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Article

Modified Suzuki-Simulation Type Contractive Mapping in Non-Archimedean Quasi Modular Metric Spaces and Application to Graph Theory

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Abstract: In this paper, we establish generalized Suzuki-simulation-type contractive mapping and prove fixed point theorems on non-Archimedean quasi modular metric spaces. As an application, we acquire graphic-type results.

Keywords: non-Archimedean quasi modular metric space; θ-contraction; Suzuki contraction; simulation contraction

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1. Introduction

In the sequel, the letter $\mathbb{R}_+$ will denote the set of all nonnegative real numbers.

Let S be a nonempty set and $V : S \to S$ be given mappings. A point $\jmath \in S$ is said to be:

i. a fixed point of V if and only if $V\jmath = \jmath$;
ii. a common fixed point of V and Z if and only if $V\jmath = Z\jmath = \jmath$.

Kosjasteh et al. [1] defined a new control function as follows.

Definition 1 ([1]). *Let $\zeta : [0,\infty)^2 \to \mathbb{R}$ be a mapping. The mapping ζ is named a simulation function satisfying the following conditions:*

ζ_1. $\zeta(0,0) = 0$,
ζ_2. $\zeta(a,b) < a - b$, *for all* $a, b > 0$,
ζ_3. *if* $\{a_k\}$ *and* $\{b_k\}$ *are sequences in* $\mathbb{R}_+$ *such that* $\lim_{k\to\infty} a_k = \lim_{k\to\infty} b_k = l$, $l \in \mathbb{R}_+$. *Thus,*

$$\limsup_{k\to\infty} \zeta(a_k, b_k) < 0.$$

Argoubi et al. [2] modified the above and so introduced it as follows.

Definition 2 ([2]). *The mapping ζ is a simulation function providing the following:*

i. $\zeta(a,b) < a - b$, *for all* $a, b > 0$,
ii. *if* $\{a_k\}$ *and* $\{b_k\}$ *are sequences in* $\mathbb{R}_+$ *such that* $\lim_{k\to\infty} a_k = \lim_{k\to\infty} b_k > 0$, *and* $a_k < b_k$, *then* $\limsup_{k\to\infty} \zeta(a_k, b_k) < 0$.

For examples and related results on simulation functions, one may refer to [1–8].

Radenovic and Chandok generalized the simulation function combining the C-class function as follows.

Definition 3 ([4]). *A mapping $G : [0,\infty)^2 \to \mathbb{R}$ is named a C-class function if it is continuous and satisfies the following conditions:*

i. $G(a,b) \leq a$,
ii. $G(a,b) = a$ *implies that either* $a = 0$ *or* $b = 0$, *for all* $a, b \in [0,\infty)$.

Definition 4 ([4]). *A* C_G*-simulation function is a mapping* $\zeta : [0,\infty)^2 \to \mathbb{R}$ *satisfying the following conditions:*

i. $\zeta(a,b) < G(a,b)$ *for all* $a, b > 0$, *where* $G : [0,\infty)^2 \to \mathbb{R}$ *is a* C*-class function,*
ii. *if* $\{a_k\}$ *and* $\{b_k\}$ *are sequences in* $(0,\infty)$ *such that* $\lim_{k\to\infty} b_k = \lim_{k\to\infty} a_k > 0$, *and* $b_k < a_k$, *then* $\limsup_{k\to\infty} \zeta(a_k, b_k) < C_G$.

Definition 5 ([4]). *A mapping* $G : [0,\infty)^2 \to \mathbb{R}$ *has the property* C_G, *if there exists a* $C_G \geq 0$ *such that:*

i. $G(a,b) > C_G$ *implies* $a > b$,
ii. $G(a,a) \leq C_G$ *for all* $a \in [0,\infty)$.

Moreover, using C-class function many researchers investigated some new results combining other control functions in different spaces [9].

Suzuki [10] proved the following fixed point theorem using a new contraction, which is known as the Suzuki contraction in literature. Furthermore, many mathematicians generalized this contraction in other spaces.

Theorem 1 ([10]). *Let* (S,d) *be a compact metric space and* $V : S \to S$ *be a mapping. Suppose that, for all* $\jmath, \ell \in S$ *with* $\jmath \neq \ell$,

$$\frac{1}{2} d(\jmath, V\jmath) < d(\jmath,\ell) \quad \Rightarrow \quad d(V\jmath, V\ell) < d(\jmath,\ell).$$

Then, V *has a unique fixed point in* S.

Bindu et al. [11] proved the commonfixed point theorem for Suzuki type mapping in a complete subspace of the partial metric space.

Theorem 2. *Let* (S,δ) *be a partial metric space and* $f, g, V, Z : S \to S$ *be mappings satisfying:*

$$\frac{1}{2} \min\{\delta(f\jmath, V\jmath), \delta(g\ell, Z\ell)\} \leq \ell(f\jmath, g\ell) \quad \Rightarrow \quad \phi(V\jmath, Z\ell) \leq \alpha(M(\jmath,\ell)) - \beta(M(\jmath,\ell)),$$

for all $\jmath, \ell \in S$, *where* $\phi, \alpha, \beta : [0,\infty) \to [0,\infty)$ *are such that* ϕ *is an altering distance function,* α *is continuous, and* β *is lower-semi continuous* $\alpha(0) = \beta(0) = 0$ *and* $\phi(t) - \alpha(t) + \beta(t) > 0$, *for all* $t > 0$ *and:*

$$M(\jmath,\ell) = \max\left\{\delta(f\jmath, g\ell), \delta(f\jmath, V\ell), \delta(g\ell, Z\ell), \frac{\delta(f\jmath, Z\ell) + \delta(g\ell, V\jmath)}{2}\right\},$$

i. $V(S) \subseteq g(S)$, $Z(S) \subseteq f(S)$;
ii. *either* $f(S)$ *or* $g(S)$ *is a complete subspace of* S;
iii. *the pairs* (f,V) *and* (g,Z) *are weakly compatible.*

Then, f, g, V, Z *have a common fixed point.*

Jleli and Samet [12] introduced a Σ-contraction and established fixed point results in generalized metric spaces. Jleli and Samet [12] also introduced a class of Θ such that $\Sigma : (0,\infty) \to (1,\infty)$ of all functions, providing the following conditions:

Σ_1. Σ is nondecreasing;
Σ_2. for any sequence $\{a_k\}$ in $(0,\infty)$, $\lim_{k\to\infty} \Sigma(a_k) = 1$ if and only if $\lim_{k\to\infty} a_k = 0$;

Σ_3. there exist $r \in (0,1)$ and $l \in (0,\infty)$ such that $\lim\limits_{k\to 0^+} \frac{\Sigma(k)-1}{k^r} = l$.

