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# Coefficient Estimates for a Subclass of Bi-Univalent Functions Defined by $q$-Derivative Operator

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**Abstract:** Recently, a number of features and properties of interest for a range of bi-univalent and univalent analytic functions have been explored through systematic study, e.g., coefficient inequalities and coefficient bounds. This study examines $\mathcal{S}_q^{\delta}(\vartheta, \eta, \rho, \nu; \psi)$ as a novel general subclass of $\Sigma$ which comprises normalized analytic functions, as well as bi-univalent functions within $\Delta$ as an open unit disk. The study locates estimates for the $|a_2|$ and $|a_3|$ Taylor–Maclaurin coefficients in functions of the class which is considered. Additionally, links with a number of previously established findings are presented.

**Keywords:** analytic functions; univalent functions; bi-univalent functions; coefficient bounds and coefficient estimates; principle of subordination; $q$-derivative operator

**MSC:** 30C45

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## 1. Introduction

Geometric function theory research has provided analysis of a number of subclasses of $\mathcal{A}$, as a class of normalised analytic function, using a range of approaches. $Q$-calculus has been widely applied in investigating a number of such subclasses within the open unit disk $\Delta$. $\partial_q$ as a $q$-derivative operator was initially applied by Ismail et al. [1] in studying a specific $q$-analogue within $\Delta$ in the starlike function class. Such $q$-operators were also approximated and their geometric properties examined by Mohammed and Darus [2] for several analytic function subclasses within compact disks. The definition of the $q$-operators involved was done through convolution normalised analytic and $q$-hypergeometric functions, and revealed a number of notable findings reported in [3,4]. Raghavendar and Swaminathan [5] studied basic $q$-close-to-convex function properties, while Aral et al. [6] identified $q$-calculus applications within operator theory. In addition, fractional $q$-derivative and fractional $q$-integral operators, among other $q$-calculus operators, have been applied in constructing a number of analytic function subclasses, as in [7–21].

The function class is denoted by $\mathcal{A}$ which represented by the following form:

$$k(\xi) = \xi + \sum_{j=2}^{\infty} a_j \xi^j, \qquad (\xi \in \Delta), \tag{1}$$

that are analytic in the region $\Delta = \{\xi \in \mathbb{C} : |\xi| < 1\}$ and satisfy the following normalization conditions:

$$k(0) = 0 \quad and \quad k'(0) = 1.$$

In addition, let $S$ be the subclass of $\mathcal{A}$ consisting of univalent function in $\Delta$.

For the two functions $k(\xi)$ and $h(\xi)$ analytic in $\Delta$, we say that $k(\xi)$ is subordinate to $h(\xi)$, usually denoted by $k(\xi) \prec h(\xi)$ $(\xi \in \Delta)$, if there exists a Schwarz function $\phi(\xi)$ within $\Delta$ with $\phi(0) = 0$ and $|\phi(\xi)| < 1$, $(\xi \in \Delta)$, such that $k(\xi) = h(\phi(\xi)), (\xi \in \Delta)$.

Especially, if the function $h$ is univalent within $\Delta$, then the above mentioned subordination is comparable to:

$$k \prec h \text{ if and only if } k(0) = h(0) \text{ and } k(\Delta) \subseteq h(\Delta).$$

It should be noted here that Koebe one-quarter theorem as described by [22] stipulates that $\Delta$ images in each univalent function $k \in \mathcal{A}$ have a disc with a $1/4$ radius, meaning that each univalent function $k$ produces $k^{-1}$ as its inverse, which is characterized as

$$k^{-1}(k(\xi)) = \xi, \qquad (\xi \in \Delta)$$

and

$$k^{-1}(k(w)) = w, \qquad (\mid w \mid < r_0(k); r_0(k) \geq \frac{1}{4}),$$

where

$$\chi(w) = k^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots \,. \tag{2}$$

A function $k \in \mathcal{A}$ is said to be bi-univalent in $\Delta$ if both $k$ and $k^{-1}$ are univalent in $\Delta$. Here, $\Sigma$ represents the bi-univalent function class which Equation (1) defines. Some of the examples of functions within the class $\Sigma$ are listed here as below (see Srivastava et al. [23]):

$$\frac{\xi}{1-\xi}, \quad -\log(1-\xi), \quad \frac{1}{2}\log\left(\frac{1+\xi}{1-\xi}\right).$$

However, the well known Koebe function is not within the class $\Sigma$. Other common examples of functions within the class $S$ such as

$$\xi - \frac{\xi^2}{2} \quad and \quad \frac{\xi}{1-\xi^2}$$

are also not within the class $\Sigma$.

For a brief history of functions in the class $\Sigma$, see [24–27]. More recent studies inspired by Srivastava et al.'s [23] ground-breaking investigations in this area examine coefficient bounds in a range of bi-univalent function subclasses (as in e.g., [8,28–33]).

This study begins with definitions of the principal terms used and in-depth concepts for the applications of $q$-calculus used. In this report, it is assumed that $0 < q < 1$. Definitions are first given for fractional $q$-calculus operators in a complex-valued function $k(\xi)$, as follows:

**Definition 1.** *Let $0 < q < 1$. The $q$-number $[j]_q$ is defined by*

$$[j]_q = \begin{cases} \dfrac{1-q^j}{1-q}, & j \in \mathbb{C}; \\ \sum_{n=0}^{m-1} q^n = 1 + q + q^2 + \cdots + q^{m-1}, & j = m \in \mathbb{N}. \end{cases}$$

**Definition 2.** *Let $0 < q < 1$. The $q$-factorial $[j]_q!$ is defined by*

$$[j]_q! = \begin{cases} [j]_q[j-1]_q \cdots\cdots [1]_q, & j = 1, 2, \ldots; \\ 1, & j = 0. \end{cases} \tag{3}$$

