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Models for MADM with Single-Valued Neutrosophic 2-Tuple Linguistic Muirhead Mean Operators

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Abstract: In this article, we expand the Muirhead mean (MM) operator and dual Muirhead mean (DMM) operator with single-valued neutrosophic 2-tuple linguistic numbers (SVN2TLNs) to propose the single-valued neutrosophic 2-tuple linguistic Muirhead mean (SVN2TLMM) operator, the single-valued neutrosophic 2-tuple linguistic weighted Muirhead mean (SVN2TLWMM) operator, the single-valued neutrosophic 2-tuple linguistic dual Muirhead mean (SVN2TLDMM) operator, and the single-valued neutrosophic 2-tuple linguistic weighted dual Muirhead mean (SVN2TLWDMM) operator. Multiple attribute decision making (MADM) methods are then proposed using these operators. Finally, we utilize an applicable example for green supplier selection in green supply chain management to prove the proposed methods.

Keywords: multiple attribute decision making (MADM); single-valued neutrosophic 2-tuple linguistic set (SVN2TLSs); SVN2TLMM operator; SVN2TLDMM operator; green supplier selection; green supply chain management

1. Introduction

In order to effectively depict the fuzziness and uncertainty information in real multiple attribute decision making (MADM) problems, Smarandache [1,2] proposed the use of neutrosophic sets (NSs), which have attracted the attention of many scholars. The main advantage of NSs is their capacity to denote inconsistent and indeterminate information. An NS has more potential power than any other fuzzy mathematical tool, such as the fuzzy set [3], the intuitionistic fuzzy set (IFS) [4], and the interval-valued neutrosophic fuzzy set (IVIFS) [5]. However, it is hard to use NSs to solve practical MADM problems. Therefore, Wang et al. [6,7] proposed the use of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), which can include much more information than fuzzy sets, IFSs, and IVIFSs. Ye [8] proposed the use of MADM with the correlation coefficients of SVNSs. Broumi and Smarandache [9] investigated the correlation coefficients of interval neutrosophic numbers (INNs). Biswas et al. [10] proposed the use of single-valued neutrosophic number TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) models. Liu et al. [11] developed the generalized Hamacher operations for SVNSs. Sahin and Liu [12] presented the maximizing deviation method using neutrosophic settings. Ye [13] defined some similarity measures of INSs. Zhang et al. [14] defined some interval neutrosophic information aggregating operators. Ye [15] proposed the use of a simplified neutrosophic set (SNS), which included SVNSs and INSs. Many researchers have given their attention to SNSs. For example, Peng et al. [16] presented some basic operational laws of simplified neutrosophic number (SNNs) and proposed the use of simplified neutrosophic aggregation operators. Additionally, Peng et al. [17] studied an outranking method to handle simplified neutrosophic information, and then Zhang et al. [18] presented an extended version of Peng's method using an interval neutrosophic environment. Liu and Liu [19] developed a generalized weighted power operator with SVNNs. Deli and Subas [20] discussed a method to rank SVNNs. Peng et al. [21] proposed the use of multi-valued neutrosophic sets and defined some power operators for multiple attribute group decision making (MAGDM). Zhang et al. [22] defined the weighted correlation coefficient for INNs. Chen and Ye [23] proposed the use of Dombi operations with SVNNs. Liu and Wang [24] proposed the use of the SVN normalized weighted Bonferroni mean (WBM). Wu et al. [25] proposed the use of prioritized operator and cross-entropy with SNSs in MADM problems. Li et al. [26] developed some SVNN Heronian mean operators in MADM problems. Xu et al. [27] proposed the use of the TODIM (an acronym in Portuguese for Interactive Multi-Criteria Decision Making) method for SVN MADM.

Even though SVNSs have been widely used in some areas, all the existing methods are unsuitable for expressing the truth-membership, indeterminacy-membership, and falsity-membership of an element to a 2-tuple linguistic term set, which can affect a decisionmaker's confidence level when they are making evaluations. In order to overcome this limit, Wu et al. [28] defined the basic concept of single-valued neutrosophic 2-tuple linguistic sets (SVN2TLSs) to cope with this problem on the basis of the SVNSs [6] and 2-tuple linguistic term set [29,30]. Therefore, how to aggregate these single-valued neutrosophic 2-tuple linguistic numbers (SVN2TLNs) is an interesting issue. To solve it, we propose the use of some Muirhead mean (MM) operators with SVN2TLNs. In order to do this, the remainder of this paper is presented as follows: In Section 2, we introduce the concept of SVN2TLSs. In Section 3, we develop some MM operators with SVN2TLNs. In Section 4, we present a numerical example to select green suppliers with SVN2TLNs in order to illustrate the method proposed. Section 5 finishes this paper with some concluding remarks.

2. Preliminaries

Wu et al. [28] proposed the use of the concept of SVN2TLSs based on the SVNSs [6] and 2-tuple linguistic term sets [29,30].

2.1. Single-Valued Neutrosophic 2-Tuple Linguistic Sets

Definition 1 ([28]). *A SVN2TLS A in X is given as follows:*

$$A = \left\{ \left(s_{\theta(x)}, \rho \right), \left(T_A(x), I_A(x), F_A(x), x \in X \right) \right\} \tag{1}$$

where $s_{\theta(x)} \in S$, $T_A(x) \in [0,1], I_A(x) \in [0,1]$, and $F_A(x) \in [0,1]$, with corresponding condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$, $\forall x \in X$. The values $T_A(x), I_A(x)$, and $F_A(x)$ represent, respectively, the truth-membership, the indeterminacy-membership, and the falsity-membership of the element x to the linguistic variable $\left(s_{\theta(x)}, \rho \right)$.

For convenience, Wu et al. [28] called $\widetilde{a} = \langle (s_a, \rho_a), (T_a, I_a, F_a) \rangle$ a single-valued neutrosophic 2-tuple linguistic number, where $T_a \in (0,1), I_a \in (0,1), F_a \in (0,1)$ $0 \leq T_a + I_a + F_a \leq 3$ $s_{\theta(x)} \in S$, and $\rho \in [-0.5, 0.5)$.

Definition 2 ([31]). *Let and $\widetilde{a}_2 = \left\langle (s_{a_2}, \rho_2), (T_{a_2}, I_{a_2}, F_{a_2}) \right\rangle$ be two SVN2TLNs, $S(\widetilde{a}_1) = \Delta\left\langle \Delta^{-1}\left(s_{\theta(a_1)}, \rho_1 \right) \frac{(2+T_{a_1}-I_{a_1}-F_{a_1})}{3} \right\rangle, S(\widetilde{a}_1) \in [0,t]$ and $S(\widetilde{a}_2) = \Delta\left\langle \Delta^{-1}\left(s_{\theta(a_2)}, \rho_2 \right) \frac{(2+T_{a_2}-I_{a_2}-F_{a_2})}{3} \right\rangle, S(\widetilde{a}_2) \in [0,t]$ be the scores values of $\widetilde{a}_1$ and $\widetilde{a}_2$, respectively, and let $H(\widetilde{a}_1) = \Delta\left\langle \Delta^{-1}\left(s_{\theta(a_1)}, \rho_1 \right)(T_{a_1} - F_{a_1}) \right\rangle, H(\widetilde{a}_1) \in [-t,t]$ and $H(\widetilde{a}_2) = \Delta\left\langle \Delta^{-1}\left(s_{\theta(a_2)}, \rho_2 \right)(T_{a_2} - F_{a_2}) \right\rangle, H(\widetilde{a}_2) \in [-t,t]$ be the accuracy degrees of $\widetilde{a}_1$ and $\widetilde{a}_2$, respectively. Then, if $S(\widetilde{a}_1) < S(\widetilde{a}_2), \widetilde{a}_1 < \widetilde{a}_2$; if $S(\widetilde{a}_1) = S(\widetilde{a}_2)$, then (1) if $H(\widetilde{a}_1) = H(\widetilde{a}_2), \widetilde{a}_1 = \widetilde{a}_2$, and (2) if $H(\widetilde{a}_1) < H(\widetilde{a}_2), \widetilde{a}_1 < \widetilde{a}_2$.*

Definition 3 ([32]). *Let and* $\widetilde{a}_2 = \left\langle (s_{a_2}, \rho_2), (T_{a_2}, I_{a_2}, F_{a_2}) \right\rangle$ *be two SVN2TLNs; then, the following is true:*

(1) $\widetilde{a}_1 \oplus \widetilde{a}_2 = \left\langle \Delta\left(\Delta^{-1}\left(s_{\theta(a_1)}, \rho_1\right) + \Delta^{-1}\left(s_{\theta(a_2)}, \rho_2\right)\right), (T_{a_1} + T_{a_2} - T_{a_1}T_{a_2}, I_{a_1}I_{a_2}, F_{a_1}F_{a_2}) \right\rangle$;

(2) $\widetilde{a}_1 \otimes \widetilde{a}_2 = \left\langle \Delta\left(\Delta^{-1}\left(s_{\theta(a_1)}, \rho_1\right)\Delta^{-1}\left(s_{\theta(a_2)}, \rho_2\right)\right), (T_{a_1}T_{a_2}, I_{a_1} + I_{a_2} - I_{a_1}I_{a_2}, F_{a_1} + F_{a_2} - F_{a_1}F_{a_2}) \right\rangle$;

(3) $\lambda\widetilde{a}_1 = \left\langle \Delta\left(\lambda\Delta^{-1}\left(s_{\theta(a_1)}, \rho_1\right)\right), \left(1 - (1 - T_{a_1})^{\lambda}, (I_{a_1})^{\lambda}, (F_{a_1})^{\lambda}\right) \right\rangle, \lambda > 0$;

(4) $(\widetilde{a}_1)^{\lambda} = \left\langle \Delta\left(\Delta^{-1}\left(s_{\theta(a_1)}, \rho_1\right)^{\lambda}\right), \left((T_{a_1})^{\lambda}, 1 - (1 - I_{a_1})^{\lambda}, 1 - (1 - F_{a_1})^{\lambda}\right) \right\rangle, \lambda > 0$,

where Δ^{-1} *is the function of converting the 2-tuple linguistic variables to the exact numbers and* Δ *is the function of converting the computing results to the 2-tuple linguistic variables.*

2.2. MM Operators

Muirhead [33] proposed the use of the MM operator. Tang et al. [34] developed some interval-valued Pythagorean fuzzy Muirhead mean operators. Wang et al. [35] proposed the use of some picture fuzzy Muirhead mean operators in MADM problems.

Definition 4 ([33]). *Let* $a_j (j = 1, 2, \ldots, n)$ *be a set of nonnegative real numbers, and* $P = (p_1, p_2, \ldots, p_n) \in R^n$ *be a vector of parameters if:*

$$\mathrm{MM}^{\mathrm{P}}(a_1, a_2, \ldots, a_n) = \left(\frac{1}{n!}\sum_{\sigma \in S_n}\prod_{j=1}^{n} a_{\sigma(j)}^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}. \quad (2)$$

MM^{P} is the MM operator, where $\sigma(j)(j = 1, 2, \ldots, n)$ is any permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$.

3. Some Muirhead Mean Operators with SVN2TLNs

3.1. The Single-Valued Neutrosophic 2-Tuple Linguistic Muirhead Mean (SVN2TLMM) Operator

This section covers MM and its fusing with SVN2TLNs and proposes the SVN2TLMM operator.

