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Some Interval-Valued Intuitionistic Fuzzy Dombi Hamy Mean Operators and Their Application for Evaluating the Elderly Tourism Service Quality in Tourism Destination

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Abstract: In this paper, we expand the Hamy mean (HM) operator and Dombi operations with interval-valued intuitionistic fuzzy numbers (IVIFNs) to propose the interval-valued intuitionistic fuzzy Dombi Hamy mean (IVIFDHM) operator, interval-valued intuitionistic fuzzy weighted Dombi Hamy mean (IVIFWDHM) operator, interval-valued intuitionistic fuzzy dual Dombi Hamy mean (IVIFDDHM) operator, and interval-valued intuitionistic fuzzy weighted dual Dombi Hamy mean (IVIFWDDHM) operator. Then the MADM models are designed with IVIFWDHM and IVIFWDDHM operators. Finally, we gave an example for evaluating the elderly tourism service quality in tourism destination to show the proposed models.

Keywords: multiple attribute decision making (MADM); interval-valued intuitionistic fuzzy numbers (IVIFNs); interval-valued intuitionistic fuzzy weighted Dombi Hamy mean (IVIFWDHM) operator; interval-valued intuitionistic fuzzy weighted dual Dombi Hamy mean (IVIFWDDHM) operator; elderly tourism service quality; tourism destination

1. Introduction

The concept of intuitionistic fuzzy sets (IFSs) [1,2] has been utilized to deal with uncertainty and imprecision. Atanassov and Gargov [3] defined the interval-valued intuitionistic fuzzy sets (IVIFSs). Xu [4] introduced a method for the comparison between two intuitionistic fuzzy numbers (IFNs) and then develop some arithmetic aggregation operators. Xu and Yager [5] proposed some new geometric aggregation operators with IFNs. Xu and Chen [6] developed some interval-valued intuitionistic fuzzy geometric operators with interval-valued intuitionistic fuzzy numbers (IVIFNs). Wei [7] proposed two new aggregation operators: the induced intuitionistic fuzzy ordered weighted geometric (I-IFOWG) operator and the induced interval-valued intuitionistic fuzzy ordered weighted geometric (I-IFOWG) operator. Wei [8] developed the gray relational analysis (GRA) for interval-valued intuitionistic fuzzy MADM with incompletely known attribute weight information. Xu and Chen [9] defined the Bonferroni mean for aggregating the IVIFNs based on the Bonferroni mean [10–17]. Chen [18] proposed the LINMAP (Linear Programming Technique for Multidimensional Analysis of Preference) model for MADM with IVIFNs. Hashemi, et al. [19] defined the multiple attribute group decision-making (MAGDM) model on the basis of the compromise ratio method with IVIFNs. Liu, et al. [20] gave the principal component analysis (IVIF-PCA) model for IVIFNs. Chen [21] proposed the interval-valued intuitionistic fuzzy preference ranking organization method for enrichment evaluations (IVIF-PROMETHEE) to deal with MADM. Dugenci [22] introduced a novel generalized distance

measure for IVIFNs and illustrated the applicability of the proposed distance measure to MAGDM. Garg [23] defined a new generalized improved score function for IVIFNs. Until now, more and more decision making theories of IFSs and IVIFSs are extended to picture fuzzy set [24–33] and Pythagorean fuzzy sets [34–44].

Although IFSs and IVIFSs have been effectively used in some areas, all the existed approaches are unsuitable to depict the interrelationships among any number of IVIFNs assigned by a variable vector. The Hamy mean (HM) operator [38,45–48] and dual Hamy mean (DHM) operator [49] are famous operators which can show interrelationships among any number of arguments assigned by a variable vector. Therefore, the HM and DHM operators can assign a robust and flexible mechanism to solve the information fusion in MADM problems. Thus, we propose some HM operator to overcome this limits. Thus, how to aggregate these IVIFNs-based the traditional HM operators based on the Dombi operations [50–53] is an interesting issue. So, the purpose of this paper is to propose some HM and DHM operators to solve the MADM for evaluating the elderly tourism service quality in tourism destination with IVIFNs. In order to do so, the rest of this paper is organized as follows. In Section 2, we introduce the IVIFNs. In Section 3, we develop some HM operators with IVIFNs based on the Dombi operations. In Section 4, we present an example for evaluating the elderly tourism service quality in tourism destination with IVIFNs. Section 5 ends this paper with some comments.

2. Preliminaries

2.1. IFSs and IVIFSs

The concept of IFSs and IVIFSs are introduced.

Definition 1 [1,2]. An IFS Q in X is designed by

$$Q = \{ \langle x, \theta_Q(x), \vartheta_Q(x) \rangle | x \in X \} \quad (1)$$

where $\theta_Q : X \rightarrow [0, 1]$ and $\vartheta_Q : X \rightarrow [0, 1]$, and $0 \leq \theta_Q(x) + \vartheta_Q(x) \leq 1, \forall x \in X$. The number $\theta_Q(x)$ and $\vartheta_Q(x)$ represents, respectively, the membership degree and non- membership degree of the element x to the set Q .

Definition 2 [3]. Let X be an universe of discourse, An IVIFS $\tilde{Q}$ over X is an object having the form as follows:

$$\tilde{Q} = \{ \langle x, \tilde{\theta}_{\tilde{Q}}(x), \tilde{\vartheta}_{\tilde{Q}}(x) \rangle | x \in X \} \quad (2)$$

where $\tilde{\theta}_{\tilde{Q}}(x) \subseteq [0, 1]$ and $\tilde{\vartheta}_{\tilde{Q}}(x) \subseteq [0, 1]$ are interval numbers, and $0 \leq \sup(\tilde{\theta}_{\tilde{Q}}(x)) + \sup(\tilde{\vartheta}_{\tilde{Q}}(x)) \leq 1, \forall x \in X$. For convenience, let $\tilde{\theta}_{\tilde{Q}}(x) = [e, f]$, $\tilde{\vartheta}_{\tilde{Q}}(x) = [g, h]$, so $\tilde{\varphi} = ([e, f], [g, h])$ is an IVIFNs.

Definition 3 [54]. Let $\tilde{\varphi} = ([e, f], [g, h])$ be an IVIFN, a score function S can be defined as follows:

$$S(\tilde{\varphi}) = \frac{e - g + f - h}{2}, S(\tilde{\varphi}) \in [-1, 1]. \quad (3)$$

Definition 4 [54]. Let $\tilde{\varphi} = ([e, f], [g, h])$ be an IVIFN, an accuracy function H can be defined as follows:

$$H(\tilde{\varphi}) = \frac{e + f + g + h}{2}, H(\tilde{\varphi}) \in [0, 1]. \quad (4)$$

To evaluate the degree of accuracy of the IVIFN $\tilde{\varphi} = ([e, f], [g, h])$.

