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A Single-Stage Manufacturing Model with Imperfect Items, Inspections, Rework, and Planned Backorders

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Abstract: Each industry prefers to sell perfect products in order to maintain its brand image. However, due to a long-run single-stage production system, the industry generally obtains obstacles. To solve this issue, a single-stage manufacturing model is formulated to make a perfect production system without defective items. For this, the industry decides to stop selling any products until whole products are ready to fulfill the order quantity. Furthermore, manufacturing managers prefer product qualification from the inspection station especially when processes are imperfect. The purpose of the proposed manufacturing model considers that the customer demands are not fulfilled during the production phase due to imperfection in the process, however customers are satisfied either at the end of the inspection process or after reworking the imperfect products. Rework operation, inspection process, and planned backordering are incorporated in the proposed model. An analytical approach is utilized to optimize the lot size and planned backorder quantities based on the minimum average cost. Numerical examples are used to illustrate and compare the proposed model with previously developed models. The proposed model is considered more beneficial in comparison with the existing models as it incorporates imperfection, rework, inspection rate, and planned backorders.

Keywords: imperfect manufacturing system; backordering; defective products; rework; inspection

1. Introduction

Economic and production order quantity models have been extensively used in real industrial life for calculation of the optimum lot size. However, these models are based on the assumption that production processes result in perfect quality products always. There are various factors, which affect the manufacturing processes to make that assumption highly unrealistic. Engineers and technical experts reprocess these imperfect products and deliver them as per customer requirements after its qualification from the quality control section. For example, manufacturing processes involved in ultra-precision manufacturing industries including the automobile sector, ship manufacturing, defense and aerospace related products manufacturing industries, and tool manufacturing industries are a few of the common areas where the production of reworked and non-reworked products cannot be eliminated at all. Such products need qualification from the quality control department of the said organization in order to ensure that the product has been manufacturing as per customer requirements. In such setups, it is very rare that the product is delivered without its confirmation from inspection stations, i.e., demands fulfillment during the production stage. This proposed model re-considers the concept of Cárdenas-Barrón [1] to develop rework and backordering for the single-stage manufacturing setup.

Jamal et al. [2] introduced the rework operation to develop an optimal batch size for a single-stage production system. Two models were developed with the assumption that the rework operation can be performed either immediately or at the end of N-cycles. The optimal batch size was calculated based on the average minimum cost function. In the model, it is assumed that customer demands are met during the production process where demands are fulfilling during the production stage although the process is imperfect. Later, Cárdenas-Barrón [1] extended this model by incorporating the concept of planned backorders for a single-stage manufacturing setup. The rework operation was performed immediately at the end of the production process. Cárdenas-Barrón [1] obtained a closed-form solution for this model based on the average system cost. The optimum lot size and backorder quantity was calculated. Recently, Sarkar et al. [3] assumed that defective items may follow three kinds of distribution functions, i.e., uniform, triangular, and beta distribution. The analytical method has been used to calculate the optimal lot size and backorder quantity. These models assume imperfect processes during the production phase on one side and customer demands fulfillment on the other side. Therefore, inventory modeling in most of the models is based on $(p(1-\gamma)-d)$. The production model based on $(p(1-\gamma)-d)$ is based on the assumption that processes deviate from ideal conditions and are unable to produce quality products each and every time. This deviation from the ideal situation results in imperfection in processes that ultimately affect the products produced (p) by these processes. This imperfection results in the production of γ that need rectification / rework. Therefore, this number of items cannot be added to the total inventory buildup rather they need rectification. Hence, the net inventory is $(p(1-\gamma)-d)$ instead of $(p-d)$, which is applicable in the case of the perfect manufacturing environment. Whereas simultaneous demands fulfillment during the production phase is relatively unrealistic. Products need qualification from an inspection station before they are delivered to the customer or warehouse especially when processes are imperfect. Optimum lot sizes were calculated for real life scenarios, resulting in extension to the basic economic order quantity (EOQ) and production quantity models. Lee and Rosenblatt [4] introduced an idea of larger investment in quality control approaches to obtain the benefit of lower defective rate, reduced setup, and holding costs. Ben-Daya and Hariga [5] developed an economic lot size model with an important realistic assumption that perfect production processes deviate at random rate and results in some non-conforming products. Salameh and Jaber [6] extended the EOQ model for electronic industry items in particular with the assumption that 100% screening is performed of all incoming units. They assumed that low quality products could be sold out at the reduced price in a single batch. Later, Goyal and Cárdenas-Barrón [7] used a simple algebraic approach for optimum lot size calculation. The results were almost similar when other approaches were utilized in an imperfect production environment.

Biswas and Sarker [8] developed an optimal lot size for a lean manufacturing system where scrap items were identified 'before', 'during' and 'after' the rework process. Shortages in the production process were met through a buffer station of the qualified finish products. Chiu et al. [9] initiated a random defective rate within an imperfect production system. Pal et al. [10] worked on the production model with a stochastic demand not a stochastic defective rate. Ojha et al. [11] considered Chiu et al.'s [9] model with a quality assurance and rework. Chiu et al. [12] made their own model [9] with service level constraint. Rahim and Al-Hajailan [13] worked on Ojha et al.'s [11] model with a time-varying defective rate. Sarkar [14] considers delay-in-payments and stock-dependent demand. Sarkar et al. [15] converted a single-stage production system into a multi-stage production system when the defective item's reworking would be after n cycles or in each cycle that would be decided, whereas they considered a constant defective rate. Sarkar et al. [16] incorporated lead-time demand in the integrated production model. Ullah and Kang [17] mathematically managed the rejections of defective products and inspection on work-in-process lot-size. Cárdenas-Barrón et al. [18–20] covered three production models with discrete deliveries, multiple shipments, and partial reworks. Wee et al. [21] proposed an alternative solution approach of the production model. Sarkar [22] worked on the improved production model with the concept of reliability without any defective products. Tayyab and Sarkar [23] extended Sarkar et al.'s [3] model with multi-stage but with constant defective products. Sarkar and Moon [24] introduced the model the under inflation and time value of money. Sarkar et al. [25] extended Sarkar's [22] model with time-dependent demand under the reliable production system. Sarkar et al. [26] developed an optimal reliable model with defective products. However, none of the researchers consider the demand after production and full inspection to fulfill the customer's demand. Wee et al. [27] used the renewal reward theorem (RRT) to calculate the average total profit for optimization of the economic production quantity lot size by considering the imperfect production environment and screening constraint. Researchers (Sana [28,29] and Chaudhuri [30]) concluded that imperfect quality items can be reworked for their practical utilization and significance.

