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Beyond MSE in Poisson Ridge Regression: New Ridge Parameter Estimators with Additional Distributional Performance Criteria

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Abstract

Despite its widespread use for mitigating multicollinearity in count data models, Poisson ridge regression (PRR) remains methodologically constrained by the choice of the ridge parameter k. Existing studies predominantly evaluate ridge parameter estimators using only the mean squared error (MSE) criterion, largely neglecting their distributional properties and estimation stability. Such a narrow evaluation framework may yield unreliable inference, particularly under high correlation and small sample sizes. This study makes two original contributions to the PRR literature. First, we conduct a comprehensive comparison of 13 commonly used ridge parameter estimators and introduce two new estimators that exhibit superior empirical performance. Second, we extend performance evaluation beyond MSE by incorporating outlier ratios and conformity to normality, thereby establishing a multidimensional framework that explicitly addresses distributional robustness and estimator stability. Monte Carlo simulations across 180 scenarios—varying the number of predictors, sample size, correlation level, and intercept value—show that several estimators deemed optimal under MSE perform poorly in terms of outlier prevalence and normality. In contrast, the proposed estimators consistently achieve a balanced performance between error minimization and distributional stability. Two real-data applications further support these findings.

Keywords: Monte Carlo simulations; multicollinearity; Poisson regression; ridge estimator

MSC: 62J07; 62J12; 62F10; 62-08; 65C05

1. Introduction

Poisson regression (PR) is a special case of generalized linear models and is widely used to model the relationship between one or more explanatory variables and a dependent variable consisting of non-negative integer counts. For instance, in mortality studies, the effects of factors such as age, sex, and lifestyle on the number of deaths can be analyzed using PR. Similarly, in insurance applications, the impact of vehicle characteristics—such as color, engine size, and prior claim history—on the number of insurance claims can be examined within the PR framework [1]. Along the same lines, Osgood [2] investigated the relationship between juvenile crime rates over a given period and socioeconomic factors, including residential instability, ethnic heterogeneity, family disruption, poverty, and unemployment, using several models, including PR.

In many empirical applications involving count data, high correlations among explanatory variables are common. In this context, Herawati et al. [3] examined maternal

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mortality during pregnancy under severe multicollinearity and identified several statistically significant predictors. Their findings further demonstrated that PRR substantially improves predictive performance compared with the classical PR model.

Parameter estimation in PR is typically carried out using the maximum likelihood (ML) method. However, multicollinearity—originally defined by Frisch [4] as the presence of strong intercorrelations among explanatory variables—poses a serious challenge for PR models. It can inflate standard errors, undermine the reliability of p-values, widen confidence intervals, and increase the likelihood of Type II errors. Consequently, some coefficients may appear statistically insignificant, parameter estimates may become unstable, and overall inferential power may deteriorate [5].

To address this issue, several remedial methods have been proposed in the literature. In the context of multiple linear regression, Hoerl and Kennard [6] introduced the biased ridge regression estimator:

$$\hat{\boldsymbol{\beta}}_{\mathbf{R}} = \left(\mathbf{X}'\mathbf{X} + k\mathbf{I}_{\mathbf{p}+1}\right)^{-1}\mathbf{X}'\mathbf{y} \tag{1}$$

where $\hat{\boldsymbol{\beta}}_{\mathbf{R}}$ denotes the $(p+1)\times 1$ parameter estimate vector, $\mathbf{X}$ is the $n\times(p+1)$ design matrix, $\mathbf{I}_{\mathbf{p}+1}$ is the identity matrix of order $(p+1)$, $\mathbf{y}$ is the $n\times 1$ response vector, and $k>0$ is the ridge parameter. By adding k to the diagonal elements of $\mathbf{X}'\mathbf{X}$, this approach shrinks the regression coefficients toward zero, reducing their variances and mitigating the effects of multicollinearity.

The ridge estimator in Equation (1) depends on a single biasing (ridge) parameter, k. Numerous methods for estimating k have been proposed, including those of Kibria [7], Muniz et al. [8], Lukman and Ayinde [9], and Göktaş et al. [10]. The extension of ridge regularization to PR was first introduced by Mansson and Shukur [11], adapting several ridge parameter estimators from the linear ridge regression (LRR) literature to the PRR model.

Among single-parameter methods, the Poisson Liu estimator extends the Liu estimator originally proposed by Kejian [12] to the PR framework [13]. The modified jackknifed estimator, developed by Türkan and Özel [14], builds on the almost unbiased jackknifed approach introduced by Singh et al. [15]. Rashad and Algamal [16] proposed a new ridge estimator specifically for PR. Lukman et al. [17] adapted the K–L estimator originally developed by Kibria and Lukman [18] to PR, while Aladeitan et al. [19] introduced a modified version of the K–L estimator. Regarding two-parameter approaches, Yang and Chang [20] proposed a linear estimator that was later generalized to PR in Asar and Genç [21]. Further developments of two-parameter ridge-type estimators for PR were presented by Alkhateeb and Algamal [22], Lukman et al. [23], and Olakunle et al. [24].

The performance of these estimators has predominantly been evaluated using the MSE:

$$MSE = \frac{1}{R}\sum_{i=1}^{R}\left(\beta_i - \hat{\beta}_i\right)^2 \tag{2}$$

where $\hat{\beta}$ denotes the estimator of β and R is the number of Monte Carlo replications. While accurate estimation of k is crucial for reliable PRR parameter estimates, a desirable ridge parameter estimator should not only achieve a low MSE but also generate stable estimates with consistent distributional properties.

From this perspective, Kibria et al. [25] adapted several ridge parameter estimators proposed for linear and logistic regression to PRR and compared their performance not only in terms of MSE but also regarding the mean and standard deviation of the estimates. They emphasized that, among estimators with similar MSE, those with smaller means and standard deviations are preferable due to greater stability. Similarly, Mermi et al. [26] examined 366 estimators proposed for multiple LRR, incorporating two additional criteria—outlier ratios and conformity to normality—alongside MSE. Their findings showed that some estimators performing well under MSE may fail regarding outlier prevalence and

normality, limiting practical applicability. Ridge parameter estimators generating excessive outliers or deviating substantially from normality pose risks for reliable inference.

The contributions of this study are twofold. First, we propose two new ridge parameter estimators that outperform the 13 most effective existing methods in terms of MSE, outlier ratios, and normality. Second, within the PRR framework, we move beyond conventional MSE-based evaluation by incorporating outlier ratios and normality as additional criteria and systematically assessing all estimators under this extended framework. To this end, an extensive Monte Carlo simulation study is conducted across 180 scenarios, varying the number of predictors (p), sample size (n), correlation among regressors (ρ), and model intercept (β_0). Thus, these additional criteria—applied for the first time in the PRR context—capture distributional properties more effectively than the mean and standard deviation measures considered by Kibria et al. [25], and lead to the identification of two new estimators that consistently outperform existing methods.

The remainder of the paper is organized as follows. Section 2 presents the PRR methodology, the proposed ridge parameter estimators, and the additional performance criteria. Section 3 reports the Monte Carlo simulation study evaluating estimator performance in terms of MSE, outlier behavior, and normality. Section 4 provides two real-data applications supporting the simulation results. Finally, Section 5 summarizes and discusses the main findings and outlines directions for future research.

2. Methodology

This section first presents the methodological framework of the PR and PRR models. It then introduces the two new ridge parameter estimators proposed in this study, along with existing estimators available in the literature. Finally, the performance evaluation criteria employed in the Monte Carlo simulation study are described.

2.1. Poisson Regression Model

In applied studies, it is common to encounter situations where the response variable consists of non-negative integer values. For such count data structures, PR is one of the most widely used modeling approaches. Accordingly, the response variable y_i is assumed to follow a Poisson distribution with parameter $\mu_i > 0$, whose probability mass function is given by

$$f(y_i) = \frac{e^{-\mu_i}\mu_i^{y_i}}{y_i!}, \qquad y_i = 0,\ 1,\ 2, \cdots; i = 1,\ 2, \cdots, n. \tag{3}$$

A fundamental property of the Poisson distribution is the equality of its mean and variance, that is, $E(y_i) = Var(y_i) = \mu_i$. In the PR framework, the canonical link function is used to express the relationship between the conditional mean of the response variable and the predictors in a linear form. Specifically, the logarithmic link function is defined as

$$g(\mu_i) = \ln(\mu_i) = \mathbf{x_i'}\boldsymbol{\beta} \tag{4}$$

where $\mathbf{x_i}$ denotes the ith row of the design matrix $\mathbf{X}$. The use of the logarithmic link function ensures that the estimated mean responses remain strictly positive. Equation (4) can equivalently be expressed using the inverse link function as follows [27]:

$$\mu_i = g^{-1}\left(\mathbf{x_i'}\boldsymbol{\beta}\right) = \exp\left(\mathbf{x_i'}\boldsymbol{\beta}\right) \tag{5}$$

The parameters of the PR model are typically estimated using the ML method. The ML estimator can be obtained via the iteratively weighted least squares (IWLS) algorithm as:

$$\hat{\boldsymbol{\beta}}_{\mathbf{ML}} = \left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}\right)^{-1}\mathbf{X}'\hat{\mathbf{W}}\hat{\mathbf{z}} \tag{6}$$

where $\hat{\mathbf{W}} = diag(\hat{\mu}_i)$ denotes the weight matrix and $\hat{z}_i = \log(\hat{\mu}_i) + (y_i - \hat{\mu}_i)/\hat{\mu}_i$. The quantities $\hat{\mathbf{W}}$ and $\hat{\mathbf{z}}$ are obtained through the Fisher scoring procedure.

The covariance matrix and asymptotic MSE of the ML estimator $\hat{\boldsymbol{\beta}}_{\mathbf{ML}}$ are given by

$$cov(\hat{\boldsymbol{\beta}}_{\mathbf{ML}}) = \left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}\right)^{-1} \tag{7}$$

$$MSE(\hat{\boldsymbol{\beta}}_{\mathbf{ML}}) = E\left[(\hat{\boldsymbol{\beta}}_{\mathbf{ML}} - \boldsymbol{\beta})'(\hat{\boldsymbol{\beta}}_{\mathbf{ML}} - \boldsymbol{\beta})\right] = tr\left[\left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}\right)^{-1}\right] = \sum_{j=1}^{p+1}\frac{1}{\lambda_j} \tag{8}$$

where λ_j denotes the jth eigenvalue of $\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}$ [17].

2.2. Poisson Ridge Regression

When a high degree of multicollinearity exists among the predictors, the weighted information matrix $\left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}\right)$ becomes ill-conditioned. In such cases, the matrix approaches singularity and its determinant tends toward zero. Consequently, the variances of the ML estimates become excessively large, leading to unstable parameter estimates.

To address this issue, Mansson and Shukur [11] proposed the PRR model, which extends LRR to the PR framework. The PRR estimator is defined as

$$\hat{\boldsymbol{\beta}}_{\mathbf{PRR}} = \left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X} + k\mathbf{I}\right)^{-1}\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}\hat{\boldsymbol{\beta}}_{\mathbf{ML}} = \mathbf{Z}\hat{\boldsymbol{\beta}}_{\mathbf{ML}} \tag{9}$$

where $\mathbf{Z} = \left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X} + k\mathbf{I}\right)^{-1}\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}$ is the transformation matrix.

The asymptotic MSE of the PRR estimator can be derived as

$$\begin{aligned} MSE(\hat{\boldsymbol{\beta}}_{\mathbf{PRR}}) &= E\left[(\hat{\boldsymbol{\beta}}_{\mathbf{PRR}} - \boldsymbol{\beta})'(\hat{\boldsymbol{\beta}}_{\mathbf{PRR}} - \boldsymbol{\beta})\right] \\ &= E\left[(\hat{\boldsymbol{\beta}}_{\mathbf{ML}} - \boldsymbol{\beta})'\mathbf{Z}'\mathbf{Z}(\hat{\boldsymbol{\beta}}_{\mathbf{ML}} - \boldsymbol{\beta})\right] + (\mathbf{Z}\boldsymbol{\beta} - \boldsymbol{\beta})'(\mathbf{Z}\boldsymbol{\beta} - \boldsymbol{\beta}) \\ &= tr\left(\mathbf{Z}'\mathbf{Z}\, Var(\hat{\boldsymbol{\beta}}_{\mathrm{ML}})\right) + \boldsymbol{\beta}'^{(\mathbf{Z}-\mathbf{I})'}(\mathbf{Z} - \mathbf{I})\boldsymbol{\beta} \\ &= tr\left[\left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X} + k\mathbf{I}\right)^{-2}\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}\right] + k^2\,\boldsymbol{\beta}'^{\left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}+k\mathbf{I}\right)^{-2}}\boldsymbol{\beta} \\ &= \sum_{j=1}^{p+1}\frac{\lambda_j}{(\lambda_j + k)^2} + k^2\,\boldsymbol{\beta}'\left(\mathbf{X}'\hat{\mathbf{W}}\mathbf{X} + k\mathbf{I}\right)^{-2}\boldsymbol{\beta}. \end{aligned} \tag{10}$$

In Equation (10), the first term represents the variance component of the parameter estimates, while the second term corresponds to the squared bias induced by ridge regularization. As in LRR, the primary objective is to select an optimal k such that the reduction in variance outweighs the increase in bias. With an appropriate choice of k, a smaller MSE than that of the ML can be achieved [6].

2.3. Proposed Shrinkage Estimators

Since Hoerl and Kennard [6], numerous estimators for the ridge parameter k have been developed, particularly in the LRR context. Many of these have been adapted to generalized linear models, including PRR. In this study, 13 ridge parameter estimators that are frequently used in PRR-based Monte Carlo simulation studies and reported to perform well are considered.

In this context, the pioneering study by Mansson and Shukur [11] introduced the PRR model and compared eight estimators originally proposed for LRR using MSE. Under various scenarios, the estimator k_1, proposed by Muniz and Kibria [28], was shown to substantially reduce MSE. This estimator is defined as

$$k_1 = max\left(\frac{1}{m_j}\right)$$

where $m_j = \sqrt{\hat{\sigma}^2/\hat{a}_j^2}$, $\hat{\sigma}^2$ denotes the variance estimate obtained from the PRR model ($\hat{\sigma}^2 = \sum_{i=1}^{n}(y_i - \hat{\mu}_i)^2/(n-p-1)$) and $\hat{\alpha}_j$ represents the jth element of the transformed coefficient vector $\mathbf{T}'\boldsymbol{\beta}_{\mathbf{ML}}$. Here, $\mathbf{T}$ denotes the matrix of eigenvectors of $\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}$. It should be noted that, unlike the LRR model, the PRR model does not include an explicit error variance parameter; therefore, $\hat{\sigma}^2$ is taken to be equal to one in the PRR framework.

