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Aircraft Trajectory Tracking Using Radar Equipment with Fuzzy Logic Algorithm

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Abstract: Radio-electronic means, including equipment for transmissions, radio-location, broadcasting, and navigation, allow the execution of various research missions and combat forces management. Determining the target coordinates and directing the armament towards them, obtaining and processing data about enemies, ensuring the navigation of ships, planes and outer atmospheric means, transmitting orders, decisions, reports and other necessary information for the armed forces; these are only some of the possibilities of radio-electronic technology. Fuzzy logic allows the linguistic description of the laws of command, operation and control of a system. When working with complex and nonlinear systems, it can often be observed that, as their complexity increases, there is a decrease in the significance of the details in describing the global behavior of the system. Even though such an approach may seem inadequate, it is often superior and less laborious than a rigorous mathematical approach. The main argument in favor of fuzzy set theory is to excel in operating with imprecise, vague notions. This article demonstrates the superiority of a fuzzy tracking system over the standard Kalman filter tracking system under the conditions of uneven accelerations and sudden change of direction of the targets, as well as in the case of failure to observe the target during successive scans. A cascading Kalman filtering algorithm was used to solve the speed ambiguity and to reduce the measurement error in real-time radar processing. The cascade filters are extended Kalman filters with controlled gain using fuzzy logic for tracking targets using radar equipment under difficult tracking conditions.

Keywords: target tracking; Kalman; fuzzy logic; radar prediction trajectory

1. Introduction

Tracking of targets represents the forecast of the possible trajectories of the target as a function of its previous positions. The prediction accuracy depends on how much precisely the previous and current positions of the object were measured. Unfortunately, due to parasitic echoes generated on the radio locator screen, the limitations of algorithms related to processing the signals coming from different sensors [1], the position of the targets can be determined with approximation. The values measured can enhance multiple variable features and may be constrained by different initial conditions depending on the flight angle [2] and on the position of the target towards the radar [3].

In [4], Zhou et al. proposed an alternative to the standard Kalman filter for target tracking in the case when the radar has a faulty functioning with model mismatching. Their solution was to use an adaptive unscented Kalman filter in which the parameters (innovation vector, covariance matrix) can be corrected in real time by using an adaptive matrix gene. The performance of the new filter proved to be higher than the standard one, being suitable for tracking missions.

A modified Kalman filter was also used by Zhang et al. in [5]. Their idea was to use variations of the cubature Kalman filter for achieving higher robustness and reducing the algorithm complexity simultaneously.

Cao et al., presented in [6], two algorithms used to detect and track moving targets in infrared aerial domain. The two algorithms presented are SFDLC (symmetric frame differencing target detection based on local clustering segmentation) used for target detection and MSDU (kernel-based mean shift target tracking based on detection updates) used for target tracking, both being based on gray-scale image processing. The use of the latter one led to an overall improvement of the tracking error.

Regarding fuzzy logic, Dahmani et al., proposed in [7] a new fuzzy α-β filter for tracking targets with high maneuvering. The coefficients were determined based on tracking index λ (index of maneuverability), having the advantages of being able to detect the starting and ending moment of a maneuver and being easy to implement through the use of the Takagi Sugeno model with few fuzzy rules. Liang-qun et al., also used fuzzy logic in [8] for tracking multiple objects.

Kalman algorithm and fuzzy logic have also been linked in [9] by Amirzadeh and Karimpour, in which they considered an interacting fuzzy fading memory based augmented Kalman filter for target tracking during maneuvers. Their solution solved the problem of unknown target acceleration.

In relation to the ideas presented in [10], the method proposed by the authors differs in many aspects. First of all, the superiority of the tracking system with extended cascade Kalman fuzzy filter over a tracking system based on a second order Kalman fuzzy filter was demonstrated, under the conditions of uneven accelerations and sudden change of target direction, as well as in the case of failure to detect the target during several successive scans. Secondly, this paper presents a new implementation method for the extended cascade Kalman fuzzy filters, which can be used practically in robotics and radar target tracking in difficult conditions. Innovation consists in using possibility distributions instead of Gaussian distributions (white Gaussian noise). Lastly, the contribution of this paper also includes a method of propagating uncertainty through both the process and the observation models. This is based on quantifying uncertainty as trapezoidal possibilities distributions.

For the Gaussian method, the noise is estimated by experimentation; an accurate model is needed, otherwise a lot of measurements will be rejected; small errors are allowed; probable data is accepted and it has to start with an accurate estimation. By comparison, in the case of the fuzzy method, the noise is estimated by approximation, an accurate model is not needed, any uncertainties will be corrected later; larger errors are allowed; only impossible data is rejected and, by default, works with uncertain estimations.

In this context, we propose a fuzzy gain filter which implements fuzzy logic for target tracking in difficult conditions and we will also show its superiority against a traditional Kalman filter.

The authors propose a cascading Kalman filtering algorithm, which is used to deal with speed ambiguity and to lower the error of measurement given by the radar processing. In comparison to the methods that use the standard Kalman filter, the gain-controlled cascaded extended Kalman filtering algorithm, using fuzzy logic, has several advantages, such as strong real-time performance, high data rate and small computation time. The results of the simulation indicate the fact that the method presented in this work can effectively deal with the ambiguity of the speed and can obtain high tracking ability. In particular, this method does not imply the transmission of a multiple or staggered pulse repetition frequency (PRF), thus, its implementation is not difficult [11].

The work was structured as follows: in the Introduction chapter, we establish the main purpose of this paper and present the state of the art regarding the use of Kalman algorithms and fuzzy logic for target tracking; the second chapter, Materials and Methods, contains the theoretical part related to the algorithms and also, the case study with four types of trajectories considered for achieving the comparison between the algorithms implemented; the third chapter, Results, contains the output of the algorithms for the given trajectories, and is followed by Discussions in which we perform a comparison of the results and establish the optimal type of algorithm, more specifically, the fuzzy gain filter [12]; finally, the Conclusions chapter summarizes the work presented in this article and establishes a solution for target tracking in difficult conditions.

2. Materials and Methods

The basis of this study is consists of documentation from the specialized scientific literature, articles in journals, papers presented at conferences [13] on using fuzzy logic for target tracking topic.

Kalman filters have been used intensively in situations requiring a good system states estimation. The tendency of using Kalman filters is due to the their properties of optimization [14–16], and also due to the fact that their implementation is easy, leading to good outputs in many situations. Estimation of states are important and interesting in many practical systems, such as vehicle tracking or state-of-health estimation for different systems. Additionally, these estimations are also used in control systems in order to implement their feedback state.

The stability of the fuzzy controller plays an important role and has been thoroughly investigated [17–19]. If a certain degree of stability cannot be achieved, it will not be implemented in applications involving delicate operations (biomedicine, aerospace) [20]. This also applies for a fuzzy estimator. Thus, if a fuzzy estimator lacks stability, the tendency of using it will decrease considerably. The solution proposed by the authors was proven to be stable for most testing conditions.

Additionally, this work makes use of reports and information regarding fuzzy logic for target tracking, documents published by the International Energy Agency, Paris, France (IEA) [21] and strategic research agenda, as well as other data from R&D institutes which is relevant to this subject.

The fact that a target with the characteristics of a military aircraft can accelerate, turn, execute quick maneuvers, thus turning away from a trajectory with constant speed due to the orders received and due to the atmospheric turbulences, makes the pursuit of a target a difficult task. In order to handle these problems, different approach models have been developed [22]. These models are usually developed based on some statistic models regarding the measurement and processing of noise and target dynamics.

