

Article

On Expansive Mappings

Marat V. Markin and Edward S. Sichel *

Department of Mathematics, California State University, Fresno 5245 N. Backer Avenue, M/S PB 108, Fresno, CA 93740-8001, USA; mmarkin@csufresno.edu

* Correspondence: edsichel@mail.fresnostate.edu

Received: 23 August 2019; Accepted: 17 October 2019; Published: 23 October 2019



Abstract: When finding an original proof to a known result describing expansive mappings on compact metric spaces as surjective isometries, we reveal that relaxing the condition of compactness to total boundedness preserves the isometry property and nearly that of surjectivity. While a counterexample is found showing that the converse to the above descriptions do not hold, we are able to characterize boundedness in terms of specific expansions we call *anticontractions*.

Keywords: Metric space; expansion; compactness; total boundedness

MSC: Primary 54E40; 54E45; Secondary 46T99

O God, I could be bounded in a nutshell, and count myself a king of infinite space - were it not that I have bad dreams.

William Shakespeare (Hamlet, Act 2, Scene 2)

1. Introduction

We take a close look at the nature of expansive mappings on certain metric spaces (compact, totally bounded, and bounded), provide a finer classification for such mappings, and use them to characterize boundedness.

When finding an original proof to a known result describing all expansive mappings on compact metric spaces as surjective isometries [1] (Problem X.5.13*), we reveal that relaxing the condition of compactness to total boundedness still preserves the isometry property and nearly that of surjectivity.

We provide a counterexample of a not totally bounded metric space, on which the only expansion is the identity mapping, demonstrating that the converse to the above descriptions do not hold.

Various examples for different types of expansions are furnished, in particular the one of a nonsurjective expansion on a totally bounded “dial set” in the complex plane which allows us to better understand the essence of the latter.

2. Preliminaries

Here, we outline certain preliminaries essential for the subsequent discourse (for more, see, e.g., [2–6]).

Definition 1 (Sequential Compactness).

A set A in a metric space (X, d) is called sequentially compact, or compact in the Bolzano-Weierstrass sense, if every sequence $(x_n)_{n \in \mathbb{N}}$ of its elements contains a subsequence convergent to an element of A .

A metric space (X, d) is said to be sequentially compact if sequentially compact is the set X .

Remark 1. In a metric space setting, the above definition of compactness is equivalent to compactness in the Heine-Borel sense defined via open covers (see, e.g., [3,5]).

It is convenient for us to use a sequential definition for total boundedness as well (see, e.g., [3,4]).

Definition 2 (Total Boundedness).

A set A in a metric space (X, d) is called totally bounded if every sequence of its elements contains a fundamental (Cauchy) subsequence.

A metric space (X, d) is said to be totally bounded if totally bounded is the set X .

Definition 3 (Boundedness).

A set A in a metric space (X, d) is said to be bounded if

$$\text{diam}(A) := \sup_{x, y \in A} d(x, y) < \infty,$$

the number $\text{diam}(A)$ being called the diameter of A .

A metric space (X, d) is said to be bounded if bounded is the set X .

Remark 2. In a metric space, a (sequentially) compact set is totally bounded and a totally bounded set is bounded but not vice versa (see, e.g., [3]).

3. Expansive Mappings

Now, we introduce and further classify the focal subject of our study, *expansive mappings* (or *expansions*).

Definition 4 (Expansive Mapping).

Let (X, d) be a metric space. A mapping $T : X \rightarrow X$ on (X, d) such that

$$\forall x, y \in X : d(Tx, Ty) \geq d(x, y)$$

is called an *expansive mapping* (or *expansion*).

It is important for our discourse to introduce a finer classification of expansions.

Definition 5 (Types of Expansions).

Let (X, d) be a metric space.

- (1) An expansion $T : X \rightarrow X$ such that

$$\forall x, y \in X : d(Tx, Ty) = d(x, y)$$

is called an *isometry*, which is the weakest form of expansive mappings.

- (2) An expansion $T : X \rightarrow X$ such that

$$\exists x, y \in X, x \neq y : d(Tx, Ty) > d(x, y)$$

we call a *proper expansion*.