Kibria et al. [29] investigated the performance of 16 different estimators for the zero-inflated PRR model using the MSE and mean absolute error criteria, and reported that the following three estimators exhibited the best performance under both criteria:

$$k_2 = max\left(\frac{1}{q_j}\right), \quad k_3 = \prod_{j=0}^{p}\left(\frac{1}{q_j}\right)^{\frac{1}{p}}, \quad k_4 = med\left(\frac{1}{q_j}\right)$$

where

$$q_j = \frac{\lambda_{max}}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{a}_j^2}.$$

The same estimators were later adapted directly to the PRR model and re-evaluated by Kibria et al. [25]. Their findings suggest that k_1 should be preferred when multicollinearity among the explanatory variables is strong, whereas k_3 and k_4 are more suitable under weak multicollinearity in terms of MSE, mean, and standard deviation criteria.

Kaçıranlar and Dawoud [30] compared the performance of three ridge parameter estimators for PRR and negative binomial ridge regression models using predictive MSE. Their results indicate that the estimator k_5, originally proposed by Hoerl et al. [31] and defined as

$$k_5 = \frac{p\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}$$

exhibits the most efficient performance in a large proportion of simulation scenarios.

Algamal and Alanaz [32] examined the MSE performance of 20 ridge parameter estimators proposed or adapted for the PRR model and recommended the use of the following two estimators:

$$k_6 = max(m_j)$$
$$k_7 = max(q_j)$$

Lukman et al. [17] proposed two new ridge parameter estimators and compared their MSE performance under different estimation methods. The estimators

$$k_8 = max\left(0, min\left(\frac{\lambda_j}{1+2\lambda_j a_j^2}\right)\right)$$
$$k_9 = \sqrt{max\left(0, min\left(\frac{\lambda_i}{1+2\lambda_j a_j^2}\right)\right)}$$

were found to perform efficiently.

Oghenekevwe et al. [33] conducted a comprehensive MSE-based comparison by adapting 50 ridge parameter estimators to the PRR model. Among these, the estimators k_{10} and k_{11}, originally proposed for the LRR model by Lukman and Ayinde [9] and Muniz and Kibria [28], were identified as performing effectively under many scenarios and are therefore included in the present study:

$$k_{10} = med\left(\frac{\lambda_j}{(n-p)+\lambda_j\hat{a}_j^2}\right),$$
$$k_{11} = \left(\prod_{j=1}^{p}\frac{\lambda_j}{(n-p)+\lambda_j\hat{a}_j^2}\right)^{\frac{1}{p}}$$

Mermi et al. [26] evaluated the performance of 366 ridge parameter estimators proposed in the LRR literature using not only MSE but also outlier ratios and normality criteria. In that study, the estimators k_{12} and k_{13}, originally proposed by Alkhamisi et al. [34] and Lukman et al. [35], were identified as the most effective across all three criteria:

$$k_{12} = max\left(\frac{\hat{\sigma}^2}{\hat{\beta}_j}\right)$$
$$k_{13} = \frac{1}{p}\sum_{j=1}^{p} \frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2+\lambda_j\hat{\alpha}_{max}^2}$$

In addition, and unlike existing approaches, two new ridge parameter estimators are proposed by combining robust dispersion measures of the transformed regression coefficients with eigenvalue-based information, the number of predictors, and sample size adjustments. The motivation is to incorporate structural factors that influence the behavior and stability of ridge-type estimators. In particular, the eigenvalue structure of $\mathbf{X}'\mathbf{W}\mathbf{X}$ plays a central role in determining the severity of multicollinearity, and several estimators rely on eigenvalue-based measures to regulate the magnitude of the ridge parameter [7,11].

At the same time, data-dependent characteristics of the regression coefficients may provide additional information for determining an appropriate level of shrinkage. Therefore, robust dispersion measures of the transformed regression coefficients are incorporated to introduce data-adaptive scaling that reflects coefficient variability while limiting the influence of extreme values. This approach aligns with the philosophy of data-driven ridge parameter estimation [9].

Accordingly, the proposed estimators jointly incorporate eigenvalue information, robust dispersion measures, model dimensionality, and sample-size adjustments to regulate the magnitude and stability of the ridge parameter under severe multicollinearity. In addition, logarithmic and exponential transformations are used to moderate the influence of large eigenvalues or dispersion, contributing to more stable shrinkage behavior across different settings.

These estimators, denoted by k_{14} and k_{15}, are defined as

$$k_{14} = \frac{\sqrt{exp(1+\hat{\alpha}_{MAD})}log(\lambda_{mean})p}{(1+\hat{\alpha}_{MAD})\left(1+\sqrt{\log(n)}\right)}$$
$$k_{15} = \frac{\exp(1+|\hat{\alpha}|_{min})log\left(\lambda_{max}^2\right)p}{\log(n)}$$

where α_{mad} denotes the median absolute deviation (MAD) of the transformed regression coefficients, defined as median $\left(\left|\hat{\alpha}_j - \text{median}\left(\hat{\alpha}_j\right)\right|\right)$.

2.4. Additional Skewness Criteria Beyond MSE

Outliers constitute a critical factor that can substantially undermine the reliability and statistical power of empirical analyses and therefore should not be overlooked. In particular, ridge parameter estimators that occasionally generate extremely small or large values of the ridge parameter k across simulation replications may produce unstable shrinkage effects. Extremely small values of k result in estimates close to the ML estimator under multicollinearity, while excessively large values may cause over-shrinkage of the regression coefficients. Both situations can lead to coefficient estimates that deviate substantially from their true values, thereby inflating the MSE. This behavior reflects the well-known bias-variance trade-off underlying ridge regression, where the magnitude of the ridge parameter directly controls the amount of shrinkage applied to the coefficient estimates [6, 36]. Moreover, the presence of a large number of outlying estimates is often associated with skewed and unstable sampling distributions.

It is important to note that the outlier ratio and normality criteria are computed based on the estimated ridge parameter values produced by each estimator across simulation replications, rather than on the regression coefficient estimates.

In this study, the skewness properties of ridge parameter estimators in the PRR framework are examined using the following inequality interval:

$$Med - 3 \times c \times MAD < x_i < Med + 3 \times c \times MAD, i = 1, 2, \cdots, n \tag{11}$$

where $MAD = Med(|x_i - Med(x_i)|)$, x denotes the $n \times 1$ vector of observations generated by a given ridge parameter estimator, and Med(.) represents the median operator. The constant c is a correction factor that takes the value 1.4826 under the assumption of normality, ensuring consistency of the MAD as a scale estimator [37]. Observations falling outside this interval are classified as outliers, and the corresponding ratio is used as an indicator of distributional skewness and instability.

In addition, to further assess whether the values produced by ridge parameter estimators exhibit skewed behavior, a normality conformity criterion is incorporated as a second skewness-related performance measure in the PRR model. Following Kim [38] and West et al. [39], an estimator is considered approximately normal when the absolute skewness is less than 2 and the absolute kurtosis is less than 7. Kim [38] notes that in large samples, normality tests based on z-statistics tend to reject normality even for minor deviations. Therefore, for sample sizes greater than 300, it is more appropriate to rely on skewness and kurtosis magnitudes rather than formal tests. Since this study evaluates distributions based on $R = 5000$ replications, these thresholds provide a practical guideline for detecting substantial deviations from normality.

Similarly, the threshold value of 0.05 adopted for the outlier ratio serves as a practical benchmark to identify ridge parameter estimators that frequently produce extreme values across simulation replications. Estimators generating outlier proportions above this level may indicate unstable behavior in the distribution of ridge parameter estimates. Accordingly, the outlier ratio and normality criteria together provide complementary indicators for assessing the distributional stability of ridge parameter estimators in the PRR framework.

By jointly evaluating MSE, outlier ratios, and normality conformity, the proposed framework provides a more comprehensive assessment of ridge parameter estimators, capturing not only estimation accuracy but also the distributional stability of the estimators, which is often overlooked in existing PRR studies.

3. Monte Carlo Simulation

In this section, since an analytical comparison among the estimators is not feasible, performance is evaluated via a Monte Carlo simulation study. First, the data generation process under different parameter configurations is described in detail. Subsequently, the performance of 15 ridge parameter estimators—including the two newly proposed estimators—is compared in terms of MSE, outliers, and normality.

To facilitate interpretation of the simulation results, the relative efficiency (RE) of each estimator with respect to the ML estimator is computed as

$$RE(\hat{\boldsymbol{\beta}}_{\mathbf{k_r}}) = \frac{MSE(\hat{\boldsymbol{\beta}}_{\mathbf{ML}})}{MSE(\hat{\boldsymbol{\beta}}_{\mathbf{r}})}, r = 1, 2, \cdots, 15. \tag{12}$$

Values greater than one indicate an improvement over the ML estimator.

3.1. Experimental Design

The response variable y_i is generated from a Poisson distribution with mean parameter μ_i, denoted by $Po(\mu_i)$, using pseudo-random numbers. The mean structure is specified as

$$\mu_i = \exp(\beta_0 + \beta_1 x_{1i} + \cdots + \beta_p x_{pi}), i = 1, 2, \cdots, n. \quad (13)$$

The regression coefficients are specified to satisfy the constraints $\sum_{j=1}^{p} \beta_j^2 = 1$ and $\beta_1 = \beta_2 = \cdots = \beta_p$, following Mansson and Shukur [11]. To generate explanatory variables with different levels of pairwise correlation ρ, the following data generation process is employed [40]:

$$x_{ij} = \sqrt{(1-\rho^2)} z_{ij} + \rho z_{i,p+1}, \quad i = 1, 2, \cdots, n, \quad j = 1, 2, \ldots, p. \quad (14)$$

Here, ρ denotes the correlation between any two explanatory variables, and z_{ij} represents mutually independent pseudo-random variables drawn from the standard normal distribution.

The simulation study considers four correlation levels ($\rho = 0.85, 0.90, 0.95, 0.99$), three values for the number of explanatory variables ($p = 2, 4, 8$), and five sample sizes ($n = 30, 60, 120, 250, 500$). Since decreasing the intercept β_0 reduces the average level of μ_i, three intercept values ($\beta_0 = -1, 0, 1$) are examined. Consequently, the simulation covers a total of 180 distinct scenarios.

For each scenario, the Poisson ML estimator in Equation (6), and the PRR estimators in Equation (9), corresponding to 15 different ridge parameter estimators, are computed. The simulation is conducted with $R = 5000$ replications per scenario. All parameter settings are summarized in Table 1.

Table 1. Monte Carlo simulation parameter values.

Simulation Parameters	Value(s)
p	2, 4, 8
n	30, 60, 120, 250, 500
ρ	0.85, 0.90, 0.95, 0.99
β_0	-1, 0, $+1$
R	5000

3.2. Evaluation in Terms of RE, Outliers, and Normality

In this section, the performance of the PRR model is comparatively evaluated for the ML estimator and 15 ridge parameter estimators. In addition to the widely used RE criterion defined in Equation (12), this study—for the first time in the PRR literature—conducts a multidimensional performance assessment by jointly considering outlier behavior and conformity to normality.

To evaluate the estimators with respect to outliers, the $R = 5000$ estimates obtained for each scenario are examined using the inequality in Equation (11), and observations classified as outliers are identified. The proportion of outliers is then computed. An upper threshold of 0.05 is adopted as the acceptable outlier ratio. It should be emphasized that the simulated datasets themselves do not contain outliers; rather, the outlier analysis is applied to the distribution of ridge parameter estimates across Monte Carlo replications to assess estimator stability.

Normality is assessed using the skewness and kurtosis criteria proposed by West et al. [39] and Kim [38]. Specifically, for each estimator and scenario, the distribution of the $R = 5000$ generated values is considered approximately normal if the absolute skewness is less than 2 and the absolute kurtosis is less than 7.

Tables 2–4 report the RE values, outlier ratios (OR), and normality results for all combinations of n and ρ under $B_0 = -1$, $p = 2$ (Table 2), $B_0 = 0$, $p = 2$ (Table 3), and $B_0 = 1$, $p = 2$ (Table 4). Scenarios satisfying the normality criterion are highlighted in blue, while those that do not are shown in orange. For each scenario, the estimator with the maximum RE is indicated in bold. Results for the remaining scenarios are provided in Appendix A.

From Table 2, for $B_0 = -1$ and $p = 2$, the estimator k_{14} achieves the maximum RE in 16 out of 20 cases. Under large sample sizes and severe multicollinearity (e.g., $n = 250$, $\rho = 0.99$ and $n = 500$, $\rho = 0.95,\ 0.99$), k_{15} becomes the most efficient estimator in terms of RE, with k_{14} ranking second. Although k_2 yields the maximum RE for $n = 30$ and $\rho = 0.99$, k_{14} outperforms all alternatives when all three criteria are jointly considered, as it satisfies both the outlier and normality conditions. Notably, k_{14} produces outlier ratios below 0.05 and approximately normal distributions across all scenarios in Table 2.

Across all 60 scenarios for $B_0 = -1$, k_{14} achieves the maximum RE in 55 cases and satisfies the normality criterion in 59 cases, failing only when $p = 8$, $n = 30$ and $\rho = 0.99$. Moreover, its outlier ratio remains below 0.05 in all scenarios. These findings indicate that k_{14} is the most effective estimator under $B_0 = -1$ when all criteria are jointly considered.

For $B_0 = 0$ and $p = 2$ (Table 3), k_{15} achieves the maximum RE in all scenarios except $n = 30$, $\rho = 0.99$, where k_{14} dominates across all three criteria. Importantly, k_{15} satisfies both the outlier and normality conditions in all scenarios reported in Table 3.

Across all 60 scenarios for $B_0 = 0$, k_{15} yields the maximum RE in 44 cases. In most remaining scenarios—typically involving larger p, moderate-to-large n, and high ρ—k_{13} attains higher RE values but fails to satisfy at least one distributional criterion. Therefore, when all three criteria are considered jointly, k_{15} is preferred due to its superior stability. Except for a few extreme cases (e.g., $p = 4,\ 8$, $n = 30$, $\rho = 0.99$), k_{15} consistently satisfies both distributional criteria.

Finally, for $B_0 = 1$ and $p = 2$ (Table 4), k_{15} outperforms all competing estimators in terms of RE across all combinations of n and ρ. It maintains an outlier ratio below 0.05 in all scenarios and satisfies the normality criterion in all but the most extreme case ($n = 30$, $\rho = 0.99$).