Several Kalman algorithms were developed [23–25], which make estimates about the noise associated with a process (P), the noise associated with the system (Q) and the noise associated with measurements (R). Some estimates [23] take into considerations the acceleration or speed constant; other estimates [24] restrict the time lapse between two acquisitions of information or impose that the detection probability to be equal to unit or to be independent in consecutive scans [25].

Due to the validity of the evaluations, many algorithms based on the statistic models have not shown their superiority over those who use a simple fixed gain filter type α-β [26]. The α-β type tracking is still often used for tracking problems, despite of the constraints they have. For example, although the optimal areas are known, the exact values of α and β are often difficult to determine. Systems with different values for α and β must be used in order to respond to accelerations or decelerations, or before, during and after a turn. In addition, for the maneuver control it is necessary to have a maneuver detector to work concurrently with a tracking system type α-β. In addition, it is necessary to make the decision regarding the moment at which the maneuver detector must be triggered.

It is difficult to make such decisions if the pursued targets have their own dynamics. Targets can pass undetected during multiple successive scans when performing quick return maneuvers. In these situations, the target will be lost due to the delay induced by the detection of the maneuver. By the time the target return maneuver was detected, this could have already been done before the tracking system was able to detect it.

In order to overcome these limitations associated with the pursuit of targets in hostile environments, if the target performs one of the following maneuvers:

- Uneven accelerations or decelerations at any point along the trajectory;
- Making sudden turns (e.g., high alpha maneuvers) for a short period of time (three, four scans);

- The impossibility of detecting the target during a number of successive scans;
- The successive positions of the target are measured with low precision or with a complete deflection in another direction with respect to the current position

We propose a new tracking method which relies on the use of a fuzzy type gain filter. The solution implements the fuzzy logic in conventional α-β filter through the use of if-then rules. Given the position error and the error change in the last prediction, these rules are used to determine the linearization of gains for α and β when making a new prediction.

Based on an α-β filter, the proposed tracking system has, first of all, the advantages of a typical α-β filter (doesn't need any evaluation related to the representation of noise or maneuvers, therefore, it is a filter easy to implement and with a low computational time). Secondly, fuzzy tracking is not affected by the limitations of the typical α-β filter, because it is capable of rapidly changing the values of sizes α and β with respect to the variations in direction and velocity, without using a detector for accelerations or maneuvers. This is why the proposed tracking system can significantly reduce any delays. In addition, using fuzzy inference, better decision can be made by taking into consideration at the same time several different situations, even conflictual ones. As a result, the number of mispredictions can be minimized, although the detection of turns is not always achieved, and the data can be sampled at longer time intervals.

Simulated data is being used in order to evaluate the behavior of various tracking methods, but, for real situations, their performance can be questionable. Therefore, for a better evaluation of the practical performances, real tracking data coming from the radar equipment were used. Using as reference a two-level Kalman filter, the performance of the proposed fuzzy tracking system was evaluated.

2.1. α-β Tracking System

An α-β tracking system is one of the most used filters having fixed coefficients. The equations below describe the filter:

$$x_s(k) = x_p(k) + \alpha\left[x_0(k) - x_p(k)\right] \tag{1}$$

$$x_p(k+1) = x_s(k) + Tv_{xs}(k) \tag{2}$$

$$v_{xs}(k) = v_{xs}(k-1) + \frac{\beta}{qT}\left[x_0(k) - x_p(k)\right] \tag{3}$$

where $x_0(k)$ is the x-coordinate of the observed target location during scanning k, $x_p(k)$ is the x-coordinate of the position evaluated during scanning k, $x_s(k)$ is the x-coordinate of the linearized position of the target during scanning k, $v_{xs}(k)$ the linearized speed of the target in the x-direction during scanning k, T is the scanning interval or the sampling time, and α and β are the parameters of the fixed coefficients filter [27–29]. Normally, the q value is defined as unit size, but, if there are unfulfilled observations, it takes the value equal to how many scans were performed starting from the previous measurements.

Usually, the initial conditions for an α-β tracking system are defined as follows [30,31]:

$$x_s(1) = x_p(2) = x_0(1) \tag{4}$$

$$v_{xs}(1) = 0 \tag{5}$$

$$v_{xs}(2) = \frac{\left|x_0(2) - x_0(1)\right|}{T} \tag{6}$$

Equations (1)–(3) are used directly when detecting a target at scanning k. However, if the detection probability is smaller than the unit size, then, for some scans, the detection of the target might be missed. In this case, the linearized position will be equal to the predicted position:

$$x_s(k) = x_p(k) \tag{7}$$

$$v_{xs}(k) = v_{xs}(k-1) \tag{8}$$

This is equivalent to making $x_0(k) = x_p(k)$. The prediction $x_p(k+1)$ for the next scanning will be calculated as above. The evaluated position of the target is a simple linear extrapolation starting from the previous position linearized with constant speed.

Linearization in an α-β filter is performed for each coordinate separately [32]. In other words, Equations (1)–(8) ca be generalized for the y-coordinate in the following way:

$$y_s(k) = y_p(k) + \alpha\Big[y_0(k) - y_p(k)\Big] \tag{9}$$

$$v_{ys}(k) = v_{ys}(k-1) + \frac{\beta}{qT}\Big[y_0(k) - y_p(k)\Big] \tag{10}$$

$$y_p(k+1) = y_s(k) + Tv_{ys}(k) \tag{11}$$

$$y_s(1) = y_p(2) = y_0(1) \tag{12}$$

$$v_{ys}(1) = 0 \tag{13}$$

$$v_{ys}(2) = \frac{\Big|y_0(2) - y_0(1)\Big|}{T} \tag{14}$$

$$y_s(k) = y_p(k) \tag{15}$$

$$v_{ys}(k) = v_{ys}(k-1) \tag{16}$$

where $y_0(k)$ is the y-coordinate for the target observed during scanning k, $y_p(k)$ is the y-coordinate of the evaluated position of the target during scanning k, and $v_{ys}(k)$ is the linearized value of the target speed in the y-direction during scanning k [32].

The accurate determination of the values of α and β can be easily done. If the coefficients have very low values, the filters can benefit from an increased noise reduction [33], but may no longer respond to the target dynamics [34]. In the case when the coefficients are high, the filter has proper tracking characteristics with the disadvantage of being noise sensitive. The α and β coefficients can be easily obtained for a target that moves straight, has the same speed during its movement and it is not subject to noise. For the situation when the target moves through a domain affected by noise and with a constantly changing flight dynamic, the coefficients cannot be fixed. The decision regarding the values that these coefficients have to take is not an easy one. At some points on the trajectory, the parameters can take high values, and in other small values. For instance, if the target diverges from the rectilinear trajectory and performs maneuvers, α and β have to be chosen from a wide range of values in order to be sure that the tracking system ensures an accurate target tracking. However, if the deviation is due to the noise induced by the measurement to a greater extent than that caused by the maneuvers of the target, α and β must be chosen rather small to ensure that the tracking system is positioned correctly [35].

Simply put, when a deviation of the target from the linear trajectory is detected, α and β can be chosen smaller or larger depending on the cause of the deviation, noise or target maneuver.

