

Single-Valued Neutrosophic Power Shapley Choquet Average Operators and Their Applications to Multi-Criteria Decision-Making

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Abstract: Single-valued neutrosophic sets (SVNSs), which involve in truth-membership, indeterminacy-membership and falsity-membership, play a significant role in describing the decision-makers' preference information. In this study, a single-valued neutrosophic multi-criteria decision-making (MCDM) approach is developed based on Shapley fuzzy measures and power aggregation operator that takes a correlative relationship among criteria into account and also simultaneously reduces the effects of abnormal preference information. Firstly, two aggregation operators, namely, generalized weighted single-valued neutrosophic power Shapley Choquet average (GWSVNPSCA) operator and generalized weighted single-valued neutrosophic power Shapley Choquet geometric (GWSVNPSCG) operator, are accordingly defined, and the corresponding properties are discussed as well. Secondly, based on the proposed aggregation operators, an integrated MCDM approach is proposed to effectively solve single-valued neutrosophic problems where the weight information is incompletely known. A programming model is constructed to obtain the optimal Shapley fuzzy measure. Next, the proposed operators are utilized to aggregate the decision-makers' preference information. Finally, a theoretical example with tourism attraction selection is provided to examine the efficacy of the developed approach, in which the results is found reasonable and credible.

Keywords: multi-criteria decision-making; single-valued neutrosophic sets; Shapley fuzzy measure; aggregation operators

1. Introduction

In real environment, a successful decision-making often requires consideration of multiple factors (criteria). This kind of problem related to decision-making simultaneously involving multiple criteria is called multi-criteria decision-making (MCDM) problem [1]. Since Zadeh put forward the theory of the fuzzy sets (FSs) in 1965 [2], MCDM based on FSs theory has been widely developed and applied to practical decision-making problems [3–11]. Due to the inherent fuzziness of human preference information, a single membership degree of FSs cannot properly describe the fuzziness and uncertainty in the decision-making process. After that, Atanassov [12] defined intuitionistic fuzzy sets (IFSs), including a membership, non-membership, and hesitation index, as the extension of FSs [2]. However, FSs and IFSs cannot deal with indeterminate information. Thus, Smarandache et al. [13,14] developed neutrosophic sets (NSs), involving truth membership, indeterminacy membership, and falsity membership, respectively, and it lies in the range of $]0^-, 1^+[$ [15].

Obviously, it is the extension of the standard interval $[0, 1]$ of IFSs. Subsequently, a single-valued neutrosophic set (SVNS) was proposed, that is an instance of NS [16–20]. In addition, other extensions of NSs, including interval neutrosophic sets [21], neutrosophic cubic sets [22,23], single-valued neutrosophic hesitant fuzzy sets (SVNHFSs) [24], rough neutrosophic sets (RNSs) [25], interval rough neutrosophic sets (IRNSs) [26], probability multi-valued neutrosophic sets [27,28], and other extensions [29,30], were proposed and applied to solve theoretical problems.

Additionally, SVNS, as a popular method to describe DMs' preference information, has greatly received scholars' attention, and also was studied from different aspects, such as aggregation operators [31–36], outranking relations [37], and information measures [38,39]. For instance, Liu and Wang [31] presented a single-valued neutrosophic normalized weighted Bonferroni mean operator; Garg [36] presented single-valued neutrosophic Frank operators; Wu et al. [39] defined a simplified neutrosophic prioritized aggregation operator. Liu et al. [28] defined some single-valued neutrosophic Schweizer-Sklar prioritized aggregation operators. To our knowledge, aggregation operators play a significant role in solving MCDM problems. Different aggregation operators are mainly involved in different functions, which can reduce the effect of abnormal data provided by DMs. Especially, power aggregation (PA) operator, defined by Yager [40], can aggregate the information by using a weighted vector based on support degree among different arguments and allow the evaluation values to be supported.

In practice, we always experience some decision-making problems with the correlation between criteria. For example, tourism attraction selection, constraints of tourists, and preferences of tourists are correlated to each other, and male tourists prefer adventurous tourism attractions. Apparently, there is a redundancy between constraints and preferences, where the sum of the weights of constraints and preferences is greater than the weight of the combination s of two criteria.

The Shapley fuzzy measure and Choquet integral [41–44] are highly applied to deal with MCDM problems where the criteria are correlated [45, 46]. For instance, Zhang et al. [44] presented some hesitant fuzzy linguistic Shapley Choquet integral operators; Meng et al. [47] developed interval-valued intuitionistic fuzzy Shapley Choquet integral operators; Qu et al. [48] defined dual hesitant fuzzy Shapley Choquet integral operators; Nie et al. [49] developed some Pythagorean fuzzy partitioned normalized weighted Bonferroni mean operator based on Shapley fuzzy measures.

From the above analysis, two shortcomings can be concluded as follows: (1) the existing single-valued neutrosophic aggregation operators only take the importance of assessment values or that of ordered position into account. However, criteria were often interdependent or interactive in the decision-making process; (2) Most of existing methods cannot handle some special MCDM problems where the weight information is incompletely known. Moreover, power aggregation operator can reduce the effects of abnormal assessment values, and Shapley fuzzy measures can simultaneously consider the importance and overall correlation among the criteria. To do this, the main contributions can be summarized as: (1) This paper proposes two new aggregation operators, namely the generalized weighted single-valued neutrosophic power Shapley Choquet average (GWSVNPSCA) operator and the generalized weighted single-valued neutrosophic power Shapley Choquet geometric (GWSVNPSCG) operator, which can avoid the first drawback. (2) This paper develops a MCDM approach based on proposed aggregation operators to solve single-valued neutrosophic problems with incomplete weight information, which can overcome the second drawback.

The rest of the study is constructed as follows. In Section 2, some related definitions are introduced. Then, the GWSVNPSCA and GWSVNPSCG operators are defined in Section 3. Next, the single-valued MCDM method accompanied with unknown weight information is constructed in Section 4. In Section 5, the selection of tourism attraction is provided to assess the effectiveness of the proposed method. Finally, we summarize the achieved results in Section 6.

2. Preliminaries

Here, some basic definitions, including Shapley fuzzy measure, PA operator, NSs, and SVNNS, are described.

2.1. Shapley Fuzzy Measure

Definition 1 [42,43]. Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite set and $P(X)$ be the power set of X . Then the function $M : (P(X) \rightarrow [0, 1])$ is defined as a fuzzy measure, satisfying:

- (1) $\mu(\Phi) = 0$ and $\mu(X) = 1$;
- (2) $\forall \alpha, \beta \in P(X)$ and $\alpha \subseteq \beta$, then $\mu(\alpha) \leq \mu(\beta)$.

To measure the influence of each condition in a game, Shapley [42] defined the Shapley function as follows:

$$\phi_s(\mu, X) = \sum_{M \subseteq X \setminus S} \frac{(n-s-m)!m!}{(n-s+1)!} (\mu(S \cup \{M\}) - \mu(M)), \forall S \in X. \quad (1)$$

Here n and m denote the cardinalities of X and M , respectively. As discussed in Zhang et al. [44], we have $\phi_i(\mu, X) \geq 0$ and $\sum_{i=1}^n \phi_i(\mu, X) = 1$, which can be true. Then, $\phi_i(\mu, X)$ is called Shapley fuzzy measure [44]. Moreover, Shapley fuzzy measures, which will be utilized in latter analysis, are additive fuzzy measures.

2.2. PA Operator

The PA operator was developed by Yager [40] in the form of nonlinear weighted average aggregation operator.

Definition 2 [40]. The PA operator is given as:

$$PA(\vartheta_1, \vartheta_2, \dots, \vartheta_n) = \frac{\sum_{i=1}^n (1 + F(\vartheta_i)) \vartheta_i}{\sum_{i=1}^n (1 + F(\vartheta_i))}. \quad (2)$$

where $\vartheta_i (i=1, 2, \dots, n)$ includes positive real numbers. $F(\vartheta_i) = \sum_{i=1, j \neq i}^n \text{Sup}(\vartheta_i, \vartheta_j)$ and $\text{Sup}(\vartheta_i, \vartheta_j)$ denote the support degree for ϑ_i from ϑ_j . After that, the support degree satisfies the properties as follows:

- (1) $\text{Sup}(\vartheta_i, \vartheta_j) \in [0, 1]$;
- (2) $\text{Sup}(\vartheta_i, \vartheta_j) = \text{Sup}(\vartheta_j, \vartheta_i)$;
- (3) $\text{Sup}(\vartheta_i, \vartheta_j) \geq \text{Sup}(\vartheta_s, \vartheta_t)$, iff $|\vartheta_i - \vartheta_j| < |\vartheta_s - \vartheta_t|$.

2.3. NSs and SVNNS

Definition 3 [13–14,50]. A NS $\tilde{\psi}$ in $X = \{x_1, x_2, \dots, x_n\}$ is characterized as: $\tilde{\psi} = \left\langle x, \tilde{T}_{\tilde{\psi}}(x), \tilde{I}_{\tilde{\psi}}(x), \tilde{F}_{\tilde{\psi}}(x) \right\rangle | x \in X$. Here $\tilde{T}_{\tilde{\psi}}(x)$, $\tilde{I}_{\tilde{\psi}}(x)$, and $\tilde{F}_{\tilde{\psi}}(x)$ denote truth-membership, indeterminacy-membership and falsity-membership respectively. Furthermore, $\tilde{T}_{\tilde{\psi}}(x)$, $\tilde{I}_{\tilde{\psi}}(x)$, and

$\tilde{F}_{\tilde{\psi}}(x)$ are subsets of $]0^-, 1^+[$, that is, $\tilde{T}_{\tilde{\psi}}(x): X \rightarrow]0^-, 1^+[$, $\tilde{I}_{\tilde{\psi}}(x): X \rightarrow]0^-, 1^+[$, and $\tilde{F}_{\tilde{\psi}}(x): X \rightarrow]0^-, 1^+[$ to satisfy the condition required for $0^- \leq \sup \tilde{T}_{\tilde{\psi}}(x) + \sup \tilde{I}_{\tilde{\psi}}(x) + \sup \tilde{F}_{\tilde{\psi}}(x) \leq 3^+$.

Since NSs can difficultly handle practical problems because of their nonstandard intervals, Ye [16] defined SVNNS based on standard intervals and developed the corresponding operations for SVNNSs.

Definition 4 [16]. An SVNNS ψ in $X = \{x_1, x_2, \dots, x_n\}$ is defined as: $\psi = \left\langle x, T_{\psi}(x), I_{\psi}(x), F_{\psi}(x) \mid x \in X \right\rangle$. Here $T_{\psi}(x)$, $I_{\psi}(x)$, and $F_{\psi}(x)$ are subsets in the standard interval of $[0,1]$, i.e., $T_{\psi}(x): X \rightarrow [0,1]$, $I_{\psi}(x): X \rightarrow [0,1]$, and $F_{\psi}(x): X \rightarrow [0,1]$. In particular, if X has only one element, then ψ is called a single-valued neutrosophic number (SVNN). For convenience, an SVNN is denoted by $\psi = \langle T_{\psi}, I_{\psi}, F_{\psi} \rangle$.

Definition 5 [16]. Let $\psi = \langle T_{\psi}, I_{\psi}, F_{\psi} \rangle$, $\psi_1 = \langle T_{\psi_1}, I_{\psi_1}, F_{\psi_1} \rangle$ and $\psi_2 = \langle T_{\psi_2}, I_{\psi_2}, F_{\psi_2} \rangle$ be three SVNNs, and with $\lambda > 0$, the following operations can be defined:

- (1) $\lambda\psi = \left\langle 1 - (1 - T_{\psi})^{\lambda}, 1 - (1 - I_{\psi})^{\lambda}, 1 - (1 - F_{\psi})^{\lambda} \right\rangle, \lambda > 0;$
- (2) $\psi^{\lambda} = \langle T_{\psi}^{\lambda}, I_{\psi}^{\lambda}, F_{\psi}^{\lambda} \rangle, \lambda > 0;$
- (3) $\psi_1 \oplus \psi_2 = \langle T_{\psi_1} + T_{\psi_2} - T_{\psi_1} \cdot T_{\psi_2}, I_{\psi_1} + I_{\psi_2} - I_{\psi_1} \cdot I_{\psi_2}, F_{\psi_1} + F_{\psi_2} - F_{\psi_1} \cdot F_{\psi_2} \rangle;$
- (4) $\psi_1 \otimes \psi_2 = \langle T_{\psi_1} \cdot T_{\psi_2}, I_{\psi_1} \cdot I_{\psi_2}, F_{\psi_1} \cdot F_{\psi_2} \rangle.$

However, the operations are unreasonable as previously outlined [32]. Then, Peng et al. [32] developed the improved operations of SVNNs and the corresponding comparison method.