Across the 60 scenarios for $B_0 = 1$, k_{15} achieves the maximum RE in 56 cases. Moreover, it yields outlier ratios below the acceptable threshold of 0.05 in all scenarios and produces estimates consistent with normality in all but the most extreme case ($n = 30$, $\rho = 0.99$). In the remaining scenarios, joint consideration of RE, outlier ratio, and normality still favors k_{15} as the most robust estimator. Overall, these results clearly indicate that, for $B_0 = 1$, k_{15} is the most effective estimator when all three criteria are jointly considered.

The findings derived from the tables are visually summarized in Figure 1. For each value of B_0, the proportions of achieving the maximum RE across 60 scenarios are presented in Figure 1a, the overall average outlier ratios are shown in Figure 1b, and the normality ratios are reported in Figure 1c.

Table 2. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = -1$ and $p = 2$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.854	2.155	1.526	1.508	1.637	1.747	2.072	1.190	1.360	1.195	1.265	2.047	2.117	**2.521**	1.201
		OR	0.014	0.042	0.049	0.071	0.035	0.172	0.078	0.051	0.017	0.015	0.026	0.126	0.077	**0.005**	0.024
30	0.90	RE	2.270	2.726	1.806	1.804	1.893	2.374	2.684	1.253	1.497	1.275	1.319	2.404	2.810	**3.539**	1.684
		OR	0.018	0.044	0.056	0.079	0.034	0.162	0.088	0.041	0.014	0.014	0.026	0.130	0.080	**0.006**	0.025
30	0.95	RE	3.590	4.647	2.633	2.614	2.456	3.770	4.239	1.381	1.827	1.480	1.447	3.565	4.824	**6.578**	2.886
		OR	0.028	0.055	0.054	0.078	0.032	0.159	0.085	0.035	0.008	0.011	0.022	0.138	0.084	**0.006**	0.030
30	0.99	RE	32.217	**40.367**	22.882	18.773	3.822	15.519	10.612	1.596	3.995	2.995	1.994	21.244	25.380	**39.742**	12.958
		OR	0.066	**0.124**	0.101	0.132	0.062	0.168	0.086	0.031	0.004	0.016	0.027	0.130	0.074	**0.008**	0.101
60	0.85	RE	1.254	1.321	1.129	1.117	1.298	1.128	1.452	1.091	1.142	1.069	1.116	1.508	1.501	**1.628**	0.785
		OR	0.008	0.024	0.025	0.034	0.017	0.171	0.068	0.035	0.014	0.012	0.032	0.104	0.069	**0.009**	0.019
60	0.90	RE	1.339	1.424	1.177	1.170	1.457	1.435	1.781	1.132	1.199	1.092	1.142	1.774	1.835	**2.075**	1.024
		OR	0.008	0.024	0.028	0.034	0.020	0.173	0.066	0.027	0.010	0.013	0.025	0.109	0.067	**0.008**	0.017
60	0.95	RE	1.781	2.016	1.453	1.456	1.925	2.231	2.804	1.243	1.397	1.196	1.218	2.436	3.037	**3.929**	1.967
		OR	0.014	0.031	0.039	0.053	0.018	0.163	0.057	0.028	0.009	0.010	0.024	0.135	0.058	**0.008**	0.033
60	0.99	RE	9.997	15.270	6.597	5.952	3.357	8.371	8.048	1.547	2.557	2.001	1.523	9.551	15.601	**26.202**	9.288
		OR	0.047	0.082	0.073	0.112	0.015	0.166	0.064	0.010	0.001	0.010	0.022	0.151	0.055	**0.006**	0.059
120	0.85	RE	1.093	1.109	1.042	1.037	1.137	0.965	1.219	1.043	1.062	1.028	1.053	1.193	1.239	**1.270**	0.653
		OR	0.002	0.011	0.011	0.018	0.012	0.166	0.055	0.024	0.010	0.018	0.032	0.057	0.058	**0.010**	0.009
120	0.90	RE	1.128	1.146	1.062	1.058	1.237	1.172	1.410	1.069	1.093	1.039	1.067	1.370	1.441	**1.574**	0.888
		OR	0.005	0.013	0.018	0.024	0.009	0.166	0.054	0.020	0.006	0.015	0.021	0.082	0.058	**0.008**	0.013
120	0.95	RE	1.263	1.308	1.136	1.130	1.510	1.588	1.945	1.136	1.186	1.079	1.103	1.875	2.036	**2.525**	1.614
		OR	0.005	0.015	0.025	0.033	0.010	0.167	0.049	0.019	0.007	0.010	0.017	0.112	0.056	**0.007**	0.038
120	0.99	RE	3.729	5.035	2.595	2.513	2.858	4.922	5.963	1.430	1.877	1.474	1.280	4.988	8.992	**16.567**	8.664
		OR	0.029	0.056	0.065	0.069	0.005	0.159	0.032	0.011	0.003	0.010	0.012	0.164	0.047	**0.006**	0.040
250	0.85	RE	1.038	1.042	1.017	1.015	1.062	0.974	1.105	1.020	1.028	1.012	1.024	1.070	1.118	**1.125**	0.666
		OR	0.001	0.004	0.005	0.010	0.009	0.127	0.046	0.022	0.008	0.024	0.034	0.033	0.039	**0.009**	0.003
250	0.90	RE	1.050	1.054	1.023	1.022	1.109	1.068	1.201	1.033	1.041	1.017	1.030	1.127	1.226	**1.283**	0.882
		OR	0.002	0.008	0.011	0.022	0.008	0.122	0.039	0.016	0.007	0.017	0.017	0.052	0.041	**0.006**	0.016
250	0.95	RE	1.100	1.108	1.046	1.045	1.257	1.356	1.528	1.068	1.085	1.034	1.049	1.358	1.523	**1.782**	1.565
		OR	0.002	0.009	0.017	0.025	0.006	0.148	0.037	0.012	0.005	0.009	0.015	0.080	0.041	**0.010**	0.028
250	0.99	RE	1.857	2.114	1.464	1.439	2.243	3.138	3.814	1.285	1.453	1.211	1.144	3.192	4.644	9.171	**9.505**
		OR	0.014	0.023	0.050	0.031	0.002	0.148	0.023	0.005	0.001	0.012	0.008	0.163	0.039	0.005	**0.067**
500	0.85	RE	1.018	1.019	1.008	1.007	1.030	0.987	1.052	1.010	1.013	1.006	1.012	1.030	1.062	**1.062**	0.749
		OR	0.003	0.004	0.003	0.008	0.006	0.094	0.039	0.018	0.012	0.033	0.024	0.025	0.035	**0.010**	0.003
500	0.90	RE	1.023	1.024	1.010	1.009	1.054	1.077	1.102	1.015	1.020	1.008	1.015	1.049	1.115	**1.147**	0.930
		OR	0.004	0.005	0.009	0.015	0.008	0.076	0.040	0.015	0.007	0.012	0.018	0.030	0.036	**0.009**	0.014

Table 2. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
500	0.95	RE	1.046	1.048	1.020	1.019	1.129	1.226	1.275	1.033	1.041	1.016	1.024	1.132	1.262	1.409	**1.462**
		OR	0.004	0.007	0.015	0.024	0.007	0.107	0.027	0.010	0.004	0.015	0.014	0.063	0.030	0.009	**0.020**
500	0.99	RE	1.333	1.393	1.156	1.147	1.736	2.567	2.712	1.166	1.231	1.098	1.077	2.487	2.659	4.764	**8.449**
		OR	0.005	0.012	0.036	0.041	0.008	0.150	0.020	0.004	0.001	0.012	0.010	0.135	0.030	0.004	**0.074**

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

Table 3. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = 0$ and $p = 2$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.246	1.285	1.179	1.207	1.327	1.222	1.558	1.111	1.161	1.212	1.140	1.645	2.127	2.133	**2.489**
		OR	0.057	0.066	0.032	0.075	0.034	0.161	0.115	0.054	0.054	0.238	0.133	0.118	0.108	0.006	**0.028**
30	0.90	RE	1.428	1.505	1.340	1.382	1.484	1.479	1.774	1.164	1.252	1.274	1.170	1.894	2.983	3.008	**3.969**
		OR	0.064	0.068	0.034	0.073	0.038	0.159	0.110	0.055	0.055	0.228	0.128	0.120	0.108	0.008	**0.036**
30	0.95	RE	1.965	2.178	1.825	1.911	1.852	2.051	2.286	1.280	1.478	1.453	1.231	2.488	5.859	5.926	**7.961**
		OR	0.083	0.084	0.037	0.087	0.024	0.170	0.131	0.068	0.076	0.225	0.123	0.137	0.105	0.009	**0.032**
30	0.99	RE	14.829	22.976	14.214	16.841	2.755	8.016	7.129	1.446	2.567	2.689	1.474	3.728	44.540	**66.951**	37.016
		OR	0.241	0.248	0.041	0.131	0.000	0.236	0.242	0.000	0.000	0.201	0.080	0.110	0.108	**0.008**	0.004
60	0.85	RE	1.081	1.086	1.041	1.055	1.135	1.012	1.348	1.041	1.058	1.122	1.069	1.182	1.520	1.441	**1.985**
		OR	0.046	0.044	0.016	0.042	0.025	0.161	0.086	0.046	0.046	0.248	0.172	0.074	0.091	0.009	**0.017**
60	0.90	RE	1.125	1.135	1.069	1.088	1.204	1.127	1.505	1.063	1.089	1.147	1.083	1.326	1.917	1.716	**2.836**
		OR	0.054	0.053	0.021	0.042	0.029	0.158	0.082	0.051	0.051	0.254	0.184	0.084	0.088	0.009	**0.020**
60	0.95	RE	1.289	1.320	1.199	1.229	1.408	1.438	1.781	1.134	1.194	1.213	1.109	1.867	3.341	2.801	**6.991**
		OR	0.070	0.062	0.033	0.061	0.046	0.156	0.089	0.063	0.065	0.239	0.159	0.111	0.090	0.008	**0.038**
60	0.99	RE	3.863	4.933	3.637	3.938	2.349	3.313	3.313	1.405	1.862	1.791	1.222	2.742	22.564	22.082	**67.940**
		OR	0.161	0.154	0.035	0.115	0.000	0.189	0.166	0.068	0.105	0.214	0.113	0.101	0.083	0.007	**0.002**
120	0.85	RE	1.030	1.032	1.012	1.019	1.054	0.986	1.173	1.015	1.022	1.080	1.035	1.053	1.226	1.176	**1.427**
		OR	0.039	0.038	0.011	0.028	0.020	0.151	0.072	0.039	0.040	0.238	0.211	0.053	0.074	0.013	**0.010**
120	0.90	RE	1.049	1.052	1.021	1.031	1.086	1.022	1.295	1.025	1.035	1.089	1.041	1.100	1.404	1.295	**1.837**
		OR	0.049	0.043	0.018	0.032	0.023	0.164	0.068	0.048	0.050	0.246	0.220	0.059	0.076	0.011	**0.015**
120	0.95	RE	1.111	1.119	1.056	1.074	1.187	1.195	1.565	1.055	1.078	1.125	1.056	1.284	2.061	1.721	**3.607**
		OR	0.060	0.052	0.033	0.051	0.046	0.162	0.064	0.055	0.056	0.231	0.202	0.076	0.073	0.013	**0.028**
120	0.99	RE	1.843	2.008	1.699	1.765	1.809	2.016	2.215	1.268	1.438	1.382	1.110	2.595	10.562	8.083	**48.156**
		OR	0.092	0.081	0.041	0.103	0.038	0.164	0.078	0.097	0.106	0.238	0.167	0.148	0.060	0.008	**0.024**
250	0.85	RE	1.012	1.013	1.004	1.007	1.022	0.932	1.078	1.006	1.009	1.045	1.016	1.019	1.097	1.074	**1.177**
		OR	0.038	0.034	0.006	0.014	0.008	0.121	0.051	0.039	0.038	0.227	0.221	0.037	0.058	0.013	**0.007**
250	0.90	RE	1.020	1.021	1.007	1.012	1.037	1.015	1.154	1.010	1.014	1.057	1.021	1.033	1.188	1.127	**1.349**
		OR	0.043	0.037	0.008	0.018	0.018	0.147	0.053	0.043	0.044	0.232	0.230	0.044	0.060	0.015	**0.007**

Table 3. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
250	0.95	RE	1.044	1.046	1.017	1.026	1.081	1.069	1.363	1.022	1.031	1.075	1.028	1.083	1.470	1.292	**1.894**
		OR	0.051	0.037	0.018	0.027	0.035	0.169	0.049	0.050	0.049	0.229	0.227	0.046	0.058	0.013	**0.015**
250	0.99	RE	1.297	1.320	1.194	1.225	1.434	1.558	1.889	1.138	1.200	1.199	1.054	2.064	4.932	3.511	**15.822**
		OR	0.066	0.051	0.050	0.086	0.077	0.161	0.043	0.074	0.075	0.223	0.198	0.109	0.056	0.011	**0.048**
500	0.85	RE	1.006	1.006	1.002	1.003	1.011	1.026	1.033	1.003	1.004	1.023	1.008	1.009	1.048	1.038	**1.092**
		OR	0.030	0.025	0.004	0.011	0.008	0.081	0.044	0.031	0.031	0.235	0.234	0.030	0.057	0.016	**0.005**
500	0.90	RE	1.009	1.010	1.003	1.005	1.018	0.995	1.076	1.005	1.007	1.032	1.011	1.015	1.095	1.063	**1.162**
		OR	0.037	0.026	0.005	0.008	0.007	0.107	0.038	0.037	0.036	0.222	0.222	0.035	0.049	0.015	**0.006**
500	0.95	RE	1.020	1.021	1.006	1.011	1.037	1.004	1.207	1.010	1.014	1.048	1.014	1.033	1.231	1.137	**1.374**
		OR	0.046	0.032	0.018	0.018	0.024	0.161	0.033	0.045	0.045	0.216	0.215	0.037	0.057	0.016	**0.010**
500	0.99	RE	1.131	1.138	1.064	1.085	1.225	1.359	1.725	1.064	1.092	1.118	1.029	1.359	2.665	2.041	**5.512**
		OR	0.055	0.044	0.048	0.066	0.073	0.155	0.018	0.065	0.066	0.213	0.211	0.076	0.043	0.014	**0.035**

