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A Bi-Level Programming Model for Optimal Bus Stop Spacing of a Bus Rapid Transit System

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Abstract: The purpose of this study is to create a bi-level programming model for the optimal bus stop spacing of a bus rapid transit (BRT) system, to ensure simultaneous coordination and consider the interests of bus companies and passengers. The top-level model attempts to optimize and determine optimal bus stop spacing to minimize the equivalent costs, including wait, in-vehicle, walk, and operator costs, while the bottom-level model reveals the relation between the locations of stops and spatial service coverage to attract an increasing number of passengers. A case study of Chengdu, by making use of a genetic algorithm, is presented to highlight the validity and practicability of the proposed model and analyze the sensitivity of the coverage coefficient, headway, and speed with different spacing between bus stops.

Keywords: bus rapid transit; stop spacing; bi-level programming model; genetic algorithms

1. Introduction

Bus rapid transit (BRT) provides a punctual, fast, safe, and comfortable service for travelers through the use of dedicated road space. Combined with the advantages of conventional public transport and rail transit systems, it has the advantages of accommodating large volumes of passengers, is fast running, and flexible. Therefore, BRT has increasingly become one of the more popular means of public transport, which is worth promoting since the cost for the development of a rail transit system is very high. BRT, which involves a lower level of investment, is quicker and more effective, has a shorter period of deployment, and has immense potential for implementation and development in many large and medium-sized cities [1,2].

Compared with conventional public transport systems, BRT has been known to be more effective in terms of passenger flow distribution. The implementation of a BRT system is generally regarded to be an effective way of mitigating the problems associated with automobiles, such as road congestion, inefficient energy consumption, and air pollution [3,4]. The advantages of BRT have been well proven in real-world services and during actual operation in cities such as Curitiba in Brazil, Bogota in Colombia, Brisbane in Australia, and Los Angeles in the United States, amongst others. It has been considered to be a low-cost efficiency solution for traffic congestion in China's cities, such as Beijing, Guangzhou, Chongqing, Dalian, and Ji'nan [5–7]. Bus stop locations are usually not placed at optimal distances and the determining factors for placing bus stops at a particular location need to be balanced between the interests of passengers as well as the bus operators.

The spacing of bus stops is correlated to the operation efficiency, costs, and service level of a BRT system. The stop spacing distance generally applied by the US transit agencies varies between 91 and 305 m in central business districts [8]. The average stop spacing distance in US cities was 198 m, whereas the optimal spacing used to be approximately 396 m [9]. Passenger flow, attraction, and accessibility increase when the bus stop spacing is relatively short, and, conversely, if bus stop spacing is wide apart and infrequent, the level of accessibility decreases, and a number of potential passengers are lost as a result. Bus stops located with short and frequent spacing shorten the walking time for passengers; however, this implies that buses will stop more frequently, thereby increasing the passenger's travel time. In contrast, if bus stops are placed further apart with larger spacing distances, the walking time for passengers increases. Mylona et al. indicated that appropriate bus stop consolidation (an 8% increment in stop spacing) had no significant effects on passenger activities (i.e., boarding and alighting) but could largely improve bus running time (i.e., a 9% reduction) [10]. This paper investigates the stop spacing problem of the BRT system, which is considered to be an important issue with respect to enhancing the attraction of the urban public transport system.

2. Overview of Solutions

At the macro level, when setting up a BRT station, it is important to take into account how convenient transfers are, passenger demand, and the nature of land use along the proposed line. At the meso-level, stop placing is related to the service radius and "reasonable" stop spacing distances. Finally, at the micro-level, stop placement is closely related to traffic conditions and the condition of the line at intersections of adjacent sections. The matter of stop spacing has long been understood and studied, yet it is still difficult to determine optimal locations. In order to minimize passenger travel times, Vuchic and Newell found that several factors influencing the optimal stop spacing for line-haul passenger transportation, for example, passenger distribution along the line, access speed, and standing time at stops [11]. Based on the automatic vehicle location systems and automatic cost acquisition systems, Tétreault and El-Geneidy studied the impact of stop location selections on trip times in transfer hub. It was found that optimized stop placing can significantly save total travel times [12].

To summarize previous studies, the key factors to consider when determining stop spacing and locations are the distribution and potential benefits for passengers, bus operators, and governments. Additionally, a number of academics have also considered the interests of passengers, bus operators, and the government to optimize stop spacing as part of their research. When considering the benefits to passengers, the bus stop spacing optimization model has been widely used for minimizing travel costs or travel times as the objective function [13–15].

