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How Does a Radio Frequency Identification Optimize the Profit in an Unreliable Supply Chain Management?

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Abstract: Competition in business is higher in the electronics sector compared to other sectors. In such a situation, the role of a manufacturer is to manage the inventory properly with optimized profit. However, the problem of unreliability within buyers still exists in real world scenarios. The manufacturer adopts the radio frequency identification (RFID) technology to manage the inventory, which can control the unreliability, the inventory pooling effect, and the investment on human labor. For detecting RFID tags, a reasonable number of readers are needed. This study investigates the optimum distance between any two readers when using the optimum number of readers. As a vendor managed inventory (VMI) policy is utilized by the manufacturer, a revenue sharing contract is adopted to prevent the loss of buyers. The aim of this study is to maximize the profits of a two-echelon supply chain management under an advanced technology system. As the life of electronic gadgets is random, it may not follow any specific type of distribution function. The distribution-free approach helps to solve this issue when the mean and the standard deviation are known. The Kuhn-Tucker methodology and classical optimization are used to find the global optimum solution. The numerical analysis demonstrates that the manufacturer can earn more profit in coordination case after utilizing revenue sharing and the optimum distance between readers optimizing cost related to the RFID system. Sensitivity analysis is performed to check the sensibility of the parameters.

Keywords: supply chain management; inventory control; distribution-free approach; revenue sharing; radio frequency identification; information asymmetry

1. Introduction

Instead of a traditional business system, supply chain management (SCM) provides different kinds of business policies in terms of inventory management. The vendor managed inventory (VMI) is one of these in which the manufacturer takes full responsibility of the existing inventory at the buyer's position. Dong and Xu [1] found opportunities where buyers received more profit than the manufacturer. The manufacturer's profit may vary according to the business policy, where the short-term and long-term VMI affects the SCM, which were decided by them. They concluded that the short-term VMI can be a competitor for coordination business policy. In any business, the forecasting uncertainty is a major issue and Guo et al. [2] developed a method to reduce the supply chain forecasting uncertainty through information sharing via macro prediction which can reduce the system robustness. However, it is possible that not all information is shared by both parties. Then, unreliability occurs in the business system due to information asymmetry (Mukhopadhyay et al. [3]; Yan and Pei [4]; Xiao and Xu [5]). An information basically flows in the upward direction of

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SCM. The lack of information of the manufacturer may cause insufficient supply of products which can affect the inventory and production process. The situation is even more complicated when an imperfect production process takes place (Sarkar [6]). The rework of defective products was considered by Cárdenas-Barrón et al. [7] for an imperfect production process. They developed an improved algorithm to find the optimum lot size and replenish the defective production system. Cleaner production can be formed by discarding defective products, which was established by Tayyab and Sarkar [8]. Those defective products were reworked up to good quality through additional investment. This work was extended by multi-stage cleaner production by Kim and Sarkar [9] using budget constraints. There are several researchers who worked on imperfect products, reworking, and deterioration (Guchhait et al. [10], Majumder et al. [11], Tiwari et al. [12]). Finally, Sarkar [13] introduced an exact duration for reworking within a multi-stage multi-cycle production system. However, there is a lack of literature regarding RFID, i.e., RFID was not used to maintain the inventory pooling effect. Reworking was considered by Sarkar et al. [14] in a material requirement planning (MRP) system.

Production quantity mainly depends upon the market demand. In reality, it cannot always be the case that data related with demand are available. If no known distribution function is followed by the demand or no data are available, then instead of taking any arbitrary probability distribution, the distribution-free (DF) approach is used (Gallego and Moon [15], Sarkar et al. [16], Guchhait et al. [17]). This method was invented by Scarf [18]. Due to the complex calculations, it was not understandable to people in the industry at that time. Later, this approach was simplified by Gallego and Moon [15]. This method is used by Sarkar et al. [19] for a consignment stock-based newsvendor model. They allowed a fixed-fee payment technique to prevent loss from any participant. There are multiple manufacturers and retailers available for a single-type of products. Based on advertisements given by the manufacturer, retailers opted to choose their manufacturers. For the random demand, the variable production rate is useful (Sarkar et al. [20]) for modeling uncertain demand. A service level can help avoid shortages (Moon et al. [21]) and backorder (Sarkar [22]) due to the uncertain random demand. Partial trade credit for deteriorating items in the inventory model was discussed by Tiwari et al. [23]. For any industry, it may be that they need to analyze their previous data. Tiwari et al. [24] provided a big data analysis of SCM from 2010 to 2016.

Competitive markets in the business industry becoming more intense everyday. To handle this situation, companies prefer to adopt smart technologies within the SCM. The fast movement of products for the electronic industry is a key feature since competition is very high in the electronics sector. The implementation of technology instead of labor-based production is helpful not only for fast production, but also to profit gain. The use of RFID technology in SCM for managing inventory has been studied by several researchers. A wireless sensing problem for coverage was first studied by Meguerdichian et al. [25]. Zhang and Hou [26] investigated how many readers need to be implemented to provide a complete coverage of a search area. The coverage area sensing radius and transmitting radius were discussed by Hefeeda and Ahmadi [27]. They established that probabilistic sensing coverage can function as deterministic coverage. Dias [28] implemented RFID for a multi-agent system. Sarac et al. [29] surveyed the literature and found several implementation and usages of RFID in different sectors of SCM. They found that inventory loss can be reduced with increased efficiency of the system and real-time information of the inventory. Kim and Glock [30] investigated the effectiveness of an RFID tracking system for container management and found that the return rate of container was increased after using RFID. A four-echelon SCM was studied by Sari [31] to examine the effects of collaboration. They found through simulation that the integrated RFID technology is more beneficial for good collaboration between participants. Besides SCM, warehouse efficiency can be improved using RFID technology (Biswal et al. [32]). In the production sector, RFID improves the efficiency and maintenance, as investigated by Chen et al. [33]. They established that operation time can be increased by up to 89% and that the labor cost is reduced significantly by using RFID. Even, remanufacturing

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companies can get benefit from RFID via just-in-time (JIT) features or transiting towards a closed-loop SCM (Tsao et al. [34]).

From literature, it is found in most of the studies that RFID is used in SCM to prevent inventory shrinkage as well as minimize the operation time of the system, reduction of lead time, and labor consumption (Ustundag and Tanyas [35]; Jaggi et al. [36]) and improve the efficiency. However, the reason behind this efficiency improvement by RFID is not discussed in the literature. This study introduces for the first time the RFID distance function f(d) based on the sensing and transmitting radii. The distance between two readers can be optimized and thus, the number of RFID readers can be found to increase the efficiency. Based on the transmitting and sensing radii, two types of readers are used by the manufacturer, namely Type 1 and Type 2. To understand the complete search capacity of a Type 1 reader, the area is divided into sub-areas that are under the coverage of Type 2 readers. This combined system may enhances the system accuracy and provides strong coverage of the sensing and transmitting areas. Table 1 gives the contribution of different authors in the literature. This study shows benefits for the buyer in the optimum order quantity, optimizes distance the between two readers, and optimizes the service given by the buyers. The rest of the study is designed as Section 2 gives the details about the mathematical model. Section 3 gives the results of the numerical experiment and Section 4 provides a discussion of results. Section 5 concludes this study. Associated references are attached in the References section.