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

Table 4. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = 1$ and $p = 2$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.132	1.142	1.048	1.047	1.291	1.348	1.694	1.072	1.097	1.116	1.129	1.450	1.694	**1.832**	1.759
		OR	0.007	0.016	0.013	0.002	0.010	0.167	0.095	0.013	0.005	0.029	0.070	0.101	0.082	**0.007**	0.012
30	0.90	RE	1.225	1.250	1.085	1.083	1.450	1.653	2.196	1.107	1.151	1.183	1.186	1.786	2.197	2.390	**2.468**
		OR	0.016	0.032	0.018	0.005	0.016	0.169	0.104	0.009	0.002	0.026	0.065	0.118	0.082	0.009	**0.007**
30	0.95	RE	1.571	1.687	1.207	1.207	1.858	2.793	4.036	1.184	1.299	1.356	1.314	2.563	3.820	4.382	**4.945**
		OR	0.024	0.048	0.025	0.015	0.014	0.168	0.106	0.003	0.000	0.012	0.053	0.138	0.082	0.007	**0.003**
30	0.99	RE	7.765	11.170	3.574	3.477	3.403	10.491	12.743	1.465	2.309	2.314	1.753	9.419	23.653	**31.159**	26.028
		OR	0.035	0.079	0.114	0.103	0.000	0.171	0.085	0.004	0.000	0.002	0.041	0.147	0.081	**0.006**	0.041
60	0.85	RE	1.040	1.039	1.013	1.015	1.112	1.042	1.284	1.029	1.036	1.043	1.052	1.128	1.281	1.326	**1.396**
		OR	0.003	0.009	0.003	0.000	0.006	0.160	0.066	0.008	0.003	0.011	0.094	0.071	0.062	0.010	**0.011**
60	0.90	RE	1.068	1.068	1.022	1.023	1.186	1.236	1.566	1.045	1.058	1.069	1.077	1.243	1.552	1.569	**1.892**
		OR	0.011	0.021	0.004	0.000	0.009	0.169	0.079	0.007	0.003	0.010	0.065	0.077	0.068	0.010	**0.005**
60	0.95	RE	1.179	1.195	1.055	1.050	1.406	1.776	2.471	1.088	1.124	1.155	1.141	1.691	2.498	2.471	**3.855**
		OR	0.021	0.040	0.010	0.001	0.016	0.168	0.084	0.002	0.002	0.008	0.056	0.111	0.060	0.008	**0.003**
60	0.99	RE	2.825	3.556	1.554	1.531	2.658	7.181	12.253	1.296	1.602	1.733	1.424	4.450	13.752	15.545	**25.580**
		OR	0.020	0.053	0.043	0.037	0.000	0.178	0.072	0.001	0.000	0.002	0.030	0.172	0.057	0.009	**0.011**
120	0.85	RE	1.015	1.014	1.005	1.007	1.046	0.836	1.136	1.012	1.015	1.018	1.023	1.038	1.124	1.139	**1.168**
		OR	0.001	0.006	0.000	0.000	0.002	0.170	0.054	0.006	0.001	0.002	0.114	0.044	0.057	0.010	**0.016**
120	0.90	RE	1.025	1.023	1.008	1.010	1.077	1.046	1.293	1.020	1.024	1.029	1.033	1.068	1.263	1.242	**1.431**
		OR	0.010	0.023	0.001	0.001	0.004	0.179	0.065	0.003	0.003	0.001	0.119	0.047	0.057	0.012	**0.005**

Table 4. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
120	0.95	RE	1.067	1.066	1.020	1.020	1.189	1.315	1.723	1.044	1.056	1.070	1.064	1.233	1.756	1.658	**2.533**
		OR	0.019	0.032	0.003	0.002	0.016	0.162	0.066	0.002	0.003	0.002	0.075	0.076	0.053	0.011	**0.001**
120	0.99	RE	1.615	1.765	1.165	1.150	1.976	4.333	7.843	1.178	1.297	1.392	1.222	2.900	7.308	7.474	**18.765**
		OR	0.022	0.049	0.010	0.008	0.039	0.177	0.052	0.000	0.000	0.001	0.022	0.158	0.043	0.009	**0.003**
250	0.85	RE	1.007	1.006	1.002	1.003	1.021	0.821	1.080	1.006	1.007	1.009	1.011	1.015	1.065	1.066	**1.082**
		OR	0.002	0.006	0.000	0.006	0.004	0.202	0.033	0.004	0.002	0.000	0.207	0.029	0.039	0.011	**0.012**
250	0.90	RE	1.011	1.009	1.003	1.005	1.035	0.819	1.147	1.009	1.011	1.014	1.016	1.026	1.126	1.113	**1.208**
		OR	0.015	0.023	0.001	0.005	0.004	0.220	0.060	0.001	0.003	0.000	0.228	0.039	0.037	0.012	**0.001**
250	0.95	RE	1.026	1.024	1.008	1.010	1.084	1.105	1.383	1.021	1.025	1.032	1.030	1.073	1.366	1.292	**1.731**
		OR	0.020	0.028	0.001	0.008	0.010	0.183	0.048	0.003	0.006	0.000	0.108	0.055	0.043	0.014	**0.000**
250	0.99	RE	1.215	1.232	1.060	1.053	1.522	2.362	3.702	1.104	1.147	1.192	1.107	2.017	3.823	3.598	**9.797**
		OR	0.020	0.035	0.001	0.000	0.048	0.150	0.030	0.000	0.000	0.002	0.019	0.118	0.038	0.010	**0.000**
500	0.85	RE	1.003	1.003	1.001	1.001	1.009	0.597	1.030	1.002	1.003	1.004	1.005	1.006	1.027	1.029	**1.032**
		OR	0.003	0.006	0.000	0.004	0.002	0.252	0.016	0.002	0.002	0.000	0.295	0.027	0.030	0.013	**0.004**
500	0.90	RE	1.005	1.004	1.002	1.002	1.017	0.826	1.076	1.005	1.005	1.007	1.008	1.012	1.065	1.056	**1.107**
		OR	0.017	0.024	0.000	0.005	0.003	0.269	0.035	0.002	0.004	0.000	0.288	0.033	0.035	0.012	**0.000**
500	0.95	RE	1.012	1.010	1.004	1.005	1.041	0.942	1.202	1.011	1.012	1.016	1.015	1.030	1.181	1.145	**1.350**
		OR	0.020	0.029	0.001	0.009	0.007	0.212	0.036	0.002	0.006	0.000	0.158	0.042	0.034	0.014	**0.000**
500	0.99	RE	1.092	1.090	1.028	1.028	1.268	1.693	2.278	1.059	1.076	1.095	1.054	1.364	2.308	2.182	**4.918**
		OR	0.024	0.040	0.002	0.001	0.053	0.150	0.018	0.000	0.004	0.000	0.003	0.093	0.027	0.007	**0.002**

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

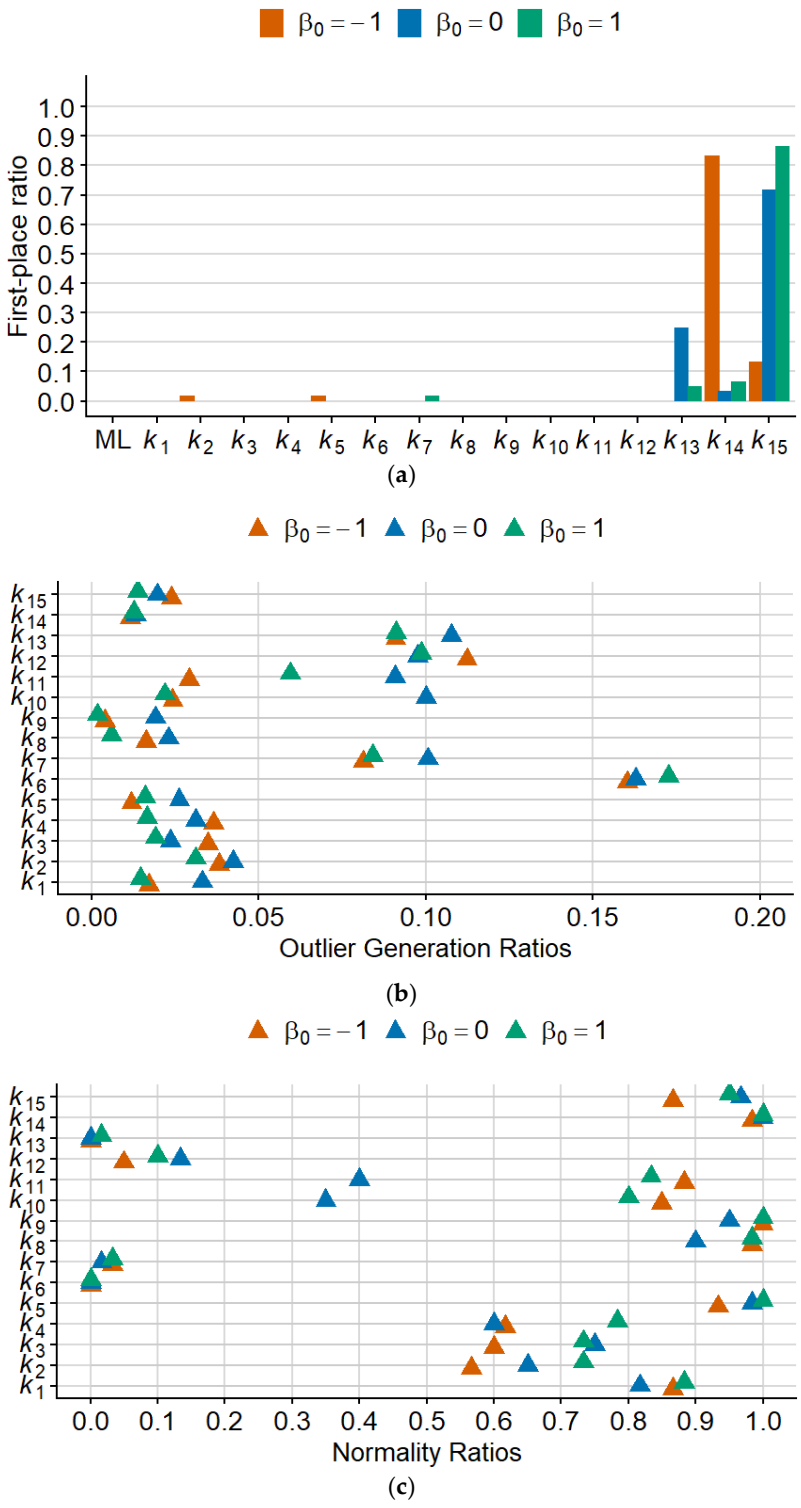


Figure 1. First-place ratios in terms of RE (**a**), overall average outlier ratios (**b**), and normality ratios (**c**) of the ML and ridge parameter estimators for $B_0 = -1,\ 0, 1$.

An examination of Figure 1a clearly indicates that the most RE-efficient estimators are k_{14} for $B_0 = -1$, and k_{15} for both $B_0 = 0$ and $B_0 = 1$. As illustrated in Figure 1b, the outlier ratios of these estimators remain below the acceptable threshold of 0.05 across all corresponding scenarios. Moreover, Figure 1c demonstrates that the normality conformity ratios of k_{14} and k_{15} are very close to unity, indicating strong performance with respect to the normality criterion.

In contrast, although Figure 1a shows that k_{13} attains the highest RE in some scenarios for $B_0 = 0$ and $B_0 = 1$, a joint evaluation of the outlier and normality criteria reveals that this estimator exhibits outlier ratios well above 0.05 for all values of B_0 and produces estimates that are almost never consistent with normality. Therefore, despite its occasional RE superiority, the estimator proposed by Lukman et al. [35] is not recommended for use within the PRR framework.

The remaining estimators can be summarized as follows. The ML estimator yields the lowest RE values in the vast majority of the 180 simulation scenarios and consistently underperforms relative to the PRR-based estimators. The estimators k_2, k_5, and k_7 achieve the maximum RE only once across all scenarios; however, even in those cases, they fail to satisfy at least one of the outlier or normality criteria. Furthermore, the estimators k_1, k_3, k_4, k_6, k_8, k_9, k_{10}, k_{11}, and k_{12} do not attain RE optimality in any scenario.

With respect to the outlier criterion, the estimators k_1, k_2, k_3, k_4, k_5, k_8, k_9, and k_{10} exhibit outlier ratios below 0.05 in the majority of scenarios. Conversely, k_6, k_7, k_{11}, k_{12}, and k_{13} demonstrate relatively poor performance, frequently producing outlier ratios exceeding the 0.05 threshold. Among these, k_6 can be regarded as the least successful estimator under this criterion, as it often yields outlier ratios above 0.10.

Regarding the normality criterion, k_5, k_8, and k_9 satisfy the normality condition in most scenarios, whereas k_6, k_7, k_{12}, and k_{13} produce approximately normal estimates only in a very limited number of cases. The estimators k_1, k_2, k_3, k_4, k_{10}, and k_{11} can be considered to exhibit moderate performance in terms of normality.

In addition, the overall trends in RE values were examined with respect to changes in B_0, p, n, and ρ. As the correlation level ρ increases, the RE of the k_1–k_{12} estimators decreases regardless of the values of B_0, n, and p. In contrast, the RE of k_{14} for $B_0 = -1$, and that of k_{15} and k_{13} for $B_0 = 0$ and $B_0 = 1$, increases as ρ increases. These estimators were already identified as MSE-efficient for the corresponding values of B_0. Thus, although the general tendency is toward decreasing RE under stronger multicollinearity, efficient estimators such as k_{14} and k_{15} exhibit greater robustness to high correlation levels.

In terms of the intercept parameter, a substantial increase in RE is observed for all estimators when moving from $B_0 = -1$ to $B_0 = 0$ and $B_0 = 1$. However, when transitioning from $B_0 = 0$ to $B_0 = 1$, the RE values of k_1, k_2, k_3, k_4, k_{13}, k_{14}, and k_{15} decrease, in contrast to the remaining estimators. As the number of regressors p increases, the RE values of the k_1–k_5 and k_8–k_{13} estimators decrease, whereas an increasing trend is observed for k_7. For the remaining estimators, RE decreases when moving from $p = 2$ to $p = 4$, followed by an increase when moving from $p = 4$ to $p = 8$. Finally, as the sample size n increases, a general improvement in MSE performance is observed for the ML estimator and all ridge parameter estimators, regardless of the values of B_0, p, and ρ.