Based on the bus operation costs and passenger travel times, Wirasinghe and Nadia, comprehensively studied the optimization issue of average stop spacing [14]. Academics have proposed a set-cover-based iterative algorithm to address this problem, which makes use of the minimum number of bus stops to maximize the stop coverage and minimize total pedestrian (walking) distances [16–20]. The studies can partly be categorized into a non-deterministic polynomial problem, which refers to the existence of non-deterministic problems that can be solved by a polynomial algorithm and belongs to a non-deterministic polynomial operation problem, and an enumeration and heuristic search algorithm are used in the process of solving the model. To better understand optimum stop spacing and the influence of different stop spacing methods on passenger and bus travel times, as well as the transit system, a multi-objective optimization model is developed to establish the relationship between stop spacing and average passenger travel times [21–23]. Stop spacing is an important determinant when planning a transit system since it has an influence on the service level, as well as the operational costs. A mathematical model is developed, and the objective function is user travel time, which is minimized by optimized stop spacing and headway [10,24,25]. The bi-level programming model is often used to optimize bus stop spacing. The objective function of the top-level programming model is to minimize the total cost incurred to passenger and bus operators while for the lower level, the problem of bus traffic assignment is addressed [26–29]. To minimize the total residence time at stops and the number of bus stops required for efficient operations, a bi-objective optimization model has been proposed by Chen et al.; furthermore, the issues of stop congestion and its effect on road traffic flow has also been considered in the model. The results showed that the formulated model could solve problems faced in real-world situations [30].

Government requirements are also an important factor in determining stop spacing, including increasing the attractiveness of public transport and maximizing social benefits, which involve a trade-off between the interests of passengers and operating companies. Tirachini discussed the influence of station spacing on the operation times of buses and constructed an analytical model for the benefit of bus operators. The objective function includes a maximum reduction in operating costs under the appropriate level of public transport services [15]. Furthermore, the objective of several studies has been to maximize social welfare, that is, consumer surplus plus government revenue minus the operating costs of the public transport companies [30,31].

Recently, many studies have been conducted on the sensitivity analysis of stop spacing related parameters. These studies have found that when the demand is high, the two variables of bus demand and stop placing are independent; however, stop placing and line length are negatively correlated [25]. Alonso et al. assume that the departure frequency is constant, and the results from their model also indicate a negative correlation between stop placing and bus trip frequency [28]. In contrast, an opposing conclusion can be drawn—that when bus fleet size is a given, there is a positive correlation between stop spacing and bus trip volume [15].

This brief review of the literature suggests that the stop spacing problem can be optimized in many aspects. Despite the increasing attention devoted to the study of the model choice of bus stop spacing, the current literature has certain gaps that are the motivation for this research.

The contributions of this paper are threefold. Most of the previous studies were conducted on the premise of ideal assumptions. For example, it is assumed that stop spacing is not related to the passenger flow. In fact, the changes in stop spacing can simultaneously cause changes in spatial and time accessibility, resulting in changes in demand for public transportation and thus affecting the setting of the spacing. In our study, situations where the passenger flow demand is affected by different stop spacing are considered, and the coverage coefficient of the stop is applied to the model.

In addition, some studies have analyzed the stop scope or stop locations separately and give less importance to the impact of a public transit network. Although passenger travel densities with simplification or a hypothesis are measured, the investigation is inadequate and the resulting stop spacing cannot be applied practically in real-world situations. In actual operation, many BRT platforms are set according to intersections, which only consider the priority given to buses. In addition, the influence of the location of an intersection on the placing of stops and spacing is not well established. Comprehensive methodologies that address the issue of stop placement in BRT systems are in short supply. This paper attempts to fill this gap in the existing body of the available research literature.

Finally, following previous studies, our approach proposes an optimization bi-level programming objective function. This function can address and solve for the optimal location of each stop. The two-layer optimization model takes full account of the influence of the location of the intersections and aims to achieve the optimal benefit for operators as well as passengers. In addition, the model can provide a reference for the optimal spacing of bus stops based on a real case study.

3. Model Development

3.1. Top-Level Model Establishment

For the top-level model, the influence of BRT station spacing on costs are analyzed, including user cost (C_u) and operator cost (C_o). Therefore, taking the minimum equivalent cost of BRT route as the objective value (f_1), the optimal stop spacing model is set up as follows:

$$\min f_1 = C_u + C_o \tag{1}$$

The passenger travel time can be classified into three categories: wait time, in-vehicle travel time, and walk time [32]. Therefore, the user one-way cost associated with these time categories can be formulated as Equation (2), shown below:

$$C_u = C_1 + C_2 + C_3 \tag{2}$$

where C_1, C_2, and C_3 represent wait cost, in-vehicle cost, and walk cost, respectively.