**Definition 3.** (see [34,35]) *Let $k \in \mathcal{A}$ and $0 < q < 1$. The $q$-derivative operator of a function $k$ is defined by*

$$\partial_q k(\xi) = \begin{cases} \dfrac{k(q\xi) - k(\xi)}{(q-1)\xi}, & \xi \neq 0; \\ k'(\xi), & \xi = 0. \end{cases} \tag{4}$$

We note from Definition 3 that

$$\lim_{q \to 1} (\partial_q k)(\xi) = \lim_{q \to 1} \frac{k(\xi q) - k(\xi)}{(q-1)\xi} = k'(\xi).$$

From Equations (1) and (4), we get

$$\partial_q k(\xi) = 1 + \sum_{j=2}^{\infty} [j]_q a_j \xi^{j-1}.$$

In 2014, Aldweby and Darus [7] introduced the $q$- analogue of Ruscheweyh Operator $\mathcal{R}_q^\delta$ by

$$\mathcal{R}_q^\delta k(\xi) = \xi + \sum_{j=2}^{\infty} \frac{[j+\delta-1]_q!}{[\delta]_q![j-1]_q!} a_j \xi^j,$$

where $\delta > -1$ and $[j]_q!$ given by Equation (3).

Moreover, as $q \longrightarrow 1$ we have

$$\begin{aligned} \lim_{q \longrightarrow 1} \mathcal{R}_q^\delta k(\xi) &= \xi + \lim_{q \longrightarrow 1} \left[ \sum_{j=2}^{\infty} \frac{[j+\delta-1]_q!}{[\delta]_q![j-1]_q!} a_j \xi^j \right] \\ &= \xi + \sum_{j=2}^{\infty} \frac{(j+\delta-1)!}{(\delta)!(j-1)!} a_j \xi^j \\ &= \mathcal{R}^\delta k(\xi), \end{aligned}$$

where $\mathcal{R}^\delta k(\xi)$ is Ruscheweyh differential operator which was introduced in [36] and a number of authors have studied it before, see for instance [37,38].

The aim of the present work is to introduce $\mathcal{S}_q^\delta(\vartheta, \eta, \rho, \nu; \psi)$ as a general subclass of $\Sigma$ as a class of bi-univalent functions. Within this, estimates are derived for initial coefficients $|a_2|$ and $|a_3|$ for functions within the general subclass. Below, various bi-univalent general subclasses are introduced.

**Definition 4.** *Let $\delta > -1, \vartheta \in \mathbb{C}/\{0\}, 0 \leq \eta \leq 1, 0 \leq \rho \leq 1$ and $0 \leq \nu \leq 1$. A function $k \in \Sigma$ is in the class $\mathcal{S}_q^\delta(\vartheta, \eta, \rho, \nu; \psi)$, if it is satisfying the following subordination conditions :*

$$1 + \frac{1}{\vartheta} \left[ \frac{\xi \partial_q (\mathcal{R}_q^\delta k(\xi)) + \eta \xi^2 \partial_q (\partial_q (\mathcal{R}_q^\delta k(\xi)))}{(1-\rho)\xi + \rho(1-\nu)\mathcal{R}_q^\delta k(\xi) + \nu \xi \partial_q (\mathcal{R}_q^\delta k(\xi))} - \frac{1}{[1+\nu(1-\rho)]} \right] \prec \psi(\xi), \tag{5}$$

*and*

$$1+\frac{1}{\vartheta}\left[\frac{w\partial_q(\mathcal{R}_q^{\delta}\chi(w))+\eta w^2\partial_q(\partial_q(\mathcal{R}_q^{\delta}\chi(w)))}{(1-\rho)w+\rho(1-\nu)\mathcal{R}_q^{\delta}\chi(w)+\nu w\partial_q(\mathcal{R}_q^{\delta}\chi(w))}-\frac{1}{[1+\nu(1-\rho)]}\right]\prec\psi(w), \tag{6}$$

*where the function $\chi$ is given by Equation (2).*

**Remark 1.** *It can clearly be seen that when parameters $\vartheta,\eta,\rho,\delta,q$ and $\nu$ are specialised, this produces a number of established $\Sigma$ subclasses, and there are a number of recent works which examine these. Examples are provided for these subclasses.*

**Example 1.** *Let $\delta = 0$ and $q \to 1$. Then the class $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,\nu;\psi)$ reduces to the class $\mathcal{S}_{\Sigma}(\vartheta,\eta,\rho,\nu;\psi)$ examined by Srivastava et al. [28] which is characterized by*

$$1+\frac{1}{\vartheta}\left[\frac{\xi k'(\xi)+\eta\xi^2k''(\xi)}{(1-\rho)\xi+\rho(1-\nu)k(\xi)+\nu\xi k'(\xi)}-\frac{1}{1+\nu(1-\rho)}\right]\prec\psi(\xi),$$

*and*

$$1+\frac{1}{\vartheta}\left[\frac{w\chi'(w)+\eta w^2\chi''(w)}{(1-\rho)w+\rho(1-\nu)\chi(w)+\nu w\chi'(w)}-\frac{1}{1+\nu(1-\rho)}\right]\prec\psi(w),$$

