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Stochastic-Petri Net Modeling and Optimization for Outdoor Patients in Building Sustainable Healthcare System Considering Staff Absenteeism

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Abstract: Sustainable healthcare systems are gaining more importance in the era of globalization. The efficient planning with sustainable resources in healthcare systems is necessary for the patient's satisfaction. The proposed research considers performance improvement along with future sustainability. The main objective of this study is to minimize the queue of patients and required resources in a healthcare unit with the consideration of staff absenteeism. It is a resource-planning model with staff absenteeism and operational utilization. Petri nets have been integrated with a mixed integer nonlinear programming model (MINLP) to form a new approach that is used as a solution method to the problem. The Petri net is the combination of graphical, mathematical technique, and simulation for visualizing and optimization of a system having both continuous and discrete characteristics. In this research study, two cases of resource planning have been presented. The first case considers the planning without absenteeism and the second incorporates planning with the absenteeism factor. The comparison of both cases showed that planning with the absenteeism factor improved the performance of healthcare systems in terms of the reduced queue of patients and improved operational sustainability.

Keywords: stochastic-petri net modeling; sustainable healthcare system; staff absenteeism; patient's queue; mixed integer nonlinear programming

1. Introduction

Sustainability was first introduced by Brundtland in 1987 [1] and it deals with economic, environmental, and social dimensions [2–6]. Efficient healthcare systems ensure social and economic sustainability. The management of healthcare systems involves the management of operations within the facility. Healthcare organizations should encourage social sustainability to set values for corporate social responsibility. To attain this, they should hunt for a balance between patient comfort, care, and economic need to guarantee sustainable development [7]. The patient's satisfaction and comfort are considered as a primary social indicator in the healthcare industry. The rapid increase in diseases has diverged the direction of researchers to sustainable healthcare systems. Promising innovations and sustainable operations are necessary for the healthcare system to achieve the patient's satisfaction level [8]. A healthcare system may be in a hospital or clinic or some temporary medical camp.

Patients visit hospitals for their treatment, but the limited resources and absence of doctors, dispensers, and nurses at healthcare systems creates issues. The severity of disease also varies from patient to patient. Therefore, sometimes, longer wait times in queue results in death. In order to avoid such problems, there should be some planning of resources by building sustainable healthcare systems for a patient's satisfaction. Since this comfort and satisfaction is a basic social aspect and pillar of sustainability. The planning of resources of the healthcare system is extensively reported in the literature [9–13].

Zeinali, et al. [14] addressed the resource planning model in the emergency department. The resources included were nurses, receptionists, and beds. However, they did not consider the physician, which are a core requirement of any healthcare system. In addition, they used a simulation model and performed a what-if analysis for resource management. However, the proposed model addresses the management of resources with the consideration of their absenteeism probability. Yousefi, et al. [15] focused the resource planning in the emergency department using a genetic algorithm. However, the outdoor patient department is not well focused. Feng, et al. [16] studied medical resource allocation in a healthcare system. They used a simulation integrated multi-objective algorithm for the allocation of resources such as beds, nurses, and other medical staff. The processes in healthcare centers are both discrete and continuous. However, simulation models only discrete events and system dynamic models focus on the continuous system. The proposed research integrates the Petri net with mixed integer nonlinear programming for modeling both discrete and continuous states in the outdoor patient department, which has not been well addressed in previous literature.

Most of the researchers have presented some static mathematical models for the planning of healthcare systems and optimized linear programming or heuristics [17–19]. However, these models might not be suitable for dynamic systems due to randomness in patient arrival, the nature of the disease, and treatment time of patients. Hence, healthcare systems should be efficient, sustainable, and flexible to incorporate the staff absenteeism factor. The dynamics of the healthcare system is modeled with either system dynamic (SD) approaches or discrete event simulation (DES). The objects in DES are different entities and discrete, while, in DS, are merged in a population to form a continuous quantity with a continuous state change [20].

A healthcare system has both characteristics DES and SD such as a number of patients in a system are discrete. However, the state change in a resource is a system dynamic. For example, a physician takes different treatment times for different patients, which is a continuous variable. SD methods are good for dynamic complexity, which makes it useful for strategic decision making and DES focuses on the detailed complexity of operational decision-making [21]. The Petri nets introduced by Adam Carl Petri (1962) combines the qualities of DES and SD. In Petri net modeling, the events, event calendar, entities, workflow, and constraints (decisional, operational) are presented in a DES pattern with a strong mathematical background [22]. The discrete event simulation (DES) has been widely used for the performance evaluation of healthcare systems such as utilization of resources, waiting time for patients, patient flow, and patient safety [23–27].

There are two types of resources in the healthcare system. One is an infrastructure of healthcare systems such as a number of beds, statures, and surgery equipment and other are staff members, doctor, nurses, and clerks. This research considers the human resource management, even though some researchers have considered the number of staff personnel required for patient's satisfaction [14,15,28]. Yalçındağ, et al. [29] addressed the planning of surgeons in the healthcare department but did not consider the absenteeism factor. Similarly, En-nahli, et al. [30] optimized the resources in the emergency department without staff absenteeism. However, this research focuses the resource management on the outdoor patient department for patient queue reduction and satisfaction. It was found that the required staff might not be available or on duty to meet the patient service. The absenteeism of staff may result in a long queue for the patient.

The concept of resource planning with absenteeism of staff has not received proper attention. Thus, the viability of such a healthcare system is necessary for sustainability and growth. The mixed integer nonlinear programming-based Petri net have been introduced to model this situation because of its discrete and continuous nature. Petri net itself combine the properties of simulation and mathematical models and it is possible to do what-if analysis. It is a time-consuming process and does not guarantee an optimal solution. However, in this research integration of Petri net with mixed integer nonlinear programming, reduces the computational time and iterations and gives optimal results. The major advantage of using mixed integer nonlinear programming Petri net model is reduced computational time, improved visualization of the flow of entities (patients), and its capability to find the best solution. The proposed mathematical solution and model approaches are useful for the sustainability of healthcare systems. The incorporation of Stochastic-Petri net approach with MINLP for real-time optimization in the scheduling model will enable the attainment of sustainable operational efficiency. The objective of the proposed model is to reduce the cycle time for the patient's satisfaction under the factor of absenteeism. In order to minimize cycle time and improve customer satisfaction, patient queues and resources should be minimized. The rest of the paper is organized as follows. The detailed literature review is in the second section. The third section includes the development of the model. The solution methodology of the research is represented in detail form as in the fourth section. A numerical example of a healthcare center is presented in the fifth section as an application of the proposed model.

2. Literature Review

The two major categories of the industries can be divided into manufacturing and service industry. Researchers are working to enhance the production, quality, and cycle time of the manufacturing firm to support the manufacturers, customers and government [31–33]. However, healthcare systems are essential for the population to provide services. Therefore, management and control is inevitable. Many operational and managerial strategies have been introduced to plan and control the healthcare systems. The planning of the resources especially human resources is critical because of the direct interaction with the patients. Healthcare models developed can be classified into four types including static models, dynamic models, simulation models, and Petri net models.

2.1. Static System Models

Bachouch, Guinet and Hajri-Gabouj [17] addressed the static strategic enterprise-resource-planning model for the healthcare systems and developed a multi-objective goal-programming model with finance, labor, revenue, capacity, and admission resources. Hans, Van Houdenhoven and Hulshof [11] proposed the scheduling model for healthcare systems for minimizing idle time of nurses in a week. Ghazalbash, et al. [34] developed a mathematical model for the scheduling of the operating rooms of teaching hospitals with the objectives of minimizing operation time and operating room idle time. Nasir and Dang [35] solved a flexible home health care routing and scheduling problem with the joint patient and nursing staff selection. Tan, et al. [36] presented a two-stage multi-objective mathematical model for the planning and scheduling of operating rooms considering the patient flow. This is clear from the literature that the static models have mostly focused the operating rooms, nurses, and operating equipment while neglecting the major functional department of healthcare systems such as an outdoor patient ward, where many patients visit on a daily basis. There are various reasons for modeling this department and the most prominent is the daily arrival of patients in healthcare systems. The static mathematical model does not provide the clarity of the dynamic behavior of the systems.