3.1.1. Wait Cost

The wait cost is defined as the product of the value of wait time (ϕ_1) and the total wait time for all the passengers (t_1), and is related to average wait time of each stop and the BRT demand of each stop. Thus, the total wait cost is the sum of the wait cost of all stops as follows:

$$C_1 = \phi_1 \times t_1 \tag{3}$$

$$t_1 = \sum_{i}^{n} q_i t_i \tag{4}$$

where n represents the total number of stops, and q_i and t_i are BRT demand and average wait time at stop i, respectively.

Given that the BRT demand is assumed to be uniformly distributed along the BRT route, q_i is equal to the BRT passenger flow attraction in relation to the stop spacing (q_s) divided by n, where n is equal to the route length (l) divided by the stop spacing (s). Here, the average wait time (t_i) can simply be considered equal to half of the BRT headway (h). Thus,

$$q_i = q_s / n \tag{5}$$

$$n = \lceil l/s \rceil + 1 \tag{6}$$

$$t_i = h/2 \tag{7}$$

3.1.2. In-Vehicle Cost

The in-vehicle cost is related to the total in-vehicle time (t_2, i.e., the unimpeded running time and the time caused by the delay) and the value of in-vehicle time (ϕ_2).

$$C_2 = \phi_2 \times t_2 \tag{8}$$

t_2 includes the time for vehicle acceleration and deceleration (t_{ad}), the dwell time at the stop station (t_s), the dwell time caused by traffic signals (t_w), and the unimpeded running time of vehicles (t_r) [33,34].

$$t_2 = t_{ad} + t_s + t_w + t_r \tag{9}$$

The acceleration and deceleration of the BRT vehicles are often caused by the intermediate stop stations and signal intersections. Therefore, the amount of vehicle acceleration and deceleration is also related to the spacing between stops and signal intersections. When the spacing between the stop platforms and signalized intersections is within 50 m, the driver can go through the intersection without stopping, whereas for instances where the spacing is in excess of 50 m, the driver may stop at the intersection. The total one-way travel time attributed to acceleration and deceleration of a BRT vehicle can be determined as follows:

$$t_{ad} = (\rho \times n_c + n - 1) \times [v_s \times (a + d) / (a \times d)] \tag{10}$$

where n_c is the number of signalized intersections which are located at a considerable distance from stops, and the spacing is at least at a distance of 50 m; ρ is the possibility that vehicles adequately stop for red lights at intersections, that is, $\rho = (T - g)/T$; the average effective green time and average cycle time of signalized intersection are g and T, respectively; v_s is the speed of stops; and a and d are the average values of acceleration and deceleration, respectively.

Considering the safety of passengers, BRT vehicles have a maximum recommended and permissible speed ($v_{\max}$) limit. When stop spacing is closer together, or signalized intersections are located close together, vehicles are unable to achieve their maximum speed; therefore, the speed of stops is expressed as follows:

$$v_s = \min\left\{v_{\max}, \sqrt{\frac{2 \times a \times d \times (x_a + x_d)}{a + d}}\right\} \tag{11}$$

where x_a is the distance travelled during acceleration of the BRT vehicle, x_d is the distance travelled during deceleration of the BRT vehicle.

The dwell time at the stop station is related to the residence time of the intersection, which includes the time when the vehicle doors open and close while passengers may alight and board. Thus,

$$t_s = n \times h \times \tau \times q_i + (n - 1) \times k \tag{12}$$

where τ is the time required per person to alight or board a bus, and k is the time for the opening and closing of doors.

The dwell time caused by traffic signals primarily occurs at signalized intersections, which is related to the mean of $(T - g)$. Therefore, the dwell time caused by traffic signals is denoted as follows:

$$t_w = 0.5 \times (T - g) \times \rho \times n_s \tag{13}$$

where n_s is the number of signalized intersections.

The unimpeded running time of vehicles is determined by

$$t_r = l/v - 0.5 \times t_{ad} \tag{14}$$

where v is the normal speed in the running time.

3.1.3. Walk Cost

The walk cost can be represented by the value of the walk time (ϕ_3) and the total walk time for all the passengers (t_3), which is related to total demand and the average walk time (t_a).

$$C_3 = \phi_3 \times t_3 \tag{15}$$

$$t_3 = q_s \times t_a \tag{16}$$

The average walk time is related to en-route access to the bus stop. For simplicity, the average walk time is a quarter of the stop spacing divided by the walk speed of passengers (v_p).

$$t_a = \frac{s}{4 \times v_p} \tag{17}$$

3.1.4. Operator Cost

The operator cost is related to the BRT headway, the round-trip BRT travel time (t_4), and the average vehicle operating cost (ϕ_4) [32]. The operator cost can be derived as follows:

$$C_o = \phi_4 \times t_4/h \tag{18}$$

The round-trip BRT travel time is as the product of the total in-vehicle time and the BRT headway. Thus,

$$t_4 = \begin{cases} t_2 + h & \text{if the BRT is loop line} \\ 2 \times (t_2 + h) & \text{otherwise} \end{cases} \tag{19}$$