Theorem 3 ([12]). *Let (S,d) be a complete generalized metric space and $V : S \to S$ be a mapping. Suppose that there exist $\Sigma \in \Theta$ and $\gamma \in (0,1)$ such that:*

$$d\,(V\jmath, V\ell) \neq 0 \quad \Rightarrow \quad \Sigma\,(d\,(V\jmath, V\ell)) \leq [\Sigma\,(d\,(\jmath,\ell))]^{\gamma},$$

for all $\jmath, \ell \in S$. Then, V has a unique fixed point.

After that, many authors generalized such a contraction in different spaces [13–17].

Liu et al. [15] modified the class of function Θ exchanging conditions. The class of functions $\tilde{\Theta}$ was defined by the set of $\Sigma : (0,\infty) \to (1,\infty)$ satisfying the following conditions:

$\tilde{\Sigma}_1$. Σ is non-decreasing and continuous,
$\tilde{\Sigma}_2$. $\inf\limits_{k\in(0,\infty)} \Sigma\,(k) = 1$.

Lemma 1 ([15]). *Let $\Sigma : (0,\infty) \to (1,\infty)$ be a non-decreasing and continuous function with $\inf\limits_{k\in(0,\infty)} \Sigma\,(k) = 1$ and $\{a_k\}$ be a sequence in $(0,\infty)$. Then, the following condition holds:*

$$\lim_{k\to\infty} \Sigma\,(a_k) = 1 \quad \Leftrightarrow \quad \lim_{k\to\infty} a_k = 0.$$

Zheng et al. [18] denoted new set functions Φ satisfying the following conditions:

Φ_1. $\varphi : [1,\infty) \to [1,\infty)$ is nondecreasing,
Φ_2. for each $k > 0$, $\lim\limits_{n\to\infty} \varphi^n\,(k) = 1$,
Φ_3. φ is continuous on $[1,\infty)$.

Lemma 2 ([18]). *If $\varphi \in \Phi$, then $\varphi(1) = 1$ and $\varphi(t) < t$ for each $t > 1$.*

Definition 6 ([18]). *Let (S,d) be a metric space and $V : S \to S$ be a mapping. V is said to be a $\Sigma - \varphi$-contraction if there exist $\Sigma \in \Theta$ and $\varphi \in \Phi$ such that for any $\jmath, \ell \in S$,*

$$\Sigma\,(d\,(V\jmath, V\ell)) \leq \varphi\,(\Sigma\,(N\,(\jmath,\ell))),$$

where:

$$N\,(\jmath,\ell) = \max\{d\,(\jmath,\ell)\,, d\,(\jmath, V\ell)\,, d\,(\jmath, V\ell)\}.$$

Theorem 4 ([18]). *Let (S,d) be a complete metric space and $V : S \to S$ be a $\Sigma - \varphi$-contraction. Then, V has a unique fixed point.*

Motivated by the above, we will establish a generalized Suzuki-simulation-type contractive mapping and obtain fixed point results.

2. Quasi Modular Metric Space

Girgin and Öztürk [19] introduced a new space, which is named a quasi modular metric space. Furthermore, they gave some topological properties. Moreover, defining non-Archimedean quasi modular metric space, they proved some fixed point theorems and obtained some applications.

Definition 7 ([19]). *A function $Q : (0,\infty) \times S \times S \to [0,\infty]$ is called a quasi modular metric on S if the following hold:*

q_1. *$\xi = \eta$ if and only if $Q_m\,(\xi,\eta) = 0$ for all $m > 0$;*

q_2. $Q_{m+n}(\xi,\eta) \leq Q_m(\xi,\nu) + Q_n(\nu,\eta)$ *for all* $m,\ n > 0$ *and* $\xi,\eta,\nu \in S$.

Then, S_Q *is a quasi modular metric space. If in the above definition, we utilize the condition:*

$q_{1'}$. $Q_m(\xi,\xi) = 0$ *for all* $m > 0$ *and* $\xi \in S$,

instead of (q_1)*, then* Q *is said to be a quasi pseudo modular metric on* S*. A quasi modular metric* Q *on* S *is called a regular if the following weaker version of* (q_1) *is satisfied:*

q_3. $\xi = \eta$ *if and only if* $Q_m(\xi,\eta) = 0$ *for some* $m > 0$.

Again, Q *is called a convex if for* $m,\ n > 0$ *and* $\xi,\eta,\nu \in S$*, the inequality holds:*

q_4. $Q_{m+n}(\xi,\eta) \leq \frac{m}{m+n} Q_m(\xi,\nu) + \frac{n}{m+n} Q_n(\nu,\eta)$.

Definition 8 ([19]). *In Definition 7, if we replace* (q_2) *by:*

q_5. $Q_{\max\{m,n\}}(\xi,\eta) \leq Q_m(\xi,\nu) + Q_n(\nu,\eta)$

for all $m,n > 0$ *and* $\xi,\eta,\nu \in S$*, then* S_Q *is called a non-Archimedean quasi modular metric space.*

Note that the function $Q_{\max\{m,n\}}$ is more general than the function $Q_{m+n}(\xi,\eta)$, so every non-Archimedean quasi modular metric space is a quasi modular metric space.