2.2. System Dynamic Models

Since the behavior of healthcare systems is dynamic and this factor should be taken into account while modeling the healthcare systems. Lane, Monefeldt and Rosenhead [13] provided the dynamic model for the accident and emergency department and planned the resources such as beds and the hospital process by minimizing the delay time of the patients. Carayon, et al. [37] improved the quality and patient safety by introducing the system engineering initiative for the patient safety (SEIPS) model, which analyzes the behavior of workers in human factor contexts. Atun [38] discussed the dynamic complexity of healthcare systems by considering the dynamic interactions between innovation and institutions. Fraher, et al. [39] developed the projection model for the forecasting of surgeons for healthcare systems for 2009 to 2028 with consideration of specialty, sex, and age to tackle the dynamic change in surgery complexity. There is plenty of literature on the system dynamic behavior of the healthcare system. The shorter review of dynamic modeling in healthcare is available in References [10,40]. The system dynamic models are helpful when a change in the state or occurrence of the event is continuous. However, it is not necessary that events are continuous in nature. The nature of events might be discrete, too. This limitation of dynamic models is not to consider the discrete change.

2.3. Discrete Event Simulation Models

A simulation is a tool for performance evaluation, planning, and scheduling of the healthcare systems because of its ability to model the discrete events. Rohleder, Lewkonia, Bischak, Duffy and Hendijani [27] reported the DES model for the performance improvement of the orthopedic outpatient clinic. The objective of their research was to minimize the waiting time of patients. Robinson, Radnor, Burgess and Worthington [26] introduced the concept of SimLean in healthcare, which is the process of performance improvement. It is a helpful tool for the planning and scheduling of healthcare systems.

Abo-Hamad and Arisha [41] presented an interactive program with simulation-based decision support systems for the optimization of the healthcare systems for optimum resource utilization. Zeltyn, et al. [42] focused the workforce staffing problems in the emergency department and developed the simulation model to evaluate different alternatives of the emergency department for customer satisfaction. The use of simulation models for the scheduling or planning of the healthcare systems is reported in the literature and some details can be found in the models of References [43–46].

2.4. Petri-Net Models

The Petri nets are a combination of dynamic modeling and simulation incorporating visualization with the help of graphs. These models deal with both discrete and continuous quantities. The planning and scheduling of the healthcare system involve both discrete and continuous variables. The number of patients, the number of physicians, and a number of nurses have a discrete nature. The examination time, medication time, and waiting time are the example of continuous variables. However, some mathematical models have been employed for scheduling they did not consider the dynamics of both discrete and continuous variables [47]. The processes within a healthcare center are random or stochastic. Therefore, in this research, stochastic Petri net modeling is chosen for realistic results.

There are very few articles on healthcare planning and scheduling using Petri nets. Mahulea, et al. [48] modeled the healthcare system by using Petri nets and evaluated the performance measure, but they did not optimize this process for customer satisfaction. Bertolini, et al. [49] analyzed the blood transfusion with component-based timed-arc Petri nets and modeled the healthcare workflow. Hicheur, et al. [50] modeled the flexible healthcare systems by using recursive and workflow Petri nets and evaluated the performance of the healthcare system by analyzing the medical properties of healthcare processes. In all of the above cases, Petri net have been used for flow modeling and process visualization and do not give us any information when the maximum number of patients will be served. However, proposed research not only models the healthcare system using a Petri net but also optimizes the process. In this regard, Petri nets have been integrated with a mixed integer nonlinear programming model that has been utilized to maximize customer satisfaction by reducing the cycle time.

Wang [51] modeled the emergency response in the emergency department by using Petri nets. The proposed methodology considered the stochastic Petri net modeling and optimization for outdoor patients in healthcare systems and the aim is to satisfy each patient with minimum waiting time while considering the staff (doctors, nurses, or cleric) unavailability. The concept of planning and scheduling the resources with consideration of staff absenteeism has not been paid proper attention.

3. Petri Nets Modeling of the Healthcare System

The Petri nets models are the graphical, simulation, and mathematical representation of the system [52]. A typical Petri net model is a bipartite directed graph, which consists of four elements called places, transitions, arcs, and token. Places and transitions are connected with each other with the help of the directed arc [53]. Figure 1 shows a basic Petri net graph and definition of each element in the Petri net is given below.

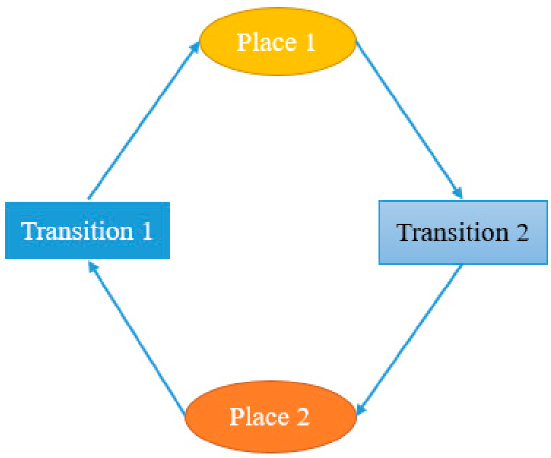

Figure 1. A Petri net graph.

a. Place

A place in the Petri net is a position with two states. A token is a process or event that occurs in place, so a place has a token or not. In a healthcare example, a physician is in a place with two states, either he/she sits idle or examines the patient. Places in Petri nets define discrete variables.

b. Transition

Transition shows the activity or process and its duration within the model. A transition represents continuous variables within the process. For example, patient examination time of the physician is a transition.

c. Arcs

Arcs are used to direct and connect the transitions with places. The sequence of a visit from registration to the medication is an example of art that shows the process plan.

d. Tokens

Tokens are the entities moving along arcs within transitions and places in a Petri net graph. The number of patients within the healthcare center are the example of tokens. The event in Petri net can be defined by introducing a transition between input and output places. The dynamic behavior analysis of Petri net involves availability or unavailability of tokens in the input or output places. If the token is available, then the condition associated with the place is true. Otherwise, it is false.

Definition 1. *A Petri net can be defined with five-tuple* $M = (\rho, \tau, I, O, \mu,)$, *where*

(1) $\rho = \{\sigma_1, \sigma_2, \ldots\ldots \sigma_m\}$ *is the finite set of places*
(2) $\tau = \{t_1, t_2, \ldots\ldots t_n\}$ *is the finite set of transitions* $\rho \cup \tau \neq \varnothing$, *and* $\rho \cap \tau = \varnothing$

(3) $I : \rho \times \tau \rightarrow M$ *is a function, which acts as an input for defining a directed arc between places and transitions and "M" is set to positive integers.*
(4) $O : \tau \times \rho \rightarrow M$ *is a function, which acts as an output for defining a directed arc between places.*
(5) $\mu : \rho \rightarrow M$ *is called an initial marking.*

The assignment of the token to the places in Petri net is called marking. The position of token keeps on changing, according to the input and output directing arcs in Petri net modeling [54].

3.1. Petri Net Graphical Representation

The Petri net models can be explained well with the help of the graph, which is a bipartite directed multigraph. There are two types of nodes in Petri net graphs. A place node, which is usually represented by a circle and transition is represented by a bar or box. To understand the graphical illustration of the Petri net considers a healthcare center. There are two types of outdoor customers or patients. The patients arrive in the healthcare system, register themselves for treatment at the registration desk, and move to the next station where physicians examine them. The doctor completes the examination of the patient. He/she is sent to a dispenser for medicines. Table 1 shows the process plan for patient treatment in a healthcare center.

Table 1. Outdoor patient treatment process plan.

	Registration	**Examination**	**Medication**
$i = 1$	t_{11}	t_{12}	t_{13}
$i = 2$	t_{22}	t_{22}	t_{23}

The bipartite directed multigraph for the Petri net model can be developed using the process plan of patient treatment, as shown in Figure 2, which is built using the concept of Figure 1 and process plan in Table 1. It is clear from Figure 2 that there are two types of overlapping circuits called the patient circuit and sequencing circuits. The distribution and number of tokens in net control of its execution. The number of tokens in the patient circuit is equal to the number of patients waiting for service. The sequencing circuit is used for the representation of servers. Figure 2 is the graphical representation of the flow of entities such as patients in a healthcare system. Rectangles in a blue color show the transitions that are used for assigning times to processes. However, places in red circles are used to represent resources. Green-colored and orange-colored tokens represent two different types of patients.