*where $\chi$ is defined by Equation (2).*

**Example 2.** *Let $\delta = 0, \rho = 0, \nu = 0$ and $q \to 1$. Then the class $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,\nu;\psi)$ reduces to the class $\Sigma(\vartheta,\eta;\psi)$ studied by Srivastava and Bansal [39] that is defined by way of*

$$1+\frac{1}{\vartheta}\left[k'(\xi)-\eta\xi k''(\xi)-1\right]\prec\psi(\xi),$$

*and*

$$1+\frac{1}{\vartheta}\left[\chi'(w)-\eta w\chi''(w)-1\right]\prec\psi(w),$$

*where $\chi$ is defined by Equation (2).*

**Example 3.** *Let $\delta = 0, \rho = 1, \nu = \eta$ and $q \to 1$. Then the class $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,\nu;\psi)$ reduces to the class $S_{\Sigma}(\vartheta,\nu;\psi)$ scrutinized by Deniz [40] that is defined by way of*

$$1+\frac{1}{\vartheta}\left[\frac{\xi k'(\xi)+\nu\xi^2k''(\xi)}{(1-\nu)k(\xi)+\nu\xi k'(\xi)}-1\right]\prec\psi(\xi),$$

*and*

$$1+\frac{1}{\vartheta}\left[\frac{w\chi'(w)+\nu w^2\chi''(w)}{(1-\nu)\chi(w)+\nu w\chi'(w)}-1\right]\prec\psi(w),$$

*where $\chi$ is defined by Equation (2).*

**Example 4.** *Let $\delta = 0, \nu = 0, \eta = 0$ and $q \to 1$. Then the class $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,\nu;\psi)$ reduces to the class $S_{\Sigma}(\vartheta,\rho;\psi)$ studied by Peng et al. [41] which is defined as*

$$1+\frac{1}{\vartheta}\left[\frac{\xi k'(\xi)}{(1-\rho)\xi+\rho k(\xi)}-1\right]\prec\psi(\xi),$$

*and*

$$1+\frac{1}{\vartheta}\left[\frac{w\chi'(w)}{(1-\rho)w+\rho\chi(w)}-1\right]\prec\psi(w),$$

*where $\chi$ is defined by Equation (2).*

**Example 5.** *Let $\delta = 0, \nu = 0, \eta = 0, \vartheta = 1$ and $q \to 1$. Then the class $\mathcal{S}_q^{\delta}(\vartheta, \eta, \rho, \nu; \psi)$ reduces to the class $S_{\Sigma}(\rho; \psi)$ examined by Magesh and Yamini [42] which is characterized as*

$$\frac{\xi k'(\xi)}{(1-\rho)\xi + \rho k(\xi)} \prec \psi(\xi),$$

*and*

$$\frac{w\chi'(w)}{(1-\rho)w + \rho\chi(w)} \prec \psi(w),$$

*where $\chi$ is defined by Equation (2).*

In order to prove our main results, we need the subsequent lemma.

**Lemma 1.** *(see [43]) Let $p \in \mathcal{P}$, then*

$$b_j \leq 2 \quad (j \in \mathbb{N}),$$

*where $\mathcal{P}$ is the family of all analytic functions $p$ in $\Delta$, for which*

$$Re(p(\xi)) > 0 \quad (\xi \in \Delta),$$

*where*

$$p(\xi) = 1 + b_1\xi + b_2\xi^2 + \dots . \quad (\xi \in \Delta).$$

Additionally, some useful work associated with inequalities and their properties can be read in [44–47].

## 2. A Set of Main Results

This section starts by establishing estimates for $\mathcal{S}_q^{\delta}(\vartheta, \eta, \rho, \nu; \psi)$ class function for coefficients $|a_2|$ and $|a_3|$.

Let $\psi$ be an analytic function with $Re(\psi(\xi)) > 0$ within $\Delta$, satisfying $\psi(0) = 1, \psi'(0) > 0$, and $\psi(\Delta)$ is symmetric with respect to the real axis. Such a function has a form:

$$\psi(\xi) = 1 + B_1\xi + B_2\xi^2 + B_3\xi^3 + B_4\xi^4 + \dots . \quad (B_1 > 0; \xi \in \Delta). \tag{7}$$

**Theorem 1.** *Let $k(\xi) \in \mathcal{S}_q^{\delta}(\vartheta, \eta, \rho, \nu; \psi)$ be of the form in Equation (1). Then*

$$|a_2| \leq \frac{\sqrt{2[2]_q}\, |\vartheta|\, B_1^{\frac{3}{2}} (\rho\nu - \nu - 1)^2}{\sqrt{\left|\vartheta B_1^2 [\delta+1]_q (\rho\nu - \nu - 1) \Psi(\eta, \rho, \nu, \delta, q) + 2[2]_q (B_1 - B_2) \Theta^2(\eta, \rho, \nu, \delta, q)\right|}}, \tag{8}$$

*and*

$$|a_3| \leq B_1 |\vartheta| (\rho\nu - \nu - 1)^2 \left( \frac{B_1 |\vartheta| (\rho\nu - \nu - 1)^2}{\Theta^2(\eta, \rho, \nu, \delta, q)} + \frac{[2]_q}{|\Upsilon(\eta, \rho, \nu, \delta, q)|} \right), \tag{9}$$

*where*

$$\Theta(\eta, \rho, \nu, \delta, q) = [\delta+1]_q \left( [2]_q - q\rho\nu + [2]_q\eta\nu - [2]_q\rho\eta\nu - \rho + [2]_q\eta \right), \tag{10}$$

$$\Upsilon(\eta,\rho,\nu,\delta,q) = [\delta+1]_q[\delta+2]_q([3]_q - q[2]_q\rho\nu + [2]_q[3]_q\eta\nu - [2]_q[3]_q\rho\eta\nu - \rho + [2]_q[3]_q\eta), \quad (11)$$