Definition 6 [32]. Let $\psi = \langle T_{\psi}, I_{\psi}, F_{\psi} \rangle$, $\psi_1 = \langle T_{\psi_1}, I_{\psi_1}, F_{\psi_1} \rangle$ and $\psi_2 = \langle T_{\psi_2}, I_{\psi_2}, F_{\psi_2} \rangle$ be three SVNNs, and $\lambda > 0$. The SVNNs operations are defined in the following:

- (1) $\lambda\psi = \left\langle 1 - (1 - T_{\psi})^{\lambda}, I_{\psi}^{\lambda}, F_{\psi}^{\lambda} \right\rangle;$
- (2) $\psi^{\lambda} = \left\langle T_{\psi}^{\lambda}, 1 - (1 - I_{\psi})^{\lambda}, 1 - (1 - F_{\psi})^{\lambda} \right\rangle;$
- (3) $\psi_1 \oplus \psi_2 = \langle T_{\psi_1} + T_{\psi_2} - T_{\psi_1} \cdot T_{\psi_2}, I_{\psi_1} \cdot I_{\psi_2}, F_{\psi_1} \cdot F_{\psi_2} \rangle;$
- (4) $\psi_1 \otimes \psi_2 = \langle T_{\psi_1} \cdot T_{\psi_2}, I_{\psi_1} + I_{\psi_2} - I_{\psi_1} \cdot I_{\psi_2}, F_{\psi_1} + F_{\psi_2} - F_{\psi_1} \cdot F_{\psi_2} \rangle.$

Example 1. Let $\psi = \langle 0.5, 0.1, 0.3 \rangle$, $\psi_1 = \langle 0.7, 0.1, 0.1 \rangle$, and $\psi_2 = \langle 0.5, 0.2, 0.3 \rangle$ be three SVNNs, and $\lambda = 2$. Based on the operations presented in Definition 6, we can get the following results:

- (1) $\lambda\psi = \left\langle 1 - (1 - 0.5)^2, 0.1^2, 0.3^2 \right\rangle = \langle 0.75, 0.01, 0.09 \rangle;$
- (2) $\psi^{\lambda} = \left\langle 0.5^2, 1 - (1 - 0.1)^2, 1 - (1 - 0.3)^2 \right\rangle = \langle 0.025, 0.19, 0.51 \rangle;$
- (3) $\psi_1 \oplus \psi_2 = \langle 0.7 + 0.5 - 0.7 \cdot 0.5, 0.1 \cdot 0.2, 0.1 \cdot 0.3 \rangle = \langle 0.85, 0.02, 0.03 \rangle;$
- (4) $\psi_1 \otimes \psi_2 = \langle 0.7 \cdot 0.5, 0.1 + 0.2 - 0.1 \cdot 0.2, 0.1 + 0.3 - 0.1 \cdot 0.3 \rangle = \langle 0.35, 0.28, 0.37 \rangle.$

Definition 7 [32]. Let $\psi_1 = \langle T_{\psi_1}, I_{\psi_1}, F_{\psi_1} \rangle$ and $\psi_2 = \langle T_{\psi_2}, I_{\psi_2}, F_{\psi_2} \rangle$ be two SVNNs. Then the method of

comparison is defined as:

- (1) If $\bar{s}(\psi_1) > \bar{s}(\psi_2)$, then ψ_1 is preferable to ψ_2 , represented by $\psi_1 \succ \psi_2$;
- (2) If $\bar{s}(\psi_1) = \bar{s}(\psi_2)$ and $\bar{a}(\psi_1) > \bar{a}(\psi_2)$, then ψ_1 is preferable to ψ_2 , represented by $\psi_1 \succ \psi_2$;
- (3) If $\bar{s}(\psi_1) = \bar{s}(\psi_2)$, $\bar{a}(\psi_1) = \bar{a}(\psi_2)$ and $\bar{c}(\psi_1) > \bar{c}(\psi_2)$, then ψ_1 is preferable to ψ_2 , represented by $\psi_1 \succ \psi_2$;
- (4) If $\bar{s}(\psi_1) = \bar{s}(\psi_2)$, $\bar{a}(\psi_1) = \bar{a}(\psi_2)$ and $\bar{c}(\psi_1) = \bar{c}(\psi_2)$, then ψ_1 is indifferent to ψ_2 , represented by $\psi_1 \sim \psi_2$.
- (5) where $\bar{s}(\psi_i) = (T_{\psi_i} + 1 - I_{\psi_i} + 1 - F_{\psi_i})/3$, $\bar{a}(\psi_i) = T_{\psi_i} - F_{\psi_i}$ and $\bar{c}(\psi_i) = T_{\psi_i}$ ($i=1,2$) denote the score function, accuracy function and certainty function of SVNNS, respectively.

Example 2. Let $\psi_1 = \langle 0.8, 0.1, 0.2 \rangle$ and $\psi_2 = \langle 0.6, 0.2, 0.3 \rangle$ be two SVNNS, then we have $\bar{s}(\psi_1) = 0.83$ and $\bar{s}(\psi_2) = 0.7$. Since $\bar{s}(\psi_1) > \bar{s}(\psi_2)$, so ψ_1 is preferable to ψ_2 , i.e., $\psi_1 \succ \psi_2$.

Definition 8. Let $\psi_1 = \langle T_{\psi_1}, I_{\psi_1}, F_{\psi_1} \rangle$ and $\psi_2 = \langle T_{\psi_2}, I_{\psi_2}, F_{\psi_2} \rangle$ be two SVNNS. The generalized normalized single-valued neutrosophic distance between two SVNNS of ψ_1 and ψ_2 can be defined as:

$$d_{gnsvn}(\psi_1, \psi_2) = \left(\frac{1}{5} \left(|T_{\psi_1} - T_{\psi_2}|^\gamma + |I_{\psi_1} - I_{\psi_2}|^\gamma + |F_{\psi_1} - F_{\psi_2}|^\gamma + |\max\{T_{\psi_1}, I_{\psi_2}\} - \max\{I_{\psi_1}, T_{\psi_2}\}|^\gamma + |\max\{T_{\psi_1}, F_{\psi_2}\} - \max\{F_{\psi_1}, T_{\psi_2}\}|^\gamma \right) \right)^{1/\gamma} \quad (\gamma > 0). \quad (3)$$

Especially, if $\gamma=1$, then $d_{gnsvn}(\psi_1, \psi_2)$ may reduce to the normalized single-valued neutrosophic Hamming distance, i.e.,

$$d_{gnsvn}(\psi_1, \psi_2) = \frac{1}{5} \left(|T_{\psi_1} - T_{\psi_2}| + |I_{\psi_1} - I_{\psi_2}| + |F_{\psi_1} - F_{\psi_2}| + |\max\{T_{\psi_1}, I_{\psi_2}\} - \max\{I_{\psi_1}, T_{\psi_2}\}| + |\max\{T_{\psi_1}, F_{\psi_2}\} - \max\{F_{\psi_1}, T_{\psi_2}\}| \right). \quad (4)$$

If $\gamma=2$, then $d_{gnsvn}(\psi_1, \psi_2)$ may reduce to the normalized single-valued neutrosophic Euclidean distance, i.e.,

$$d_{gnsvn}(\psi_1, \psi_2) = \left(\frac{1}{5} \left(|T_{\psi_1} - T_{\psi_2}|^2 + |I_{\psi_1} - I_{\psi_2}|^2 + |F_{\psi_1} - F_{\psi_2}|^2 + |\max\{T_{\psi_1}, I_{\psi_2}\} - \max\{I_{\psi_1}, T_{\psi_2}\}|^2 + |\max\{T_{\psi_1}, F_{\psi_2}\} - \max\{F_{\psi_1}, T_{\psi_2}\}|^2 \right) \right)^{1/2} \quad (\gamma > 0). \quad (5)$$

Example 3. Based on Example 2, and let $\gamma=1$, then we have

$$d_{gnsvn}(\psi_1, \psi_2) = \frac{1}{5} (|0.8 - 0.6| + |0.1 - 0.2| + |0.2 - 0.3| + |\max\{0.8, 0.2\} - \max\{0.1, 0.6\}| + |\max\{0.8, 0.3\} - \max\{0.2, 0.6\}|) = 0.16.$$

3. Generalized Weighted Single-Valued Neutrosophic Power Shapley Choquet Operators

Based on the PA operator and Shapley fuzzy measure, the GWSVNPSCA operator and GWSVNPSCG operator are defined respectively, and the corresponding properties are discussed as well.

3.1. Generalized Weighted Single-Valued Neutrosophic Power Shapley Choquet Averaging Operator

Definition 9. Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ be a group of SVNNS, and $\phi(u, X)$ be a Shapley fuzzy measure. Then, GWSVNPSCA operator with dimension n is a mapping GWSVNPSCA: $SVNN^n \rightarrow SVNN$, and

$$GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) = \left(\frac{\bigoplus_{j=1}^n (\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X)) (1 + S(\psi_{\sigma(j)})) \psi_{\sigma(j)}^\lambda}{\sum_{j=1}^n (\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X)) (1 + S(\psi_{\sigma(j)}))} \right)^{1/\lambda}. \quad (6)$$

Here, $\lambda > 0$, $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(j=1, 2, \dots, n)$, while it can be presented as $\psi_{\sigma(1)} \leq \psi_{\sigma(2)} \leq \dots \leq \psi_{\sigma(n)}$, $B_{\sigma(j)} = \{\sigma(j), \sigma(j+1), \dots, \sigma(n)\}$, $B_{\sigma(n+1)} = \Phi$, and $\phi_{B_{\sigma(n+1)}}(u, X) = 0$.

Moreover, $S(\psi_{\sigma(j)}) = \sum_{i=1, j \neq i}^n (\phi_{B_{\sigma(j)}}(\mu, X) - \phi_{B_{\sigma(j+1)}}(\mu, X)) \text{Sup}(\psi_{\sigma(j)}, \psi_{\sigma(i)})$, and $\text{Sup}(\psi_{\sigma(j)}, \psi_{\sigma(i)}) = 1 - d(\psi_{\sigma(j)}, \psi_{\sigma(i)})$ presents the support for $\psi_{\sigma(j)}$ from $\psi_{\sigma(i)}$, that satisfies the conditions:

- (1) $\text{Sup}(\psi_{\sigma(i)}, \psi_{\sigma(j)}) \in [0, 1]$;
- (2) $\text{Sup}(\psi_{\sigma(i)}, \psi_{\sigma(j)}) = \text{Sup}(\psi_{\sigma(j)}, \psi_{\sigma(i)})$;
- (3) $\text{Sup}(\psi_{\sigma(i)}, \psi_{\sigma(j)}) \geq \text{Sup}(\psi_{\sigma(p)}, \psi_{\sigma(q)})$ iff $d(\psi_{\sigma(i)}, \psi_{\sigma(j)}) < d(\psi_{\sigma(p)}, \psi_{\sigma(q)})$, here d is the distance measure as defined in Definition 8.

Theorem 1. Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ be a group of SVNNS and $\phi(u, X)$ be the Shapley fuzzy measure. Then the aggregated results by utilizing the GWSVNPSCA operator is also an SVNNS, i.e.,

$$GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) = \left\langle \left(1 - \prod_{j=1}^n \left(1 - T_{\psi_{\sigma(j)}}^\lambda \right) \right)^{1/\lambda}, \right. \\ \left. 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right) \right)^{1/\lambda}, \right. \\ \left. 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right) \right)^{1/\lambda} \right\rangle. \quad (7)$$

Here, $S(\psi_{\sigma(j)}) = \sum_{i=1, j \neq i}^n (\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X)) \text{Sup}(\psi_{\sigma(j)}, \psi_{\sigma(i)})$ and satisfies the conditions mentioned in Definition 9.

Proof. To simplify the process, let $\omega_{\sigma(j)} = \frac{(\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X))(1 + S(\psi_{\sigma(j)}))}{\sum_{j=1}^n (\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X))(1 + S(\psi_{\sigma(j)}))}$ in the

following proof.

By using the mathematical induction method, if $n=2$, based on the operation laws in Definition 6, we have:

$$\begin{aligned} \psi_{\sigma(1)}^\lambda &= \left\langle T_{\psi_{\sigma(1)}}^\lambda, 1 - \left(1 - I_{\psi_{\sigma(1)}}\right)^\lambda, 1 - \left(1 - F_{\psi_{\sigma(1)}}\right)^\lambda \right\rangle, \\ \psi_{\sigma(2)}^\lambda &= \left\langle T_{\psi_{\sigma(2)}}^\lambda, 1 - \left(1 - I_{\psi_{\sigma(2)}}\right)^\lambda, 1 - \left(1 - F_{\psi_{\sigma(2)}}\right)^\lambda \right\rangle, \\ \omega_{\sigma(1)} \psi_{\sigma(1)}^\lambda &= \left\langle 1 - \left(1 - T_{\psi_{\sigma(1)}}^\lambda\right)^{\omega_{\sigma(1)}}, \left(1 - \left(1 - I_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}}, \left(1 - \left(1 - F_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}} \right\rangle, \\ \omega_{\sigma(2)} \psi_{\sigma(2)}^\lambda &= \left\langle 1 - \left(1 - T_{\psi_{\sigma(2)}}^\lambda\right)^{\omega_{\sigma(2)}}, \left(1 - \left(1 - I_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}}, \left(1 - \left(1 - F_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}} \right\rangle, \\ \omega_{\sigma(1)} \psi_{\sigma(1)}^\lambda \oplus \omega_{\sigma(2)} \psi_{\sigma(2)}^\lambda &= \left\langle 1 - \left(1 - T_{\psi_{\sigma(1)}}^\lambda\right)^{\omega_{\sigma(1)}} + 1 - \left(1 - T_{\psi_{\sigma(2)}}^\lambda\right)^{\omega_{\sigma(2)}} - \left(1 - \left(1 - T_{\psi_{\sigma(1)}}^\lambda\right)^{\omega_{\sigma(1)}}\right) \left(1 - \left(1 - T_{\psi_{\sigma(2)}}^\lambda\right)^{\omega_{\sigma(2)}}\right), \right. \\ &\quad \left. \left(1 - \left(1 - I_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}} \cdot \left(1 - \left(1 - I_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}}, \left(1 - \left(1 - F_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}} \cdot \left(1 - \left(1 - F_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}} \right\rangle \\ &= \left\langle 1 - \left(1 - T_{\psi_{\sigma(1)}}^\lambda\right)^{\omega_{\sigma(1)}} \left(1 - T_{\psi_{\sigma(2)}}^\lambda\right)^{\omega_{\sigma(2)}}, \left(1 - \left(1 - I_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}} \cdot \left(1 - \left(1 - I_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}}, \right. \\ &\quad \left. \left(1 - \left(1 - F_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}} \cdot \left(1 - \left(1 - F_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}} \right\rangle, \\ (\omega_{\sigma(1)} \psi_{\sigma(1)}^\lambda \oplus \omega_{\sigma(2)} \psi_{\sigma(2)}^\lambda)^{1/\lambda} &= \left\langle \left(1 - \left(1 - T_{\psi_{\sigma(1)}}^\lambda\right)^{\omega_{\sigma(1)}} \left(1 - T_{\psi_{\sigma(2)}}^\lambda\right)^{\omega_{\sigma(2)}}\right)^{1/\lambda}, 1 - \left(1 - \left(1 - I_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}} \cdot \left(1 - \left(1 - I_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}} \right. \\ &\quad \left. 1 - \left(1 - \left(1 - F_{\psi_{\sigma(1)}}\right)^\lambda\right)^{\omega_{\sigma(1)}} \cdot \left(1 - \left(1 - F_{\psi_{\sigma(2)}}\right)^\lambda\right)^{\omega_{\sigma(2)}} \right\rangle^{1/\lambda}. \end{aligned}$$

Apparently, if $n=2$, then Equation (7) holds.