In order to prevent false alarms [36] about approaching a target, as long as a precise tracking of the maneuvers carried out by the target is performed, the selection of optimal values of α and β is not done using the true-false logic. The limits of selection must be of type fuzzy in order to minimize the target trajectory loss by reducing the transient amplitude given by a sudden change of decisions.

2.2. Fuzzy Gain Filter

In order to define the limits of decision for tracking a target in a hostile environment, a gain filter based on fuzzy logic is described, identified by a set of fuzzy rules. Given the error and its variation from the previous prediction, these rules output the solutions for α and β to be used in the further predictions. Changes in the dynamic of the target or in the tracking environment will cause changes in the values of α and β.

The performance of the fuzzy system, evaluated according to the prediction error and the number of missed targets, and compared to that of a system using a two stage Kalman filter, demonstrates the effectiveness of the fuzzy system, which has a high degree of error tolerance and a better return rate. As is the case with rule-based approaches, generalizing the use of the fuzzy system for any type of target depends on the development of automated approaches in order to determine the most appropriate rules and membership functions.

2.2.1. System Variables

Two sets of variables (input and output) are used for expressing the set of fuzzy rules associated with the suggested filter. The variables used as input, $E'(k)$ and $\Delta E'(k)$, are expressed according to the error of prediction $E(k)$ and the error variation $\Delta E(k)$ during scanning k, where:

$$E(k) = \sqrt{E_x^2(k) + E_y^2(k)} \tag{17}$$

$$\Delta E(k) = E(k) - E(k-1) \tag{18}$$

$$E_x(k) = x_0(k) - x_p(k) \tag{19}$$

$$E_y(k) = y_0(k) - y_p(k) \tag{20}$$

The range of values [$E_{min}(k)$, $E_{max}(k)$] for $E(k)$ and [$\Delta E_{min}(k)$, $\Delta E_{max}(k)$] for $\Delta E(k)$ are proportional with how big is the difference between the position of the target measured during scanning k and the predicted position. $E_{min}(k)$ and $\Delta E_{min}(k)$ can actually be null, and $E_{max}(k)$ and $\Delta E_{max}(k)$ could higher than the maximum speed of the target. As long as different targets have different speeds, the values for $E_{max}(k)$ and $\Delta E_{max}(k)$ can differ from one target to another. For the architecture of the tracking system, based on fuzzy logic, that can be implemented for various targets, x and y coordinates are normalized for $E(k)$ and $\Delta E(k)$, such that instead of $E_x(k)$ and $E_y(k)$, $E'(k)$ and $\Delta E'(k)$ are computed, where:

$$E'_x(k) = \begin{cases} \frac{x_0(k)-x_p(k)}{x_0(k)-x_0(k-1)}, & if\ |x_0(k)-x_p(k)| < |x_0(k)-x_0(k-1)| \\ \frac{x_0(k)-x_p(k)}{x_0(k)-x_p(k-1)}, & if\ |x_0(k)-x_p(k)| > |x_0(k)-x_0(k-1)| \\ 0, & if\ x_0(k)-x_p(k) = x_0(k)-x_0(k-1) = 0 \end{cases} \tag{21}$$

$$E'_y(k) = \begin{cases} \frac{y_0(k)-y_p(k)}{y_0(k)-y_0(k-1)}, & if\ |y_0(k)-y_p(k)| < |y_0(k)-y_0(k-1)| \\ \frac{y_0(k)-y_p(k)}{y_0(k)-y_p(k-1)}, & if\ |y_0(k)-y_p(k)| > |y_0(k)-y_0(k-1)| \\ 0, & if\ y_0(k)-y_p(k) = y_0(k)-y_0(k-1) = 0 \end{cases} \tag{22}$$

The values for $E'_x(k)$ and $E'_y(k)$ are in the range [−1, 1]. Similar to the definitions for $E'_x(k)$ and $E'_y(k)$, $\Delta E'_x(k)$ and $\Delta E'_y(k)$ are defined as follows:

$$\Delta E'_x(k) = \begin{cases} \frac{E'_x(k)-E'_x(k-1)}{E'_x(k-1)}, & if\ |E'_x(k)-E'_x(k-1)| < |E'_x(k-1)| \\ \frac{E'_x(k)-E'_x(k-1)}{|E'_x(k)-E'_x(k-1)|}, & if\ |E'_x(k)-E'_x(k-1)| > |E'_x(k-1)| \\ 0, & if\ E'_x(k)-E'_x(k-1) = E'_x(k-1) = 0 \end{cases} \tag{23}$$

$$\Delta E'_y(k) = \begin{cases} \frac{E'_y(k)-E'_y(k-1)}{E'_y(k-1)}, & if\ |E'_y(k)-E'_y(k-1)| < |E'_y(k-1)| \\ \frac{E'_y(k)-E'_y(k-1)}{|E'_y(k)-E'_y(k-1)|}, & if\ |E'_y(k)-E'_y(k-1)| > |E'_y(k-1)| \\ 0, & if\ E'_y(k)-E'_y(k-1) = E'_y(k-1) = 0 \end{cases} \tag{24}$$

Thus, the domain in which $E'(k)$ and $\Delta E'(k)$ can take values is the range [0, 1], without taking into account the target and its velocity. The values for $E'(k)$ and $\Delta E'(k)$ within this interval are the fixed values for $E'(k)$ and $\Delta E'(k)$.

2.2.2. Membership Functions

Fixed values are applied to the fuzzy sets [37,38] described in the definition domain of $E'(k)$ and $\Delta E'(k)$. The sets are described with the following notions: zero (ZE), small positive (SP), medium positive (MP) and large positive (LP).

The functions corresponding to each set are used to determine the significance of the previous mentioned notions. The membership functions take valued in the definition domain of $E'(k)$ and $\Delta E'(k)$ by the following functions with trapezoidal representation shown in Figure 1. The degrees of membership are associated to the variable values of $E'(k)$ and $\Delta E'(k)$ through the previous mentioned functions.

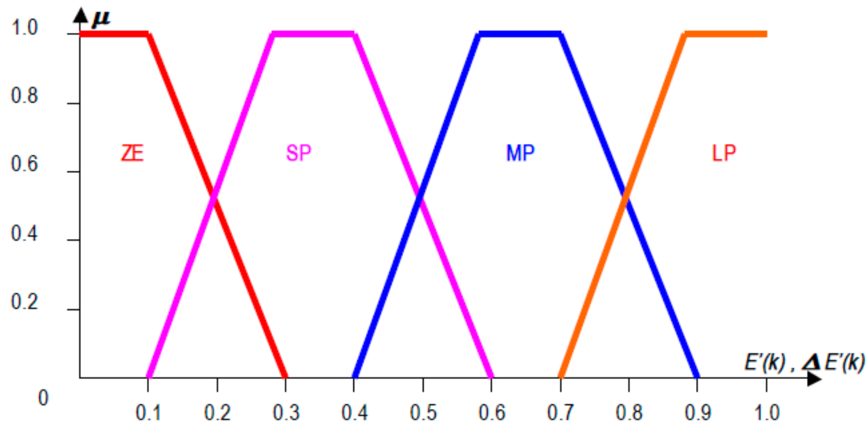

Figure 1. Membership functions for the fuzzy sets corresponding to $E'(k)$ and $\Delta E'(k)$ (ZE = zero, SP = small positive, MP = medium positive, LP = large positive).