The basic objective of the study is to convert and model the imperfect production system into a mathematical form. The purpose of the proposed production model considers the scenario where the customer demands are not fulfilled during the production phase due to imperfection in the process, and customers are satisfied either at the end of the inspection process or after reworking the imperfect products. The rework operation, inspection process, and planned backordering are incorporated in the proposed model. An analytical approach is utilized to optimize the lot size and planned backorder quantities based on the minimum average cost. The introduction has been given in this section. The rest of the paper is modeled as follows: The next section is related to the past and current research studies related to the imperfection in the production model. Section 3 describes the model development with notation, assumptions, and equations required for optimal solutions. In Section 4, numerical examples are used to evaluate and discuss the proposed model's results. Section 5 includes the results and discussion of the numerical experiments performed using the proposed methodology. The sensitivity analysis has been given in detailed form in Section 6. Conclusions and future directions of the research are presented in the last section.

2. Literature Review

Most of the research carried out in the past concentrated on the production process itself. Defective products are produced in an out-of-control status of the process. These products are either scrapped or reworked in most of the research literature. One major aspect of the production setup is the product qualification via the inspection process. Industries including aerospace, automobile, and pharmaceutical cannot deliver the product until and unless it is qualified by the inspection stage. Inspection is considered as a process that consumes resources (time, material, techniques) in a similar way as are consumed by other processes. Relatively less attention has been given to the inspection rate in conjunction with the manufacturing process. However, in most of the inventory related literature, it is assumed that inspection takes either minimal time or is performed in parallel to the manufacturing processes. Whereas inspection rate importance is much required in environments where production processes are imperfect as the probability of non-conforming products increases. Ben-Salem et al. [31,32] considered an environmental issue and sub-control issues within two alternative production maintenance systems where the production degradation is considered without stopping the demanded products for the market. Purohit et al. [33] extended a basic production model with maintenance planning within an integrated approach, even though they considered production and demand in parallel. Ullah et al. [34] explained the inspection errors within the inspection system within a tradition production system. Shin et al. [35] extended Ullah et al.'s [34] model with trade-credit financing. Kim and Sarkar [36] converted the basic single-stage production system to the multi-stage production system where they did not consider initial production and then demand. Diabat et al. [37] extended the basic production model with partial downstream down payment. Kang et al. [38] explained the effect of smart manufacturing technology based on the human quality control system. Sarkar et al. [39] extended the field of the imperfect production with a distribution free approach to calculate shortages during lead-time demand. Omair et al. [40] and Sarkar et al. [41] extended the production model by considering the imperfect production with the inspection process, rework, and rejected products. Kang et al. [42] extended Cárdenas-Barrón's [1] model with safety stock and planned backorders whereas they did not think about imperfect products.

Researchers highlighted the impact of the inspection process on the optimum lot size during the last few decades. Zhang and Gerchak [43] introduced a model based on the joint lot size in addition with the inspection policy. The single period manufacturing problem was considered with uncertainty in demand and inconstant yield. Inspection errors effect on total cost and lot size was highlighted by Ben-Daya and Rahim [44] in a multistage production setup. They assumed that inspection is performed at the end of every stage. Ben-Daya et al. [45] developed models for an integrated system highlighting the impact of different inspection policies including 'no inspection', 'sample based inspection', and 100 percent inspection. Khan et al. [46] presented an extensive review of EOQ models focusing on the model developed by the Salameh and Jaber model [6]. The review classified various extended models based on imperfect items, shortages backordering, quality, and supply chain. Razaei [47] incorporated backorder into the inventory model for an imperfect manufacturing setup. Analytical approaches were used to get the optimal lot size and shortages level. Wee et al. [48] extended Salameh and Jaber's model [6] for the imperfect production setup incorporating the backorders. Taleizadeh et al. [49] developed an economic production order quantity model with disturbance in processes that result in scrap and rework. They considered the backorder with cycle length and optimal backorder quantity as the decision variable. Hu et al. [50] introduced an inventory model that highlighted that the backorder cost per unit is increasing linearly with shortage time. Ganguly et al. [51] considered partial backorders in supply chain management to meet the demand in the market.

Table 1 highlights contributions made during the last several years in the field of inventory management focusing on imperfect production processes. It can be observed in Table 1 that the literature assumes imperfect production processes for a production quantity model along with rework operation, inspection process, and backorder process. Most of the considered research works focuses on the objective to minimize the cost of the production system under the optimization process except the work of Wee et al. [27] (maximizing total profit of the production system). All these extended models assume that customer demands are fulfilled in the production phase. However, products need qualification from an inspection station before they are delivered to the customer especially when the process is imperfect. Therefore, it is possible that demands fulfillment during the production phase is not appropriate as the probability of delivering imperfect quality products to the customer rises. Research gap exists in the available literature, to the best of our knowledge, to ensure qualified products delivery to the end user while working in random imperfect production setup. This paper is an approach towards fulfillment of this gap in the existing literature. From Table 1, it is found that several authors considered production and supply in parallel when imperfect products are in the system, which is impractical to maintain the brand image of the industry. The most benefitted way is that during production, the products supply should not be allowed to meet the demand as imperfect products are there. This is a major research gap within all studies in this direction. Thus, this proposed model is considered to solve this issue.