- (3) An expansion $T : X \rightarrow X$ such that

$$\forall x, y \in X, x \neq y : d(Tx, Ty) > d(x, y)$$

we call a *strict expansion*.

- (4) Finally, an expansion $T : X \rightarrow X$ such that

$$\exists E > 1 \forall x, y \in X : d(Tx, Ty) \geq Ed(x, y)$$

we call an *anticontraction* with expansion constant E .

Remark 3. Clearly, any anticontraction is necessarily a strict expansion, which in turn is also a proper expansion. However, as the following examples demonstrate, the converse statements are not true.

Example 1.

1. On $\mathbb{C}$ with the standard metric, the mapping

$$g(z) := e^i z,$$

i.e., the counterclockwise rotation by one radian, is an isometry which is not a proper expansion.

2. On the space ℓ_∞ of all real- or complex-termed bounded sequences with its standard supremum metric

$$\ell_\infty \ni x := (x_k)_{k \in \mathbb{N}}, y := (y_k)_{k \in \mathbb{N}} \mapsto d_\infty(x, y) := \sup_{k \in \mathbb{N}} |x_k - y_k|,$$

the right shift mapping

$$\ell_\infty \ni (x_1, x_2, x_3 \dots) \mapsto T(x_1, x_2, x_3 \dots) := (0, x_1, x_2, x_3 \dots) \in \ell_\infty$$

is also an isometry which is not a proper expansion.

3. On ℓ_∞ , the mapping

$$\ell_\infty \ni (x_1, x_2, x_3 \dots) \mapsto T(x_1, x_2, x_3 \dots) := (x_1, x_1^2, x_2, x_2^2, \dots) \in \ell_\infty$$

is a proper expansion that is not strict, since, for $x := (1, 0, 0, \dots), y := (1/2, 0, 0, \dots) \in \ell_\infty$,

$$d_\infty(Tx, Ty) = 3/4 > 1/2 = d_\infty(x, y),$$

but, for $x := (1, 0, 0, \dots), y := (0, 0, 0, \dots) \in \ell_\infty$,

$$d_\infty(Tx, Ty) = 1 = d_\infty(x, y).$$

4. In the space $L_2(0, \infty)$, consider the set of the equivalence classes $\{f_n\}_{n \in \mathbb{N}}$ represented by the functions

$$f_n(x) := \sqrt{n} \chi_{[0, 1/n]}(x), \quad n \in \mathbb{N}, x \in (0, \infty),$$

($\chi(\cdot)$ is the characteristic function of a set), which is a subset of the unit sphere

$$S(0, 1) := \{f \in L_2(0, \infty) \mid d_2(f, 0) = \|f\|_2 = 1\}.$$

For any $m, n \in \mathbb{N}$ with $n > m$, we have:

$$\begin{aligned} d_2(f_n, f_m) &= \|f_n - f_m\|_2 = \left[\int_0^\infty |f_n(x) - f_m(x)|^2 dx \right]^{1/2} \\ &= \left[\int_0^\infty \left| (\sqrt{n} - \sqrt{m}) \chi_{[0, 1/n]}(x) - \sqrt{m} \chi_{(1/n, 1/m]}(x) \right|^2 dx \right]^{1/2} \\ &= \left[\int_0^{1/n} (\sqrt{n} - \sqrt{m})^2 dx + \int_{1/n}^{1/m} \sqrt{m}^2 dx \right]^{1/2} \\ &= \left[\frac{m - 2\sqrt{m}\sqrt{n} + n}{n} + m \left(\frac{1}{m} - \frac{1}{n} \right) \right]^{1/2} = \left[2 - 2\sqrt{\frac{m}{n}} \right]^{1/2}. \end{aligned}$$

The map $Tf_n := f_{kn}$, $n \in \mathbb{N}$, with an arbitrary fixed $k \in \mathbb{N}$ is an isometry on $\{f_n\}_{n \in \mathbb{N}}$ since, for any $m, n \in \mathbb{N}$ with $n > m$,

$$\begin{aligned} d_2(Tf_n, Tf_m) &= \|Tf_n - Tf_m\|_2 = \|f_{kn} - f_{km}\|_2 = \left[2 - 2\sqrt{\frac{km}{kn}}\right]^{1/2} \\ &= \left[2 - 2\sqrt{\frac{m}{n}}\right]^{1/2} = \|f_m - f_n\|_2 = d_2(f_n, f_m). \end{aligned}$$