Definition 5. *Let* $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of SVN2TLNs. The SVN2TLMM operator is*

$$\mathrm{SVN2TLMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n) = \left(\frac{1}{n!}\left(\underset{\sigma \in S_n}{\oplus}\left(\overset{n}{\underset{j=1}{\otimes}} \widetilde{a}_{\sigma(j)}^{p_j}\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}. \quad (3)$$

Theorem 1. *Let $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ be a set of SVN2TLNs. The aggregated value by using SVN2TLMM operators is also a SVN2TLN where*

$$\begin{aligned}
&\mathrm{SVN2TLMM}^P(\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n) \\
&= \left(\frac{1}{n!} \left(\underset{\sigma \in S_n}{\oplus} \left(\overset{n}{\underset{j=1}{\otimes}} \widetilde{a}_{\sigma(j)}^{p_j} \right) \right) \right)^{\frac{1}{\sum_{j=1}^n p_j}} \\
&= \left\{ \begin{array}{l}
\Delta\left(\frac{1}{n!} \left(\sum\limits_{\sigma \in S_n} \left(\prod\limits_{j=1}^{n} \left(\Delta^{-1}\left(s_j, \rho_j\right) \right)^{p_j} \right) \right)^{\frac{1}{\sum_{j=1}^n p_j}} \right), \\
\left(\begin{array}{l}
\left(1 - \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(T_{\sigma(j)}\right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}, 1 - \left(1 - \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - I_{\sigma(j)}\right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}, \\
1 - \left(1 - \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - F_{\sigma(j)}\right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}
\end{array} \right)
\end{array} \right\}
\end{aligned} \tag{4}$$

Proof. Based on the exponential operation laws of SVN2TLNs, we can derive

$$\widetilde{a}_{\sigma(j)}^{p_j} = \left\{ \Delta\left(\left(\Delta^{-1}\left(s_j, \rho_j\right) \right)^{p_j} \right), \left(\left(T_{\sigma(j)}\right)^{p_j}, 1 - \left(1 - I_{\sigma(j)}\right)^{p_j}, 1 - \left(1 - F_{\sigma(j)}\right)^{p_j} \right) \right\}. \tag{5}$$

□

Therefore, by utilizing the multiplication operation laws of SVN2TLNs, the $\overset{n}{\underset{j=1}{\otimes}} \widetilde{a}_{\sigma(j)}^{p_j}$ can be derived as follows:

$$\overset{n}{\underset{j=1}{\otimes}} \widetilde{a}_{\sigma(j)}^{p_j} = \left\{ \begin{array}{l}
\Delta\left(\prod\limits_{j=1}^{n} \left(\Delta^{-1}\left(s_j, \rho_j\right) \right)^{p_j} \right), \\
\left(\begin{array}{l}
\prod\limits_{j=1}^{n} \left(T_{\sigma(j)}\right)^{p_j}, 1 - \prod\limits_{j=1}^{n} \left(1 - I_{\sigma(j)}\right)^{p_j}, \\
1 - \prod\limits_{j=1}^{n} \left(1 - F_{\sigma(j)}\right)^{p_j}
\end{array} \right)
\end{array} \right\}. \tag{6}$$

Therefore, according to the addition operation laws of SVN2TLNs, we can obtain

$$\begin{aligned}
&\underset{\sigma \in S_n}{\oplus} \left(\overset{n}{\underset{j=1}{\otimes}} \widetilde{a}_{\sigma(j)}^{p_j} \right) \\
&= \left\{ \begin{array}{l}
\Delta\left(\sum\limits_{\sigma \in S_n} \left(\prod\limits_{j=1}^{n} \left(\Delta^{-1}\left(s_j, \rho_j\right) \right)^{p_j} \right) \right), \\
\left(\begin{array}{l}
1 - \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(T_{\sigma(j)}\right)^{p_j} \right), \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - I_{\sigma(j)}\right)^{p_j} \right), \\
\prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - F_{\sigma(j)}\right)^{p_j} \right)
\end{array} \right)
\end{array} \right\}
\end{aligned} \tag{7}$$

Furthermore, based on the scalar-multiplication operation of SVN2TLNs, we can derive

$$\frac{1}{n!}\left(\underset{\sigma\in S_n}{\oplus}\left(\overset{n}{\underset{j=1}{\otimes}}\widetilde{a}_{\sigma(j)}^{p_j}\right)\right) = \left\{\begin{array}{l} \Delta\left(\frac{1}{n!}\left(\sum_{\sigma\in S_n}\left(\prod_{j=1}^{n}\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)\right), \\ \left(\begin{array}{l} 1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}, \prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-I_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}, \\ \prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-F_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}} \end{array}\right) \end{array}\right\} \tag{8}$$

Therefore, the aggregated value by using SVN2TLMM operators can be listed as follows:

$$\begin{aligned} &\mathrm{SVN2TLMM}^{P}(\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_n) \\ &= \left(\frac{1}{n!}\left(\underset{\sigma\in S_n}{\oplus}\left(\overset{n}{\underset{j=1}{\otimes}}\widetilde{a}_{\sigma(j)}^{p_j}\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n}p_j}} \\ &= \left\{\begin{array}{l} \Delta\left(\frac{1}{n!}\left(\sum_{\sigma\in S_n}\left(\prod_{j=1}^{n}\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right), \\ \left(\begin{array}{l} \left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}, 1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-I_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}, \\ 1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-F_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}} \end{array}\right) \end{array}\right\} \end{aligned} \tag{9}$$

Therefore, Equation (4) is kept. In Equations (4)–(9), the Δ^{-1} is the function of converting the 2-tuple linguistic variables to the exact numbers and Δ is the function of converting the computing results to the 2-tuple linguistic variables.

Then, we need to prove that Equation (4) is a SVN2TLN. We need to prove two conditions as follows:

$$0 \le T \le 1, 0 \le I \le 1, 0 \le F \le 1 \quad 0 \le T + I + F \le 3$$

Let

$$T = \left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$I = 1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-I_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$F = 1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-F_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

Proof. ① Because $0 \le T_{\sigma(j)} \le 1$, we get

$$0 \le \left(T_{\sigma(j)}\right)^{p_j} \le 1 \textit{ and } 0 \le 1-\prod_{j=1}^{n}\left(T_{\sigma(j)}\right)^{p_j} \le 1. \tag{10}$$

Then,

$$0 \le \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(T_{\sigma(j)} \right)^{p_j} \right)^{\frac{1}{n!}} \le 1 \tag{11}$$

$$0 \le \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(T_{\sigma(j)} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \le 1. \tag{12}$$

That means $0 \le T \le 1$, so ① is maintained. Similarly, we can find that $0 \le I \le 1, 0 \le F \le 1$.

$$\text{Because } 0 \le T \le 1, 0 \le I \le 1, 0 \le F \le 1, 0 \le T + I + F \le 3.$$

□

Example 1. *Let* $\langle (s_3, 0), (0.7, 0.5, 0.3) \rangle, \langle (s_4, 0), (0.8, 0.6, 0.2) \rangle, \langle (s_2, 0), (0.6, 0.7, 0.1) \rangle$ *be three SVN2TLNs, and* $\mathrm{P} = (0.3, 0.4, 0.2)$*; then, according to Equation (4), we have*

$$\begin{aligned}
&\mathrm{SVN2TLMM}^{(0.3,0.4,0.2)} \begin{pmatrix} \langle (s_3, 0), (0.7, 0.5, 0.3) \rangle, \langle (s_4, 0), (0.8, 0.6, 0.2) \rangle, \\ \langle (s_2, 0), (0.6, 0.7, 0.1) \rangle \end{pmatrix} \\
&= \left\{ \begin{array}{l} \Delta \left(\frac{1}{n!} \left(\sum_{\sigma \in S_n} \left(\prod_{j=1}^{n} \left(\Delta^{-1} \left(s_j, \rho_j \right) \right)^{p_j} \right) \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \right), \\ \left(\begin{array}{l} \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(T_{\sigma(j)} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, 1 - \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - I_{\sigma(j)} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - F_{\sigma(j)} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \end{array} \right) \end{array} \right\} \\
&= \left(\begin{array}{l} \Delta \left(\frac{1}{3!} \begin{pmatrix} 3^{0.3} \times 4^{0.4} \times 2^{0.2} + 3^{0.3} \times 2^{0.4} \times 4^{0.2} + 4^{0.3} \times 3^{0.4} \times 2^{0.2} \\ + 4^{0.3} \times 2^{0.4} \times 3^{0.2} + 2^{0.3} \times 3^{0.4} \times 4^{0.2} + 2^{0.3} \times 4^{0.4} \times 3^{0.2} \end{pmatrix}^{\frac{1}{0.3+0.4+0.2}} \right), \\
\left(\begin{array}{l} \left(1 - \begin{pmatrix} \left(1 - 0.7^{0.3} \times 0.8^{0.4} \times 0.6^{0.2} \right) \times \left(1 - 0.7^{0.3} \times 0.6^{0.4} \times 0.8^{0.2} \right) \\ \times \left(1 - 0.8^{0.3} \times 0.7^{0.4} \times 0.6^{0.2} \right) \times \left(1 - 0.8^{0.3} \times 0.6^{0.4} \times 0.7^{0.2} \right) \\ \times \left(1 - 0.6^{0.3} \times 0.7^{0.4} \times 0.8^{0.2} \right) \times \left(1 - 0.6^{0.3} \times 0.8^{0.4} \times 0.7^{0.2} \right) \end{pmatrix}^{\frac{1}{3!}} \right)^{\frac{1}{0.3+0.4+0.2}}, \\
1 - \left(1 - \begin{pmatrix} \left(1 - (1-0.5)^{0.4} \times (1-0.6)^{0.3} \times (1-0.7)^{0.2} \right) \\ \times \left(1 - (1-0.5)^{0.4} \times (1-0.7)^{0.3} \times (1-0.6)^{0.2} \right) \\ \times \left(1 - (1-0.6)^{0.4} \times (1-0.5)^{0.3} \times (1-0.7)^{0.2} \right) \\ \times \left(1 - (1-0.6)^{0.4} \times (1-0.7)^{0.3} \times (1-0.5)^{0.2} \right) \\ \times \left(1 - (1-0.7)^{0.4} \times (1-0.5)^{0.3} \times (1-0.6)^{0.2} \right) \\ \times \left(1 - (1-0.7)^{0.4} \times (1-0.6)^{0.3} \times (1-0.5)^{0.2} \right) \end{pmatrix}^{\frac{1}{3!}} \right)^{\frac{1}{0.3+0.4+0.2}}, \\
1 - \left(1 - \begin{pmatrix} \left(1 - (1-0.3)^{0.4} \times (1-0.2)^{0.3} \times (1-0.1)^{0.2} \right) \\ \times \left(1 - (1-0.3)^{0.4} \times (1-0.1)^{0.3} \times (1-0.2)^{0.2} \right) \\ \times \left(1 - (1-0.2)^{0.4} \times (1-0.3)^{0.3} \times (1-0.1)^{0.2} \right) \\ \times \left(1 - (1-0.2)^{0.4} \times (1-0.1)^{0.3} \times (1-0.3)^{0.2} \right) \\ \times \left(1 - (1-0.1)^{0.4} \times (1-0.3)^{0.3} \times (1-0.2)^{0.2} \right) \\ \times \left(1 - (1-0.1)^{0.4} \times (1-0.2)^{0.3} \times (1-0.3)^{0.2} \right) \end{pmatrix}^{\frac{1}{3!}} \right)^{\frac{1}{0.3+0.4+0.2}} \end{array} \right) \end{array} \right) \\
&= \langle (s_4, -0.475), (0.6958, 0.6080, 0.2034) \rangle.
\end{aligned}$$

Then, we can identify some properties of the SVN2TLMM operator.