This model assumes that products can only be consumed at a demand rate when they are either qualified or processes are perfect. The management delivers the manufactured products to the inspection station where products are classified as either qualified, rejected or to be reworked. It is assumed that imperfect products are rewardable. However, the probability of rejected products is low and can be ignored. It may further be added that the backordering process has also been taken into consideration to calculate the optimum backorder quantity in an imperfect manufacturing environment. It is hoped that this model integrates all major aspects (rework, inspection rate and backordering) of the production setup to help managers in making decisions regarding the inventory level and backorder quantity for an imperfect production setup focusing on qualified products delivery to the customer.

Table 1. Author contribution in the production model on the basis of process, rework, inspection, backordering and demand.

Author (s)	Process			Rework Process			Inspection Process			Backordering Process			Demands Fulfillment (*d*)	
	Imperfect	Re-Workable	Scrap	Perfect	Not Allowed	Imperfect	No	Sampling	100%	No Back-Ordering	Lost Sale	Back-Ordering	During Production Phase	After Inspection
Cárdenas-Barrón [1]	√	√		√					√		√	√	√	
Salameh and Jaber [6]	√		√		√		√			√				
Ojha et al. [11]	√	√		√					√	√				
Chiu et al. [12]	√	√	√			√	√					√	√	
Sarkar et al. [15]	√	√		√			√				√		√	
Wee et al. [27]	√		√		√				√			√	√	
Sana [29]	√	√		√			√			√				
Ben-Daya and Rahim [44]	√		√	√				√				√	√	
Ben-Salem et al. [31]	√		√		√									
Ullah et al. [34]	√	√					√			√	√			
Kim and Sarkar [36]	√			√								√	√	
Kang et al. [38]	√			√			√			√			√	
Sarkar et al. [39]	√	√	√	√	√	√	√	√	√	√	√	√	√	
This Paper	√	√	√	√					√			√		√

3. Mathematical Model

The problem starts with the assumption that the optimal lot size and backorder quantity are derived for an imperfect manufacturing environment. Lot size Q arrives at the manufacturing station during each cycle. Due to process imperfection, the γ percent of total products needs rectification as the process goes out-of-control status during the production phase. All these imperfect products are reworkable. Due to imperfection in the process, manufactured products are not delivered to the customer directly, rather these products are processed through an inspection stage with a known inspection rate (M) in order to know about product qualification. On the other side, all reworkable products are re-processed after the inspection process. It is assumed that reworked products are qualified after reprocessing and no re-inspection is required.

Other assumptions are as follows:

1. Demand and production rates are known and constant. During production, no product is to be sold. After production completion, the demand is started as the production system contains the defective product.
2. Production rate is higher than the demand rate.
3. Reworkable products produced during the manufacturing phase are known in percentage.
4. Imperfect products produced during the production processes are re-processed on the same machine with 100% qualification.
5. Customer receives qualified perfect products only to maintain the good brand image of the industry.
6. Qualified products are delivered to the customer after the inspection process.
7. Planned backordering is allowed and the inventory storage space is unlimited.

The following notation has been used to develop the model.

Parameters	
d	demand rate (units/unit time)
p	production rate (units/unit time)
h	inventory holding cost per unit time ($/unit/unit time)
t_1	time required for planned backorder units to be fulfilled when process starts again.
t_2	time required by machining station to manufacture lot size Q
t_3	time required to inspect manufactured units
t_4	time required to rework imperfect units
t_5	time required to consume on hand inventory
t_6	time required to build the backorder inventory
I_1	average inventory developed during machining of lot size Q (units/unit time)
I_2	average inventory produced during rework operation (units/unit time)
I_{max}	maximum average inventory produced during one cycle time (units/unit time)
γ	percentage of imperfect products produced during the production phase (%)
Z	backorder cost per unit product per unit time ($/unit backorder/ unit time)
c	cost of manufacturing per unit product ($/unit)
k	setup cost per lot size ($/setup)
C_i	inventory holding cost per unit of time ($/unit time)
C_b	backorder cost per unit time ($/unit time)
C_s	setup cost per unit time ($/unit time)
C_m	manufacturing cost per unit time ($/unit time)
$TC(Q,B)$	total cost per unit time ($/unit time)
Variables	
Q	lot size (units)
B	backorder quantity (units)

The inventory over time for an economic order production quantity model for a complete cycle is shown below in Figure 1. The process considers planned backorders, manufacturing of lot size, inspection process, and rework process of imperfect products. As stated earlier, customer demands are not fulfilled during the production phase due to the process imperfection. The objective is to ensure qualified products for delivery to the customer. The inventory builds up during the production phase. Demands are not fulfilled in the production phase. Customer demands are fulfilled at the rate d during the rework process only. Some percentages of products are qualified at the rework stage and can be delivered to the customer in order to minimize inventory cost.

