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Distributed Event-Triggered Fixed-Time Time-Varying Formation Control for Multi-Agent Systems

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Abstract

This paper investigates the distributed event-triggered fixed-time time-varying formation control problem for a class of nonlinear multi-agent systems subject to model uncertainties and unknown time-varying disturbances. To address issues in traditional formation control methods, such as convergence time dependence on initial states and high communication resource consumption, a distributed cooperative control scheme integrating fixed-time control, event-triggered mechanisms, and dynamic surface control is proposed. Firstly, a fixed-time disturbance observer is designed to accurately estimate the agents' lumped disturbances within a fixed time independent of initial conditions. Secondly, by incorporating dynamic surface control techniques, a distributed event-triggered formation control law is constructed, effectively reducing communication and computational resource usage. Furthermore, using Lyapunov stability theory, the closed-loop system is proven to exhibit practical fixed-time stability, and the existence of a positive lower bound for triggering intervals precludes Zeno behavior. Finally, numerical simulations validate the superiority of the proposed method in terms of convergence speed, control accuracy, and resource efficiency. This research provides an efficient, robust, and resource-friendly solution for cooperative control of multi-agent systems in complex environments.

Keywords: multi-agent systems (MASs); fixed-time control; event trigger; time-varying formation control; nonlinear functions

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1. Introduction

Multi-agent system (MAS) cooperative control constitutes a current research hotspot at the intersection of control theory, robotics, and artificial intelligence. It demonstrates broad application prospects in fields such as cooperative search with UAV swarms [1], intelligent traffic management [2], and distributed sensor networks [3]. Through local perception and communication among individuals, multi-agent systems can accomplish complex cooperative tasks—including formation, encircling, and coverage—in a distributed manner, exhibiting significant advantages in robustness, flexibility, and scalability [4,5]. Among numerous cooperative control problems, formation control serves as the foundation for achieving cooperative operations, with its core objective being to guide the agent group to form and maintain specific spatial or functional configurations [6].

Traditional formation control research primarily focuses on static formations, where the desired configuration remains fixed [7]. However, facing increasingly complex application scenarios (e.g., dynamic obstacle avoidance, terrain adaptation, target tracking),

static formations often fail to meet mission requirements, drawing widespread attention to the time-varying formation control problem [8,9]. In time-varying formations, the desired configuration can undergo translation, rotation, scaling, or more complex deformations over time. This demands that the control system not only achieves asymptotic convergence but also possesses precise tracking capabilities for time-varying trajectories, imposing higher requirements on control algorithm design [10].

In practical applications, agents often exhibit model uncertainties and unknown time-varying disturbances (e.g., gust disturbances for UAVs, ocean current interference for underwater vehicles) [11,12]. To overcome these challenges, a series of robust and adaptive control methods has been proposed. For instance, ref. [13] employs adaptive neural networks to approximate system uncertain nonlinearities and designs a robust controller based on backstepping; ref. [14] constructs a disturbance observer to estimate lumped disturbances and achieves precise tracking via dynamic surface control; ref. [15] designs a prediction-based fuzzy state observer, effectively enhancing the system's anti-interference capability. While these methods address uncertainty issues, most only guarantee asymptotic or finite-time convergence, with convergence time dependent on initial conditions.

Convergence speed is a key performance indicator for formation control. Although finite-time control achieves rapid convergence, its convergence time upper bound depends on initial conditions [16]. Consequently, fixed-time stability theory is proposed [17], whose most significant feature is that the convergence time upper bound can be predetermined by controller parameters and is independent of initial conditions. This characteristic significantly enhances system reliability and predictability under unknown initial conditions. Based on this, ref. [18] designs a fixed-time disturbance observer, achieving fast and accurate estimation of lumped disturbances; ref. [19] combines a fixed-time extended state observer with nonsingular terminal sliding mode to propose a fixed-time formation control scheme. However, most existing fixed-time control methods rely on centralized communication architectures and global information exchange, limiting their application in large-scale systems.

Compared with centralized control, distributed control relies only on local information exchange between an agent and its neighbors, offering higher reliability and scalability [20]. Ref. [21] utilizes super-twisting observers and sliding mode techniques to achieve distributed finite-time formation; ref. [22] addresses distributed formation predictive control under communication delays; ref. [23] designs a distributed prescribed performance controller based on data-driven methods. Nevertheless, these studies predominantly adopt time-triggered mechanisms, requiring periodic sampling, communication, and control updates, which can lead to unnecessary waste of communication bandwidth and computational resources when facing resource-constrained hardware platforms.