In order to determine the membership functions for each of these fuzzy sets, the following intervals were chosen, in which the different membership functions have the maximum value: [0, 0.1] for ZE, [0.3, 0.4] for SP, [0.6, 0.7] for MP and [0.9, 1.0] for LP. The intervals in which no function of membership takes the maximum value represent the boundary intervals. All of the membership functions graphs have the same form [39,40]. Such a choice for defining the membership functions ensure equal chances of belonging to one of the fuzzy sets defined for all possible input values and possibilities of triggering practically equal rules. While the premises (the background) of the fuzzy rules are expressed according to $E'(k)$ and $\Delta E'(k)$, the consequences of rules are expressed relatively to $\alpha(k)$ and $\beta(k)$, which represent the α-β filter coefficients during scanning k. Even though, theoretically, $\alpha(k)$ and $\beta(k)$ can take values fitting into a wide interval, it is rarely fully used. For the proposed tracking system, the domain of definition for both coefficients can be considered the interval [0, 1].

The fuzzy sets of $\alpha(k)$ and $\beta(k)$ are described by the previously defined notions: ZE, SP, MP, LP and, in addition, VP (very positive) and EP (extremely positive). The functions of membership specific to each fuzzy set offer the significance of each notion and can take values in the domain of $\alpha(k)$ and $\beta(k)$ through triangular shaped functions, as in Figure 2.

Unlike for $E'(k)$ and $\Delta E'(k)$, the definition domain is divided into six fuzzy sets, and their maximum values are not evenly distributed. The central area is covered more consistently by four fuzzy sets, SP, MP, LP and VP, due to the fact that, on one hand, the area in which takes optimal values is found in the interval [0.3, 0.8], and, on the other hand, in order to avoid sudden output variations. In addition, experts in defense systems agree with such a partitioning of the definition domain which allows a faster decision of α and β values, when the specific values are available for $E'(k)$ and $\Delta E'(k)$.

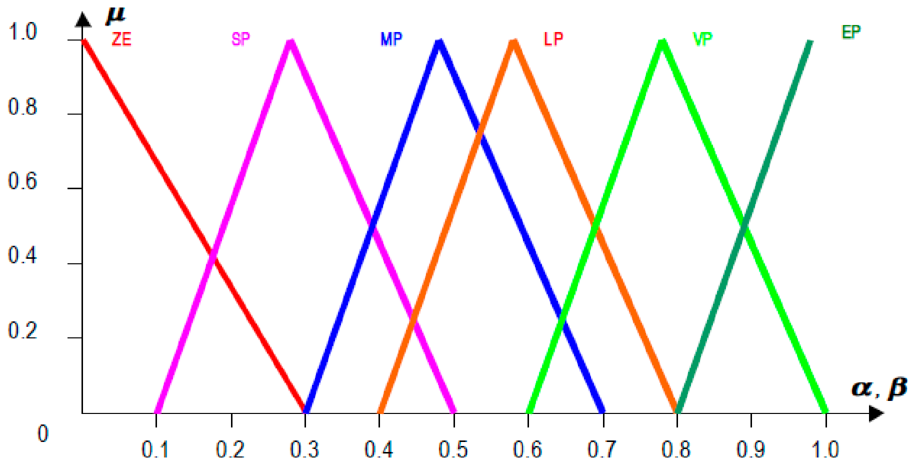

Figure 2. Membership functions for the fuzzy sets of α and β.

2.3. Fuzzy Rules

The inclusion of fuzzy logic in conventional alpha-beta filters determines the dynamic change of the coefficients according to the conclusions of the fuzzy rules set. These rules determine alpha and beta depending on the size of the last prediction error and on the error change. Whenever it is necessary to evaluate several variants simultaneously, multiple rules can be triggered. The levels at which these rules are triggered are quantitively combined to determine the final values for alpha and beta.

The fuzzy rules are formulated in the following way, given the definition of the input and output variables:

$$\textbf{IF } E'(k) = e\prime \textbf{ AND, THEN } \alpha(k) = \alpha \textbf{ AND } \beta(k) = \beta \tag{25}$$

where e', $\Delta e'$, α and β are fuzzy sets taking values in the domain of $E'(k)$, $\Delta E'(k)$, $\alpha(k)$ and $\beta(k)$. Due to the fact that each of these rules is used by the fuzzy tracking system, it can be regarded as a connection that explains a fuzzy relationship [41], they can be briefly described as language table or as a fuzzy associative memory, like in Tables 1 and 2.

Table 1. Fuzzy associations for $\alpha(k)$.

		$E'(k)$			
	$\alpha(k)$	**ZE**	**SP**	**MP**	**LP**
$\Delta E'(k)$	**ZE**	VP	SP	EP	EP
	SP	LP	LP	VP	VP
	MP	EP	VP	MP	MP
	LP	VP	ZE	MP	EP

Table 2. Fuzzy associations for $\beta(k)$.

		$E'(k)$			
	$\beta(k)$	**ZE**	**SP**	**MP**	**LP**
$\Delta E'(k)$	**ZE**	VP	SP	ZE	EP
	SP	ZE	ZE	ZE	ZE
	MP	ZE	ZE	LP	VP
	LP	ZE	LP	MP	SP

The rules [42,43] are thus capable of generating the most appropriate response related to any flight dynamic which includes acceleration, deceleration, maneuvers, lack of detection and high noise areas. Therefore, a lot of fuzzy associations were finalized (for example, the entries in Tables 1 and 2) which include the following characteristics:

- The diagonal entries on the bottom left of both tables are respectively VP and ZE (except the one in the upper right corner). These inputs are mainly used to solve the cases in which the target maneuvers are of small amplitude. Having α = VP and β = ZE, the tracking system can describe the target trajectory with high precision, without the need to bring the estimated speed to zero. Due to the fact that the time required for adapting to a new speed model is quite large, changes in β are reduced as much as it can be done in order to not have false alarms. The value of β can stay approximately constant as long as the deviation from the real value represents a small amplitude maneuver.
- Values in the right upper corner are generally used for restarting the tracking if the target was lost. In such cases, assigning the EP value to α and β can significantly increase the determination of the new dynamic of the target.
- Values in the left upper corner are used for solving the cases of lack of observation. Assigning the VP value to α and β allows the tracking system to update its target dynamics after the observation is reestablished.
- All other values from the first column deal with the situations when the previous predictions were correct. As long as α has to be large in order to be able to pursue the target closely and to prepare the system for any unexpected maneuver, β takes the ZE value to prevent a significant change of the predicted speed value. This choice must prevent the occurrence of false alarms resulting from observations accompanied by noise
- Entries from positions two and three in the first row are used for the cases of smooth maneuvers. In such cases, in order to be able to track the target, the tracking systems has to perform a slight modification in speed and a bigger modification in location.
- Entries from the second row imply minor changes of the prediction errors. These are due to a noisy observation or due to the beginning of a target maneuver. In the second case, the speed of the tracking system should not be significantly modified (as long as the maneuver is only in the initial stage). If the divergence is related to a noisy observation, the adjustment in speed must be avoided, thus, β must take the ZE value.
- The four values in the last part of the tables may be related to a possible loss of pursuit. This can be explained by the cumulative effect generated by wrong decisions or unexpected changes in the dynamic of the target. If the flight direction is correctly estimated, the tracking system will return immediately to a correct pursuit.