Definition 5 [54]. Let $\tilde{\varphi}_1 = ([e_1, f_1], [g_1, h_1])$ and $\tilde{\varphi}_2 = ([e_2, f_2], [g_2, h_2])$ be two IVIFNs, $s(\tilde{\varphi}_1) = \frac{e_1 - f_1 + g_1 - h_1}{2}$ and $s(\tilde{\varphi}_2) = \frac{e_2 - f_2 + g_2 - h_2}{2}$ be the scores of $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$, respectively, and let $H(\tilde{\varphi}_1) = \frac{e_1 + f_1 + g_1 + h_1}{2}$

and $H(\tilde{\varphi}_2) = \frac{e_2+f_2+g_2+h_2}{2}$ be the accuracy degrees of $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$, respectively, then if $S(\tilde{\varphi}_1) < S(\tilde{\varphi}_2)$, then $\tilde{\varphi}_1 < \tilde{\varphi}_2$; if $S(\tilde{\varphi}_1) = S(\tilde{\varphi}_2)$, then (1) if $H(\tilde{\varphi}_1) = H(\tilde{\varphi}_2)$, then $\tilde{\varphi}_1 = \tilde{\varphi}_2$; (2) if $H(\tilde{\varphi}_1) < H(\tilde{\varphi}_2)$, then $\tilde{\varphi}_1 < \tilde{\varphi}_2$.

Definition 6 [54]. For two IVIFNs $\tilde{\varphi}_1 = ([e_1, f_1], [g_1, h_1])$ and $\tilde{\varphi}_2 = ([e_2, f_2], [g_2, h_2])$, the following operational laws are defined as follows:

- (1) $\tilde{\varphi}_1 \oplus \tilde{\varphi}_2 = ([e_1 + e_2 - e_1 e_2, f_1 + f_2 - f_1 f_2], [g_1 g_2, h_1 h_2]);$
- (2) $\tilde{\varphi}_1 \otimes \tilde{\varphi}_2 = ([e_1 e_2, f_1 f_2], [g_1 + g_2 - g_1 g_2, h_1 + h_2 - h_1 h_2]);$
- (3) $\lambda \tilde{\varphi}_1 = \left(\left[1 - (1 - e_1)^\lambda, 1 - (1 - f_1)^\lambda \right], [g_1^\lambda, h_1^\lambda] \right), \lambda > 0;$
- (4) $(\tilde{\varphi}_1)^\lambda = \left([e_1^\lambda, f_1^\lambda], \left[1 - (1 - g_1)^\lambda, 1 - (1 - h_1)^\lambda \right] \right), \lambda > 0.$

2.2. HM Operator

Hara, Uchiyama and Takahasi [48] proposed the HM operator.

Definition 7 [48]. The HM operator is defined as follows:

$$HM^{(x)}(\varphi_1, \varphi_2, \dots, \varphi_n) = \frac{\sum_{1 \leq i_1 < \dots < i_x \leq n} \left(\prod_{j=1}^x \varphi_{i_j} \right)^{\frac{1}{x}}}{C_n^x} \quad (5)$$

where x is a parameter, $x = 1, 2, \dots, n$, $i_1, i_2, \dots, i_x$ are x integer values taken from the set $\{1, 2, \dots, n\}$ of k integer values, C_n^x is the binomial coefficient, $C_n^x = \frac{n!}{x!(n-x)!}$.

2.3. Dombi Operations of IVIFNs

Definition 8 [50]. Dombi [50] proposed a generator to produce Dombi T-norm and T-conorm which are shown as follows:

$$D(q, r) = \frac{1}{1 + \left(\left(\frac{1-q}{q} \right)^\beta + \left(\frac{1-r}{r} \right)^\beta \right)^{1/\beta}} \quad (6)$$

$$D^c(q, r) = 1 - \frac{1}{1 + \left(\left(\frac{q}{1-q} \right)^\beta + \left(\frac{r}{1-r} \right)^\beta \right)^{1/\beta}} \quad (7)$$

where $\beta > 0$, $(q, r) \in [0, 1]$.

Based on the Dombi T-norm and T-conorm, we can give the operational rules of IVIFNs.

Definition 9. For two IVIFNs $\tilde{\varphi}_1 = ([e_1, f_1], [g_1, h_1])$ and $\tilde{\varphi}_2 = ([e_2, f_2], [g_2, h_2])$, $\lambda > 0$, the Dombi operational laws are defined as follows:

$$(1) \tilde{\varphi}_1 \oplus \tilde{\varphi}_2 = \left(\left[\frac{1 - \frac{1}{1 + \left(\left(\frac{e_1}{1-e_1} \right)^\lambda + \left(\frac{e_2}{1-e_2} \right)^\lambda \right)^{\frac{1}{\lambda}}}}{1}, \frac{1 - \frac{1}{1 + \left(\left(\frac{f_1}{1-f_1} \right)^\lambda + \left(\frac{f_2}{1-f_2} \right)^\lambda \right)^{\frac{1}{\lambda}}}}{1} \right], \left[\frac{1}{1 + \left(\left(\frac{1-g_1}{g_1} \right)^\lambda + \left(\frac{1-g_2}{g_2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\left(\frac{1-h_1}{h_1} \right)^\lambda + \left(\frac{1-h_2}{h_2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right);$$

$$\begin{aligned}
(2) \quad \tilde{\varphi}_1 \otimes \tilde{\varphi}_2 &= \left(\left[\frac{1}{1 + \left(\left(\frac{1-e_1}{e_1} \right)^\lambda + \left(\frac{1-e_2}{e_2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\left(\frac{1-f_1}{f_1} \right)^\lambda + \left(\frac{1-f_2}{f_2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right], \left[1 - \frac{1}{1 + \left(\left(\frac{g_1}{1-g_1} \right)^\lambda + \left(\frac{g_2}{1-g_2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\left(\frac{h_1}{1-h_1} \right)^\lambda + \left(\frac{h_2}{1-h_2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right); \\
(3) \quad n\tilde{\varphi}_1 &= \left(\left[1 - \frac{1}{1 + \left(n \left(\frac{e_1}{1-e_1} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(n \left(\frac{f_1}{1-f_1} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right], \left[\frac{1}{1 + \left(n \left(\frac{1-g_1}{g_1} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(n \left(\frac{1-h_1}{h_1} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right); \\
(4) \quad (\tilde{\varphi}_1)^n &= \left(\left[\frac{1}{1 + \left(n \left(\frac{1-e_1}{e_1} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(n \left(\frac{1-f_1}{f_1} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right], \left[1 - \frac{1}{1 + \left(n \left(\frac{g_1}{1-g_1} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(n \left(\frac{h_1}{1-h_1} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right).
\end{aligned}$$

