the results of traffic assignment for different low-mobility groups are discussed.

Keywords: traffic assignment; multimodal transport; practicality; low-mobility group

1. Introduction

Generally, Low-Mobility Individuals (LMIs) are considered as individuals who cannot have access to transportation services. They can be classified into three groups—elderly people, low-income individuals, and people with disabilities [1]. The seminal study performed by Jansuwan et al. [1] is widely used as a reference for difficulties in travel faced by the LMIs. The transportation demands of LMIs in Wenling city, China were analyzed by Ren et al. [2], and a model of transit route (i.e., public transport or bus) network design for the LMIs was presented by Zhang et al. [3]. The transportation network design of other trip modes related to the LMIs, such as walking, non-vehicles that include regular and electric bikes in this paper, and private cars should also be studied. As the traffic assignment forms the foundation of transportation network design, this paper presents a practical traffic assignment model for multimodal transport system considering low-mobility groups.

Traffic assignment is an importation part of traffic network optimization and is also one of the key technologies involved in traffic management. Traffic assignment can be described as the assignment of origin–destination (O-D) travel demands to a traffic network based on certain rules to select paths, resulting in traffic volume gain in the traffic network. Since Wardrop proposed the principle of user equilibrium (UE) for the first time in 1952 [4], different researchers have studied the UE assignment problem in detail and proposed objective traffic assignment models for private cars [5–7]. However, economic development and advancement of urbanization has gradually resulted



in a comprehensive transportation network, including private cars, buses, metros, taxis, bicycles and pedestrians. Consequently, the traffic assignment problem for multimodal networks has been a subject of interest for many researchers [8–30].

The multimodal assignment models proposed initially were based on splitting travel demands for each trip mode using the logit function [8–12]. However, these models considered mode choice equilibrium rather than route choice equilibrium, i.e., assigning travel demands of different trip modes into the multimodal traffic network [13]. To deal with this shortcoming, combined models that incorporated equilibriums of both modal and route choices together were proposed in numerous studies [14–16,21–28]. These combined models could be regarded as convex optimization problems where the travel cost structures were either separable or symmetric [31,32]. The multimodal assignment models were formulated as different problems, e.g., mathematical programming [19,33–36], dynamic simulation [23,37,38], fixed point [20,21], variational inequality [22,24,27,39–44], etc. However, the variational inequality problem is most widely used by researchers because it can reflect the asymmetric interaction of cost structures effectively, resulting from the assumption of separable or symmetric travel cost structures.

As the travel demands of different trip modes can be obtained from the results presented in previous studies [1,2], this paper formulates the equilibrium of route choice for each trip mode described by the logit function as a variational inequality problem, instead of considering the equilibrium of modal choice. Furthermore, this paper extends previous studies by taking into account different low-mobility groups and provides the basis for the design and optimization of multimodal traffic strategies for the LMIs.

In the economics domain, the generalized travel cost is typically used to evaluate the transportation services associated with different properties of the transportation modes, such as travel time, travel expense, and convenience. Monotonically increasing link impedance functions in the generalized travel cost are used to derive the equilibrium of route choice for each trip mode based on the UE principle. Meanwhile, it is worth noting that the generalized travel cost of a link depends on an entire path load pattern, where a path contains many links, rather than on a single link flow [22,24,45,46]. However, previous studies mostly presented the theoretical generalized travel costs and did not consider the viability of generalized travel costs. This paper considers the generalized travel cost from the point of view of path selection and improves the generalized travel cost function for different trip modes in the following three ways:

- (a) Unlike existing references, the generalized travel cost of a path is calculated by summing the travel costs of all links and intersections present in a path. Furthermore, the intersection can be classified into signalized and unsignalized, and, therefore, two different computing methods to calculate the travel time of an intersection in each trip mode are proposed.
- (b) In the private car mode, a traveler may choose a path with a longer travel time compared to the other paths because the fuel cost involved in travelling the path is the lowest. Hence, the link travel cost of a private car is calculated by a subjective weighting of the travel time and fuel cost. On the contrary, the walking and non-vehicle modes only consider the travel time because they are pollution-free and do not involve any fuel costs.
- (c) The existing time cost functions only consider the link impedance related to traffic flows rather than the actual situations. In this paper, the influence of traffic barricades present between different lanes is considered in the calculation of link travel costs of different trip modes. To effectively show the significance of this influence, we do not consider the traffic barricades present in the vehicle lanes because that only involves a single mode. The influence considered has implications in the following cases: (1) If there are no traffic barricades present between walking and non-vehicle lanes, the travel time of the former type of lanes can increase, and (2) if there are no traffic barricades between vehicle and non-vehicle lanes, the travel times of walking, non-vehicle, and private car can all increase.