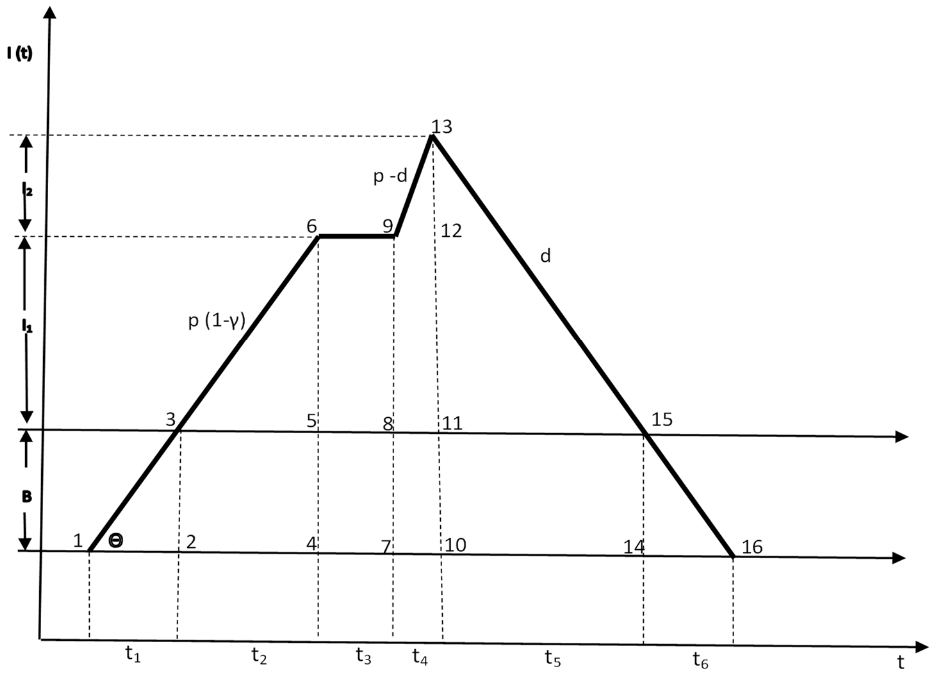

Figure 1. Inventory over time in an imperfect manufacturing environment with inspection and planned backordering.

Considering Figure 1, it is found that the time required to produce lot size Q at the production rate p is the sum of the time required by different processes to produce Q units. These processes include the time required for planned backorder units to be fulfilled when the process starts again (t_1), time required to manufacture lot size Q (t_2), time required to inspect the manufactured units (t_3), and time required to rework imperfect units (t_4), i.e.,

$$t_p = t_1 + t_2 + t_3 + t_4 \tag{1}$$

Obtaining these times in terms of the demand rate (d) and inventory lot size Q,

From Figure 1, the area of Δ 146 is the accumulated by production and backorder inventory, one can obtain

$$\tan\,\theta = \frac{I_1 + B}{t_1 + t_2}$$

$$p(1-\gamma) = \frac{I_1 + B}{t_1 + t_2}$$

$$t_1 + t_2 = \frac{I_1 + B}{p(1-\gamma)} \tag{2}$$

The time required for the inspection of manufactured products (t_3) is given by the following relation at the rate M,

$$t_3 = \frac{I_1 + B}{M} \tag{3}$$

Similarly, t_4 is the time required to rework imperfect products,

$$t_4 = \frac{\gamma Q}{p}$$

In addition, the area of Δ 91213 is the accumulated inventory during reworking, then the time can be obtained as follows:

$$t_4 = \frac{I_2}{(p-d)}$$

i.e.,

$$I_2 = \gamma Q\left(1-\frac{d}{p}\right) \tag{4}$$

Therefore, (1) becomes

$$\frac{Q}{p} = \frac{I_1 + B}{p(1-\gamma)} + \frac{I_1 + B}{M} + \frac{\gamma Q}{p}$$

After some algebraic rearrangements and simplification, one can obtain

$$I_1 = \frac{MQ(1-\gamma)^2}{M + p(1-\gamma)} - B \tag{5}$$

Therefore, from (4) and (5), the maximum inventory is given by

$$I_{max} = I_1 + I_2 = \frac{QM\,(1-\gamma)^2}{M + p(1-\gamma)} - B + \gamma Q\left(1-\frac{d}{p}\right) \tag{6}$$

Assuming

$$\frac{(1-\gamma)^2}{M + p(1-\gamma)} = \theta_1 \text{ and } (1-\frac{d}{p}) = \theta_2, \text{ then}$$

$I_1 = MQ\theta_1 - B$
$I_2 = \gamma Q\theta_2$
$I_{max} = MQ\theta_1 + \gamma Q\theta_2 - B$

The average inventory can be found by taking the sum of the area of triangles and rectangles as shown in Figure 1 divided by the cycle time. The cycle time is defined as the time in which Q units inventory is consumed at the demand rate (d).

From the area Δ 356, the inventory for accruing Q quantity during inspection, area A_1 is given by

$$A_1 = \frac{I_1 \times t_2}{2}$$

where $t_2 = \frac{I_1}{p(1-\gamma)}$, putting in the above equation

$$A_1 = \frac{(MQ\theta_1 - B)^2}{2p(1-\gamma)} \tag{7}$$

The area of Δ 5698 is the area for holding inventory Q quantity during inspection, A_2 is as follows:

$$A_2 = I_1 \times t_3$$

where $t_3 = \frac{(I_1+B)}{M}$, thus

$$A_2 = \frac{(MQ\theta_1 - B)^2 + (B(MQ\theta_1 - B))}{M} \tag{8}$$

Similarly,

$A_3 = I_1 \times t_4$, where $t_4 = \frac{\gamma Q}{p}$

$$A_3 = \left(\frac{MQ^2\theta_1\gamma}{p} - \frac{BQ\gamma}{p} \right) \tag{9}$$

The area of Δ 91213 is the area for reworking, which gives the building of inventory. One can have

$$A_4 = \frac{I_2 \times t_4}{2}$$

$$A_4 = \frac{Q^2\gamma^2\theta_2}{2p} \tag{10}$$

The area A_5 of Δ111315 is the area for demand of products without any production. Thus, the area can be found as follows:

$$A_5 = \frac{I_{max} \times t_5}{2}, \text{ where } t_5 = \frac{I_{max}}{d}$$

Therefore,

$$A_5 = \frac{(MQ\theta_1 + \gamma Q\theta_2 - B)^2}{2d} \tag{11}$$

Therefore, the average inventory $\left(I_{avg}\right)$ per unit cycle time is the summation of (7–10), and (11) divided by the cycle time (t), i.e.,

$$I_{avg} = \left\{ \frac{\left(\frac{(MQ\theta_1 - B)^2}{2p(1-\gamma)}\right) + \left(\frac{(MQ\theta_1 - B)^2 + (B(MQ\theta_1 - B))}{M}\right) + \left(\frac{MQ^2\theta_1\gamma}{p} - \frac{BQ\gamma}{p}\right) + \left(\frac{Q^2\gamma^2\theta_2}{2p}\right) + \left(\frac{(MQ\theta_1 + \gamma Q\theta_2 - B)^2}{2d}\right)}{\frac{Q}{d}} \right\} \tag{12}$$