Property 1. *(Idempotency) If* $\widetilde{a}_{\sigma(j)} = \left\langle \left(s_j, \rho_j\right), \left(T_{\sigma(j)}, I_{\sigma(j)}, F_{\sigma(j)}\right) \right\rangle (j = 1, 2, \ldots, n)$ *are equal, then*

$$\mathrm{SVN2TLMM}^P(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) = \widetilde{a}. \tag{13}$$

Proof. Because $\widetilde{a}_{\sigma(j)} = \widetilde{a} = \langle (s, \rho), (T, I, F) \rangle$, then

$$\begin{aligned}
&\mathrm{SVN2TLMM}^P(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \\
&= \left\{ \begin{array}{l} \Delta\left(\frac{1}{n!} \left(\sum\limits_{\sigma \in S_n} \left(\prod\limits_{j=1}^{n} \left(\Delta^{-1}(s, \rho) \right)^{p_j} \right) \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \right), \\ \left(\begin{array}{l} \left(1 - \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} T^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, 1 - \left(1 - \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} (1 - I)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} (1 - F)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \end{array} \right) \end{array} \right\} \\
&= \left\{ \begin{array}{l} \Delta\left(\frac{1}{n!} \left(n! \left(\left(\Delta^{-1}(s, \rho) \right)^{\sum_{j=1}^{n} p_j} \right) \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \right), \\ \left(\begin{array}{l} \left(1 - \left(\left(1 - T^{\sum_{j=1}^{n} p_j} \right)^{\frac{1}{n!}} \right)^{n!} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, 1 - \left(1 - \left(\left(1 - (1 - I)^{\sum_{j=1}^{n} p_j} \right)^{\frac{1}{n!}} \right)^{n!} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \left(\left(1 - (1 - F)^{\sum_{j=1}^{n} p_j} \right)^{\frac{1}{n!}} \right)^{n!} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \end{array} \right) \end{array} \right\}. \\
&= \left\langle \Delta\left(\Delta^{-1}(s, \rho) \right), (T, I, F) \right\rangle = \widetilde{a}
\end{aligned}$$

□

Property 2. *(Monotonicity) Let* $\widetilde{a}_j = \left\langle \left(s_{a_j}, \rho_{a_j}\right), \left(T_{a_j}, I_{a_j}, F_{a_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *and* $\widetilde{b}_j = \left\langle \left(s_{b_j}, \rho_{b_j}\right), \left(T_{b_j}, I_{b_j}, F_{b_j}\right) \right\rangle$ $(j = 1, 2, \ldots, n)$ *be two sets of SVN2TLNs. If* $\Delta^{-1}\left(S_{a_j}, \rho_{a_j}\right) \leq \Delta^{-1}\left(S_{b_j}, \rho_{b_j}\right)$ *and* $T_{a_j} \leq T_{b_j}$ *and* $I_{a_j} \geq I_{b_j}$ *and* $F_{a_j} \geq F_{b_j}$ *hold for all j, then*

$$\mathrm{SVN2TLMM}^P(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \mathrm{SVN2TLMM}^P\left(\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_n\right). \tag{14}$$

Proof. Let $\mathrm{SVN2TLMM}^P(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) = \langle (s_a, \rho_a), (T_a, I_a, F_a) \rangle$ and $\mathrm{SVN2TLMM}^P\left(\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_n\right) = \langle (s_b, \rho_b), (T_b, I_b, F_b) \rangle$. Given that $\Delta^{-1}\left(S_{a_i}, \rho_{a_i}\right) \leq \Delta^{-1}\left(S_{b_i}, \rho_{b_i}\right)$, we can obtain

$$\prod_{j=1}^{n} \left(\Delta^{-1}\left(S_{a_i}, \rho_{a_i}\right) \right)^{p_j} \leq \prod_{j=1}^{n} \left(\Delta^{-1}\left(S_{b_i}, \rho_{b_i}\right) \right)^{p_j} \tag{15}$$

$$\sum_{\sigma \in S_n} \left(\prod_{j=1}^{n} \left(\Delta^{-1}\left(S_{a_i}, \rho_{a_i}\right) \right)^{p_j} \right) \leq \sum_{\sigma \in S_n} \left(\prod_{j=1}^{n} \left(\Delta^{-1}\left(S_{b_i}, \rho_{b_i}\right) \right)^{p_j} \right). \tag{16}$$

Therefore,

$$\Delta\left(\frac{1}{n!} \left(\sum_{\sigma \in S_n} \left(\prod_{j=1}^{n} \left(\Delta^{-1}\left(S_{a_i}, \rho_{a_i}\right) \right)^{p_j} \right) \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \right) \leq \Delta\left(\frac{1}{n!} \left(\sum_{\sigma \in S_n} \left(\prod_{j=1}^{n} \left(\Delta^{-1}\left(S_{b_i}, \rho_{b_i}\right) \right)^{p_j} \right) \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \right). \tag{17}$$

That means $(S_a, \rho_a) \le (S_b, \rho_b)$. Given that $T_{a_i} \le T_{b_i}$, we can also obtain

$$\prod_{j=1}^{n} T_{a_i}^{p_j} \le \prod_{j=1}^{n} T_{b_i}^{p_j} \tag{18}$$

$$\prod_{\sigma \in S_n}\left(1-\prod_{j=1}^{n} T_{a_i}^{p_j}\right)^{\frac{1}{n!}} \ge \prod_{\sigma \in S_n}\left(1-\prod_{j=1}^{n} T_{b_i}^{p_j}\right)^{\frac{1}{n!}} \tag{19}$$

$$\left(1-\prod_{\sigma \in S_n}\left(1-\prod_{j=1}^{n} T_{a_i}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \le \left(1-\prod_{\sigma \in S_n}\left(1-\prod_{j=1}^{n} T_{b_i}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}. \tag{20}$$

That is $T_a \le T_b$. Similarly, we can obtain $I_a \ge I_b$ and $F_a \ge F_b$.
If $\Delta^{-1}\left(S_{a_i}, \rho_{a_i}\right) < \Delta^{-1}\left(S_{b_i}, \rho_{b_i}\right)$ and $T_{a_i} \le T_{b_i}$ and $I_{a_i} \ge I_{b_i}$ and $F_{a_i} \ge F_{b_i}$,
$\text{SVN2TLMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) < \text{SVN2TLMM}^{P}\left(\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_n\right)$.
If $\Delta^{-1}\left(S_{a_i}, \rho_{a_i}\right) = \Delta^{-1}\left(S_{b_i}, \rho_{b_i}\right)$ and $T_{a_i} < T_{b_i}$ and $I_{a_i} > I_{b_i}$ and $F_{a_i} > F_{b_i}$,
$\text{SVN2TLMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) < \text{SVN2TLMM}^{P}\left(\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_n\right)$.
If $\Delta^{-1}\left(S_{a_i}, \rho_{a_i}\right) = \Delta^{-1}\left(S_{b_i}, \rho_{b_i}\right)$ and $T_{a_i} = T_{b_i}$ and $I_{a_i} = I_{b_i}$ and $F_{a_i} = F_{b_i}$,
$\text{SVN2TLMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) = \text{SVN2TLMM}^{P}\left(\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_n\right)$.
So, Property 2 is correct. □

Property 3. *(Boundedness) Let* $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right)\right\rangle (j = 1, 2, \ldots, n)$ *be a set of SVN2TLNs. If* $\widetilde{a}_i^{+} = (\max_i(S_i, \rho_i), (\max_i(T_i), \min_i(I_i), \min_i(F_i)))$ *and* $\widetilde{a}_i^{-} = (\min_i(S_i, \rho_i), (\min_i(T_i), \max_i(I_i), \max_i(F_i)))$, *then*

$$\widetilde{a}^{-} \le \text{SVN2TLMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \le \widetilde{a}^{+}. \tag{21}$$

From Property 1:

$$\text{SVN2TLMM}^{P}(\widetilde{a}^{-}{}_1, \widetilde{a}^{-}{}_2, \cdots, \widetilde{a}^{-}{}_n) = \widetilde{a}^{-}$$
$$\text{SVN2TLMM}^{P}(\widetilde{a}^{+}{}_1, \widetilde{a}^{+}{}_2, \cdots, \widetilde{a}^{+}{}_n) = \widetilde{a}^{+}$$

From Property 2:

$$\widetilde{a}^{-} \le \text{SVN2TLMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \le \widetilde{a}^{+}.$$

3.2. The Single-Valued Neutrosophic 2-Tuple Linguistic Weighted Muirhead Mean (SVN2TLWMM) Operator

In actual MADM, it is important to consider attribute weights. This section proposes the use of a SVN2TLWMM operator as follows:

Definition 6. *Let* $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right)\right\rangle (j = 1, 2, \ldots, n)$ *be a set of SVN2TLNs with a weight vector of* $w_i = (w_1, w_2, \ldots, w_n)^T$, *thereby satisfying* $w_i \in [0, 1]$ *and* $\sum_{i=1}^{n} w_i = 1$, *and let* $P = (p_1, p_2, \ldots, p_n) \in R^n$ *be a vector of parameters if*

$$\text{SVN2TLWMM}_{nw}^{P}(\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n) = \left(\frac{1}{n!}\left(\underset{\sigma \in S_n}{\oplus}\left(\overset{n}{\underset{j=1}{\otimes}} nw_{\sigma(j)}\widetilde{a}_{\sigma(j)}^{p_j}\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}. \tag{22}$$

SVN2TLWMM_{w}^{P} is the single-valued neutrosophic 2-tuple linguistic MM, where $\sigma(j) (j = 1, 2, \ldots, n)$ is any permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$.