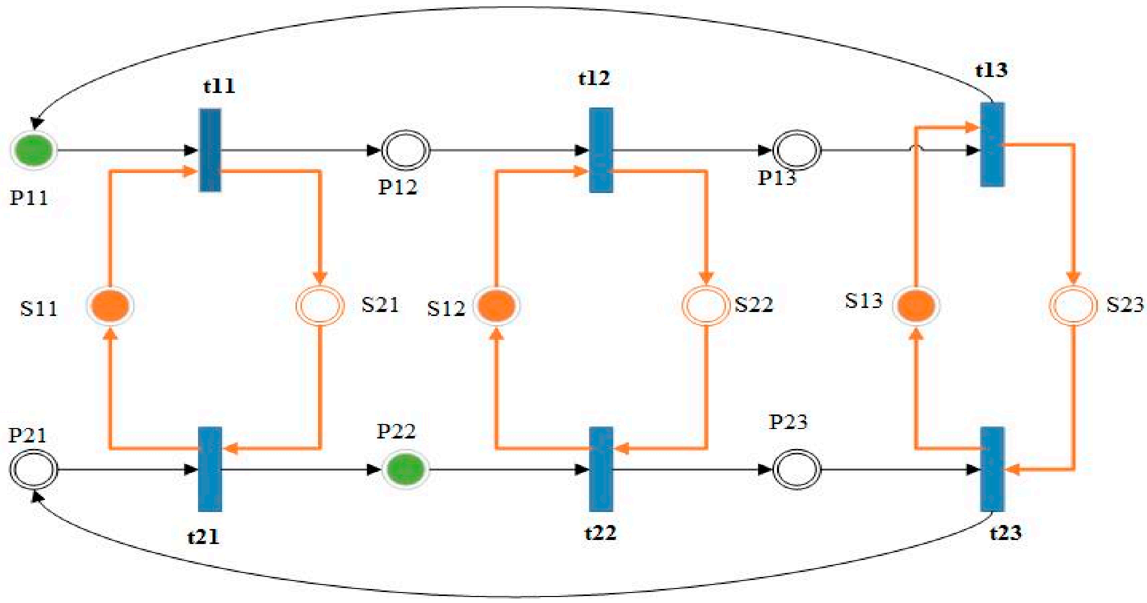

Figure 2. Petri net model of the healthcare system with a process plan.

3.1.1. Model Assumptions

To develop a mathematical model, the following assumptions must be taken into account.

1. The service times are stochastic.
2. One server can serve only one patient at a time.
3. Transitions and places cannot be together so, during any loop, this rule should not be violated.
4. Server interruption by another patient is not allowed when a server is in service of another patient.
5. The patient keeps arriving at the healthcare system randomly.

3.1.2. Mathematical Model

In the sequence loop, the number of tokens is invariants. The total transition time $\tau(\zeta)$ is equal to the sum of transition firing times in a sequence circuit. t_{ij} is the time taken by the server j to serve patient i.

$$\tau(\zeta) = \sum_{i=1}^{I}\sum_{j=1}^{J} t_{ij} \tag{1}$$

The total sum of tokens in a sequential loop or server loop $M(\zeta)$ is in Equation (2). In this case, S_{ij} shows a place of server j for serving patient i.

$$M(\zeta) = \sum_{j=1}^{J} S_{ij}\ \forall_i \tag{2}$$

Aziz, et al. [55] computed cycle time of each sequential circuit $C(\zeta)$ using Equation (3).

$$C(\zeta) = \frac{\tau(\zeta)}{M(\zeta)} \tag{3}$$

However, this cycle time does not consider the absenteeism of the server from the service stations. If there is some probability of absenteeism, then Equation (3) becomes PA, which represents the probability of absenteeism.

$$C(\zeta) = \frac{\tau(\zeta)}{PA \times M(\zeta)} \tag{4}$$

$\tau(\zeta_s)$ is the total service time in one service cycle by only one server to provide the service to a single patient. If the number of the server at each service station is known, then the service rate SR can be calculated using Equation (5).

$$SR = \frac{1}{C(\zeta)} \tag{5}$$

$$SR \times \tau(\zeta_s) = \frac{\tau(\zeta_s)}{C(\zeta_s)} \text{ and } M(\zeta_s) \geq \frac{\tau(\zeta_s)}{C(\zeta_s)} \tag{6}$$

After a finite transition, the system starts to operate at a steady state. As the system achieves the steady state, the service cycle time becomes equal to the maximum cycle time of the servers. The service cycle time reduces with the introduction of more tokens. The number of tokens in the server circuit is normally kept fixed per the capacity of healthcare centers. However, the number of patient tokens depends upon the arrival of patients and planning horizon. Equation (7) shows the required cycle time of the patient circuit $C(R\zeta_p)$,

$$C(R\zeta_p) = \frac{Planning\ horizon}{Number\ of\ patients} = \frac{PH}{NP}, \tag{7}$$

while

$$M(\zeta_s) \geq \frac{\tau(\zeta_s)}{C(R\zeta_p)} \tag{8}$$

Bottleneck server cycle time $C(B\zeta_s)$ and $C(R\zeta_p)$ have two possible cases as follows:

1. $C(B\zeta_s) \geq C(R\zeta_p)$
2. $C(B\zeta_s) < C(R\zeta_p)$

The performance of the system will be influenced because of the maximum number of bottleneck server cycle times $C(B\zeta_s)$ and required cycle times for the patient circuit $C(R\zeta_p)$ [56].

$$C(\zeta_{sMAX}) = Max\left\{C(B\zeta_s), C(R\zeta_p)\right\} \tag{9}$$

By considering the server bottleneck circuit as well as cycle time as the maximum cycle time, then the Petri net model is formulated as a mixed integer nonlinear programming problem [57].

$$Minimize \sum_{i=1}^{I}\sum_{j=1}^{j} P_{ij} \tag{10}$$

Subject to

$$C(\zeta) \leq C(\zeta_{sMAX})\ with\ C(\zeta) > C(R\zeta_p) \tag{11}$$

Equation (12) does not consider the absenteeism while Equation (13) considers it.

$$\tau(\zeta) = C(\zeta_{sMAX})\ \times M(\zeta) \tag{12}$$

$$\tau(\zeta) = C(\zeta_{sMAX}) \times PA \times M(\zeta) \tag{13}$$

$$C(\zeta_p) \leq C(R\zeta_p)\ \forall\ \zeta_p \quad \text{with } C(\zeta_p) > C(R\zeta_p) \tag{14}$$

$$\forall\ P_{ij}\ ,\ S_{ij}\quad \text{Integer} \ \geq 0$$

The objective function of this model is to minimize the service cycle time. Equation (10) is the objective function. The constraint in Equation (11) ensures that the cycle must be within critical limits defined by the bottleneck cycle time or required cycle time for the patient loop. Equation (12) considers the effect of critical or maximum cycle time on total transition time without considering the absenteeism of the server. However, Equation (13) takes this factor into account. Equation (14) shows patient service achievement.

4. Solution Methodology

The method explained in Section 3 has been explained with the help of a numerical example of a hypothetical healthcare center.

4.1. Numerical Example

The numerical example of the proposed model is demonstrated as follows.

4.1.1. Problem Statement

Consider a healthcare center that provides services to the patients. Patients visiting the healthcare center have to process three steps including registration, medical examination, and medication. There are two servers at the registration boot, three physicians for the treatment of patients, and two servers in the dispensary. There are three possibilities due to the absenteeism of servers.

1. The server at the registration booth may be absent and this situation results in the long queue of patients at the registration booth.
2. The absenteeism of physicians may result in the long queue of patients and the station becomes a bottleneck.
3. If any of the servers from the last station is absent, then it also creates a longer wait time at the last station, which is also the reason for the bottleneck.

Table 2 shows the probability of absenteeism at registration, medical examination, and dispensary. The probability of absenteeism is computed using past data. The data has been collected through a brief interview and brainstorming session with healthcare managers. He suggested the following probability of absenteeism for each resource. Following one-week probability can be projected for one year.

Table 2. Absenteeism probability of servers at each step in the healthcare center.

	Absenteeism Probability		
Days	**Registration**	**Medical Examination**	**Medication**
Monday	~0.03	~0.07	~0.04
Tuesday	~0.05	~0.07	~0.05
Wednesday	~0.05	~0.08	~0.05
Thursday	~0.05	~0.10	~0.05
Friday	~0.12	~0.14	~0.10
	Average = 0.060	Average = 0.092	Average = 0.058

There are two types of patients. Type 1 patients have severe diseases and type 2 patients have less severe diseases. The processing time at all stations is stochastic, which can be modeled with the help of some distribution. Processing time of each server at each station is measured and data is analyzed to find the best fit distribution. In distribution analysis, data was analyzed on a set of distributions such as uniform, lognormal, binomial, and normal distribution. In this data analysis, uniform distribution was selected because it had the highest value of Log-likelihood and minimum value of Akaike information criterion. Table 3 shows the processing time of each process at each step in the healthcare center. Here, U represents un-firm distribution, in the case of medical examination U (3,7), which means that a physician takes three to seven minutes to examine a patient. Data analysis also showed that the number of patients visiting the hospital follows the Poisson distribution. The number of patients visiting in the next month has been generated randomly using a convolution theorem. Table 4 shows the number of patients visiting each month.

Table 3. Outdoor patient treatment process plan.