2.4. Kalman Tracking System

The Kalman filter presented in Section 2 was chosen taken into account the following considerations and objectives:

- Identification of the limitations of an alpha-beta tracking system and the need to use fuzzy logic in order to extend its tracking capability.
- Establishing a method of evaluating the performance of the filter with the gain controlled by means of fuzzy logic, by comparing them with the performances of a good Kalman estimator.
- Demonstrating the superiority of a fuzzy tracking system over the Kalman filter tracking system under the conditions of uneven accelerations and sudden change of target direction, as well as in the case of failing to observe the target during several successive scans, using the simultaneous computer simulation of the tracking through the two methods.

The execution of the fuzzy tracking system was put against a performant Kalman estimator, which takes into account the situations when the target shifts and accelerates quickly [44]. The filter can be explained with the equations below:

$$K_P = \Phi P(k|k-1)H^T\left[HP(k|k-1)H^T + R_c\right]^{-1} \tag{26}$$

$$\hat{x}(k+1|k) = \Phi\hat{x}(k|k-1) + K_p(k)[y(k) - H\hat{x}(k|k-1)] \tag{27}$$

$$P(k+1|k) = \left[\Phi - K_p(k)H\right]P(k|k-1)\Phi^T + Q \tag{28}$$

where $\hat{x}$ is the state vector of the target, y is the vector of measurements, Φ is the matrix of transition, $P(k|k-1)$ is the noise matrix associated to the process during scanning k based on scanning $k-1$, $K_p(k)$ represent the gain during scanning k, Q is the covariance matrix of the system noise, H is the measurement matrix and R_c is the covariance of the measurement noise. The variables used for the representation of the target state vector and the vector of measurements have a lot in common with those of the x and y coordinates for the positions measured or predicted in the previous chapters, but are not linked to them. This is due to the fact that they are often used in the field.

The average speed of the pursued targets is 370 m/s and the sampling period T is 10s. Thus, the average dislocation is 0.36 T = 3.6 km. Initially, x and v_x are considered zero. For a model with reduced states, the following parameters can be considered for relations (29)–(31):

$$\Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{29}$$

$$H = [1 \;\; 0]^T \tag{30}$$

$$\hat{x} = \begin{bmatrix} x \\ v_x \end{bmatrix} \tag{31}$$

In order for a maneuver model to be able to track targets with accelerations up to 6g, σ_m must take the value 2 g and τ_{m} must take the value of 50 s. The following parameters result:

$$P = \begin{bmatrix} 4 & 0.4 & 0.08 \\ 0.4 & 0.4 & 0.008 \\ 0.08 & 0.008 & 0.0016 \end{bmatrix} \tag{32}$$

$$\hat{x} = \begin{bmatrix} x \\ v_x \\ a_x \end{bmatrix} \tag{33}$$

$$Q = \frac{2\sigma_m^2}{\tau_m}\begin{bmatrix} \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} \\ \frac{T^4}{8} & \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^3}{6} & \frac{T^2}{2} & T \end{bmatrix} \tag{34}$$

2.5. Case Study

In order to assess the execution of a fuzzy tracking system, real data sets acquired by the defense department of Canada and the United States of America were used. Data were collected during experiments with tracking systems conducted at a Canadian Armed Forces site. A few F-18 planes were used as targets, and the data were recorded simultaneously by a traffic control radar and by a radar in the air defense system, using two multichannel analog recorders with magnetic tape. The radars used worked on the 1300 MHz frequency, with a scanning period of 10 s. The probability of target detection is 0.8, the lack of detection being possible even for rectilinear flight trajectories. The directivity characteristic of the radar is 2°, the radar being unable to distinguish very close targets when they perform maneuvers, and thus for some target measurements, may be lost. The flight took an hour and a half. The altitude was 7000 m and speed was 900 km/h. The turns were characterized by accelerations between 1 g and 6 g.

The tracking method based on the extremum seeking control proposed in [45], which was tested in [45,46] for tracking the global extremes, may be used for the aircraft trajectory tracking considering the advantage of adaptive tracking loop.

3. Results

Using real data sets, the execution of the suggested fuzzy tracking system and the extend Kalman filter tracking system were assessed taking into account the number of tracking losses (general indicator of reliability and robustness of the tracking system) and the average of the prediction error (indicator for the ability of the system to follow the targets closely). It was also investigated the way in which the recovery time after incorrect actions and the initial data affect the performance of the tracking system [45].

The simulation is based on a program developed in MATLAB, which performs both the simulation of the behavior of a Kalman estimator and the simulation of the behavior of an α-β estimator with variable coefficients established by fuzzy methods.

The program has four sets of coordinates of four real aircraft trajectories, providing the charts of the trajectories estimated by both methods, together with the representation of the real trajectory, the graphs of estimation errors for each representation, as well as files containing the estimation errors. The figures presented below show the result of the simulations.

In the first situation (Figures 3 and 4), a target accelerates rapidly and executes rapid turns. As it can be observed, the Kalman filter performs several erroneous actions in series, which increases the displacement between the current position and the predicted one by accumulating the errors from one scan to another. This low rate of response to unexpected changes in the dynamic of flight is due to the time required by the extend Kalman estimator's response (usually during a few scans). For the target executing a 360° turn over six scans, a delay of two scans may represent a 120° angular deviation. Thus, the tendency of the Kalman system to overestimate the next position of the target is not surprising, thus resulting in loss of the target.

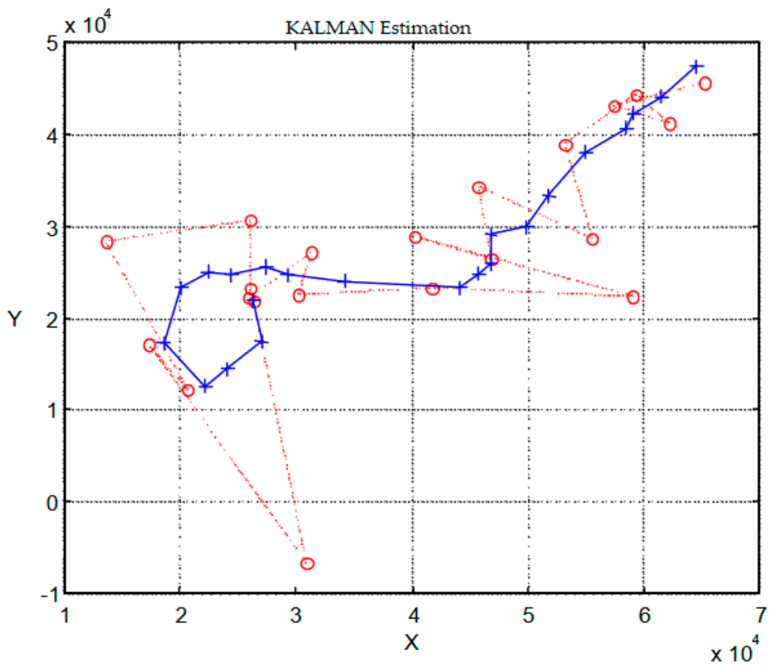

Figure 3. Kalman filter estimation (red—Real target position, blue—Estimated target position).

Figures 5 and 6 show the performances of the two systems in another situation. A target is being tracked on an approximate linear trajectory with constant speed. There are no significant losses of the target. It can be noticed that the predictions of the Kalman system are placed on a curve in a zig-zag pattern. The system is thus dependent of noise, even if the noise level is minimum.