On the other hand, the map $Sf_n := f_{n^2}$, $n \in \mathbb{N}$, is a strict expansion on $\{f_n\}_{n \in \mathbb{N}}$ since, for any $m, n \in \mathbb{N}$ with $n > m$,

$$\begin{aligned} d_2(Sf_n, Sf_m) &= \|Sf_n - Sf_m\|_2 = \|f_{n^2} - f_{m^2}\|_2 = \left[2 - 2\sqrt{\frac{m^2}{n^2}}\right]^{1/2} \\ &= \left[2 - 2\frac{m}{n}\right]^{1/2} > \left[2 - 2\sqrt{\frac{m}{n}}\right]^{1/2} = \|f_n - f_m\|_2 = d_2(f_n, f_m), \end{aligned}$$

which is not an anticontraction since

$$\frac{d_2(Sf_{n^2}, Sf_n)}{d_2(f_{n^2}, f_n)} = \frac{\left[2 - \frac{2}{n}\right]^{1/2}}{\left[2 - \frac{2}{\sqrt{n}}\right]^{1/2}} \rightarrow 1, \quad n \rightarrow \infty.$$

5. On $\mathbb{R}$ with the standard metric, the mapping

$$f(x) = 2x$$

is an anticontraction with expansion constant $E = 2$. However, the same mapping, when considered on $\mathbb{R}$ equipped with the metric

$$\mathbb{R} \ni x, y \mapsto \rho(x, y) := \frac{|x - y|}{|x - y| + 1},$$

turning $\mathbb{R}$ into a bounded space (see, e.g., [3]), is merely a strict expansion, which is not an anticontraction since

$$\frac{\rho(f(x), f(0))}{\rho(x, 0)} = \frac{\rho(2x, 0)}{\rho(x, 0)} = \frac{\frac{|2x|}{|2x|+1}}{\frac{|x|}{|x|+1}} \rightarrow 1, \quad x \rightarrow \infty.$$

4. Expansions on Compact Metric Spaces

Theorem 1 (Expansions on Compact Metric Spaces [1] (Problem X.5.13*)).

An expansive mapping T on a compact metric space (X, d) is a surjection, i.e.,

$$T(X) = X,$$

and an isometry, i.e.,

$$\forall x, y \in X : d(Tx, Ty) = d(x, y).$$

Proof. For an arbitrary point $x \in X$, and an increasing sequence $(n(k))_{k \in \mathbb{N}}$ of natural numbers, consider the sequence

$$(x_{n(k)} := T^{n(k)}x)_{k \in \mathbb{N}}$$

in (X, d) .

Since the space (X, d) is compact, there exists a convergent subsequence $(x_{n(k(j))})_{j \in \mathbb{N}}$, which is necessarily fundamental.

Remark 4. Subsequently, we use only the fundamentality, and not the convergence of the subsequence, and hence, only the total boundedness and not the compactness of the underlying space (Remark 2).

By the fundamentality of $(x_{n(k(j))})_{j \in \mathbb{N}}$, without loss of generality, we can regard the indices $n(k(j))$, $j \in \mathbb{N}$, chosen sparsely enough so that

$$d(x_{n(k(j))}, x_{2n(k(j))}) \leq \frac{1}{j}, \quad j \in \mathbb{N}.$$

Since T is an expansion,

$$d(x, x_{n(k(j))}) \leq d(T^{n(k(j))}x, T^{n(k(j))}x_{n(k(j))}) = d(x_{n(k(j))}, x_{2n(k(j))}) \leq \frac{1}{j}, \quad j \in \mathbb{N}.$$