Algebraic simplification results in the following expression:

$$\begin{aligned} I_{avg} = \{ (& \frac{(M^2Q^2\theta_1^2 + B^2 - 2\,BMQ\theta_1)}{2p(1-\gamma)} + MQ^2\theta_1^2 - BQ\theta_1 + \frac{MQ^2\theta_1\gamma}{p} + \frac{Q^2\gamma^2\theta_2}{2p} - \frac{BQ\gamma}{p} + \\ & \frac{1}{2d}(M^2Q^2\theta_1^2 + B^2 + Q^2{\theta_2}^2\gamma^2 - 2BMQ\theta_1 - 2\,BQ\gamma\theta_2 + 2MQ^2\theta_1\theta_2\gamma))/(\tfrac{Q}{d})\} \end{aligned} \tag{13}$$

Hence, the total inventory holding cost per unit cycle time is given by

$$\begin{aligned} C_i = h\{ (& \frac{(M^2Q^2\theta_1^2 + B^2 - 2\,BMQ\theta_1)}{2p(1-\gamma)} + MQ^2\theta_1^2 - BQ\theta_1 + \frac{MQ^2\theta_1\gamma}{p} + \frac{Q^2\theta_2\gamma^2}{2p} - \frac{BQ\gamma}{p} \\ & + \frac{1}{2d}(M^2Q^2\theta_1^2 + B^2 + Q^2{\theta_2}^2\gamma^2 - 2BMQ\theta_1 - 2\,BQ\gamma\theta_2 + 2MQ^2\theta_1\theta_2\gamma))/(\tfrac{Q}{d})\} \end{aligned} \tag{14}$$

The average backordering cost can be computed by calculating the area of Δ 123 and Δ 141516 which represent the average inventory in the form of backordered products per unit time (t).

Therefore,

$$\begin{aligned} \text{Backorder cost } (C_b) &= (\text{Average backorder inventory}) \times (\text{Unit cos t per unit backorder product per unit of time}) \\ &= \frac{(\text{Area of } (\Delta\ 123) + \text{Area of}(\Delta\ 141516))}{t} \times (z) \\ &= \frac{\left(\frac{B\times t_1}{2} + \frac{B\times t_6}{2}\right)}{\frac{Q}{d}}(z) \end{aligned}$$

where $t_1 = \frac{B}{p(\ 1-\gamma)}$, $t_6 = \frac{B}{d}$, which gives

$$C_b = \left(\frac{B^2(p(1-\gamma)+d)}{2p(1-\gamma)Q}\right)(z) \tag{15}$$

Other important costs taken into consideration are the setup cost (C_s) and the manufacturing cost (C_m).

$$C_s = \frac{kd}{Q} \tag{16}$$

$$C_m = cd(1+\gamma) \tag{17}$$

Therefore, following the relation for all associated costs including the inventory-holding cost, setup cost, backorder cost and manufacturing cost per unit of time.

$$\begin{gathered} TC\ (Q,B) = \ h\{(\frac{M^2Q^2\theta_1^2+B^2-2\ BMQ\theta_1}{2p(1-\gamma)} + MQ^2\theta_1^2 - BQ\theta_1 + \frac{MQ^2\theta_1\gamma}{p} + \frac{Q^2\theta_2\gamma^2}{2p} - \frac{BQ\gamma}{p} + \\ \frac{1}{2d}(M^2Q^2\theta_1^2 + B^2 + Q^2{\theta_2}^2\gamma^2 - 2BMQ\theta_1\ - 2\ BQ\gamma\theta_2 + \ 2MQ^2\theta_1\theta_2\gamma))/(\frac{Q}{d})\} \\ + (\frac{B^2(p(1-\gamma)+d)}{2p(1-\gamma)Q})(z) + cd(1+\gamma) + \frac{kd}{Q} \end{gathered} \tag{18}$$

After algebraic simplification, one obtains:

$$TC\ (Q,B) = \left(\frac{dhM^2\theta_1^2}{2p(1-\gamma)} + dhM\theta_1^2 + \frac{dh\theta_2\gamma^2}{2p} + \frac{dhM\theta_1\gamma}{p} + \frac{M^2h{\theta_1}^2}{2} + \frac{h{\theta_2}^2\gamma^2}{2} + \ hM\theta_1\theta_2\gamma\right)Q$$

$$\left(\frac{dh}{p(1-\gamma)} + h + \frac{(p(1-\gamma)+d)z}{p(1-\gamma)}\right)\frac{B^2}{2Q} - \left(\frac{dhM\theta_1}{p(1-\gamma)} + dh\theta_1 + \frac{dh\gamma}{p} + hM\theta_1 + h\theta_2\gamma\right)B\ + cd(1+\gamma) + \frac{kd}{Q}$$

The following symbols are used to simplify the above expression,

$$R_1 = \left(\frac{dhM^2\theta_1^2}{2p(1-\gamma)} + dhM\theta_1^2 + \frac{dh\theta_2\gamma^2}{2p} + \frac{dhM\theta_1\gamma}{p} + \frac{M^2h{\theta_1}^2}{2} + \frac{h{\theta_2}^2\gamma^2}{2} + \ hM\theta_1\theta_2\gamma\right)$$

$$R_2 = \left(\frac{dh}{p(1-\gamma)} + h + \frac{(p(1-\gamma)+d)z}{p(1-\gamma)}\right)$$

$$R_3 = \left(\frac{dhM\theta_1}{p(1-\gamma)} + dh\theta_1 + \frac{dh\gamma}{p} + hM\theta_1 + h\theta_2\gamma\right)$$

thus,

$$TC\ (Q,B) = \ Q(R_1) + \left(\frac{B^2}{2Q}\right)(R_2) - B(R_3) + \frac{kd}{Q}\ + \ cd(1+\gamma) \tag{19}$$

The optimum lot size (Q^*) and backorder quantity (B^*) can be found by minimization of the total cost function assuming Q and B as continues function.