In conclusion, when RE, outlier, and normality criteria are jointly considered, k_{14} for $B_0 = -1$ and k_{15} for $B_0 = 0$ and $B_0 = 1$ are identified as the most effective estimators within the PRR framework.

4. Empirical Applications

In this section, the performance of the proposed estimators is evaluated using two real-life datasets. The first application considers a dataset with a relatively small number of predictors, while the second application examines a more complex setting with a larger number of predictors and a stronger multicollinearity structure.

4.1. Application 1: Recreation Demand Dataset

In this section, the performance of the Poisson ML estimator and the ridge parameter estimators is empirically compared using the low-dimensional Recreation Demand dataset.

The analysis is based on a dataset presented in Cameron and Trivedi [41], which has also been used by Mansson and Kibria [42] and Abonazel and Dawoud [43].

The dataset originates from a survey conducted in 1980 to measure the demand for recreational boating trips to Somerville Lake in the East Texas region. The response variable y_i represents the number of recreational boating trips undertaken by individuals. The explanatory variables are defined as the cost of visiting Lake Conroe (X_1), the cost of visiting Lake Somerville (X_2), and the cost of visiting Lake Houston (X_3).

The original dataset consists of 201 observations. However, observations with absolute standardized residuals greater than 2 were identified as outliers and excluded. Consequently, all analyses were conducted using a reduced sample of 143 observations. This step was taken to limit the influence of extreme observations on coefficient estimation. The primary objective of the empirical application is to compare the behavior of ridge parameter estimators under multicollinearity rather than to assess robustness to observation-level outliers. Prior to estimation, the predictors were standardized to avoid potential scale effects on the eigenvalues involved in the ridge-type estimators.

The Pearson correlation coefficients among the explanatory variables are computed as

$$\rho_{X_1,X_2} = 0.975, \rho_{X_1,X_3} = 0.984, \rho_{X_2,X_3} = 0.946.$$

These high pairwise correlations clearly indicate the presence of severe multicollinearity in the dataset. Accordingly, the maximum correlation value of 0.9842, observed between X_1 and X_3, is taken as the representative correlation level in the subsequent analyses.

To formally assess multicollinearity, the eigenvalues λ_j of the matrix $\mathbf{X}'\hat{\mathbf{W}}\mathbf{X}$ and the corresponding condition index $C_i = \sqrt{\lambda_{max}/\lambda_{min}}$ are computed. The estimated eigenvalues are $\lambda_1 = 1101.485$, $\lambda_2 = 452.677$, $\lambda_3 = 24.262$, and $\lambda_4 = 3.867$, yielding a condition index value of $C_i = 284.842$. Since this value exceeds the commonly accepted threshold of 30, it confirms the presence of strong multicollinearity among the predictors.

Table 5 reports the performance rankings of the ML and ridge parameter estimators based on the MSE criterion, along with the estimated ridge parameters $\hat{k}_r$ and the corresponding model parameter estimates. Since the true regression coefficients are unknown in the real data application, the asymptotic MSE values are computed using Equation (10). Moreover, to facilitate a direct comparison between simulation and empirical results, the table also includes the MSE-based rankings obtained from a Monte Carlo simulation with $p = 3$, $n = 143$, $\rho = 0.984$, and $B_0 = 1$, based on $R = 5000$ replications.

According to Table 5, the most efficient estimators in the empirical application, in terms of MSE, are k_{15}, k_{14}, k_7, k_{13}, and k_6, respectively. In contrast, the poorest-performing estimators are the ML estimator, k_4, k_3,k_{11}, and k_8. In the simulation results, the leading estimators are k_{15}, k_{13}, k_7, k_{14}, and k_6, whereas the least efficient estimators are the ML estimator,k_3, k_4,k_{11}, and k_{10}.

In practice, since β_0 is unknown, practitioners may prefer k_{14} for datasets characterized by relatively low counts and k_{15} for moderate to high counts, thereby aligning the choice of estimator with the scale of the response variable.

Overall, these findings indicate strong agreement between the performance rankings obtained from the simulation study and those observed in the empirical application. Therefore, it can be concluded that the Monte Carlo simulation results are well supported by the real data analysis and that the empirical findings are largely consistent with the simulation-based evidence.

Table 5. MSE-based performance rankings of ridge parameter estimators in simulation and empirical application: Recreation demand dataset.

Simulation Results		**Application Results**						
$p = 3, n = 143, \rho = 0.984$		$p = 3, n = 143, \rho = 0.984$						
k_r	**MSE**	k_r	**MSE**	$\hat{k}_r$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
k_{15}	0.017	k_{15}	0.024	39.546	1.071	−0.085	0.042	−0.104
k_{13}	0.023	k_{14}	0.051	10.439	1.147	−0.148	0.139	−0.131
k_7	0.037	k_7	0.088	5.251	1.160	−0.199	0.185	−0.125
k_{14}	0.042	k_{13}	0.105	4.062	1.163	−0.219	0.200	−0.119
k_6	0.074	k_6	0.107	3.945	1.163	−0.221	0.201	−0.118
k_{12}	0.087	k_{12}	0.117	3.422	1.164	−0.232	0.209	−0.115
k_5	0.113	k_5	0.141	2.493	1.166	−0.257	0.225	−0.106
k_1	0.155	k_2	0.182	1.456	1.169	−0.294	0.247	−0.090
k_2	0.155	k_1	0.199	1.153	1.170	−0.307	0.255	−0.084
k_9	0.160	k_9	0.237	0.613	1.171	−0.336	0.270	−0.071
k_8	0.166	k_{10}	0.257	0.392	1.171	−0.350	0.278	−0.064
k_{10}	0.167	k_8	0.259	0.376	1.171	−0.351	0.278	−0.063
k_{11}	0.172	k_{11}	0.261	0.352	1.171	−0.352	0.279	−0.063
k_4	0.177	k_3	0.262	0.347	1.171	−0.353	0.279	−0.063
k_3	0.178	k_4	0.274	0.232	1.172	−0.361	0.283	−0.059
ML	0.188	ML	0.303	-	1.172	−0.378	0.292	−0.050

4.2. Application 2: Fertil2 Dataset

In this section, unlike Application 1, the performance of the proposed estimators under multicollinearity is further examined using a second real dataset with a larger number of regressors. The analysis is conducted using the *Fertil2* dataset available in the *Wooldridge* R package (version 4.5.3), which is originally derived from Botswana's 1988 Demographic and Health Survey [44].

The dataset consists of 4361 observations and 27 variables, including fertility outcomes (e.g., number of children) as well as various socioeconomic and demographic characteristics. In line with the objective of the study, the number of children (y_i) is taken as the dependent variable, while the explanatory variables included in the PRR model are age (X_1), years of education (X_2), age at first birth (X_3), year of first marriage (X_4), age at first marriage (X_5), ideal number of children (X_6), husband's years of education (X_7), urban $\times$ education (X_8), and age squared (X_9).

After removing observations with missing values, the sample size decreased to 1766. From this subset, a random sample of 250 observations was selected without replacement. To mitigate the influence of extreme observations, data points with absolute standardized residuals greater than 2 were identified based on an initial regression model and subsequently excluded. As a result, the final analysis was conducted on 233 observations using standardized predictors.

The Pearson correlation coefficients among the explanatory variables are reported in Table 6.

Table 6. Pearson correlation coefficients among predictors.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
X_2	−0.155 *							
X_3	0.325 **	0.215 **						
X_4	−0.736 **	0.163 *	−0.023					
X_5	0.315 **	0.012	0.400 **	0.407 **				
X_6	0.222 **	−0.264 **	−0.073	−0.156 *	0.084			
X_7	−0.140 *	0.647 **	0.090	0.131 *	−0.009	−0.243 **		
X_8	−0.135 *	0.724 **	0.198 **	0.144 *	0.012	−0.275 **	0.580 **	
X_9	0.993 **	−0.148 *	0.335 **	−0.729 **	0.316 **	0.219 **	−0.136 *	−0.122

* Significant at the 0.01–0.05 level. ** Significant at the 0.01 level.

According to Table 6, the presence of statistically significant correlations at varying levels indicates the existence of multicollinearity in the dataset. For subsequent analyses, the highest observed correlation, namely 0.993 between X_1 and X_9, is taken as the representative correlation level.

To further assess multicollinearity, the eigenvalues of the matrix $\mathbf{X}^{'}\hat{\mathbf{W}}\mathbf{X}$ were computed as

$$\lambda_1 = 3204.869,\ \lambda_2 = 1914.989,\ \lambda_3 = 1549.213,\ \lambda_4 = 817.315,\ \lambda_5 = 745.764,$$
$$\lambda_6 = 517.868,\ \lambda_7 = 7.178,\ \lambda_8 = 1.632,\ \lambda_9 = 220.717, \text{ and } \lambda_{10} = 320.805.$$

The corresponding condition index is calculated as $C_i = 1963.768$, which is substantially higher than commonly accepted threshold values, thereby providing strong evidence of severe multicollinearity.

Table 7 presents the performance rankings of the estimators based on the asymptotic MSE values for the real dataset, along with the estimated regression coefficients. In addition, to ensure comparability with the empirical setting, Monte Carlo simulation results based on $R = 5000$ replications are also reported for the parameter configuration $p = 9$, $n = 233$, $\rho = 0.993$, and $\beta_0 = 1$.

Table 7. MSE-based performance rankings of ridge parameter estimators in simulation and empirical application: Fertil2 dataset.

Simulation				**Application**										
p **= 9,** n **= 233,** ρ **= 0.993**				p **= 9,** n **= 233,** ρ **= 0.993**										
k_r	**MSE**	k_r	**MSE**	$\hat{k}_r$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$
k_{15}	0.013	k_{15}	0.036	82.085	1.117	0.201	−0.016	−0.138	−0.137	0.079	0.066	−0.022	−0.073	0.044
k_7	0.016	k_6	0.045	51.615	1.166	0.233	−0.010	−0.142	−0.155	0.095	0.063	−0.021	−0.072	−0.009
k_{13}	0.021	k_{14}	0.077	28.950	1.202	0.292	−0.006	−0.145	−0.189	0.122	0.061	−0.021	−0.070	−0.101
k_{14}	0.034	k_{12}	0.118	17.783	1.219	0.361	−0.005	−0.145	−0.226	0.150	0.060	−0.022	−0.067	−0.203
k_6	0.056	k_7	0.138	14.294	1.224	0.397	−0.004	−0.145	−0.244	0.163	0.060	−0.022	−0.066	−0.254
k_{12}	0.101	k_{13}	0.225	4.601	1.237	0.622	−0.005	−0.145	−0.324	0.221	0.058	−0.024	−0.060	−0.545
k_5	0.120	k_5	0.273	2.528	1.238	0.750	−0.005	−0.144	−0.339	0.232	0.057	−0.025	−0.058	−0.681
k_9	0.214	k_2	0.306	1.883	1.238	0.811	−0.006	−0.144	−0.337	0.231	0.057	−0.025	−0.057	−0.738
k_8	0.215	k_{10}	0.319	1.715	1.238	0.829	−0.006	−0.144	−0.335	0.230	0.057	−0.025	−0.056	−0.754
k_1	0.220	k_1	0.353	1.347	1.238	0.875	−0.006	−0.144	−0.329	0.225	0.057	−0.025	−0.056	−0.792
k_2	0.226	k_{11}	0.431	0.830	1.237	0.956	−0.007	−0.144	−0.311	0.213	0.057	−0.026	−0.055	−0.853
k_{11}	0.231	k_9	0.510	0.515	1.237	1.021	−0.007	−0.144	−0.290	0.198	0.056	−0.026	−0.054	−0.896
k_{10}	0.236	k_8	0.605	0.265	1.236	1.084	−0.007	−0.144	−0.266	0.181	0.056	−0.026	−0.054	−0.934
k_4	0.247	k_3	0.622	0.228	1.236	1.094	−0.007	−0.144	−0.262	0.178	0.056	−0.026	−0.053	−0.940
k_3	0.249	k_4	0.630	0.213	1.236	1.099	−0.007	−0.144	−0.260	0.177	0.056	−0.026	−0.053	−0.943
ML	0.252	ML	0.766	-	1.236	1.168	−0.008	−0.144	−0.228	0.155	0.056	−0.026	−0.053	−0.980

According to Table 7, the best-performing estimators in the empirical application, in terms of asymptotic MSE, are k_{15}, k_6, k_{14}, k_{12}, and k_7, respectively. In contrast, the poorest-performing estimators are the ML estimator, k_4, k_3, k_8, and k_9.

The simulation results indicate that the most efficient estimators are k_{15}, k_7, k_{13}, k_{14}, and k_6, whereas the least efficient ones are the ML estimator, k_3, k_4, k_{10}, and k_{11}.

Overall, these findings show a substantial agreement between the empirical and simulation results, even in the presence of a larger number of predictors. The relatively greater variation in ranking compared to Application 1 may be attributed to the more complex correlation structure in the real dataset, where both positive and negative relationships exist at varying magnitudes.

In conclusion, the results demonstrate that the proposed estimators maintain strong and consistent performance not only in low-dimensional settings but also in models with a larger number of predictors and more complex correlation structures.

5. Results and Discussion

Ridge regression is a widely used and powerful approach that enables more reliable and stable estimation of model parameters in the presence of multicollinearity. The accurate estimation of the ridge parameter k, which lies at the core of this method, is of critical importance not only for the LRR model but also for the PRR model designed for count data. However, in the existing literature, the performance of ridge parameter estimators for both LRR and PRR models has predominantly been evaluated solely based on the MSE criterion.

In contrast, the distributional properties of the estimates produced by ridge parameter estimators—particularly their tendency to generate outliers and the degree of skewness in their distributions—are equally important for obtaining accurate and reliable coefficient estimates. An estimator that appears successful in terms of MSE may still undermine the reliability of statistical inference if it exhibits high outlier ratios or substantial deviations from normality.

The contribution of this study is twofold. First, by considering 13 ridge parameter estimators widely used in the literature for LRR and PRR models, two new estimators (k_{14} and k_{15}) are proposed, both of which demonstrate superior performance compared to existing estimators as well as the classical ML estimator. Second, within the PRR framework, estimator performance is systematically evaluated—for the first time—not only in terms of MSE but also with respect to two additional distribution-based criteria: outlier ratios and conformity to normality.

To this end, an extensive Monte Carlo simulation study is conducted across 180 distinct scenarios defined by different combinations of key model parameters, including the number of explanatory variables, sample size, correlation level, and intercept term. The results clearly show that the performance of ridge parameter estimators varies substantially not only in terms of MSE but also with respect to their distributional characteristics.