*and*

$$\begin{aligned}\Psi(\eta,\rho,\nu,\delta,q) &= -2[3]_q[\delta+2]_q + \left(4[3]_q[\delta+2]_q - 4[2]_q^2[\delta+1]_q\right)\rho\nu \\ &+ \left(2[2]_q^3[\delta+1]_q - 4[2]_q[3]_q[\delta+2]_q\right)\eta\nu + \left(2q[2]_q[\delta+1]_q - 2q[2]_q[\delta+2]_q\right)\rho^2\nu^2 \\ &+ 2[2]_q^2[\delta+1]_q\rho\eta - 2[2]_q[\delta+1]_q\rho^2 - \left(2[2]_q(q-1)[\delta+1]_q + 2[\delta+2]_q\right)\rho^2\nu \\ &+ \left(2[2]_q^3[\delta+1]_q - 2[2]_q[3]_q[\delta+2]_q\right)\eta\nu^2 + \left(2[2]_q^2[\delta+1]_q - 2[2]_q[3]_q[\delta+2]_q\right)\rho^2\eta\nu^2 \\ &+ \left(4[2]_q[3]_q[\delta+2]_q - 2[2]_q(1+[2]_q)[\delta+1]_q\right)\rho\eta\nu^2 + 4[2]_q[3]_q[\delta+2]_q\rho\eta\nu \\ &+ \left(2[2]_q^3[\delta+1]_q - 2[3]_q[\delta+2]_q\right)\nu - 2[2]_q^2[\delta+1]_q\rho^2\eta\nu - 2[2]_q[3]_q[\delta+2]_q\eta \\ &+ \left(2[2]_q^2[\delta+1]_q + 2[\delta+2]_q\right)\rho + \left(2q[2]_q^2[\delta+1]_q - 2q[2]_q[\delta+2]_q\right)\rho\nu^2. \quad (12)\end{aligned}$$

**Proof.** Let $k \in \mathcal{S}_q^\delta(\vartheta,\eta,\rho,\nu;\psi)$ and $\chi = k^{-1}$. Then there are analytic functions $u, v : \Delta \to \Delta$ with $u(0) = v(0) = 0$, satisfying the following conditions:

$$1 + \frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^\delta k(\xi)) + \eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^\delta k(\xi)))}{(1-\rho)\xi + \rho(1-\nu)\mathcal{R}_q^\delta k(\xi) + \nu\xi\partial_q(\mathcal{R}_q^\delta k(\xi))} - \frac{1}{[1+\nu(1-\rho)]}\right] = \psi(u(\xi)), \quad (\xi \in \Delta), \quad (13)$$

and

$$1 + \frac{1}{\vartheta}\left[\frac{w\partial_q(\mathcal{R}_q^\delta \chi(w)) + \eta w^2\partial_q(\partial_q(\mathcal{R}_q^\delta \chi(w)))}{(1-\rho)w + \rho(1-\nu)\mathcal{R}_q^\delta \chi(w) + \nu w\partial_q(\mathcal{R}_q^\delta \chi(w))} - \frac{1}{[1+\nu(1-\rho)]}\right] = \psi(v(w)), \quad (w \in \Delta). \quad (14)$$

Define the functions $p$ and $h$ by

$$p(\xi) = \frac{1+u(\xi)}{1-u(\xi)} = 1 + p_1\xi + p_2\xi^2 + p_3\xi^3 + \ldots, \quad (15)$$

and

$$h(\xi) = \frac{1+v(\xi)}{1-v(\xi)} = 1 + h_1\xi + h_2\xi^2 + h_3\xi^3 + \ldots. \quad (16)$$

Then $p$ and $h$ are analytic within $\Delta$ with $p(0) = h(0) = 1$. By reason of $u, v : \Delta \to \Delta$, the functions $p$ and $h$ has a positive real part in $\Delta$. Thus, by using Lemma 1, we have

$$|p_j| \le 2 \quad \textit{and} \quad |h_j| \le 2, \quad (j \in \mathbb{N} = \{1,2,3,\ldots\}).$$

It follows from Equations (15) and (16) that

$$u(\xi) = \frac{p(\xi)-1}{p(\xi)+1} = \frac{1}{2}\left[p_1\xi + \left(p_2 - \frac{p_1^2}{2}\right)\xi^2\right] + \ldots \quad , \quad (\xi \in \Delta), \quad (17)$$

and

$$v(\xi) = \frac{h(\xi)-1}{h(\xi)+1} = \frac{1}{2}\left[h_1\xi + \left(h_2 - \frac{h_1^2}{2}\right)\xi^2\right] + \ldots \quad , \quad (\xi \in \Delta). \quad (18)$$

Clearly, substituting Equations (17) and (18) into Equations (13) and (14), respectively, in the event that we make use of Equation (7), we get

$$1+\frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^\delta k(\xi))+\eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^\delta k(\xi)))}{(1-\rho)\xi+\rho(1-\nu)\mathcal{R}_q^\delta k(\xi)+\nu\xi\partial_q(\mathcal{R}_q^\delta k(\xi))}-\frac{1}{[1+\nu(1-\rho)]}\right]=$$
$$\psi\left(\frac{p(\xi)-1}{p(\xi)+1}\right)=1+\frac{B_1p_1}{2}\xi+\left[\frac{1}{2}\left(p_2-\frac{p_1^2}{2}\right)B_1+\frac{1}{4}p_1^2B_2\right]\xi^2+\ldots, \tag{19}$$

and

$$1+\frac{1}{\vartheta}\left[\frac{w\partial_q(\mathcal{R}_q^\delta \chi(w))+\eta w^2\partial_q(\partial_q(\mathcal{R}_q^\delta \chi(w)))}{(1-\rho)w+\rho(1-\nu)\mathcal{R}_q^\delta \chi(w)+\nu w\partial_q(\mathcal{R}_q^\delta \chi(w))}-\frac{1}{[1+\nu(1-\rho)]}\right]=$$
$$\psi\left(\frac{h(\xi)-1}{h(\xi)+1}\right)=1+\frac{B_1h_1}{2}w+\left[\frac{1}{2}\left(h_2-\frac{h_1^2}{2}\right)B_1+\frac{1}{4}h_1^2B_2\right]w^2+\ldots. \tag{20}$$