Theorem 1. *$TC^*(Q,B)$ will have global maximum values at Q^* and B^* if* $Q^* = \sqrt{\frac{2kd(R_2)}{2R_1R_2-(R_3)^2}}$ *and* $B^* = \frac{R_3}{R_2}\left(\sqrt{\frac{2kd(R_2)}{2R_1R_2-(R_3)^2}}\right)$.

Proof. The following conditions need to be fulfilled in order to verify that (19) is a convex function.

$$\begin{gathered} \text{I)}\quad \frac{\partial^2 TC\ (Q,\ B)}{\partial Q^2} > \ 0,\ \frac{\partial^2 TC\ (Q,\ B)}{\partial B^2}\ > \ 0 \\ \text{II)}\quad \left(\frac{\partial^2 TC\ (Q,\ B)}{\partial Q^2}\right)\left(\frac{\partial^2 TC\ (Q,\ B)}{\partial B^2}\right) - \left(\frac{\partial^2 TC\ (Q,\ B)}{\partial Q \partial B}\right)^2 > \ 0 \end{gathered} \tag{20}$$

□

Assuming Q and B as continues, first partial derivatives with respect to Q and B of (19) are shown in (21) and (22), respectively.

$$\frac{\partial TC\ (Q,\ B)}{\partial Q} = (R_1) - \left(\frac{B^2}{2Q^2}\right)(R_2) - \frac{kd}{Q^2} \tag{21}$$

$$\frac{\partial TC\ (Q,\ B)}{\partial B} = \left(\frac{B}{Q}\right)(R_2) - (R_3) \tag{22}$$

Similarly,

$$\frac{\partial^2 TC\ (Q,\ B)}{\partial Q^2} = \left(\frac{B^2}{Q^3}\right)(R_2) + \frac{2kd}{Q^3} > 0 \tag{23}$$

$$\frac{\partial^2 TC\ (Q,\ B)}{\partial B^2} = \left(\frac{R_2}{Q}\right) > 0 \tag{24}$$

$$\left(\frac{\partial^2 TC\ (Q,\ B)}{\partial Q^2}\right)\left(\frac{\partial^2 TC\ (Q,\ B)}{\partial B^2}\right) - \left(\frac{\partial^2 TC\ (Q,\ B)}{\partial Q \partial B}\right)^2 = \frac{2kd}{Q^4}(R_2) \tag{25}$$

The necessary and sufficient conditions for (19) to be convex have been fulfilled as shown in (23), (24), and (25). Therefore, an optimal solution exists at which the total cost function will be minimum. Thus, (21) and (22) are simultaneously solved to obtain the optimal lot size (Q) and backorder quantity (B) which are given by:

$$Q^* = \sqrt{\frac{2kd(R_2)}{2R_1R_2 - (R_3)^2}} \tag{26}$$

$$B^* = \frac{R_3}{R_2}\left(\sqrt{\frac{2kd(R_2)}{2R_1R_2 - (R_3)^2}}\right) \tag{27}$$

The optimal total cost can be obtained by substituting (26) and (27) in (19) as follows:

$$TC^*(Q^*, B^*) = \sqrt{\frac{2kd\left(2R_1R_2 - R_3^2\right)}{R_2}} + cd(1+\gamma)$$

Therefore, $TC^*(Q^*, B^*)$ is the global minimum total cost at Q^* and B^*.

This proposed model could be reduced to the basic economic production quantity model (EPQ) if processes are assumed perfect, demand is fulfilled during the production phase and shortages are not expected. In addition, the optimum lot size and optimum backorder quantity obtained by (26) and (27) can be reduced to the mathematical model developed by Cárdenas-Barrón [1] if the assumption of meeting customer demands during the production phase is made.

4. Numerical Examples

Two numerical examples have been used to understand the proposed model utilization for practical engineering problems and its comparison with previously developed models. Data has been taken from the Cárdenas-Barrón's model [1]. Numerical examples illustrate the effect of imperfect products and the inspection rate on the optimum lot size and backorder quantity. The total cost, optimal lot size and optimal backorder quantity at different values of the defect rate and inspection rate are calculated.

4.1. Example 1

The parametric data used for the mathematical experiment of the proposed model include the demand rate (d) = 300 units per year, inspection rate (M) = 550 units per year, production rate (p) = 550 units per year, holding cost (h) = \$50 per unit per year, shortage cost per unit (z) = \$10 per unit short per year, manufacturing cost (c) = \$7 per unit, setup cost (k) = \$50 per unit per year.

4.2. Example 2

The similar type of analysis is performed for another example whose data set is relatively for higher demand and production rate. In this example, the demand rate (d) = 4800 units per year, production rate (p) = 24,000 units per year, setup cost (k) = \$120 per lot size, holding cost (h) = \$0.6 per unit per year, shortages cost (z) = \$14 per unit short per year and manufacturing cost (c) = \$3 per unit. The inspection rate is assumed as 36,000 units per year and defective products produced during the process are varied gradually over an interval (0, 40%).