3.2. Construction of a Bottom-Level Model

The bottom-level model investigates the impact of the BRT passenger flow attraction in relation to stop spacing (q_s). The coverage coefficient of the bus stops caused by stop spacing ($f_2(s)$) has a linear relationship with the attracted traffic under the hypothesis that the volume of traffic on a road is constant, $q_s = q_0 f_2(s)$, where q_0 denotes potential bus traffic and $f_2(s)$ has a value between 0 and 1. Therefore, in order to calculate the maximum impact coefficient of BRT bus station coverage, the optimal stop spacing model is established as follows:

$$\max f_2(s) \tag{20}$$

Some academics have found that the walking distance (d_w) of residents around a BRT line to stations determines the probability of choosing BRT, from which the coverage coefficient of bus stops is obtained [31]. Some studies show that residents within 300 meters of a bus station have a higher probability of taking the bus and are more stable, while the probability of taking the bus more than 300 meters gradually decreases [24,27,35,36]. BRT is one of the urban public transportation systems that has great advantages in attracting passengers. Therefore, assuming that passengers will choose BRT when the d_w ranges from 0 to 300 m, the probability of passengers choosing a BRT ($p(d_w)$) is a decreasing function when the d_w ranges between 300 to 700 m, expressed as the following:

$$p(d_w) = \begin{cases} 1 & 0 \leq d_w \leq 300m \\ -\frac{1}{400} d_w + \frac{7}{4} & 300m \leq d_w \leq 700m \end{cases} \tag{21}$$

According to formula (22), the service area of a BRT can be divided into two categories. Suppose D_1 represents an area within 300 m from a BRT stop and D_2 represents an area where the distance ranges between 300 and 700 m to a stop. When BRT stop spacing approaches a value of 0, the probability (p) of passengers in D_1 choosing BRT is 1 and the probability (p) of the passengers within D_2 choosing BRT is between 0 and 1. The probability of passengers choosing to travel by BRT is mathematically integrated in the heterogeneous region D_1 and D_2.

It is assumed that the service level between up-run and down-run direction is symmetrical to the BRT line. It is only possible to analyze the up-run direction of the line to produce robust calculations. The actual service area of the up-run direction of the line is C_{up}. When the stop spacing approaches a value of 0, the ideal stop service area is C_{id}, and includes the maximum values of D_1 ($D_{1\max}$) and D_2 ($D_{2\max}$).

The ratio of the integral value of the rapid transit station to the maximum value of the integral, as the coverage factor of the BRT stop has been quoted. For robust and convenient calculations, only the up-run direction of the line needs to be considered. Therefore, the formula obtained by integration is as follows [20]:

$$f_2(s) = \frac{C_{up}}{C_{id}} = \frac{\iint\limits_{D_1+D_2} p\Big(|x| + \big|y\big|\Big)dxdy}{\iint\limits_{D_{1\max}+D_{2\max}} p\Big(|x| + \big|y\big|\Big)dxdy} = \begin{cases} \frac{a_0+(n_t-1)\cdot a_1}{a_0+250l} s \leq 600m \\ \frac{a_0+(n_t-1)\cdot a_2}{a_0+250l} 600m < s \leq 1400m \\ \frac{n_t \cdot a_0}{a_0+250l} s > 1400m \end{cases} \tag{22}$$

a_0 is a non-repeated service area between adjacent stops; and a_1 and a_2 are repeat service areas between adjacent stops in an area where stop spacing is between 0–300 m and 300–700 m, respectively. a_0, a_1, and a_2 can be calculated separately as follows:

$$a_0 = \int_0^{300}\int_0^{\sqrt{300^2-x^2}} dxdy + \int_0^{700}\int_{\sqrt{300^2-x^2}}^{700-x} \frac{700-x-y}{400}dxdy \tag{23}$$

$$a_1 = \int_0^{s/2}\int_0^{\sqrt{300^2-x^2}} dxdy + \int_0^{s/2}\int_{\sqrt{300^2-x^2}}^{700-x} \frac{700-x-y}{400}dxdy \tag{24}$$

$$a_2 = \int_0^{300}\int_0^{\sqrt{300^2-x^2}} dxdy + \int_0^{s/2}\int_{\sqrt{300^2-x^2}}^{700-x} \frac{700-x-y}{400}dxdy \tag{25}$$

The ideal stop service area C_{id} is as expressed as follows:

$$C_{id} = \iint_{D_{1\max}+D_{2\max}} p(|x| + |y|)dxdy = 2a_0 + \int_0^{l}\int_0^{300} dxdy + \int_0^{l}\int_{300}^{700} \frac{700-\mathrm{y}}{400}dxdy \tag{26}$$