Example 1 ([19]). *Let* $S = [0,\infty)$ *and* Q *be defined by:*

$$Q_m(\xi,\eta) = \begin{cases} \frac{\xi-\eta}{m} & \text{if } \xi \geq \eta \\ 1 & \text{if } \xi < \eta. \end{cases}$$

Then, S_Q *is a quasi modular metric space with* $m = \frac{1}{3}$ *and* $n = \frac{2}{3}$*, but is not modular metric space since* $Q_m(0,1) = 1$ *and* $Q_m(1,0) = \frac{1}{3}$.

Remark 1 ([19]). *From the above definitions we deduce that:*

i. *For a quasi modular metric* Q *on* S*, the conjugate quasi modular metric* Q^{-1} *on* S *of* Q *is defined by* $Q_m^{-1}(\xi,\eta) = Q_m(\eta,\xi)$.
ii. *If* Q *is a* T_0*-quasi pseudo modular metric on* S*, then the function* Q^E *defined by* $Q^E = Q^{-1} \vee Q$*, that is* $Q_m^E(\xi,\eta) = \max\{Q_m(\xi,\eta), Q_m(\eta,\xi)\}$*, defines a modular metric space.*

Now, we discuss some topological properties of quasi modular metric spaces.

Definition 9 ([19]). *A sequence* $\{\xi_p\}$ *in* S_Q *converges to* ξ *and is called:*

a. Q*-convergent or left convergent if* $\xi_p \to \xi \iff Q_m(\xi,\xi_p) \to 0$.
b. Q^{-1}*-convergent or right convergent if* $\xi_p \to \xi \iff Q_m(\xi_p,\xi) \to 0$.
c. Q^E*-convergent if* $Q_m(\xi,\xi_p) \to 0$ *and* $Q_m(\xi_p,\xi) \to 0$.

Definition 10 ([19]). *A sequence* $\{\xi_p\}$ *in a quasi modular metric space* S_Q *is called:*

d. *left (right) Q-K-Cauchy if for every* $\varepsilon > 0$*, there exists* $p_\varepsilon \in N$ *such that* $Q_m(\xi_r,\xi_p) < \varepsilon$ *for all* $p,r \in N$ *with* $p_\varepsilon \leq r \leq p$ $(p_\varepsilon \leq p \leq r)$ *and for all* $m > 0$.
e. Q^E*-Cauchy if for every* $\varepsilon > 0$*, there exists* $p_\varepsilon \in N$ *such that* $Q_m(\xi_p,\xi_r) < \varepsilon$ *for all* $p,r \in N$ *with* $p,r \geq p_\varepsilon$.

Remark 2 ([19]). *From the above definitions, we deduce that:*

i. *a sequence is left Q-K-Cauchy with respect to* Q *if and only if it is right Q-K-Cauchy with respect to* Q^{-1};

ii. *a sequence is Q^E-Cauchy if and only if it is left and right Q-K-Cauchy.*

Definition 11 ([19])**.** *A quasi modular metric space S_Q is called:*

i. *left Q-K-complete if every left Q-K-Cauchy is Q-convergent.*
ii. *Q-Smyth-complete if every left Q-K-Cauchy sequence is Q^E-convergent.*

3. Common Fixed Point Results

In the sequel, Q is regular and convex and T_Z denotes the family of all C_G-simulation functions $\zeta : [0,\infty)^2 \to \mathbb{R}$.

Definition 12. *Let S_Q be a non-Archimedean quasi modular metric space and $V : S_Q \to S_Q$ be a mapping. We say that V is a generalized Suzuki-simulation-type contractive mapping if there exist $\Sigma \in \tilde{\Theta}$, $\varphi \in \Phi$ and $\zeta \in T_Z$ such that:*

$$\begin{aligned} &\tfrac{1}{2}Q_m(\xi, V\xi) \le Q_m(\xi, \eta) \quad \text{implies} \\ &\zeta(\Sigma(Q_m(V\xi, V\eta)), \varphi(\Sigma(P(\xi,\eta)))) \ge C_G \end{aligned} \tag{1}$$

where:

$$P(\xi,\eta) = \max\{Q_m(\xi,\eta), Q_m(\xi, V\xi), Q_m(\eta, V\eta)\}$$

for all $\xi, \eta \in S_Q$.

Theorem 5. *Let S_Q be a Q-Smyth-complete non-Archimedean quasi modular metric space and V be the generalized Suzuki-simulation-type contractive mapping. Then, V has a unique fixed point.*

Proof. Define a sequence $\{\xi_k\}$ in S_Q by:

$$\xi_{k+1} = V\xi_k, \tag{2}$$

for all $k \in \mathbb{N}$. If there exists an k_0 such that $\xi_{k_0} = \xi_{k_0+1}$, then $z = \xi_{k_0}$ becomes a fixed point of V. Consequently, we shall assume that $\xi_k \ne \xi_{k+1}$ for all $k \in \mathbb{N}$. Therefore, we have:

$$Q_m(\xi_k, \xi_{k+1}) > 0, \quad \text{for all } n = 0, 1, 2\ldots. \tag{3}$$

Hence, we have:

$$\begin{aligned} &\frac{1}{2}Q_m(\xi_k, V\xi_k) < Q_m(\xi_k, V\xi_k) = Q_m(\xi_k, \xi_{k+1}) \quad \text{implies,} \\ &C_G \le \zeta(\Sigma(Q_m(V\xi_k, V\xi_{k+1})), \varphi(\Sigma(P(\xi_k, \xi_{k+1})))) \\ &= \zeta(\Sigma(Q_m(\xi_{k+1}, \xi_{k+2})), \varphi(\Sigma(P(\xi_k, \xi_{k+1})))) \\ &< G(\varphi(\Sigma(P(\xi_k, \xi_{k+1}))), \Sigma(Q_m(\xi_{k+1}, \xi_{k+2}))), \end{aligned} \tag{4}$$

by Definition 5, we get that:

$$\Sigma(Q_m(\xi_{k+1}, \xi_{k+2})) < \varphi(\Sigma(P(\xi_k, \xi_{k+1}))), \tag{5}$$

where:

$$\begin{aligned} P(\xi_k, \xi_{k+1}) &= \max\{Q_m(\xi_k, \xi_{k+1}), Q_m(\xi_k, V\xi_k), Q_m(\xi_{k+1}, V\xi_{k+1})\} \\ &= \max\{Q_m(\xi_k, \xi_{k+1}), Q_m(\xi_k, \xi_{k+1}), Q_m(\xi_{k+1}, \xi_{k+2})\} \\ &= \max\{Q_m(\xi_k, \xi_{k+1}), Q_m(\xi_{k+1}, \xi_{k+2})\}. \end{aligned} \tag{6}$$