Author(s)	Model Type	Business Policy	Unreliability	RFID
Dong and Xu [1]	stochastic	VMI	NA	NA
Guo et al. [2]	stochastic	macro prediction market	NA	NA
Mukhopadhyay et al. [3]	deterministic	mixed channel	information	NA
Yan and Pei [4]	deterministic	mixed channel	information	NA
Xiao and Xu [5]	deterministic	VMI	NA	NA
Sarkar [6]	stochastic	production model	reliable	NA
Guchhait et al. [10]	deterministic	traditional	NA	NA
Majumder et al. [11]	deterministic	traditional	NA	NA
Gallego and Moon [15]	stochastic (DF)	inventory model	NA	NA
Scarf [18]	stochastic (DF)	inventory model	NA	NA
Sarkar et al. [19]	stochastic (DF)	CP	NA	NA
Moon et al. [21]	stochastic (DF)	inventory model	NA	NA
Tiwari et al. [23]	deterministic	SCM	NA	NA
Meguerdicihian et al. [25]	networking	NA	NA	sensing
Zhang and Hou [26]	networking	NA	NA	sensing
Hefeeda and Ahmadi [27]	networking	NA	NA	coverage
Dias et al. [28]	survey	SCM	NA	survey
Sarac et al. [29]	value chain	survey	NA	survey
Kim and Glock [30]	stochastic	closed-loop	NA	tracking
Shin et al. [37]	stochastic (DF)	inventory	NA	NA
This model	stochastic (DF)	VMI	information	distance and readers

Table 1. Comparison of author's contribution.

2. Problem Definition, Notation, and Assumptions

This section describes the problem definition for this study. Associated assumptions and notation are given here.

2.1. Problem Definition

A two-echelon supply chain model is considered under the newsvendor framework where participants are in a VMI contract. The inventory of the whole system is controlled by the manufacturer. Controlling the inventory manually by human labor is a time consuming task, as the manufacturer takes full responsibility of the full business of all buyers. To do this, the manufacturer installs smart RFID technology. The number of RFID readers is needed by the manufacturer such that the inventory can be controlled in a proper way within a minimum time duration. The number of readers depends

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on the sensing distance between two readers. Thus, the distance between readers is optimized for RFID investment. Buyers are not reliable with respect to the manufacturer's business. Buyers provide services to the customers, and therefore an unreliable SCM is formed as a single-manufacturer multi-buyer. The goal of the newsvendor model is to maximize profit for the buyer without incurring any storage or redundancy costs. However, the buyer is unable to decide on the optimum order quantity, where there should not be any understock or overstock costs. For that, the manufacturer takes the full responsibility of the buyers to for profits through the VMI strategy. Even though the manufacturer tries their best to help the buyer, the buyer is unreliable in nature and may provide wrong information regarding the demand to manufacturer. To mitigate this matter, the RFID technology is installed allowing the manufacturer to obtain more profit.

2.2. Notation

The following notation (Table 2) is used in the present study.

Table 2. Notation in this study.

Index	
i	number of buyers i , $i = 1, 2,, n$
Decision variables	description
$egin{array}{c} \delta_i \ q_i \ d \end{array}$	service by buyer i order quantity of buyer i per cycle (units/cycle), $Q = \sum_{i=1}^{n} q_i$ distance between two RFID readers
Parameters	description
p_i d_i μ_i σ_i l, b S_t S_s ρ c_1, c_2 λ	selling price of buyer i per unit under RFID effect (\$/unit) demand of buyer i per cycle (unit/cycle) mean value of demand d_i , $\mu = \sum_{i=1}^n \mu_i$ standard deviation length and breadth of the search area (m) transmission radius of Type 1 reader (m) sensing radius of Type 1 reader (m) decay parameter for sensing costs of Type 1 and Type 2 reader per unit (\$/unit) maximum threshold value of Type 1 reader threshold parameter $(0 < \theta < 1)$
$w \\ \pi_m \\ \pi_{ri} \\ \eta_i \\ \delta_i \\ \zeta_i \\ h_m \\ h_{ri}$	purchasing/wholesale cost per unit under the RFID effect ($\$$ /unit) goodwill lost cost of manufacturer per unit with RFID ($\$$ /unit) buyer i 's goodwill lost cost per unit under consideration of RFID ($\$$ /unit) service investment of buyer i ($\$$) service by buyer i customer satisfaction cost of buyer i ($\$$ /unit) holding cost of manufacturer under RFID effect ($\$$ /unit/unit time) buyer i 's holding cost with RFID ($\$$ /unit/unit time)
Others	description
$f(d)$ $E(\cdot)$ ETP ETP_r ETP_m	cost of RFID per cycle (\$/cycle) expected value expected total profit of the coordinate case per cycle (\$/cycle) buyer's expected total profit per cycle (\$/cycle) manufacturer's expected total profit per cycle (\$/cycle)

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2.3. Assumptions

The following assumptions are used for this model.

1. A two-echelon SCM is considered for a single-type of electronic products, where the inventory is managed by a manufacturer through a VMI contract. To ensure the profit of the buyers, a revenue sharing policy for coordination case is used by the manufacturer. The finished products are sent to the *n* buyers.

- 2. Buyers are not reliable enough and they are not sharing data to the manufacturer. It forms an information asymmetry in the business system. The manufacturer losses some information about market and installs the RFID system to solve the unreliability issue.
- As VMI recommends that the supreme controlling authority is the manufacturer and the manufacturer decides to use RFID technology for controlling the unreliability issues. Hence, the manufacturer decides the whole deployment for the design of installing RFID reader, which can be done by the third-party. As the manufacturer cannot reach to the retailer's place in each and every moment, the technology will support to solve the issue of the unreliability. Those support will be taken from the third-party by investing some fixed cost. That fixed cost is inserted within the cost of Type 1 and Type 2 reader. Therefore, the RFID reader deployment cannot be specified within the modelling part of the manufacturer. However, the design of RFID reader can be added for the entering gate or any other place, but it depends on the third-party who is dealing with the whole area for covering the RFID. Therefore, through VMI, it is not the responsibility for the manufacturer to check the design for the installed RFID readers as this is a paid service from the third-party. Two types of reader are used to give a complete coverage of the search area. The total search area is divided into subareas and each subarea is covered by Type 1 readers, based on a disk sensing model. Each subarea is again divided into small search areas that are covered by Type 2 readers, based on an exponential coverage protocol. The frequency range of the readers is measured for usual road transport.
- 4. It may not be possible that the demand pattern always follows some distribution function. As data are random, it is assumed that the market demand is uncertain and does not follow any particular type of distribution. The known mean is μ_i and the standard deviation is σ_i (Shin et al. [37]).
- 5. The planning horizon is [0,T] and the lead time is negligible.

