mathematics

Correction

Correction: Wang, M. and Yin, S. Some Liouville Theorems on Finsler Manifolds. *Mathematics*, 2019, 7, 351

Songting Yin [1,*] and Minqiu Wang [2]

1 Department of Mathematics and Computer Science, Tongling University, Tongling 244000, China
2 Department of Mathematics and Physics, Hefei University, Hefei 230601, China
* Correspondence: yst419@163.com

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The authors are sorry to report that the proof of case III of Theorem 1.2 in their recently published paper [1] was incorrect. Upon revising the manuscript, they mistakenly thought that u_k is in H^2 and $\Delta u_k = 0$ holds on some particular subset of the manifold M. Consequently, the authors wish to make the following corrections to the paper at this time:

We first remark that the proof of case III is different from the one in [2] because there is a mistake there. For every $k \in \mathbb{R}^+$, set

$$u_k = \begin{cases} k, & u \geq k; \\ u, & u < k. \end{cases}$$

In what follows, we will follow the arguments in [3] (p. 178) with some modifications. Let β be a symmetric, convex, and bounded smooth function with $|\beta'| < 1$ and $|s| < \beta < \epsilon + |s|$, where $0 < \epsilon < 1$ is such that $u - \epsilon > 0$. Define

$$\tilde{u}_k = \frac{u+k}{2} - \frac{\beta(u-k)}{2}.$$

Then, for any positive integer k, it holds that $\tilde{u}_k > \frac{u+k}{2} - \frac{\epsilon+|u-k|}{2} > 0$. Moreover, $\tilde{u}_k$ is a superharmonic function in a weak sense. Indeed, by definition, we have $d\tilde{u}_k = \frac{1}{2}(1-\beta')du$, which yields $\nabla\tilde{u}_k = \frac{1}{2}(1-\beta')\nabla u$ by Legendre transformation. As $\tilde{u}_k \in H^2_{loc}$ and thus $\Delta\tilde{u}_k = 0$ a.e. on $M \backslash M_u$, for ψ defined in Case I, we have

$$\begin{aligned} 2\int_{B^-_{x_0}(R)} \psi\Delta\tilde{u}_k d\mu &= -2\int_{B^-_{x_0}(R)} d\psi(\nabla\tilde{u}_k)d\mu = -\int_{B^-_{x_0}(R)} (1-\beta')d\psi(\nabla u)d\mu \\ &= -\int_{B^-_{x_0}(R)} d[(1-\beta')\psi](\nabla u)d\mu - \int_{B^-_{x_0}(R)} \beta''\psi F(\nabla u)^2 d\mu \\ &\leq -\int_{B^-_{x_0}(R)} d[(1-\beta')\psi](\nabla u)d\mu = \int_{B^-_{x_0}(R)} (1-\beta')\psi\Delta u d\mu \\ &\leq 0. \end{aligned}$$

The last step holds because $(1-\beta')\psi$ is differentiable almost everywhere on $B^-_{x_0}(R)$ with bounded differential, and u is superharmonic. Moreover, $\tilde{u}_k$ is smooth on the open subset M_u and is also superharmonic, in the classical sense, on M_u. Notice that ψ is differentiable almost everywhere on

$B^-_{x_0}(R)$ with bounded differential. Hence, by similar arguments, we can also obtain (4) (see [1], p. 6) for $\tilde{u}_k$ on $B^-_{x_0}(R)$ as in case I. Set $v_k = \tilde{u}_k^{\frac{q}{2}}$ for any $q \in (0,1)$. Then we have (5) (see [1], p. 7) as follows:

$$
\begin{aligned}
&(1-\frac{1}{q})^2 \int_{B^-_{x_0}(R)} \psi^2 F(\nabla v_k)^2 d\mu \\
\leq &\widehat{C} \left(\int_{B^-_{x_0}(R)\setminus \bar{B}^-_{x_0}(r_0)} v_k^2 \right)^{\frac{1}{2}} \left((1-\frac{1}{q})^2 \int_{B^-_{x_0}(R)\setminus \bar{B}^-_{x_0}(r_0)} \psi^2 F(\nabla v_k)^2 d\mu \right)^{\frac{1}{2}} \\
= &\widehat{C} (V_q(R) - V_q(r_0))^{\frac{1}{2}} \left((1-\frac{1}{q})^2 \int_{B^-_{x_0}(R)\setminus \bar{B}^-_{x_0}(r_0)} \psi^2 F(\nabla v_k)^2 d\mu \right)^{\frac{1}{2}}.
\end{aligned}
$$

On the other hand, note that $\tilde{u}_k \leq k$, and thus

$$
\int_{B^-_{x_0}(R)} \tilde{u}_k^q d\mu \leq \int_{B^-_{x_0}(R)} k^q d\mu = k^q V(R),
$$

which implies that

$$
\int_1^\infty \frac{r}{V_q(r)} dr = \infty.
$$

Then by the same discussion in the proof of (2) (see [1], pp. 5–7) and Case I of (1) (see [1], p. 7), we show that this $\tilde{u}_k$ is constant. Take then a sequence β_n (such that each β_n satisfies the same properties as β) uniformly converging to the absolute value function. Every $\tilde{u}_{k,n}$ is then constant. These constants are bounded (they are in $(0,k)$). Thus, up to pass to a subsequence $\tilde{u}_{k,n}$ converges uniformly to u_k and to a constant at the same time. Hence, u_k must be constant. k being arbitrary, u is also constant.

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