To conserve system resources, event-triggered control mechanisms emerge [24]. This mechanism performs communication and control updates only when specific triggering conditions are met (e.g., tracking error exceeds a threshold), significantly reducing system energy consumption and communication load. Ref. [25] combines event-triggering with adaptive observers to achieve multi-agent distributed formation; ref. [26] designs an event-triggered controller based on a fixed-time fuzzy observer, reducing triggering frequency; ref. [27] employs reinforcement learning to solve the adaptive optimal formation problem under communication link failures. However, research integrating event-triggered mechanisms, fixed-time stability, and distributed time-varying formation still faces challenges: firstly, the nonlinear coupling of time-varying trajectories, fixed-time convergence, and event-triggering makes system analysis and design exceptionally complex; secondly, Zeno behavior in event-triggering must be strictly avoided; thirdly, achieving fixed-time

convergence within a distributed framework requires constructing novel Lyapunov functions and designing triggering conditions that depend solely on local information.

Based on the above analysis, the main contributions of this paper are summarized as follows:

1. Compared with the multi-agent system in [28], to handle unknown nonlinear functions and unknown time-varying disturbances in the system dynamics, a novel fixed-time disturbance observer is designed. This observer accurately estimates the agents' lumped disturbances within a fixed time independent of initial states, effectively overcoming the limitation of traditional asymptotic or finite-time observers whose convergence time depends on initial conditions, thereby enhancing system reliability and response speed under unknown initial conditions.
2. Different from the multi-agent system framework in [29], the system studied in this paper incorporates time-varying characteristics. As the relative positions or velocities between agents vary over time, the formation control strategy dynamically updates to regulate agent motion and maintain the target formation state. A real-time feedback mechanism enables continuous adaptation to time-varying requirements.
3. Compared with the multi-agent system in [30], this work integrates event-triggered communication mechanisms, dynamic surface control techniques, and fixed-time control theory to construct a distributed formation control law that relies solely on local information. This strategy ensures high-precision tracking of time-varying formation trajectories while significantly reducing communication and computational resource consumption, making it suitable for resource-constrained practical applications.

To clearly illustrate the advancements of this work, Table 1 provides a comprehensive comparative analysis with the closely related studies [28–30]. The comparison is conducted from some key perspectives: system model characteristics, control objectives, methodological approaches, and performance guarantees.

Table 1. Comparative analysis with references [28–30].

Aspect	Reference [28]	Reference [29]	Reference [30]	This Work
System Complexity	Only random disturbances	Only unknown nonlinearities	Only unknown nonlinearities	Both random disturbances and unknown nonlinearities, and the model can be used for coordinate transformation
Disturbance and Nonlinearity Handling	Disturbance observer	Adaptive	Adaptive	Disturbance observer
Convergence Time	Finite-time	Asymptotic	Fixed-time	Fixed-time
Triggering Mechanism	Time-triggered	Event-triggered	Time-triggered	Event-triggered
Formation Type	Static formation	Static formation	Static formation	Time-varying formation
Computational Complexity	No processing	No processing	No processing	Dynamic surface technique

As evidenced in Table 1, these advancements collectively offer a more comprehensive and practical solution for MAS cooperative control in complex environments.

2. Preliminaries and Problem Formulation

2.1. Graph Theory

Consider a graph $\Xi = (V, \varepsilon)$ consisting of nodes and edges, where $V = \{v_1, v_2, \dots, v_n\}$ represents the set of nodes and $\varepsilon \subseteq \{(v_i, v_j) | v_i, v_j \subseteq V\}$ represents the set of edges. In an undirected graph $\Xi = (V, \varepsilon)$, an edge from node v_i to node v_j is denoted as $v_{ij} = (v_i, v_j), i \neq j$, which implies that nodes v_i and v_j can exchange information. Consequently, v_j is a neighbor of v_i , and the neighbor set of node v_i is defined as $N_i = \{v_j \in V | (v_i, v_j) \subseteq \varepsilon\}$.

The adjacency matrix of graph $\Xi = (V, \varepsilon)$ is denoted by $A = [a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} \geq 0$ represents the weight of each edge. The weights are designed as follows:

$$\begin{cases} a_{ij} = a_{ji}, & (v_i, v_j) \in \varepsilon, \\ a_{ij} = 0, & (v_i, v_j) \notin \varepsilon. \end{cases}$$

The Laplacian matrix of graph $\Xi = (V, \varepsilon)$ is given by $L = [l_{ij}] = D - A \in \mathbb{R}^{n \times n}$, where $D = \text{diag}\{d_1, d_2, \dots, d_n\}$ and $d_i = \sum_{j=1}^n a_{ij}$.

The communication matrix between followers and the leader is defined as $B = \text{diag}\{b_1, b_2, \dots, b_n\} \in \mathbb{R}^{n \times n}$. If follower i exchanges information with the leader, then $b_i > 0$; otherwise, $b_i = 0$.

Assumption 1. The graph $\Xi = (V, \varepsilon)$ is an undirected, connected graph without loops or repeated edges.

Remark 1. The proposed scheme is inherently extensible because each agent only requires local information from its neighbors. The analysis assumes an undirected and fixed communication graph (Assumption 1). Extending it to directed or switching topologies is a promising direction for future research, which would require additional conditions such as graph balance or joint connectivity, and might involve constructing more complex Lyapunov functions.