Theorem 2. *Let* $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of SVN2TLNs. The aggregated value by using SVN2TLWMM operators is also a SVN2TLN where*

$$\mathrm{SVN2TLWMM}^{p}_{nw}(\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n) = \left(\frac{1}{n!}\left(\underset{\sigma\in S_n}{\oplus}\left(\overset{n}{\underset{j=1}{\otimes}} nw_{\sigma(j)}\widetilde{a}^{p_j}_{\sigma(j)}\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}$$
$$= \left\{ \begin{array}{l} \Delta\left(\left(\frac{1}{n!}\left(\sum\limits_{\sigma\in S_n}\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\Delta^{-1}\left(s_j, \rho_j\right)\right)^{p_j}\right)\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}\right), \\ \left(\begin{array}{l} \left(1 - \left(\prod\limits_{\sigma\in S_n}\left(1 - \prod\limits_{j=1}^{n}\left(1 - \left(1 - T^{p_j}_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \left(\prod\limits_{\sigma\in S_n}\left(1 - \prod\limits_{j=1}^{n}\left(1 - \left(1 - (1 - I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \left(\prod\limits_{\sigma\in S_n}\left(1 - \prod\limits_{j=1}^{n}\left(1 - \left(1 - (1 - F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \end{array} \right) \end{array} \right\}. \tag{23}$$

Proof. From the exponential operation of the SVN2TLNs, we can derive

$$\widetilde{a}^{p_j}_{\sigma(j)} = \left\langle \Delta\left(\left(\Delta^{-1}\left(s_j, \rho_j\right)\right)^{p_j}\right), \left(T^{p_j}_{\sigma(j)}, 1 - (1 - I_{\sigma(j)})^{p_j}, 1 - (1 - F_{\sigma(j)})^{p_j}\right) \right\rangle. \tag{24}$$

Therefore, by utilizing the scalar-multiplication operation laws of the SVN2TLNs, the $nw_{\sigma(j)}\widetilde{a}^{p_j}_{\sigma(j)}$ can be derived as

$$nw_{\sigma(j)}\widetilde{a}^{p_j}_{\sigma(j)} = \left\{ \begin{array}{l} \Delta\left(nw_{\sigma(j)}\left(\Delta^{-1}\left(s_j, \rho_j\right)\right)^{p_j}\right), \\ \left(\begin{array}{l} 1 - \left(1 - T^{p_j}_{\sigma(j)}\right)^{nw_{\sigma(j)}}, \left(1 - (1 - I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}, \\ \left(1 - (1 - F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}} \end{array} \right) \end{array} \right\} \tag{25}$$

Therefore, according to the multiplication operation of the SVN2TLNs, we can obtain

$$\overset{n}{\underset{j=1}{\otimes}} nw_{\sigma(j)}\widetilde{a}^{p_j}_{\sigma(j)} = \left\{ \begin{array}{l} \Delta\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\Delta^{-1}\left(s_j, \rho_j\right)\right)^{p_j}\right)\right), \\ \left(\begin{array}{l} \prod\limits_{j=1}^{n}\left(1 - \left(1 - T^{p_j}_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right), 1 - \prod\limits_{j=1}^{n}\left(1 - \left(1 - (1 - I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right), \\ 1 - \prod\limits_{j=1}^{n}\left(1 - \left(1 - (1 - F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right). \end{array} \right) \end{array} \right\} \tag{26}$$

Therefore, by utilizing the addition operation of the SVN2TLNs, we can get

$$\begin{aligned}
&\underset{\sigma\in S_n}{\oplus}\left(\underset{j=1}{\overset{n}{\otimes}} nw_{\sigma(j)}\widetilde{a}_{\sigma(j)}^{p_j}\right)\\
&=\left\{\begin{array}{l}
\Delta\left(\sum\limits_{\sigma\in S_n}\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)\right),\\
\left(\begin{array}{l}
1-\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\right),\\
\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right),\\
\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right).
\end{array}\right)
\end{array}\right\}
\end{aligned}\tag{27}$$

Furthermore, based on the scalar-multiplication operation of the SVN2TLNs, we can derive

$$\begin{aligned}
&\frac{1}{n!}\left(\underset{\sigma\in S_n}{\oplus}\left(\underset{j=1}{\overset{n}{\otimes}} nw_{\sigma(j)}\widetilde{a}_{\sigma(j)}^{p_j}\right)\right)\\
&=\left\{\begin{array}{l}
\Delta\left(\frac{1}{n!}\left(\sum\limits_{\sigma\in S_n}\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)\right)\right),\\
\left(\begin{array}{l}
1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}},\\
\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}},\\
\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}
\end{array}\right)
\end{array}\right\}
\end{aligned}\tag{28}$$

Therefore, the aggregated value by using SVN2TLWMM operators can be listed as follows:

$$\begin{aligned}
&\mathrm{SVN2TLWMM}_{nw}^{P}(\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_n)\\
&=\left(\frac{1}{n!}\left(\underset{\sigma\in S_n}{\oplus}\left(\underset{j=1}{\overset{n}{\otimes}} nw_{\sigma(j)}\widetilde{a}_{\sigma(j)}^{p_j}\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\\
&=\left\{\begin{array}{l}
\Delta\left(\left(\frac{1}{n!}\left(\sum\limits_{\sigma\in S_n}\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right),\\
\left(\begin{array}{l}
\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}},\\
1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}},\\
1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}
\end{array}\right)
\end{array}\right\}
\end{aligned}\tag{29}$$

Therefore, Equation (23) is kept. In Equations (23)–(29), Δ^{-1} is the function of converting the 2-tuple linguistic variables to the exact numbers and Δ is the function of converting the computing results to the 2-tuple linguistic variables.

Then we need to prove that Equation (23) is a SVN2TLN.

$$0\le T\le 1, 0\le I\le 1, 0\le F\le 1 \qquad 0\le T+I+F\le 3.$$

□

Proof. Let

$$T=\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$I=1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-(1-I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$F=1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-(1-F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

① Because $0\le T_{\sigma(j)}\le 1$, we get

$$0\le T_{\sigma(j)}^{p_j}\le 1\ and\ 0\le 1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\le 1. \tag{30}$$

Then,

$$0\le 1-\prod_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\le 1 \tag{31}$$

$$0\le\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\le 1 \tag{32}$$

$$0\le\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\le 1. \tag{33}$$

That means $0\le T\le 1$, so ① is maintained.
Similarly, we can get $0\le I\le 1, 0\le F\le 1$,
② Because $0\le T\le 1, 0\le I\le 1, 0\le F\le 1, 0\le T+I+F\le 3$. □

Example 2. *Let* $\langle (s_3,0),(0.7,0.5,0.3)\rangle, \langle (s_4,0),(0.8,0.6,0.2)\rangle$, *and* $\langle (s_2,0),(0.6,0.7,0.1)\rangle$ *be three SVN2TLNs,* $\mathrm{P}=(0.3,0.4,0.2)$ *and* $w=(0.4,0.3,0.3)$*, then according to (23) we have:*

$$
\begin{aligned}
&\mathrm{SVN2TLWMM}^{(0.3,0.4,0.2)}_{(0.4,0.3,0.3)}\begin{pmatrix} \langle (s_3,0),(0.7,0.5,0.3)\rangle, \langle (s_4,0),(0.8,0.6,0.2)\rangle, \\ \langle (s_2,0),(0.6,0.7,0.1)\rangle \end{pmatrix} \\
&= \left\{ \begin{array}{l}
\Delta\left(\left(\frac{1}{n!}\left(\sum\limits_{\sigma\in S_n}\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right), \\
\left(\begin{array}{l}
\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-T_{\sigma(j)}^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}, \\
1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}, \\
1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}
\end{array}\right)
\end{array}\right\}. \\
&\left(\begin{array}{l}
\Delta\left(\frac{1}{3!}\begin{pmatrix} 1.2\times 3^{0.3}\times 0.9\times 4^{0.4}\times 0.9\times 2^{0.2}+1.2\times 3^{0.3}\times 0.9\times 2^{0.4}\times 0.9\times 4^{0.2} \\ +0.9\times 4^{0.3}\times 1.2\times 3^{0.4}\times 0.9\times 2^{0.2}+0.9\times 4^{0.3}\times 0.9\times 2^{0.4}\times 1.2\times 3^{0.2} \\ +0.9\times 2^{0.3}\times 1.2\times 3^{0.4}\times 0.9\times 4^{0.2}+0.9\times 2^{0.3}\times 0.9\times 4^{0.4}\times 1.2\times 3^{0.2} \end{pmatrix}^{\frac{1}{0.3+0.4+0.2}}\right), \\
\left(1-\begin{pmatrix}
\left(1-\left(\left(1-\left(1-0.7^{0.3}\right)^{3\times 0.4}\right)\times\left(1-\left(1-0.8^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.6^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-0.7^{0.3}\right)^{3\times 0.4}\right)\times\left(1-\left(1-0.6^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.8^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-0.8^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.7^{0.4}\right)^{3\times 0.4}\right)\times\left(1-\left(1-0.6^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-0.8^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.6^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.7^{0.2}\right)^{3\times 0.4}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-0.6^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.7^{0.4}\right)^{3\times 0.4}\right)\times\left(1-\left(1-0.8^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-0.6^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.8^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-0.7^{0.2}\right)^{3\times 0.4}\right)\right)\right)
\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.3+0.4+0.2}},
\end{array}\right) \\
&= \left(\begin{array}{l}
1-\left(1-\begin{pmatrix}
\left(1-\left(\left(1-\left(1-(1-0.5)^{0.4}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.6)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.7)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.5)^{0.4}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.7)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.6)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.6)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.5)^{0.3}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.7)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.6)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.7)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.5)^{0.2}\right)^{3\times 0.4}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.7)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.5)^{0.3}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.6)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.7)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.6)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.5)^{0.2}\right)^{3\times 0.4}\right)\right)\right)
\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.3+0.4+0.2}}, \\
1-\left(1-\begin{pmatrix}
\left(1-\left(\left(1-\left(1-(1-0.3)^{0.4}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.2)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.1)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.3)^{0.4}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.1)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.2)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.2)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.3)^{0.3}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.1)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.2)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.1)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.3)^{0.2}\right)^{3\times 0.4}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.1)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.3)^{0.3}\right)^{3\times 0.4}\right)\times\left(1-\left(1-(1-0.2)^{0.2}\right)^{3\times 0.3}\right)\right)\right) \\
\times\left(1-\left(\left(1-\left(1-(1-0.1)^{0.4}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.2)^{0.3}\right)^{3\times 0.3}\right)\times\left(1-\left(1-(1-0.3)^{0.2}\right)^{3\times 0.4}\right)\right)\right)
\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.3+0.4+0.2}}
\end{array}\right) \\
&= \langle (s_3,0.415),(0.6827,0.7900,0.2568)\rangle
\end{aligned}
$$

We will now discuss some properties of the SVN2TLWMM operator.

Property 4. *(Monotonicity) Let* $\widetilde{a}_j=\left\langle\left(s_{a_j},\rho_{a_j}\right),\left(T_{a_j},I_{a_j},F_{a_j}\right)\right\rangle (j=1,2,\ldots,n)$ *and* $\widetilde{b}_j=\left\langle\left(s_{b_j},\rho_{b_j}\right),\left(T_{b_j},I_{b_j},F_{b_j}\right)\right\rangle$ $(j=1,2,\ldots,n)$ *be two sets of SVN2TLNs. If* $\Delta^{-1}\left(S_{a_j},\rho_{a_j}\right)\leq\Delta^{-1}\left(S_{b_j},\rho_{b_j}\right)$ *and* $T_{a_j}\leq T_{b_j}$ *and* $I_{a_j}\geq I_{b_j}$ *and* $F_{a_j}\geq F_{b_j}$ *hold for all j, then:*

$$
\mathrm{SVN2TLWMM}^{P}_{nw}(\widetilde{a}_1,\widetilde{a}_2,\cdots,\widetilde{a}_n)\leq\mathrm{SVN2TLWMM}^{P}_{nw}\left(\widetilde{b}_1,\widetilde{b}_2,\cdots,\widetilde{b}_n\right) \tag{34}
$$

The proof is similar to SVN2TLMM. It is omitted here.