5. Results and Discussion

Table 2 shows the impact of the defective rate on the optimal lot size (Q) and optimal backorder quantity (B). It can be observed that lot size increases with an increase in the percentage of the defective rate. Similarly, the total cost variation is also significant. Total cost is increased with an increase in imperfection during the manufacturing process. However, an increase in the optimal backorder quantity level is relatively low with an increase in the defective rate. Comparing these results with the results obtained by Cárdenas-Barrón [1] (Example 1), it can be observed that the total cost of our proposed model is comparable for the range of imperfection (0%, 40%). However, the lot size has been increased significantly. The backorder quantity increases up to 35% of the defective rate and then decreases with increases in defective products.

Table 2. Change in optimal lot size and backorder quantity with variation in the defective rate (Example 1).

γ(%)	θ_1	Total Cost (\$/Year)	Q*(Units)	B*(Units)
0	0.000909091	2423.44	93	52
1	0.000895477	2437.49	95	53
5	0.000841492	2493.71	104	57
10	0.00077512	2564.18	118	62
15	0.000710074	2635.20	136	69
20	0.000646465	2707.40	160	79
25	0.000584416	2782.06	191	90
30	0.000524064	2861.69	228	104
35	0.000465565	2950.74	259	113
40	0.000409091	3054.67	262	109

It can be observed in Table 3 that the total cost and optimal lot size increase with an increase in the defective rate significantly at higher demand and production rate. The higher the defective products produced, the higher will be the lot size to be ordered. However, the change in backorder units is less significant for higher production rate and demand rate when the holding cost per unit product is comparatively low. Comparing these results with the results obtained by Cárdenas-Barrón [1] (Example 2), the total cost of our proposed model has not been increased significantly although the inspection process and carrying inventory over a longer period have been incorporated. Relatively larger lot sizes have been proposed by our model.

The change in optimal lot size with the change in the inspection rate has been shown in Figure 2 below. Example 2 data has been used for further analysis. It can be observed that the optimal lot size goes on decreasing with an increase in the inspection rate. It may be noted that the inspection rate is increased from 24,000 units (equal to the production rate) to 42,000 units per year (one and half time higher than the production rate). Defective products rate remain fixed at 20%. All other data remain the same. The total cost and backorder quantity level has been shown in Table 4. It may be noted that the backorder quantity level doesn't change with significant change in the inspection rate at the fixed level of defective products produced.

Table 3. Change in optimal lot size and backorder quantity with variation in the defective rate (Example 2).

γ(%)	θ_1	Total Cost ($/Year)	Q^* (Units)	B^*(Units)
0	0.000016	14,991.78	1947	52
1	0.000016	15,133.44	1954	52
5	0.000015	15,700.49	1985	52
10	0.000014	16,410.33	2020	52
15	0.000012	17,121.43	2052	52
20	0.000011	17,833.88	2080	52
25	0.000010	18,547.80	2103	51
30	0.000009	19,263.32	2120	51
35	0.000008	19,980.57	2131	51
40	0.000007	20,699.69	2135	50

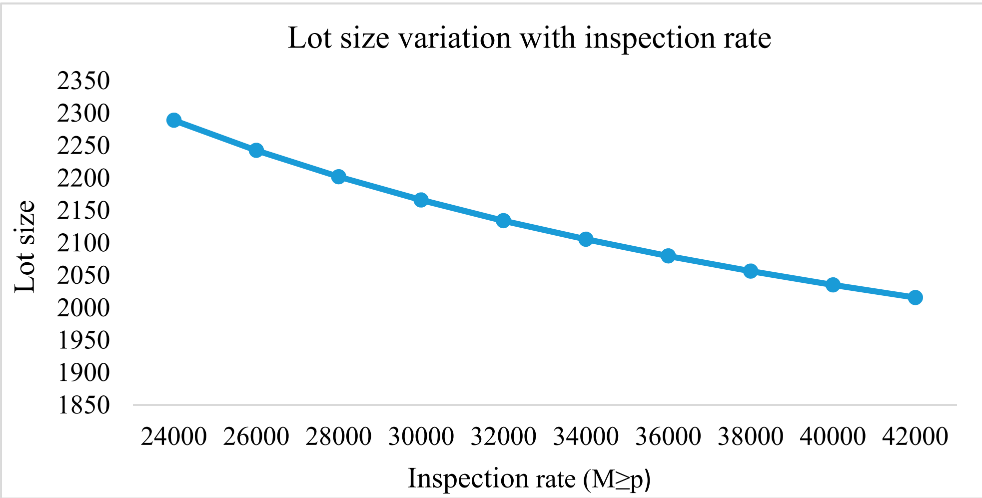

Figure 2. Change in optimal lot size with change in the inspection rate.

Table 4. Change in optimal lot size and backorder quantity with variation in the inspection rate (Example 2).

M (Units Per Year)	θ_1	Total Cost ($/Year)	Q^*(Units)	B^* (Units)
24,000	0.0000148	17,783.27	2289	52
26,000	0.0000141	17,793.67	2243	52
28,000	0.0000135	17,803.15	2202	52
30,000	0.0000130	17,811.81	2166	52
32,000	0.0000125	17,819.77	2134	52
34,000	0.0000120	17,827.10	2106	52
36,000	0.0000115	17,833.88	2080	52
38,000	0.0000112	17,840.17	2057	52
40,000	0.0000108	17,846.02	2035	52
42,000	0.0000104	17,851.47	2016	52

At the end, both the inspection rate and defective products production rate are changed simultaneously with an ascending order and their impact on the lot size, backorder level, and total cost is evaluated. The data has been shown in Table 5. It can be observed that total cost increases with an increase in the inspection and defective rate. Moreover, it can be observed that for higher inspection rate relative to the production process (M>>>p), the optimal lot size calculated by our proposed model approaches the lot size calculated by using the economic order quantity model under ideal conditions.

Table 5. Change in optimal lot size and backorder quantity with variation in the defective rate and inspection rate (Example 2).