If $n=k$, Equation (7) holds, i.e.,

$$\begin{aligned} &\omega_{\sigma(1)} \psi_{\sigma(1)}^\lambda \oplus \omega_{\sigma(2)} \psi_{\sigma(2)}^\lambda, \dots, \oplus \omega_{\sigma(k)} \psi_{\sigma(k)}^\lambda \\ &= \left\langle 1 - \prod_{j=1}^k \left(1 - T_{\psi_{\sigma(j)}}^\lambda\right)^{\omega_{\sigma(j)}}, \prod_{j=1}^k \left(1 - \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda\right)^{\omega_{\sigma(j)}}, \prod_{j=1}^k \left(1 - \left(1 - F_{\psi_{\sigma(j)}}\right)^\lambda\right)^{\omega_{\sigma(j)}} \right\rangle \end{aligned}$$

and

$$GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_k) \\ = \left\langle \left(1 - \prod_{j=1}^k \left(1 - T_{\psi_{\sigma(j)}}^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^k \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^k \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \right\rangle.$$

Then for $n = k + 1$, we have

$$\begin{aligned} & \omega_{\sigma(1)} \psi_{\sigma(1)}^\lambda \oplus \omega_{\sigma(2)} \psi_{\sigma(2)}^\lambda \oplus \dots \oplus \omega_{\sigma(k)} \psi_{\sigma(k)}^\lambda \oplus \omega_{\sigma(k+1)} \psi_{\sigma(k+1)}^\lambda = \left\langle \left(1 - \prod_{j=1}^k \left(1 - T_{\psi_{\sigma(j)}}^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda}, \prod_{j=1}^k \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right. \\ & \left. \prod_{j=1}^k \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right\rangle \oplus \left\langle 1 - \left(1 - T_{\psi_{\sigma(k+1)}}^\lambda \right)^{\omega_{\sigma(k+1)}}, \left(1 - \left(1 - I_{\psi_{\sigma(k+1)}} \right)^\lambda \right)^{\omega_{\sigma(k+1)}}, \left(1 - \left(1 - F_{\psi_{\sigma(k+1)}} \right)^\lambda \right)^{\omega_{\sigma(k+1)}} \right\rangle \\ & = \left\langle 1 - \prod_{j=1}^k \left(1 - T_{\psi_{\sigma(j)}}^\lambda \right)^{\omega_{\sigma(j)}} + 1 - \left(1 - T_{\psi_{\sigma(k+1)}}^\lambda \right)^{\omega_{\sigma(k+1)}} - \left(1 - \prod_{j=1}^k \left(1 - T_{\psi_{\sigma(j)}}^\lambda \right)^{\omega_{\sigma(j)}} \right) \left(1 - \left(1 - T_{\psi_{\sigma(k+1)}}^\lambda \right)^{\omega_{\sigma(k+1)}} \right), \right. \\ & \left. \prod_{j=1}^k \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \left(1 - \left(1 - I_{\psi_{\sigma(k+1)}} \right)^\lambda \right)^{\omega_{\sigma(k+1)}}, \prod_{j=1}^k \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \left(1 - \left(1 - F_{\psi_{\sigma(k+1)}} \right)^\lambda \right)^{\omega_{\sigma(k+1)}} \right\rangle \\ & = \left\langle 1 - \prod_{j=1}^{k+1} \left(1 - T_{\psi_{\sigma(j)}}^\lambda \right)^{\omega_{\sigma(j)}}, \prod_{j=1}^{k+1} \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}}, \prod_{j=1}^{k+1} \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right\rangle. \end{aligned}$$

Thus,

$$\begin{aligned} GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_{k+1}) &= \left(\omega_{\sigma(1)} \psi_{\sigma(1)}^\lambda \oplus \omega_{\sigma(2)} \psi_{\sigma(2)}^\lambda \oplus \dots \oplus \omega_{\sigma(k)} \psi_{\sigma(k)}^\lambda \oplus \omega_{\sigma(k+1)} \psi_{\sigma(k+1)}^\lambda \right)^{1/\lambda} \\ &= \left\langle \left(1 - \prod_{j=1}^{k+1} \left(1 - T_{\psi_{\sigma(j)}}^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^{k+1} \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^{k+1} \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \right\rangle. \end{aligned}$$

Thus, if $n = k + 1$, Equation (7) is held. In the other words, for any positive real number n , Equation (7) is held. $\square$

Example 4. Let $\psi_1 = \langle 0.7, 0.1, 0.3 \rangle$ and $\psi_2 = \langle 0.8, 0.2, 0.2 \rangle$ be two SVNNS, then we have $\bar{s}(\psi_1) = 0.77$ and $\bar{s}(\psi_2) = 0.8$, i.e., $\psi_{\sigma(1)} = \psi_1$ and $\psi_{\sigma(2)} = \psi_2$. Assume $\mu(\emptyset) = 0$, $\mu(\{\psi_1\}) = 0.6$, $\mu(\{\psi_2\}) = 0.3$, and $\mu(\{\psi_1, \psi_2\}) = 1$, then $\phi_{\psi_1}(\mu, X) = 0.65$, and $\phi_{\psi_2}(\mu, X) = 0.35$. Based on the single-valued neutrosophic normalized Hamming distance presented in Definition 8, we can get $Sup(\psi_1, \psi_2) = Sup(\psi_1, \psi_2) = 1 - d_{gsvnn}(\psi_1, \psi_2) = 0.9$. Thus, let $\lambda = 1$, from Theorem 1, we have:

$$\begin{aligned}
GWSVNPSCA(\psi_1, \psi_2) &= \left\langle \left(1 - (1 - T_{\psi_1})^{\frac{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1))}{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1)) + \left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}} \right) \right. \\
&\quad \left. (1 - T_{\psi_2})^{\frac{\left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1)) + \left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}} \right) \right. \\
&\quad \left. 1 - \left(1 - I_{\psi_1}^{\frac{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1))}{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1)) + \left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}} \right) I_{\psi_2}^{\frac{\left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1)) + \left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}} \right) \right. \\
&\quad \left. 1 - \left(1 - F_{\psi_1}^{\frac{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1))}{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1)) + \left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}} \right) F_{\psi_2}^{\frac{\left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}{\left(\phi_{\psi_1}^s(\mu, X) - \phi_{\psi_2}^s(\mu, X) \right) (1+S(\psi_1)) + \left(\phi_{\psi_2}^s(\mu, X) - 0 \right) (1+S(\psi_2))}} \right) \right) \Bigg\rangle \\
&= \left\langle \left(1 - (1 - 0.7)^{\frac{(0.65-0.35)(1+0.27)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} (1 - 0.8)^{\frac{(0.35-0)(1+0.315)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} \right) \right. \\
&\quad \left. 1 - \left(1 - 0.1^{\frac{(0.65-0.35)(1+0.27)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} 0.2^{\frac{(0.35-0)(1+0.315)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} \right) \right. \\
&\quad \left. 1 - \left(1 - 0.3^{\frac{(0.65-0.35)(1+0.27)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} 0.2^{\frac{(0.35-0)(1+0.315)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} \right) \right) \Bigg\rangle = \langle 0.7597, 0.1461, 0.2403 \rangle.
\end{aligned}$$

In the following, a number of properties of GWSVNPSCA operator are discussed.

Theorem 2 (Idempotency). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle$ ($j = 1, 2, \dots, n$) be a group of SVNNS. If $\psi_1 = \psi_2 = \dots = \psi_n = \psi = \langle T_\psi, I_\psi, F_\psi \rangle$, then $GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) = \psi$.

Proof. Since $\psi_1 = \psi_2 = \dots = \psi_n = \psi = \langle T_\psi, I_\psi, F_\psi \rangle$, thus we have

$$\begin{aligned}
GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) &= GWSVNPSCA(\psi, \psi, \dots, \psi) = \left\langle \left(1 - (1 - T_\psi^\lambda)^{\frac{\sum_{j=1}^n \frac{\left(\phi_{\psi_{\sigma(j)}}^s(\mu, X) - \phi_{\psi_{\sigma(j+1)}}^s(\mu, X) \right) (1+S(\psi))}{\sum_{j=1}^n \left(\phi_{\psi_{\sigma(j)}}^s(\mu, X) - \phi_{\psi_{\sigma(j+1)}}^s(\mu, X) \right) (1+S(\psi))}} \right)^{1/\lambda} \right. \\
&\quad \left. 1 - \left(1 - (1 - I_\psi^\lambda)^{\frac{\sum_{j=1}^n \frac{\left(\phi_{\psi_{\sigma(j)}}^s(\mu, X) - \phi_{\psi_{\sigma(j+1)}}^s(\mu, X) \right) (1+S(\psi))}{\sum_{j=1}^n \left(\phi_{\psi_{\sigma(j)}}^s(\mu, X) - \phi_{\psi_{\sigma(j+1)}}^s(\mu, X) \right) (1+S(\psi))}} \right)^{1/\lambda} \right. \\
&\quad \left. 1 - \left(1 - (1 - F_\psi^\lambda)^{\frac{\sum_{j=1}^n \frac{\left(\phi_{\psi_{\sigma(j)}}^s(\mu, X) - \phi_{\psi_{\sigma(j+1)}}^s(\mu, X) \right) (1+S(\psi))}{\sum_{j=1}^n \left(\phi_{\psi_{\sigma(j)}}^s(\mu, X) - \phi_{\psi_{\sigma(j+1)}}^s(\mu, X) \right) (1+S(\psi))}} \right)^{1/\lambda} \right) \right) \\
&= \left\langle (1 - 1 + T_\psi^\lambda)^{1/\lambda}, 1 - (1 - 1 + (1 - I_\psi^\lambda)^{1/\lambda})^{1/\lambda}, 1 - (1 - 1 + (1 - F_\psi^\lambda)^{1/\lambda})^{1/\lambda} \right\rangle \\
&= \left\langle (T_\psi^\lambda)^{1/\lambda}, 1 - ((1 - I_\psi^\lambda)^{1/\lambda})^{1/\lambda}, 1 - ((1 - F_\psi^\lambda)^{1/\lambda})^{1/\lambda} \right\rangle = \langle T_\psi, I_\psi, F_\psi \rangle.
\end{aligned}$$

Thus, $GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) = \psi$ can be true. $\square$

Theorem 3 (Permutability). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ be a group of SVNNS. If $\bar{\psi}_j = (\bar{T}_{\bar{\psi}_j}, \bar{I}_{\bar{\psi}_j}, \bar{F}_{\bar{\psi}_j}) (j=1, 2, \dots, n)$ accompanies with any permutation of $\psi_j = (T_{\psi_j}, I_{\psi_j}, F_{\psi_j}) (j=1, 2, \dots, n)$, then,

$$GWSVNPSCA(\bar{\psi}_1, \bar{\psi}_2, \dots, \bar{\psi}_n) = GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n).$$

Proof. Since $\bar{\psi}_j = \langle \bar{T}_{\bar{\psi}_j}, \bar{I}_{\bar{\psi}_j}, \bar{F}_{\bar{\psi}_j} \rangle (j=1, 2, \dots, n)$ is an any permutation of $\psi_j = (T_{\psi_j}, I_{\psi_j}, F_{\psi_j}) (j=1, 2, \dots, n)$, therefore $\bar{\psi}_{\sigma(j)} = \psi_{\sigma(j)} (j=1, 2, \dots, n)$, i.e., $\bar{T}_{\bar{\psi}_{\sigma(j)}} = T_{\psi_{\sigma(j)}}$, $\bar{I}_{\bar{\psi}_{\sigma(j)}} = I_{\psi_{\sigma(j)}}$ and $\bar{F}_{\bar{\psi}_{\sigma(j)}} = F_{\psi_{\sigma(j)}}$. Thus,

$$\begin{aligned} GWSVNPSCA(\bar{\psi}_1, \bar{\psi}_2, \dots, \bar{\psi}_n) &= \left\langle \left(1 - \prod_{j=1}^n \left(1 - \bar{T}_{\bar{\psi}_{\sigma(j)}}^{\lambda} \right) \right)^{\frac{1}{\lambda}}, \right. \\ &\quad \left. 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - \bar{I}_{\bar{\psi}_{\sigma(j)}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}, \right. \right. \\ &\quad \left. \left. 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - \bar{F}_{\bar{\psi}_{\sigma(j)}} \right)^{\lambda} \right)^{\frac{1}{\lambda}} \right) \right)^{\frac{1}{\lambda}} \right\rangle \\ &= \left\langle \left(1 - \prod_{j=1}^n \left(1 - T_{\psi_{\sigma(j)}}^{\lambda} \right) \right)^{\frac{1}{\lambda}}, \left(1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}, \right. \right. \\ &\quad \left. \left. 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^{\lambda} \right)^{\frac{1}{\lambda}} \right) \right)^{\frac{1}{\lambda}} \right\rangle \\ &= GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n). \quad \square \end{aligned}$$

Theorem 4 (Boundedness). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ be a group of SVNNS. If

$$\psi^- = \left\langle \min_j \{T_{\psi_j}\}, \max_j \{I_{\psi_j}\}, \max_j \{F_{\psi_j}\} \right\rangle \quad \text{and} \quad \psi^+ = \left\langle \max_j \{T_{\psi_j}\}, \min_j \{I_{\psi_j}\}, \min_j \{F_{\psi_j}\} \right\rangle, \quad \text{then}$$

$$\psi^- \leq GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) \leq \psi^+.$$

Proof. For simplicity, let $\omega_{\sigma(j)} = \frac{(\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X))(1 + S(\psi_{\sigma(j)}))}{\sum_{j=1}^n (\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X))(1 + S(\psi_{\sigma(j)}))}$ in the following

proof.