Property 5. *(Boundedness) Let* $\widetilde{a_i} = \langle (s_i, \rho_i), (T_i, I_i, F_i) \rangle (i = 1, 2, \ldots, n)$ *be a set of SVN2TLNs. If* $\widetilde{a_i}^+ = (\max_i(S_i, \rho_i), (\max_i(T_i), \min_i(I_i), \min_i(F_i)))$ *and* $\widetilde{a_i}^- = (\min_i(S_i, \rho_i), (\min_i(T_i), \max_i(I_i), \max_i(F_i)))$ *then:*

$$\widetilde{a}^- \leq \mathrm{SVN2TLWMM}^P_{nw}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \widetilde{a}^+ \tag{35}$$

From Theorem 2, we get:

$$\begin{aligned}&\mathrm{SVN2TLWMM}^P_{nw}(\widetilde{a}^-{}_1, \widetilde{a}^-{}_2, \cdots, \widetilde{a}^-{}_n)\\ &= \left\{\begin{array}{l} \Delta\left(\left(\frac{1}{n!}\left(\sum\limits_{\sigma\in S_n}\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\min\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)\right)\right)^{\frac{1}{\sum_{j=1}^n p_j}}\right),\\ \left(\begin{array}{l}\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-\min T^{p_j}_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}},\\ 1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-\max I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}},\\ 1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-\max F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}\end{array}\right)\end{array}\right\}\end{aligned} \tag{36}$$

$$\begin{aligned}&\mathrm{SVN2TLWMM}^P_{nw}(\widetilde{a}^+{}_1, \widetilde{a}^+{}_2, \cdots, \widetilde{a}^+{}_n)\\ &= \left\{\begin{array}{l} \Delta\left(\left(\frac{1}{n!}\left(\sum\limits_{\sigma\in S_n}\left(\prod\limits_{j=1}^{n}\left(nw_{\sigma(j)}\left(\max\Delta^{-1}\left(s_j,\rho_j\right)\right)^{p_j}\right)\right)\right)\right)^{\frac{1}{\sum_{j=1}^n p_j}}\right),\\ \left(\begin{array}{l}\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-\max T^{p_j}_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}},\\ 1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-\min I_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}},\\ 1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\left(1-(1-\min F_{\sigma(j)})^{p_j}\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}\end{array}\right)\end{array}\right\}\end{aligned} \tag{37}$$

From Property 4, we get:

$$\widetilde{a}^- \leq \mathrm{SVN2TLWMM}^P_{nw}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \widetilde{a}^+ \tag{38}$$

It is obvious that SVN2TLMM operators lacks the property of idempotency.

3.3. The Single-Valued Neutrosophic 2-Tuple Linguistic Dual Muirhead Mean (SVN2TLDMM) Operator

Qin and Liu [36] proposed the use of the dual Muirhead mean (DMM) operator.

Definition 7. *Let* $a_j (j = 1, 2, \ldots, n)$ *be a set of non-negative real numbers and* $P = (p_1, p_2, \ldots, p_n) \in R^n$ *be a vector of parameters if:*

$$\mathrm{DMM}^{\mathrm{P}}(a_1, a_2, \ldots, a_n) = \frac{1}{\sum_{j=1}^n p_j}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n} p_j a_{\sigma(j)}\right)^{\frac{1}{n!}} \tag{39}$$

DMM^{P} is the dual Muirhead mean (DMM) operator, where $\sigma(j)(j = 1, 2, \ldots, n)$ is any permutation of $\{1, 2, \ldots, n\}$, and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$.

In this section we will propose the SVN 2-tuple linguistic DMM (SVN2TLDMM) operator.

Definition 8. *Let $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ be a set of SVN2TLNs and let $P = (p_1, p_2, \ldots, p_n) \in R^n$ be a vector of parameters if:*

$$\text{SVN2TLDMM}^P(\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n) = \frac{1}{\sum_{j=1}^n p_j} \left(\underset{\sigma \in S_n}{\otimes} \left(\overset{n}{\underset{j=1}{\oplus}} \left(p_j \widetilde{a}_{\sigma(j)} \right) \right) \right)^{\frac{1}{n!}} \tag{40}$$

where $\sigma(j)(j = 1, 2, \ldots, n)$ is any permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$.

Theorem 3. *Let $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ be a set of SVN2TLNs. The aggregated value by using the SVN2TLDMM operators is also a SVN2TLN where:*

$$\begin{aligned} &\text{SVN2TLDMM}^P(\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n) \\ &= \frac{1}{\sum_{j=1}^n p_j} \left(\underset{\sigma \in S_n}{\otimes} \left(\overset{n}{\underset{j=1}{\oplus}} \left(p_j \widetilde{a}_{\sigma(j)} \right) \right) \right)^{\frac{1}{n!}} \\ &= \left\{ \begin{array}{l} \Delta \left(\frac{1}{\sum_{j=1}^n p_j} \prod_{\sigma \in S_n} \left(\sum_{j=1}^n \left(p_j \Delta^{-1}\left(s_j, \rho_j\right) \right) \right)^{\frac{1}{n!}} \right), \\ \left(\begin{array}{l} 1 - \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n \left(1 - T_{\sigma(j)} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}, \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n I_{\sigma(j)}^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}, \\ \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n F_{\sigma(j)}^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}. \end{array} \right) \end{array} \right\} \end{aligned} \tag{41}$$

Proof. From the multiplication operation laws of SVN2TLNs depicted in Definition 3, we can obtain:

$$p_j a_{\sigma(j)} = \left\langle \Delta\left(p_j \Delta^{-1}\left(s_j, \rho_j\right) \right), \left(1 - \left(1 - T_{\sigma(j)} \right)^{p_j}, I_{\sigma(j)}^{p_j}, F_{\sigma(j)}^{p_j} \right) \right\rangle \tag{42}$$

Therefore, according to the addition operation of SVN2TLNs, we can derive:

$$\overset{n}{\underset{j=1}{\oplus}} \left(p_j a_{\sigma(j)} \right) = \left\langle \Delta\left(\sum_{j=1}^n \left(p_j \Delta^{-1}\left(s_j, \rho_j\right) \right) \right), \left(1 - \prod_{j=1}^n \left(1 - T_{\sigma(j)} \right)^{p_j}, \prod_{j=1}^n I_{\sigma(j)}^{p_j}, \prod_{j=1}^n F_{\sigma(j)}^{p_j} \right) \right\rangle \tag{43}$$

Therefore, based on the multiplication operation of SVN2TLNs, we can get:

$$\underset{\sigma \in S_n}{\otimes} \left(\overset{n}{\underset{j=1}{\oplus}} \left(p_j a_{\sigma(j)} \right) \right) = \left\{ \begin{array}{l} \Delta\left(\prod_{\sigma \in S_n} \left(\sum_{j=1}^n \left(p_j \Delta^{-1}\left(s_j, \rho_j\right) \right) \right) \right), \\ \left(\begin{array}{l} \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n \left(1 - T_{\sigma(j)} \right)^{p_j} \right), 1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n I_{\sigma(j)}^{p_j} \right), \\ 1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n F_{\sigma(j)}^{p_j} \right) \end{array} \right) \end{array} \right\} \tag{44}$$

Furthermore, by utilizing the exponential operation of SVN2TLNs we can derive:

$$\left(\underset{\sigma\in S_n}{\otimes}\left(\overset{n}{\underset{j=1}{\oplus}}\left(p_j a_{\sigma(j)}\right)\right)\right)^{\frac{1}{n!}}=\left\{\begin{array}{l}\Delta\left(\prod\limits_{\sigma\in S_n}\left(\sum\limits_{j=1}^{n}\left(p_j\Delta^{-1}\left(s_j,\rho_j\right)\right)\right)^{\frac{1}{n!}}\right),\\ \left(\begin{array}{l}\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}},1-\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}I_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}},\\ 1-\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}F_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}}\end{array}\right)\end{array}\right\} \tag{45}$$

Therefore, the aggregated results using the SVN2TLDMM operator can be shown as:

$$\begin{array}{l}\mathrm{SVN2TLDMM}^P(\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_n)\\ =\frac{1}{\sum_{j=1}^n p_j}\left(\underset{\sigma\in S_n}{\otimes}\left(\overset{n}{\underset{j=1}{\oplus}}\left(p_j\widetilde{a}_{\sigma(j)}\right)\right)\right)^{\frac{1}{n!}}\\ =\left\{\begin{array}{l}\Delta\left(\frac{1}{\sum_{j=1}^n p_j}\prod\limits_{\sigma\in S_n}\left(\sum\limits_{j=1}^{n}\left(p_j\Delta^{-1}\left(s_j,\rho_j\right)\right)\right)^{\frac{1}{n!}}\right),\\ \left(\begin{array}{l}1-\left(1-\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}},\left(1-\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}I_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}},\\ \left(1-\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}F_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}.\end{array}\right)\end{array}\right\}\end{array} \tag{46}$$

Therefore, (41) is kept. In Equations (41)–(46), the symbol "Δ^{-1}" is the function of converting the 2-tuple linguistic variables to the exact numbers and "Δ" is the function of converting the computing results to the 2-tuple linguistic variables.

We now need to prove that (41) is a SVN2TLN. We need to prove the two conditions:

$$0\le T\le 1, 0\le I\le 1, 0\le F\le 1 \quad 0\le T+I+F\le 3$$

Let

$$T=1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}$$

$$I=\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}I_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}$$

$$F=\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}F_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}$$

① Since $0\le T_{\sigma(j)}\le 1$, we get:

$$0\le\left(1-T_{\sigma(j)}\right)^{p_j}\le 1 \; and \; 0\le 1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}\right)^{p_j}\le 1 \tag{47}$$

$$0\le 1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\le 1 \tag{48}$$

Then:

$$0 \leq 1 - \left(1 - \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \leq 1 \tag{49}$$

That means $0 \leq T \leq 1$, so ① is maintained.

Similarly, we can get $0 \leq I \leq 1, 0 \leq F \leq 1$

② Because $0 \leq T \leq 1, 0 \leq I \leq 1, 0 \leq F \leq 1, 0 \leq T + I + F \leq 3$.

Example 3. *Let* $\langle (s_3, 0), (0.7, 0.5, 0.3) \rangle, \langle (s_4, 0), (0.8, 0.6, 0.2) \rangle, \langle (s_2, 0), (0.6, 0.7, 0.1) \rangle$ *be three SVN2TLNs, and* $\mathrm{P} = (0.3, 0.4, 0.2)$*; then, according to Equation (41), we have*