The minimum value of stop spacing ($s_{\min}$) should not be lower than the minimum value of BRT stop spacing (s_p) specified in the general specification and should not be lower than the range of attraction. This can be shown as follows: $s_{\min} = \min[700, s_p]$. The maximum value of stop spacing ($s_{\max}$) should not exceed the maximum s_p and twice the range of non-attraction. It can be shown as follows: $s_{\max} = \min[1400, 2 \times s_p]$. Assuming that the spacing is extremely wide apart, this will increase the walk times for passengers while the travelling time for passengers will increase if spacing is very close together as a result of stopping and frequent parking. Therefore, stop spacing must be within a reasonable range and as follows:

$$s_{\min} \le s \le s_{\max} \tag{27}$$

4. Case Study

A BRT operation loop line K1 of Chengdu has been determined and a circuit diagram is presented in Figure 1 below. The route of K1 with 23 intersections was 28.3 km, the number of stops is 29 and the average stop spacing is about 1 km. In 2017, the line K1 and reverse line K2 were equipped with 270 vehicles with a top speed of 45 km/h and an average operating speed of 25 km/h. The BRT headway is 40 s during peak passenger hours and the shortest BRT headway is 28 s. The average daily passenger load is 290,000 persons, with a maximum of 350,000 persons. For the identified BRT line K1, the variables and parameter values in the hypothetical model were calculated and are shown in Table 1.

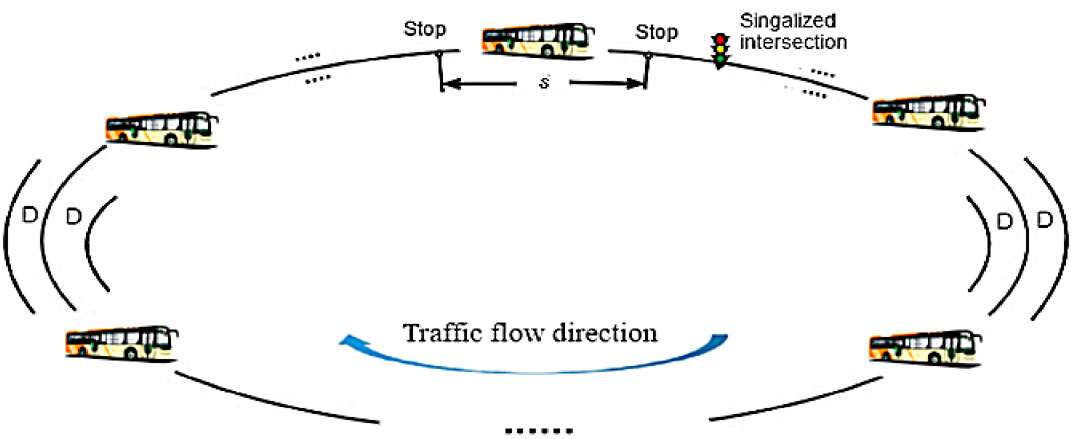

Figure 1. BRT line schematic.

Table 1. Variables and parameters.

Symbol	Definitions	Unit	Value
f_1	Equivalent cost (objective value of the top-level model)	¥	
C_u	User cost	¥	
C_o	Operator cost	¥	
C_1	Wait cost	¥	
C_2	In-vehicle cost	¥	
C_3	Walk cost	¥	
ϕ_1	Value of wait time	¥/s	0.2
ϕ_2	Value of in-vehicle time	¥/s	0.1
ϕ_3	Value of walk time	¥/s	0.2
ϕ_4	Average vehicle operating cost	¥/(vehicle-s)	0.5
t_1	Total wait time for all the passengers	s	
t_2	Total in-vehicle time	s	
t_3	Total walk time for all the passengers	s	
t_4	Round-trip BRT travel time	s	
t_{ad}	Time for the vehicle acceleration and deceleration	s	
t_s	Dwell time at the stop station	s	
t_w	Dwell time caused by traffic signals	s	
t_r	Unimpeded running time of vehicles	s	
n	Total number of stops	stops	
q_i	BRT passenger flow at stop i	persons	
t_i	Average wait time at stop i	s	
q_s	BRT demand attraction in relation to stop spacing	persons	
l	BRT route length	km	28.3
s	BRT stop spacing	m	
t_i	Average wait time	s	
h	BRT headway	s	
n_c	Number of signalized intersections which are located at a considerable distance from stops, and the spacing is at least at a distance of 50 m	intersections	
ρ	Possibility that vehicles adequately stop for red lights at intersections		0.5
T	Average cycle time of signalized intersection	s	80
g	Average cycle time of signalized intersection	s	40
v_p	Walk speed of passengers	m/s	1.2
a	Average value of acceleration	m/s^2	0.6
d	Average value of deceleration	m/s^2	1
v	Normal speed	m/s	
v_s	Speed of stops	m/s	
$v_{\max}$	Maximum recommended and permissible speed	m/s	60
x_a	Distance travelled during acceleration of the BRT vehicle	m	
x_d	Distance travelled during deceleration of the BRT vehicle	m	