	Registration	Examination	Medication
$i = 1$	U (1,5)	U (3,7)	U (1,2)
$i = 2$	U (1,5)	U (4,6)	U (1,2)

Table 4. Number of patients visiting the healthcare center each month.

January	February	March	April	May	June	July	August	September	October	November	December
2979	2726	1798	2630	2809	1736	2208	1629	1750	1643	2334	2841

The hospital is interested in the planning of the outdoor patient department (OPD) for minimizing the number of patients waiting in the queue at all servers by determining the number of servers considering their absenteeism.

4.1.2. Problem Solution

The station in the system becomes a bottleneck when there is physician absenteeism at the medical examination center. Considering the server capacity at each station, the service cycle time can be calculated using the equation below.

$$Service\ Cycle\ time = \frac{total\ proces\sin g\ time}{Number\ of\ servers} \tag{15}$$

Service rate is computed using equation as follows.

$$Service\ rate = \frac{1}{Service\ cycle\ time} \tag{16}$$

In this case, the processing time is uniformly distributed. Therefore, the cycle time of each station will also be uniformly distributed. The planning horizon of the hospital is one month and there are 26 working days and, there are 12 operating hours. Therefore, the available time is 18,720 min. Considering the number of patients visiting the hospital from Table 4 and available time in each month, the required cycle time for the patient circuit can be computed using Equation (7). Table 5 shows the required cycle time of a patient circuit.

Table 5. Required patient circuit cycle time for each month.

Month#	January	February	March	April	May	June	July	August	September	October	November	December
$C(R\zeta_p)$	6.3	6.9	10.4	7.1	6.7	10.8	8.5	11.5	10.7	11.4	8.0	6.6

This actual cycle time is not according to the required cycle time. Therefore, the system should be redesigned in such a way that there should be a minimum queue of patients at the station. Mixed integer nonlinear programming model is solved for each required cycle time without considering the absenteeism.

4.1.3. Case for a Numerical Example

Two cases for the numerical example have been presented. The first case is monthly planning without absenteeism and the second case considers the absenteeism.

Case 1: Human Resource Planning without Absenteeism

In this case, the Petri net have been used to optimize and compute the number of servers and queues using a mathematical model consisting of Equations (10)–(12) and Equation (14). The processing time has been generated using Table 3. The required cycle time of each month shown in Table 6 is used for cycle time constraint in the Petri net model.

Table 6. Required resources and Queue length with absenteeism.

Month	$C(R\zeta_p)$	Patients in Queue			Number of Servers Required		
		Station 1	Station 2	Station 3	Station 1	Station 2	Station 3
January	6.3	2	2	2	1	2	2
February	6.9	1	2	1	1	1	2
March	10.4	1	1	1	0	1	1
April	6.71	1	2	1	1	2	2
May	6.70	1	2	1	1	2	2
June	10.8	1	1	1	0	2	1
July	8.50	1	1	1	1	2	1
August	11.5	1	1	1	1	1	0
September	10.7	1	1	1	1	1	0
October	11.4	1	1	1	1	1	1
November	8.0	1	2	1	1	1	1
December	6.60	2	2	2	1	2	2
Average		1.2	1.5	1.2	0.8	1.1	1.3

Case 2: Human Resource Planning with Absenteeism

In this case, the factor of absenteeism is considered for the planning of human resources. Equations (10), (11), (13) and (14) have been used in the model. The processing times are the same in either case.

5. Results and Discussion

Considering Figure 1 and the mathematical model in Section 3.1.2, an excel spreadsheet program is generated and solved using a personal computer with 4 GB RAM and 2.67 GHz processor.

5.1. Results of Case 1

It can be seen from Table 1 that there are two types of patients and three types of servers and service processes consist of three processes so the total number of places is 12 and the total number of transitions is six because there are three processes and two types of patients. There are three types of loops in the Petri net model, which are the patient loop, a sequential loop, and a mixed loop. There are three sequential loops because the number of processes of service is three. The number of patient loops is equal to the type of patients who visit the hospital. In each elementary circuit, the availability and unavailability of each token (server, patient) are represented by binary (0, 1) digits. It is clear from Table 6 that the required cycle time for the month of January is 6.3 min per patient. The problem is solved by using MINLP for the required cycle time of January.

Number of patients waiting at registration = $P11 + P21 = 0 + 1 = 1$

Number of patients waiting at the medical examination center = $P12 + P22 = 0 + 1 = 1$

Number of patients waiting at the dispensary = $P13 + P23 = 2 + 0 = 2$

Similarly, the required number of the server at each station can be calculated using the following formula.

Number of servers at the registration booth = $S11 + S21 = 0 + 2 = 2$

Number of servers at the medical examination center = $S12 + S22 = 1 + 1 = 2$

Required servers at the dispensary = $S13 + S23 = 1 + 0 = 1$

Similarly, the MINLP model is run for each cycle time in Table 5 for each month and the required number of servers and number of patients waiting in a queue obtained from Petri net-MINLP at each station are given in Table 7.

Table 7. Required resources and Queue length without absenteeism.

Month	$C(R\zeta_p)$	Patients in Queue			Number of Servers Required		
		Station 1	**Station 2**	**Station 3**	**Station 1**	**Station 2**	**Station 3**
January	6.3	1	2	2	3	2	1
February	6.9	1	2	2	2	1	2
March	10.4	1	1	1	1	0	1
April	7.1	1	2	2	1	1	2
May	6.70	1	2	2	2	2	1
June	10.8	1	1	1	1	1	1
July	8.50	1	1	1	1	1	1
August	11.5	1	1	1	1	1	1
September	10.7	1	1	1	1	1	1
October	11.4	1	1	1	1	1	1
November	8.0	1	2	1	1	1	1
December	6.60	1	2	2	2	2	1
Average		1	1.5	1.4	1.4	1.2	1.2

It is clear from Table 8 that the average queue length at station 1, station 2, and station 3 are 1, 1.5, and 1.4. However, the average number of servers at each station is 1.4, 1.2, and 1.2, respectively.

5.2. Results of Case 2

The aim of this research to design a healthcare center with a factor of resource absenteeism. The same numerical example is used for the planning of the healthcare center with the factor of absents. The objective is to determine the optimal number of resources at each station with a minimum queue length. To do so, the Petri net-mixed integer nonlinear programming (MINLP) model included Equation (13) instead of Equation (12). The Petri net-MINLP is run for each cycle for each month and the required number of resources and queue generated at the station is recorded. Table 6 shows that the average queue length of each station has been reduced while the required number of average resources has increased.

The results of resource planning with absenteeism are more robust and ensure patient satisfaction with reduced queue length. The Petri net-MINLP model used the process plan from Table 1 and employed Figure 2 for determining sequential, patient, and mixed loops. Since there are three stations, it contains three sequential loops. A number of patient loops are two because two types of patients visit the healthcare center. The mixed loops depend on the sequential and patient loops. These loops can be computed using the following Equation (17).

$$\textit{Mixed loops} = \textit{Patient loops} \times \textit{sequential loops} \tag{17}$$

Table 8 shows markings of the circuit with cycle time for the Petri net model for the healthcare system. The binary digits show the availability and unavailability of the token in places, hiring, and firing from the transitions. Two types of cycle time have been used for this Petri net model. Circuit cycle time is known from the process plan and Petri net diagram. However, for the required cycle, the Petri net model is hybridized with MINLP to get the optimal number of servers and queue length.

Table 8. Marking of the circuit with cycle time for the Petri net model for the healthcare system in the month of January without absenteeism.

Transition Time	U (1,1.5)	U (3,7)	U (1,2)	U (1,1.5)	U (4,6)	U (1,2)	U (1,1.5)	U (1,1.5)	U (3,7)	U (4,6)	U (1,2)	U (1,2)				
Transition Name	t11	t12	t13	t21	t22	t23	t11	t21	t12	t22	t13	t23				
Place Name	P11	P12	P13	P21	P22	P23	S11	S21	S12	S22	S13	S23				
No.	1	2	3	4	5	6	7	8	9	10	11	12	$\tau(\zeta)$	L.T	$M(\zeta)lp$	$C(\zeta)lp$
1	0	0	0	0	0	0	1	1	0	0	0	0	8	S.C	1.27	6.30
2	0	0	0	0	0	0	0	0	1	1	0	0	12	S.C	1.90	6.30
3	0	0	0	0	0	0	0	0	0	0	1	1	11	S.C	1.75	6.30
4	1	1	1	0	0	0	0	0	0	0	0	0	15	P.C	2.38	6.30
5	0	0	0	1	1	1	0	0	0	0	0	0	16	P.C	2.54	6.30
6	1	0	0	0	1	1	0	1	0	0	1	0	27	MX	4.29	6.30
7	0	1	1	1	0	0	1	0	0	0	0	1	23	MX	3.65	6.30
8	1	1	0	0	0	1	0	0	0	1	1	0	24	MX	3.81	6.30
9	0	1	0	1	0	1	1	0	0	1	0	0	21	MX	3.33	6.30
10	1	0	1	0	1	0	0	1	1	0	0	0	30	MX	4.76	6.30
11	0	0	1	1	1	0	0	0	1	0	0	1	30	MX	4.76	6.30
Marking	1	1	1	1	1	0	0	1	1	1	1	1	9.8			

L.T: Loop type. S.C: Sequencing circuit. P.C: Patient circuit. MX: Mixed circuit of sequencing and patient. $M(\zeta)lp$: Sum of tokens in circuits in the mixed integer nonlinear programming model (MINLP). $C(\zeta)lp$: Cycle time of the circuit for MI.