A large number methods have been proposed to solve multimodal traffic assignment models, such as Frank–Wolfe [33,36], the Gauss Seidel iteration [39,42], the sensitivity analysis-based method [19,22], the method of successive averages (MSA) [29,34,35,37], etc. MSA is the most widely accepted method as it is easy to program and based on a simple principle and shows good convergence and results. However, the convergence rate of the traditional MSA slows down with increasing number of iterations, particularly near an optimal solution. To overcome this limitation, Meng proposed an improved MSA, called the method of successive weight averages (MSWA), and verified that its convergence and results are better than those of MSA [24]. Hence, in this paper, MSWA is used to solve the multimodal traffic assignment problem.

The objective of this study is to develop upon the previous studies of the authors [2,3]. Different concerns related to the multimodal traffic assignment are addressed, including the following:

- (a) Describing the equilibrium of route choice for each trip mode using the logit function based on the existing studies [22–30] and research requirements. This is formulated as the variational inequality problem considering low-mobility groups.
- (b) Improving the practicality of generalized travel costs, considering the travel times of both links and signalized and unsignalized intersections, the travel times and fuel costs of private cars, and the influence of traffic barricades present between different lanes in a path.
- (c) Using the MSWA to solve the proposed multimodal traffic assignment problem. To verify the model and the algorithm, a real case study is performed. Furthermore, the sensitivity of adjustment parameters related to travel costs is analyzed, the practicality of the proposed model is explored, and the results of traffic assignments obtained for different low-mobility groups are discussed.

2. Model Development

In this paper, a directed graph G = (V, E) is used to represent the traffic network, where V is the set of nodes or intersections and E is the set of links. A directed link l connects nodes i and j (l = i, $j; l \in E; i, j \in V; i \neq j$), ij denotes an O-D pair, IJ is the set of O-D pairs, and p denotes a candidate path. Note that there are several candidate paths between an O-D pair. This study only considers the traffic assignment of walking, non-vehicles, and private cars because public transportation has already been investigated by the authors [3].

2.1. Equilibrium Analysis

According to stochastic user equilibrium (SUE), travelers adhere to the famous Wardrop's first principle. Considering the road impedance as a random variable, they select the shortest path between each O-D pair. In each mode, the traffic volume on a candidate path and the expected cost of traveling through it are inversely proportional. Here, we need to presume that all travelers cannot reduce the path travel cost once the traffic network reaches the balanced condition with respect to travel demands of different trip modes. Correspondingly, we can describe the travel preference for different candidate paths in different trip modes based on the logistic regression. This is known as the SUE model and described in Equation (1) as follows:

$$q_{ij,p}^{m} = d_{ij}^{m} \times \frac{\exp\left(-\theta \times C_{ij,p}^{m}\right)}{\sum\limits_{p \in P_{ij}^{m}} \exp\left(-\theta \times C_{ij,p}^{m}\right)},\tag{1}$$

where $q_{ij,p}^m$ is the traffic volume in mode *m* on path *p* between O-D pair *ij*, and *m* can represent walking (wal), non-vehicle (bik), or private car (car). The travel demand by mode *m* between O-D pair *ij* is represented by d_{ij}^m , $\theta \in [0, 1]$ is the cost adjustment parameter, and $C_{ij,p}^m$ and P_{ij}^m are the generalized travel cost and the set of valid paths between O-D pair *ij* in mode *m*, respectively.

Furthermore, the computational process to achieve the research objective related to traffic assignment of low-mobility groups is presented by Equation (2). This process is based on the proportional relationship between different groups. Based on the comparison between the results of real and assigned traffic volumes in the case study, it is supposed that a private car can accommodate up to three people.

$$q_{ij,p}^{m,g} = q_{ij,p}^{m} \times \frac{d_{ij,g}^{m}}{d_{ij}^{m}}.$$
(2)

In Equation (2), $q_{ij,p}^{m,g}$ and $d_{ij,g}^m$ are the traffic volume and travel demands of group *g* using the mode *m* on path *p* between O-D pair *ij*, respectively, where *g* can represent low-mobility groups, such as elderly people, people with disabilities, low-income individuals.

The constraint related to travel demands and traffic volumes on different paths between O-D pair *ij* is given as follows:

$$\sum_{p \in P_{ij}^m} q_{ij,p}^m = d_{ij}^m, ij \in IJ.$$
(3)

This constraint signifies that the sum of traffic volumes of mode *m* on different candidate paths is equal to the travel demand of mode m between O-D pair *ij*.