Table 1. *Cont.*

Symbol	Definitions	Unit	Value
τ	Time required per person to alight or board a bus	s	3
k	Time for the opening and closing of doors	s	3
n_s	Number of signalized intersections	intersections	23
$f_2(s)$	The coverage coefficient of bus stops caused by stop spacing		
q_0	Potential bus traffic	persons	320,000
d_w	Walking distance	m	
$p(d_w)$	Probability of passengers choosing a BRT		
D_1	Area within 300 m from a BRT stop	m^2	
D_2	Area where the distance ranges between 300 and 700 m to a stop	m^2	
p	Probability of passengers in D_1 and D_2 choosing BRT		
C_{up}	Actual service area of the up-run direction of the line	m^2	
C_{id}	Ideal stop service area	m^2	
$D_{1\max}$	Maximum value of D_1	m^2	
$D_{2\max}$	Maximum value of D_2	m^2	
a_0	Non-repeated service area between adjacent stops	m^2	
a_1	Repeat service areas between adjacent stops in an area where stop spacing is between 0 to 300 m	m^2	
a_2	Repeat service areas between adjacent stops in an area where stop spacing is between 300 to 700 m	m^2	
$s_{\min}$	Minimum value of stop spacing	m	
$s_{\max}$	Maximum value of stop spacing	m	
s_p	BRT stop spacing	m	

4.1. Model Verification

Each parameter value was substituted into the case study in the program using the genetic algorithm to solve the proposed model. According to the model's solution needs, the iterations of the genetic algorithm were set as 19. To better verify the proposed model, the BRT headway and normal speed were 40 s and 45 km/h, respectively, which were taken from current K1 line data; the different values of stop spacing were considered ranging from 300 to 2100 m with a step of 100 m, and the substitution value of n_c varied directly as the stop spacing.

Figure 2 shows that the changes in the solution of the equivalent cost and coverage coefficient of bus stops, which indicates that the algorithm was guaranteed to be convergent. Hence, the change trend of the equivalent cost and coverage coefficient of bus stops in the solution can reveal the effectiveness of the up-level model and the bottom-level model in the proposed model, respectively.

Figure 3 shows the relationships between different costs (in-vehicle, walk, operator, and equivalent costs) and the bus stop spacing. It can be seen that with an increase in stop spacing, the wait cost remains unchanged, the walk cost increases rapidly, and the in-vehicle and operator costs both decrease gradually. These findings correspond with the anticipated results based on the proposed model formula, and are in accordance with the factual operational status of BRT. Furthermore, when the stop spacing increases, the equivalent cost reduces quickly and increases slowly within the stop spacing range of 300–800 m and 800–2100 m, respectively. This indicates that the impact on equivalent costs is greater when stop spacing is closer together. In addition, the optimal stop spacing is 800 to 900 m (the minimum value of equivalent costs) and nearly equal to the current stop spacing of the line KI.

To summarize, the proposed model applied on the bus stop spacing problem of BRT can obtain a reasonable optimal solution.

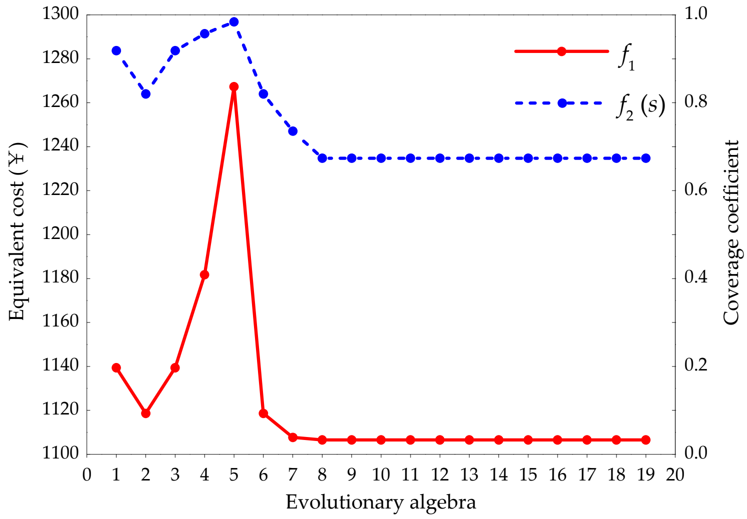

Figure 2. Iterative trend curve.