If:

$$\max\{Q_m(\xi_k, \xi_{k+1}), Q_m(\xi_{k+1}, \xi_{k+2})\} = Q_m(\xi_{k+1}, \xi_{k+2})$$

for some $k \in \mathbb{N}$, then it follows from (5) and Lemma 2 that we get:

$$\Sigma(Q_m(\xi_{k+1}, \xi_{k+2})) < \varphi(\Sigma(Q_m(\xi_{k+1}, \xi_{k+2}))) < \Sigma(Q_m(\xi_{k+1}, \xi_{k+2}))$$

which is a contradiction. Therefore, we have:

$$P(\xi_k, \xi_{k+1}) = Q_m(\xi_k, \xi_{k+1})$$

for each $k \in \mathbb{N}$. Also, by (5), we have

$$\Sigma(Q_m(\xi_{k+1}, \xi_{k+2})) < \varphi(\Sigma(Q_m(\xi_k, \xi_{k+1}))).$$

Repeating this step, we conclude that:

$$\begin{aligned} \Sigma(Q_m(\xi_{k+1}, \xi_{k+2})) &< \varphi(\Sigma(Q_m(\xi_k, \xi_{k+1}))) \\ &< \varphi^2(\Sigma(Q_m(\xi_{k-1}, \xi_k))) \\ &\vdots \\ &< \varphi^k(\Sigma(Q_m(\xi_1, \xi_2))), \end{aligned}$$

for all $k \in \mathbb{N}$. Taking the limit $k \to \infty$ above, by the definition of φ and property Θ_2, we have:

$$\lim_{k\to\infty} \varphi^k(Q_m(\xi_1, \xi_2)) = 1. \tag{7}$$

Thus, from Lemma 1, it follows that:

$$\lim_{k\to\infty} Q_m(\xi_{k+1}, \xi_{k+2}) = 0, \tag{8}$$

for all $k \in \mathbb{N}$. Now, we show that $\{\xi_k\}$ is a left Q-K-Cauchy sequence. Assume the contrary. There exists $\varepsilon > 0$ such that we can find two subsequences $\{t_k\}$ and $\{s_k\}$ of positive integers satisfying the following inequalities:

$$Q_m(\xi_{t_k}, \xi_{s_k}) \geq \varepsilon, \quad \text{and} \quad Q_m(\xi_{t_k-1}, \xi_{s_k}) < \varepsilon. \tag{9}$$

From (9) and (q_5), it follows that:

$$\begin{aligned} \varepsilon \leq Q_m(\xi_{t_k}, \xi_{s_k}) &= Q_{\max\{m,m\}}(\xi_{t_k}, \xi_{s_k}) \\ &\leq Q_m(\xi_{t_k}, \xi_{t_k-1}) + Q_m(\xi_{t_k-1}, \xi_{s_k}) \\ &< \varepsilon + Q_m(\xi_{t_k}, \xi_{t_k-1}). \end{aligned} \tag{10}$$

On taking the limit as $k \to \infty$ in the above relation, we obtain that:

$$\lim_{k\to\infty} Q_m(\xi_{t_k}, \xi_{s_k}) = \varepsilon. \tag{11}$$

Also, from (9) and (q_5), it follows that:

$$\begin{aligned}
Q_m\left(\xi_{t_k+1},\xi_{s_k+1}\right) &= Q_{\max\{m,m\}}\left(\xi_{t_k+1},\xi_{s_k+1}\right) \\
&\leq Q_m\left(\xi_{t_k+1},\xi_{t_k}\right)+Q_m\left(\xi_{t_k},\xi_{s_k+1}\right) \\
&= Q_m\left(\xi_{t_k+1},\xi_{t_k}\right)+Q_{\max\{m,m\}}\left(\xi_{t_k},\xi_{s_k+1}\right) \\
&\leq Q_m\left(\xi_{t_k},\xi_{t_k-1}\right)+Q_m\left(\xi_{t_k-1},\xi_{s_k+1}\right)+Q_m\left(\xi_{t_k+1},\xi_{t_k}\right) \\
&= Q_m\left(\xi_{t_k},\xi_{t_k-1}\right)+Q_m\left(\xi_{t_k+1},\xi_{t_k}\right)+Q_{\max\{m,m\}}\left(\xi_{t_k-1},\xi_{s_k+1}\right) \\
&\leq Q_m\left(\xi_{t_k-1},\xi_{s_k}\right)+Q_m\left(\xi_{s_k},\xi_{s_k+1}\right) \\
&\quad +Q_m\left(\xi_{t_k},\xi_{t_k-1}\right)+Q_m\left(\xi_{t_k+1},\xi_{t_k}\right) \\
&< \varepsilon+Q_m\left(\xi_{s_k},\xi_{s_k+1}\right)+Q_m\left(\xi_{t_k},\xi_{t_k-1}\right) \\
&\quad +Q_m\left(\xi_{t_k+1},\xi_{t_k}\right).
\end{aligned} \tag{12}$$

Next, we claim that:

$$\frac{1}{2}Q_m\left(\xi_{t_k},V\xi_{t_k}\right)\leq Q_m\left(\xi_{t_k},\xi_{s_k}\right).$$