3. Mathematical Modelling

A VMI contract policy for the electronic industry is discussed for a single-manufacturer and multi-buyer newsvendor model. The optimum number of RFID readers, which can cover the optimized distance, can provide maximum profit to the supply chain for a long time. As implementation of RFID requires a huge investment, a reasonable demand rate is expected for the manufacturer. However, the market demand (d_i) for buyer i is uncertain, it cannot be predicted. The demand (d_i) for buyer i can be represented by a random variable where the mean is (μ_i) and (σ_i) is the standard deviation which both are known. As d_i does not follow any specific distribution function, this problem can be solved using the DF approach. The surplus and shortage amount can be calculated by the lemma of Gallego and Moon [15]. The required surplus amount is

$$E(q_i - d_i)^+ \le \frac{1}{2} \left[\sqrt{\sigma_i^2 (\mu_i - q_i)^2} + (q_i - \mu_i) \right], \mu_i < q_i$$
 (1)

and the shortage amount is

$$E(d_i - q_i)^+ = \frac{1}{2} \left[\sqrt{\sigma_i^2 (\mu_i - q_i)^2} + (\mu_i - q_i) \right], q_i < \mu_i, \text{ for } F \in \mathbb{F}.$$
 (2)

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3.1. Structure of the Proposed RFID System

The total search area is covered by the RFID tracking system. The cost regarding RFID depends on the number of readers. The concept of VMI is that the manufacturer will manage the whole inventory of the retailer as some unreliable issues are coming from retailer's side. To overcome these issues, the manufacturer introduces RFID technology with the minimum investment for it. Therefore, within the total area of the retailer, how much inventory are these, that should be verified by RFID readers. Therefore, it is not essential to use always powerful RFID readers like as Type 1 or similarly it is not recommended also that always low powerful Type 2 reader should be used. Hence, an optimization is needed to optimize the optimum number of Type 1 and Type 2 reader within the whole area. That is why, this model recommended two types of RFID reader for the sensing and coverage model: the disk sensing model and the exponential coverage model. The entire search area is divided into subareas which are covered by the Type 1 reader. This Type 1 reader has a higher sensing power for coverage, which uses the disk sensing model. Each subarea is divided into subareas those are covered by two Type 2 readers. Type 2 readers have low sensing power and use an exponential coverage protocol system. The connectivity between the sensing radius and transmitting radius is given by the condition $2S_s \leq S_t$ (for instance, see Zhang and Hou [26]).

If l_1 is the length and b_1 is the breadth of each subdivided area, then from the properties of right-angled triangle (Figure 1), it is follows that

$$l_1^2 + b_1^2 = c^2$$
, i.e., $l_1^2 + b_1^2 = 4S_t^2$, i.e., $S_t = \sqrt{\frac{l_1^2 + b_1^2}{4}}$.

For each square foot area, $l_1 = b_1$, which implies that

$$S_t = \frac{l_1}{\sqrt{2}}$$
, i.e., $l_1 = \sqrt{2}S_t$.

Therefore, if the length and the breadth of the total search area are l and b, respectively, the total number of Type 1 reader is $\left[\frac{l}{\sqrt{2}S_t}\right]\left[\frac{b}{\sqrt{2}S_t}\right]$.

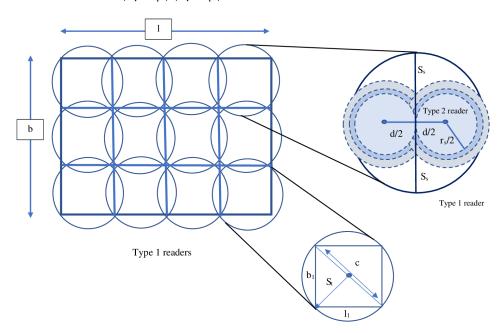


Figure 1. Execution of Type 1 and Type 2 readers for a search area.

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Now, each subdivided area of sensing radius S_s is divided into two areas with sensing radius r_s . The maximum distance between two Type 2 readers is d, i.e., $r_s = \frac{d}{2}$. Now, from the exponential coverage protocol (Hefeeda and Ahmadi [27]), the maximum distance d between two Type 2 readers is

smaller than
$$\sqrt{3}\left(\lambda - \frac{Log\left[1 - \sqrt[3]{1-\theta}\right]}{\rho}\right)$$
, i.e.,

$$d \leq \sqrt{3} \left(\lambda - \frac{Log \left[1 - \sqrt[3]{1 - \theta} \right]}{\rho} \right).$$

The area of the circle for sensing radius S_s is πS_s^2 . The area of circle of sensing radius r_s is $\pi r_s^2 = \frac{\pi d^2}{4}$. Therefore, the number of Type 2 readers for each subdivided area of Type 1 reader is $\left[\frac{\pi S_s^2}{\frac{\pi d^2}{4}}\right] = \left[\frac{4S_s^2}{d^2}\right]$. Hence, the total number of Type 2 readers for all Type 1 readers is

$$\left\lceil \frac{4S_s^2}{d^2} \right\rceil \left\lceil \frac{l}{\sqrt{2}S_t} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_t} \right\rceil.$$

3.2. Manufacturer's Model

In reality, it is not always the case that all buyers are reliable enough to share all information to the manufacturer. To prevent the piracy on the inventory inaccuracy, the manufacturer invests in RFID technology even though this may reduce the profit margins. However, there may be long-term benefits compensate the shrinkage of inventory. Still, there may be some ambiguity regarding information due to information asymmetry.

3.2.1. RFID Cost

The total area is covered by $\left\lceil \frac{l}{\sqrt{2}S_t} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_t} \right\rceil$ Type 1 readers. This area is again subdivided and is covered by Type 2 readers. If c_1 is the cost of each Type 1 reader and c_2 is for each Type 2. A fixed cost is included within c_1 and c_2 which the manufacturer pays as an investment. Then the required RFID cost is given by

$$f(d) = c_1 \left\lceil \frac{l}{\sqrt{2}S_t} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_t} \right\rceil + c_2 \left\lceil \frac{4S_s^2}{d^2} \right\rceil \left\lceil \frac{l}{\sqrt{2}S_t} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_t} \right\rceil$$

subject to the conditions

$$d \le \sqrt{3} \left(\lambda - \frac{Log \left[1 - \sqrt[3]{1 - \theta} \right]}{\rho} \right)$$

$$S_t \ge 2S_s. \tag{3}$$

Therefore, the RFID cost per cycle is $\frac{f(d)\mu}{Q}$, where $D = \sum_i d_i$ and $\mu = E(D)$.