The simulation findings indicate that, for $\beta_0 = -1$, the proposed estimator k_{14} yields higher RE values than all competing estimators in nearly all scenarios, while simultaneously maintaining low outlier ratios and largely satisfying the normality criterion. This suggests that k_{14} is particularly effective in count data settings characterized by low mean levels, offering advantages in both accuracy and stability. In contrast, for $B_0 = 0$ and $B_0 = 1$, the estimator k_{15} emerges as the most efficient estimator in terms of RE and also exhibits consistently strong performance with respect to the outlier and normality criteria.

Although certain estimators in the literature (e.g., k_{13}) perform well in terms of RE under specific scenarios, they exhibit high outlier ratios and fail to satisfy the normality criterion in many cases. This finding clearly demonstrates that evaluations based

solely on MSE may not adequately reflect the reliability of ridge parameter estimators in practical applications.

The simulation findings are further supported by the empirical analyses on two real datasets. In both applications, k_{14} and k_{15} are consistently ranked among the top estimators in terms of MSE, while the overall ranking patterns closely align with the simulation results. Notably, the second application—with more predictors and a more complex multicollinearity structure—confirms that the proposed estimators remain effective and stable under moderately high-dimensional settings. Overall, these results demonstrate that the proposed estimators are effective not only in simulation settings but also in real-world applications, highlighting their practical reliability.

In conclusion, this study demonstrates that the MSE criterion alone is insufficient for evaluating ridge parameter estimators within the PRR framework. Distribution-based criteria, such as outlier behavior and normality, should also be taken into account. Based on the results, k_{14} for $\beta_0 = -1$ and k_{15} for $B_0 = 0$ and $B_0 = 1$ are identified as the most effective and reliable estimators when all criteria are considered jointly.

In the simulation literature, multicollinear predictors are typically generated using formulations that impose a uniform correlation structure, where all variables are strongly correlated at a predetermined level. However, in real-world datasets, the strength of correlations among regressors usually varies across low, moderate, and high levels. This heterogeneity makes it difficult to represent the overall dependence structure using a single correlation level. Moreover, such variability is likely to influence the performance of ridge parameter estimators. Therefore, an important direction for future research is the development of data generation processes that more realistically reflect the correlation patterns observed in real data.

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Data Availability Statement: The datasets presented in this study are openly available in https://doi.org/10.1002/cpe.6979 [43] and https://www.cengage.com/cgi-wadsworth/course_products_wp.pl?fid=M20b&product_isbn_issn=9781111531041 (accessed on 24 March 2026) [44].

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Conflicts of Interest: The author declares no conflicts of interest.

Appendix A

Table A1. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = -1$ and $p = 4$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.911	2.089	1.344	1.355	2.201	2.876	3.100	1.254	1.442	1.281	1.326	2.835	3.137	**4.026**	2.337
		OR	0.027	0.059	0.064	0.054	0.008	0.173	0.109	0.018	0.003	0.028	0.030	0.156	0.114	**0.009**	0.018
30	0.90	RE	2.454	2.804	1.589	1.607	2.518	3.884	4.247	1.298	1.588	1.353	1.385	3.502	4.586	**6.202**	3.595
		OR	0.029	0.064	0.067	0.063	0.010	0.167	0.106	0.021	0.004	0.021	0.035	0.151	0.123	**0.007**	0.017
30	0.95	RE	4.170	5.166	2.206	2.272	3.141	7.668	8.961	1.382	1.931	1.402	1.463	6.252	9.414	**13.394**	7.061
		OR	0.043	0.086	0.083	0.090	0.011	0.172	0.110	0.025	0.004	0.025	0.034	0.149	0.120	**0.005**	0.025
30	0.99	RE	34.582	55.169	12.942	13.338	4.287	33.187	30.386	1.529	3.765	1.526	1.693	32.130	60.299	**95.164**	35.149
		OR	0.044	0.109	0.115	0.118	0.066	0.163	0.125	0.042	0.007	0.057	0.045	0.152	0.126	**0.013**	0.079
60	0.85	RE	1.202	1.213	1.053	1.055	1.508	1.442	1.545	1.112	1.149	1.092	1.121	1.719	1.563	**2.011**	1.152
		OR	0.009	0.028	0.023	0.019	0.005	0.170	0.096	0.015	0.003	0.026	0.035	0.135	0.100	**0.012**	0.014
60	0.90	RE	1.293	1.315	1.075	1.079	1.723	1.942	2.132	1.147	1.203	1.108	1.138	2.059	2.121	**2.773**	1.658
		OR	0.013	0.034	0.028	0.021	0.004	0.169	0.096	0.012	0.002	0.020	0.032	0.140	0.103	**0.009**	0.015
60	0.95	RE	1.614	1.688	1.178	1.192	2.225	3.315	4.038	1.230	1.354	1.140	1.179	2.974	4.029	**5.334**	3.238
		OR	0.019	0.048	0.044	0.037	0.003	0.169	0.091	0.014	0.003	0.023	0.036	0.140	0.099	**0.011**	0.019
60	0.99	RE	6.731	9.168	2.937	3.144	3.794	14.897	20.538	1.458	2.271	1.190	1.323	12.157	26.950	**41.287**	18.001
		OR	0.033	0.078	0.083	0.085	0.010	0.164	0.097	0.021	0.002	0.047	0.030	0.165	0.104	**0.007**	0.045
120	0.85	RE	1.057	1.054	1.012	1.013	1.201	0.928	1.021	1.046	1.055	1.034	1.047	1.248	1.062	**1.355**	0.784
		OR	0.004	0.013	0.009	0.004	0.005	0.165	0.076	0.011	0.003	0.022	0.031	0.078	0.082	**0.015**	0.015
120	0.90	RE	1.081	1.075	1.017	1.019	1.311	1.203	1.345	1.066	1.077	1.041	1.055	1.420	1.383	**1.727**	1.128
		OR	0.002	0.012	0.009	0.005	0.003	0.163	0.076	0.008	0.003	0.020	0.031	0.104	0.086	**0.015**	0.017
120	0.95	RE	1.174	1.168	1.040	1.044	1.622	1.957	2.492	1.122	1.151	1.054	1.074	1.927	2.502	**2.978**	2.231
		OR	0.009	0.022	0.018	0.008	0.005	0.162	0.075	0.013	0.003	0.022	0.041	0.130	0.096	**0.013**	0.014
120	0.99	RE	2.432	2.714	1.443	1.507	2.999	7.450	13.241	1.352	1.634	1.073	1.151	5.403	16.580	**19.538**	13.464
		OR	0.028	0.058	0.060	0.054	0.001	0.168	0.071	0.013	0.002	0.041	0.039	0.168	0.087	**0.011**	0.028
250	0.85	RE	1.021	1.018	1.004	1.005	1.083	0.767	0.874	1.019	1.022	1.014	1.020	1.073	0.932	**1.145**	0.736
		OR	0.000	0.007	0.003	0.000	0.004	0.169	0.055	0.007	0.002	0.021	0.026	0.043	0.072	**0.013**	0.018
250	0.90	RE	1.029	1.025	1.005	1.006	1.130	0.883	1.051	1.028	1.031	1.017	1.023	1.123	1.096	**1.314**	0.966
		OR	0.001	0.008	0.002	0.000	0.004	0.160	0.052	0.006	0.001	0.018	0.032	0.063	0.072	**0.013**	0.019
250	0.95	RE	1.057	1.048	1.011	1.013	1.279	1.346	1.782	1.057	1.063	1.022	1.031	1.345	1.846	**1.897**	1.819
		OR	0.002	0.012	0.004	0.001	0.003	0.168	0.056	0.006	0.001	0.019	0.045	0.091	0.075	**0.016**	0.017
250	0.99	RE	1.424	1.432	1.104	1.121	2.235	4.139	8.391	1.227	1.309	1.029	1.069	3.079	10.979	8.666	**11.404**
		OR	0.015	0.028	0.034	0.021	0.001	0.166	0.062	0.008	0.001	0.034	0.032	0.149	0.074	0.017	**0.027**

Table A1. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
500	0.85	RE	1.009	1.008	1.002	1.002	1.036	0.625	0.855	1.009	1.010	1.006	1.009	1.027	0.923	**1.061**	0.797
		OR	0.000	0.003	0.000	0.000	0.004	0.166	0.040	0.004	0.000	0.016	0.027	0.032	0.058	**0.018**	0.022
500	0.90	RE	1.012	1.010	1.002	1.002	1.057	0.745	0.969	1.012	1.013	1.008	1.011	1.042	1.030	**1.144**	0.963
		OR	0.000	0.003	0.001	0.000	0.005	0.155	0.046	0.003	0.000	0.018	0.030	0.039	0.062	**0.013**	0.022
500	0.95	RE	1.023	1.018	1.004	1.005	1.129	1.108	1.425	1.028	1.028	1.010	1.014	1.110	1.476	1.422	**1.504**
		OR	0.000	0.003	0.001	0.000	0.005	0.170	0.043	0.002	0.000	0.024	0.047	0.063	0.063	0.016	**0.018**
500	0.99	RE	1.158	1.142	1.034	1.039	1.680	2.637	5.425	1.130	1.153	1.013	1.032	2.029	7.194	4.399	**8.182**
		OR	0.005	0.017	0.013	0.005	0.011	0.173	0.039	0.008	0.001	0.034	0.031	0.136	0.063	0.015	**0.018**

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

Table A2. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = -1$ and $p = 8$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	2.975	3.349	1.569	1.663	3.381	8.043	8.531	1.422	1.824	1.732	1.754	6.142	8.416	**11.608**	8.359
		OR	0.049	0.094	0.092	0.091	0.002	0.168	0.135	0.020	0.002	0.035	0.035	0.158	0.157	**0.008**	0.013
30	0.90	RE	4.191	4.867	2.009	2.155	4.113	11.070	11.770	1.471	2.133	1.923	1.951	10.090	12.802	**18.759**	13.326
		OR	0.057	0.101	0.102	0.101	0.002	0.161	0.159	0.022	0.003	0.038	0.038	0.155	0.161	**0.009**	0.013
30	0.95	RE	8.731	11.249	3.415	3.832	4.660	22.693	26.436	1.552	2.779	2.145	2.133	16.194	30.459	**48.245**	31.549
		OR	0.062	0.112	0.116	0.117	0.017	0.166	0.158	0.039	0.005	0.040	0.034	0.165	0.169	**0.011**	0.022
30	0.99	RE	64.566	121.041	18.440	22.225	5.371	99.857	119.141	1.604	4.999	2.315	2.387	73.725	165.988	**342.905**	164.436
		OR	0.055	0.125	0.121	0.120	0.092	0.159	0.187	0.067	0.016	0.060	0.048	0.160	0.174	**0.021**	0.047
60	0.85	RE	1.191	1.172	1.026	1.031	1.917	2.169	1.756	1.162	1.187	1.175	1.199	2.247	1.718	**2.832**	1.915
		OR	0.018	0.035	0.029	0.024	0.000	0.174	0.127	0.013	0.001	0.036	0.028	0.151	0.136	**0.011**	0.011
60	0.90	RE	1.297	1.280	1.042	1.052	2.254	3.089	2.591	1.206	1.255	1.211	1.239	2.891	2.447	**4.166**	2.900
		OR	0.024	0.045	0.033	0.026	0.001	0.169	0.134	0.012	0.002	0.029	0.027	0.152	0.145	**0.010**	0.013
60	0.95	RE	1.631	1.641	1.107	1.132	3.018	5.733	5.291	1.307	1.437	1.275	1.303	4.751	4.722	**8.649**	6.105
		OR	0.026	0.055	0.049	0.045	0.000	0.166	0.133	0.009	0.001	0.026	0.034	0.159	0.145	**0.009**	0.011
60	0.99	RE	5.450	6.823	2.110	2.336	4.676	25.777	33.558	1.478	2.269	1.388	1.466	17.326	26.785	**65.134**	38.708
		OR	0.044	0.093	0.085	0.085	0.015	0.165	0.142	0.035	0.008	0.054	0.041	0.155	0.146	**0.009**	0.020
120	0.85	RE	1.038	1.028	1.004	1.005	1.302	1.082	0.686	1.055	1.053	1.058	1.069	1.306	0.675	**1.490**	0.927
		OR	0.011	0.030	0.006	0.006	0.002	0.166	0.114	0.009	0.002	0.027	0.027	0.105	0.127	**0.014**	0.012
120	0.90	RE	1.054	1.041	1.005	1.007	1.427	1.435	0.929	1.074	1.073	1.067	1.079	1.468	0.863	**1.927**	1.311
		OR	0.009	0.026	0.011	0.005	0.009	0.167	0.116	0.007	0.001	0.028	0.030	0.117	0.124	**0.012**	0.011
120	0.95	RE	1.115	1.093	1.012	1.015	1.798	2.437	1.853	1.132	1.138	1.092	1.108	2.026	1.601	**3.429**	2.739
		OR	0.012	0.032	0.014	0.006	0.008	0.160	0.118	0.010	0.001	0.026	0.030	0.139	0.121	**0.013**	0.014