We thus conclude that

$$x_{n(k(j))} = T^{n(k(j))}x \rightarrow x, \quad j \rightarrow \infty,$$

which implies that the range $T(X)$ is dense in (X, d) , i.e.,

$$\overline{T(X)} = X.$$

Now, let $x, y \in X$ be arbitrary. Then, for the sequence $(x_n := T^n x)_{n \in \mathbb{N}}$, we can, by the above argument, select a subsequence $(x_{n(k)})_{k \in \mathbb{N}}$ such that

$$x_{n(k)} \rightarrow x, \quad k \rightarrow \infty,$$

and then, in turn, for the sequence $(y_{n(k)} := T^{n(k)}y)_{k \in \mathbb{N}}$, we choose a subsequence $(y_{n(k(j))})_{j \in \mathbb{N}}$ for which

$$y_{n(k(j))} \rightarrow y, \quad j \rightarrow \infty.$$

Since $(x_{n(k(j))})_{j \in \mathbb{N}}$ is a subsequence of $(x_{n(k)})_{k \in \mathbb{N}}$, we also have:

$$\lim_{j \rightarrow \infty} x_{n(k(j))} = \lim_{k \rightarrow \infty} x_{n(k)} = x.$$

Then, in view of the expansiveness of T , for any $j \in \mathbb{N}$,

$$d(x, y) \leq d(Tx, Ty) \leq d(T^{n(k(j))}x, T^{n(k(j))}y) = d(x_{n(k(j))}, y_{n(k(j))}).$$

Whence, passing to the limit as $j \rightarrow \infty$, by joint continuity of metric, we arrive at

$$d(x, y) \leq d(Tx, Ty) \leq d(x, y),$$

which implies that

$$\forall x, y \in X : d(Tx, Ty) = d(x, y),$$

i.e., T is an isometry.

Remark 5. Thus far, only the total boundedness and not the compactness of the underlying space has been utilized (Remark 2).

Being an isometry, the mapping T is *continuous*, whence, since X is *compact*, we infer that the image $T(X)$ is compact as well, and therefore *closed* in (X, d) (see, e.g., [3]).

In view of the *denseness* and the *closedness* of $T(X)$, we conclude that

$$T(X) = \overline{T(X)} = X,$$

i.e., T is also a *surjection*, as desired, which completes the proof.

Remark 6. For the surjectivity of T , the requirement of the compactness of the underlying space is essential, as we rely on the fact the continuous image of a compact set is compact. Example 2 demonstrates that this requirement cannot be relaxed even to total boundedness.

□

5. Expansions on Totally Bounded Metric Spaces

We proceed now to demonstrate that relaxing the condition of the compactness of the underlying space to total boundedness yields a slightly weaker result, in which expansions emerge as “presurjective” isometries.

Theorem 2 (Expansions on Totally Bounded Metric Spaces).

An expansive mapping T on a totally bounded metric space (X, d) has a dense range, i.e.,

$$\overline{T(X)} = X$$

(“presurjection”), and is an isometry, i.e.,

$$\forall x, y \in X : d(Tx, Ty) = d(x, y).$$

Proof. As is shown in the corresponding part of the proof of Theorem 1 (see Remarks 4 and 5), the image $T(X)$ is *dense* in (X, d) , i.e.,

$$\overline{T(X)} = X,$$

and T is an *isometry*. □

As is mentioned in Remark 6, the compactness of the underlying space is essential for the *surjectivity* of expansions, the following example demonstrating that, when compactness is relaxed to total boundedness, surjectivity is not guaranteed.

Example 2 (Dial Set).

Let

$$D := \{e^{in}\}_{n \in \mathbb{Z}_+} \subset \{z \in \mathbb{C} \mid |z| = 1\}$$

($\mathbb{Z}_+$ is the set of nonnegative integers) be a dial set in the complex plane $\mathbb{C}$ with the usual distance, which is bounded in $\mathbb{C}$, and hence, totally bounded (see, e.g., [3]), and

$$D \ni e^{in} \mapsto Te^{in} := e^{i(n+1)} \in D, \quad n \in \mathbb{Z}_+,$$

be the counterclockwise rotation by one radian, which is, clearly, an isometry (see Examples 1) but not a surjection on D since, as is easily seen,

$$D \ni 1 = e^{0i} \notin T(D).$$

Remark 7.