3.2.2. Production Cost and Wholesale Price

If the manufacturer produces a lot size Q per cycle then the production cost of those products is given by cQ. When the manufacturer sells products as a wholesale price w per unit, then the wholesale price is given by wQ.

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3.2.3. Holding Cost

The situation of holding products is created when the demand (d_i) is less than the ordered quantity wq_i . If h_{ri} is the unit holding cost of buyer i, the holding cost is $h_{ri}E(q_i-d_i)^+$, $d_i \leq q_i$. As the manufacturer pays both the holding cost of the buyers and the manufacturer (h_m) , the total holding cost of the manufacturer is given by $\sum_i E(q_i-d_i)^+ \left(h_{ri}+h_m\right)$, $d_i \leq q_i$.

3.2.4. Goodwill Lost Cost

A goodwill lost cost (π_m) is allowed since the manufacturer takes the responsibility for the products for the whole supply chain, where shortage affects the goodwill of manufacturer. The cost expression for goodwill loss is given by $\sum_i \pi_m E(q_i - d_i)^+$, $q_i < d_i$.

Including the RFID cost, the expected total profit of the manufacturer is given by the following expression

$$ETP_{m}(q_{i},d) = (w-c)Q - \sum_{i} \frac{1}{2} (h_{r_{i}} + h_{m}) \left[\sqrt{\sigma_{i}^{2} + (\mu_{i} - q_{i})^{2}} + (q_{i} - d_{i}) \right] - \frac{\pi_{m}}{2} \sum_{i} \left[\sqrt{\sigma_{i}^{2} + (\mu_{i} - q_{i})^{2}} + (d_{i} - q_{i}) \right] - \frac{f(d)\mu}{Q}$$

$$(4)$$

subject to the conditions

$$d \le \sqrt{3} \left(\lambda - \frac{Log \left[1 - \sqrt[3]{1 - \theta} \right]}{\rho} \right)$$

$$S_t \ge 2S_s.$$

3.3. Buyer's Model

Buyers are unreliable resulting in information asymmetry. As this is a dependent business policy and the manufacturer is responsible for both inventory supervision and holding inventory for buyers, all information should be known to the manufacturer. However, today's business systems are very complex and buyers are unreliable at sharing information their own business strategy. Buyer *i* buys the electronic products from the manufacturer and sells them in the market. To increase market demand, the buyers provide facilities to the customers without telling the manufacturer meaning that an unreliable supply chain system is formulated.

3.3.1. Revenue

 p_i is the unit selling price of the electronic products. Now, two types of situation may arise, where the demand (d_i) is more than the ordered quantity (q_i) or vice-versa. Then the selling price can be found as

$$\begin{cases}
p_i d_i & d_i \leq q_i \\
p_i q_i & q_i < d_i.
\end{cases}$$

3.3.2. Purchasing Cost and Goodwill Lost Cost

If w is the unit purchasing cost for the ordered quantity q_i , then the purchasing cost is given by wq_i . When the reverse situation arises i.e., the demand is more than the ordered quantity, backordering occurs, meaning that some goodwill for buyer i is lost. The goodwill lost cost is given by $\pi_{ri}E(d_i-q_i)^+$, $q_i < d_i$ where π_{ri} is the unit goodwill lost cost of buyer i.

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3.3.3. Service Cost

The buyer provides extra services (δ_i) to attract customers, which requires extra money to invests (η_i) . Customer satisfaction is involved in this situation. If the service is appropriate and satisfactory to the customers, the purpose of giving service is fulfilled. On the other hand, if some customers are not happy with the given service or buyer is incapable to give the standard service, customers may not want to buy products from that buyer as customers have multiple choices to buy the same product. This is the opposite situation of the service, i.e., $(1 - \delta_i)$. Thus, it creates some monetary loss to the buyer, which is indicated as customer satisfaction cost. It has the inverse relation with the provided service. Whenever the service increases, the customer satisfaction increases and thus the cost $(1 - \delta_i)^2 \zeta_i$, related to the customer satisfaction decreases. If η_i is the service cost and ζ_i is the customer satisfaction cost, the relative cost is given by $\frac{\eta_i \delta_i^2}{2} + (1 - \delta_i)^2 \zeta_i$. Therefore, the expected total profit of buyer i is

$$ETP_{ri} = \begin{cases} p_i \mu_i - w q_i - \frac{\eta_i \delta_i^2}{2} - (1 - \delta_i)^2 \zeta_i, & d_i \le q_i \\ p_i q_i - w q_i - \pi_{ri} E(d_i - q_i)^+ - \frac{\eta_i \delta_i^2}{2} - (1 - \delta_i)^2 \zeta_i, & q_i < d_i. \end{cases}$$
(5)

The total profit of buyer is given by

$$ETP_{r}(q_{i}, \delta_{i}) = \sum p_{i}(\mu_{i} + q_{i}) - wQ - \frac{1}{2} \sum \pi_{r_{i}} \left(\sqrt{\sigma i^{2} + (\mu_{i} - d_{i})^{2}} + \mu_{i} - q_{i} \right) - \sum \frac{\eta_{i} \delta_{i}^{2}}{2} - \sum (1 - \delta_{i}^{2}) \zeta_{i}.$$
 (6)

Therefore, the expected total profit of SCM is given by

$$ETP(q_{i}, \delta_{i}, d) = \sum_{i} p_{i}(\mu_{i} + q_{i}) - cQ - \frac{1}{2} \sum_{i} (h_{r_{i}} + h_{m}) \left[\sqrt{\sigma_{i}^{2} + (\mu_{i} - q_{i})^{2}} + (q_{i} - \mu_{i}) \right] - \frac{1}{2} \sum_{i} (\pi_{r_{i}} + \pi_{m}) \left[\sqrt{\sigma_{i}^{2} + (\mu_{i} - q_{i})^{2}} + (\mu_{i} - q_{i}) \right] - \sum_{i} \frac{\eta_{i} \delta_{i}^{2}}{2} - \sum_{i} (1 - \delta_{i})^{2} \zeta_{i} - \frac{f(d)\mu}{Q}$$

$$(7)$$

subject to the conditions

$$d \le \sqrt{3} \left(\lambda - \frac{Log \left[1 - \sqrt[3]{1 - \theta} \right]}{\rho} \right)$$
$$S_t \ge 2S_s.$$