3. Some Dombi Hamy Mean Operators with IVIFNs

3.1. The IVIFDHM Operator

Based on the HM operator and Dombi operation rules, the IVIFDHM operator is defined as follows:

Definition 10. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ be a set of IVIFNs. The IVIFDHM operator is

$$\text{IVIFDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \frac{1 \leq i_1 < \dots < i_x \leq n \left(\bigotimes_{j=1}^x \tilde{\varphi}_{i_j} \right)^{\frac{1}{x}}}{C_n^x} \quad (8)$$

Theorem 1. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ be a set of IVIFNs. The fused value by the IVIFDHM operators is also an IVIFN where

$$\begin{aligned}
&\text{IVIFDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \frac{1 \leq i_1 < \dots < i_x \leq n \left(\bigotimes_{j=1}^x \tilde{\varphi}_{i_j} \right)^{\frac{1}{x}}}{C_n^x} \\
&= \left(\left[\frac{1 - \frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{1-e_{i_j}}{e_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{1-f_{i_j}}{f_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right], \left[\frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{g_{i_j}}{1-g_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{h_{i_j}}{1-h_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right] \right) \quad (9)
\end{aligned}$$

Proof.

$$\bigotimes_{j=1}^x \tilde{\varphi}_{ij} = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{1-e_{ij}}{e_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{1-f_{ij}}{f_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ 1 - \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{g_{ij}}{1-g_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right] \right) \quad (10)$$

Thus,

$$\left(\bigotimes_{j=1}^x \tilde{\varphi}_{ij} \right)^{\frac{1}{x}} = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{1-e_{ij}}{e_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{1-f_{ij}}{f_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{g_{ij}}{1-g_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{h_{ij}}{1-h_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right] \right) \quad (11)$$

Thereafter,

$$\bigoplus_{1 \leq i_1 < \dots < i_x \leq n} \left(\bigotimes_{j=1}^x \tilde{\varphi}_{ij} \right)^{\frac{1}{x}} = \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1-e_{ij}}{e_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1-f_{ij}}{f_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \\ \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{g_{ij}}{1-g_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{h_{ij}}{1-h_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right) \quad (12)$$

Therefore,

$$\begin{aligned} \text{IVIFDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) &= \frac{\bigoplus_{1 \leq i_1 < \dots < i_x \leq n} \left(\bigotimes_{j=1}^x \tilde{\varphi}_{ij} \right)^{\frac{1}{x}}}{C_n^x} \\ &= \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{1-e_{ij}}{e_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{1-f_{ij}}{f_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \\ \frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{g_{ij}}{1-g_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{x}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{1}{\sum_{j=1}^x \left(\frac{h_{ij}}{1-h_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right) \end{aligned} \quad (13)$$

Thus Equation (9) is right. $\square$

Example 1. Let $\tilde{\varphi}_1 = ([0.2, 0.4], [0.3, 0.6])$, $\tilde{\varphi}_2 = ([0.1, 0.3], [0.2, 0.5])$, $\tilde{\varphi}_3 = ([0.3, 0.5], [0.1, 0.2])$ and $\tilde{\varphi}_4 = ([0.1, 0.4], [0.3, 0.5])$ be four IVIFNs, and $x=2, \lambda=3$,

$$\frac{1-e_{ij}}{e_{ij}} = (4.0000, 9.0000, 2.3333, 9.0000), \frac{1-f_{ij}}{f_{ij}} = (1.5000, 2.3333, 1.0000, 1.5000)$$

$$\frac{g_{ij}}{1-g_{ij}} = (0.4286, 0.2500, 0.1111, 0.4286), \frac{h_{ij}}{1-h_{ij}} = (1.5000, 1.0000, 0.2500, 1.0000)$$

Then according to Equation (9), we have

$$\text{IFDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \frac{1 \leq i_1 < \dots < i_x \leq n}{C_n^x} \left(\bigoplus_{j=1}^x \tilde{\varphi}_{i_j} \right)^{\frac{1}{x}}$$

$$= \left[\begin{array}{c} \left[1 - 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{4.0000^3 + 9.0000^3} + \frac{1}{4.0000^3 + 2.3333^3} + \frac{1}{4.0000^3 + 9.0000^3} \right) \right)^{\frac{1}{3}} \right), \right. \\ \left. 1 - 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{1.5000^3 + 2.3333^3} + \frac{1}{1.5000^3 + 1.0000^3} + \frac{1}{1.5000^3 + 1.5000^3} \right) \right)^{\frac{1}{3}} \right) \right]^{\frac{1}{3}}, \\ \left[1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{0.4286^3 + 0.2500^3} + \frac{1}{0.4286^3 + 0.1111^3} + \frac{1}{0.4286^3 + 0.4286^3} \right) \right)^{\frac{1}{3}} \right), \right. \\ \left. 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{1.5000^3 + 1.0000^3} + \frac{1}{1.5000^3 + 0.2500^3} + \frac{1}{1.5000^3 + 1.0000^3} \right) \right)^{\frac{1}{3}} \right) \right]^{\frac{1}{3}} \end{array} \right]$$

$$= ([0.1560, 0.3919], [0.2306, 0.4941])$$

Then we list some properties of IVIFDHM operator.

Property 1. (Idempotency) If $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n) = \tilde{\varphi}$ are equal, then

$$\text{IVIFDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \tilde{\varphi} \quad (14)$$

Property 2. (Monotonicity) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ and $\tilde{\theta}_j = ([r_j, s_j], [m_j, n_j]) (j = 1, 2, \dots, n)$ be two sets of IVIFNs. If $e_j \leq r_j, f_j \leq s_j$ and $g_j \geq m_j, h_j \geq n_j$ hold for all j , then

$$\text{IVIFDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \text{IVIFDHM}^{(x)}(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n) \quad (15)$$

Property 3. (Boundedness) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ be a set of IVIFNs. If $\tilde{\varphi}_i^+ = (([\max_i(e_j), \max_i(f_j)], [\min_i(g_j), \min_i(h_j)]))$ and $\tilde{\varphi}_i^- = ([\min_i(e_j), \min_i(f_j)], [\max_i(g_j), \max_i(h_j)])$ then

$$\tilde{\varphi}^- \leq \text{IVIFDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \tilde{\varphi}^+ \quad (16)$$

3.2. The IVIFWDHM Operator

In real MADM, it is important to pay attention to attribute weights. Thus we propose the interval-valued intuitionistic fuzzy weighted Dombi Hamy mean (IVIFWDHM) operator.