2.2. Equivalent Transformation

To better describe and solve the proposed model, this section transforms the SUE model into the equivalent variational inequality, which is given by Equation (4). The transformation is based on the constraint given by Equation (3), where the variable $q_{ii,p}^{m*}$ represents the equilibrium solution.

$$\sum_{ij\in I} \sum_{p\in P_{ij}^m} \left[(\ln\sum_{p\in P_{ij}^m} \exp(\theta \times C_{ij,p}^m) + \frac{1}{\theta} \times (\ln d_{ij}^m - \ln q_{ij,p}^{m*}) - C_{ij,p}^m \right] \times (q_{ij,p}^m) - q_{ij,p}^{m*}) \ge 0.$$
(4)

Equation (5) can be deduced using the Karush–Kuhn–Tucker (KKT) condition of Equation (4). Other derivation processes are given in Equations (5)–(9). It can be observed that Equation (9) is the same as Equation (1), which indicates the equivalence between the variational inequality and the SUE model.

$$\ln \sum_{p \in \mathcal{P}_{ij}^{m}} \exp(\theta \times C_{ij,p}^{m}) + \frac{1}{\theta} \times (\ln d_{ij}^{m} - \ln q_{ij,p}^{m}) - C_{ij,p}^{m} = 0,$$
(5)

$$\frac{1}{\theta} \times (\ln d_{ij}^m - \ln q_{ij,p}^m) = C_{ij,p}^m - \ln \sum_{p \in P_{ij}^m} \exp(\theta \times C_{ij,p}^m), \tag{6}$$

$$\exp\left[\frac{1}{\theta} \times \left(\ln d_{ij}^m - \ln q_{ij,p}^m\right)\right] = \exp\left[C_{ij,p}^m - \ln \sum_{p \in P_{ij}^m} \exp\left(\theta \times C_{ij,p}^m\right)\right],\tag{7}$$

$$\frac{d_{ij}^m}{q_{ij,p}^m} = \frac{\exp(\theta \times C_{ij,p}^m)}{\sum\limits_{p \in P_{ij}^m} \exp(\theta \times C_{ij,p}^m)},\tag{8}$$

$$q_{ij,p}^{m} = d_{ij}^{m} \times \frac{\exp(-\theta \times C_{ij,p}^{m})}{\sum\limits_{p \in P_{ij}^{m}} \exp(-\theta \times C_{ij,p}^{m})}.$$
(9)

Equation (4) is a continuous function because its constraint condition, i.e., Equation (3), is a closed convex set. Therefore, Equation (4) can be solved based on the Brouwer's fixed point theorem. However, the monotonicity of travel cost function in each mode is not guaranteed due to the interrelationships

between walking, non-vehicles, and private cars. Hence, it cannot be ensured that the solution of Equation (4) is unique.

3. Travel Cost Function of Different Trip Modes

To strengthen the practicality of the proposed model, this section integrates three features into the generalized travel cost function for each trip mode. These features include

- (a) travel times to traverse links and intersections in a path;
- (b) travel times and fuel costs of private cars;
- (c) the influence of traffic barricades between different lanes.

The generalized travel cost of a path is given by Equation (10), which is the sum of travel costs of links and intersections in a path, which are independent of each other.

$$C_{ij,p}^{m} = \sum_{l \in p} \sum_{z \in p} (C_{l}^{m} + C_{z}^{m}),$$
(10)

where *z* represents an intersection corresponding to a node, and C_l^m and C_z^m are the travel costs of mode *m* on link *l* and intersection *z*, respectively.

For the sake of convenience, a few variables and parameters are summarized as follows: σ_m is the stability influential factor of mode m, t_l^m is the travel time of mode m on link l, as shown in Equation (11), r_l^m is the traffic saturation of mode m on link l, as given by Equation (12), φ is the adjusted coefficient, w_s^m is the standard lane-width of mode m, and w_l^m is the lane-width of mode m on link l.

$$t_l^m = L_l / v_l^m, \tag{11}$$

$$r_l^m = q_l^m / c_l^m. (12)$$

Furthermore, L_l , v_l^m , q_l^m and c_l^m are the length of link *l*, and speed, traffic volume, and traffic capacity of mode *m* on link *l*, respectively.