If:

$$\begin{aligned}
&\tfrac{1}{2}Q_m\left(\xi_{t_k},V\xi_{t_k}\right) > Q_m\left(\xi_{t_k},\xi_{s_k}\right) \\
&= \tfrac{1}{2}Q_m\left(\xi_{t_k},\xi_{t_k+1}\right) > Q_m\left(\xi_{t_k},\xi_{s_k}\right),
\end{aligned} \tag{13}$$

then letting $k\to\infty$ in (13), from (11) and (8), we have that $0>\varepsilon$ is a contradiction. Hence,

$$\frac{1}{2}Q_m\left(\xi_{t_k},V\xi_{t_k}\right)\leq Q_m\left(\xi_{t_k},\xi_{s_k}\right).$$

From the generalized Suzuki-simulation-type contractive mapping, we get:

$$\begin{aligned}
C_G &\leq \zeta\left(\Sigma\left(Q_m\left(V\xi_{t_k},V\xi_{s_k}\right)\right),\varphi\left(\Sigma\left(P\left(\xi_{t_k},\xi_{s_k}\right)\right)\right)\right) \\
&= \zeta\left(\Sigma\left(Q_m\left(\xi_{t_k+1},\xi_{s_k+1}\right)\right),\varphi\left(\Sigma\left(P\left(\xi_{t_k},\xi_{s_k}\right)\right)\right)\right),
\end{aligned} \tag{14}$$

where:

$$\begin{aligned}
P\left(\xi_{t_k},\xi_{s_k}\right) &= \max\left\{Q_m\left(\xi_{t_k},\xi_{s_k}\right),Q_m\left(\xi_{t_k},V\xi_{t_k}\right),Q_m\left(\xi_{s_k},V\xi_{s_k}\right)\right\} \\
&= \max\left\{Q_m\left(\xi_{t_k},\xi_{s_k}\right),Q_m\left(\xi_{t_k},\xi_{t_k+1}\right),Q_m\left(\xi_{s_k},\xi_{s_k+1}\right)\right\}.
\end{aligned} \tag{15}$$

Taking the limit $k\to\infty$ using (8), (11), and (12) in (14) and (15), we obtain:

$$C_G\leq\zeta\left(\Sigma\left(\varepsilon\right),\varphi\left(\Sigma\left(\varepsilon\right)\right)\right)<G\left(\varphi\left(\Sigma\left(\varepsilon\right)\right),\Sigma\left(\varepsilon\right)\right).$$

From Definition 5, we get:

$$\Sigma\left(\varepsilon\right)<\varphi\left(\Sigma\left(\varepsilon\right)\right)<\Sigma\left(\varepsilon\right).$$

It follows that $\Sigma\left(\varepsilon\right)<\Sigma\left(\varepsilon\right)$, a contradiction. Hence, $\{\xi_k\}$ is a left Q-K-Cauchy sequence. As S_Q is a Q-Smyth-complete non-Archimedean quasi modular metric space, there exists $u\in S_Q$ such that:

$$\lim_{k\to\infty}{Q_m}^E\left(\xi_k,u\right)=0.$$

Thus, we have:

$$\lim_{k\to\infty}Q_m\left(\xi_k,u\right)=0 \qquad \text{and} \qquad \lim_{k\to\infty}Q_m\left(u,\xi_k\right)=0.$$

Now, we show that u is a fixed point of V. Assume that $Q_m\left(Vu,u\right)>0$. We claim that for each $k\geq 0$, the following holds:

$$\frac{1}{2}Q_m\left(\xi_k,V\xi_k\right)\leq Q_m\left(\xi_k,u\right).$$

On the contrary, suppose that:

$$\frac{1}{2}Q_m\left(\xi_k, V\xi_k\right) > Q_m\left(\xi_k, u\right) = \frac{1}{2}Q_m\left(\xi_k, \xi_{k+1}\right) > Q_m\left(\xi_k, u\right). \tag{16}$$

Taking the limit as $k \to \infty$ in (16), we obtain $0 > 0$, a contradiction. Thus, the claim is true, and so, from the generalized Suzuki-simulation-type contractive mapping, we get:

$$\begin{aligned} C_G &\leq \zeta\left(\Sigma\left(Q_m\left(V\xi_k, Vu\right)\right), \varphi\left(\Sigma\left(P\left(\xi_k, u\right)\right)\right)\right) \\ &= \zeta\left(\Sigma\left(Q_m\left(\xi_{k+1}, Vu\right)\right), \varphi\left(\Sigma\left(P\left(\xi_k, u\right)\right)\right)\right) \\ &< G\left(\varphi\left(\Sigma\left(P\left(\xi_k, u\right)\right)\right), \Sigma\left(Q_m\left(\xi_{k+1}, Vu\right)\right)\right). \end{aligned} \tag{17}$$

By Definition 5,

$$\Sigma\left(Q_m\left(\xi_{k+1}, Vu\right)\right) < \varphi\left(\Sigma\left(P\left(\xi_k, u\right)\right)\right), \tag{18}$$

where:

$$\begin{aligned} P\left(\xi_k, u\right) &= \max\left\{Q_m\left(\xi_k, u\right), Q_m\left(\xi_k, V\xi_k\right), Q_m\left(u, Vu\right)\right\} \\ &= \max\left\{Q_m\left(\xi_k, u\right), Q_m\left(\xi_k, \xi_{k+1}\right), Q_m\left(u, Vu\right)\right\}. \end{aligned} \tag{19}$$

Letting $k \to \infty$ in (17)–(19), we have:

$$\Sigma\left(Q_m\left(u, Vu\right)\right) < \varphi\left(\Sigma\left(Q_m\left(u, Vu\right)\right)\right) < \Sigma\left(Q_m\left(u, Vu\right)\right).$$