Definition 11. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ be a set of IVIFNs with their weight vector $w_i = (w_1, w_2, \dots, w_n)^T$, thereby satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Then the IVIFWDHM operator is as follows:

$$\text{IVIFWDHM}_w^{(x)}(\varphi_1, \varphi_2, \dots, \varphi_n) = \frac{1 \oplus_{1 \leq i_1 < \dots < i_x \leq n} \left(\bigotimes_{j=1}^x (\tilde{\varphi}_{i_j})^{w_{i_j}} \right)^{\frac{1}{x}}}{C_n^x} \quad (17)$$

Theorem 2. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ be a set of IVIFNs. The fused value by IVIFWDHM operators is also an IVIFN where

$$\begin{aligned} \text{IVIFWDHM}_w^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) &= \frac{1 \oplus_{1 \leq i_1 < \dots < i_x \leq n} \left(\bigotimes_{j=1}^x (\tilde{\varphi}_{i_j})^{w_{i_j}} \right)^{\frac{1}{x}}}{C_n^x} \\ &= \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{1-e_{i_j}}{e_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{1-f_{i_j}}{f_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right], \left[\begin{array}{cc} \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{g_{i_j}}{1-g_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{h_{i_j}}{1-h_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right). \end{aligned} \quad (18)$$

Proof.

$$\left(\tilde{\varphi}_{i_j} \right)^{w_{i_j}} = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(w_{i_j} \left(\frac{1-e_{i_j}}{e_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(w_{i_j} \left(\frac{1-f_{i_j}}{f_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right], \left[\begin{array}{cc} 1 - \frac{1}{1 + \left(w_{i_j} \left(\frac{g_{i_j}}{1-g_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(w_{i_j} \left(\frac{h_{i_j}}{1-h_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right] \right). \quad (19)$$

Thus,

$$\bigotimes_{j=1}^x \left(\tilde{\varphi}_{i_j} \right)^{w_{i_j}} = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{j=1}^x w_{i_j} \left(\frac{1-e_{i_j}}{e_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{j=1}^x w_{i_j} \left(\frac{1-f_{i_j}}{f_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right], \left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\sum_{j=1}^x w_{i_j} \left(\frac{g_{i_j}}{1-g_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^x w_{i_j} \left(\frac{h_{i_j}}{1-h_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right] \right). \quad (20)$$

Therefore,

$$\left(\bigotimes_{j=1}^x (\tilde{\varphi}_{i_j})^{w_{i_j}} \right)^{\frac{1}{x}} = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x w_{i_j} \left(\frac{1-e_{i_j}}{e_{i_j}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x w_{i_j} \left(\frac{1-f_{i_j}}{f_{i_j}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \\ 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x w_{i_j} \left(\frac{g_{i_j}}{1-g_{i_j}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x w_{i_j} \left(\frac{h_{i_j}}{1-h_{i_j}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right)^{\frac{1}{x}}. \quad (21)$$

Thereafter,

$$\begin{aligned} & \bigoplus_{1 \leq i_1 < \dots < i_x \leq n} \left(\bigotimes_{j=1}^x (\tilde{\varphi}_{i_j})^{w_{i_j}} \right)^{\frac{1}{x}} \\ &= \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{1-e_{i_j}}{e_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{1-f_{i_j}}{f_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}} \\ \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{g_{i_j}}{1-g_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{h_{i_j}}{1-h_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right)^{\frac{1}{x}}. \quad (22) \end{aligned}$$

Therefore,

$$\begin{aligned} & \text{IVIFWDHM}_w^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \frac{\bigoplus_{1 \leq i_1 < \dots < i_x \leq n} \left(\bigotimes_{j=1}^x (\tilde{\varphi}_{i_j})^{w_{i_j}} \right)^{\frac{1}{x}}}{C_n^x} \\ &= \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{1-e_{i_j}}{e_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{1-f_{i_j}}{f_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}} \\ \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{g_{i_j}}{1-g_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x w_{i_j} \left(\frac{h_{i_j}}{1-h_{i_j}} \right)^{\lambda}} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right)^{\frac{1}{x}}. \quad (23) \end{aligned}$$

Hence, Equation (18) is kept. $\square$

Example 2. Let $\tilde{\varphi}_1 = ([0.2, 0.4], [0.3, 0.6])$, $\tilde{\varphi}_2 = ([0.1, 0.3], [0.2, 0.5])$, $\tilde{\varphi}_3 = ([0.3, 0.5], [0.1, 0.2])$ and $\tilde{\varphi}_4 = ([0.1, 0.4], [0.3, 0.5])$ be four IVIFNs, and $x=2, \lambda=3, w=(0.4, 0.1, 0.3, 0.2)$,

$$\frac{1-e_{ij}}{e_{ij}} = (4.0000, 9.0000, 2.3333, 9.0000), \quad \frac{1-f_{ij}}{f_{ij}} = (1.5000, 2.3333, 1.0000, 1.5000)$$

$$\frac{g_{ij}}{1-g_{ij}} = (0.4286, 0.2500, 0.1111, 0.4286), \quad \frac{h_{ij}}{1-h_{ij}} = (1.5000, 1.0000, 0.2500, 1.0000)$$