Moreover,

$$1+\frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^\delta k(\xi))+\eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^\delta k(\xi)))}{(1-\rho)\xi+\rho(1-\nu)\mathcal{R}_q^\delta k(\xi)+\nu\xi\partial_q(\mathcal{R}_q^\delta k(\xi))}-\frac{1}{[1+\nu(1-\rho)]}\right]=$$
$$1+\left\{\frac{[\delta+1]_q\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^2}\right\}a_2\xi+$$
$$\left\{\frac{[\delta+1]_q[\delta+2]_q\left([3]_q-q[2]_q\rho\nu+[2]_q[3]_q\eta\nu-[2]_q[3]_q\rho\eta\nu-\rho+[2]_q[3]_q\eta\right)}{\vartheta[2]_q(\rho\nu-\nu-1)^2}a_3-\right.$$
$$\left.\frac{[\delta+1]_q^2(\rho\nu-[2]_q\nu-\rho)\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^3}a_2^2\right\}\xi^2+\ldots, \tag{21}$$

and

$$1+\frac{1}{\vartheta}\left[\frac{w\partial_q(\mathcal{R}_q^\delta \chi(w))+\eta w^2\partial_q(\partial_q(\mathcal{R}_q^\delta \chi(w)))}{(1-\rho)w+\rho(1-\nu)\mathcal{R}_q^\delta \chi(w)+\nu w\partial_q(\mathcal{R}_q^\delta \chi(w))}-\frac{1}{[1+\nu(1-\rho)]}\right]=$$
$$1-\left\{\frac{[\delta+1]_q\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^2}\right\}a_2w+$$
$$\left\{\frac{[\delta+1]_q[\delta+2]_q\left([3]_q-q[2]_q\rho\nu+[2]_q[3]_q\eta\nu-[2]_q[3]_q\rho\eta\nu-\rho+[2]_q[3]_q\eta\right)}{\vartheta[2]_q(\rho\nu-\nu-1)^2}(2a_2^2-a_3)-\right.$$
$$\left.\frac{[\delta+1]_q^2(\rho\nu-[2]_q\nu-\rho)\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^3}a_2^2\right\}w^2+\ldots.. \tag{22}$$

Now, equating the coefficients in Equations (19)–(22), we get

$$\frac{[\delta+1]_q\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^2}a_2=\frac{B_1p_1}{2}, \tag{23}$$

and

$$\frac{[\delta+1]_q[\delta+2]_q\left([3]_q-q[2]_q\rho\nu+[2]_q[3]_q\eta\nu-[2]_q[3]_q\rho\eta\nu-\rho+[2]_q[3]_q\eta\right)}{\vartheta[2]_q(\rho\nu-\nu-1)^2}a_3 \\ -\frac{[\delta+1]_q^2(\rho\nu-[2]_q\nu-\rho)\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^3}a_2^2 \\ =\frac{1}{2}\left(p_2-\frac{p_1^2}{2}\right)B_1+\frac{1}{4}p_1^2B_2. \tag{24}$$

Moreover, we have

$$-\frac{[\delta+1]_q\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^2}a_2=\frac{B_1h_1}{2}, \tag{25}$$

and

$$\frac{[\delta+1]_q[\delta+2]_q\left([3]_q-q[2]_q\rho\nu+[2]_q[3]_q\eta\nu-[2]_q[3]_q\rho\eta\nu-\rho+[2]_q[3]_q\eta\right)}{\vartheta[2]_q(\rho\nu-\nu-1)^2}(2a_2^2-a_3) \\ -\frac{[\delta+1]_q^2(\rho\nu-[2]_q\nu-\rho)\left([2]_q-q\rho\nu+[2]_q\eta\nu-[2]_q\rho\eta\nu-\rho+[2]_q\eta\right)}{\vartheta(\rho\nu-\nu-1)^3}a_2^2 \\ =\frac{1}{2}\left(h_2-\frac{h_1^2}{2}\right)B_1+\frac{1}{4}h_1^2B_2. \tag{26}$$

From Equations (23) and (25), we find that

$$p_1=-h_1. \tag{27}$$

By adding Equations (24) and (26), and then using Equation (27), we obtain

$$\frac{[\delta+1]_q}{\vartheta[2]_q(\rho\nu-\nu-1)^3}\Big\{-2[3]_q[\delta+2]_q+\left(4[3]_q[\delta+2]_q-4[2]_q^2[\delta+1]_q\right)\rho\nu \\ +\left(2[2]_q^3[\delta+1]_q-4[2]_q[3]_q[\delta+2]_q\right)\eta\nu+\left(2q[2]_q[\delta+1]_q-2q[2]_q[\delta+2]_q\right)\rho^2\nu^2 \\ +2[2]_q^2[\delta+1]_q\rho\eta-2[2]_q[\delta+1]_q\rho^2-\left(2[2]_q(q-1)[\delta+1]_q+2[\delta+2]_q\right)\rho^2\nu \\ +\left(2[2]_q^3[\delta+1]_q-2[2]_q[3]_q[\delta+2]_q\right)\eta\nu^2+\left(2[2]_q^2[\delta+1]_q-2[2]_q[3]_q[\delta+2]_q\right)\rho^2\eta\nu^2 \\ +\left(4[2]_q[3]_q[\delta+2]_q-2[2]_q(1+[2]_q)[\delta+1]_q\right)\rho\eta\nu^2+4[2]_q[3]_q[\delta+2]_q\rho\eta\nu \\ +\left(2[2]_q^3[\delta+1]_q-2[3]_q[\delta+2]_q\right)\nu-2[2]_q^2[\delta+1]_q\rho^2\eta\nu-2[2]_q[3]_q[\delta+2]_q\eta \\ +\left(2[2]_q^2[\delta+1]_q+2[\delta+2]_q\right)\rho+\left(2q[2]_q^2[\delta+1]_q-2q[2]_q[\delta+2]_q\right)\rho\nu^2\Big\}a_2^2 \\ =\frac{p_1^2}{2}(B_2-B_1)+\frac{B_1}{2}(p_2+h_2). \tag{28}$$