$$\begin{aligned}
&\mathrm{SVN2TLDMM}^{(0.3,0.4,0.2)}\begin{pmatrix} \langle (s_3,0),(0.7,0.5,0.3)\rangle, \langle (s_4,0),(0.8,0.6,0.2)\rangle, \\ \langle (s_2,0),(0.6,0.7,0.1)\rangle \end{pmatrix} \\
&= \left\{ \begin{array}{l} \Delta\left(\frac{1}{\sum_{j=1}^{n} p_j} \prod_{\sigma \in S_n}\left(\sum_{j=1}^{n}\left(p_j \Delta^{-1}(s_j, \rho_j)\right)\right)^{\frac{1}{n!}}\right), \\ \left(\begin{array}{l} 1 - \left(1 - \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - T_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \left(1 - \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n} I_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ \left(1 - \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n} F_{\sigma(j)}^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}. \end{array} \right) \end{array} \right\}. \\
&= \left\{ \begin{array}{l} \Delta\left(\frac{1}{0.3+0.4+0.2}\begin{pmatrix} (0.3 \times 3 + 0.4 \times 4 + 0.2 \times 2) \times (0.3 \times 3 + 0.4 \times 2 + 0.2 \times 4) \\ \times (0.3 \times 4 + 0.4 \times 3 + 0.2 \times 2) \times (0.3 \times 4 + 0.4 \times 2 + 0.2 \times 3) \\ \times (0.3 \times 2 + 0.4 \times 3 + 0.2 \times 4) \times (0.3 \times 2 + 0.4 \times 4 + 0.2 \times 3) \end{pmatrix}^{\frac{1}{3!}}\right) \\ \left(\begin{array}{l} 1 - \left(1 - \begin{pmatrix} \left(1 - (1-0.7)^{0.4} \times (1-0.8)^{0.3} \times (1-0.6)^{0.2}\right) \times \left(1 - (1-0.7)^{0.4} \times (1-0.6)^{0.3} \times (1-0.8)^{0.2}\right) \\ \times \left(1 - (1-0.8)^{0.4} \times (1-0.7)^{0.3} \times (1-0.6)^{0.2}\right) \times \left(1 - (1-0.8)^{0.4} \times (1-0.6)^{0.3} \times (1-0.7)^{0.2}\right) \\ \times \left(1 - (1-0.6)^{0.4} \times (1-0.7)^{0.3} \times (1-0.8)^{0.2}\right) \times \left(1 - (1-0.6)^{0.4} \times (1-0.8)^{0.3} \times (1-0.7)^{0.2}\right) \end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.3+0.4+0.2}}, \\ \left(1 - \begin{pmatrix} \left(1 - 0.5^{0.4} \times 0.6^{0.3} \times 0.7^{0.2}\right) \times \left(1 - 0.5^{0.4} \times 0.7^{0.3} \times 0.6^{0.2}\right) \times \left(1 - 0.6^{0.4} \times 0.5^{0.3} \times 0.7^{0.2}\right) \\ \times \left(1 - 0.6^{0.4} \times 0.7^{0.3} \times 05^{0.2}\right) \times \left(1 - 0.7^{0.4} \times 0.5^{0.3} \times 0.6^{0.2}\right) \times \left(1 - 0.7^{0.4} \times 0.6^{0.3} \times 0.5^{0.2}\right) \end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.3+0.4+0.2}}, \\ \left(1 - \begin{pmatrix} \left(1 - 0.3^{0.4} \times 0.2^{0.3} \times 0.1^{0.2}\right) \times \left(1 - 0.3^{0.4} \times 0.1^{0.3} \times 0.2^{0.2}\right) \times \left(1 - 0.2^{0.4} \times 0.3^{0.3} \times 0.1^{0.2}\right) \\ \times \left(1 - 0.2^{0.4} \times 0.1^{0.3} \times 0.3^{0.2}\right) \times \left(1 - 0.1^{0.4} \times 0.3^{0.3} \times 0.2^{0.2}\right) \times \left(1 - 0.1^{0.4} \times 0.2^{0.3} \times 0.3^{0.2}\right) \end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.3+0.4+0.2}} \end{array} \right) \end{array} \right\}. \\
&= \langle (s_3, -0.004), (0.7110, 0.5949, 0.1825) \rangle.
\end{aligned}$$

Similar to the SVN2TLMM operator, we can get the properties as follows:

Property 6. *(Idempotency) If* $\widetilde{a}_{\sigma(j)} = \left\langle \left(s_j, \rho_j\right), \left(T_{\sigma(j)}, I_{\sigma(j)}, F_{\sigma(j)}\right) \right\rangle (j = 1, 2, \ldots, n)$ *are equal, then*

$$\mathrm{SVN2TLDMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) = \widetilde{a}. \tag{50}$$

Property 7. *(Monotonicity) Let* $\widetilde{a}_j = \left\langle \left(s_{a_j}, \rho_{a_j}\right), \left(T_{a_j}, I_{a_j}, F_{a_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *and* $\widetilde{b}_j = \left\langle \left(s_{b_j}, \rho_{b_j}\right), \left(T_{b_j}, I_{b_j}, F_{b_j}\right) \right\rangle$ $(j = 1, 2, \ldots, n)$ *be two sets of SVN2TLNs. If* $\Delta^{-1}\left(S_{a_j}, \rho_{a_j}\right) \leq \Delta^{-1}\left(S_{b_j}, \rho_{b_j}\right)$ *and* $T_{a_j} \leq T_{b_j}$ *and* $I_{a_j} \geq I_{b_j}$ *and* $F_{a_j} \geq F_{b_j}$ *hold for all j, then*

$$\mathrm{SVN2TLDMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \mathrm{SVN2TLDMM}^{P}\left(\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_n\right). \tag{51}$$

Property 8. *(Boundedness) Let* $\widetilde{a}_i = \langle (s_i, \rho_i), (T_i, I_i, F_i) \rangle (i = 1, 2, \ldots, n)$ *be a set of SVN2TLNs. If* $\widetilde{a}_i^{+} = (\max_i(S_i, \rho_i), (\max_i(T_i), \min_i(I_i), \min_i(F_i)))$ *and* $\widetilde{a}_i^{-} = (\min_i(S_i, \rho_i), (\min_i(T_i), \max_i(I_i), \max_i(F_i)))$, *then*

$$\widetilde{a}^{-} \leq \mathrm{SVN2TLDMM}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \widetilde{a}^{+}. \tag{52}$$

3.4. The Single-Valued Neutrosophic 2-Tuple Linguistic Weighted Dual Muirhead Mean (SVN2TLWDMM) Operator

In actual MADM, it is important to consider attribute weights. This section proposes the use of a SVN2TLWDMM operator.

Definition 9. *Let* $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of SVN2TLNs with a weight vector of* $w_i = (w_1, w_2, \ldots, w_n)^T$*, thereby satisfying* $w_i \in [0, 1]$ *and* $\sum_{i=1}^{n} w_i = 1$*, and let* $P = (p_1, p_2, \ldots, p_n) \in R^n$ *be a vector of parameters if*

$$\text{SVN2TLWDMM}_{nw}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) = \frac{1}{\sum_{j=1}^{n} p_j} \left(\underset{\sigma \in S_n}{\otimes} \left(\overset{n}{\underset{j=1}{\oplus}} \left(p_j a_{\sigma(j)}^{nw_{\sigma(j)}} \right) \right) \right)^{\frac{1}{n!}} \tag{53}$$

where $\sigma(j)(j = 1, 2, \ldots, n)$ *is any permutation of* $\{1, 2, \ldots, n\}$ *and* S_n *is the set of all permutations of* $\{1, 2, \ldots, n\}$.

Theorem 4. *Let* $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of SVN2TLNs. The aggregated value by using SVN2TLWDMM operators is also a SVN2TLN where*

$$\begin{aligned}
&\text{SVN2TLWDMM}_{nw}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \\
&= \frac{1}{\sum_{j=1}^{n} p_j} \left(\underset{\sigma \in S_n}{\otimes} \left(\overset{n}{\underset{j=1}{\oplus}} \left(p_j a_{\sigma(j)}^{nw_{\sigma(j)}} \right) \right) \right)^{\frac{1}{n!}} \\
&= \left\{ \begin{array}{l}
\Delta\left(\frac{1}{\sum_{j=1}^{n} p_j} \left(\prod_{\sigma \in S_n} \left(\sum_{j=1}^{n} \left(p_j \left(\Delta^{-1}\left(s_j, \rho_j\right)\right)^{nw_{\sigma(j)}} \right) \right) \right)^{\frac{1}{n!}} \right), \\
\left(\begin{array}{l}
1 - \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - T_{\sigma(j)}^{nw_{\sigma(j)}} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - I_{\sigma(j)}\right)^{nw_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - F_{\sigma(j)}\right)^{nw_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}
\end{array} \right)
\end{array} \right\}
\end{aligned} \tag{54}$$

Proof. From the exponential operation laws of SVN2TLNs depicted in Definition 3, we can ascertain that

$$a_{\sigma(j)}^{nw_{\sigma(j)}} = \left\langle \Delta\left(\left(\Delta^{-1}\left(s_j, \rho_j\right)\right)^{nw_{\sigma(j)}} \right), \left(T_{\sigma(j)}^{nw_{\sigma(j)}}, 1 - \left(1 - I_{\sigma(j)}\right)^{nw_{\sigma(j)}}, 1 - \left(1 - F_{\sigma(j)}\right)^{nw_{\sigma(j)}} \right) \right\rangle. \tag{55}$$

Then, based on the scalar-multiplication operation laws of SVN2TLNs, we can derive

$$p_j a_{\sigma(j)}^{nw_{\sigma(j)}} = \left\{ \begin{array}{l}
\Delta\left(p_j \left(\Delta^{-1}\left(s_j, \rho_j\right)\right)^{nw_{\sigma(j)}} \right), \\
\left(\begin{array}{l}
1 - \left(1 - T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j}, \left(1 - \left(1 - I_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}, \\
\left(1 - \left(1 - F_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}
\end{array} \right)
\end{array} \right\}. \tag{56}$$

Therefore, according to the addition operation laws of SVN2TLNs, we can get

$$\underset{j=1}{\overset{n}{\oplus}}\left(p_j a_{\sigma(j)}^{nw_{\sigma(j)}}\right)=\left\{\begin{array}{l}\Delta\left(\sum_{j=1}^{n}\left(p_j\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{nw_{\sigma(j)}}\right)\right),\\ \left(\begin{array}{l}1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j},\prod_{j=1}^{n}\left(1-\left(1-I_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j},\\ \prod_{j=1}^{n}\left(1-\left(1-F_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\end{array}\right)\end{array}\right\}. \tag{57}$$

Therefore, by utilizing the multiplication operation laws of SVN2TLNs, we can derive

$$\begin{array}{l}\underset{\sigma\in S_n}{\otimes}\left(\underset{j=1}{\overset{n}{\oplus}}\left(p_j a_{\sigma(j)}^{nw_{\sigma(j)}}\right)\right)\\ =\left\{\begin{array}{l}\Delta\left(\prod_{\sigma\in S_n}\left(\sum_{j=1}^{n}\left(p_j\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{nw_{\sigma(j)}}\right)\right)\right),\\ \left(\begin{array}{l}\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j}\right),\\ 1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-I_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right),\\ 1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-F_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right)\end{array}\right)\end{array}\right\}\end{array} \tag{58}$$

Furthermore, by using the exponential operation laws of SVN2TLNs, we can get

$$\begin{array}{l}\left(\underset{\sigma\in S_n}{\otimes}\left(\underset{j=1}{\overset{n}{\oplus}}\left(p_j a_{\sigma(j)}^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\\ =\left\{\begin{array}{l}\Delta\left(\left(\prod_{\sigma\in S_n}\left(\sum_{j=1}^{n}\left(p_j\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right),\\ \left(\begin{array}{l}\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j}\right)^{\frac{1}{n!}},\\ 1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-I_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right)\right)^{\frac{1}{n!}},\\ 1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-F_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}.\end{array}\right)\end{array}\right\}\end{array} \tag{59}$$

Therefore, the fused results using the SVN2TLWDMM operator can be shown as follows:

$$\begin{aligned}
&\mathrm{SVN2TLWDMM}_{nw}^{p}(\widetilde{a}_1,\widetilde{a}_2,\cdots,\widetilde{a}_n)\\
&=\frac{1}{\sum_{j=1}^{n}p_j}\left(\underset{\sigma\in S_n}{\otimes}\left(\overset{n}{\underset{j=1}{\oplus}}\left(p_j a_{\sigma(j)}^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\\
&=\left\{\begin{array}{l}
\Delta\left(\frac{1}{\sum_{j=1}^{n}p_j}\left(\prod_{\sigma\in S_n}\left(\sum_{j=1}^{n}\left(p_j\left(\Delta^{-1}\left(s_j,\rho_j\right)\right)^{nw_{\sigma(j)}}\right)\right)\right)^{\frac{1}{n!}}\right),\\
\left(\begin{array}{l}
1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}},\\
\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-I_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}},\\
\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-F_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}
\end{array}\right)
\end{array}\right\}
\end{aligned} \tag{60}$$

Therefore, Equation (54) is kept. In Equations (54)–(60), Δ^{-1} is the function of converting the 2-tuple linguistic variables to the exact numbers and Δ is the function of converting the computing results to the 2-tuple linguistic variables.