5.3. Comparison of Results of Case 1 and Case 2 with the Existing System

In this section, a detailed analysis of servers and queue is presented and results are compared with the existing system. Figure 3 shows a healthcare center and its process plan.

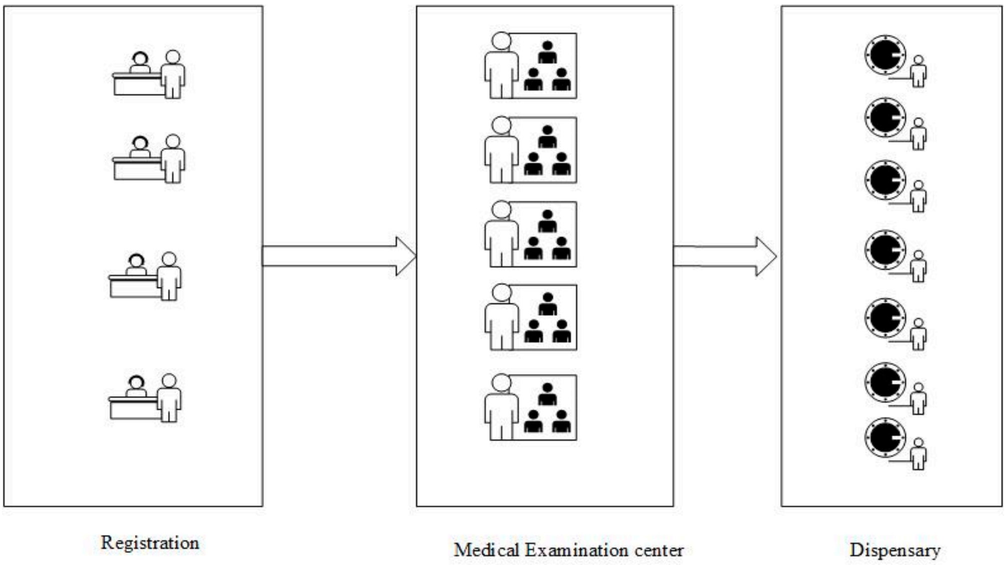

Figure 3. Process plan of a healthcare center.

5.3.1. Analysis of Servers

The healthcare center operated the overutilization of servers. The existing healthcare center contained two servers at station 1, three servers at station 2, and two servers at station 3. Figure 4 shows the number of servers required in both cases with and without the absenteeism case. It is clear from Figure 4 that the existing number of servers at the registration booth are in access. However, the required number of servers in both cases is reduced for the effective utilization of resources. Figure 5 shows the analysis of a number of servers at the medical examination center. It is shown that the required number of servers, in either case, is less than the existing servers.

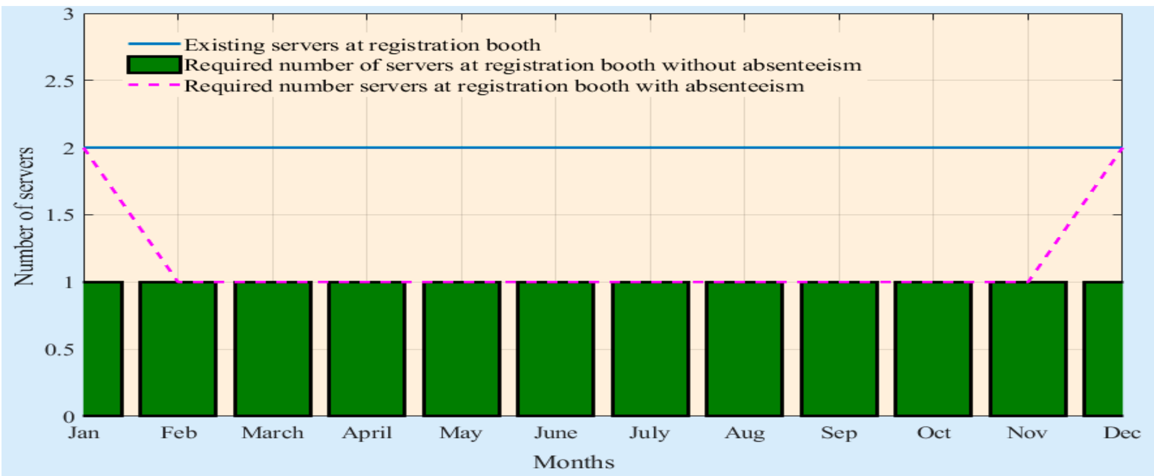

Figure 4. Number of servers at the registration booth during each month.

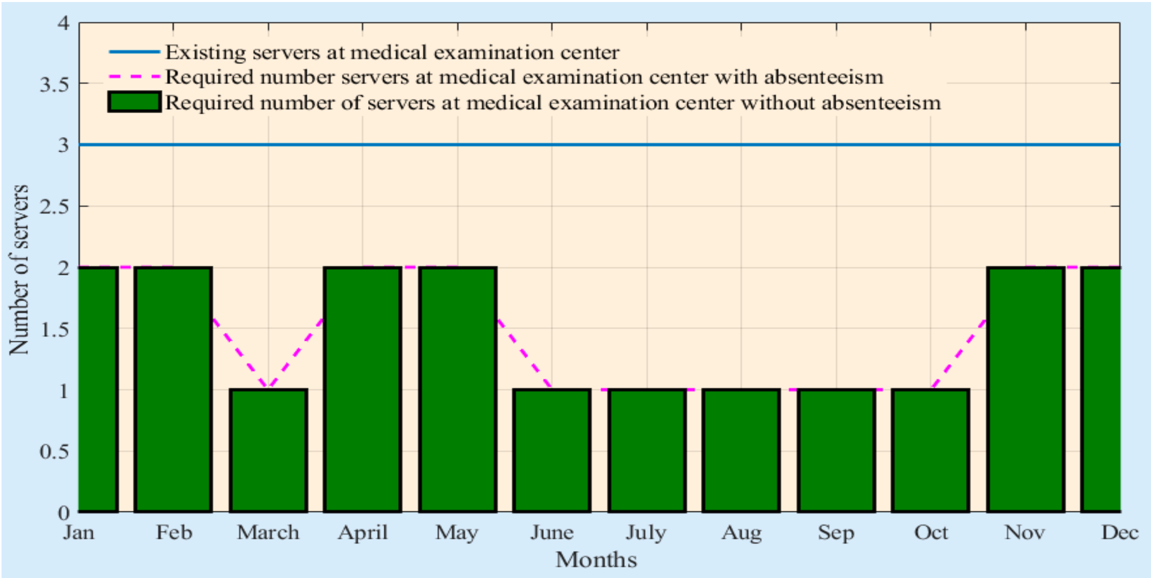

Figure 5. Number of servers at a medical examination center for each month.

Figure 6 shows the number of servers at the dispensary. It can be seen that a number of servers with an absenteeism case are higher from April to June.