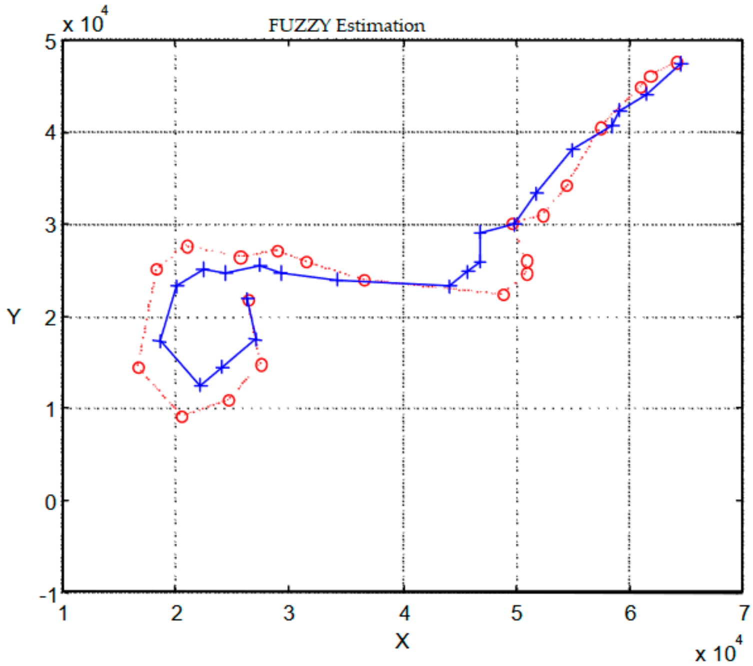

Figure 4. Fuzzy filter estimation (red—Real target position, blue—Estimated target position).

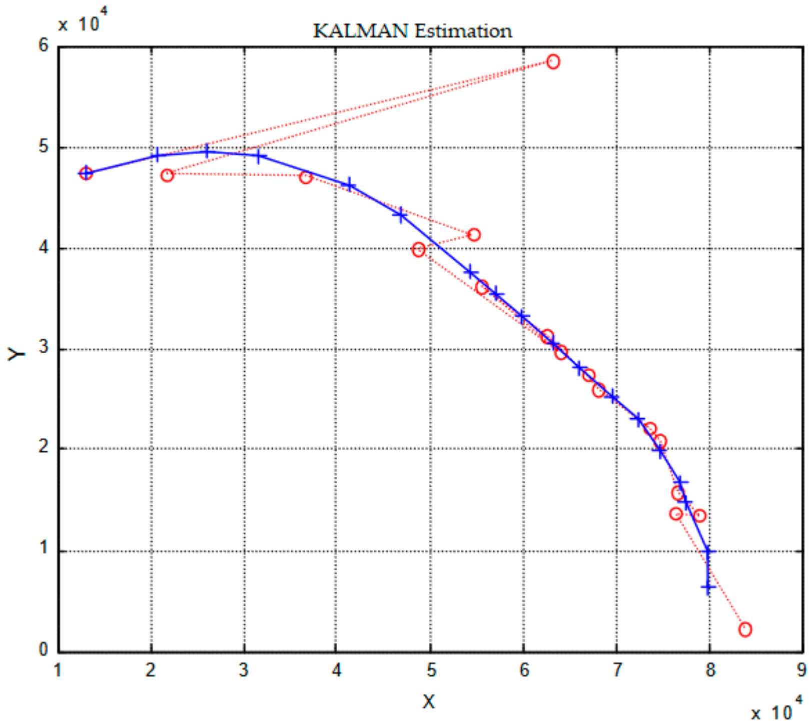

Figure 5. Kalman filter estimation (red—Real target position, blue—Estimated target position).

Figures 7 and 8 illustrate the system's capacity to follow a rapidly accelerating target over a linear trajectory. The difficulty of the tracking conditions is low given the fact that, besides the rapid acceleration of the target, there are no significant target losses, and the noise level is also low. As it can be seen in the figures, the Kalman system committed multiple errors leading to the loss of trajectory.

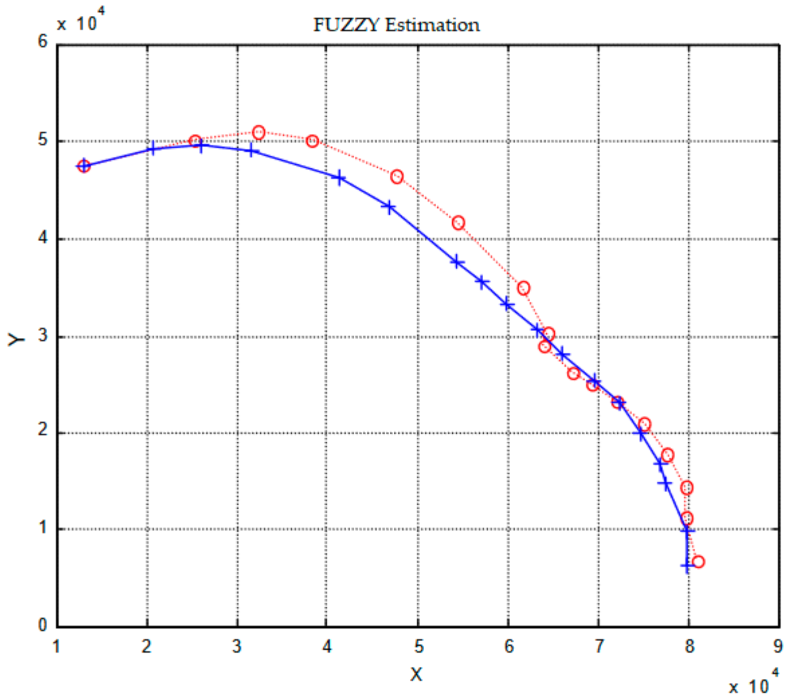

Figure 6. Fuzzy filter estimation (red—Real target position, blue—Estimated target position).

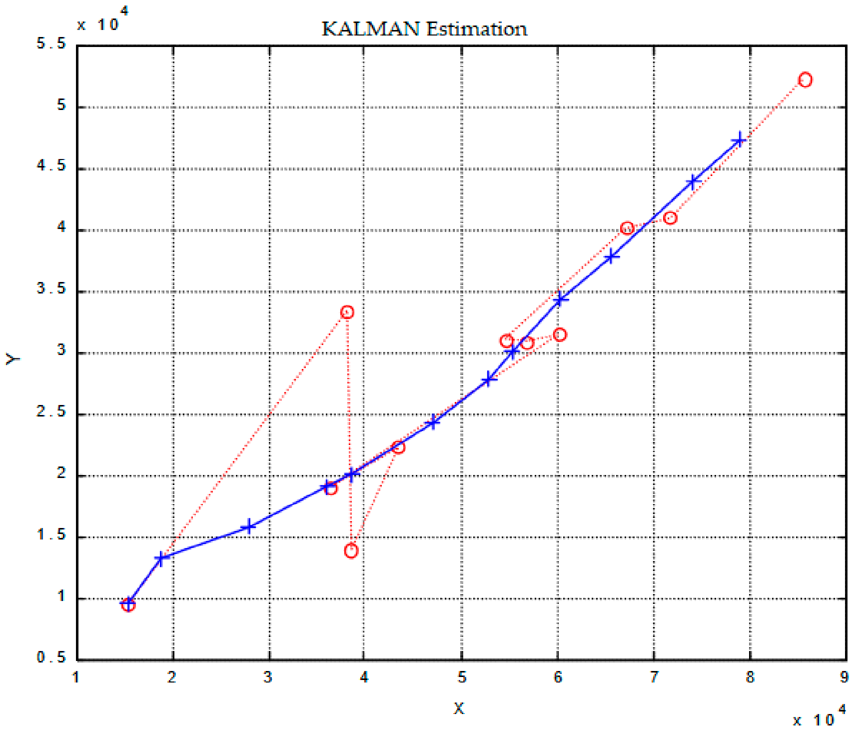

Figure 7. Kalman filter estimation (red—Real target position, blue—Estimated target position).