$$\text{Since} \quad \min_j \{T_{\psi_j}\} \leq T_{\psi_{\sigma(j)}} \leq \max_j \{T_{\psi_j}\}, \quad \text{i.e.,} \quad \min_j \{T_{\psi_j}^\lambda\} \leq T_{\psi_{\sigma(j)}}^\lambda \leq \max_j \{T_{\psi_j}^\lambda\},$$

$$\text{Then} \quad -\max_j \{T_{\psi_j}^\lambda\} \leq -T_{\psi_{\sigma(j)}}^\lambda \leq -\min_j \{T_{\psi_j}^\lambda\} \Leftrightarrow 1 - \max_j \{T_{\psi_j}^\lambda\} \leq 1 - T_{\psi_{\sigma(j)}}^\lambda \leq 1 - \min_j \{T_{\psi_j}^\lambda\}$$

$$\Leftrightarrow \left(1 - \max_j \{T_{\psi_j}^\lambda\}\right)^{\omega_{\sigma(j)}} \leq \left(1 - T_{\psi_{\sigma(j)}}^\lambda\right)^{\omega_{\sigma(j)}} \leq \left(1 - \min_j \{T_{\psi_j}^\lambda\}\right)^{\omega_{\sigma(j)}}$$

$$\Leftrightarrow 1 - \max_j \{T_{\psi_j}^\lambda\} = \prod_{j=1}^n \left(1 - \max_j \{T_{\psi_j}^\lambda\}\right)^{\omega_{\sigma(j)}} \leq \prod_{j=1}^n \left(1 - T_{\psi_{\sigma(j)}}^\lambda\right)^{\omega_{\sigma(j)}} \leq \prod_{j=1}^n \left(1 - \min_j \{T_{\psi_j}^\lambda\}\right)^{\omega_{\sigma(j)}} = 1 - \min_j \{T_{\psi_j}^\lambda\}$$

$$\Leftrightarrow \min_j \{T_{\psi_j}^\lambda\} - 1 \leq -\prod_{j=1}^n \left(1 - T_{\psi_{\sigma(j)}}^\lambda\right)^{\omega_{\sigma(j)}} \leq \max_j \{T_{\psi_j}^\lambda\} - 1 \Leftrightarrow \min_j \{T_{\psi_j}^\lambda\} \leq 1 - \prod_{j=1}^n \left(1 - T_{\psi_{\sigma(j)}}^\lambda\right)^{\omega_{\sigma(j)}} \leq \max_j \{T_{\psi_j}^\lambda\}$$

$$\Leftrightarrow \min_j \{T_{\psi_j}\} = \left(\min_j \{T_{\psi_j}^\lambda\}\right)^{1/\lambda} \leq \left(1 - \prod_{j=1}^n \left(1 - T_{\psi_{\sigma(j)}}^\lambda\right)^{\omega_{\sigma(j)}}\right)^{1/\lambda} \leq \left(\max_j \{T_{\psi_j}^\lambda\}\right)^{1/\lambda} = \max_j \{T_{\psi_j}\}$$

$$\text{Moreover,} \quad \min_j \{I_{\psi_j}\} \leq I_{\psi_{\sigma(j)}} \leq \max_j \{I_{\psi_j}\} \Leftrightarrow 1 - \max_j \{I_{\psi_j}\} \leq 1 - I_{\psi_{\sigma(j)}} \leq 1 - \min_j \{I_{\psi_j}\}$$

$$\Leftrightarrow \left(1 - \max_j \{I_{\psi_j}\}\right)^\lambda \leq \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda \leq \left(1 - \min_j \{I_{\psi_j}\}\right)^\lambda$$

$$\Leftrightarrow 1 - \left(1 - \min_j \{I_{\psi_j}\}\right)^\lambda \leq 1 - \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda \leq 1 - \left(1 - \max_j \{I_{\psi_j}\}\right)^\lambda$$

$$\Leftrightarrow \left(1 - \left(1 - \min_j \{I_{\psi_j}\}\right)^\lambda\right)^{\omega_{\sigma(j)}} \leq \left(1 - \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda\right)^{\omega_{\sigma(j)}} \leq \left(1 - \left(1 - \max_j \{I_{\psi_j}\}\right)^\lambda\right)^{\omega_{\sigma(j)}}$$

$$\Leftrightarrow \prod_{j=1}^n \left(1 - \left(1 - \min_j \{I_{\psi_j}\}\right)^\lambda\right)^{\omega_{\sigma(j)}} \leq \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda\right)^{\omega_{\sigma(j)}} \leq \prod_{j=1}^n \left(1 - \left(1 - \max_j \{I_{\psi_j}\}\right)^\lambda\right)^{\omega_{\sigma(j)}}$$

$$\Leftrightarrow \left(1 - \left(1 - \min_j \{I_{\psi_j}\}\right)^\lambda\right)^{\sum_{j=1}^n \omega_{\sigma(j)}} \leq \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda\right)^{\omega_{\sigma(j)}} \leq \left(1 - \left(1 - \max_j \{I_{\psi_j}\}\right)^\lambda\right)^{\sum_{j=1}^n \omega_{\sigma(j)}}$$

$$\Leftrightarrow 1 - \left(1 - \min_j \{I_{\psi_j}\}\right)^\lambda \leq \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda\right)^{\omega_{\sigma(j)}} \leq 1 - \left(1 - \max_j \{I_{\psi_j}\}\right)^\lambda$$

$$\Leftrightarrow \left(1 - \max_j \{I_{\psi_j}\}\right)^\lambda \leq 1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}}\right)^\lambda\right)^{\omega_{\sigma(j)}} \leq \left(1 - \min_j \{I_{\psi_j}\}\right)^\lambda$$

$$\begin{aligned}
&\Leftrightarrow \left(\left(1 - \max_j \{ I_{\psi_j} \} \right)^\lambda \right)^{1/\lambda} \leq \left(1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \leq \left(\left(1 - \min_j \{ I_{\psi_j} \} \right)^\lambda \right)^{1/\lambda} \\
&\Leftrightarrow 1 - \max_j \{ I_{\psi_j} \} \leq \left(1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \leq 1 - \min_j \{ I_{\psi_j} \} \\
&\Leftrightarrow \min_j \{ I_{\psi_j} \} = 1 - 1 + \min_j \{ I_{\psi_{\sigma(j)}} \} \leq 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \leq 1 - 1 + \max_j \{ I_{\psi_j} \} = \max_j \{ I_{\psi_j} \} \\
&\min_j \{ F_{\psi_j} \} \leq 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \leq \max_j \{ F_{\psi_j} \}
\end{aligned}$$

Similarly,

Thus, we have

$$\begin{aligned}
&\Leftrightarrow \frac{1}{3} \left(\min_j \{ T_{\psi_j} \} + 1 - \max_j \{ I_{\psi_j} \} + 1 - \max_j \{ F_{\psi_j} \} \right) \leq \frac{1}{3} \left(\left(1 - \prod_{j=1}^n \left(1 - T_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} + 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - I_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \right. \\
&\quad \left. + 1 - \left(1 - \prod_{j=1}^n \left(1 - \left(1 - F_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\omega_{\sigma(j)}} \right)^{1/\lambda} \right) \leq \frac{1}{3} \left(\max_j \{ T_{\psi_j} \} + 1 - \min_j \{ I_{\psi_j} \} + 1 - \min_j \{ F_{\psi_j} \} \right)
\end{aligned}$$

i.e., $\bar{s}(\psi^-) \leq s(GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n)) \leq \bar{s}(\psi^+)$.

Hence, we can obtain $\psi^- \leq GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) \leq \psi^+$. $\square$

Theorem 5 (Monotonicity). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j = 1, 2, \dots, n)$ and $\bar{\psi}_j = \langle \bar{T}_{\bar{\psi}_j}, \bar{I}_{\bar{\psi}_j}, \bar{F}_{\bar{\psi}_j} \rangle (j = 1, 2, \dots, n)$ be two groups of SVNNS. If $T_{\psi_j} \geq \bar{T}_{\bar{\psi}_j}, I_{\psi_j} \leq \bar{I}_{\bar{\psi}_j}$ and $F_{\psi_j} \leq \bar{F}_{\bar{\psi}_j}$ for all $j = 1, 2, \dots, n$, then

$$GWSVNPSCA(\psi_1, \psi_2, \dots, \psi_n) \geq GWSVNPSCA(\bar{\psi}_1, \bar{\psi}_2, \dots, \bar{\psi}_n).$$

Proof. According to Theorem 4, Theorem 5 can be confirmed. Therefore, the process of treatment is omitted here. $\square$

Besides, GWSVNPSCA operator, which defined by combing the advantages of Shapley fuzzy measure and PA aggregation operator, can reduce the effects of abnormal data and simultaneously consider an interrelationship among data and criteria. It is more appropriate in aggregation data because of different parameter.

3.2. Generalized Weighted Single-Valued Neutrosophic Power Shapley Choquet Geometric Operator

Definition 10. Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j = 1, 2, \dots, n)$ be a group of SVNNS, and $\phi(u, X)$ be the Shapley fuzzy measure. Then GWSVNPSCG operator of dimension n is a mapping $GWSVNPSCG: SVNNS^n \rightarrow SVNNS$, and

$$GWSVNPSCG(\psi_1, \psi_2, \dots, \psi_n) = \frac{1}{\lambda} \left(\frac{\bigotimes_{j=1}^n (\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X)) (1 + S(\psi_{\sigma(j)})) \lambda \psi_{\sigma(j)}}{\sum_{j=1}^n (\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X)) (1 + S(\psi_{\sigma(j)}))} \right). \quad (8)$$

where $\lambda > 0$, $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(j = 1, 2, \dots, n)$ satisfying

$$\psi_{\sigma(1)} \leq \psi_{\sigma(2)} \leq \dots \leq \psi_{\sigma(n)}, \quad B_{\sigma(j)} = \{\sigma(j), \sigma(j+1), \dots, \sigma(n)\} \quad \text{and} \quad B_{\sigma(n+1)} = \Phi, \phi_{B_{\sigma(n+1)}}(u, X) = 0.$$

Theorem 6. Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j = 1, 2, \dots, n)$ be a group of SVNNS, and $\phi(u, X)$ be the Shapley fuzzy measure. Next, the aggregated results by utilizing GWSVNPSCG operator are also an SVNNS, i.e.,

$$GWSVPSCG(\psi_1, \psi_2, \dots, \psi_n) = \left\langle 1 - \prod_{j=1}^n \left(1 - \left(1 - T_{\psi_{\sigma(j)}} \right)^\lambda \right)^{\frac{\left\{ \phi_{\theta_{\sigma(j)}}^{\delta}(\mu, X) - \phi_{\theta_{\sigma(j+1)}}^{\delta}(\mu, X) \right\} 1 + S(\psi_{\sigma(j)})}}{\sum_{j=1}^n \left\{ \phi_{\theta_{\sigma(j)}}^{\delta}(\mu, X) - \phi_{\theta_{\sigma(j+1)}}^{\delta}(\mu, X) \right\} 1 + S(\psi_{\sigma(j)})}} \right\rangle^{\frac{1}{\lambda}},$$

Proof. The proof is omitted here. $\square$

Example 5. Based on Example 4 and Theorem 6, the following results can be obtained:

$$\begin{aligned}
GMS/NPSCG(\psi_1, \psi_2) &= \left\langle 1 - \left(1 - T_{\psi_1} \frac{\frac{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1))}{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1)) + (\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))}}{T_{\psi_2} \frac{(\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))}{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1)) + (\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))}} \right) \right. \\
&\quad \left(1 - \left(1 - I_{\psi_1} \frac{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1))}{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1)) + (\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))} \right) \left(1 - I_{\psi_2} \frac{(\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))}{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1)) + (\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))} \right) \right) \\
&\quad \left. \left(1 - \left(1 - F_{\psi_1} \frac{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1))}{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1)) + (\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))} \right) \left(1 - F_{\psi_2} \frac{(\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))}{(\hat{\phi}_{\psi_1}(\mu, X) - \hat{\phi}_{\psi_2}(\mu, X))(1+S(\psi_1)) + (\hat{\phi}_{\psi_2}(\mu, X) - 0)(1+S(\psi_2))} \right) \right) \right) \\
&= \left\langle 1 - \left(1 - 0.7^{\frac{(0.65-0.35)(1+0.27)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} 0.8^{\frac{(0.35-0)(1+0.315)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} \right) \right. \\
&\quad \left(1 - (1-0.1)^{\frac{(0.65-0.35)(1+0.27)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} (1-0.2)^{\frac{(0.35-0)(1+0.315)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} \right) \\
&\quad \left. \left(1 - (1-0.3)^{\frac{(0.65-0.35)(1+0.27)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} (1-0.2)^{\frac{(0.35-0)(1+0.315)}{(0.65-0.35)(1+0.27)+(0.35-0)(1+0.315)}} \right) \right) = \langle 0.753, 0.1561, 0.247 \rangle.
\end{aligned}$$

Similarly, GWSVNPSCG operator has the following properties.

Theorem 7 (Idempotency). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ be a group of SVNNS. If $\psi_1 = \psi_2 = \dots = \psi_n = \psi = \langle T_\psi, I_\psi, F_\psi \rangle$, then $GWSVNPSCG(\psi_1, \psi_2, \dots, \psi_n) = \psi$.

Theorem 8 (Permutability). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ be a group of SVNNS. If $\bar{\psi}_j = \langle \bar{T}_{\bar{\psi}_j}, \bar{I}_{\bar{\psi}_j}, \bar{F}_{\bar{\psi}_j} \rangle (j=1, 2, \dots, n)$ is any permutation of $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$, then,

$$GWSVNPSCG(\bar{\psi}_1, \bar{\psi}_2, \dots, \bar{\psi}_n) = GWSVNPSCG(\psi_1, \psi_2, \dots, \psi_n).$$

Theorem 9 (Boundedness). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ be a group of SVNNS. If $\psi^- = \langle \min_j \{T_{\psi_j}\}, \max_j \{I_{\psi_j}\}, \max_j \{F_{\psi_j}\} \rangle$ and $\psi^+ = \langle \max_j \{T_{\psi_j}\}, \min_j \{I_{\psi_j}\}, \min_j \{F_{\psi_j}\} \rangle$, thus, $\psi^- \leq GSVNPSCG(\psi_1, \psi_2, \dots, \psi_n) \leq \psi^+$.

Theorem 10 (Monotonicity). Let $\psi_j = \langle T_{\psi_j}, I_{\psi_j}, F_{\psi_j} \rangle (j=1, 2, \dots, n)$ and $\bar{\psi}_j = \langle \bar{T}_{\bar{\psi}_j}, \bar{I}_{\bar{\psi}_j}, \bar{F}_{\bar{\psi}_j} \rangle (j=1, 2, \dots, n)$ be two groups of SVNNS. If $T_{\psi_j} \geq \bar{T}_{\bar{\psi}_j}, I_{\psi_j} \leq \bar{I}_{\bar{\psi}_j}$ and $F_{\psi_j} \leq \bar{F}_{\bar{\psi}_j}$ for all $j=1, 2, \dots, n$, then

$$GWSVNPSCG(\psi_1, \psi_2, \dots, \psi_n) \geq GWSVNPSCG(\bar{\psi}_1, \bar{\psi}_2, \dots, \bar{\psi}_n).$$

4. A Single-Valued Neutrosophic MCDM Approach with Incomplete Weight Information

Assume $\psi = \{\psi_1, \psi_2, \dots, \psi_n\}$ is a group of alternatives and $C = \{c_1, c_2, \dots, c_m\}$ includes corresponding criteria. $R = (\psi_{ij})_{n \times m}$ is the single-valued neutrosophic decision matrix, and $\psi_{ij} = \langle T_{\psi_{ij}}, I_{\psi_{ij}}, F_{\psi_{ij}} \rangle (i=1, 2, \dots, n; j=1, 2, \dots, m)$ can be provided by the DM with respect to ψ_i for criterion c_j being expressed by SVNNS. If the weight of criteria for the Shapley fuzzy measure would be known, then the corresponding aggregation operators can be directly used to obtain the aggregated values; otherwise, if it would be partly known or fully unknown, next, the weights of criteria should be initially determined. The scheme of proposed method can be presented in Figure 1, and the procedure aiming to determine the optimal alternative(s) is provided in the following.