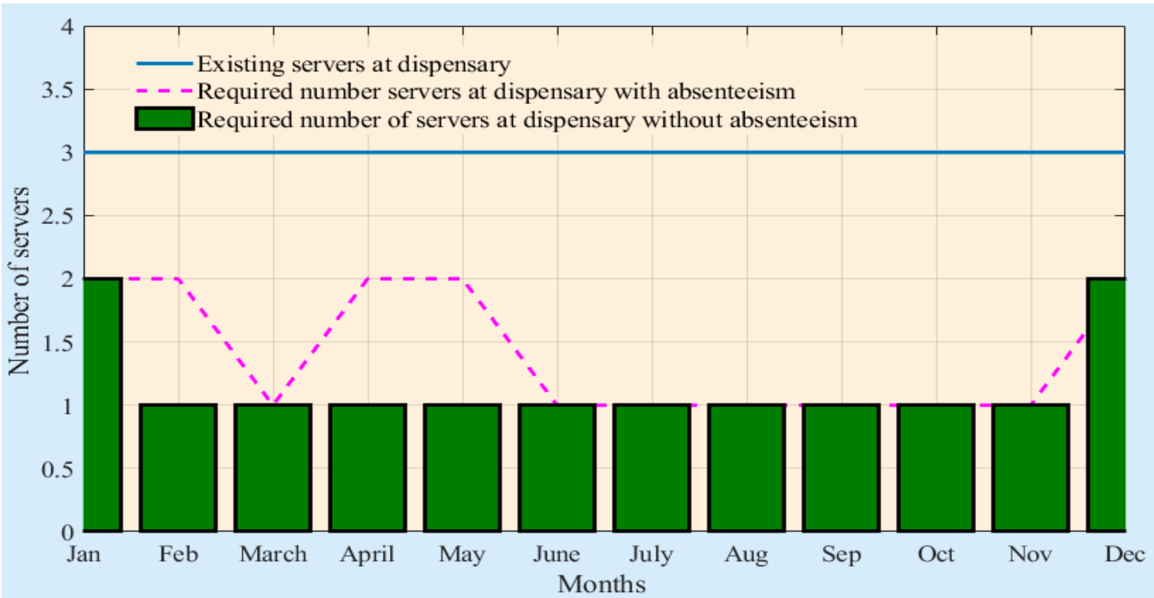

Figure 6. Number of servers at a dispensary for each month.

5.3.2. Analysis of Queue

Considering the case of server absenteeism reduced the queues of patients at all booths in a healthcare center. Figure 7 shows the comparison of queues for both cases of absenteeism and without absenteeism.

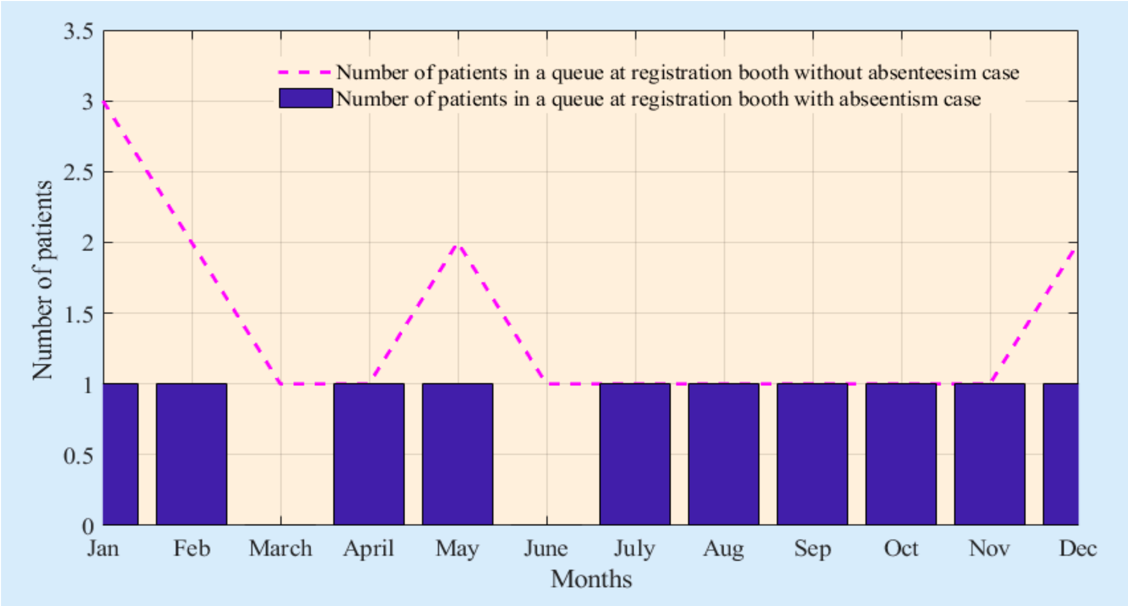

Figure 7. Number of patients waiting for service at the registration booth.

Figure 8 shows the number of patients waiting for medical treatment at the medical examination center. Planning resources as the factor of absenteeism reduced the queue length at the examination center.

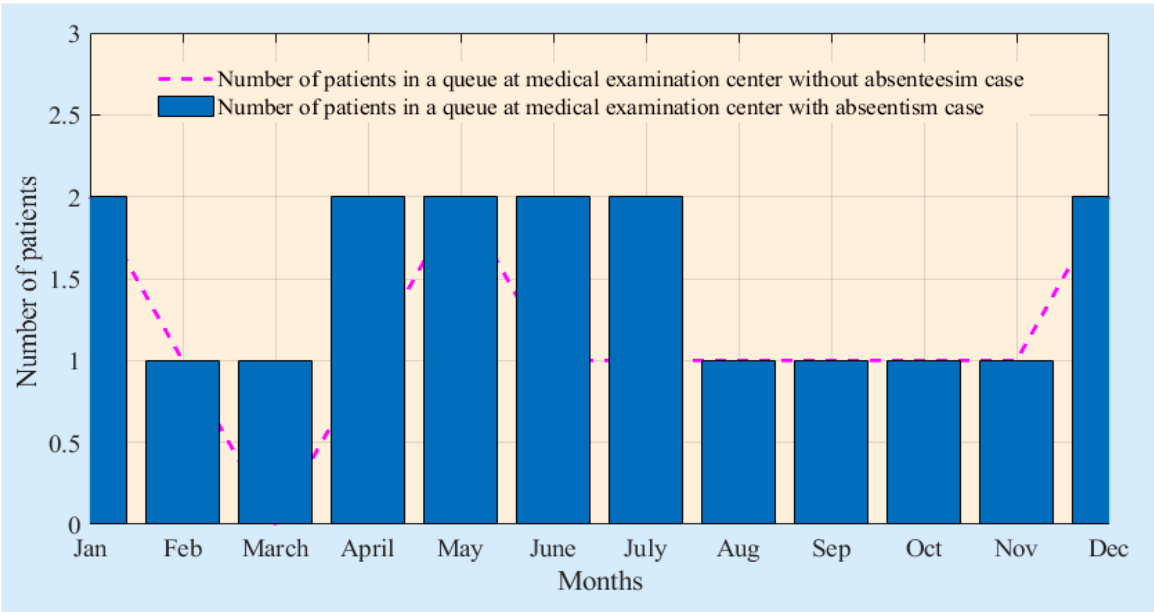

Figure 8. Number of patients waiting for service at the medical examination center.

It is prominent from Figures 7–9 that the length of queue reduced by determining the number of servers at each station. Therefore, this model provides the best approach for the planning of resources in a healthcare system. The consideration of server absenteeism in the planning of resources reduces queue length substantially, which increases resource utilization and patient satisfaction.

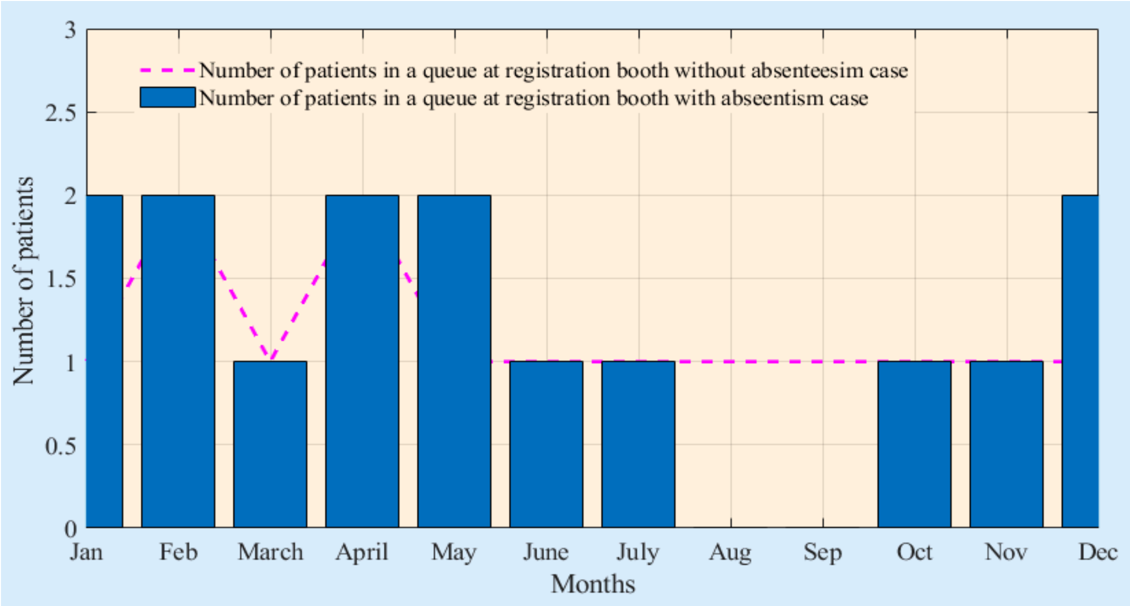

Figure 9. Number of patients waiting for service at the dispensary.