3.4. Solution Methodology

The solution is found for both the coordination and non-coordination cases. The model is solved by using classical optimization techniques. The necessary conditions give the optimum results for the corresponding decision variables and the sufficient conditions give the stability of the solutions. The constraint function of the manufacturer is modified and transferred into an unconstrained function using the Kuhn-Tucker (KT) method. The modified function is given by

$$LETP_{m} = (w - c)Q - \sum_{i} (h_{r_{i}} + h_{m})E(q_{i} - d_{i})^{+} - \pi_{m} \sum_{i} E(d_{i} - q_{i})^{+} - \frac{\mu}{Q} \left(c_{1} \left\lceil \frac{l}{\sqrt{2}S_{t}} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_{t}} \right\rceil \right] + c_{2} \left\lceil \frac{4S_{s}^{2}}{d^{2}} \right\rceil \left\lceil \frac{l}{\sqrt{2}S_{t}} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_{t}} \right\rceil + \lambda_{1} \left\lceil \sqrt{3} \left(\lambda - \frac{\log(1 - \sqrt[3]{1 - \theta})}{\rho} \right) - d \right\rceil + \lambda_{2} (2S_{s} - S_{t}).$$

$$(8)$$

Then, the total profit of the entire SCM is

$$LETP = \sum_{i} p_{i}(\mu_{i} + q_{i}) - cQ - \sum_{i} (h_{r_{i}} + h_{m})E(q_{i} - d_{i})^{+} - \sum_{i} (\pi_{m} + \pi_{r_{i}})E(d_{i} - q_{i})^{+} - \frac{\mu}{Q} \left(c_{1} \left\lceil \frac{l}{\sqrt{2}S_{t}} \right\rceil \right)$$

$$\left\lceil \frac{b}{\sqrt{2}S_{t}} \right\rceil + c_{2} \left\lceil \frac{4S_{s}^{2}}{d^{2}} \right\rceil \left\lceil \frac{l}{\sqrt{2}S_{t}} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_{t}} \right\rceil \right) + \lambda_{1} \left[\sqrt{3} \left(\lambda - \frac{\log(1 - \sqrt[3]{1 - \theta})}{\rho} \right) - d \right]$$

$$+ \lambda_{2} (2S_{s} - S_{t}).$$

$$(9)$$

3.4.1. Non-Coordination Case

The necessary conditions of optimization provide the optimum values of the decision variable for the manufacturer. The value of the decision variable (q_i) is computed by

$$\frac{\partial LETP_{m}}{\partial q_{i}} = (w - c) + \frac{\mu_{i} - q_{i}}{2\sqrt{\sigma_{i}^{2} + (\mu_{i} - q_{i})^{2}}} (h_{r_{i}} + h_{m} + \pi_{m}) - \frac{1}{2} (h_{r_{i}} + h_{m} - \pi_{m}) + \frac{f(d)\mu}{Q^{2}} = 0,$$
i.e., $q_{i} = \mu_{i} \pm \frac{\sigma_{i}\Gamma_{1}}{\sqrt{1 - \Gamma_{1}^{2}}}$, (10)

where

$$\Gamma_1 = rac{2\Big(w - c + rac{f(d)\mu}{Q^2}\Big) - h_{r_i} - h_m + \pi_m}{h_{r_i} + h_m + \pi_m}.$$

Therefore, $Q = \sum_i q_i$ (from Equation (10)) gives the optimum order quantity for the manufacturer. The optimum distance is given by the following value of d.

$$\frac{\partial LETP_m}{\partial d} = -\lambda_1 + \frac{8c_2S_s^2}{d^3} \left[\frac{l}{\sqrt{2}S_t} \right] \left[\frac{b}{\sqrt{2}S_t} \right] \frac{\mu}{Q} = 0,$$

$$\lambda_1 = \frac{8\mu c_2S_s^2 \left[\frac{l}{\sqrt{2}S_t} \right] \left[\frac{b}{\sqrt{2}S_t} \right]}{Qd^3 \left(\lambda - \frac{\log(1 - \sqrt[3]{1 - \theta})}{\rho} \right)}.$$

$$\frac{\partial LETP_m}{\partial \lambda_1} = \sqrt{3} \left(\lambda - \frac{\log(1 - \sqrt[3]{1 - \theta})}{\rho} \right) - d = 0,$$
i.e., $d = \sqrt{3} \left(\lambda - \frac{\log(1 - \sqrt[3]{1 - \theta})}{\rho} \right).$
(11)

Equation (11) provides the optimum distance between readers. The sufficient conditions prove that the above results represent global solutions.

$$\begin{split} \frac{\partial^2 LETP_m}{\partial q_i^2} &= -\frac{1}{2} \frac{(h_{r_i} + h_m + \pi_m)\sigma_i^2}{3} - \frac{2f(d)\mu}{Q^3} < 0, \\ & \left(\sigma_i^2 + (\mu_i - q_i)^2\right)^{\frac{1}{2}} \\ \frac{\partial^2 LETP_m}{\partial d^2} &= -\frac{24c_2S_s^2\mu}{Qd^4} \left\lceil \frac{l}{\sqrt{2}S_t} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_t} \right\rceil < 0, \\ \frac{\partial^2 LETP_m}{\partial q_i d_i} &= -\frac{\mu\lambda_1}{Q^2} < 0, \end{split}$$

and
$$\begin{pmatrix} \frac{\partial LETP_m}{\partial q_i^2} & \frac{\partial LETP_m}{\partial q_i d_i} \\ \frac{\partial LETP_m}{\partial d_i q_i} & \frac{\partial LETP_m}{\partial d_i^2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \frac{(h_{r_i} + h_m + \pi_m)\sigma_i^2}{3} \\ \left(\sigma_i^2 + (\mu_i - q_i)^2\right)^{\frac{3}{2}} + \frac{2f(d)\mu}{Q^3} \end{pmatrix} \begin{pmatrix} \frac{24c_2S_s^2\mu}{Qd^4} \left\lceil \frac{l}{\sqrt{2}S_t} \right\rceil \left\lceil \frac{b}{\sqrt{2}S_t} \right\rceil \right) \\ + \frac{\mu\lambda_1}{Q^2} > 0.$$

All criterion for the sufficient conditions of a Hessian matrix are satisfied proving the stability of the optimum solution. Therefore, the values of the decision variables are the optimum for the manufacturer.