Finally, Figures 9 and 10 show the issues related to target tracking under lack of observation conditions. The tracking conditions were characterized by a low noise level. The target did not accelerate and did not perform sudden changes in the flight direction. The Kalman system did not behave properly in this case either. The maneuver of small successive changes of the target direction before the tracking loss determined the system to interpret it as a maneuver of significant magnitude, thus being determined to follow a non-existing trajectory. On the other hand, the fuzzy system did not react to the perturbation, due to its abilities to consider multiple possibilities at the same moment (multiple rules can be enabled simultaneously).

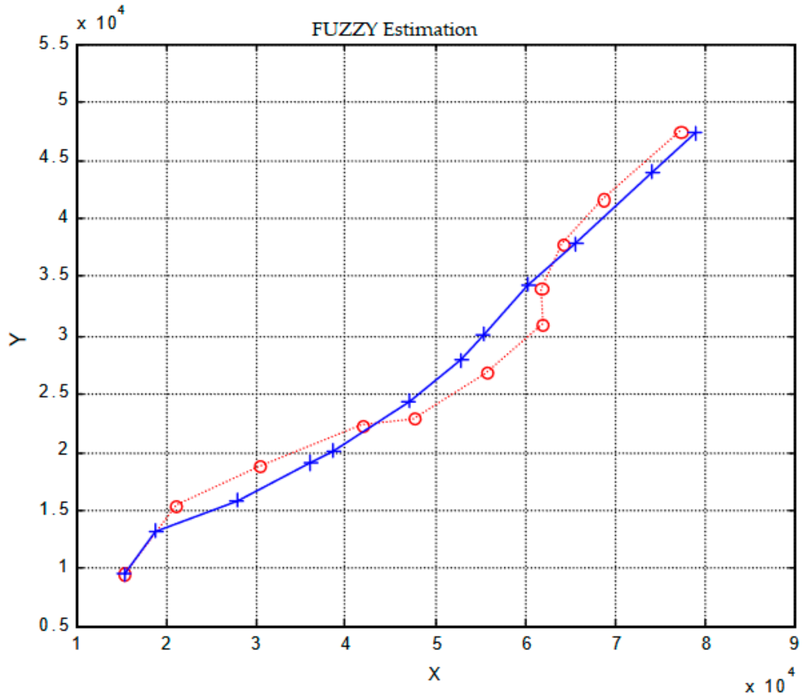

Figure 8. Fuzzy filter estimation (red—Real target position, blue—Estimated target position).

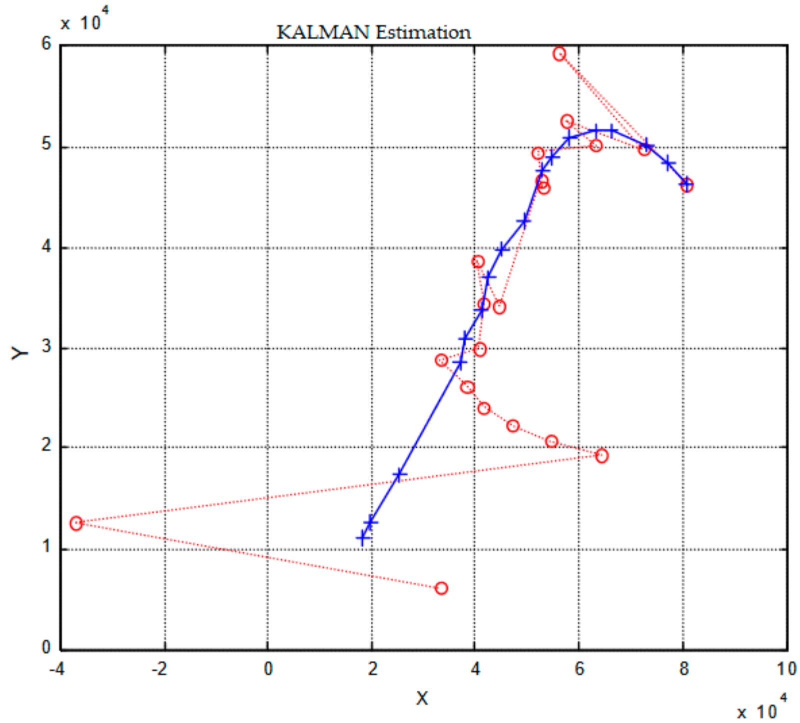

Figure 9. Kalman filter estimation (red—Real target position, blue—Estimated target position).

It should be noted that the probability of losing a target over the course of several successive scans is actually quite high. This increases as the aircraft is more maneuverable. Due to the flight angle and the ability to make sudden changes of direction, these aircraft are much more difficult to detect in the conditions in which they perform maneuvers [45–47].

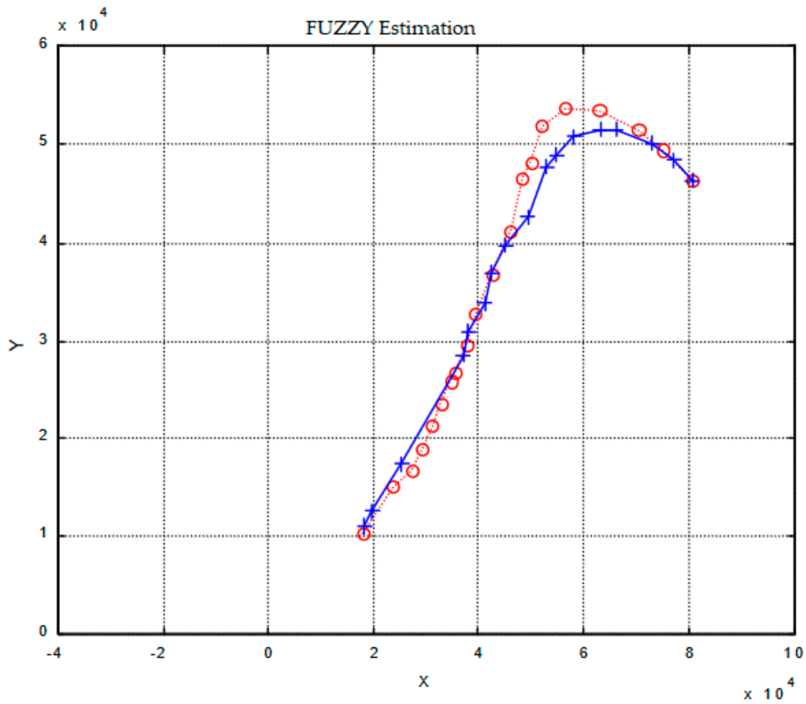

Figure 10. Fuzzy filter estimation (red—Real target position, blue—Estimated target position).