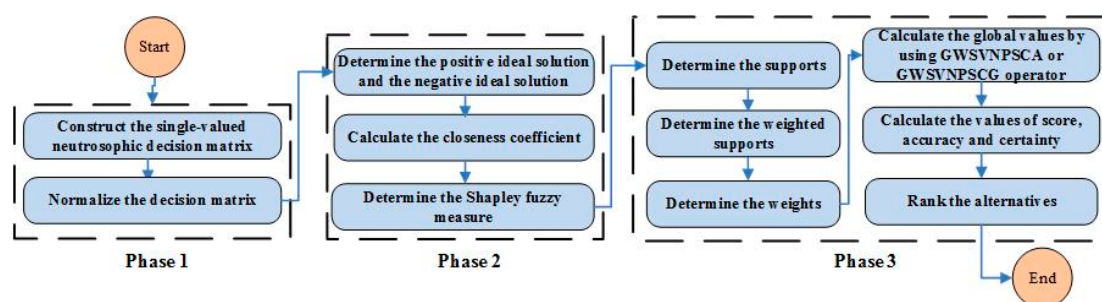


Figure 1. The scheme of proposed method.

Phase 1. Construction and normalization of the decision matrix

In this phase, the criteria for each alternative are assessed by DMs. The decision-matrix can be constructed accordingly. Moreover, the criteria always involves benefit type and cost type in MCDM problems. If the criteria are benefit-based, then, there is no need to normalize the decision matrix; otherwise, the cost-based criteria should be transformed into corresponding benefit-based criteria as:

$$\bar{\psi}_{ij} = \begin{cases} \psi_{ij}, & \text{for benefit criteria } c_j \\ (\psi_{ij})^c, & \text{for cost criteria } c_j \end{cases}, \quad (i=1,2,\dots,n; j=1,2,\dots,m). \quad (100)$$

where, $(\psi_{ij})^c = \langle F_{ij}, 1 - I_{ij}, T_{ij} \rangle$ is the complement of ψ_{ij} .

Then, the normalized decision matrix $\bar{R} = (\bar{\psi}_{ij})_{n \times m}$ can be obtained.

Phase 2. Determinations of the Shapley fuzzy measures

Step 2.1. Determination of ideal solutions

Let $\bar{\psi}^+ = (\bar{\psi}_1^+, \bar{\psi}_2^+, \dots, \bar{\psi}_n^+)$ be positive ideal solution, and $\bar{\psi}^- = (\bar{\psi}_1^-, \bar{\psi}_2^-, \dots, \bar{\psi}_n^-)$ be negative ideal solution, respectively. Here, $\bar{\psi}_j^+ = (\max_i \bar{T}_{ij}, \min_i \bar{I}_{ij}, \min_i \bar{F}_{ij})$ and $\bar{\psi}_j^- = (\min_i \bar{T}_{ij}, \max_i \bar{I}_{ij}, \max_i \bar{F}_{ij})$ ($i=1,2,\dots,n; j=1,2,\dots,m$).

Step 2.2. Calculation of closeness coefficients

Based on the technique presented for order preference by similarity to ideal solution (TOPSIS) method [51], the closeness coefficient of candidate from positive ideal solution can be calculated as follows:

$$D_{ij}^+ (\bar{\psi}_{ij}, \bar{\psi}^+) = \frac{d_{ij} (\bar{\psi}_{ij}, \bar{\psi}^+)}{d_{ij} (\bar{\psi}_{ij}, \bar{\psi}^+) + d_{ij} (\bar{\psi}_{ij}, \bar{\psi}^-)} \quad (i=1,2,\dots,n; j=1,2,\dots,m). \quad (11)$$

where, $d_{ij} (\bar{\psi}_{ij}, \bar{\psi}^+)$ can be obtained by using Equation (3).

Step 2.3. Determination of Shapley fuzzy measures

According to TOPSIS method [52], the smaller $D_{ij}^+ (\bar{\psi}_{ij}, \bar{\psi}^+)$, the better $\bar{\psi}_{ij}$ is. If the weight information of criteria would be partly known, then, an optimal model based on the Shapley fuzzy measure can be constructed as:

$$\begin{aligned} & \min \sum_{j=1}^n D_{ij}^+ (\bar{\psi}_{ij}, \bar{\psi}^+) \phi_{c_j} (\mu, C) \\ & s.t. \begin{cases} \mu(C) = 1 \\ \mu(E) \leq \mu(F), \forall E, F \in C \text{ and } E \subseteq F \\ \mu(C_j) \in G_j, \mu(C_j) \geq 0, j=1,2,\dots,n \end{cases} \end{aligned} \quad (12)$$

where, $\phi_{c_j} (\mu, C)$ denotes the integral weight of the criterion c_j , and G_j represents the weight information.

Next, fuzzy measure and corresponding the Shapley fuzzy measure can be obtained by solving the mode (12).

Phase 3. Aggregation the preference information

Step 3.1. Determination of the supports

From Definition 9, we can achieve the supports $Sup(\bar{\psi}_{ij}, \bar{\psi}_{ip})$ as:

$$Sup(\bar{\psi}_{ij}, \bar{\psi}_{ip}) = 1 - d(\bar{\psi}_{ij}, \bar{\psi}_{ip}), \quad i = 1, 2, \dots, n; j, p = 1, 2, \dots, m; j \neq p. \quad (133)$$

where, $Sup(\bar{\psi}_{ij}, \bar{\psi}_{ip})$ represents the support for $\bar{\psi}_{ij}$ from $\bar{\psi}_{ip}$, satisfying the conditions provided in Definition 9. Besides, $d(\bar{\psi}_{ij}, \bar{\psi}_{ip})$ can be calculated by using the distance presented in Definition 8.

Step 3.2. Determination the weighted supports

The weighted support $S(\bar{\psi}_{ij})$ of $\bar{\psi}_{ij}$ from $\bar{\psi}_{ip}$ ($p = 1, 2, \dots, m$ and $p \neq j$) can be calculated using the Shapley fuzzy measure of the criterion c_j ($j = 1, 2, \dots, m$).

$$S(\bar{\psi}_{ij}) = \sum_{p=1, p \neq j}^m \left(\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X) \right) Sup(\bar{\psi}_{ij}, \bar{\psi}_{ip}) \quad (p = 1, 2, \dots, m). \quad (144)$$

Step 3.3. Determination of the weights

From Step 3.2, we can determine the weights π_{ij} ($j = 1, 2, \dots, m$) associated with $\bar{\psi}_{ij}$ ($j = 1, 2, \dots, m$) as:

$$\pi_{ij} = \frac{\left(\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X) \right) (1 + S(\bar{\psi}_{ij}))}{\sum_{j=1}^m \left(\phi_{B_{\sigma(j)}}^s(\mu, X) - \phi_{B_{\sigma(j+1)}}^s(\mu, X) \right) (1 + S(\bar{\psi}_{ij}))} \quad (j = 1, 2, \dots, m). \quad (155)$$

Here $\pi_{ij} \geq 0$ ($j = 1, 2, \dots, m$) and $\sum_{j=1}^m \pi_{ij} = 1$.

Step 3.4. Calculation of the global values

By using GWSVNPSCW operator, i.e., Equation (8), the global value α_i ($i = 1, 2, \dots, n$) of candidate $\bar{\psi}_i$ ($i = 1, 2, \dots, n$) can be obtained.

Step 3.5. Calculation of the values of score, accuracy, and certainty

Based on Definition 7, the values of score $\bar{s}(\alpha_i)$, accuracy $\bar{a}(\alpha_i)$, and certainty $\bar{c}(\alpha_i)$ of $\bar{\psi}_i$ ($i = 1, 2, \dots, n$) can be achieved.

Step 3.6. Ranking the alternatives

According to Step 3.5, all candidates $\bar{\psi}_i$ ($i = 1, 2, \dots, n$) can be ranked and the optimal one can be selected.

5. Tourism Attraction Selection

In this section, a MCDM problem under single-valued environment is presented to illustrate the efficacy of the proposed method by selecting an appropriate tourism attraction. With the

widespread popularity of the mobile Internet and the increasing demand of tourists regarding the quality of tourism services, online tourism, mobile tourism, and other services have gradually risen. Personalized tourism recommendations can provide tourism products for a user, reflecting his/her needs and preferences and help him/her promptly make tourism decisions. In the personalized tourism recommendation system, accurate and effective recommendations are provided mainly through four criteria related to preference information, including constraints, tourism resource bank, user's tourism demand, and user's preference for tourism products.

c_1 : Constraints are described as some objective conditions that affect the user's decision making about tourism, including the age or physical condition of user, the gender or income of user, and weather conditions related to tourist attractions, carrying children and the elderly, etc.

c_2 : The tourist resource bank contains six aspects of the tourist products, including food, residence, tourism, entertainment, and purchase. It also contains the data related to various kinds of tourism products. The tourist resource bank is of great importance to analyze, produce, and make up the recommended results. Generally speaking, tourist attractions' recommendation can collect the names of scenic spots and tickets, categories of scenic spots, grades, longitude and latitude directions, and so on.

c_3 : Tourism demand refers to the user's demand for tourism products that they would like to purchase, that is, a subjective condition, such as the user's tourism days, hotel's condition, etc.

c_4 : The user's preference for tourism products can be obtained by analyzing the tourist's historical tourism data (such as the tourist photos uploaded by the user, the user's comments or rating on the tourism products), the pages on the tourism website, purchase or transaction records of tourism products, etc.

According to the four criteria presented above, users should select the best tourist attraction according to their own and actual situation of the tourist attractions. Apparently, four criteria are correlated with each other. Suppose five tourist attractions (candidates) are taken into consideration, which are represented by $\psi_i (i=1,2,3,4,5)$. During assessment, four criteria, namely: c_1 : constraints; c_2 : the tourist resource bank; c_3 : tourism demand; c_4 : preferences are considered. The corresponding weights of these four criteria are partly known, given as $H = \{0.10 \leq w_1 \leq 0.15, 0.10 \leq w_2 \leq 0.20, 0.15 \leq w_3 \leq 0.35, 0.20 \leq w_4 \leq 0.45\}$. The evaluation of five candidates $\psi_i (i=1,2,3,4,5)$ is performed using SVNNS by the DM based on criterion $c_k (k=1,2,3,4)$. Then, the single-valued neutrosophic decision matrix $R = (\psi_{kl})_{5 \times 4}$ is established and shown in Table 1.

Table 1. Decision-making matrix.

	c_1	c_2	c_3	c_4
ψ_1	(0.75,0.15,0.20)	(0.68,0.13,0.25)	(0.64,0.11,0.15)	(0.65,0.09,0.18)
ψ_2	(0.62,0.21,0.17)	(0.75,0.10,0.23)	(0.58,0.20,0.22)	(0.82,0.10,0.09)
ψ_3	(0.68,0.05,0.24)	(0.71,0.20,0.17)	(0.61,0.23,0.21)	(0.74,0.2,0.05)
ψ_4	(0.72,0.11,0.21)	(0.79,0.10,0.23)	(0.73,0.14,0.2)	(0.67,0.13,0.10)
ψ_5	(0.76,0.23,0.19)	(0.65,0.16,0.22)	(0.77,0.13,0.20)	(0.71,0.06,0.22)

5.1. The Decision-Making Process

The decision-making process, by using the proposed method, is as follows.

Phase 1. Construction and normalization of the decision matrix

Since the assessment values are provided by DMs in form of SVNNS and the criteria are benefit-based, as a result, the normalized decision matrix $\bar{R} = R = (\psi_{ij})_{n \times m}$ is obtained.

Phase 2. Determination of Shapley fuzzy measures

Step 2.1. Determination of ideal solutions

From Table 1, we can achieve positive ideal solution $\bar{\psi}^+ = (\bar{\psi}_1^+, \bar{\psi}_2^+, \dots, \bar{\psi}_n^+)$ and negative ideal solution $\bar{\psi}^- = (\bar{\psi}_1^-, \bar{\psi}_2^-, \dots, \bar{\psi}_n^-)$, as displayed in Table 2.

Table 2. Positive ideal solution and negative ideal solution.

	$\bar{\psi}^+$	$\bar{\psi}^-$
c_1	(0.76, 0.05, 0.17)	(0.62, 0.23, 0.24)
c_2	(0.79, 0.10, 0.17)	(0.65, 0.10, 0.17)
c_3	(0.67, 0.11, 0.15)	(0.58, 0.23, 0.22)
c_4	(0.82, 0.06, 0.05)	(0.64, 0.20, 0.22)

Step 2.2. Calculation of the closeness coefficient

Let $\lambda = 2$, the closeness coefficient of alternative from positive ideal solution can be determined as given in Table 3.