That is, $\Sigma\left(Q_m\left(u, Vu\right)\right) < \Sigma\left(Q_m\left(u, Vu\right)\right)$, a contradiction. Thus, u is a fixed point of V. Suppose that there is an another fixed point u^* of V such that $Vu^* = u^*$ and $u \neq u^*$. Then, $Q_m\left(Vu, Vu^*\right) = Q_m\left(u, u^*\right) > 0$, and:

$$0 = \frac{1}{2}Q_m\left(u, Vu\right) \leq Q_m\left(u, u^*\right).$$

By the generalized Suzuki-simulation-type contractive mapping, we have:

$$\begin{aligned} C_G &\leq \zeta\left(\Sigma\left(Q_m\left(Vu, Vu^*\right)\right), \varphi\left(\Sigma\left(P\left(u, u^*\right)\right)\right)\right) \\ &= \zeta\left(\Sigma\left(Q_m\left(u, u^*\right)\right), \varphi\left(\Sigma\left(P\left(u, u^*\right)\right)\right)\right) \\ &< G\left(\varphi\left(\Sigma\left(P\left(u, u^*\right)\right)\right), \Sigma\left(Q_m\left(u, u^*\right)\right)\right). \end{aligned} \tag{20}$$

From the property of G,

$$\Sigma\left(Q_m\left(u, u^*\right)\right) < \varphi\left(\Sigma\left(P\left(u, u^*\right)\right)\right), \tag{21}$$

where:

$$P\left(u, u^*\right) = \max\left\{Q_m\left(u, u^*\right), Q_m\left(u, Vu\right), Q_m\left(u^*, Vu^*\right)\right\} = Q_m\left(u, u^*\right). \tag{22}$$

From (20)–(22), we attain the following ordering:

$$\Sigma\left(Q_m\left(u, u^*\right)\right) < \varphi\left(\Sigma\left(Q_m\left(u, u^*\right)\right)\right) < \Sigma\left(Q_m\left(u, u^*\right)\right),$$

which is a contradiction. Hence, u is a unique fixed point of V. □

Now, we give some corollaries that are directly acquired from our main results.

Corollary 1. *Let S_Q be a Q-Smyth-complete non-Archimedean quasi modular metric space and $V : S_Q \to S_Q$ be a mapping. If there exists $\Sigma \in \tilde{\Theta}$, $\varphi \in \Phi$, and $\zeta \in T_Z$ such that:*

$$\frac{1}{2}Q_m(\jmath, V\jmath) \leq Q_m(\jmath, \ell) \qquad \text{implies,}$$

$$\zeta\left(\Sigma\left(Q_m\left(V\jmath, V\ell\right)\right), \varphi\left(\Sigma\left(Q_m\left(\jmath, \ell\right)\right)\right)\right) \geq C_G,$$

for all $\jmath, \ell \in S_Q$, then V has a unique fixed point.

Corollary 2. *Let S_Q be a Q-Smyth-complete non-Archimedean quasi modular metric space and $V : S_Q \to S_Q$ be a mapping. If there exists $\Sigma \in \tilde{\Theta}$, $\varphi \in \Phi$, and $\zeta \in T_Z$ such that:*

$$\zeta\left(\Sigma\left(Q_m\left(V\jmath, V\ell\right)\right), \varphi\left(\Sigma\left(P\left(\jmath, \ell\right)\right)\right)\right) \geq C_G$$

where:

$$P(\jmath, \ell) = \max\left\{Q_m(\jmath, \ell), Q_m(\jmath, V\jmath), Q_m(\ell, V\ell)\right\},$$

for all $\jmath, \ell \in S_Q$, then V has a unique fixed point.

Corollary 3. *Let S_Q be a Q-Smyth-complete non-Archimedean quasi modular metric space and $V : S_Q \to S_Q$ be a mapping. If there exists $\Sigma \in \tilde{\Theta}$ and $\varphi \in \Phi$ such that:*

$$\frac{1}{2}Q_m(\jmath, V\jmath) \leq Q_m(\jmath, \ell) \qquad \text{implies,}$$

$$\Sigma\left(Q_m\left(V\jmath, V\ell\right)\right) \leq \varphi\left(\Sigma\left(P\left(\jmath, \ell\right)\right)\right)$$

where:

$$P(\jmath, \ell) = \max\left\{Q_m(\jmath, \ell), Q_m(\jmath, V\jmath), Q_m(\ell, V\ell)\right\},$$

for all $\jmath, \ell \in S_Q$, then V has a unique fixed point.

Corollary 4. *Let S_Q be a Q-Smyth-complete non-Archimedean quasi modular metric space and $V : S_Q \to S_Q$ be a mapping. If there exists $\Sigma \in \tilde{\Theta}$ and $\varphi \in \Phi$ such that:*

$$\Sigma\left(Q_m\left(V\jmath, V\ell\right)\right) \leq \varphi\left(\Sigma\left(P\left(\jmath, \ell\right)\right)\right)$$

where:

$$P(\jmath, \ell) = \max\left\{Q_m(\jmath, \ell), Q_m(\jmath, V\jmath), Q_m(\ell, V\ell)\right\},$$

for all $\jmath, \ell \in S_Q$, then V has a unique fixed point.

Corollary 5. *Let S_Q be a Q-Smyth-complete non-Archimedean quasi modular metric space and $V : S_Q \to S_Q$ be a mapping. If there exists $\Sigma \in \tilde{\Theta}$ and $\varphi \in \Phi$ such that:*

$$\Sigma\left(Q_m\left(V\jmath, V\ell\right)\right) \leq \varphi\left(\Sigma\left(Q_m\left(\jmath, \ell\right)\right)\right),$$

for all $\jmath, \ell \in S_Q$, then V has a unique fixed point.

4. Application to a Graph Structure

Let S_Q be a non-Archimedean quasi modular metric space and $\Delta = \{(\jmath, \jmath) : \jmath \in S_Q\}$ denote the diagonal of $S_Q \times S_Q$. Let H be a directed graph such that the set $C(H)$ of its vertices coincides with S_Q and $B(H)$ is the set of edges of the graph such that $\Delta \subseteq B(H)$. H is determined with the pair $(C(H), B(H))$.

If $\jmath$ and ℓ are vertices of H, then a path in H from $\jmath$ to ℓ of length $p \in \mathbb{N}$ is a finite sequence $\{\jmath_p\}$ of vertices such that $\jmath = \jmath_0, ..., \jmath_p = \eta$ and $(\jmath_{i-1}, \jmath_i) \in B(H)$ for $i \in \{1, 2, ..., p\}$.