In the case of applications that require increased computing speed, analog processors are often the only solution, which can lead to decreased flexibility and a number of membership functions that can overlap at certain intervals. However, it is demonstrated that, using circuits in MOS-FET technology, simple analog fuzzification and inference circuits can be made, thus developing specialized processors [48].

The prediction errors of the two systems are compared and presented in the Section 4.

4. Discussion

As shown in Table 3 below, the Kalman tracking system does not have the performance of the fuzzy system, both in terms of preventing loss of tracking and closely tracking the target. The differences are significant both when the average estimation errors and the behavior of the systems for reducing the tracking intervals are considered [48,49]. For the fuzzy system, reducing the tracking range from 550 to 400 ms has resulted in an increase in the number of lost tracks from 3 to 9 for trajectory 1, from 7 to 10 for trajectory 2, from 6 to 8 for trajectory 3 and from 0 to 10 for trajectory 4. In the Kalman tracking system, the same reduction in the tracking interval led to an increase in the number of lost tracking from 9 to 12 for trajectory 1, from 7 to 10 for trajectory 2, from 6 to 8 for trajectory 3 and from 7 to 10 for trajectory 4, as it can be seen in Table 4. A clearer picture is presented in Table 5, which represents the percentage of lost tracking from the total measurements made. The difference between the two systems is also significant in terms of prediction error [48].

Table 3. Comparison regarding the number of lost tracking for the Kalman and Fuzzy system.

TRAJECTORY NO.	TRACKING INTERVAL			
	400 ms		550 ms	
	KALMAN	FUZZY	KALMAN	FUZZY
1	12	9	9	3
2	10	4	7	1
3	8	5	6	4
4	10	5	7	0

Table 4. Comparison of prediction errors for the Kalman and Fuzzy systems.

SCAN NO.	TRAJECTORY 1		TRAJECTORY 2		TRAJECTORY 3		TRAJECTORY 4	
	KALMAN	FUZZY	KALMAN	FUZZY	KALMAN	FUZZY	KALMAN	FUZZY
1	4455	4455	7885	7885	5093	5093	4294	4294
2	22,188	3340	37,887	937	20,343	6917	19,095	1916
3	6728	3012	10,179	2036	5689	5748	6419	4213
4	5556	8345	4834	4962	5185	3758	5727	1964
5	8309	9350	8035	3213	11,854	1549	4962	3163
6	6718	4188	6240	3982	8057	3087	3006	4238
7	2294	4501	8937	4510	1644	6719	1663	3113
8	3639	2015	5364	5344	6453	1578	5104	4210
9	3183	2385	1094	1810	2722	1388	5712	1716
10	4194	3346	1272	2342	3810	6015	2927	281
11	2439	7595	1457	437	8159	1748	1040	2010
12	13,545	3922	1487	231			2792	1481
13	7384	4185	1136	1122			3794	2456
14	2642	4981	856	1287			F	F
15	5988	2889	1822	2331			F	F
16	5889	2285	5229	1559			F	F
17	2046	4006	5561	1128			F	F
18	3864	1111					38,781	2114
19	3198	3135					56,989	4616
20	4243	2114					15,816	642
21	1925	492						
AVERAGE ERROR	5018	3402	6428	2654	7183	3964	11,133	2652

Table 5. Comparison regarding the percentage of lost tracking for Kalman and Fuzzy systems.

TRAJECTORY NO.	TRACKING INTERVAL			
	400 ms		550 ms	
	KALMAN	FUZZY	KALMAN	FUZZY
1	57%	43%	43%	14%
2	59%	24%	41%	6%
3	73%	45%	55%	36%
4	63%	31%	44%	0%

The performance deterioration of the Kalman tracking system is quite significant in some cases presented in Table 4. In the case of Trajectory 4, which is a difficult trajectory for a tracking process, the average error on a Kalman system is five times higher than on a system based on fuzzy rules. Careful examination of the data leads to the conclusion that the main cause is the reduced possibility of the Kalman system to resume tracking after erroneous actions. When the tracking interval was relatively large, the Kalman system was able to track the target, but when reducing the interval, it needed more time to correct its errors. This is because the Kalman filter requires a longer duration to ensure a reliable and stable prediction. Unlike the Kalman system, the fuzzy system is able to return much faster to a stable prediction, which explains the large difference between the number of lost targets of the two systems [50].

Proving the performance of the extended cascade fuzzy Kalman system, evaluated according to the prediction error and the number of missed targets, and compared to that of a system using a two stage second order Kalman fuzzy filter, demonstrates the effectiveness of the proposed fuzzy system, which has a high degree of error tolerance and a better return rate. As is the case with rule-based approaches, generalizing the use of the fuzzy system for any type of target depends on the development of automated approaches to determine the most appropriate rules and membership functions.

It was also shown the possibility of implementing the fuzzy logic in a real target tracking application in difficult conditions. As an initial approach, a classical alpha-beta system can be modified in order to determine alpha and beta coefficients by fuzzy logic. Based on the fuzzy gain filter presented in the article, an automated structure can be developed that provides the tracking system with real-time updated values for alpha and beta coefficients.

The components of a fuzzy system were defined: a rigid-fuzzy transformation component and rigid-fuzzy transformation component. The notion of rigid set was introduced, regarding the sets of elements that do not accept the nuanced definition of the degree of membership, but only recognize total membership or non-membership.

The system variables were defined and also the universe of speech and the membership functions necessary for the rigid-fuzzy transformation for a controlled gain filter that uses fuzzy logic for tracking targets using radar equipment under difficult tracking conditions.

5. Conclusions

The performance of the fuzzy system, evaluated according to the error of prediction and the missed targets, and measured against a system using a two-stage Kalman filter, demonstrates the effectiveness of the fuzzy system, which has a high tolerance of error and rate of better returns. As in the case of rule-based approaches, the generalization of using the fuzzy system for any type of target depends on the development of automated approaches in order to determine the most appropriate rules and membership functions.

This paper demonstrates the possibility of implementing fuzzy logic for tracking targets in difficult conditions such as high noise levels, but also uneven acceleration of targets, sharp turns and failure to observe the target during several successive scans during maneuvers. The inclusion of fuzzy logic in a usual α-β filter determines the dynamic modification of the coefficients depending on the output sets of the fuzzy rules suggested. The rules are used to find the values of α and β depending on the size of the previous error of prediction and the error modification. Whenever it is necessary to evaluate several variants simultaneously, several rules can be triggered. The levels where the rules are triggered are quantitatively used to obtain the overall values for α and β.

The efficiency of the fuzzy tracking system was computed based on some cases with different levels of difficulty. The performance of the fuzzy system, evaluated based on the error of prediction and the number of missed targets is compared to that of a system that uses a two-stage Kalman filter. The outputs indicate the fact that the fuzzy system has a high error tolerance and a better return rate.

The development and implementation of this new filtering algorithm can solve the three-dimensional radar measurements in the proposed case, allowing the direct adjustment of the parameters for the artificial and mean covariance. The simulation results show that the algorithm is efficient in the computation of nonlinear measurement, compared to the standard Kalman filter, having an error value from 0.77% to 1.15%.

The main directions for further research are as follows: the implementation of fuzzy logic in a real target tracking application under difficult conditions. As an initial approach, a classical alpha-beta system can be modified in order to determine alpha and beta coefficients by fuzzy logic. Based on the fuzzy gain filter presented in this article, an automated structure can be developed, which provides the tracking system with real-time updated values for alpha and beta coefficients.

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