3.1. Private Car

The link travel cost of a private car can be obtained by Equation (13) as follows:

$$C_l^{\text{car}} = W_1 \times \sigma_{\text{car}} \times t_l^{\text{car}} \times \left[1 + \alpha \times \left(r_l^{\text{car}} \right)^{\beta} \right] + W_2 \times \eta \times \rho \times L_l, \tag{13}$$

where α and β are retardation parameters that adjust the traffic saturation, η is a factor representing conversion between money and time, and ρ is the fuel fee per unit length. In Equation (13), the first part, i.e., the link travel time of a private car, can be described using the bureau of public road (BPR) function because it is influenced by the traffic volume and traffic capacity of a vehicle. The second part is the time cost converted from the fuel cost. The trade-off between these two parts is achieved by the weight coefficients W_1 and W_2 .

Furthermore, the stability influential factor of a private car is introduced to consider the traffic barricade between different lanes, given by Equation (14) as follows:

$$\sigma_{\rm car} = 1 + \varphi \times \psi_{l,\rm bik}^{\rm car} \times r_l^{\rm car} \times r_l^{\rm bik} \times \frac{w_{\rm s}^{\rm car}}{w_l^{\rm car}},\tag{14}$$

where

$$\psi_{l,\text{bik}}^{\text{car}} = \begin{cases} 0 \text{ if traffic barricade between vehicle and non-vehicle lanes exists} \\ 1 \text{ otherwise} \end{cases}$$
(15)

is the existing coefficient of traffic barricade between vehicle and non-vehicle lanes.

According to the above equations, if there is no traffic barricade between vehicle and non-vehicle lanes, the change in travel impedance is directly proportional to both traffic saturations of private car and non-vehicle, and inversely proportional to the width of vehicle lane. If the traffic barricade exists, the travel impedance is 0.

The travel cost of a private car to traverse the intersection only considers the travel time and not the fuel cost due to the short distance. The travel cost at the signalized intersection considering the length of the car is calculated using the physics queuing theory, as shown in Equation (16). The travel cost at signalized intersection without considering the length of the car is calculated based on the point queuing theory, as shown in Equation (17). Note that Equation (18) represents the constraint condition between a link and an intersection, i.e., the intersection *z* is the corresponding intersection of link *l*.

$$C_z^{\text{car}} = \vartheta_l \times \left(\frac{T_l}{2} + r_l^{\text{car}} \times \frac{N_l^{\text{car}} \times L_{\text{car}}}{v_l^{\text{car}}} + \frac{L_l^z}{v_l^{\text{car}}}\right),\tag{16}$$

$$C_z^{\text{car}} = \vartheta_l \times \frac{L_l^z}{v_l^{\text{car}}} \times (1 + r_l^{\text{car}}), \tag{17}$$

$$l = (i, j), z = j.$$
 (18)

In the above equations, T_l and N_l^{car} are the length of red light and the number of private cars on the corresponding intersection of link l, respectively. The standard length of private cars is given by L_{car} . L_l^z is the forward crossing length of the corresponding intersection of link l. ϑ_l is the coefficient related to driving direction on the corresponding intersection of link l. The values of the coefficient include 1, 1.5, and 0.5, corresponding to going forward, turning left, and turning right, respectively.

3.2. Non-Vehicle

The link travel cost of a non-vehicle can be obtained by Equation (19), which only considers travel time and can also be described using the BPR function due to the reason mentioned in Section 3.1.

$$C_l^{\text{bik}} = \sigma_{\text{bik}} \times t_l^{\text{bik}} \times \left[1 + \alpha \times \left(r_l^{\text{bik}} \right)^{\beta} \right].$$
(19)

The stability influential factor of a non-vehicle is also introduced to consider the traffic barricade between different lanes. It is given by Equation (20), and its function can be defined as similar to that of Equation (14), i.e., only changing the width of a vehicle lane into that of a non-vehicle lane.

$$\sigma_{\rm bik} = 1 + \varphi \times \psi_{l,\rm bik}^{\rm car} \times r_l^{\rm bik} \times r_l^{\rm car} \times \frac{w_{\rm s}^{\rm bik}}{w_l^{\rm bik}}.$$
(20)

We assume that the crossing times of non-vehicle and walking are similar because the crossing time of walking is related to the crossing speed of pedestrians, the length and width of crosswalks, and the number of crossing pedestrians. Therefore, the travel cost of a non-vehicle at a signalized intersection can be calculated using Equation (21). The travel cost of a non-vehicle at an unsignalized intersection is also calculated based on the point queuing theory in Equation (22). Furthermore, Equation (23) has the same significance as Equation (18).

$$C_z^{\text{bik}} = \vartheta_l \times \left(\frac{T_l}{2} + \frac{L_l^z}{v_l^{\text{bik}}} + \gamma \times \frac{N_l^{\text{bik}}}{w_l^{\text{bik}}}\right),\tag{21}$$

$$C_z^{\text{bik}} = \vartheta_l \times \frac{L_l^z}{v_l^{\text{bik}}} \times (1 + r_l^{\text{bik}}), \tag{22}$$

$$l = (i, j), z = j.$$
 (23)

In the above equations, N_l^{bik} is the number of non-vehicles on the corresponding intersection of link *l* and γ is the adjustment parameter.