Recall that H is connected if there is a path between any two vertices, and it is weakly connected if $\widetilde{H}$ is connected, where $\widetilde{H}$ defines the undirected graph obtained from H by ignoring the direction of edges. Define by H^{-1} the graph obtained from H by reversing the direction of edges. Thus,

$$B\left(H^{-1}\right) = \left\{(\jmath, \ell) \in S_Q \times S_Q : (\ell, \jmath) \in B(H)\right\}.$$

It is more convenient to treat $\widetilde{H}$ as a directed graph for which the set of its edges is symmetric, under this convention; we have that:

$$B(\widetilde{H}) = B(H) \cup B(H^{-1}).$$

Let $H_\jmath$ be the component of H consisting of all the edges and vertices that are contained in some way in H starting at $\jmath$. We denote the relation (R) in the following way:

We have $\jmath(R)\ell$ if and only if, there is a path in H from $\jmath$ to ℓ, for $\jmath, \ell \in C(H)$.

If H is such that $B(H)$ is symmetric, then for $\jmath \in C(H)$, the equivalence class $[\jmath]_G$ in $V(G)$ described by the relation (R) is $C(H_\jmath)$.

Let S_Q be a non-Archimedean quasi modular metric space endowed with a graph H and $\hbar : S_Q \to S_Q$. We set:

$$S_\hbar = \left\{\jmath \in S_Q : (\jmath, \hbar\jmath) \in B(H)\right\}.$$

Definition 13 ([20]). *(S, d) is a metric space, and $\hbar : S \to S$ is a mapping. Then, $\hbar$ is called a Banach H-contraction if the following hold:*

B_1. *$\hbar$ preserves edges of H, i.e., for all $\jmath, \ell \in S$,*

$$(\jmath, \ell) \in B(H) \quad \Rightarrow \quad (\hbar\jmath, \hbar\ell) \in B(H),$$

B_2. *there exists $\delta \in (0, 1)$ such that:*

$$d(\hbar\jmath, \hbar\ell) \leq \delta d(\jmath, \ell)$$

for all $(\jmath, \ell) \in B(H)$.

After that, many fixed point researchers investigated fixed point results improving the Jachymski fixed point theorems in [17,21–23].

Now, motivated by [24–26], we generate a new contraction and obtain fixed point results using a graph structure.

Definition 14. *Let S_Q be a non-Archimedean quasi modular metric space and $\hbar : S_Q \to S_Q$ be a mapping. Then, we say that $\hbar$ is a generalized Suzuki-simulation-H-type contractive mapping if the following conditions hold:*

H_1. *$\hbar$ preserves edges of G;*
H_2. *there exists $\Sigma \in \tilde{\Theta}$, $\varphi \in \Phi$ and $\zeta \in T_Z$ such that:*

$$\begin{aligned} &\tfrac{1}{2} Q_m(\jmath, \hbar\jmath) \leq Q_m(\jmath, \ell) \quad \text{implies}, \\ &\zeta\left(\Sigma\left(Q_m(\hbar\jmath, \hbar\ell)\right), \varphi\left(\Sigma\left(P(\jmath, \ell)\right)\right)\right) \geq C_G, \end{aligned} \tag{23}$$

where

$$P(\jmath, \ell) = \max\left\{Q_m(\jmath, \ell), Q_m(\jmath, \hbar\jmath), Q_m(\ell, \hbar\ell)\right\}$$

for all $\jmath, \ell \in B(H)$ and for all $m > 0$.

Remark 3. *Let S_Q be a non-Archimedean quasi modular metric space with a graph H and $\hbar : S_Q \to S_Q$ be a generalized Suzuki-simulation-H-type contractive mapping. If there exists $\jmath_0 \in S_Q$ such that $\hbar\jmath_0 \in [\jmath_0]_{\tilde{H}}$, then:*

R_1. *$\hbar$ is both a generalized Suzuki-simulation-H^{-1}-type contractive mapping and a generalized Suzuki-Simulation-$\tilde{H}$-type contractive mapping.*

R_2. *$[\jmath_0]_{\tilde{H}}$ is $\hbar$-invariant, and $\hbar\Big|_{[\jmath_0]_{\tilde{H}}}$ is a generalized Suzuki-simulation-$\tilde{H}_{\jmath_0}$-type contractive mapping.*

Theorem 6. *Let S_Q be a Q-Smyth-complete non-Archimedean quasi modular metric space with a graph H and $\hbar : S_Q \to S_Q$ be a mapping.*

i. *there exists $\jmath_0 \in S_\hbar$;*
ii. *$\hbar$ is the generalized Suzuki-simulation-$\tilde{H}$-type contractive mapping;*
iii. *H is weakly connected;*
iv. *if $\{\jmath_k\}$ is a sequence in S_Q such that $\lim_{k\to\infty} {Q_m}^E(\jmath_k, u) = 0$ and $(\jmath_k, \jmath_{k+1}) \in B(H)$, then there exists a subsequence $\{\jmath_{k_s}\}$ of $\{\jmath_k\}$ such that $(\jmath_{k_s}, u) \in B(H)$.*

Then, $\hbar$ has a unique fixed point.