5.4. Petrinet Based Mixed Integer Nonlinear Programming (Petrinet-MINLP)

Most of the real-life problems have a nonlinear nature because of their complexity and involvement of many dependent decision variables. An optimization problem is said to be nonlinear if it contains a nonlinear equation either in the objective function or in a constraint. There are different software packages used for nonlinear optimization such as MATLAB, CPL EX, GAMS, and LINGO. However, in all these packages, an interior point algorithm is used for the optimization in mixed integer nonlinear programming. Mixed integer nonlinear programming is an exact method and guarantees the global optima in the optimization problem (Tawarmalani, Sahinidis, & Sahinidis, 2002). Traditionally, MINLP was used for mathematical optimization.

Petri net is a combination of simulation, mathematical, and graphical methods that are used to represent the real systems in both dynamic and static ways. Traditionally, Petri nets have been used for what-if analysis and could not optimize the complete process in a dynamic environment (Jensen, 2013). The optimization of the Petri net has not been well focused. In this research, we made an attempt to integrate the Petri net with mixed integer nonlinear programming for the optimization of real systems in a dynamic way. In this research, the Petri net-based mixed integer nonlinear programming is used to solve a healthcare resource planning problem. However, this proposed method can be applied to all business problems for their dynamic analysis and optimization.

5.5. Performance Evaluation of Petri Net-Based Mixed-Integer Nonlinear Programming

In this research study, a nonlinear mixed integer nonlinear programming model has been integrated with a Petri net for the dynamic behavior of patients flow in a healthcare center. In order to evaluate the performance of a proposed method, Petri nets have been integrated with genetic algorithm, a pattern search method, and case, which considers that the absenteeism has been solved using a Petri net integrated genetic algorithm and a patient integrated pattern search algorithm. Details of each method is given below.

5.5.1. Petri Net Based Genetic Algorithm

Genetic algorithm is a heuristic optimization method based on the theory of evaluation by Darwin. According to the theory of Darwin, only the fittest individuals survive in the next generation. In a genetic algorithm, an initial solution for an initial population is randomly generated. A randomly generated solution is called chromosome or individuals. The chromosome is composed of genes.

In optimization, a decision variable is treated as a gene and a chromosome is treated as an initial feasible solution. A set of feasible solutions is called a population.

The structure of a chromosome is given below in Figure 10. Crossover and mutation are the two genetic operators. The crossover occurs between two individuals or chromosomes, which is done by exchanging the genes. In this research, we used a two-point cross over. A mutation is because of change in genes in an individual chromosome. Mutation introduces the diversity while chromosomes transfer parental characteristics to the children or generations.

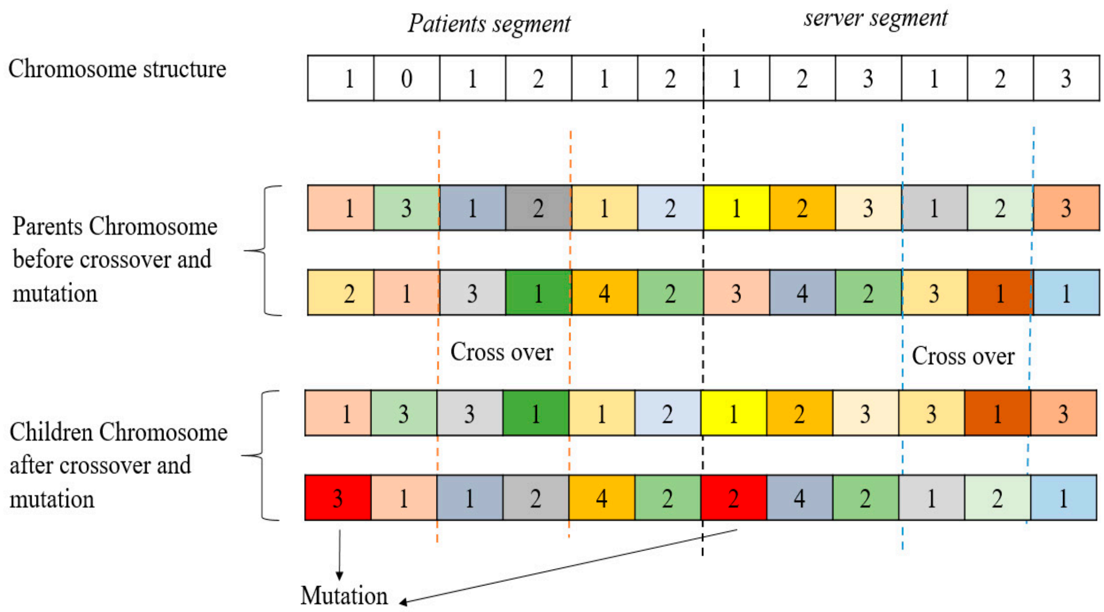

Figure 10. Chromosome structure and genetic operators.

The fittest chromosome in a population is called elite chromosome. Genetic algorithm is a heuristic method and it may or may not give the global optima [58]. In addition, it follows the random search method. Therefore, its computation time is higher than the mixed integer nonlinear programming. In solving the numerical example, the Petri net is integrated with a genetic algorithm [59]. In a Petri net-based genetic algorithm, the population consisted of 200 chromosomes. The mutation function was constraint-dependent. Elitism was $0.25 \times \textit{population size}$ with 80% crossover probability, which was used for the diversity in next generations, and a stochastic uniform selection method was adopted for the selection of individuals or chromosomes for the next or upcoming generation. Termination criteria for the genetic algorithm were the repetition of the same elite in 20 successive generations.

5.5.2. Petri Net-Based Pattern Search Method

The pattern search is a derivative-free search method and its independent of a gradient. In this method, the initial solution is provided and it follows the basic procedure of other optimization methods. This method was introduced in 1961 and emerged in two domains such as the exploratory search and pattern move. The exploratory search follows the direction along the gradient and the pattern move keeps improving its search. The pattern search method involves some parameters that affect its performance for a different problem data set. Kang, Li, and Li (2013) explained the pattern move with the help of Figure 11.

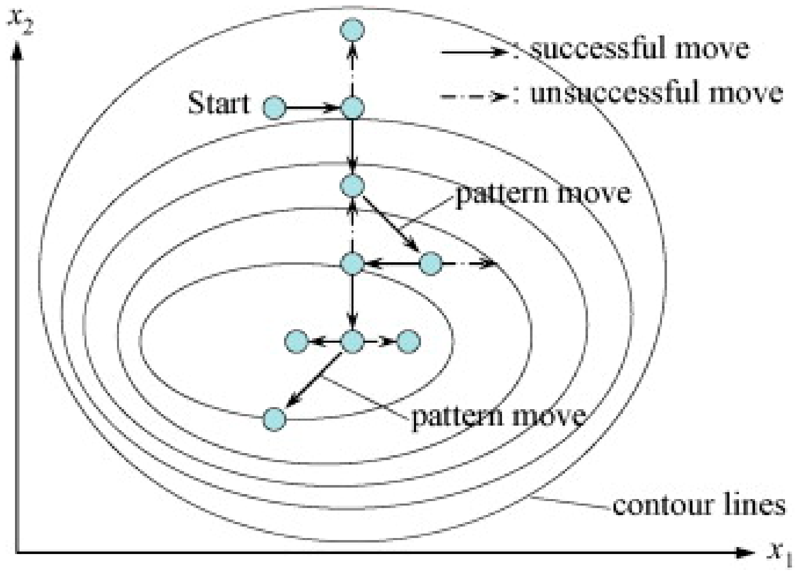

Figure 11. Pattern search illustration.

It is clear that the pattern search method initiates its move to find an appropriate direction for one variable at a time along with the coordinates of the individual from the base point. Once the exploratory search is over, the pattern move starts accelerating along the predetermined direction by the exploratory search. The expansion and contraction during the move are the most influencing parameters in the pattern search. In order to optimize the numerical example, the Petri net was integrated with a pattern search method [60]. In the patient-pattern search method, the same termination criteria were used as in the Petri net-genetic algorithm, expansion, and contraction factors, which were restricted to 2 and 0.5.