3.3. Walking

The link travel cost of walking only considers the travel time because the walking travel time is not limited by the traffic volume and traffic capacity due to the small footprint of pedestrians. It can be simply represented by Equation (24).

$$C_l^{\text{wal}} = (1 + \sigma_{\text{wal}}) \times t_l^{\text{wal}}.$$
(24)

The stability influential factor of walking is also introduced to consider the traffic barricade between different lanes and can be written as follows:

$$\sigma_{\text{wal}} = (1 + \varphi \times \psi_{l,\text{bik}}^{\text{wal}} \times r_l^{\text{wal}} \times r_l^{\text{bik}} \times \frac{w_s^{\text{wal}}}{w_l^{\text{wal}}}) \times (1 + \varphi \times \psi_{l,\text{bik}}^{\text{car}} \times \psi_{l,\text{bik}}^{\text{wal}} \times r_l^{\text{wal}} \times r_l^{\text{car}} \times \frac{w_s^{\text{wal}}}{w_l^{\text{wal}}}), \quad (25)$$

where

$$\psi_{l,\text{bik}}^{\text{wal}} = \begin{cases} 0 \text{ if traffic barricade between walking and non-vehicle lanes exists} \\ 1 \text{ otherwise} \end{cases}$$
(26)

is the existing coefficient of traffic barricade between walking and non-vehicle lanes.

Note that, according to Equation (25), if there is no traffic barricade between walking and non-vehicle lanes, the change in travel is directly proportional to both traffic saturations of walking and non-vehicle, and inversely proportional to the width of vehicle lane. If there are no traffic barricades, the change in travel impedance is also directly proportional to the traffic saturation of private cars. If traffic barricades between walking, non-vehicle, and vehicle lanes exist, the travel impedance is zero.

The travel cost of walking at a signalized intersection is given by Equation (27), which is similar to that of a non-vehicle at a signalized intersection. The travel cost of a non-vehicle at an unsignalized intersection can be calculated simply using Equation (28) due to the small footprint of pedestrians. Equation (29) has the same significance as Equation (18).

$$C_z^{\text{wal}} = \vartheta_l \times \left(\frac{T_l}{2} + \frac{L_l^z}{v_l^{\text{wal}}} + \gamma \times \frac{N_l^{\text{wal}}}{w_l^{\text{wal}}}\right),\tag{27}$$

$$C_z^{\text{wak}} = \vartheta_l \times \frac{L_l^z}{v_l^{\text{wak}}},\tag{28}$$

$$l = (i, j), z = j.$$
 (29)

In Equation (27), N_l^{wal} is the number of pedestrians crossing the corresponding intersection of link *l*.

4. Proposed Solution

This section solves the proposed model using the MSWA. The flow chart of the proposed solution is shown in Figure 1, which consists of the following steps:

- 1. Load the travel demands of walking, non-vehicle, and private cars into the multimodal traffic network.
- 2. Initialize the iterations to n = 0, and the traffic volume of mode *m* on link *l* in the zeroth iteration to $q_{l(0)}^m = 0$.
- 3. Determine different candidate paths between the O-D pair *ij* using the k-shortest path algorithm.

- 4. In the *n*th iteration, calculate the travel cost of mode *m* on all links $C_{l(n)}^m$ and intersections $C_{z(n)}^m$ based on Equations (13), (16), (17), (19), (21), (22), (24), (27) and (28).
- 5. In the *n*th iteration, determine the travel cost $C_{ij,p(n)}^m$ of mode *m* on candidate path *p* between the O-D pair *ij* using Equation (10) and select the shortest path travel cost $C_{ij(n)}^{m,\min}$ of mode *m* between the O-D pair *ij* from among the different candidate paths.
- 6. In the *n*th iteration, select valid paths from the candidate paths based on the decision condition, given by the following equation:

$$C_{ij,p(n)}^{m} \le (1+\omega) \times C_{ij(n)}^{m,\min},$$
(30)

where ω is the decision coefficient.