Proof. Define a sequence $\{\jmath_k\}$ in S_Q by:

$$\jmath_{k+1} = \hbar\jmath_k, \tag{24}$$

for all $k \in \mathbb{N}$. Let $\jmath_0$ be a given point in $S_\hbar$; thus, $(\jmath_0, \hbar\jmath_0) = (\jmath_0, \jmath_1) \in B(H)$. Because $\hbar$ preserves the edges of H,

$$(\jmath_0, \jmath_1) \in B(H) \qquad \Rightarrow \qquad (\hbar\jmath_0, \hbar\jmath_1) \in B(H).$$

Continuing this way, we get:

$$(\jmath_k, \jmath_{k+1}) \in B(H).$$

Then from Theorem 5, we get that $\{\jmath_k\}$ is a left Q-K-Cauchy sequence in S_Q. By the Q-Smyth-completeness of S_Q, there exists $u \in S_Q$ such that:

$$\lim_{k\to\infty} Q_m^E(\jmath_k, u) = 0. \tag{25}$$

Thus, we have:

$$\lim_{k\to\infty} Q_m(\jmath_k, u) = 0 \ \text{ and } \ \lim_{k\to\infty} Q_m(u, \jmath_k) = 0. \tag{26}$$

Now, we show that u is a fixed point of $\hbar$. Using (iv), we get $(\jmath_{k_s}, u) \in B(H)$. We claim that:

$$\frac{1}{2} Q_m(\jmath_{k_s}, \hbar\jmath_{k_s}) \le Q_m(\jmath_{k_s}, u). \tag{27}$$

If

$$\tfrac{1}{2} Q_m(\jmath_{k_s}, \hbar\jmath_{k_s}) > Q_m(\jmath_{k_s}, u) = \tfrac{1}{2} Q_m(\jmath_{k_s}, \jmath_{k_s+1}) > Q_m(\jmath_{k_s}, u) \tag{28}$$

and taking the limit $s \to \infty$ in (28), we get $0 > 0$, a contradiction. Hence, the claim is true. Since $\hbar$ is a generalized Suzuki-simulation-$\tilde{H}$-type contractive mapping, we have:

$$\begin{aligned} C_G &\le \zeta\left(\Sigma\left(Q_m(\hbar\jmath_{k_s}, \hbar u)\right), \varphi\left(\Sigma\left(P(\jmath_{k_s}, u)\right)\right)\right) \\ &\le \zeta\left(\Sigma\left(Q_m(\hbar\jmath_{k_s}, \hbar u)\right), \varphi\left(\Sigma\left(P(\jmath_{k_s}, u)\right)\right)\right) \\ &\le G\left(\varphi\left(\Sigma\left(P(\jmath_{k_s}, u)\right)\right), \Sigma\left(Q_m(h\jmath_{k_s}, hu)\right)\right), \end{aligned} \tag{29}$$

from Definition 5, we get:

$$\Sigma\left(Q_m\left(\hbar\jmath_{k_s},\hbar u\right)\right),\varphi\left(\Sigma\left(P\left(\jmath_{k_s},u\right)\right)\right), \tag{30}$$

where:

$$\begin{aligned} P\left(\jmath_{k_s},u\right) &= \max\left\{Q_m\left(\jmath_{k_s},u\right),Q_m\left(\jmath_{k_s},\hbar\jmath_{k_s}\right),\ Q_m\left(u,\hbar u\right)\right\} \\ &= \max\left\{Q_m\left(\jmath_{k_s},u\right),Q_m\left(\jmath_{k_s},\jmath_{k_s+1}\right),\ Q_m\left(u,\hbar u\right)\right\}. \end{aligned} \tag{31}$$

Taking the limit as $s\to\infty$ in (29)–(31), we get:

$$\Sigma\left(Q_m\left(u,hu\right)\right)<\varphi\left(\Sigma\left(Q_m\left(u,hu\right)\right)\right)<\Sigma\left(Q_m\left(u,hu\right)\right).$$

It follows that $\Sigma\left(Q_m\left(u,hu\right)\right)<\Sigma\left(Q_m\left(u,hu\right)\right)$, a contradiction. Therefore, we get $Q_m\left(u,\hbar u\right)=0$, that is $u=\hbar u$ since Q is regular.

Next, we show that u is a unique fixed point of $\hbar$. On the contrary, we suppose that u^* is another fixed point of $\hbar$, i.e., $u^*=\hbar u^*$ and $u\neq u^*$. Then, there exist $\sigma\in S_Q$ such that $(u,\sigma)\in B(H)$ and $(\sigma,u^*)\in B(H)$. Using (iii), we get that $(u,u^*)\in B(\tilde{H})$. Furthermore,

$$0=\frac{1}{2}Q_m\left(u,hu\right)<Q_m\left(u,u^*\right). \tag{32}$$

From the generalized Suzuki-Simulation-$\tilde{H}$-type contractive mapping we have:

$$\begin{aligned} C_G &\leq \zeta\left(\Sigma\left(Q_m\left(hu,hu^*\right)\right),\varphi\left(\Sigma\left(P\left(u,u^*\right)\right)\right)\right) \\ &\leq \zeta\left(\Sigma\left(Q_m\left(u,u^*\right)\right),\varphi\left(\Sigma\left(P\left(u,u^*\right)\right)\right)\right) \\ &\leq G\left(\varphi\left(\Sigma\left(P\left(u,u^*\right)\right)\right),\Sigma\left(Q_m\left(hu,hu^*\right)\right)\right). \end{aligned} \tag{33}$$

Using Definition 5, we get:

$$\Sigma\left(Q_m\left(u,u^*\right)\right)<\varphi\left(\Sigma\left(P\left(u,u^*\right)\right)\right) \tag{34}$$

where:

$$\begin{aligned} P\left(u,u^*\right) &= \max\left\{Q_m\left(u,u^*\right),Q_m\left(u,\hbar u\right),\ Q_m\left(u^*,\hbar u^*\right)\right\} \\ &= \max\left\{Q_m\left(u,u^*\right),\ 0\right\}=Q_m\left(u,u^*\right). \end{aligned} \tag{35}$$

From (33)–(35), it follows that:

$$\Sigma\left(Q_m\left(u,u^*\right)\right)<\varphi\left(\Sigma\left(Q_m\left(u,u^*\right)\right)\right)<\Sigma\left(Q_m\left(u,u^*\right)\right).$$

This is an incorrect statement. Hence, $u=u^*$. □

5. Conclusions

First, motivated by [4,10,15], we established a new contractive mapping, which is called the generalized Suzuki-simulation-type contractive mapping. Second, in [19], we constituted a new quasi metric space, which is named the non-Archimedean quasi modular metric space, and so using this, we attained fixed point theorems via generalized Suzuki-simulation-type contractive mapping. Finally, we acquired graphical fixed point results in non-Archimedean quasi modular metric spaces.

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