Then, we need to prove that Equation (54) is a SVN2TLN. We need to prove the two conditions:

$$0 \le T \le 1, 0 \le I \le 1, 0 \le F \le 1 \qquad (0 \le T + I + F \le 3$$

□

Proof. Let

$$T = 1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$I = \left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-I_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$F = \left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-F_{\sigma(j)}\right)^{nw_{\sigma(j)}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

① Because $0 \le T_{\sigma(j)} \le 1$, we can get

$$0 \le \left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j} \le 1 \; and \; 0 \le 1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j} \le 1. \tag{61}$$

Then,

$$0 \le \prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j}\right)^{\frac{1}{n!}} \le 1 \tag{62}$$

$$0 \le 1-\left(1-\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-T_{\sigma(j)}^{nw_{\sigma(j)}}\right)^{p_j}\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}} \le 1. \tag{63}$$

That means $0 \le T \le 1$, so ① is maintained.

Similarly, we can get $0 \le I \le 1, 0 \le F \le 1$.
② Because $0 \le T \le 1, 0 \le I \le 1, 0 \le F \le 1, 0 \le T + I + F \le 3$. □

Example 4. *Let* $\langle (s_3, 0), (0.7, 0.5, 0.3) \rangle, \langle (s_4, 0), (0.8, 0.6, 0.2) \rangle, \langle (s_2, 0), (0.6, 0.7, 0.1) \rangle$ *be three SVN2TLNs, and* $P = (0.3, 0.4, 0.2)$ *and* $w = (0.4, 0.3, 0.3)$*; then, according to Equation (54), we have*

$$\mathrm{SVN2TLWDMM}^{(0.3,0.4,0.2)}_{(0.4,0.3,0.3)}\begin{pmatrix} \langle (s_3,0),(0.7,0.5,0.3)\rangle, \langle (s_4,0),(0.8,0.6,0.2)\rangle, \\ \langle (s_2,0),(0.6,0.7,0.1)\rangle \end{pmatrix}$$

$$= \left\{ \begin{array}{l} \Delta\left(\frac{1}{\sum_{j=1}^n p_j} \left(\prod_{\sigma \in S_n} \left(\sum_{j=1}^n \left(p_j \left(\Delta^{-1}(s_j, \rho_j) \right) \right)^{n w_{\sigma(j)}} \right) \right)^{\frac{1}{n!}} \right), \\ \left(\begin{array}{l} 1 - \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n \left(1 - T_{\sigma(j)}^{n w_{\sigma(j)}} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}, \\ \left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\sigma(j)} \right)^{n w_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}}, \\ \left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^n \left(1 - \left(1 - F_{\sigma(j)} \right)^{n w_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^n p_j}} \end{array} \right) \end{array} \right\}.$$

$$= \left(\begin{array}{l} \Delta\left(\frac{1}{0.3+0.4+0.2} \left(\begin{array}{l} \left(0.3 \times 3^{1.2} + 0.4 \times 4^{0.9} + 0.2 \times 2^{0.9}\right) \times \left(0.3 \times 3^{1.2} + 0.4 \times 2^{0.9} + 0.2 \times 4^{0.9}\right) \\ \times \left(0.3 \times 4^{0.9} + 0.4 \times 3^{1.2} + 0.2 \times 2^{0.9}\right) \times \left(0.3 \times 4^{0.9} + 0.4 \times 2^{0.9} + 0.2 \times 3^{1.2}\right) \\ \times \left(0.3 \times 2^{0.9} + 0.4 \times 3^{1.2} + 0.2 \times 4^{0.9}\right) \times \left(0.3 \times 2^{0.9} + 0.4 \times 4^{0.9} + 0.2 \times 3^{1.2}\right) \end{array} \right)^{\frac{1}{3!}} \right) \\ \left(\begin{array}{l} 1 - \left(1 - \left(\begin{array}{l} \left(1 - \left(1 - 0.7^{3\times0.4}\right)^{0.3} \times \left(1 - 0.8^{3\times0.3}\right)^{0.4} \times \left(1 - 0.6^{3\times0.3}\right)^{0.2}\right) \\ \times \left(1 - \left(1 - 0.7^{3\times0.4}\right)^{0.3} \times \left(1 - 0.6^{3\times0.3}\right)^{0.4} \times \left(1 - 0.8^{3\times0.3}\right)^{0.2}\right) \\ \times \left(1 - \left(1 - 0.8^{3\times0.3}\right)^{0.3} \times \left(1 - 0.7^{3\times0.4}\right)^{0.4} \times \left(1 - 0.6^{3\times0.3}\right)^{0.2}\right) \\ \times \left(1 - \left(1 - 0.8^{3\times0.3}\right)^{0.3} \times \left(1 - 0.6^{3\times0.3}\right)^{0.4} \times \left(1 - 0.7^{3\times0.4}\right)^{0.2}\right) \\ \times \left(1 - \left(1 - 0.6^{3\times0.3}\right)^{0.3} \times \left(1 - 0.7^{3\times0.4}\right)^{0.4} \times \left(1 - 0.8^{3\times0.3}\right)^{0.2}\right) \\ \times \left(1 - \left(1 - 0.6^{3\times0.3}\right)^{0.3} \times \left(1 - 0.8^{3\times0.3}\right)^{0.4} \times \left(1 - 0.7^{3\times0.4}\right)^{0.2}\right) \end{array} \right)^{\frac{1}{3!}} \right)^{\frac{1}{0.3+0.4+0.2}} \\ \left(1 - \left(\begin{array}{l} \left(1 - \left(1 - \left(1 - (1-0.5)^{3\times0.4}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.6)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.7)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.5)^{3\times0.4}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.7)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.6)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.6)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.5)^{3\times0.4}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.7)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.6)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.7)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.5)^{3\times0.4}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.7)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.5)^{3\times0.4}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.6)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.7)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.6)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.5)^{3\times0.4}\right)^{0.2}\right)\right) \end{array} \right)^{\frac{1}{3!}} \right)^{\frac{1}{0.3+0.4+0.2}} \\ \left(1 - \left(\begin{array}{l} \left(1 - \left(1 - \left(1 - (1-0.3)^{3\times0.4}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.2)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.1)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.3)^{3\times0.4}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.1)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.2)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.2)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.3)^{3\times0.4}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.1)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.2)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.1)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.3)^{3\times0.4}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.1)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.3)^{3\times0.4}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.2)^{3\times0.3}\right)^{0.2}\right)\right) \\ \times \left(1 - \left(1 - \left(1 - (1-0.1)^{3\times0.3}\right)^{0.4}\right) \times \left(1 - \left(1 - (1-0.2)^{3\times0.3}\right)^{0.3}\right) \times \left(1 - \left(1 - (1-0.3)^{3\times0.4}\right)^{0.2}\right)\right) \end{array} \right)^{\frac{1}{3!}} \right)^{\frac{1}{0.3+0.4+0.2}} \end{array} \right) \end{array} \right)$$

$$= \langle (s_3, 0.024), (0.7135, 0.0013, 0.0356) \rangle$$

We will now discuss some properties of the SVN2TLWMM operator.

Property 9. *(Monotonicity) Let* $\widetilde{a}_j = \left\langle \left(s_{a_j}, \rho_{a_j}\right), \left(T_{a_j}, I_{a_j}, F_{a_j}\right) \right\rangle (j = 1, 2, \ldots, n)$ *and* $\widetilde{b}_j = \left\langle \left(s_{b_j}, \rho_{b_j}\right), \left(T_{b_j}, I_{b_j}, F_{b_j}\right) \right\rangle$ $(j = 1, 2, \ldots, n)$ *be two sets of SVN2TLNs. If* $\Delta^{-1}\left(S_{a_j}, \rho_{a_j}\right) \leq \Delta^{-1}\left(S_{b_j}, \rho_{b_j}\right)$ *and* $T_{a_j} \leq T_{b_j}$ *and* $I_{a_j} \geq I_{b_j}$ *and* $F_{a_j} \geq F_{b_j}$ *hold for all j, then*

$$\mathrm{SVN2TLWDMM}_{nw}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \mathrm{SVN2TLWDMM}_{nw}^{P}\left(\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_n\right). \tag{64}$$

The proof is similar to SVN2TLWMM. It is omitted here.

Property 10. *(Boundedness) Let* $\widetilde{a}_j = \left\langle \left(s_j, \rho_j\right), \left(T_j, I_j, F_j\right) \right\rangle (j = 1, 2, \ldots, n)$ *be a set of SVN2TLNs. If* $\widetilde{a}_i^{+} = (\max_i(S_i, \rho_i), (\max_i(T_i), \min_i(I_i), \min_i(F_i)))$ *and* $\widetilde{a}_i^{-} = (\min_i(S_i, \rho_i), (\min_i(T_i), \max_i(I_i), \max_i(F_i)))$, *then*

$$\widetilde{a}^{-} \leq \mathrm{SVN2TLWDMM}_{nw}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \widetilde{a}^{+}. \tag{65}$$

From Theorem 4:

$$\begin{aligned}
&\mathrm{SVN2TLWDMM}_{nw}^{P}(\widetilde{a}^{-}{}_1, \widetilde{a}^{-}{}_2, \cdots, \widetilde{a}^{-}{}_n) \\
&= \left\{ \begin{array}{l}
\Delta\left(\frac{1}{\sum_{j=1}^{n} p_j} \left(\prod_{\sigma \in S_n} \left(\sum_{j=1}^{n} \left(p_j \left(\min \Delta^{-1}\left(s_j, \rho_j\right) \right)^{nw_{\sigma(j)}} \right) \right) \right)^{\frac{1}{n!}} \right), \\
\left(\begin{array}{l}
1 - \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \min T_{\sigma(j)}^{nw_{\sigma(j)}} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - \max I_{\sigma(j)} \right)^{nw_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - \max F_{\sigma(j)} \right)^{nw_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}
\end{array} \right)
\end{array} \right\}
\end{aligned} \tag{66}$$

$$\begin{aligned}
&\mathrm{SVN2TLWDMM}_{nw}^{P}(\widetilde{a}^{+}{}_1, \widetilde{a}^{+}{}_2, \cdots, \widetilde{a}^{+}{}_n) \\
&= \left\{ \begin{array}{l}
\Delta\left(\frac{1}{\sum_{j=1}^{n} p_j} \left(\prod_{\sigma \in S_n} \left(\sum_{j=1}^{n} \left(p_j \left(\max \Delta^{-1}\left(s_j, \rho_j\right) \right)^{nw_{\sigma(j)}} \right) \right) \right)^{\frac{1}{n!}} \right), \\
\left(\begin{array}{l}
1 - \left(1 - \prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \max T_{\sigma(j)}^{nw_{\sigma(j)}} \right)^{p_j} \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - \min I_{\sigma(j)} \right)^{nw_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod_{\sigma \in S_n} \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - \min F_{\sigma(j)} \right)^{nw_{\sigma(j)}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}
\end{array} \right)
\end{array} \right\}
\end{aligned} \tag{67}$$

From Property 9:

$$\widetilde{a}^{-} \leq \mathrm{SVN2TLWDMM}_{nw}^{P}(\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_n) \leq \widetilde{a}^{+}. \tag{68}$$

It is obvious that the SVN2TLWDMM operator lacks the property of idempotency.