Then according to Equation (18), we have

$$\begin{aligned} \text{IVIFWDHM}_w^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) &= \frac{\bigoplus_{1 \leq i_1 < \dots < i_x \leq n} \left(\bigotimes_{j=1}^x (\tilde{\varphi}_{i_j})^{w_{i_j}} \right)^{\frac{1}{x}}}{C_n^x} \\ &= \left[\begin{array}{l} 1 - 1/\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4 \times 4.0000^3 + 0.1 \times 9.0000^3} + \frac{1}{0.4 \times 4.0000^3 + 0.3 \times 2.3333^3}}{\frac{1}{0.4 \times 4.0000^3 + 0.2 \times 9.0000^3} + \frac{1}{0.1 \times 9.0000^3 + 0.3 \times 2.3333^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}}, \\ 1 - 1/\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4 \times 1.5000^3 + 0.1 \times 2.3333^3} + \frac{1}{0.4 \times 1.5000^3 + 0.3 \times 1.0000^3}}{\frac{1}{0.4 \times 1.5000^3 + 0.2 \times 1.5000^3} + \frac{1}{0.1 \times 2.3333^3 + 0.3 \times 1.0000^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}}, \\ 1/\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4 \times 0.4286^3 + 0.1 \times 0.2500^3} + \frac{1}{0.4 \times 0.4286^3 + 0.3 \times 0.1111^3}}{\frac{1}{0.4 \times 0.4286^3 + 0.2 \times 0.4286^3} + \frac{1}{0.1 \times 0.2500^3 + 0.3 \times 0.1111^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}}, \\ 1/\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4 \times 1.5000^3 + 0.1 \times 1.0000^3} + \frac{1}{0.4 \times 1.5000^3 + 0.3 \times 0.2500^3}}{\frac{1}{0.4 \times 1.5000^3 + 0.2 \times 1.0000^3} + \frac{1}{0.1 \times 1.0000^3 + 0.3 \times 0.2500^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}} \end{array} \right] \\ &= ([0.2257, 0.5165], [0.1392, 0.3476]) \end{aligned}$$

Then we list some properties of the IVIFWDHM operator.

Property 4. (Monotonicity) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j])$ ($j = 1, 2, \dots, n$) and $\tilde{\theta}_j = ([r_j, s_j], [m_j, n_j])$ ($j = 1, 2, \dots, n$) be two sets of IVIFNs. If $e_j \leq r_j, f_j \leq s_j$ and $g_j \geq m_j, h_j \geq n_j$ hold for all j , then

$$\text{IVIFWDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \text{IVIFWDHM}^{(x)}(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n) \quad (24)$$

The proof is similar to IVIFDHM, thus, it is omitted here.

Property 5. (Boundedness) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j])$ ($j = 1, 2, \dots, n$) be a set of IVIFNs. If $\tilde{\varphi}_i^+ = (([\max_i(e_j), \max_i(f_j)], [\min_i(g_j), \min_i(h_j)]))$ and $\tilde{\varphi}_i^- = ([\min_i(e_j), \min_i(f_j)], [\max_i(g_j), \max_i(h_j)])$ then

$$\tilde{\varphi}^- \leq \text{IVIFWDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \tilde{\varphi}^+ \quad (25)$$

3.3. The IVIFDDHM Operator

Wu, Wang, Wei, and Wei [49] proposed the dual HM (DHM) operator.

Definition 12 [49]. The DHM operator is as follows:

$$\text{DHM}^{(x)}(\varphi_1, \varphi_2, \dots, \varphi_n) = \left(\prod_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\sum_{j=1}^x \varphi_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}} \quad (26)$$

where x is a parameter and $x = 1, 2, \dots, n$, $i_1, i_2, \dots, i_x$ are x integer values taken from the set $\{1, 2, \dots, n\}$ of k integer values, C_n^x denotes the binomial coefficient and $C_n^x = \frac{n!}{x!(n-x)!}$.

In this section, we will propose the interval-valued intuitionistic fuzzy Dombi DHM (IVIFDDHM) operator.

Definition 13. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j])$ ($j = 1, 2, \dots, n$) be a set of IVIFNs. The IVIFDDHM operator is as follows:

$$\text{IVIFDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x \tilde{\varphi}_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}} \quad (27)$$

Theorem 3. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j])$ ($j = 1, 2, \dots, n$) be a set of IVIFNs. The fused value by IVIFDDHM operators is also an IVIFN where

$$\begin{aligned} \text{IVIFDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) &= \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x \tilde{\varphi}_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}} \\ &= \left(\left[\begin{aligned} &\frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{e_{i_j}}{1 - e_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{f_{i_j}}{1 - f_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \\ &1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1 - g_{i_j}}{g_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1 - h_{i_j}}{h_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{aligned} \right] \right)^{\frac{1}{C_n^x}} \end{aligned} \quad (28)$$

Proof.

$$\bigoplus_{j=1}^x \tilde{\varphi}_{ij} = \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{e_{ij}}{1-e_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{f_{ij}}{1-f_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right], \right. \\ \left. \left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{1-g_{ij}}{g_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{j=1}^x \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right] \right) \quad (29)$$

Thus,

$$\frac{\bigoplus_{j=1}^x \tilde{\varphi}_{ij}}{x} = \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{e_{ij}}{1-e_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{f_{ij}}{1-f_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right], \right. \\ \left. \left[\begin{array}{cc} \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{1-g_{ij}}{g_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^x \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right] \right) \quad (30)$$

Therefore,

$$\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x \tilde{\varphi}_{ij}}{x} \right) = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{e_{ij}}{1-e_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{f_{ij}}{1-f_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right], \right. \\ \left. \left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1-g_{ij}}{g_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right] \right] \right) \quad (31)$$

Therefore,

$$\text{IVIFDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x \tilde{\varphi}_{ij}}{x} \right) \right)^{\frac{1}{C_n^x}} \\ = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{e_{ij}}{1-e_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{f_{ij}}{1-f_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right], \right. \\ \left. \left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1-g_{ij}}{g_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^x \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right] \right] \right) \quad (32)$$

Thus, Equation (28) is right. $\square$

Example 3. Let $\tilde{\varphi}_1 = ([0.2, 0.4], [0.3, 0.6])$, $\tilde{\varphi}_2 = ([0.1, 0.3], [0.2, 0.5])$, $\tilde{\varphi}_3 = ([0.3, 0.5], [0.1, 0.2])$ and $\tilde{\varphi}_4 = ([0.1, 0.4], [0.3, 0.5])$ be four IVIFNs, and $x=2, \lambda=3$,

$$\frac{e_{ij}}{1-e_{ij}} = (0.2500, 0.1111, 0.4286, 0.1111), \quad \frac{f_{ij}}{1-f_{ij}} = (0.6667, 0.4286, 1.0000, 0.6667)$$

$$\frac{1-g_{ij}}{g_{ij}} = (2.3333, 4.0000, 9.0000, 2.3333), \quad \frac{1-h_{ij}}{h_{ij}} = (0.6667, 1.0000, 4.0000, 1.0000)$$

Then according to Equation (28), we have

$$\text{IVIFDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x \tilde{\varphi}_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}}$$

$$= \left\{ \left[\begin{array}{l} 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{0.2500^3 + 0.1111^3} + \frac{1}{0.2500^3 + 0.4286^3} + \frac{1}{0.2500^3 + 0.1111^3} \right) \right)^{\frac{1}{3}} \right), \\ 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{0.6667^3 + 0.4286^3} + \frac{1}{0.6667^3 + 1.0000^3} + \frac{1}{0.6667^3 + 0.6667^3} \right) \right)^{\frac{1}{3}} \right), \\ 1 - 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{2.3333^3 + 4.0000^3} + \frac{1}{2.3333^3 + 9.0000^3} + \frac{1}{2.3333^3 + 2.3333^3} \right) \right)^{\frac{1}{3}} \right), \\ 1 - 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{0.6667^3 + 1.0000^3} + \frac{1}{0.6667^3 + 4.0000^3} + \frac{1}{0.6667^3 + 1.0000^3} \right) \right)^{\frac{1}{3}} \right) \end{array} \right] \right\}$$

$$= ([0.1523, 0.4052], [0.2217, 0.4699])$$

The IVIFDDHM operator has the following properties.