For the purpose of brevity, we will utilize the notations given in Equations (10)–(12). Now, making use of the notations defined above and combining Equations (23) and (28), we get

$$a_2^2=\frac{[2]_q\vartheta^2B_1^3(\rho\nu-\nu-1)^4(p_2+h_2)}{2\left[\vartheta B_1^2[\delta+1]_q(\rho\nu-\nu-1)\Psi(\eta,\rho,\nu,\delta,q)+2[2]_q(B_1-B_2)\Theta^2(\eta,\rho,\nu,\delta,q)\right]}. \tag{29}$$

Applying Lemma 1 to the coefficients $p_2$ and $h_2$, we find that

$$a_2^2 \leq \frac{2[2]_q \, |\vartheta|^2 \, B_1^3 (\rho\nu - \nu - 1)^4}{\left|\vartheta B_1^2 [\delta+1]_q (\rho\nu - \nu - 1) \Psi(\eta,\rho,\nu,\delta,q) + 2[2]_q (B_1 - B_2) \Theta^2(\eta,\rho,\nu,\delta,q)\right|}, \tag{30}$$

so that

$$|a_2| \leq \frac{\sqrt{2[2]_q} \, |\vartheta| \, B_1^{\frac{3}{2}} (\rho\nu - \nu - 1)^2}{\sqrt{\left|\vartheta B_1^2 [\delta+1]_q (\rho\nu - \nu - 1) \Psi(\eta,\rho,\nu,\delta,q) + 2[2]_q (B_1 - B_2) \Theta^2(\eta,\rho,\nu,\delta,q)\right|}}, \tag{31}$$

where $\Psi(\eta,\rho,\nu,\delta,q)$ and $\Theta(\eta,\rho,\nu,\delta,q)$ are given by Equations (10) and (12), respectively.

Similarly, upon subtracting Equation (26) from Equation (24) and then using Equation (27), we get

$$\frac{\Upsilon(\eta,\rho,\nu,\delta,q)}{[2]_q \vartheta (\rho\nu - \nu - 1)^2} (a_3 - a_2^2) = \frac{B_1}{4} (p_2 - h_2), \tag{32}$$

where $\Upsilon(\eta,\rho,\nu,\delta,q)$ is defined by Equation (11). It follows from Equations (23) and (32) that

$$a_3 = \frac{B_1^2 \vartheta^2 (\rho\nu - \nu - 1)^4 p_1^2}{4\Theta^2(\eta,\rho,\nu,\delta,q)} + \frac{[2]_q B_1 \vartheta (\rho\nu - \nu - 1)^2 (p_2 - h_2)}{4\Upsilon(\eta,\rho,\nu,\delta,q)}. \tag{33}$$

Finally, applying the Lemma 1 once more for the coefficients $p_1$, $p_2$ and $h_2$, we have

$$|a_3| \leq B_1 \, |\vartheta| \, (\rho\nu - \nu - 1)^2 \left( \frac{B_1 \, |\vartheta| \, (\rho\nu - \nu - 1)^2}{\Theta^2(\eta,\rho,\nu,\delta,q)} + \frac{[2]_q}{|\Upsilon(\eta,\rho,\nu,\delta,q)|} \right). \tag{34}$$

This completes the proof of Theorem 1. □

## 3. Applications of the Main Result

This part of the paper presents certain distinctive cases within the broader results, provided as corollaries. First of all, by letting

$$\psi(\xi) = \frac{1 + C\xi}{1 + D\xi} = 1 + (C - D)\xi + D(D - C)\xi^2 + \ldots \quad (-1 \leq D < C \leq 1),$$

in Definition 4 of the class $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,\nu;\psi)$, we obtain a new class $\mathcal{S}_q^{1,\delta}(\vartheta,\eta,\rho,\nu;C,D)$ given by Definition 5.

**Definition 5.** *Let $\delta > -1, \vartheta \in \mathbb{C}/\{0\}, 0 \leq \eta \leq 1, 0 \leq \rho \leq 1$ and $0 \leq \nu \leq 1$. A function $k \in \Sigma$ is said to be in the class $\mathcal{S}_q^{1,\delta}(\vartheta,\eta,\rho,\nu;C,D)$, if each of the following subordination condition holds true:*

$$1 + \frac{1}{\vartheta} \left[ \frac{\xi \partial_q (\mathcal{R}_q^{\delta} k(\xi)) + \eta \xi^2 \partial_q (\partial_q (\mathcal{R}_q^{\delta} k(\xi)))}{(1-\rho)\xi + \rho(1-\nu)\mathcal{R}_q^{\delta} k(\xi) + \nu \xi \partial_q (\mathcal{R}_q^{\delta} k(\xi))} - \frac{1}{[1 + \nu(1-\rho)]} \right] \prec \frac{1 + C\xi}{1 + D\xi},$$

*and*

$$1 + \frac{1}{\vartheta} \left[ \frac{w \partial_q (\mathcal{R}_q^{\delta} \chi(w)) + \eta w^2 \partial_q (\partial_q (\mathcal{R}_q^{\delta} \chi(w)))}{(1-\rho)w + \rho(1-\nu)\mathcal{R}_q^{\delta} \chi(w) + \nu w \partial_q (\mathcal{R}_q^{\delta} \chi(w))} - \frac{1}{[1 + \nu(1-\rho)]} \right] \prec \frac{1 + Cw}{1 + Dw},$$

*where the function $\chi$ is given by Equation (2).*

Utilizing the parameters setting of Definition 5 within the Theorem 1, we obtain the following result.