The optimum values of the decision variables for the buyer are given by the following necessary conditions for optimization.

$$\frac{\partial ETP_r}{\partial q_i} = 0,$$
i.e., $q_i = \mu_i \pm \frac{\sigma_i \Gamma_2}{\sqrt{1 - \Gamma_2^2}},$
(12)

where

$$\Gamma_2 = \frac{2(p_i - w) + \pi_{r_i}}{\pi_{r_i}}.$$

The optimum order quantity for the buyer i is given by Equation (12). Equation (13) gives the optimum service provided by the buyer i to customers.

$$\frac{\partial ETP_r}{\partial \delta_i} = -\eta_i \delta_i + 2\zeta_i (1 - \delta_i) = 0,$$
i.e., $\delta_i = \frac{2\zeta_i}{\eta_i + 2\zeta_i}$. (13)

This sufficient condition proves the global nature of the solution.

$$\begin{split} \frac{\partial ETP_r}{\partial q_i^2} &= -\frac{\pi_{r_i}\sigma_i^2}{2\left(\sigma_i^2 + (\mu_i - q_i)^2\right)^{\frac{3}{2}}} < 0, \\ & 2\left(\sigma_i^2 + (\mu_i - q_i)^2\right)^{\frac{3}{2}} \end{split}$$

$$\frac{\partial ETP_r}{\partial \delta_i^2} &= -\eta_i - 2\zeta_i < 0, \\ \frac{\partial^2 ETP_r}{\partial \delta_i q_i} &= 0, \\ \text{i.e., } \begin{pmatrix} \frac{\partial ETP_r}{\partial q_i^2} & \frac{\partial ETP_r}{\partial q_i \delta_i} \\ \frac{\partial ETP_r}{\partial \delta_i q_i} & \frac{\partial ETP_r}{\partial \delta_i^2} \end{pmatrix} = \frac{\pi_{r_i}\sigma_i^2 \left(\eta_i + 2\zeta_i\right)}{2\left(\sigma_i^2 + (\mu_i - q_i)^2\right)^{\frac{3}{2}}} > 0. \end{split}$$

The Algorithm 1 is developed to find the numerical results from theory. The following steps help to solve the model numerically.

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Algorithm 1:	
Step 1	Input all values of all relevant parameters. Set the value of <i>i</i> .
Step 2	Set the initial values of q_i for manufacturer and buyers.
Step 3	Write down the values of q_i from the Equation (10) and d from the Equation (11) for manufacturer.
	For buyers, the values of q_i and δ_i are given by Equation (12) and (13), respectively.
Step 4	Find the value of q_i , δ_i , and d using the values from Step 1 and Step 2.
Step 4.a	If $q_i \ge q_{i+1}$ and $\delta_i \ge \delta_{i+1}$, then terminate the process. The optimum values are obtained as q_i^* δ_i^* , and d^* .
Step 4.b	Else if $q_i < q_{i+1}$ and $\delta_i < \delta_{i+1}$, go to Step 4.
Step 4.c	Increment of i as $i = i + 1$.
Step 5	Stop.

3.4.2. Coordination Case

The results for the joint profit of the entire SCM are given by the following necessary conditions.

$$\frac{\partial ETP}{\partial q_{i}} = p_{i} - c + \frac{f(d)\mu}{Q^{2}} + \frac{\mu_{i} - q_{i}}{2\sqrt{\sigma_{1}^{2} + (\mu_{i} - q_{i})^{2}}} (h_{r_{i}} + h_{m} + \pi_{r_{i}} + \pi_{m}) = 0$$

$$- \frac{1}{2} (h_{r_{i}} + h_{m} - \pi_{r_{i}} - \pi_{m}),$$
i.e., $q_{i} = \mu_{i} \pm \frac{\sigma_{i} \Gamma_{3}}{\sqrt{1 - \Gamma_{3}^{2}}},$
(14)

where

$$\Gamma_{3} = \frac{2\left(p_{i}-c+\frac{f(d)\mu}{Q^{2}}\right) - h_{r_{i}} - h_{m} + \pi_{r_{i}} + \pi_{m}}{h_{r_{i}} + h_{m} + \pi_{r_{i}} + \pi_{m}},$$
and
$$\frac{\partial ETP}{\partial \delta_{i}} = -\eta_{i}\delta_{i} - 2(1-\delta_{i})\zeta_{i}(-1) = 0,$$
i.e.,
$$\delta_{i} = \frac{2\zeta_{i}}{\eta_{i} + 2\zeta_{i}}.$$

$$(15)$$

The optimum order quantity is given by Equation (14) and service is given by Equation (15). Using the necessary conditions, one has

$$\frac{\partial LETP}{\partial \lambda_{1}} = \sqrt{3} \left(\lambda - \frac{\log(1 - \sqrt[3]{1 - \theta})}{\rho} \right) - d = 0,$$
i.e., $d = \sqrt{3} \left(\lambda - \frac{\log(1 - \sqrt[3]{1 - \theta})}{\rho} \right),$

$$\frac{\partial LETP}{\partial d} = -\lambda_{1} + \frac{8c_{2}S_{s}^{2}}{d^{3}} \left\lceil \frac{l}{\sqrt{2S_{t}}} \right\rceil \left\lceil \frac{b}{\sqrt{2S_{t}}} \right\rceil \frac{\mu}{Q} = 0,$$
i.e., $\lambda_{1} = \frac{8c_{2}S_{s}^{2}\mu \left\lceil \frac{l}{\sqrt{2S_{t}}} \right\rceil \left\lceil \frac{b}{\sqrt{2S_{t}}} \right\rceil}{Qd^{3}}.$
(16)

Equation (16) gives the optimum distance between two RFID readers. From the sufficient conditions, it can be concluded that since the second order derivatives are negative definite and the values of the Hessian matrix alternate, the required values of the decision variables are global.

$$\frac{\partial^{2}ETP}{\partial q_{i}^{2}} = -\frac{1}{2} \frac{(h_{r_{i}} + h_{m} + \pi_{r_{i}} + \pi_{m})\sigma_{i}^{2}}{3} - \frac{2f(d)\mu}{Q^{3}} < 0,$$

$$\left(\sigma_{i}^{2} + (\mu_{i} - q_{i})^{2}\right)^{\frac{3}{2}}$$

$$\frac{\partial^{2}ETP}{\partial \delta_{i}^{2}} = -\eta_{i}^{2} - 2\zeta_{i} < 0,$$

$$\frac{\partial^{2}ETP}{\partial d^{2}} = -\frac{24c_{2}S_{s}^{2}}{d^{4}} \left[\frac{l}{\sqrt{2}S_{t}}\right] \left[\frac{b}{\sqrt{2}S_{t}}\right] \frac{\mu}{Q} < 0.$$