Table 3. Closeness coefficient.

	c_1	c_2	c_3	c_4
ψ_1	0.3040	0.7055	0.5648	0.7342
ψ_2	0.7996	0.3132	0.9207	0.1437
ψ_3	0.4276	0.5658	0.8533	0.4606
ψ_4	0.3197	0.1857	0.2473	0.6565
ψ_5	0.4225	0.8359	0.1352	0.5942

Step 2.3. Determination of the Shapley fuzzy measures

Then the optimal Shapley fuzzy measure model on the basis of criteria can be constructed as:

$$\begin{aligned}
 & \min -0.0914\mu(c_1) - 0.0359\mu(c_1, c_2) - 0.0139\mu(c_1, c_2, c_3) - 0.0579\mu(c_1, c_2, c_4) - 0.0167\mu(c_1, c_3) \\
 & - 0.0195\mu(c_1, c_3, c_4) - 0.00387\mu(c_1, c_4) + 0.0195\mu(c_2) + 0.0387\mu(c_2, c_3) + 0.0914\mu(c_2, c_3, c_4) + \\
 & 0.0167\mu(c_2, c_4) + 0.0579\mu(c_3) + 0.0359\mu(c_3, c_4) + 0.0139\mu(c_4) + 2.5475 \\
 & s.t. \begin{cases} \mu(c_1, c_2, c_3, c_4) = 1 \\ \mu(E) \leq \mu(F), \forall E, F \in C \text{ and } E \subseteq F \\ 0.1 \leq \mu(c_1) \leq 0.15 \\ 0.10 \leq \mu(c_2) \leq 0.2 \\ 0.15 \leq \mu(c_3) \leq 0.35 \\ 0.25 \leq \mu(c_4) \leq 0.45 \end{cases}
 \end{aligned}$$

With the help of MATLAB software, the mathematical programming model presented above can be solved, and the fuzzy measure on the basis of the criteria is $\mu(c_1) = \mu(c_2) = \mu(c_1, c_2) = 0.1$,

$$\mu(c_2, c_3) = \mu(c_3, c_4) = \mu(c_2, c_3, c_4) = \mu(c_1, c_3, c_4) = \mu(c_1, c_2, c_3) = \mu(c_1, c_2, c_3, c_4) = 1,$$

$$\mu(c_4) = \mu(c_1, c_4) = \mu(c_2, c_4) = \mu(c_1, c_2, c_4) = 0.20, \quad \mu(c_3) = \mu(c_1, c_3) = 0.35.$$

Form Definition 1, the corresponding Shapley fuzzy measure can be calculated.

$$\begin{aligned} \phi_{\{c_1\}}^s &= 0.025, \phi_{\{c_2\}}^s = 0.1333, \phi_{\{c_3\}}^s = 0.6583, \phi_{\{c_4\}}^s = 0.1833, \phi_{\{c_1, c_2\}}^s = 0.1417, \phi_{\{c_1, c_3\}}^s = 0.6667, \\ \phi_{\{c_1, c_4\}}^s &= 0.1917, \phi_{\{c_2, c_3\}}^s = 0.8833, \phi_{\{c_2, c_4\}}^s = 0.4083, \phi_{\{c_3, c_4\}}^s = 0.9333, \phi_{\{c_1, c_2, c_3\}}^s = 0.9000, \\ \phi_{\{c_1, c_2, c_4\}}^s &= 0.4250, \phi_{\{c_1, c_3, c_4\}}^s = 0.9500, \phi_{\{c_2, c_3, c_4\}}^s = 0.9500, \phi_{\{c_1, c_2, c_3, c_4\}}^s = 1. \end{aligned}$$

Phase 3. Aggregation of the preference information

Step 3.1. Determination of the supports

From Definition 9, we can achieve the supports $Sup(\psi_{ij}, \psi_{ip})$ as follows:

$$\begin{aligned} Sup(\psi_{11}, \psi_{12}) &= Sup(\psi_{12}, \psi_{11}) = 0.6485, Sup(\psi_{11}, \psi_{13}) = Sup(\psi_{13}, \psi_{11}) = 0.6316, \\ Sup(\psi_{11}, \psi_{14}) &= Sup(\psi_{14}, \psi_{11}) = 0.6322, Sup(\psi_{12}, \psi_{13}) = Sup(\psi_{13}, \psi_{12}) = 0.6840, \\ Sup(\psi_{12}, \psi_{14}) &= Sup(\psi_{14}, \psi_{12}) = 0.6854, Sup(\psi_{13}, \psi_{14}) = Sup(\psi_{14}, \psi_{13}) = 0.6794, \\ Sup(\psi_{21}, \psi_{22}) &= Sup(\psi_{22}, \psi_{21}) = 0.6552, Sup(\psi_{21}, \psi_{23}) = Sup(\psi_{23}, \psi_{21}) = 0.7422, \\ Sup(\psi_{21}, \psi_{24}) &= Sup(\psi_{24}, \psi_{21}) = 0.5870, Sup(\psi_{22}, \psi_{23}) = Sup(\psi_{23}, \psi_{22}) = 0.6502, \\ Sup(\psi_{22}, \psi_{24}) &= Sup(\psi_{24}, \psi_{22}) = 0.5779, Sup(\psi_{23}, \psi_{24}) = Sup(\psi_{24}, \psi_{23}) = 0.5928, \\ Sup(\psi_{31}, \psi_{32}) &= Sup(\psi_{32}, \psi_{31}) = 0.6808, Sup(\psi_{31}, \psi_{33}) = Sup(\psi_{33}, \psi_{31}) = 0.7053, \\ Sup(\psi_{31}, \psi_{34}) &= Sup(\psi_{34}, \psi_{31}) = 0.6525, Sup(\psi_{32}, \psi_{33}) = Sup(\psi_{33}, \psi_{32}) = 0.6860, \\ Sup(\psi_{32}, \psi_{34}) &= Sup(\psi_{34}, \psi_{32}) = 0.6445, Sup(\psi_{33}, \psi_{34}) = Sup(\psi_{34}, \psi_{33}) = 0.6581, \\ Sup(\psi_{41}, \psi_{42}) &= Sup(\psi_{42}, \psi_{41}) = 0.6047, Sup(\psi_{41}, \psi_{43}) = Sup(\psi_{43}, \psi_{41}) = 0.6480, \\ Sup(\psi_{41}, \psi_{44}) &= Sup(\psi_{44}, \psi_{41}) = 0.6470, Sup(\psi_{42}, \psi_{43}) = Sup(\psi_{43}, \psi_{42}) = 0.6147, \\ Sup(\psi_{42}, \psi_{44}) &= Sup(\psi_{44}, \psi_{42}) = 0.6047, Sup(\psi_{43}, \psi_{44}) = Sup(\psi_{44}, \psi_{43}) = 0.6415, \\ Sup(\psi_{51}, \psi_{52}) &= Sup(\psi_{52}, \psi_{51}) = 0.6564, Sup(\psi_{51}, \psi_{53}) = Sup(\psi_{53}, \psi_{51}) = 0.6460, \\ Sup(\psi_{51}, \psi_{54}) &= Sup(\psi_{54}, \psi_{51}) = 0.6522, Sup(\psi_{52}, \psi_{53}) = Sup(\psi_{53}, \psi_{52}) = 0.6284, \\ Sup(\psi_{52}, \psi_{54}) &= Sup(\psi_{54}, \psi_{52}) = 0.6665, Sup(\psi_{53}, \psi_{54}) = Sup(\psi_{54}, \psi_{53}) = 0.6203. \end{aligned}$$

Step 3.2. Determination of the weighted supports

The weighted support of each assessment value could be obtained, as shown in Table 4.

Table 4. Weighted supports.

	c_1	c_2	c_3	c_4
ψ_1	0.6168	0.2604	0.6917	0.7351
ψ_2	0.5908	0.4907	0.8300	0.3231
ψ_3	0.5727	0.7460	0.7015	0.3391
ψ_4	0.5672	0.4969	0.8333	0.3664
ψ_5	0.2288	0.4747	0.8603	0.6942

Step 3.3. Determination of the weights

Next, the weights π_{ij} associated with the ψ_{ij} on the basis of the criterion c_j could be achieved, as presented in Table 5.

Table 5. Weights.

	c_1	c_2	c_3	c_4
ψ_1	0.0215	0.7062	0.2323	0.0400
ψ_2	0.1623	0.2126	0.0128	0.6122
ψ_3	0.1695	0.0069	0.1994	0.6242
ψ_4	0.1218	0.2629	0.0130	0.6023
ψ_5	0.7043	0.2452	0.0118	0.0388

Step 3.4. Calculation of the global values

By using GWSVNPSCW operator and $\lambda=2$, the global assessment value $\alpha_i (i=1,2,\dots,n)$ of candidate $\bar{\psi}_i (i=1,2,\dots,n)$ can be obtained as:

$$\begin{aligned}
 \alpha_1 &= GSVNPSCOA(\psi_{11}, \psi_{12}, \psi_{13}, \psi_{14}) = \left\langle \left(1 - \prod_{j=1}^4 \left(1 - T_{\psi_{1\sigma(j)}}^2 \right)^{\pi_{1j}} \right)^{\frac{1}{2}}, \right. \\
 &\quad \left. 1 - \left(1 - \prod_{j=1}^4 \left(1 - \left(1 - I_{\psi_{1\sigma(j)}} \right)^{\lambda} \right)^{\pi_{1j}} \right)^{\frac{1}{2}}, \right. \\
 &\quad \left. 1 - \left(1 - \prod_{j=1}^4 \left(1 - \left(1 - F_{\psi_{1\sigma(j)}} \right)^{\lambda} \right)^{\pi_{1j}} \right)^{\frac{1}{2}} \right\rangle \\
 &= \left\langle \left(1 - \left(1 - T_{11}^2 \right)^{\pi_{11}} \left(1 - T_{13}^2 \right)^{\pi_{13}} \left(1 - T_{14}^2 \right)^{\pi_{14}} \left(1 - T_{12}^2 \right)^{\pi_{12}} \right)^{\frac{1}{2}}, \right. \\
 &\quad \left. 1 - \left(1 - \left(1 - \left(1 - I_{11} \right)^2 \right)^{\pi_{11}} \left(1 - \left(1 - I_{13} \right)^2 \right)^{\pi_{13}} \left(1 - \left(1 - I_{14} \right)^2 \right)^{\pi_{14}} \left(1 - \left(1 - I_{12} \right)^2 \right)^{\pi_{12}} \right)^{\frac{1}{2}}, \right. \\
 &\quad \left. 1 - \left(1 - \left(1 - \left(1 - F_{11} \right)^2 \right)^{\pi_{11}} \left(1 - \left(1 - F_{13} \right)^2 \right)^{\pi_{13}} \left(1 - \left(1 - F_{14} \right)^2 \right)^{\pi_{14}} \left(1 - \left(1 - F_{12} \right)^2 \right)^{\pi_{12}} \right)^{\frac{1}{2}} \right\rangle \\
 &= \langle 0.6719, 0.1236, 0.2173 \rangle. \\
 \alpha_2 &= \langle 0.7814, 0.1134, 0.1224 \rangle; \alpha_3 = \langle 0.7088, 0.1608, 0.0862 \rangle; \alpha_4 = \langle 0.7147, 0.1189, 0.1365 \rangle; \\
 \alpha_5 &= \langle 0.7357, 0.1975, 0.1981 \rangle.
 \end{aligned}$$

Step 3.5. Calculation of the values of score, accuracy, and certainty

According to Definition 7, the values of score $\bar{s}(\alpha_i) (i=1,2,3,4,5)$ can be calculated as follows:

$$\bar{s}(\alpha_1) = 0.7770; \bar{s}(\alpha_2) = 0.8485; \bar{s}(\alpha_3) = 0.8206; \bar{s}(\alpha_4) = 0.8198; \bar{s}(\alpha_5) = 0.7800.$$

Since the values of score are different, thus there is no need to calculate the values of accuracy and certainty.

Step 3.6. Ranking the candidates

Apparently, $\bar{s}(\alpha_2) > \bar{s}(\alpha_3) > \bar{s}(\alpha_4) > \bar{s}(\alpha_5) > \bar{s}(\alpha_1)$, thus, we have $\psi_2 \succ \psi_3 \succ \psi_4 \succ \psi_5 \succ \psi_1$. The optimal attraction is ψ_2 while the worst one is ψ_1 .

If the GWSVNPSCG operator is used in Step 3.4 to aggregate the preference information of each candidates, then, the final ranking is $\psi_2 \succ \psi_3 \succ \psi_4 \succ \psi_5 \succ \psi_1$. The best attraction is ψ_2 as well.

5.2. Sensitivity Analysis

In order to discuss the influence of different values γ and λ on the final ranking, sensitivity analysis is carried out. If GWSVNPSCW and GWSVNPSCG operators are used respectively in the Step 3.4 and $\gamma, \lambda \in (0, 10]$, then the results of the two cases are shown in Tables 6 and 7 and Figures 2–6.

For GWSVNPSCW operator, if $\gamma = \lambda = 1, 2, 10$, accordingly, the final ranking is $\psi_2 \succ \psi_3 \succ \psi_4 \succ \psi_5 \succ \psi_1$; however, if $\gamma = \lambda = 4, 6, 8$, the final ranking is $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_5 \succ \psi_1$, as shown in Table 6. Although the position of ψ_3 and ψ_4 is different as two parameters changes, the optimal attraction is always ψ_2 , while the worst one is ψ_1 . Moreover, from the score values presented in Table 6, it can be seen that the difference between the first position and the second position is greater as two parameters change, demonstrating the rationality of the alternative ψ_2 as an optimal scheme. For GWSVNPSCG operator, if $\gamma = \lambda = 1, 2$, therefore, the final ranking is $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1 \succ \psi_5$; if $\gamma = \lambda = 4, 6$, as a result, the final ranking is $\psi_2 \succ \psi_4 \succ \psi_5 \succ \psi_1 \succ \psi_3$; however, if $\gamma = \lambda = 8, 10$, thus, the final ranking is $\psi_2 \succ \psi_4 \succ \psi_1 \succ \psi_5 \succ \psi_3$, as shown in Table 7. Although the position of ψ_1, ψ_3 , and ψ_5 is different, the optimal attraction is ψ_2 . Besides, in order to further understand the influence of parameters on final rankings, the trend of score values of five candidates are presented with two parameters in the range of $(0, 1]$ by using GWSVNPSCW and GWSVNPSCG operators, as shown in Figures 1–5. It can be seen that the fluctuation tendency of score values, as displayed in Figure 3, indicates that the candidate ψ_2 is always the best one.

Table 6. Score values by using GWSVNPSCW operator.