**Corollary 1.** *Let $k(\xi) \in \mathcal{S}_q^{1,\delta}(\vartheta,\eta,\rho,\nu;C,D)$ be of the form in Equation (1). Then*

$$|a_2| \leq \frac{\sqrt{2[2]_q}\, |\vartheta|\, (C-D)(\rho\nu-\nu-1)^2}{\sqrt{\left|\vartheta(C-D)[\delta+1]_q(\rho\nu-\nu-1)\Psi(\eta,\rho,\nu,\delta,q)+2[2]_q(1+D)\Theta^2(\eta,\rho,\nu,\delta,q)\right|}},$$

*and*

$$|a_3| \leq (C-D)\, |\vartheta|\, (\rho\nu-\nu-1)^2 \left( \frac{(C-D)\, |\vartheta|\, (\rho\nu-\nu-1)^2}{\Theta^2(\eta,\rho,\nu,\delta,q)} + \frac{[2]_q}{|\Upsilon(\eta,\rho,\nu,\delta,q)|} \right),$$

*where* $\Theta(\eta,\rho,\nu,\delta,q)$, $\Upsilon(\eta,\rho,\nu,\delta,q)$ *and* $\Psi(\eta,\rho,\nu,\delta,q)$ *are given by Equations (10)–(12).*

Next, if we set

$$\psi(\xi) = \left(\frac{1+\xi}{1-\xi}\right)^{\sigma} = 1 + 2\sigma\xi + 2\sigma^2\xi^2 + \ldots \quad (0 < \sigma \leq 1)$$

in Definition 4 of the class $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,\nu;\psi)$, we get a new class $\mathcal{S}_q^{2,\delta}(\vartheta,\eta,\rho,\nu;\sigma)$ given as

**Definition 6.** *Let* $\delta > -1, \vartheta \in \mathbb{C}/\{0\}, 0 \leq \eta \leq 1, 0 \leq \rho \leq 1$ *and* $0 \leq \nu \leq 1$. *A function* $k \in \Sigma$ *is said to be in the class* $\mathcal{S}_q^{2,\delta}(\vartheta,\eta,\rho,\nu;\sigma)$, *if each of the following subordination condition holds true:*

$$\left|\arg\left\{1+\frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi))+\eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^{\delta}k(\xi)))}{(1-\rho)\xi+\rho(1-\nu)\mathcal{R}_q^{\delta}k(\xi)+\nu\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi))}-\frac{1}{[1+\nu(1-\rho)]}\right]\right\}\right| < \frac{\sigma\pi}{2},$$

*and*

$$\left|\arg\left\{1+\frac{1}{\vartheta}\left[\frac{w\partial_q(\mathcal{R}_q^{\delta}\chi(w))+\eta w^2\partial_q(\partial_q(\mathcal{R}_q^{\delta}\chi(w)))}{(1-\rho)w+\rho(1-\nu)\mathcal{R}_q^{\delta}\chi(w)+\nu w\partial_q(\mathcal{R}_q^{\delta}\chi(w))}-\frac{1}{[1+\nu(1-\rho)]}\right]\right\}\right| < \frac{\sigma\pi}{2},$$

*where the function* $\chi$ *is given by Equation (2).*

Utilizing the parameters setting of Definition 6 within the Theorem 1, we obtain the following result

**Corollary 2.** *Let* $k(\xi) \in \mathcal{S}_q^{2,\delta}(\vartheta,\eta,\rho,\nu;\sigma)$ *be of the form in Equation (1). Then*

$$|a_2| \leq \frac{2\sqrt{2[2]_q}\, |\vartheta|\, \sigma(\rho\nu-\nu-1)^2}{\sqrt{\left|2\vartheta\sigma[\delta+1]_q(\rho\nu-\nu-1)\Psi(\eta,\rho,\nu,\delta,q)+2[2]_q(1-\sigma)\Theta^2(\eta,\rho,\nu,\delta,q)\right|}},$$

*and*

$$|a_3| \leq 2\sigma\, |\vartheta|\, (\rho\nu-\nu-1)^2 \left( \frac{2\sigma\, |\vartheta|\, (\rho\nu-\nu-1)^2}{\Theta^2(\eta,\rho,\nu,\delta,q)} + \frac{[2]_q}{|\Upsilon(\eta,\rho,\nu,\delta,q)|} \right),$$

*where* $\Theta(\eta,\rho,\nu,\delta,q)$, $\Upsilon(\eta,\rho,\nu,\delta,q)$ *and* $\Psi(\eta,\rho,\nu,\delta,q)$ *are given by Equations (10)–(12).*

Finally, if we set

$$\psi(\xi) = \frac{1+(1-2\alpha)\xi}{1-\xi} = 1 + 2(1-\alpha)\xi + 2(1-\alpha)\xi^2 + \ldots \quad (0 < \alpha \leq 1),$$

in Definition 4 of the class $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,\nu;\psi)$, we get a new class $\mathcal{S}_q^{3,\delta}(\vartheta,\eta,\rho,\nu;\alpha)$ given as

**Definition 7.** *Let $\delta > -1, \vartheta \in \mathbb{C}/\{0\}, 0 \leq \eta \leq 1, 0 \leq \rho \leq 1$ and $0 \leq \nu \leq 1$. A function $k \in \Sigma$ is said to be in the class $\mathcal{S}_q^{3,\delta}(\vartheta, \eta, \rho, \nu; \alpha)$, if each of the following subordination condition holds true:*

$$Re\left\{1 + \frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^\delta k(\xi)) + \eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^\delta k(\xi)))}{(1-\rho)\xi + \rho(1-\nu)\mathcal{R}_q^\delta k(\xi) + \nu\xi\partial_q(\mathcal{R}_q^\delta k(\xi))} - \frac{1}{[1+\nu(1-\rho)]}\right]\right\} > \alpha,$$