- 7. In the *n*th iteration, assign travel demands of mode *m* into valid paths based on Equation (1), and obtain the supplementary traffic volume $y_{l(n)}^m$ of mode *m* on link *l*.
- 8. In the (*n*+1)th iteration, calculate the traffic volume $q_{l(n+1)}^m$ of mode *m* on link *l* using (31).

$$q_{l(n+1)}^{m} = q_{l(n)}^{m} + \frac{y_{l(n)}^{m} - q_{l(n)}^{m}}{1 + 2 + 3 + \dots + n}.$$
(31)

9. Check for convergence by calculating the error value of mode *m* in the *n*th iteration $G_{(n)}^m$ using Equation (32). Note that $G_{(n)}^m \le \varepsilon$, where ε is the error parameter and $q_{ij,p}^m q_{l(n+1)}^m$ is the equilibrium solution of mode *m*.

$$G_{(n)}^{m} = \sqrt{\sum_{l \in E} (q_{l(n+1)}^{m} - q_{l(n)}^{m})^{2}} \times (\sum_{l \in E} q_{l(n)}^{m})^{-1}.$$
(32)

If the equilibrium solution is obtained for walking, non-vehicles, and private cars, calculate the output results $q_{l(n+1)}^m$ and the traffic volume of group g using mode m on link l in the (n+1)th iteration $q_{l(n+1)}^{m,g}$ using Equation (2). Otherwise, increase n to n + 1, and return to Step 4.



Figure 1. Flow chart of MSWA.

5. Case Study

The proposed model was implemented in MATLAB and executed on a Windows 10 PC with an Intel 32 GHz processor. Similar to the previous studies [2,3], we apply the proposed model to the Chinese city of Wenling in order to demonstrate its validity and correctness. Subsequently, we analyzed the sensitivity of cost adjustment parameter θ , explored the effectiveness of the presented practicalities, and finally we discuss the assignment results for different low-mobility groups.

5.1. Scenarios

As shown in Figure 2, the traffic network in Wenling city has 259 nodes, i.e., intersections, where each number corresponds to one node. There are a total of 406 links whose lengths can be obtained based on the scale and 240 links controlled by traffic signals lights whose signal timings are assigned in the supplementary material provided with this manuscript. The travel demand O-D matrices consisting of 259 rows and 259 columns, which correspond to walking, non-vehicles, and private cars

of different groups ,can be obtained using our previous survey results [2] and the similar generation method mentioned in our previous work [3]. The road (link) is divided into four classes: urban expressway, arterial road, sub-arterial road, and branch road, whose details are given in Table 1. Details



Figure 2. Layout of traffic network in Wenling, China.

Туре	Index	Urban Expressway	Arterial Road	Sub-Arterial Road	Branch Road
	Walking (m)	0.9	2.5	1.5	0.75
Lane-width	Non-vehicle (m)	3.5	3	2.5	1.5
	Vehicle (m)	3.75	3.5	3	3
Number of	Walking	2	2	2	2
lanes	Non-vehicle	2	2	2	2
(two-way)	Vehicle	6	6	4	4
Traffic capacity	Walking (persons/h)	1800	6000	4000	1500
(one-way)	Non-vehicle (vehicles/h)	2400	2200	2000	800
	Vehicle (vehicles/h)	3900	3000	1400	800

Туре	Index	Urban Expressway	Arterial Road	Sub-Arterial Road	Branch Road	
	Pedestrian (km/h)	5.4	5.4	5.4	5.4	
Travel speed	Regular bike (km/h)	10	10	10	8	
	Electric bike (km/h)	20	20	20	15	
	Private car (km/h)	80	50	40	30	
Is there a traffic bar and non-v	ricade between vehicle rehicle lanes?	Yes	Yes	Yes	No	
Is there a traffic barricade between walking and non-vehicle lanes?		Yes	Yes	Yes	No	

Table 1. Cont.

Note that the travel speeds of non-vehicles can be calculated based on the mixing ratio of traffic volumes for regular and electric bikes.

 Table 2. Relevant variable units and parameter values.

Symbol	Description		Unit	Value
		Walking	persons/h	
$q^m_{ij,p}$	Traffic volume on path p between O-D –	Non-vehicle	vehicles/h	
	pan ij	Private car	vehicles/h	
θ	Cost adjustment parameter	r		1
		Walking	persons/h	
d_{ij}^m	Travel demand between O-D pair <i>ij</i>	Non-vehicle	vehicles/h	
		Private car	vehicles/h	
	Generalized travel cost between O-D pair ij	Walking	h	
$C^m_{ij,p}$		Non-vehicle	min	
		Private car	min	
	Travel time on link <i>l</i>	Walking	h	
t_l^m		Non-vehicle	min	
		Private car	min	
φ	Adjusted coefficient			0.25
		Walking	m	1.5
$w^m_{ m s}$	Standard lane-width	Non-vehicle	m	1.5
		Private car	m	3.75
w_l^m	Lane-width of mode <i>m</i> on link <i>l</i>		m	
L_l	Length of link <i>l</i>	Length of link <i>l</i>		
		Walking	persons/h	
c_l^m	Traffic capacity on link <i>l</i>	Non-vehicle	vehicles/h	
	_	Private car	vehicles/h	
α	Retardation parameter			0.15
β	Retardation parameter			4
η	Factor representing conversion between money and time		min/¥	1.89
ρ	Fuel cost per unit length		¥/km	0.75
T_l	Length of red light on the corresponding intersection of link <i>l</i>		S	
N _l car	Number of private cars on the corresponding	vehicles		