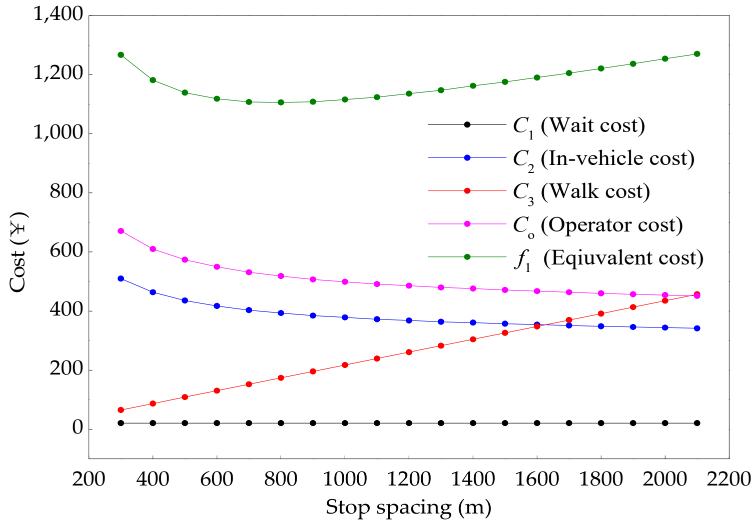

Figure 3. Different costs versus stop spacing.

4.2. Sensitivity Analysis

In this section, we analyzed the sensitivity of the coverage coefficient of bus stops, BRT headway, and normal speed. While the headway and speed changed, the trends of different costs (including wait, in-vehicle, walk, operator, and equivalent costs) almost remained unchanged with the increase of stop spacing. Therefore, we only discussed the equivalent costs in sensitivity analysis of BRT headway and normal speed.

4.2.1. Coverage Coefficient of Bus Stops

Figure 4 shows that the coverage coefficient of bus stops decreases as the bus stop spacing increases, and this relationship is inversely proportional. However, to ensure that BRT stops serve more passengers, simply shortening stop spacing is unrealistic. With reference to the trend of the coverage coefficient in Figure 2, the optimal stop spacing is not the minimum value because interests of bus operators must also be considered while sparing no effort to serve more passengers at a lower cost.

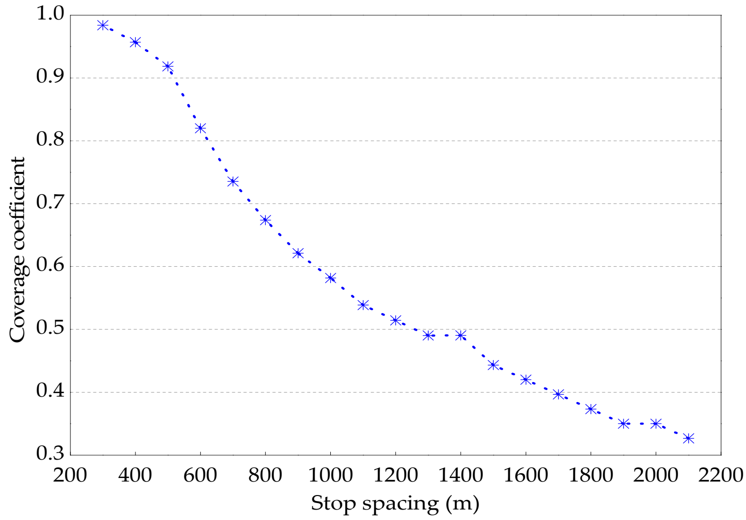

Figure 4. Coverage coefficient of stop spacing.

4.2.2. BRT Headway

Based on the actual headway of the line KI (h = 40 s), different headway values (30, 35, 40, 45, and 50 s) were extracted to study the sensitivity of BRT headway. Figure 5 shows that the larger the headway is, the less the equivalent cost is, which indicates that the operator cost has a decisive effect on the equivalent cost as the headway value is changed. As the headway values is increasing, the optimal stop spacing is decreasing (30 s is 900–1000 m, 35 s and 40 s are 800–900 m, and 45 s and 50 s are 700–800 m). Likewise, to reduce the operator cost (i.e., increase the headway value), simply shortening stop spacing is unrealistic, because the optimal stop spacing is also decided by the wait cost and other factors (e.g., regional road planning and passenger flow distribution).