- This, in particular, implies that, by Theorem 1, the dial set D is not compact, and hence, not closed, in $\mathbb{C}$ (see, e.g., [3]).
- Thus, on a totally bounded, in particular compact, metric space, any expansion is not proper but is an isometry which may fall a little short of being surjective.

By Theorem 2, the range $T(D)$ is *dense* in the dial set D , which is not closed, relative to the usual distance. This allows us to “turn the tables” on the dial set and derive the following rather interesting immediate corollary.

Corollary 1. *Let*

$$D := \left\{ e^{in} \right\}_{n \in \mathbb{Z}_+}.$$

Then,

(1) *for an arbitrary $n \in \mathbb{Z}_+$, there exists an increasing sequence $(n(k))_{k \in \mathbb{N}}$ of natural numbers such that*

$$e^{in(k)} \rightarrow e^{in}, k \rightarrow \infty;$$

(2) *there exists a $\theta \in \mathbb{R} \setminus \mathbb{Z}_+$ for which there is an increasing sequence $(n(k))_{k \in \mathbb{N}}$ of natural numbers such that*

$$e^{in(k)} \rightarrow e^{i\theta}, k \rightarrow \infty.$$

Proof.

- (1) Part (1) immediately follows from the fact that, by Theorem 2, the range $T(D) = \{e^{in}\}_{n \in \mathbb{N}}$ is *dense* in D .
- (2) Part (2) follows from the fact that the set D , being *not closed* (see Remark 7), has at least one limit point not belonging to D , which, by continuity of metric, is located on the *unit circle* $\{z \in \mathbb{C} \mid |z| = 1\}$, i.e., is of the form $e^{i\theta}$ with some $\theta \in \mathbb{R} \setminus \mathbb{Z}_+$.

□

Remark 8. *If posed as a problem, the prior statement, although simply stated, might be quite challenging to be proved exclusively via the techniques of classical analysis.*

6. Are the Converse Statements True?

Now, there are two natural questions to ask.

- If every expansive map T on a metric space (X, d) is a surjective isometry, is the space compact?
- If every expansive map T on a metric space (X, d) is a presurjective isometry (see Theorem 2), is the space totally bounded?

In other words, do the converse statements to Theorems 1 and 2 hold?

The following example answers both questions in the negative.

Example 3. *In the space ℓ_∞ , consider the bounded set $\{x_n\}_{n \in \mathbb{N}}$ defined by*

$$x_n := \left(0, \dots, 0, \underbrace{1 + \frac{1}{n}}_{n\text{th term}}, 0, \dots \right), n \in \mathbb{N},$$

and let T be an arbitrary expansion on $\{x_n\}_{n \in \mathbb{N}}$. First, we note that, for any expansion, if

$$\exists m, n \in \mathbb{N}, m \neq n : Tx_m = Tx_n,$$

then

$$0 = d(Tx_m, Tx_n) < d(x_m, x_n)$$

contradicting the expansiveness of T . Thus, the mapping T is injective.

Observe that

$$\forall m, n = 2, 3, \dots : d(x_m, x_n) < 2 = d(x_1, x_n).$$

Assume

$$Tx_1 \neq x_1. \quad (1)$$

Then

$$Tx_1 = x_k$$

with some $k \in \mathbb{N}$, $k \geq 2$. Let $n \in \mathbb{N}$, $n \geq 2$, be arbitrary.

There are two possibilities: either

$$Tx_n \neq x_1$$

or

$$Tx_n = x_1.$$

In the first case, we have:

$$d(Tx_1, Tx_n) = d(x_k, Tx_n) < 2 = d(x_1, x_n).$$

contradicting the expansiveness of T .

In the second case, for any $m \in \mathbb{N}$, $m \neq n$, by the injectivity of T ,

$$Tx_m \neq x_1,$$

and hence,

$$d(Tx_1, Tx_m) = d(x_k, Tx_m) < 2 = d(x_1, x_m),$$

which again contradicts the expansiveness of T .