Symbol	Description		Unit	Value
L _{car}	Standard length of private cars		m	7.2
L_l^z	Forward crossing length of the corresponding intersection of link <i>l</i>		m	
ϑ_l	Coefficient related to driving direction on the corresponding intersection of link <i>l</i>	Turning left		1.5
		Going forward		1
		Turning right		0.5
$N_l^{\rm bik}$	Number of non-vehicles on the corresponding intersection of link <i>l</i>		vehicles	
γ	Adjustment parameter			2.09
$N_l^{ m wal}$	Number of pedestrians crossing the corresponding intersection of link <i>l</i> pers		persons	
ω	Decision coefficient			3
ε	Error parameter			0.01

Table	2.	Cont.
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5.2. Model Validation

The travel demands of walking, non-vehicles, and private cars are assigned to multimodal traffic networks using MSWA for a total of 10 instances. The average running times of ten computations for walking, non-vehicles, and private cars are 1.5 s, 4.3 s, and 35.8 s, respectively. An average number of four iterations were required during the computations. Similar assignment results were obtained for ten computations in each trip mode. These findings demonstrate the computational efficiency of the MSWA. Figure 3a shows part of the real traffic volume of private cars obtained from an actual survey, and Figure 3b shows the traffic volume of private cars assigned by the proposed model. It is evident that the traffic volume distributions in Figure 3a,b are identical. Minor differences between the traffic volume distributions in Figure 3a,b exist because the proposed model ignores the vehicles outside the case area. These findings show the optimization quality of MSWA. The above analysis shows the feasibility and practicability of the MSWA for solving the proposed model.



Figure 3. Traffic volume of private cars.

5.3. Sensitivity Analysis

This section introduces the similarity coefficient to analyze the sensitivity of cost adjustment parameter θ . This coefficient can describe the similarity degree of two assignment matrices obtained by the proposed model, and is given as follows:

$$r(A,B) = \begin{cases} 1, & \text{if } A = B\\ \frac{\left|\sum\limits_{x} \sum\limits_{y} (A_{xy} - \overline{A}) \times (B_{xy} - \overline{B})\right|}{\sqrt{\left[\sum\limits_{x} \sum\limits_{y} (A - \overline{A})\right]^2 \times \left[\sum\limits_{x} \sum\limits_{y} (B - \overline{B})\right]^2}}, & \text{if } A \neq B \end{cases}$$
(33)

where r(A, B) is the similarity coefficient of matrices *A* and *B*, and *x* and *y* are the number of rows and columns of the matrices, respectively. Note that the similarity coefficient is equal to one or zero when the two matrices are the same or completely different, respectively. The value of this coefficient varies between zero and one, where a larger value signifies a higher similarity degree.

We denote the assignment results obtained using the proposed model as A when θ is equal to one. Assignment results for values of θ equal to from 0.1 to one with the increasement of 0.1, were denoted as B and used to study the sensitivity of θ . Figure 4a–c show curves of r(A, B) versus different values of cost adjustment parameters for walking, non-vehicles and private cars, respectively. It can be observed that the variation in the values of r(A, B) is directly proportional to the value of the cost adjustment parameter. Accordingly, the cost adjustment parameter θ used to generate matrix A is greater than zero and less than one. We can deduce that the change in r(A, B) is directly proportional to the cost adjustment parameter used to generate matrix B, if it is in the range of $[0.1, \theta)$. The change in r(A, B) is inversely proportional to the cost adjustment parameter used to generate matrix B, if it is in the range of $[\theta, 1]$. Furthermore, the variability of r(A, B) for non-vehicle is the most obvious, whereas the variability values for walking and private car are quite similar.



Figure 4. Cont.



(c) Private car

Figure 4. Similarity coefficient versus different cost adjustment parameters in three trip modes.