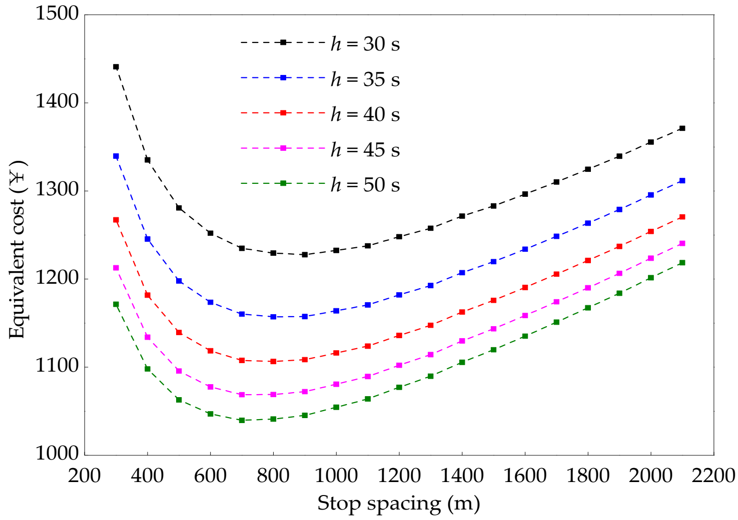

Figure 5. Equivalent cost versus stop spacing with different BRT headways.

4.2.3. Normal Speed

Different normal speed values (35, 40, 45, 50, and 55 km/h) were extracted to study the sensitivity of normal speed according to the actual running speed of the line KI (v = 45 km/h). Figure 6 shows that the larger the speed is, the less the equivalent cost is, which indicates that the in-vehicle cost plays an important role in the equivalent cost as the speed value is changed. When the normal speed was set to different values, there is a similar optimal stop spacing (800–900 m). Beyond all doubt, the increase of the speed can improve the operation state of BRT, but it is limited to the road safety, vehicle performance, and passenger comfort, which is another challenge for researchers and planners who are interested in optimization problems of BRT.

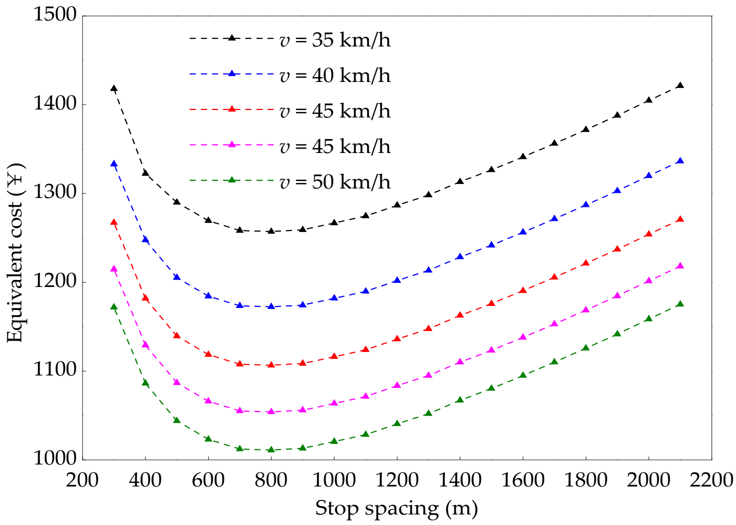

Figure 6. Equivalent cost versus stop spacing with different normal speeds.

5. Conclusions

The purpose of this study is to create a bi-level programming model for the optimal bus stop spacing of a BRT system, in order to ensure the simultaneous coordination and consider the interests of bus companies and passengers.

Firstly, this paper summarized previous studies and analyzed the advantages and disadvantages of their findings. Then, a bi-level programming model, including the top-level model and micro-level model, was established to obtain the optimal stop spacing for a BRT system to realize the lowest cost for operators and provide maximum benefit to passengers. In the top-level model, for considering travel cost of passengers, a desired objective is presented to minimize the weighted sum of running time of the line, the time at the stop station, and the construction costs of the stop. In the micro-level model, another objective is presented to maximize the coverage coefficient of stops to make BRT services meet more passengers. Finally, the case study of BRT in Chengdu is presented to highlight the performance and benefits of the proposed model, where we have analyzed the relationship and sensitivity between model parameters and optimal stop spacing.

Compared with other models, based on the premise of stop spacing and stop selection, the factors affecting the determination of stop spacing have been fully considered. During this process, equivalent costs are introduced to simplify the calculation process. This also introduced a new variable, the coverage of stop service scope, to fully ensure that the interests of passengers have been considered, thereby increasing the practical value of the BRT optimal stop spacing model.

However, BRT stop spacing optimization involves a number of complicated factors, and the method proposed in this paper is primarily from a theoretical perspective. Although the model considers the interests of bus operators and passengers, its objective function is selective as there are many influencing factors. In some aspects, there are some inevitable drawbacks, whereby the model is only suitable for studying the influence of certain parameters on stop spacing. This reduces the scope of the model and its research value. Therefore, we will further our research from two aspects in future work:

The developed model can be enhanced as another stop spacing optimization problem by considering the alternative lines of normal bus transit and metro. With that, the stop site and spacing can be influenced to avoid the repeated stops, which is already in increasing accordance with the truth.

The proposed model should be applied in more BRT lines of different cities. With that, the related parameters can be better calibrated, which can enhance the practical application value of the proposed model.

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