The obtained contradictions making assumption (1) false, we conclude that

$$Tx_1 = x_1.$$

Therefore, by the injectivity of T , we can restrict the expansion T to the subset $\{x_n\}_{n \geq 2}$. Applying the same argument, one can show that

$$Tx_2 = x_2.$$

Continuing inductively, we see that

$$\forall n \in \mathbb{N} : Tx_n = x_n,$$

i.e. T is the identity map, which is both a surjection and an isometry, even though the set $\{x_n\}_{n \in \mathbb{N}}$ is not totally bounded, let alone compact (see Remark 2), as

$$\forall m, n \in \mathbb{N}, m \neq n : d_\infty(x_m, x_n) > 1.$$

Remark 9. Thus, a metric space with the property that every expansion on it is a presurjective isometry need not be totally bounded. Such spaces, which, by Theorems 1 and 2, encompass compact and totally bounded, can be called nonexpansive.

7. A Characterization of Boundedness

Although bounded sets support strict expansions (see Example 1, 4 and 5). Any attempt to produce an anticontraction on a bounded set would be futile, the following characterization explaining why.

Theorem 3 (Anticontraction Characterization of Boundedness).

A metric space (X, d) is bounded iff no subset of X supports an anticontraction.

Proof. The case of a singleton being trivial, suppose that X consists of at least two distinct elements.

“Only if” part. We proceed *by contradiction*, assuming that X is bounded and there exists a subset $A \subseteq X$ supporting an anticontraction $T : A \rightarrow A$ with expansion constant E . Then

$$\forall x, y \in A, x \neq y \forall n \in \mathbb{N} : T^n x, T^n y \in A,$$

which implies

$$\text{diam}(A) \geq d(T^n x, T^n y) \geq E^n d(x, y) \rightarrow \infty, n \rightarrow \infty.$$

Hence, A is unbounded, and since $A \subseteq X$, this *contradicts* the boundedness of X , the obtained contradiction proving the “only if” part.

“If” part. Here, we proceed *by contrapositive* assuming X to be *unbounded* and showing that there exists a subset of X which supports an anti-contraction.

Since X is unbounded, we can select two distinct points $x_1, x_2 \in X$, and subsequently pick x_3 so that

$$\min_{1 \leq i \leq 2} d(x_3, x_i) > 2 \max_{1 \leq i, j \leq 2} d(x_i, x_j)$$

Continuing inductively in this fashion, we construct a countably infinite subset $S := \{x_n\}_{n \in \mathbb{N}}$ of X such that

$$\min_{1 \leq i \leq n} d(x_{n+1}, x_i) > 2 \max_{1 \leq i, j \leq n} d(x_i, x_j).$$

Let If we then define $T : \{x_n\}_{n \in \mathbb{N}} \rightarrow \{x_n\}_{n \in \mathbb{N}}$ by:

$$S \ni x_n \mapsto Tx_n := x_{n+1} \in S, n \in \mathbb{N}.$$

Then, for any $m, n \in \mathbb{N}$ with $n > m$,

$$\begin{aligned} d(Tx_n, Tx_m) &= d(x_{n+1}, x_{m+1}) \geq \min_{1 \leq i \leq n} d(x_{n+1}, x_i) \\ &> 2 \max_{1 \leq i, j \leq n} d(x_i, x_j) \geq 2d(x_n, x_m), \end{aligned}$$

which implies that T is an anti-contraction with expansion constant $E = 2$ on $S \subseteq X$ completing the proof of the “if” part and the entire statement. $\square$

Reformulating equivalently, we arrive at

Theorem 4 (Anticontraction Characterization of Unboundedness).

A metric space (X, d) is unbounded iff there exists a subset of X which supports an anticontraction.

Author Contributions: Conceptualization, M.V.M., E.S.S.; methodology, M.V.M., E.S.S.; validation, M.V.M., E.S.S.; formal analysis, M.V.M., E.S.S.; investigation, M.V.M., E.S.S.; writing—original draft preparation, M.V.M., E.S.S.; writing—review and editing, M.V.M., E.S.S.; supervision, M.V.M.

Funding: This research received no external funding.

Acknowledgments: The authors would like to express their appreciation to Michael Bishop, Przemyslaw Kajetanowicz, and other members of the Functional Analysis and Mathematical Physics Interdepartmental Research Group (FAMP) of California State University, Fresno for insightful questions and stimulating discussions.

Conflicts of Interest: The authors declare no conflict of interest.

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