4. Numerical Example and Comparative Analysis

4.1. Numerical Example

The green supplier selection is a classic MADM problem [37–39]. Therefore, in this section we use a numerical example to select green suppliers in green supply chain management with SVN2TLNs in order to show the proposed method. There are five possible green suppliers $A_i(i = 1, 2, 3, 4, 5)$ to be selected. We selected four attributes to assess these possible green suppliers: G_1 is the product quality factor, G_2 is the environmental factor, G_3 is the delivery factor, and G_4 is the price factor. These five possible green suppliers $A_i(i = 1, 2, 3, 4, 5)$ are to be assessed with SVN2TLNs by the decisionmaker using the above four attributes, whose weighting vectors $\omega = (0.2, 0.3, 0.4, 0.1)$ are listed in Table 1.

Table 1. Single-valued neutrosophic 2-tuple linguistic number (SVN2TLN) decision matrix.

	G_1	G_2	G_3	G_4
A_1	<(s_6,0),(0.7,0.4,0.6)>	<(s_4,0),(0.4,0.5,0.6)>	<(s_3,0),(0.4,0.5,0.6)>	<(s_4,0),(0.6,0.5,0.3)>
A_2	<(s_3,0),(0.5,0.4,0.2)>	<(s_5,0),(0.6,0.5,0.4)>	<(s_2,0),(0.6,0.4,0.2)>	<(s_3,0),(0.8,0.3,0.5)>
A_3	<(s_2,0),(0.5,0.6,0.2)>	<(s_3,0),(0.7,0.5,0.6)>	<(s_4,0),(0.5,0.6,0.3)>	<(s_5,0),(0.4,0.5,0.2)>
A_4	<(s_5,0),(0.8,0.4,0.6)>	<(s_4,0),(0.7,0.4,0.3)>	<(s_6,0),(0.6,0.5,0.3)>	<(s_4,0),(0.6,0.4,0.6)>
A_5	<(s_1,0),(0.5,0.6,0.4)>	<(s_5,0),(0.7,0.4,0.7)>	<(s_1,0),(0.6,0.5,0.2)>	<(s_3,0),(0.8,0.6,0.8)>

We can now use the approach developed for selecting green suppliers in green supply chain management.

Step 1. According to SVN2TLNs $r_{ij}(i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$, we can aggregate all SVN2TLNs r_{ij} by using the SVN2TLWMM (SVN2TLWDMM) operator to get the SVN2TLNs $A_i(i = 1, 2, 3, 4, 5)$ of the green suppliers A_i. Supposing that $P = (1, 1, 1, 0)$, the aggregating results are shown in Table 2.

Table 2. The aggregating results of the green suppliers by the single-valued neutrosophic 2-tuple linguistic dual Muirhead mean (SVN2TLWMM) (single-valued neutrosophic 2-tuple linguistic weighted dual Muirhead mean (SVN2TLWDMM)) operator.

	SVN2TLWMM	SVN2TLWDMM
A_1	<(s_4, −0.4633),(0.4692,0.5201,0.5716)>	<(s_5, −0.2958),(0.5785,0.4324,0.4843)>
A_2	<(s_3, −0.3366),(0.5757,0.4534,0.3992)>	<(s_3, −0.0273),(0.6525,0.3702,0.2761)>
A_3	<(s_3, −0.1294),(0.4853,0.5894,0.3953)>	<(s_4, −0.2162),(0.5754,0.4996,0.2846)>
A_4	<(s_4, −0.0021),(0.6189,0.4719,0.5075)>	<(s_7, −0.0573),(0.7143,0.3868,0.3947)>
A_5	<(s_2, −0.1995),(0.6013,0.5649,0.6052)>	<(s_2, −0.2488),(0.6804,0.4747,0.4541)>

Step 2. In accordance with the aggregating results in Table 2, the score values of the green suppliers are shown in Table 3.

Table 3. The score values of the green suppliers.

	SVN2TLWMM	SVN2TLWDMM
A_1	(s_2, −0.3762)	(s_3, −0.3943)
A_2	(s_2, −0.4702)	(s_2, −0.0122)
A_3	(s_1, 0.4358)	(s_2, 0.2592)
A_4	(s_2, 0.1847)	(s_4, 0.4730)
A_5	(s_1, −0.1410)	(s_1, 0.0225)

Step 3. According to the score values listed in Table 3, the order of the green suppliers are listed in Table 4. The best green supplier is A_4.

Table 4. Ordering of the green suppliers.

	Ordering
SVN2TLWMM	$A_4 > A_1 > A_2 > A_3 > A_5$
SVN2TLWDMM	$A_4 > A_1 > A_3 > A_2 > A_5$

4.2. Influence of the Parameter on the Final Result

In order to show the effects on the ranking results by altering the parameters of P in the SVN2TLWMM (SVN2TLWDMM) operators, the results are listed in Tables 5 and 6.

Table 5. Ranking results for different parameters of the SVN2TLWMM operator.

P	$s(A_1)$	$s(A_2)$	$s(A_3)$	$s(A_4)$	$s(A_5)$	Ordering
(1,0,0,0)	$(s_1, 0.3096)$	$(s_1, 0.2931)$	$(s_1, 0.2036)$	$(s_2, -0.1905)$	$(s_1, -0.1063)$	A4 > A1 > A2 > A3 > A5
(1,1,0,0)	$(s_2, -0.4419)$	$(s_2, -0.4987)$	$(s_1, 0.4121)$	$(s_2, 0.1192)$	$(s_1, -0.0706)$	A4 > A1 > A2 > A3 > A5
(1,1,1,0)	$(s_2, -0.3762)$	$(s_2, -0.4702)$	$(s_1, 0.4358)$	$(s_2, 0.1847)$	$(s_1, -0.1410)$	A4 > A1 > A2 > A3 > A5
(1,1,1,1)	$(s_2, -0.3628)$	$(s_2, -0.4811)$	$(s_1, 0.4095)$	$(s_2, 0.1700)$	$(s_1, -0.2159)$	A4 > A1 > A2 > A3 > A5
(2,2,2,2)	$(s_2, -0.3628)$	$(s_2, -0.4811)$	$(s_1, 0.4095)$	$(s_2, 0.1700)$	$(s_1, -0.2159)$	A4 > A1 > A2 > A3 > A5
(2,0,0,0)	$(s_2, -0.2505)$	$(s_2, -0.2515)$	$(s_2, -0.3744)$	$(s_2, 0.3621)$	$(s_1, 0.3912)$	A4 > A1 > A2 > A3 > A5
(3,0,0,0)	$(s_2, -0.0101)$	$(s_2, 0.0035)$	$(s_2, -0.1482)$	$(s_3, -0.3787)$	$(s_2, -0.2759)$	A4 > A1 > A2 > A3 > A5

Table 6. Ranking results for different parameters of the SVN2TLWDMM operator.

P	$s(A_1)$	$s(A_2)$	$s(A_3)$	$s(A_4)$	$s(A_5)$	Ordering
(1,0,0,0)	$(s_2, 0.2756)$	$(s_2, 0.0001)$	$(s_2, -0.0497)$	$(s_4, -0.4654)$	$(s_1, 0.0155)$	A4 > A1 > A2 > A3 > A5
(1,1,0,0)	$(s_2, 0.3874)$	$(s_2, -0.0806)$	$(s_2, 0.0341)$	$(s_4, -0.0287)$	$(s_1, -0.0515)$	A4 > A1 > A3 > A2 > A5
(1,1,1,0)	$(s_3, -0.3943)$	$(s_2, -0.0122)$	$(s_2, 0.2592)$	$(s_4, 0.4730)$	$(s_1, 0.0225)$	A4 > A1 > A3 > A2 > A5
(1,1,1,1)	$(s_3, -0.2159)$	$(s_1, 0.2825)$	$(s_1, 0.2287)$	$(s_2, -0.4771)$	$(s_1, -0.0165)$	A1 > A4 > A2 > A3 > A5
(2,2,2,2)	$(s_3, -0.2159)$	$(s_1, 0.2825)$	$(s_1, 0.2287)$	$(s_2, -0.4771)$	$(s_1, -0.0165)$	A1 > A4 > A2 > A3 > A5
(2,0,0,0)	$(s_2, 0.2100)$	$(s_2, -0.0916)$	$(s_2, -0.1583)$	$(s_3, 0.2577)$	$(s_1, -0.1059)$	A4 > A1 > A2 > A3 > A5
(3,0,0,0,)	$(s_2, 0.3086)$	$(s_2, 0.1445)$	$(s_2, 0.0336)$	$(s_3, 0.4540)$	$(s_1, 0.0299)$	A4 > A1 > A2 > A3 > A5

4.3. Comparative Analysis

We can now compare our proposed method with TOPSIS methods using the single-valued neutrosophic linguistic numbers (SVNLNs) proposed by Ye [32]. The comparative results are listed in Table 7.

Table 7. Ordering of the green suppliers.

	Ordering
TOPSIS with SVNLNs	$A_4 > A_1 > A_2 > A_3 > A_5$

In Table 7, we can see that we get the same best green supplier, and only two of the methods' ranking results are slightly different. This shows that the method we proposed is reasonable and effective. However, the existing TOPSIS methods with SVNLNs [32] do not consider the relationship information among the arguments being aggregated and therefore cannot eliminate the influence of unfair arguments on decision results. Our proposed SVN2TLWMM and SVN2TLWDMM operators consider the relationship information among the arguments being aggregated.

5. Conclusions

In this paper, we investigated MADM problems with SVN2TLNs. Then, we utilized the Muirhead mean operator and dual Muirhead mean operator to develop some Muirhead mean operators with SVN2TLNs: the SVN2TLMM operator, the SVN2TLWMM operator, the SVN2TLDMM operator, and the SVN2TLWDMM operator. The main properties of these proposed operators were investigated. We then used these operators to propose some models for MADM problems with SVN2TLNs.

The case study for green supplier selection shows that the proposed MADM method is practical and effective. The advantages of the proposed approach are as follows:

(1) The proposed approach is based on SVN2TLNs, which are suitable to be used in real life situations. SVN2TLNs have the capacity to deal with imprecise and vague information. They are suitable for expressing the truth-membership, indeterminacy-membership, and falsity-membership of an element to a 2-tuple linguistic term, which can affect the decisionmaker's confidence level when they are making the evaluation. Therefore, the decisionmakers may find it more flexible and convenient to express their opinions as SVN2TLNs. The existing operation rules and comparison rules were contrasted and discussed.

(2) Some Muirhead mean operators with SVN2TLNs were developed: the SVN2TLMM operator, the SVN2TLWMM operator, the SVN2TLDMM operator, and the SVN2TLWDMM operator. The main characteristics of these proposed operators were investigated.

(3) Some methods were established to solve MADM problems with SVN2TLNs, and the evaluation results turned out to be reasonable.

(4) MADM methods based on such operators with SVN2TLNs are novel decision making methods, and these methods were applied to green supplier selection in this study. Furthermore, MADM methods based on such operators with SVN2TLNs are not only easy to calculate, but can also realize the reasonable and stable ranking of alternatives.

In future studies, the application of the proposed aggregating operators of SVN2TLNs need to be studied in many other uncertain and fuzzy environments [40–43] and extended to other application domains [44,45].

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