Now, the calculation of the principal minors gives

$$H_{1} = \begin{pmatrix} \frac{\partial^{2}ETP}{\partial q_{i}^{2}} & \frac{\partial^{2}ETP}{\partial q_{i}\partial\delta_{i}} \\ \frac{\partial^{2}ETP}{\partial\delta_{i}\partial q_{i}} & \frac{\partial^{2}ETP}{\partial\delta_{i}^{2}} \end{pmatrix} = \begin{bmatrix} \frac{\left(h_{r_{i}} + h_{m} + \pi_{r_{i}} + \pi_{m}\right)\sigma_{i}^{2}}{3} + \frac{2f(d)\mu}{Q^{3}} \end{bmatrix} \begin{bmatrix} \eta_{i}^{2} + 2\zeta_{i} \end{bmatrix} > 0,$$

$$H_{2} = \begin{pmatrix} \frac{\partial^{2}ETP}{\partial q_{i}^{2}} & \frac{\partial^{2}ETP}{\partial q_{i}\partial\delta_{i}} & \frac{\partial^{2}ETP}{\partial q_{i}\partial\delta_{i}} \\ \frac{\partial^{2}ETP}{\partial\delta_{i}\partial q_{i}} & \frac{\partial^{2}ETP}{\partial\delta_{i}^{2}} & \frac{\partial^{2}ETP}{\partial\delta_{i}\partial d} \\ \frac{\partial^{2}ETP}{\partial d\partial q_{i}} & \frac{\partial^{2}ETP}{\partial d\partial\delta_{i}} & \frac{\partial^{2}ETP}{\partial d^{2}} \end{pmatrix} = -\left(\eta_{i} + 2\zeta_{i}\right) \begin{bmatrix} \frac{\left(h_{r_{i}} + h_{m} + \pi_{r_{i}} + \pi_{m}\right)\sigma_{i}^{2}}{3} + \frac{2f(d)\mu}{Q^{3}} \\ 2\left(\sigma_{i}^{2} + (\mu_{i} - q_{i})^{2}\right)^{\frac{1}{2}} \end{pmatrix}$$

$$\frac{24c_{2}S_{s}^{2}}{d^{4}} \begin{bmatrix} \frac{l}{\sqrt{2}S_{t}} \end{bmatrix} \begin{bmatrix} \frac{b}{\sqrt{2}S_{t}} \end{bmatrix} - \left\{\frac{8\mu c_{2}S_{s}^{2}}{Q^{2}d^{3}} \begin{bmatrix} \frac{l}{\sqrt{2}S_{t}} \end{bmatrix} \begin{bmatrix} \frac{b}{\sqrt{2}S_{t}} \end{bmatrix}^{2} \right\} < 0.$$

Lemma 1. The values of the coordinated case are optimum if the Hessian matrix of third order (H_2) has a value less than zero, i.e., $H_2 < 0$. The required criteria is given by

$$\left(\frac{(h_{r_i}+h_m+\pi_{r_i}+\pi_m)Q^2\sigma_i^2}{\frac{3}{2}}+\frac{2f(d)}{Q}\right)\frac{3}{d}>\left\lceil\frac{l}{\sqrt{2}S_t}\right\rceil\left\lceil\frac{b}{\sqrt{2}S_t}\right\rceil.$$

$$2\mu\left(\sigma_i^2+(\mu_i-q_i)^2\right)^{\frac{3}{2}}$$

This Algorithm 2 helps to find the numerical results. The following steps are required as follows.

Algorithm 2:	
Step 1	Input all parametric values. Set the value of <i>i</i> .
Step 2	Set the initial values of q_i .
Step 3	Write down the values of q_i from the Equation (14), δ_i from Equation (15), and d from the
	Equation (16).
Step 4	Find the value of q_i , δ_i , and d using the values from Step 1 and Step 2.
Step 4.a	If $q_i \ge q_{i+1}$ and $\delta_i \ge \delta_{i+1}$, then terminate the process. The optimum values are obtained as q_i^* δ_i^* , and d^* .
Step 4.b	Else $q_i < q_{i+1}$ and $\delta_i < \delta_{i+1}$, go to Step 4.
Step 4.c	i as $i = i + 1$.
Step 5	Stop.

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3.5. Revenue Sharing (RS)

Instead of a traditional policy, the manufacturer and multiple buyers are involved in a VMI contract. It is the manufacturer's role to support buyer such that the buyers so that they do not face losses due to the contract. Thus, a revenue sharing policy for coordinated supply chain is incurred by the manufacturer. If α (0 < α < 1) is the sharable revenue by the manufacturer from the total profit, then the sharing mechanism for the coordinated case is αETP . The rest of the profit is accounted for by the manufacturer as he invests more in the business.

4. Numerical Experiment

Numerical experiments are used to validate this study numerically. Supportive data are taken from Sarkar et al. [19] and Xiao and Xu [5]. Some data are taken from an industry visit in West Bengal, India, which justifies the industry using this policy for their business. Two examples are provided here.

Example 1. Table 3 gives all input values of the related parameters and Table 4 provides the optimum results for Example 1.

Parameters	Values	Parameters	Values	Parameters	Values
п	2	(π_{r_1}, π_{r_2})	\$(10, 11)/unit	π_m	\$20/unit
(p_1, p_2)	\$(33, 34) /unit	(σ_1, σ_2)	(200, 202)	h_m	\$0.33c/unit/year
(μ_1, μ_2)	(200, 210)unit/year	(η_1, η_2)	\$(2, 3)	c	\$19/unit
(h_{r_1}, h_{r_2})	\$(0.21c, 0.23c)/unit/year	(ζ_1,ζ_2)	(0.7, 0.8)	w	\$30/unit
(c_1, c_2)	\$(140, 90)/reader	ρ	0.032	λ	10
1	200 m	b	200 m	S_s	50 m

Table 3. Input values of the parameters for Example 1.

Table 4. Optimum results from the numerical analysis for Example 1.

Coordination	Case				
Variables	Optimum Values	Number of Readers	Optimum Values	Results	Optimum Values
$ \begin{array}{c} (q_1^*, q_2^*) \\ (s_1^*, s_2^*) \end{array} $	(201.05, 210.82) unit (0.41, 0.35)	Type 1 Type 2	4 8	d* ETP RFID cost	85.56 m \$19,783.46/cycle \$293.48/cycle
Non-Coordin	ation Case				
Manufacture	1				
$\frac{(q_1^*, q_2^*)}{\text{RFID cost}}$	(212.86, 251.57) unit \$82.62/cycle	Type 1 Type 2	4 8	d^* ETP_m	85.56 m \$4431.10/cycle
Buyers					
(q_1^*, q_2^*)	(200, 210) unit	(s_1, s_2)	(0.41, 0.35)	ETP_r	\$15,179.07/cycle

Therefore, \$19,783.46 is the total profit of the entire supply chain. After gaining profit from the business, the manufacturer shares the revenue $\alpha=0.45$ (Xiao and Xu [5]) of the total profit with the buyers, i.e., the manufacturer shares \$8902.56 with the two buyers. Thus, a (\$19,783.46 - \$8902.56) = \$10,880.90 profit is earned by the manufacturer from the VMI contract policy. The required number of Type 1 readers is 4 and the number of Type 2 readers is 8, which cover the total search area.

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Example 2. Table 5 gives all input values of the related parameters and Table 6 provides the optimum results for Example 2.

Parameters	Values	Parameters	Values	Parameters	Values
п	2	(π_{r_1}, π_{r_2})	\$(6, 8)/unit	π_m	\$12/unit
(p_1, p_2)	\$(32, 30) /unit	(σ_1, σ_2)	(200, 202)	h_m	\$0.30c/unit/year
(μ_1,μ_2)	(190, 195) unit/year	(η_1, η_2)	\$(1.8, 1.5)	C	\$18/unit
(h_{r_1}, h_{r_2})	\$(0.18c, 0.19c)/unit/year	(ζ_1,ζ_2)	(0.6, 0.5)	w	\$27/unit
(c_1, c_2)	\$(138, 100)/reader	ρ	0.059	λ	12
ì	210 m	b	190 m	S_{s}	45 m

Table 5. Input values of the parameters for Example 2.