Parameter	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	Final Rankings
$\gamma = \lambda = 1$	0.7769	0.8360	0.8351	0.8217	0.7808	$\psi_2 \succ \psi_3 \succ \psi_4 \succ \psi_5 \succ \psi_1$
$\gamma = \lambda = 2$	0.7770	0.8485	0.8206	0.8198	0.7800	$\psi_2 \succ \psi_3 \succ \psi_4 \succ \psi_5 \succ \psi_1$
$\gamma = \lambda = 4$	0.7818	0.8547	0.8093	0.8135	0.8045	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_5 \succ \psi_1$
$\gamma = \lambda = 6$	0.7825	0.8561	0.8120	0.8144	0.8062	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_5 \succ \psi_1$
$\gamma = \lambda = 8$	0.7832	0.8574	0.8148	0.8153	0.8080	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_5 \succ \psi_1$
$\gamma = \lambda = 10$	0.7839	0.8587	0.8176	0.8163	0.8096	$\psi_2 \succ \psi_3 \succ \psi_4 \succ \psi_5 \succ \psi_1$

Table 7. Score values by using GWSVNPSCG operator.

Parameter	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	Final Rankings
$\gamma = \lambda = 1$	0.7778	0.8219	0.8126	0.8137	0.7720	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1 \succ \psi_5$
$\gamma = \lambda = 2$	0.7728	0.8284	0.7876	0.8072	0.7727	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1 \succ \psi_5$
$\gamma = \lambda = 4$	0.7743	0.8205	0.7661	0.7981	0.7763	$\psi_2 \succ \psi_4 \succ \psi_5 \succ \psi_1 \succ \psi_3$
$\gamma = \lambda = 6$	0.7717	0.8044	0.7572	0.7935	0.7718	$\psi_2 \succ \psi_4 \succ \psi_5 \succ \psi_1 \succ \psi_3$
$\gamma = \lambda = 8$	0.7696	0.7906	0.7510	0.7904	0.7687	$\psi_2 \succ \psi_4 \succ \psi_1 \succ \psi_5 \succ \psi_3$
$\gamma = \lambda = 10$	0.7679	0.7882	0.7464	0.7798	0.7662	$\psi_2 \succ \psi_4 \succ \psi_1 \succ \psi_5 \succ \psi_3$

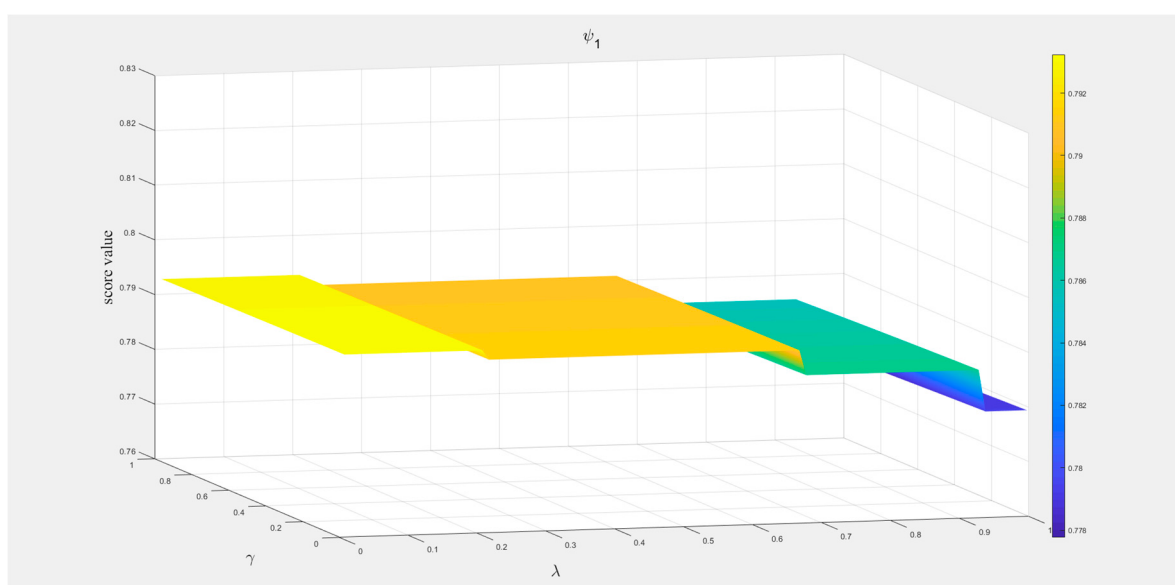
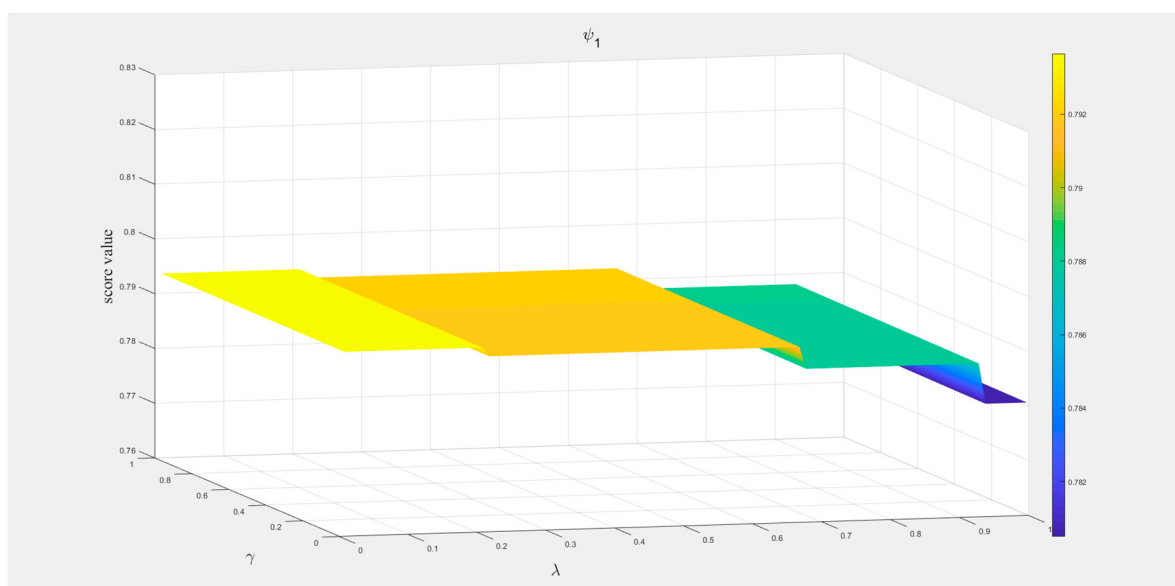
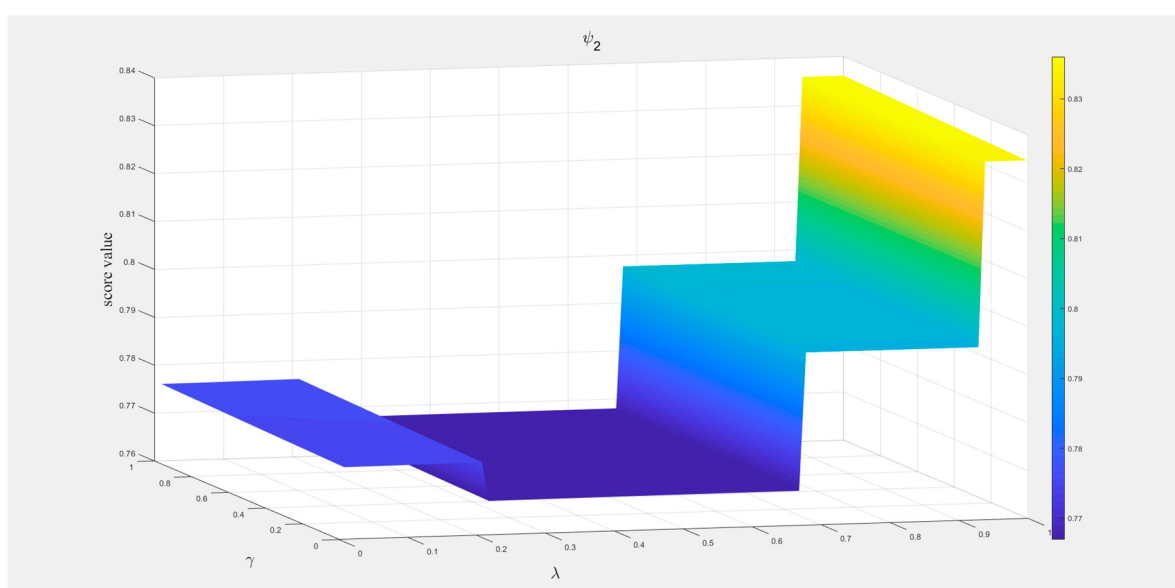


Figure 2. Score values of ψ_1 with $\gamma, \lambda \in (0,1]$ by using two proposed operators.



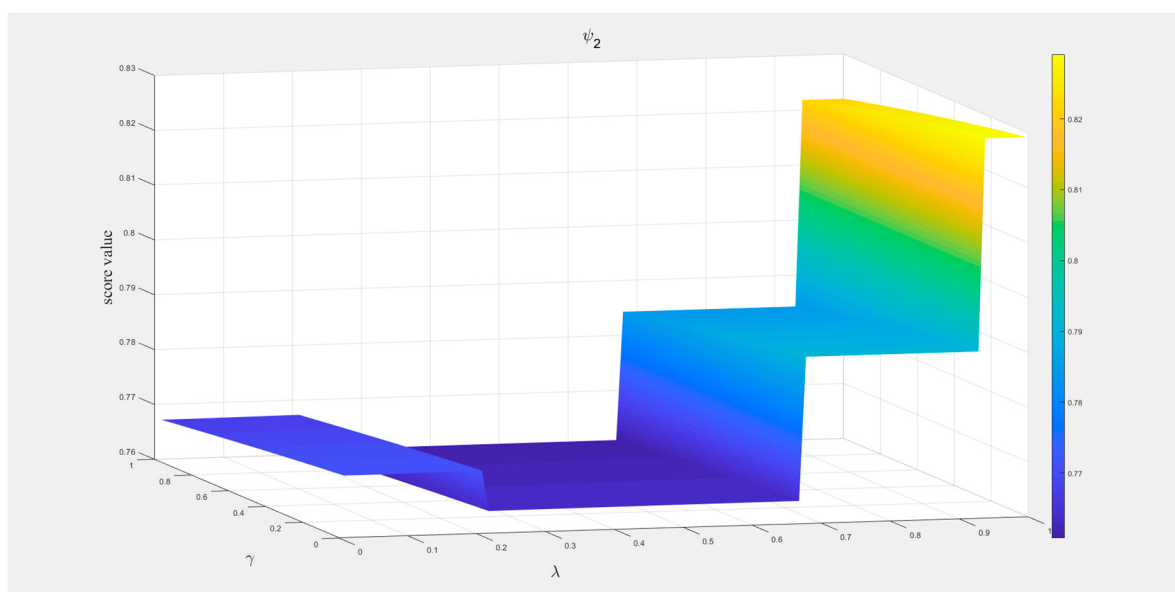


Figure 3. Score values of ψ_2 with $\gamma, \lambda \in (0, 1]$ by using two proposed operators.

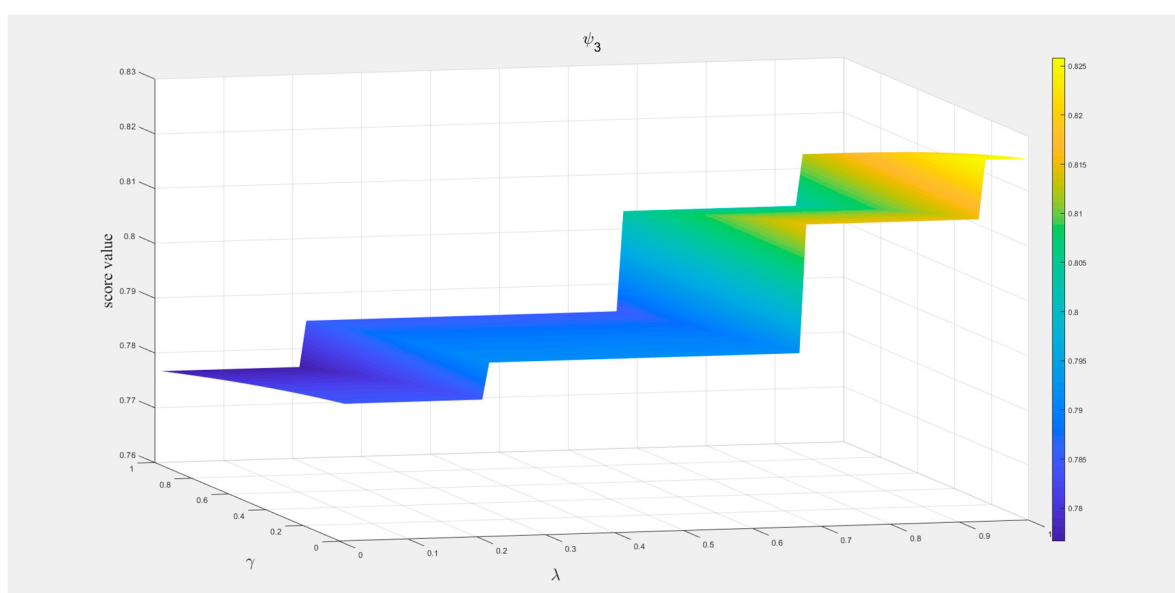
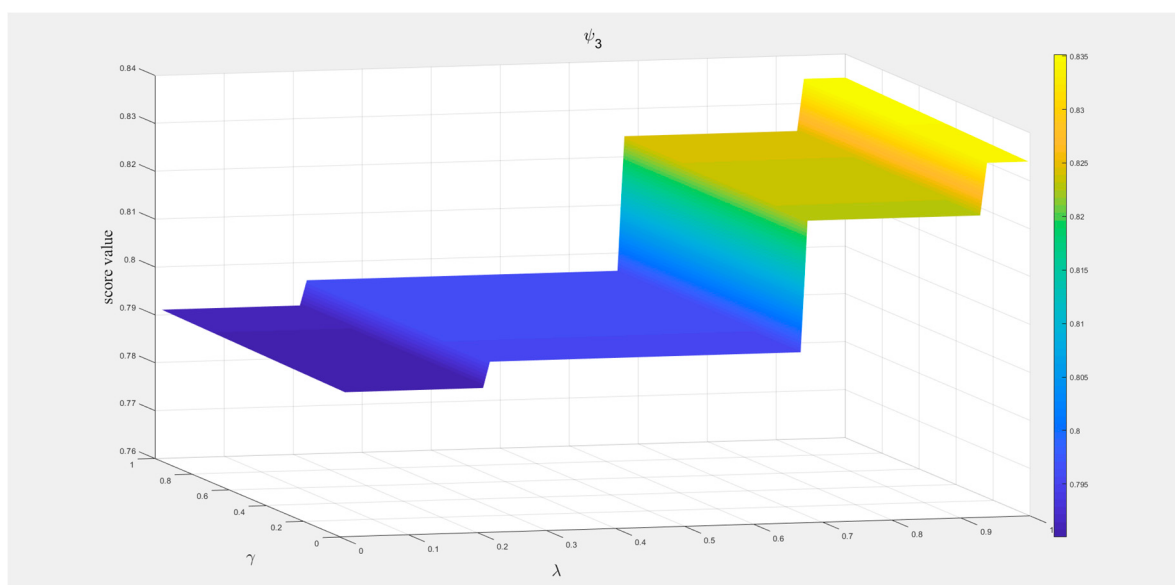


Figure 4. Score values of ψ_3 with $\gamma, \lambda \in (0,1]$ by using two proposed operators.