Property 6. (Idempotency) If $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j])$ ($j = 1, 2, \dots, n$) are equal, then

$$\text{IVIFDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \tilde{\varphi} \quad (33)$$

Property 7. (Monotonicity) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j])$ ($j = 1, 2, \dots, n$) and $\tilde{\theta}_j = ([r_j, s_j], [m_j, n_j])$ ($j = 1, 2, \dots, n$) be two sets of IVIFNs. If $e_j \leq r_j, f_j \leq s_j$ and $g_j \geq m_j, h_j \geq n_j$ hold for all j , then

$$\text{IVIFDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \text{IVIFDDHM}^{(x)}(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n) \quad (34)$$

Property 8. (Boundedness) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j])$ ($j = 1, 2, \dots, n$) be a set of IVIFNs. If $\tilde{\varphi}_i^+ = (([\max_i(e_j), \max_i(f_j)], [\min_i(g_j), \min_i(h_j)]))$ and $\tilde{\varphi}_i^- = ([\min_i(e_j), \min_i(f_j)], [\max_i(g_j), \max_i(h_j)])$ then

$$\tilde{\varphi}^- \leq \text{IVIFDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \tilde{\varphi}^+ \quad (35)$$

3.4. The IVIFWDDHM Operator

In practical MADM, it is important to pay attention to attribute weights; we propose the interval-valued intuitionistic weighted Dombi DHM (IVIFWDDHM) operator.

Definition 14. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ be a set of IVIFNs with their weight vector be $w_i = (w_1, w_2, \dots, w_n)^T$, thereby satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$.

$$\text{IVIFWDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x w_{i_j} \tilde{\varphi}_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}} \quad (36)$$

Theorem 4. Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j = 1, 2, \dots, n)$ be a set of IVIFNs. The fused value by IVIFWDDHM operators is also an IVIFN where

$$\begin{aligned} \text{IVIFWDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) &= \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x w_{i_j} \tilde{\varphi}_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}} \\ &= \left(\left[\begin{array}{l} \left[\frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{i_j} \left(\frac{e_{i_j}}{1-e_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{i_j} \left(\frac{f_{i_j}}{1-f_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right], \right. \\ \left. \left[1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{i_j} \left(\frac{1-g_{i_j}}{g_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{i_j} \left(\frac{1-h_{i_j}}{h_{i_j}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \right] \right] \right)^{\frac{1}{\lambda}} \end{array} \right) \quad (37)$$

Proof.

$$w_{i_j} \tilde{\varphi}_{i_j} = \left(\left[\begin{array}{l} \left[1 - \frac{1}{1 + \left(w_{i_j} \left(\frac{e_{i_j}}{1-e_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(w_{i_j} \left(\frac{f_{i_j}}{1-f_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right], \right. \\ \left. \left[\frac{1}{1 + \left(w_{i_j} \left(\frac{1-g_{i_j}}{g_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(w_{i_j} \left(\frac{1-h_{i_j}}{h_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right] \right) \quad (38)$$

Then,

$$\bigoplus_{j=1}^x w_{i_j} \tilde{\varphi}_{i_j} = \left(\left[\begin{array}{l} \left[1 - \frac{1}{1 + \left(\sum_{j=1}^n w_{i_j} \left(\frac{e_{i_j}}{1-e_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^n w_{i_j} \left(\frac{f_{i_j}}{1-f_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right], \right. \\ \left. \left[\frac{1}{1 + \left(\sum_{j=1}^n w_{i_j} \left(\frac{1-g_{i_j}}{g_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{j=1}^n w_{i_j} \left(\frac{1-h_{i_j}}{h_{i_j}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \right] \right) \quad (39)$$

Thus,

$$\frac{\bigoplus_{j=1}^x w_{ij} \tilde{\varphi}_{ij}}{x} = \left(\left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^n w_{ij} \left(\frac{e_{ij}}{1-e_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^n w_{ij} \left(\frac{f_{ij}}{1-f_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right], \right. \\ \left. \left[\begin{array}{cc} \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^n w_{ij} \left(\frac{1-g_{ij}}{g_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{x} \sum_{j=1}^n w_{ij} \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right] \right) \quad (40)$$

Therefore,

$$\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x w_{ij} \tilde{\varphi}_{ij}}{x} \right) \\ = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{e_{ij}}{1-e_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{f_{ij}}{1-f_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right], \right. \\ \left. \left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{1-g_{ij}}{g_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right) \quad (41)$$

Therefore,

$$\text{IVIFWDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x w_{ij} \tilde{\varphi}_{ij}}{x} \right) \right)^{\frac{1}{C_n^x}} \\ = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{e_{ij}}{1-e_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{f_{ij}}{1-f_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right], \right. \\ \left. \left[\begin{array}{cc} 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{1-g_{ij}}{g_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{C_n^x} \sum_{1 \leq i_1 < \dots < i_x \leq n} \frac{x}{\sum_{j=1}^n w_{ij} \left(\frac{1-h_{ij}}{h_{ij}} \right)^\lambda} \right)^{\frac{1}{\lambda}}} \end{array} \right] \right) \quad (42)$$