Table 6. Optimum results from the numerical analysis for Example 2.

Coordination	Case				
Variables	Optimum Values	Number of Readers	Optimum Values	Results	Optimum Values
(q_1^*, q_2^*)	(190.07, 195.19) unit	Type 1	4	d*	63.37 m
(s_1^*, s_2^*)	(0.40, 0.40)	Type 2	12	ETP	\$17,122.01/cycle
				RFID cost	\$434.65/cycle
Non-Coordin	ation Case				
Manufacture	r				
(q_1^*, q_2^*)	(216.36, 277.90) unit	Type 1	4	d*	63.37 m
RFID cost	\$82.45/cycle	Type 2	12	ETP_m	\$3439.95/cycle
Buyers					
(q_1^*, q_2^*)	(190, 195) unit	(s_1, s_2)	(0.40, 0.40)	ETP_r	\$13,464.34/cycle

\$17,122.01 is the total profit of the entire supply chain for Example 2. The manufacturer shares the revenue $\alpha=0.45$ (Xiao and Xu [5]) of the total profit with the buyers, i.e., the manufacturer shares \$7704.90 with the two buyers for the coordination business policy. Thus, a (\$17,122.01 - \$7704.90) = \$9417.11 profit is earned by manufacturer from the VMI contract policy. The total search area is covered by 4 number of Type 1 readers and 12 number of Type 2 readers.

Comparative Study of the Coordination and Non-Coordination Cases

From Table 7, it is seen that, manufacturer and buyer's profit in the coordination case are higher than the non-coordination case for both of the examples. The results conclude that the coordination VMI is more beneficial for both business participants. It is seen that the coordination policy is beneficial for both the manufacturer and the total supply chain profit, whereas buyers get more profit in the non-coordination policy than then coordination case. The shared revenue to the buyers in the coordinated case is less than the profit earned from the non-coordination case. As in the non-coordination policy, buyers can move freely according to their surrounding phenomenon, but in the coordination policy, the joint profit for the entire supply chain is more important for a long-term business rather than an individual one. Even though the profit of buyers is less in the coordination case, they do not face any loss from the business. In both cases of coordination and non-coordination policy, the manufacturer needs same number of readers as the area of the manufacturer is fixed for both of the cases.

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	Example 1		Example 2	
Participant(s)	Coordination Case	Non-Coordination Case	Coordination Case	Non-Coordination Case
Manufacturer	\$10,880.90	\$4431.10	\$9417.11	\$3439.95
Buyers	\$8902.56	\$15,179.07	\$7704.9	\$13,464.34
SCM	\$19,783.46	\$19,610.17	\$17,122.01	\$16,904.29

Table 7. Comparative study between the coordination and non-coordination cases.

5. Discussion

Service is provided to the customers by buyers. This extra service makes an effect to the customers of satisfaction that they are happy and satisfied after buying products from that buyer. Whenever the service level increases, the satisfaction increases.

The sensitivities of the cost parameters of Example 1 over the total profit are depicted in Table 8. It is found that the manufacturing cost c is the most profit sensitive parameter relative to the others. Positive percentage changes of the parameter are more sensitive than negative changes, i.e., profit loss will be more whenever the cost increases. For the holding cost of the manufacturer (h_m) , whenever h_m decreases and increases, the total profit decreases and increases, respectively. Negative percentage changes of h_m result in a smaller q_i , which leads to an increased RFID cost, i.e., decreasing h_m increases the radio frequency cost per cycle. The holding cost of the buyers and the shortage costs of the manufacturer and buyers have the same type of positive and negative changes. The service investment of the buyers has the usual impact on total profit, where increasing the investment causes less profit and vice-versa.

Parameters	Percentage Changes	Changes in Profit (%)	Parameters	Percentage Changes	Changes in Profit (%)
	-20	0.008		-20	0.02
	-10	0.005		-10	0.011
h_{r_1}	+10	-0.008	h_{r_2}	+10	-0.13
•	+20	-0.02	-	+20	-0.03
	-20	-2.52		-20	0.002
	-10	-2.41		-10	0.001
1.	+10	-0.04	π_m	+10	-0.002
h_m	+20	-0.09		+20	-0.005
	-20	0.002		-20	0.0008
	-10	0.001		-10	0.0003
_	+10	-0.001		+10	-0.0001
π_{r_1}	+20	-0.002	π_{r_2}	+20	-0.0003
	-20	0.0002		-20	0.0002
	-10	0.00007		-10	0.00008
	+10	-0.0001		+10	-0.0001
η_1	+20	-0.0002	η_2	+20	-0.0002
	-20	4.75			
	-10	1.14			

Table 8. Sensitivity analysis of the key parameters of Example 1.

6. Conclusions and Future Recommendations

+10

+20

The measurement of the distance between two RFID readers could lead an SCM towards sustainability, which not only helps to prevent inventory shrinkage, but also helps to collect used products via RFID tags and readers. The distance between two readers was optimized, and based on this an industry manager can decide how many readers are needed to cover the whole search

-4.29

-26.88

area. Results confirmed that RFID could be profitable for a VMI contract. This business policy was shown to be beneficial for the entire supply chain for the coordinated case. Besides that, a non-coordinated business policy provided profit to both the manufacturer and the buyers. This study ensured that the manufacturer need not be worried about the installation of smart technology by themselves. The manufacturer was benefited from a third-party provider and can mitigate the problems of unreliability within the SCM. Implementation of an RFID system was beneficial for the electronics industry by reducing e-waste and reusing products and parts. However, this study did not consider the reuse of tags of used products, which can be an immediate extension for waste reduction. Within this study, it was assumed that the coverage area for Type 1 and Type 2 readers is perfectly circular. In general, it may not be circular always. Using any other geometrical shape or any non-geometrical shape, the number of the readers can be increased or decreased. Those will be further extensions of this model. This study did not consider any obstacles and interference sources within the range of the RFID readers. Therefore, using one or more obstacles or interference can change the number of Type 1 and Type 2 readers as Type 1 readers are more powerful than Type 2 readers. This study can be extended by optimizing the utilization of human labor and a comparative study can be made of human labor over autonomation. Another realistic scenario is imperfect production for which an autonomation policy can help reduce the unclear scarp faster than human labor.

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Abbreviations

The following abbreviations are used in this manuscript:

SCM Supply chain management VMI Vendor managed inventory

RFID Radio frequency identification

DF Distribution-free RS Revenue sharing JIT Just-in-time

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