*and*

$$Re\left\{1 + \frac{1}{\vartheta}\left[\frac{w\partial_q(\mathcal{R}_q^\delta \chi(w)) + \eta w^2\partial_q(\partial_q(\mathcal{R}_q^\delta \chi(w)))}{(1-\rho)w + \rho(1-\nu)\mathcal{R}_q^\delta \chi(w) + \nu w\partial_q(\mathcal{R}_q^\delta \chi(w))} - \frac{1}{[1+\nu(1-\rho)]}\right]\right\} > \alpha,$$

*where the function $\chi$ is given by Equation (2).*

Utilizing the parameter setting of Definition 7 within Theorem 1, we obtain the following result

**Corollary 3.** *Let $k(\xi) \in \mathcal{S}_q^{3,\delta}(\vartheta, \eta, \rho, \nu; \alpha)$ be of the form in Equation (1). Then*

$$|a_2| \leq \frac{2\sqrt{[2]_q}\,|\vartheta|\,(1-\alpha)(\rho\nu - \nu - 1)^2}{\sqrt{\left|\vartheta B_1^2[\delta+1]_q(\rho\nu - \nu - 1)\Psi(\eta, \rho, \nu, \delta, q)\right|}},$$

*and*

$$|a_3| \leq 2(1-\alpha)\,|\vartheta|\,(\rho\nu - \nu - 1)^2\left(\frac{2(1-\alpha)\,|\vartheta|\,(\rho\nu - \nu - 1)^2}{\Theta^2(\eta, \rho, \nu, \delta, q)} + \frac{[2]_q}{|\Upsilon(\eta, \rho, \nu, \delta, q)|}\right),$$

*where $\Theta(\eta, \rho, \nu, \delta, q)$, $\Upsilon(\eta, \rho, \nu, \delta, q)$ and $\Psi(\eta, \rho, \nu, \delta, q)$ are given by Equations (10)–(12).*

**Remark 2.** *Taking $\delta = 0$ and $q \to 1$ in Theorem 1, we obtain the following result.*

**Corollary 4** ( [28]). *Let $k(\xi) \in \mathcal{S}_\Sigma(\vartheta, \eta, \rho, \nu; \psi)$ be of the form in Equation (1). Then*

$$|a_2| \leq \frac{|\vartheta|\, B_1^{\frac{3}{2}}(\rho\nu - \nu - 1)^2}{\sqrt{\left|\vartheta B_1^2(\rho\nu - \nu - 1)\Psi(\eta, \rho, \nu) + (B_1 - B_2)\Theta^2(\eta, \rho, \nu)\right|}},$$

*and*

$$|a_3| \leq B_1\,|\vartheta|\,(\rho\nu - \nu - 1)^2\left(\frac{B_1\,|\vartheta|\,(\rho\nu - \nu - 1)^2}{\Theta^2(\eta, \rho, \nu)} + \frac{1}{|\Upsilon(\eta, \rho, \nu)|}\right),$$

*where*

$$\Theta(\eta, \rho, \nu) = 2 - \rho\nu + 2\eta\nu - 2\eta\rho\nu - \rho + 2\eta,$$
$$\Upsilon(\eta, \rho, \nu) = 3 - 2\rho\nu + 6\eta\nu - 6\eta\rho\nu - \rho + 6\eta,$$

*and*

$$\Psi(\eta, \rho, \nu) = -3 + 2\rho\nu - 8\eta\nu - \rho^2\nu^2 + 2\rho\eta - \rho^2 - \rho^2\nu - 2\eta\nu^2 + 6\rho\eta\nu^2 - 4\rho^2\eta\nu^2 + 12\rho\eta\nu + \nu - 2\rho^2\eta\nu + 3\rho - 6\eta,$$

*where the class $\mathcal{S}_\Sigma(\vartheta, \eta, \rho, \nu; \psi)$ is defined in Example 1.*

**Remark 3.** *Setting $\rho = \nu = \delta = 0$ and $q \to 1$ in Theorem 1, we get the following corollary*

**Corollary 5** ([39])**.** *Let $k(\xi) \in \Sigma(\vartheta, \eta; \psi)$ be of the form in Equation (1). Then*

$$|a_2| \leq \frac{|\vartheta|\, B_1^{\frac{3}{2}}}{\sqrt{\left|3\vartheta B_1^2(1+2\eta) + 4(B_1 - B_2)(1+\eta)^2\right|}},$$

*and*

$$|a_3| \leq B_1\,|\vartheta| \left( \frac{B_1\,|\vartheta|}{4(1+\eta)^2} + \frac{1}{3(1+2\eta)} \right),$$

*where the class $\Sigma(\vartheta, \eta; \psi)$ is defined in Example 2.*

## 4. Conclusions

The $q$-calculus is a wide field and is applicable to many areas of physics and mathematics and, as well, to other areas, for example, in differential equations, the theory of special functions, analytic number theory, combinatorics, quantum theory, quantum group, numerical analysis, special polynomials, operator theory and other related theories. This paper principally aims to derive the two initial Taylor–Maclaurin coefficient estimates of functions within the novel subclass $\mathcal{S}_q^{\delta}(\vartheta, \eta, \rho, \nu; \psi)$ for analytic/bi-univalent functions within $\Delta$ as an open unit disk. This is achieved by using the Ruscheweyh $q$-calculus operator. Further, using corollaries and consequences, as discussed earlier through appropriate specialisation of the $\delta, \rho, \eta, \nu$ and $q$ parameters, the paper also demonstrates that the findings here can enhance and generalise some work recently published.

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