Table A2. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
120	0.99	RE	1.807	1.834	1.130	1.167	3.352	9.368	11.255	1.345	1.516	1.138	1.188	6.399	8.342	**19.679**	17.913
		OR	0.033	0.062	0.053	0.043	0.000	0.163	0.106	0.007	0.001	0.042	0.035	0.161	0.130	**0.011**	0.013
250	0.85	RE	1.010	1.007	1.001	1.001	1.090	0.729	0.364	1.017	1.015	1.021	1.024	1.066	0.370	**1.115**	0.704
		OR	0.009	0.024	0.003	0.002	0.003	0.168	0.091	0.005	0.001	0.022	0.027	0.057	0.115	**0.019**	0.015
250	0.90	RE	1.013	1.009	1.001	1.001	1.137	0.921	0.466	1.024	1.022	1.024	1.029	1.105	0.425	**1.297**	0.925
		OR	0.008	0.024	0.002	0.002	0.007	0.172	0.089	0.005	0.001	0.020	0.026	0.070	0.112	**0.020**	0.014
250	0.95	RE	1.028	1.020	1.003	1.003	1.284	1.391	0.886	1.047	1.043	1.034	1.040	1.251	0.731	**1.890**	1.723
		OR	0.008	0.023	0.004	0.003	0.010	0.171	0.091	0.005	0.001	0.025	0.028	0.094	0.110	**0.020**	0.014
250	0.99	RE	1.184	1.153	1.019	1.026	2.159	4.185	4.983	1.186	1.203	1.052	1.072	2.691	3.488	7.022	**10.052**
		OR	0.014	0.036	0.017	0.011	0.014	0.166	0.099	0.008	0.002	0.036	0.030	0.151	0.117	0.014	**0.015**
500	0.85	RE	1.003	1.002	1.000	1.000	**1.032**	0.615	0.277	1.006	1.006	1.009	1.010	1.018	0.299	1.024	0.730
		OR	0.011	0.028	0.001	0.000	**0.004**	0.166	0.075	0.004	0.001	0.018	0.026	0.039	0.096	0.018	0.017
500	0.90	RE	1.004	1.003	1.000	1.000	1.050	0.710	0.327	1.009	1.008	1.010	1.012	1.028	0.307	**1.107**	0.877
		OR	0.004	0.020	0.002	0.001	0.007	0.165	0.070	0.002	0.001	0.022	0.026	0.048	0.108	**0.022**	0.022
500	0.95	RE	1.009	1.006	1.001	1.001	1.104	1.047	0.548	1.017	1.015	1.014	1.017	1.067	0.443	**1.355**	1.331
		OR	0.006	0.021	0.003	0.001	0.007	0.156	0.076	0.005	0.001	0.024	0.028	0.066	0.110	**0.026**	0.021
500	0.99	RE	1.056	1.040	1.005	1.007	1.551	2.413	3.201	1.089	1.084	1.023	1.032	1.548	2.159	3.445	**5.558**
		OR	0.009	0.024	0.004	0.002	0.028	0.172	0.090	0.004	0.001	0.030	0.038	0.129	0.106	0.020	**0.017**

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

Table A3. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = 0$ and $p = 4$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.353	1.381	1.105	1.125	1.781	2.507	4.005	1.162	1.233	1.232	1.240	2.304	4.394	3.873	**5.031**
		OR	0.014	0.025	0.029	0.019	0.018	0.173	0.133	0.013	0.002	0.044	0.051	0.142	0.134	0.008	**0.014**
30	0.90	RE	1.569	1.637	1.176	1.212	2.047	3.413	6.122	1.209	1.327	1.261	1.280	2.879	7.528	5.882	**9.160**
		OR	0.022	0.041	0.036	0.026	0.007	0.172	0.139	0.013	0.002	0.044	0.056	0.154	0.138	0.008	**0.015**
30	0.95	RE	2.354	2.632	1.465	1.567	2.593	5.877	11.789	1.301	1.570	1.309	1.340	4.184	21.391	13.573	**26.221**
		OR	0.046	0.077	0.060	0.051	0.000	0.167	0.138	0.008	0.002	0.046	0.055	0.143	0.134	0.005	**0.019**
30	0.99	RE	14.264	22.399	5.767	7.221	3.658	20.151	25.893	1.469	2.778	1.379	1.467	10.081	**280.952**	157.894	190.044
		OR	0.048	0.105	0.108	0.099	0.008	0.166	0.152	0.026	0.002	0.066	0.068	0.154	**0.150**	0.009	0.057
60	0.85	RE	1.081	1.078	1.017	1.020	1.275	1.467	2.150	1.057	1.070	1.078	1.088	1.390	2.257	1.915	**2.599**
		OR	0.001	0.005	0.007	0.000	0.029	0.175	0.106	0.008	0.001	0.039	0.063	0.098	0.115	0.013	**0.014**
60	0.90	RE	1.128	1.125	1.028	1.033	1.404	1.841	3.029	1.082	1.105	1.093	1.107	1.619	3.563	2.519	**4.233**
		OR	0.003	0.007	0.008	0.002	0.029	0.163	0.112	0.013	0.002	0.030	0.052	0.116	0.114	0.011	**0.015**

Table A3. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
60	0.95	RE	1.288	1.299	1.068	1.085	1.738	2.860	5.739	1.146	1.203	1.113	1.141	2.294	8.748	4.554	**10.893**
		OR	0.012	0.020	0.016	0.007	0.026	0.165	0.109	0.011	0.003	0.033	0.058	0.143	0.122	0.012	**0.013**
60	0.99	RE	3.441	4.169	1.847	2.100	2.905	8.723	18.612	1.353	1.780	1.149	1.220	5.481	127.009	36.351	**175.075**
		OR	0.050	0.088	0.071	0.066	0.000	0.170	0.129	0.006	0.001	0.052	0.059	0.151	0.112	0.010	**0.033**
120	0.85	RE	1.024	1.022	1.004	1.005	1.097	1.073	1.363	1.020	1.023	1.030	1.037	1.085	1.411	1.321	**1.536**
		OR	0.000	0.000	0.000	0.000	0.024	0.177	0.084	0.008	0.001	0.035	0.056	0.059	0.103	0.014	**0.018**
120	0.90	RE	1.038	1.035	1.007	1.009	1.152	1.291	1.792	1.031	1.037	1.037	1.044	1.163	1.961	1.541	**2.123**
		OR	0.000	0.001	0.002	0.000	0.035	0.166	0.088	0.009	0.001	0.030	0.054	0.070	0.098	0.015	**0.019**
120	0.95	RE	1.087	1.081	1.017	1.021	1.311	1.766	3.127	1.063	1.077	1.047	1.061	1.447	4.378	2.290	**4.400**
		OR	0.001	0.003	0.003	0.000	0.048	0.164	0.091	0.008	0.001	0.024	0.056	0.104	0.102	0.015	**0.015**
120	0.99	RE	1.632	1.699	1.176	1.236	2.170	4.446	11.399	1.231	1.361	1.060	1.102	3.288	**52.906**	10.995	47.359
		OR	0.023	0.040	0.033	0.028	0.002	0.174	0.101	0.014	0.004	0.042	0.056	0.144	**0.109**	0.013	0.020
250	0.85	RE	1.009	1.008	1.001	1.002	1.036	0.894	1.090	1.007	1.009	1.012	1.016	1.025	1.119	1.125	**1.202**
		OR	0.000	0.000	0.001	0.000	0.017	0.163	0.053	0.007	0.001	0.035	0.049	0.037	0.091	0.019	**0.028**
250	0.90	RE	1.013	1.011	1.002	1.003	1.056	1.064	1.273	1.011	1.013	1.015	1.019	1.042	1.317	1.203	**1.396**
		OR	0.000	0.000	0.000	0.000	0.024	0.171	0.071	0.005	0.000	0.030	0.055	0.045	0.092	0.020	**0.024**
250	0.95	RE	1.029	1.025	1.005	1.007	1.124	1.314	1.945	1.026	1.030	1.019	1.028	1.113	**2.364**	1.488	2.175
		OR	0.000	0.001	0.000	0.000	0.045	0.164	0.070	0.004	0.000	0.032	0.055	0.060	**0.084**	0.014	0.023
250	0.99	RE	1.193	1.186	1.042	1.055	1.595	2.541	6.180	1.120	1.156	1.025	1.047	2.014	**20.967**	4.385	12.984
		OR	0.002	0.006	0.008	0.002	0.034	0.163	0.074	0.012	0.003	0.040	0.048	0.125	**0.091**	0.013	0.018
500	0.85	RE	1.003	1.003	1.000	1.001	1.014	0.774	0.971	1.003	1.003	1.005	1.007	1.009	1.015	1.051	**1.074**
		OR	0.000	0.000	0.000	0.000	0.011	0.155	0.040	0.004	0.000	0.030	0.049	0.025	0.075	0.022	**0.030**
500	0.90	RE	1.005	1.005	1.001	1.001	1.024	0.919	1.061	1.005	1.006	1.007	1.009	1.015	1.098	1.088	**1.165**
		OR	0.000	0.000	0.000	0.000	0.019	0.159	0.043	0.004	0.000	0.025	0.043	0.033	0.073	0.022	**0.030**
500	0.95	RE	1.011	1.009	1.002	1.003	1.051	1.146	1.395	1.011	1.012	1.008	1.013	1.035	**1.554**	1.201	1.449
		OR	0.000	0.000	0.000	0.000	0.033	0.158	0.049	0.003	0.001	0.026	0.041	0.042	**0.076**	0.016	0.029
500	0.99	RE	1.074	1.065	1.014	1.018	1.292	1.824	3.773	1.060	1.071	1.011	1.023	1.375	**9.230**	2.410	5.100
		OR	0.000	0.000	0.001	0.000	0.047	0.166	0.056	0.006	0.001	0.028	0.039	0.098	**0.072**	0.017	0.022

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

Table A4. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = 0$ and $p = 8$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.850	1.887	1.188	1.234	3.183	7.318	14.279	1.335	1.522	1.599	1.620	5.421	14.852	12.137	**15.975**
		OR	0.044	0.072	0.061	0.052	0.000	0.168	0.146	0.007	0.000	0.041	0.044	0.156	0.170	0.008	**0.013**
30	0.90	RE	2.374	2.508	1.324	1.401	3.693	10.775	26.699	1.393	1.703	1.692	1.719	7.691	30.382	21.147	**32.471**
		OR	0.043	0.079	0.070	0.066	0.000	0.168	0.155	0.007	0.000	0.041	0.037	0.168	0.169	0.009	**0.012**
30	0.95	RE	4.193	4.816	1.861	2.055	4.485	19.826	64.176	1.468	2.131	1.825	1.852	12.787	105.481	54.641	**107.030**
		OR	0.057	0.103	0.100	0.095	0.000	0.166	0.167	0.018	0.001	0.051	0.046	0.162	0.177	0.007	**0.012**
30	0.99	RE	27.428	47.008	8.263	10.177	5.513	67.825	164.871	1.558	3.927	2.026	2.067	36.587	**1902.055**	597.966	1352.926
		OR	0.053	0.116	0.107	0.114	0.064	0.160	0.195	0.061	0.016	0.067	0.062	0.168	**0.183**	0.010	0.046
60	0.85	RE	1.095	1.078	1.010	1.013	1.630	2.400	4.155	1.105	1.111	1.156	1.172	1.851	4.190	3.243	**4.583**
		OR	0.008	0.021	0.010	0.006	0.031	0.163	0.133	0.005	0.001	0.029	0.032	0.138	0.155	0.012	**0.015**
60	0.90	RE	1.151	1.128	1.017	1.023	1.882	3.348	7.349	1.146	1.162	1.186	1.205	2.273	7.870	4.655	**7.933**
		OR	0.013	0.026	0.015	0.012	0.028	0.165	0.130	0.006	0.001	0.033	0.034	0.145	0.163	0.012	**0.010**
60	0.95	RE	1.381	1.364	1.058	1.076	2.499	5.762	19.845	1.234	1.304	1.248	1.275	3.841	**26.342**	9.851	22.802
		OR	0.026	0.048	0.030	0.025	0.008	0.161	0.141	0.007	0.002	0.034	0.037	0.159	**0.159**	0.012	0.014
60	0.99	RE	3.507	4.013	1.595	1.764	4.157	19.544	107.794	1.440	1.951	1.334	1.385	11.964	**541.444**	72.312	324.929
		OR	0.047	0.096	0.078	0.077	0.000	0.161	0.153	0.020	0.004	0.051	0.046	0.163	**0.170**	0.010	0.021
120	0.85	RE	1.020	1.014	1.002	1.002	1.183	1.362	1.651	1.031	1.029	1.051	1.058	1.164	1.638	1.608	**1.882**
		OR	0.005	0.019	0.002	0.001	0.035	0.171	0.120	0.005	0.001	0.028	0.029	0.084	0.134	0.014	**0.012**
120	0.90	RE	1.030	1.022	1.003	1.004	1.269	1.685	2.674	1.045	1.043	1.063	1.071	1.261	**2.731**	1.969	2.718
		OR	0.005	0.020	0.002	0.001	0.045	0.169	0.127	0.004	0.000	0.029	0.031	0.100	**0.143**	0.016	0.014
120	0.95	RE	1.068	1.053	1.007	1.010	1.519	2.448	6.740	1.087	1.088	1.087	1.099	1.589	**8.476**	3.245	6.048
		OR	0.008	0.023	0.006	0.003	0.047	0.159	0.132	0.008	0.001	0.030	0.029	0.120	**0.139**	0.013	0.012
120	0.99	RE	1.483	1.477	1.070	1.098	2.704	7.262	45.991	1.267	1.360	1.126	1.157	4.439	**168.398**	15.347	53.916
		OR	0.034	0.054	0.036	0.028	0.003	0.160	0.134	0.006	0.001	0.038	0.033	0.159	**0.136**	0.013	0.018
250	0.85	RE	1.005	1.004	1.000	1.001	1.055	1.036	0.838	1.009	1.009	1.019	1.022	1.032	0.824	1.186	**1.203**
		OR	0.004	0.022	0.001	0.000	0.021	0.163	0.099	0.003	0.000	0.025	0.034	0.055	0.126	0.017	**0.017**
250	0.90	RE	1.007	1.005	1.001	1.001	1.077	1.163	1.196	1.013	1.012	1.022	1.025	1.047	1.148	1.280	**1.423**
		OR	0.005	0.019	0.002	0.000	0.033	0.171	0.103	0.003	0.000	0.026	0.034	0.056	0.127	0.018	**0.018**
250	0.95	RE	1.015	1.011	1.001	1.002	1.157	1.468	2.610	1.026	1.024	1.031	1.035	1.123	**2.973**	1.650	2.285
		OR	0.004	0.021	0.001	0.000	0.056	0.166	0.112	0.002	0.000	0.024	0.035	0.079	**0.131**	0.019	0.018
250	0.99	RE	1.101	1.080	1.010	1.014	1.716	3.313	16.990	1.119	1.124	1.047	1.061	1.854	**52.717**	4.737	11.367
		OR	0.007	0.022	0.007	0.004	0.038	0.169	0.124	0.004	0.000	0.031	0.039	0.137	**0.128**	0.020	0.018

Table A4. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
500	0.85	RE	1.002	1.001	1.000	1.000	1.019	0.862	0.571	1.003	1.003	1.007	1.008	1.009	0.597	**1.063**	1.047
		OR	0.004	0.025	0.003	0.000	0.011	0.174	0.083	0.003	0.000	0.030	0.030	0.040	0.102	**0.022**	0.017
500	0.90	RE	1.002	1.002	1.000	1.000	1.026	0.970	0.733	1.004	1.004	1.009	1.010	1.013	0.683	1.100	**1.131**
		OR	0.008	0.029	0.001	0.000	0.023	0.171	0.095	0.002	0.000	0.023	0.035	0.051	0.111	0.025	**0.017**
500	0.95	RE	1.005	1.003	1.000	1.001	1.055	1.198	1.414	1.009	1.008	1.013	1.015	1.031	1.368	1.240	**1.445**
		OR	0.003	0.021	0.001	0.000	0.047	0.167	0.105	0.003	0.000	0.023	0.035	0.062	0.125	0.023	**0.016**
500	0.99	RE	1.031	1.021	1.003	1.004	1.302	1.957	6.973	1.051	1.047	1.020	1.027	1.264	**19.948**	2.417	4.209
		OR	0.004	0.015	0.002	0.001	0.057	0.164	0.119	0.002	0.000	0.031	0.041	0.105	**0.120**	0.025	0.018