5.3. Comparative Analysis

In order to examine the effectiveness of the developed method, results achieved by using GWSVNPSCA operator are compared to a number of existing methods based on aggregation operators [16,31,35–36,39,52]. Since the majority of available methods cannot truly resolve situations where the weight information is partly known, the weight is determined as $w = (0.15, 0.20, 0.30, 0.35)^T$ for the comparative analysis.

For the proposed method, the determined weights could be utilized to aggregate preference information in Phase 3 and $\lambda = 2$. For the method based on prioritized operator [39], we assume that the prioritization of the criteria satisfies $c_1 \succ c_2 \succ c_3 \succ c_4$. For the methods based on Bonferroni mean [31], Frank operator [36], and Hamacher operator [52], the corresponding parameters are determined as $p = q = 1$ and $\lambda = 2$, respectively. Then, the results could be obtained as shown in Figure 7.

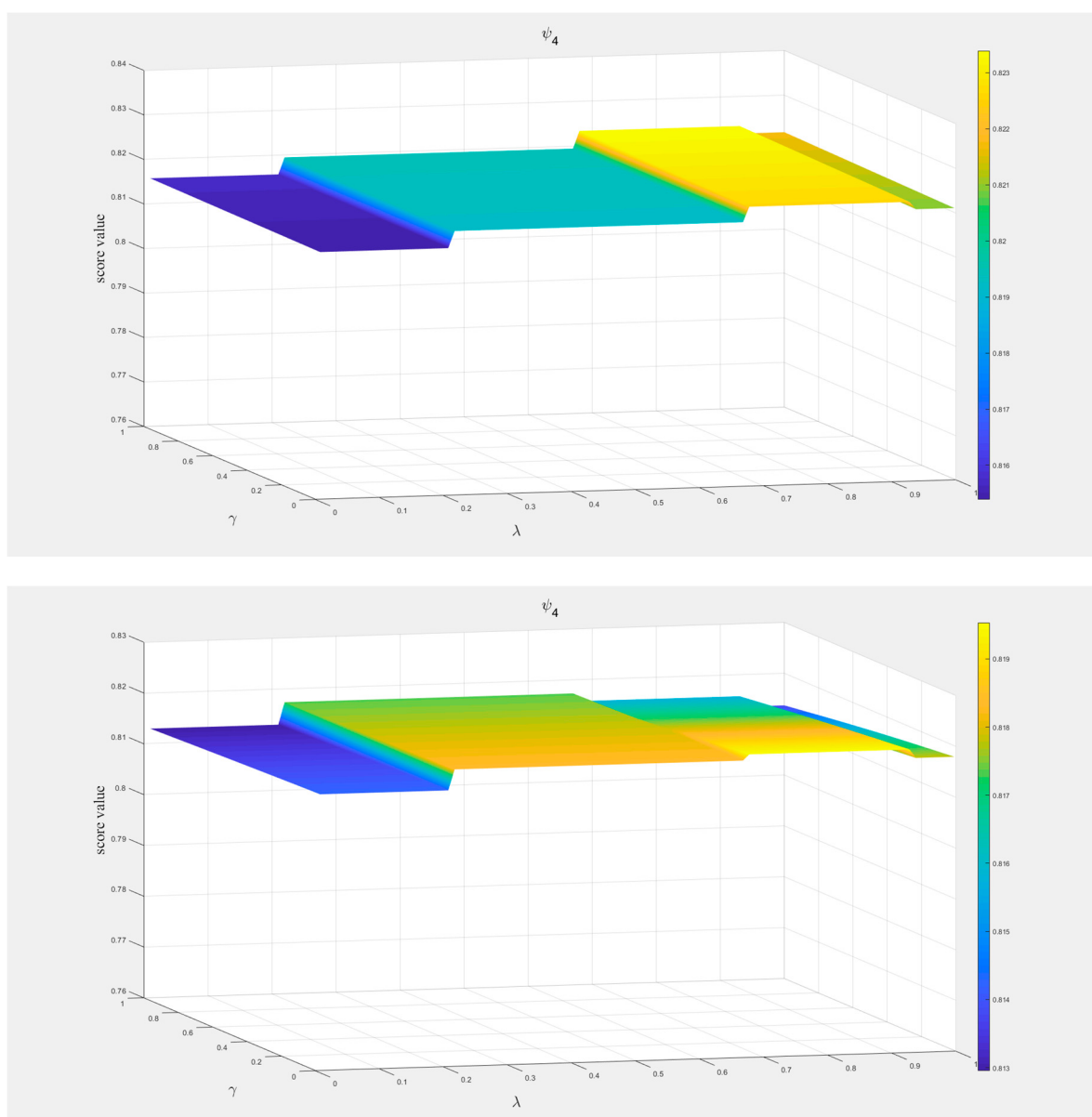


Figure 5. Score values of ψ_4 with $\gamma, \lambda \in (0,1]$ by using two proposed operators.

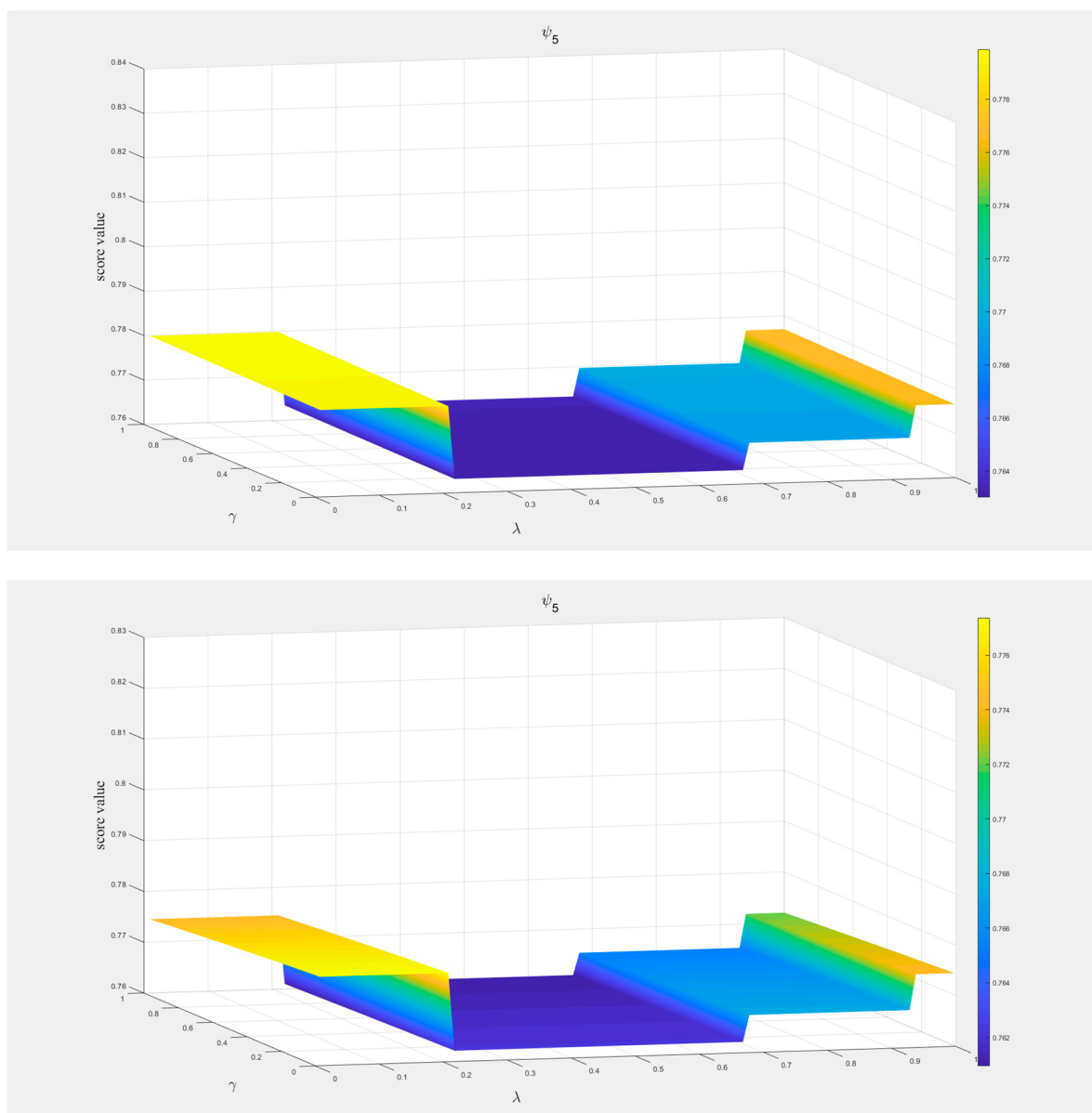


Figure 6. Score values of ψ_5 with $\gamma, \lambda \in (0, 1]$ by using two proposed operators.

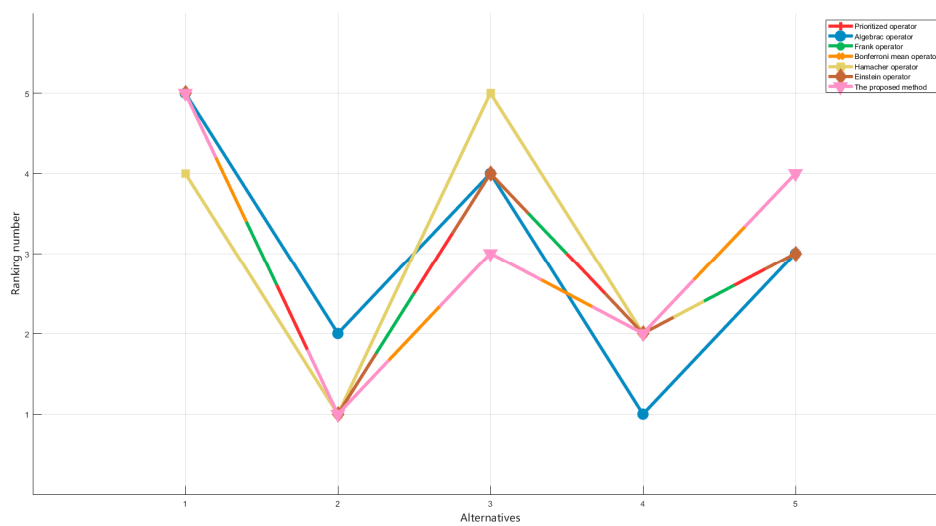


Figure 7. Comparison analysis.

According to the results presented in Figure 7, we can conclude the following achievements. Firstly, the final ranking by using the developed method is the same as those previously presented methods [31], and the final order is $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_5 \succ \psi_1$. Moreover, the developed method includes PA operator that can highly reduce the effects of abnormal assessment values. Secondly, although the positions of ψ_3 , ψ_5 , and ψ_1 based on the proposed method are different from other previously reported methods [35–36,39,52], the optimal alternative is always ψ_2 . Finally, the result based on algebraic operator [16] is $\psi_4 \succ \psi_2 \succ \psi_5 \succ \psi_3 \succ \psi_1$, which is not consistent with other methods. The main reason for this discrepancy is that the rules of corresponding operation are unreasonable as previously discussed [35].

5.4. Managerial Implications

In this study, an MCDM method based on the single-valued neutrosophic aggregation operators is proposed to handle tourism attraction selection problems. Moreover, this study investigates a sensitivity analysis and comparison analysis to prove the validity of the developed method. The results indicated that ψ_2 is the optimal one. The management implications of the developed method can be concluded in the following. First, the evaluation criteria of tourism attraction selection consist of constraints, tourism resource bank, user's tourism demand, and user's preference for tourism products, which can meet the requirements of personalized tourism recommendation. Second, DMs can express their preference information from three perspectives, i.e., truth, indeterminacy, and falsity, which can precisely describe the uncertain information. Third, the proposed aggregation operators consider the interrelationship among criteria, which can make the final results more in accordance with the actual situation. For example, the constraints c_1 , tourism demand c_3 , and tourism preference c_4 are correlated with each other in the real decision-making environment. If the correlation cannot be considered in the decision-making process, then the accuracy of recommendations will be attenuated to some extent. Last, it is difficult for DMs to provide the weight information of criteria in the form of crisp values directly. Thus, the proposed method with incompletely weight information has important theoretical significance for managers to improve the accuracy of recommendations.

6. Conclusions

In this study, a single-valued neutrosophic MCDM problem with interdependent characteristics is investigated. Firstly, according to the power aggregation operator and Shapley fuzzy measure, the GWSVNPSCA and GWSVNPSCG operators are accordingly defined, and the corresponding special cases are discussed as well. Secondly, based on the developed aggregation operators, an integrated MCDM approach is proposed to effectively solve single-valued neutrosophic problems where the weights of the criteria were incompletely known. A mathematical programming model based on fuzzy measure is established to obtain the optimal Shapley fuzzy measure. Then, the developed operators are used to aggregate the preference information. Eventually, a theoretical example is presented to testify the effectiveness of the developed approach, in which it is revealed that the results are reasonable and credible. The main advantages of this research can be concluded in two aspects. On the one hand, the developed aggregation operators can take the interrelationship among criteria into account and also simultaneously reduce the effects of abnormal assessment values. On the other hand, the proposed MCDM methods can solve single-valued neutrosophic problems where the weight information is incompletely known or completely known. The limitation of this paper is that it cannot consider the multiple DMs in the decision-making process. In the future, the other fuzzy measures of SVNns and their corresponding group decision-making methods should be further investigated.

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designed the study. Z.Z. and J.W. analyzed the data. All authors approved the publication work.

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