5.4. Comparison Analysis

This section defines the following six models: (1) model-all, i.e., the proposed model, considers travel times through intersections, traffic barricades between different lanes, and fuel costs of private cars in the generalized travel cost function; (2) model-1 only considers intersections; (3) model-2 only considers traffic barricades; (4) model-3 only considers fuel costs of private cars; (5) model-no does not consider the practicality mentioned in the Introduction; (6) model-nl uses the average distribution to describe the travel preference for different candidate paths without considering logistic regression.

In a sub-section, when θ is 1, we consider the assignment result of model-no as matrix A. The assignment results of the first five models are considered as matrix B to better discuss the practicality of the proposed model in walking, non-vehicle, and private car modes. Figure 5a–c shows the values of r(A, B) for different models used to generate B in walking, non-vehicle, and private car modes. The smaller the value of r(A, B), the smaller the influence of practicality. The influence of practicality related to the fuel costs of private cars is not discussed for walking and non-vehicle modes. It is found that the practicalities related to travel times of intersections, traffic barricades between different lanes, and fuel costs of private cars applied in the generalized travel cost function can affect the assignment results. The practicality related to intersections for the assignment results has the largest influence in each trip mode. Assignment results in Model-all are affected the most in the walking mode, followed by non-vehicle and private car modes. These findings imply that the practicalities applied in the generalized travel cost function of the proposed model are effective and feasible.



Figure 5. Similarity coefficients between real traffic volume and assignment results for different models.

In another sub-section, we consider the real traffic volume (real-volume) of private cars shown in Figure 3a as matrix *A*. The real-volume and assignment results of model-all and model-nl are considered as matrix *B* to highlight the improvement offered by logistic regression for the proposed model in private car mode, when θ is 1. Figure 5d shows that the value of r(A, B) for model-all used to generate *B* is higher than that for model-nl used to generate *B* in private car mode. This implies that the proposed model considering logistic regression can describe the travel preference better and obtain more real assignment results than that without logistic regression.

5.5. Discussion of Low-Mobility Groups

This section considers the elderly people and people with disabilities as a low-mobility group, i.e., older-disabled group, as they have similar limitations in walking due to their health conditions. Figure 6a shows the walking assignment result of this group. By referring to the research on the Wenling presented in previous studies [2,3], we can see that the walking traffic volume of the older-disabled group in the central business district (CBD) area is greater than that in the other areas. This behavior is coincident with the findings reported in previous studies, i.e., most of the walking trips by elderly people are in urban or smooth traffic areas [1,2,47]. Figure 6b shows the non-vehicle assignment result of the low-income group. Again referring to the research presented in previous studies [2,3], we can observe that the non-vehicle traffic volume of low-income group is concentrated in suburbs. This behavior is coincident with the findings reported by Ren et al. [2], who found that most of the low-income individuals do not dwell in the CBD area. These two findings indicate that the proposed model can be used to accurately analyze the traffic volume of low-mobility groups and provide the research basis for multimodal transport systems of LMIs.



(a) Walking assignment of older-disabled group (persons/h)

(**b**) Non-vehicle assignment of low-income group (vehicles/h)

Figure 6. Assignment results in terms of traffic volume.

6. Conclusions

In this paper, a practical traffic assignment model for multimodal transport system considering low-mobility groups was presented. The model analyzed the equilibrium of route choice for each trip mode using the logit function and transformed the stochastic user equilibrium model into the equivalent variational inequality. The practicality of the proposed model was strengthened by the improvement of generalized travel cost function, and an appropriate solution method (MSWA) was proposed.

The MSWA applied to the proposed model was illustrated by a real case study of Wenling, China. The proposed model and solution method were validated by analyzing the iterative process and comparing real traffic volume with assignment results of private cars. The variation regularity of assignment results with respect to the change of cost adjustment parameter was analyzed. It was observed that the practicality related to travel time through intersections, traffic barricades between different lanes, and fuel costs of private cars applied in the proposed model affected the assignment results. Furthermore, the assignment results of older, disabled, and low-income groups were obtained, which were coincident with the findings reported in existing studies.

In conclusion, the proposed model and solution method are effective, feasible, and practical, and can be used to assign travel demands into links and also provide the research basis for multimodal transport systems of LMIs. However, the proposed model and solution method have a few limitations. These limitations include (1) ignoring the modal choice equilibrium and the uncertainty of travel demands as they are beyond the scope of this research, (2) lack of detailed analysis related to the assignment results of low-mobility groups due to insufficient survey data, and (3) ignoring the methodology to build a dynamic route choice model [38,48], solve the traffic congestion problems [44,49,50], and develop a traffic assignment simulation system using the SUMO (Simulation of Urban Mobility) tool [50–52]. Future work can extend the correlation analysis to overcome these limitations.

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