M (Units Per Year)	γ(%)	θ_1	Total Cost ($/Year)	Q^* (Units)	B^* (Units)
24,000	0	0.000020	14,914.97	2237	52
26,000	5	0.000018	15,644.61	2196	52
28,000	10	0.000016	16,371.63	2167	52
30,000	15	0.000014	17,096.52	2147	52
32,000	20	0.000125	17,819.77	2134	52
34,000	25	0.000011	18,541.88	2126	52
36,000	30	0.0000092	19,263.32	2120	51
38,000	35	0.0000078	19,984.59	2115	50
40,000	40	0.0000066	20,706.15	2109	50
42,000	45	0.0000054	21,428.45	2100	49

6. Sensitivity Analysis

In order to evaluate the impact of important parameters on the optimal lot size and optimal backorder quantity as proposed by the developed model, sensitive analysis is carried out. It is based on the data shown in Example 2.

Table 6 summarizes the effect of key parameters on the optimal lot size, optimal backorder quantity, and total cost function. Parameters' values have been changed from −50% to +50%. It can be observed that the lot size increases with the increase in values for parameters k, d, p, and γ. Furthermore, the lot size decreases for increased values for parameters h M, *and* z. Similarly, the optimal backorder quantity increases for increased values of parameters k, d, h, and p. The impact of parameters' M, γ, and z is reverse, i.e., optimal backorder quantity decreases for higher values of M and z. Furthermore, the impact of positive change in parameters k, d, h, M, γ, and z values increases the total cost function value. However, the total cost function value decreases with increased values for parameter p. The setup cost and holding cost are key costs for any production system. If all costs are fixed and these costs are increased, the total cost must be increased. Within these costs, it is found that the setup cost is more sensitive than the holding cost. It means that the industry can reduce further the total cost by reducing the setup cost using some initial investment. It is found that the production rate is increased whereas the total cost is reduced. In the proposed model, the production rate is constant and it's increasing value gives decreasing total cost. This implies that the industry can think about the controllable production rate within a certain limit of minimum production rate and maximum production rate. Then, the total cost of the production system can be reduced more. As it is a traditional production system, thus the inspection cost is controlled through human inspection. With the increasing value of this cost, the total cost increases; this implies that the industry manager should think about the smart production system where all inspections controlled through online by the machine and production rate is flexible. The rejection cost is the most sensitive cost as all types of efforts are there with all procedures but there are no products that can be sold to obtain revenue. Thus, the machinery system should be perfect always to produce perfect product always such that there should not be any rejection cost. The shortage cost should not be in the production system as the production rate is greater than the demand but as during production no demanded products are given to the market. Thus, this cost has to incorporate. Even though, the shortage cost is there, the total cost is increasing very less comparing to the other costs.

Table 6. Sensitivity analysis (Example 2).

Parameter	Changes (%)	Parameter Value	Q (Units)	B (Units)	Total Cost ($/Year)
Setup cost (k)	−50%	60	1471	37	17,671.65
	−25%	90	1801	45	17,759.68
	+25%	150	2325	58	17,899.26
	+50%	180	2547	63	17,958.36
Holding cost (h)	−50%	0.3	2910	37	17,675.81
	−25%	0.45	2389	45	17,762.20
	+25%	0.75	1870	58	17,896.04
	+50%	0.9	1716	63	17,951.37
Production rate (p)	−50%	12,000	1720	47	17,949.81
	−25%	18,000	1915	50	17,881.72
	+25%	30,000	2229	53	17,796.91
	+50%	36,000	2366	54	17,766.95
Inspection rate (M)	−50%	18,000	2479	53	17,744.79
	−25%	27,000	2222	52	17,798.52
	+25%	45,000	1990	51	17,858.99
	+50%	54,000	1927	51	17,877.76
Rejection (%)	−50%	0.1	2020	52	16,410.33
	−25%	0.15	2052	52	17,121.43
	+25%	0.25	2103	51	18,547.80
	+50%	0.3	2120	51	19,263.32
Shortage cost (z)	−50%	7.2	2123	102	17,822.63
	−25%	10.8	2094	69	17,830.06
	+25%	18	2071	42	17,836.21
	+50%	21.6	2065	35	17,837.78

7. Conclusions

In this paper, a single stage manufacturing model for the imperfect production setup with rework, inspection, and backordering has been developed. It is assumed that inventory is carried over a longer period of time in the production phase, as the process is imperfect. Due to process imperfection, products are not delivered directly to the customer as observed in most of the conventional production order quantity models. This model optimized the lot size and backorder quantity in an imperfect manufacturing setup when demands are fulfilled after the production process. This approach is against the conventional production order quantity models where processes are imperfect and demands are fulfilled simultaneously. Moreover, products were passed through an inspection station for their qualification. The results were compared via numerical examples to the previously developed model from literature. The numerical examples highlighted the importance of the proposed model as compared to the previous model by considering the quality of the product through the inspection process, however, the cost is slightly been increased. Furthermore, as the demand rate and production rate were increased, the total cost of this model has been reduced in a significant way. The model provides insights to manufacturing engineers working in an imperfect manufacturing setup. It can be best utilized when manufacturing industries deliver products after the inspection process. Total cost is increased when demand is not fulfilled during the production process due to process imperfection under given conditions. Furthermore, the proposed model approaches the production quantity model to calculate the optimal lot size, when the inspection rate is much higher in comparison to the production rate under ideal situations. The model can be extended to incorporate the partial backordering and multistage manufacturing environment to obtain an optimal lot size and backorder level. The consideration of the suppliers and buyers with the imperfection process and sample based inspection can be the extension of the study with the assumption that demands are fulfilled after the inspection process. This production can be incorporated within an integrated inventory model or within a supply chain model to extend it further. The production rate can be considered as variable to make it a smart production system to produce smart products.

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wrote the original draft; M.S. analyzed the data; M.O. contributed by computer software tool to get numerical results. All authors worked equally in reporting revisions and updating the manuscript.

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