5.5.3. Comparison of Methods

The optimization problems can be classified into three categories based on the search method, which include the heuristic method, the direct search method, and the exact method. In this research study, the Petri net has been integrated with each kind of method. In order to evaluate the relative performance of these methods, the same numerical problem presented in Section 4.1.1 is solved with all methods. All methods have been coded in MATLAB (2017) Ra on a personal computer with 8 GB RAM and a 2.0 GHz Processor. The objective function in Equation (10) has been optimized subject to constraints in Equations (11)–(14). Table 9 shows the results of all the algorithms. In order to simplify the results, an average number of servers, and the average number of queues of all stations is considered.

The problem is solved for all required cycle times using both methods and an average number of servers and queues are given in Table 9 for all months of the relevant years. The computational power of methods has been measured in terms of a number of iterations and computation time. The relative performance of each method is computed using a gap analysis. Equation (18) has been used for the computation of the percentage gap.

$$\%Gap = \frac{Achieved\ value - Best\ value}{Achieved\ value} \times 100 \tag{18}$$

Table 9. Results of different algorithms for a case II with an absenteeism factor.

	Petri Net-Genetic Algorithm				**Petri Net-Pattern Search**				**Petri Net-Mixed Integer Nonlinear Programming**			
	Average Queue	**Average Servers**	**Iterations**	**CPU Time (s)**	**Average Queue**	**Average Servers**	**Iterations**	**CPU Time (s)**	**Average Queue**	**Average Servers**	**Iterations**	**CPU Time (s)**
January	3	4	59	7.12	4	3	4	5.34	2	2	254	2.33
February	3	4	55	6.35	4	4	5	4.99	1	1	258	2.78
March	4	4	416	10.12	5	4	5	4.54	1	1	268	3.1
April	3	3	92	7.87	4	3	5	4.46	1	2	253	2.35
May	3	3	75	7.23	4	4	5	4.25	1	2	265	2.78
Jun	3	3	69	7.15	4	4	5	4.21	1	1	259	3.32
July	3	3	74	7.31	3	3	5	4.29	1	1	270	3.42
August	3	4	89	7.67	3	3	5	4.23	1	1	257	2.45
September	3	3	52	6.45	4	3	5	4.19	1	1	267	2.43
October	4	3	68	7.78	4	2	5	4.89	1	1	268	2.35
November	4	4	67	7.53	5	3	5	4.56	1	1	255	2.34
December	3	4	57	7.23	4	3	5	4.45	2	2	254	2.31
Average	3	4	98	7.48	4	3	5	4.53	1	1	261	2.66

In the case of minimization/maximization problems, the best will be a minimum/maximum value of the specific performance measure. It can be seen from Table 9 that a number of iterations are greater in a Petri net mixed integer nonlinear program.

MINLP is an exact method and evaluates one solution in one iteration. Therefore, a number of iterations are evaluated in a short period of time. However, in the case of the Genetic algorithm and pattern search, it takes more time to evaluate one iteration. In one generation, multiple solutions are evaluated. Average queue length, the average number of servers at the station, the number of iterations, and computational time has been considered as performance measures of the proposed method. Table 10 shows the average performance measures computed from three different methods.

Table 10. Average performance measurement of algorithms.

	Monthly Average Queue	Monthly Average Servers	Average Iterations	Average CPU Time
Petri net-Genetic algorithm	3	4	98	7.5
Petri net-Pattern search	4	3	5	4.5
Petri net-Mixed-integer nonlinear programming	1	1	261	2.7

Using Equation (18) and Table 10, the relative percentage gap has been computed, which is given in Table 11. In order to understand the percentage gap, consider the monthly average queue in Table 10. The best value of the monthly queue is one patient. Therefore, using Equation (18), the percentage gap of the Petri net-genetic algorithm can be computed as follows. Similarly, performance measures of rest of the algorithms are given in Table 11.

$$\%\ Gap = \frac{3-1}{3} \times 100 = 66.67\% \tag{19}$$

Table 11. Relative percentage gap in different methods.

	Monthly Average Queue	Monthly Average Servers	Average Iterations	Average CPU Time	Sum
Petri net-Genetic algorithm	66.67	75.00	94.90	64.00	300.56
Petri net-Pattern Search	75.00	66.67	0.00	40.00	181.67
Petri net-MINLP	0.00	0.00	98.08	0.00	98.08

It is clear that the total percentage gap is minimum in case of Petri net-based mixed integer nonlinear programming as compared to the other methods. The Petri net-genetic algorithm and Petri net-paterren search are the heuristics and follow a random search pattern. However, in mixed integer non-linear programming, the branch and bound algorithm is used in the exact solution method.

5.6. Advantages and Disadvantages of Petri Net Integrated Mixed Integer Non-Linear Programming

The Petri net combines the characteristics of graphics, simulation, and mathematical modeling for the imitation of real systems. Petri nets are good tools for analyzing the dynamic behavior of systems with both continuous and discrete state changes. Integration of optimization with Petri nets provides the following benefits.

1. It enables real-time optimization of systems for more practical results. In traditional literature, a mathematical model is optimized and results are then used in a simulation model for the validation. However, in this research, simulation and optimization takes place at the same time. Therefore, the word real-time optimization is more suitable for this type of optimization.

2. It visualizes the patterns and their flow within the program and highlights the state changes in a system at a different process.
3. It combines the computational power of simulation and mathematics for more robust results.
4. Integration of Petri net with exact optimization reduces computational time and improves the performance.

In addition to the advantages, Petri nets have some disadvantages as well. Complex systems involving more than three processes and entities are hard to model. In Petri nets, model construction is a cumbersome process and needs computational and graphical efforts.

5.7. Managerial Insights

This research is useful for operation managers in healthcare for efficient planning of resources such as physicians and other medical staff. The consideration of the factor of absenteeism makes it more robust for long-term decision-making in determining the optimal number of resources. This planning model can be useful in determining the more accurate number of resources even in case of seasonal diseases or some disease caused by viral viruses or bacteria.

6. Conclusions

This paper presented a Petri net based mixed integer nonlinear programming model for the outdoor patient department (OPD) of the sustainable healthcare center. The integration of the Stochastic-Petri net approach with MINLP for real-time optimization in the planning model enabled the realization of sustainable operational utilization with a long-term competitive advantage. The objective of the research was to minimize the queue length of patients and the required number of servers for a sustainable future. The reason for the queue is the limited resources including unavailability or absenteeism of servers at their stations. The introduction of absenteeism in sustainable planning made this research a novel. A numerical example of a sustainable healthcare center is presented as a case study. The example provided resource planning for the period of one year. The forecast of a number of patients visiting the healthcare system is used to determine the required cycle time for each month. The Petri net mixed integer nonlinear programming is used to solve the model for determining an optimal number of servers at each station of OPD for two cases. The first case considered the determination of resources without absenteeism and the second case considered the determination of resources with absenteeism. It was observed that long queues (two to three patients) are generated if we plan the resource without the absenteeism factor and queue is substantially reduced to one patient only when absenteeism factors are considered. The reduced queue has various benefits associated with it such as maximum resource utilization and improved efficiency of the system with the highest degree of patient satisfaction. In order to evaluate the performance of a Petri net-based mixed integer nonlinear programming model, the same numerical example is solved with other methods such as the Petri net-genetic algorithm and the Petri net-pattern search method. The minimum gap of 98.08% showed that the performance of Petri net-MINLP is better in terms of computation time and function value. This research is useful for the operation manager for resource planning in a healthcare center. This is a planning model for the period of one year and estimates the required number of servers at each station. However, it does not provide any information for controlling the mechanism of servers if the server suddenly leaves the system without serving the patients on a daily basis. The sudden absenteeism control of servers is included in future research.

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Abbreviations

The notations and abbreviations used in this model are given below.

Indices:	
i	patient type $i = 1, 2 \ldots, I$
j	server type $j = 1, 2 \ldots, J$
Legends:	
$\rightarrow$	directed arc shows the flow of token between transition and place
$\circ$	token (patient)
Parameters:	
t_{ij}	attendance time of patient "i" at server "j".
ρ_{ij}	a place for waiting of patient "i" to be served by server "j".
S_{ij}	place of server "j" for serving patient "i".
ζ_p	overlapping circuit for each patient
ζ_s	overlapping sequencing circuits
$\tau(\zeta)$	total transition time
$C(\zeta)$	the cycle time of each sequential loop
PA_j	the probability of absenteeism of server "j"
SR	service rate
PH	planning horizon
NP	the expected number of patients
$C(\zeta_{sMAX})$	the maximum cycle time of the bottleneck server or required cycle time for a patient
Decision Variables:	
NQ_{ij}	Number of patients " " waiting for service at server "j"
NS_{ij}	Number of servers "j" serving the patient "i"

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