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

Table A5. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = 1$ and $p = 4$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.255	1.259	1.071	1.076	1.735	2.315	3.527	1.148	1.195	1.203	1.232	2.224	3.515	3.490	**4.062**
		OR	0.015	0.027	0.030	0.015	0.021	0.166	0.110	0.010	0.002	0.029	0.035	0.145	0.117	0.009	**0.016**
30	0.90	RE	1.394	1.411	1.107	1.119	1.994	3.150	5.153	1.194	1.272	1.245	1.283	2.786	5.213	4.904	**6.379**
		OR	0.019	0.040	0.032	0.021	0.015	0.163	0.113	0.011	0.003	0.029	0.040	0.148	0.115	0.007	**0.011**
30	0.95	RE	1.972	2.092	1.304	1.345	2.612	5.766	10.960	1.294	1.501	1.357	1.407	4.368	12.593	10.899	**15.648**
		OR	0.030	0.057	0.047	0.043	0.000	0.164	0.119	0.013	0.002	0.034	0.039	0.158	0.124	0.007	**0.013**
30	0.99	RE	9.739	13.833	3.805	4.057	4.002	21.701	38.030	1.477	2.570	1.495	1.670	17.890	95.889	92.633	**101.886**
		OR	0.044	0.097	0.104	0.105	0.001	0.175	0.135	0.020	0.003	0.057	0.042	0.152	0.126	0.006	**0.041**
60	0.85	RE	1.052	1.046	1.009	1.010	1.247	1.356	1.775	1.049	1.055	1.062	1.076	1.307	1.765	1.768	**2.083**
		OR	0.001	0.006	0.004	0.002	0.015	0.176	0.095	0.008	0.001	0.027	0.042	0.095	0.101	0.012	**0.009**
60	0.90	RE	1.086	1.076	1.017	1.018	1.377	1.728	2.485	1.074	1.085	1.082	1.097	1.550	2.566	2.294	**3.157**
		OR	0.002	0.007	0.007	0.002	0.023	0.164	0.091	0.008	0.002	0.026	0.039	0.115	0.098	0.013	**0.015**
60	0.95	RE	1.205	1.196	1.042	1.051	1.723	2.659	4.869	1.138	1.173	1.123	1.152	2.200	5.338	4.061	**6.977**
		OR	0.007	0.018	0.012	0.011	0.021	0.176	0.090	0.007	0.001	0.030	0.039	0.137	0.098	0.014	**0.015**
60	0.99	RE	2.738	3.119	1.469	1.547	3.153	9.876	24.117	1.349	1.682	1.181	1.307	6.702	40.995	28.468	**57.957**
		OR	0.030	0.063	0.069	0.056	0.000	0.166	0.092	0.011	0.002	0.049	0.037	0.157	0.096	0.009	**0.022**
120	0.85	RE	1.016	1.013	1.003	1.003	1.088	1.029	1.186	1.017	1.019	1.023	1.029	1.072	1.209	1.277	**1.369**
		OR	0.000	0.001	0.001	0.000	0.008	0.168	0.076	0.005	0.001	0.028	0.038	0.060	0.091	0.011	**0.019**
120	0.90	RE	1.026	1.022	1.004	1.005	1.142	1.216	1.535	1.028	1.030	1.032	1.039	1.138	1.583	1.480	**1.819**
		OR	0.000	0.002	0.002	0.000	0.017	0.169	0.066	0.005	0.000	0.019	0.044	0.071	0.078	0.013	**0.021**

Table A5. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
120	0.95	RE	1.061	1.052	1.010	1.012	1.305	1.707	2.743	1.059	1.066	1.047	1.061	1.388	2.936	2.159	**3.489**
		OR	0.001	0.003	0.003	0.001	0.026	0.166	0.066	0.003	0.000	0.030	0.041	0.098	0.088	0.014	**0.018**
120	0.99	RE	1.477	1.503	1.093	1.108	2.289	5.096	13.881	1.217	1.312	1.071	1.142	3.224	22.187	10.338	**28.340**
		OR	0.018	0.042	0.029	0.020	0.002	0.165	0.070	0.007	0.001	0.042	0.028	0.152	0.084	0.012	**0.022**
250	0.85	RE	1.006	1.005	1.001	1.001	1.032	0.877	0.966	1.006	1.007	1.009	1.012	1.020	1.024	1.103	**1.118**
		OR	0.000	0.000	0.000	0.000	0.006	0.167	0.047	0.003	0.001	0.025	0.042	0.038	0.067	0.016	**0.021**
250	0.90	RE	1.009	1.007	1.001	1.002	1.052	0.972	1.142	1.010	1.011	1.013	1.016	1.035	1.186	1.181	**1.301**
		OR	0.000	0.001	0.000	0.000	0.007	0.164	0.043	0.002	0.000	0.016	0.047	0.039	0.069	0.016	**0.017**
250	0.95	RE	1.021	1.016	1.003	1.004	1.123	1.294	1.762	1.025	1.026	1.019	1.024	1.102	1.919	1.461	**2.005**
		OR	0.000	0.003	0.001	0.000	0.021	0.170	0.047	0.003	0.000	0.031	0.047	0.068	0.068	0.017	**0.019**
250	0.99	RE	1.157	1.143	1.027	1.034	1.655	2.793	7.119	1.119	1.143	1.028	1.062	2.011	**12.523**	4.475	11.693
		OR	0.010	0.021	0.009	0.006	0.027	0.171	0.055	0.006	0.001	0.039	0.033	0.131	**0.077**	0.016	0.013
500	0.85	RE	1.002	1.002	1.000	1.000	1.014	0.862	0.940	1.003	1.003	1.004	1.005	1.008	1.004	1.046	**1.050**
		OR	0.000	0.000	0.000	0.000	0.004	0.166	0.035	0.001	0.000	0.023	0.047	0.025	0.052	0.014	**0.021**
500	0.90	RE	1.004	1.003	1.001	1.001	1.022	0.854	1.007	1.005	1.005	1.006	1.007	1.013	1.058	1.077	**1.119**
		OR	0.000	0.009	0.000	0.000	0.006	0.165	0.035	0.002	0.000	0.013	0.039	0.025	0.062	0.018	**0.025**
500	0.95	RE	1.009	1.007	1.001	1.002	1.054	1.109	1.366	1.011	1.011	1.008	1.011	1.034	**1.451**	1.205	1.433
		OR	0.000	0.003	0.000	0.000	0.015	0.171	0.033	0.004	0.000	0.023	0.041	0.048	**0.065**	0.020	0.025
500	0.99	RE	1.059	1.049	1.010	1.012	1.321	1.855	4.078	1.062	1.067	1.013	1.028	1.392	**7.040**	2.496	5.148
		OR	0.001	0.006	0.002	0.001	0.047	0.169	0.034	0.002	0.000	0.032	0.032	0.094	**0.062**	0.017	0.023

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

Table A6. RE values, outlier ratios, and normality classification of ridge parameter estimators for $B_0 = 1$ and $p = 8$.

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
30	0.85	RE	1.764	1.781	1.175	1.203	3.134	7.035	12.933	1.333	1.502	1.632	1.650	5.399	12.992	11.596	**14.721**
		OR	0.038	0.065	0.057	0.051	0.000	0.168	0.150	0.008	0.003	0.035	0.030	0.162	0.156	0.006	**0.010**
30	0.90	RE	2.165	2.244	1.271	1.328	3.644	10.414	21.975	1.397	1.662	1.737	1.751	7.537	22.282	18.500	**26.061**
		OR	0.052	0.083	0.075	0.064	0.000	0.177	0.147	0.008	0.000	0.029	0.034	0.155	0.170	0.008	**0.009**
30	0.95	RE	3.768	4.216	1.739	1.876	4.430	18.629	46.199	1.474	2.072	1.931	1.952	12.823	59.896	45.808	**72.090**
		OR	0.055	0.104	0.102	0.098	0.000	0.159	0.160	0.014	0.001	0.043	0.041	0.163	0.166	0.010	**0.016**
30	0.99	RE	22.347	35.486	6.735	7.884	5.839	69.314	181.957	1.592	3.837	2.243	2.337	49.427	470.025	386.391	**545.405**
		OR	0.054	0.116	0.109	0.123	0.059	0.166	0.191	0.056	0.015	0.055	0.050	0.158	0.171	0.010	**0.032**

Table A6. *Cont.*

n	ρ		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
60	0.85	RE	1.076	1.059	1.008	1.010	1.598	2.348	3.315	1.098	1.098	1.140	1.158	1.791	3.210	3.014	**3.951**
		OR	0.011	0.027	0.011	0.007	0.024	0.163	0.124	0.007	0.001	0.030	0.028	0.141	0.132	0.011	**0.011**
60	0.90	RE	1.123	1.101	1.013	1.017	1.837	3.210	5.380	1.134	1.143	1.178	1.199	2.200	5.166	4.303	**6.464**
		OR	0.013	0.030	0.016	0.010	0.026	0.161	0.127	0.007	0.001	0.030	0.031	0.146	0.148	0.011	**0.013**
60	0.95	RE	1.285	1.258	1.038	1.048	2.415	5.214	11.767	1.221	1.265	1.250	1.271	3.485	11.447	8.301	**15.024**
		OR	0.021	0.042	0.029	0.023	0.012	0.164	0.137	0.007	0.002	0.029	0.033	0.154	0.161	0.012	**0.014**
60	0.99	RE	3.026	3.328	1.426	1.548	4.245	20.164	72.515	1.434	1.867	1.375	1.450	12.454	80.309	58.560	**130.398**
		OR	0.045	0.086	0.075	0.075	0.000	0.168	0.146	0.018	0.002	0.054	0.042	0.167	0.149	0.008	**0.015**
120	0.85	RE	1.016	1.011	1.001	1.002	1.171	1.282	1.297	1.028	1.025	1.045	1.052	1.147	1.261	1.556	**1.730**
		OR	0.004	0.018	0.002	0.002	0.026	0.171	0.110	0.003	0.000	0.024	0.031	0.079	0.118	0.015	**0.013**
120	0.90	RE	1.023	1.016	1.002	1.003	1.250	1.594	1.857	1.041	1.038	1.055	1.064	1.220	1.749	1.874	**2.387**
		OR	0.007	0.022	0.002	0.003	0.028	0.165	0.107	0.005	0.000	0.024	0.031	0.093	0.128	0.017	**0.013**
120	0.95	RE	1.053	1.038	1.005	1.006	1.491	2.286	3.918	1.081	1.077	1.084	1.094	1.518	3.583	2.972	**4.854**
		OR	0.007	0.021	0.005	0.003	0.042	0.167	0.118	0.004	0.000	0.023	0.027	0.122	0.132	0.015	**0.013**
120	0.99	RE	1.400	1.375	1.054	1.072	2.740	7.160	24.697	1.266	1.338	1.137	1.175	4.272	22.633	14.127	**37.912**
		OR	0.028	0.052	0.035	0.029	0.004	0.166	0.118	0.008	0.001	0.036	0.038	0.158	0.137	0.016	**0.017**
250	0.85	RE	1.004	1.003	1.000	1.000	1.051	0.992	0.656	1.008	1.007	1.016	1.018	1.027	0.667	**1.167**	1.146
		OR	0.008	0.025	0.002	0.002	0.012	0.151	0.090	0.003	0.000	0.023	0.029	0.047	0.107	**0.016**	0.013
250	0.90	RE	1.006	1.004	1.000	1.001	1.072	1.085	0.853	1.012	1.010	1.019	1.022	1.044	0.784	1.256	**1.343**
		OR	0.008	0.025	0.002	0.000	0.022	0.169	0.097	0.004	0.001	0.023	0.027	0.061	0.120	0.023	**0.020**
250	0.95	RE	1.013	1.008	1.001	1.001	1.153	1.406	1.661	1.025	1.022	1.029	1.034	1.116	1.451	1.615	**2.136**
		OR	0.008	0.026	0.002	0.001	0.032	0.174	0.110	0.002	0.000	0.024	0.032	0.090	0.117	0.017	**0.019**
250	0.99	RE	1.090	1.068	1.009	1.012	1.746	3.225	9.608	1.123	1.122	1.051	1.066	1.851	8.148	4.705	**10.626**
		OR	0.013	0.030	0.009	0.005	0.043	0.170	0.109	0.004	0.001	0.032	0.034	0.128	0.126	0.021	**0.015**
500	0.85	RE	1.001	1.001	1.000	1.000	1.017	0.827	0.463	1.003	1.002	1.006	1.007	1.008	0.519	**1.053**	1.017
		OR	0.008	0.024	0.001	0.001	0.007	0.173	0.075	0.002	0.000	0.022	0.033	0.041	0.092	**0.020**	0.018
500	0.90	RE	1.002	1.001	1.000	1.000	1.025	0.968	0.537	1.004	1.004	1.008	1.009	1.012	0.523	1.090	**1.098**
		OR	0.007	0.023	0.004	0.001	0.011	0.167	0.080	0.003	0.000	0.021	0.031	0.044	0.108	0.020	**0.014**
500	0.95	RE	1.004	1.003	1.000	1.000	1.056	1.111	0.933	1.009	1.008	1.012	1.014	1.029	0.797	1.233	**1.402**
		OR	0.007	0.021	0.002	0.001	0.022	0.162	0.091	0.003	0.000	0.022	0.032	0.063	0.111	0.023	**0.017**
500	0.99	RE	1.027	1.018	1.002	1.003	1.313	1.962	**4.777**	1.052	1.046	1.021	1.028	1.272	4.360	2.429	4.168
		OR	0.007	0.024	0.002	0.001	0.053	0.168	**0.099**	0.002	0.000	0.029	0.037	0.104	0.113	0.017	0.018

Note: Bold: Maximum RE in each scenario; Blue: Normality criterion satisfied; Orange: Normality criterion violated; RE: Relative efficiency; OR: Outlier ratio.

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