Thus, Equation (37) is right. $\square$

Example 4. Let $\tilde{\varphi}_1 = ([0.2, 0.4], [0.3, 0.6])$, $\tilde{\varphi}_2 = ([0.1, 0.3], [0.2, 0.5])$, $\tilde{\varphi}_3 = ([0.3, 0.5], [0.1, 0.2])$ and $\tilde{\varphi}_4 = ([0.1, 0.4], [0.3, 0.5])$ be four IVIFNs, and $x=2, \lambda=3, w=(0.4, 0.1, 0.2, 0.3)$,

$$\frac{e_{ij}}{1-e_{ij}} = (0.2500, 0.1111, 0.4286, 0.1111), \quad \frac{f_{ij}}{1-f_{ij}} = (0.6667, 0.4286, 1.0000, 0.6667)$$

$$\frac{1-g_{ij}}{g_{ij}} = (2.3333, 4.0000, 9.0000, 2.3333), \quad \frac{1-h_{ij}}{h_{ij}} = (0.6667, 1.0000, 4.0000, 1.0000)$$

Then according to Equation (37), we have

$$\text{IVIFWDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) = \left(\bigotimes_{1 \leq i_1 < \dots < i_x \leq n} \left(\frac{\bigoplus_{j=1}^x w_{i_j} \tilde{\varphi}_{i_j}}{x} \right) \right)^{\frac{1}{C_n^x}}$$

$$= \left[\begin{array}{l} \left[\begin{array}{l} 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{\frac{0.4 \times 0.2500^3 + 0.1 \times 0.1111^3}{1} + \frac{0.4 \times 0.2500^3 + 0.2 \times 0.4286^3}{1} + \frac{0.4 \times 0.2500^3 + 0.3 \times 0.1111^3}{1} + \frac{0.1 \times 0.1111^3 + 0.2 \times 0.4286^3}{1} + \frac{0.1 \times 0.1111^3 + 0.3 \times 0.1111^3}{1} + \frac{0.2 \times 0.4286^3 + 0.3 \times 0.1111^3}{1}} \right) \right)^{\frac{1}{3}} \right) \\ 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{\frac{0.4 \times 0.6667^3 + 0.1 \times 0.4286^3}{1} + \frac{0.4 \times 0.6667^3 + 0.2 \times 1.0000^3}{1} + \frac{0.4 \times 0.6667^3 + 0.3 \times 0.6667^3}{1} + \frac{0.1 \times 0.4286^3 + 0.2 \times 1.0000^3}{1} + \frac{0.1 \times 0.4286^3 + 0.3 \times 0.6667^3}{1} + \frac{0.2 \times 1.0000^3 + 0.3 \times 0.6667^3}{1}} \right) \right)^{\frac{1}{3}} \right) \end{array} \right] \\ 1 - 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{\frac{0.4 \times 2.3333^3 + 0.1 \times 4.0000^3}{1} + \frac{0.4 \times 2.3333^3 + 0.2 \times 9.0000^3}{1} + \frac{0.4 \times 2.3333^3 + 0.3 \times 2.3333^3}{1} + \frac{0.1 \times 4.0000^3 + 0.2 \times 9.0000^3}{1} + \frac{0.1 \times 4.0000^3 + 0.3 \times 2.3333^3}{1} + \frac{0.2 \times 9.0000^3 + 0.3 \times 2.3333^3}{1}} \right) \right)^{\frac{1}{3}} \right) \\ 1 - 1 / \left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{1}{\frac{0.4 \times 0.6667^3 + 0.1 \times 1.0000^3}{1} + \frac{0.4 \times 0.6667^3 + 0.2 \times 4.0000^3}{1} + \frac{0.4 \times 0.6667^3 + 0.3 \times 1.0000^3}{1} + \frac{0.1 \times 1.0000^3 + 0.2 \times 4.0000^3}{1} + \frac{0.1 \times 1.0000^3 + 0.3 \times 1.0000^3}{1} + \frac{0.2 \times 4.0000^3 + 0.3 \times 1.0000^3}{1}} \right) \right)^{\frac{1}{3}} \right) \end{array} \right] \\ = ([0.0917, 0.2962], [0.3283, 0.6086])$$

Then we give some properties of the IVIFWDDHM operator.

Property 9. (Monotonicity) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j=1, 2, \dots, n)$ and $\tilde{\theta}_j = ([r_j, s_j], [m_j, n_j]) (j=1, 2, \dots, n)$ be two sets of IVIFNs. If $e_j \leq r_j, f_j \leq s_j$ and $g_j \geq m_j, h_j \geq n_j$ hold for all j , then

$$\text{IVIFWDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \text{IVIFWDDHM}^{(x)}(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n) \quad (43)$$

The proof is similar to IVIFWDHM, thus, it is omitted here.

Property 10. (Boundedness) Let $\tilde{\varphi}_j = ([e_j, f_j], [g_j, h_j]) (j=1, 2, \dots, n)$ be a set of IVIFNs. If $\tilde{\varphi}_i^+ = (([\max_i(e_j), \max_i(f_j)], [\min_i(g_j), \min_i(h_j)]))$ and $\tilde{\varphi}_i^- = ([\min_i(e_j), \min_i(f_j)], [\max_i(g_j), \max_i(h_j)])$ then

$$\tilde{\varphi}^- \leq \text{IVIFWDDHM}^{(x)}(\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_n) \leq \tilde{\varphi}^+ \quad (44)$$

4. Example and Comparison

4.1. Numerical Example

With the development of the economy and society and the deepening of aging, "Senior tourism" continues to heat up. Elderly tourism has become a tourist market which cannot be ignored at present

and in the future, and contains a tremendous potential for development. The tourism destinations are also actively involved in the development of this market. However, there are still many issues in the tourism service for the elderly tourists in the tourism destination. Although some researches on elderly tourism in China has been on the rise in recent years, many researches are conducted from the points of the consumption behavior of the elderly, the development of the elderly tourism market and the development of elderly tourism products, but the research on elderly tourism services, especially the quality of elderly tourism services, is relatively scarce. Additionally, the problems of evaluating the elderly tourism service quality in tourism destination are classical MADM problems [55–62]. Thus, we give an example to solve the MADM problems for evaluating the elderly tourism service quality in tourism destination with IVIFNs. There are five possible tourism scenic spots $A_i (i = 1, 2, 3, 4, 5)$ to assess. The experts use the four attributes to assess the five tourism scenic spots: ① G_1 is the resource safety value; ② G_2 is the infrastructure construction value; ③ G_3 is the income distribution value; ④ G_4 is the promotion employment value. The five possible tourism scenic spots are to be assessed with IVIFNs (whose weighting vector $\omega = (0.4, 0.1, 0.3, 0.2)$), as shown in the Table 1.

Table 1. The IVIFN decision matrix.

	G_1	G_2	G_3	G_4
A_1	$([0.5, 0.6], [0.1, 0.2])$	$([0.4, 0.6], [0.2, 0.4])$	$([0.2, 0.3], [0.1, 0.4])$	$([0.3, 0.5], [0.1, 0.3])$
A_2	$([0.3, 0.4], [0.2, 0.5])$	$([0.4, 0.5], [0.1, 0.2])$	$([0.6, 0.7], [0.2, 0.3])$	$([0.3, 0.4], [0.2, 0.4])$
A_3	$([0.7, 0.8], [0.1, 0.2])$	$([0.6, 0.8], [0.1, 0.2])$	$([0.4, 0.7], [0.1, 0.3])$	$([0.5, 0.6], [0.1, 0.4])$
A_4	$([0.6, 0.7], [0.1, 0.3])$	$([0.2, 0.3], [0.6, 0.7])$	$([0.4, 0.6], [0.2, 0.4])$	$([0.1, 0.3], [0.4, 0.5])$
A_5	$([0.4, 0.5], [0.1, 0.3])$	$([0.1, 0.2], [0.5, 0.7])$	$([0.3, 0.4], [0.5, 0.6])$	$([0.5, 0.7], [0.1, 0.2])$

Then, we use the approach developed for selecting the best tourism scenic spots.

Step 1. According to IVIFNs $r_{ij} (i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$, we fuse all IVIFNs r_{ij} by the IVIFWDHM (IVIFWDDHM) operator to have the IVIFNs $A_i (i = 1, 2, 3, 4, 5)$ of the tourism scenic spots A_i . Let $x = 2$, then the fused results are in Table 2.

Table 2. The fused results of the tourism scenic spots by the IVIFWDHM (IVIFWDDHM) operator.

	IVIFWDHM	IVIFWDDHM
A_1	$([0.4739, 0.631], [0.0767, 0.2244])$	$([0.2328, 0.3929], [0.1613, 0.4415])$
A_2	$([0.5148, 0.6181], [0.1262, 0.2546])$	$([0.2666, 0.3582], [0.2507, 0.4565])$
A_3	$([0.6774, 0.8127], [0.0639, 0.1873])$	$([0.4166, 0.6356], [0.1531, 0.3644])$
A_4	$([0.4237, 0.6055], [0.1902, 0.3538])$	$([0.1483, 0.2887], [0.4442, 0.5720])$
A_5	$([0.4192, 0.5342], [0.1182, 0.3106])$	$([0.2591, 0.3607], [0.4856, 0.6062])$

Step 2. Using Table 2, the score values of the tourism scenic spots are in Table 3.

Table 3. The score values of tourism scenic spots.

Alternatives	IVIFWDHM	IVIFWDDHM
A_1	0.4019	0.0115
A_2	0.3760	−0.0412
A_3	0.6194	0.2673
A_4	0.2426	−0.2896
A_5	0.2623	−0.2360

Step 3. Using Table 3, the order of tourism scenic spots is listed in Table 4. Additionally, the best tourism scenic spot is A_3 .

Table 4. The order of the tourism scenic spots.

Methods	Order
IVIFWDHM	$A_3 > A_1 > A_2 > A_5 > A_4$
IVIFWDDHM	$A_3 > A_1 > A_2 > A_5 > A_4$

4.2. Influence Analysis

In order to depict the effects on the ordering by altering parameters of x in the IVIFWDHM (IVIFWDDHM) operators, the analysis results are listed in Tables 5 and 6.

Table 5. The ordering results for the IVIFWDHM operator with different parameters.

	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	$S(A_5)$	Order
$x = 1$	0.4791	0.5436	0.6885	0.4243	0.4936	$A_3 > A_2 > A_5 > A_1 > A_4$
$x = 2$	0.4019	0.3760	0.6194	0.2426	0.2623	$A_3 > A_1 > A_2 > A_5 > A_4$
$x = 3$	0.3373	0.2970	0.5703	0.0878	0.0506	$A_3 > A_1 > A_2 > A_4 > A_5$
$x = 4$	0.2643	0.2651	0.5438	−0.0055	−0.0284	$A_3 > A_2 > A_1 > A_4 > A_5$

Table 6. The ordering results for the IVIFWDDHM operator with different parameters.

	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	$S(A_5)$	Order
$x = 1$	−0.1388	−0.1036	0.1650	−0.5660	−0.5671	$A_3 > A_2 > A_1 > A_4 > A_5$
$x = 2$	0.0115	−0.0412	0.2673	−0.2896	−0.2360	$A_3 > A_1 > A_2 > A_5 > A_4$
$x = 3$	0.0694	0.0206	0.3143	0.0232	0.0740	$A_3 > A_5 > A_1 > A_4 > A_2$
$x = 4$	0.1354	0.1212	0.3661	0.1496	0.1163	$A_3 > A_4 > A_1 > A_2 > A_5$

4.3. Comparative Analysis

We compare the IVIFWDHM and IVIFWDDHM operators with the IVIFWA operator [54], IVIFWG operator [5], gray relational analysis method [8], and the correlation coefficient [63]. The results are listed in Table 7.

Table 7. The order of the tourism scenic spots.

Methods	Order
IVIFWA operator [54]	$A_3 > A_1 > A_4 > A_2 > A_5$
IVIFWG operator [5]	$A_3 > A_1 > A_2 > A_4 > A_5$
Gray Relational Analysis Method [8]	$A_3 > A_5 > A_1 > A_2 > A_4$
Correlation Coefficient [63]	$A_3 > A_1 > A_2 > A_4 > A_5$

From above, we can get the same best tourism scenic spots and the four methods' ranking results are slightly different. However, the existing methods with IVIFNs do not consider the relationship information among the arguments. Our proposed IVIFWDHM and IVIFWDDHM operators consider the relationship among the aggregated arguments.

Additionally, Xu and Chen [9] defined the interval-valued intuitionistic fuzzy Bonferroni mean for aggregating the IVIFNs. However, these Bonferroni mean for aggregating the IVIFNs only consider the relationship information between two arguments and do not consider the relationship information among more than two arguments.

5. Conclusions

In this paper, we investigate the MADM problems with IVIFNs. Then, we utilize the HM operator and Dombi operations to design some HM operators with IVIFNs: IVIFDHM operator, IVIFWDHM operator, IVIFDDHM operator and IVIFWDDHM operator. The main characteristic of

these operators are investigated. Then, we have used the IVIFWDHM and IVIFWDDHM operators to propose two models for MADM problems with IVIFNs. Finally, a real example for evaluating the elderly tourism service quality in the tourism destination is used to show the developed approach. In the subsequent studies, the extension and application of IVIFNs needs to be studied in many other uncertain environments and other applications.

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