2.2. Important Lemmas

Lemma 1 ([31]). Consider a nonlinear system described by $\dot{y} = f(y, t), y(0) = y_0$, where $y \in \mathbb{R}^n$ is the system state variable and $f : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous function satisfying $f(0) = 0$. If there exists a continuous function $V(t) \geq 0$ such that $\dot{V}(t) \leq -a(V(t))^p - b(V(t))^q$, where $a > 0, b > 0, 0 < p < 1$, and $q > 1$, then the system is fixed-time stable. It converges to the origin within a bounded time independent of initial conditions, and the convergence time satisfies $T \leq T_{\max} = \frac{1}{a(1-p)} + \frac{1}{b(q-1)}$.

Lemma 2 ([31]). Consider a nonlinear system described by $\dot{y} = f(y, t), y(0) = y_0$, where $y \in \mathbb{R}^n$ is the system state variable and $f : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous function satisfying $f(0) = 0$. If there exists a continuous function $V(t) \geq 0$ such that $\dot{V}(t) \leq -a(V(t))^p - b(V(t))^q + c$, where $a > 0, b > 0, 0 < p < 1, q > 1$, and $0 < c < \infty$, then the system is practically fixed-time stable. The convergence time satisfies $T \leq T_{\max} = \frac{1}{a\varepsilon(1-p)} + \frac{1}{b\varepsilon(q-1)}$, where $0 < \varepsilon < 1$, and $\left\{ \lim_{t \rightarrow T} x | V(x) \leq \min \left\{ \left(\frac{c}{a(1-\varepsilon)} \right)^{\frac{1}{p}}, \left(\frac{c}{b(1-\varepsilon)} \right)^{\frac{1}{q}} \right\} \right\}$.

Remark 2. Lemma 1 provides a theoretical basis for the analysis of fixed-time stability. Lemma 2 extends Lemma 1 to actual fixed-time stability, capable of handling approximate errors (represented by c). Lemma 1 is used for the proof of fixed-time convergence of the disturbance observer, while Lemma 2 is used for the final overall fixed-time stability proof, which is crucial for our control system as it takes into account the estimation error of the disturbance observer and the event-triggered error.

This lemma ensures that the system state converges to a small residual set near the origin within a fixed time, which is consistent with our control objective.

Lemma 3 ([26]). If $z_1, \dots, z_n \geq 0$, then the following inequalities hold:

$$\begin{cases} \sum_{i=1}^n z_i^\alpha \geq (\sum_{i=1}^n z_i)^\alpha, & 0 < \alpha < 1, \\ \sum_{i=1}^n z_i^\alpha \geq n^{1-\alpha} (\sum_{i=1}^n z_i)^\alpha, & \alpha \geq 1. \end{cases}$$

2.3. Dynamic Model Description

Consider a continuous-time multi-agent system composed of multiple agents, including N follower agents and one leader agent. The system employs a homogeneous second-order model, meaning all agents share identical dynamic characteristics described by the same form of second-order differential equations. This model typically includes position and velocity states influenced by control inputs and is commonly used to characterize collective behaviors requiring coordinated motion or consensus objectives.

The dynamic model of the i -th follower agent ($i = 1, \dots, N$) is given by:

$$\begin{cases} \dot{x}_i = P_i v_i + Q_i, \\ \dot{v}_i = M_i u_i + f_i(\cdot) + d_i. \end{cases} \quad (1)$$

where x_i is the position vector, v_i is the velocity vector, u_i is the control input vector, Q_i is a known function vector, P_i and M_i are known function matrices, $f_i(\cdot)$ is an unknown nonlinear function vector, and d_i is an external disturbance vector (e.g., wind resistance). Here, $x_i, v_i, u_i, Q_i, f_i(\cdot), d_i \in \mathbb{R}^n$, and $P_i, M_i \in \mathbb{R}^{n \times n}$.

Assumption 2. The functions $f_i(\cdot)$ and d_i are unknown and time-varying, and both $\|d_i\|$ and $\|\dot{d}_i\|$ are bounded.

The dynamic model of the leader agent is described by:

$$\begin{cases} \dot{x}_0 = v_0, \\ \dot{v}_0 = f_0(t). \end{cases} \quad (2)$$

where x_0 is the position vector, v_0 is the velocity vector, and $f_0(t)$ is a known function vector dependent on time t .

Assumption 3. The trajectory of the virtual leader is known, and its components along with their first and second derivatives are smooth and bounded.

3. Formation Control Design

3.1. Fixed-Time Disturbance Observer Design

Let $W_i = f_i(\cdot) + d_i$ denote the lumped disturbance of the i -th follower agent. From Assumption 2, there exists a constant $\Omega_i > 0$ such that $\|W_i\| \leq \Omega_i$. Then we have:

$$\dot{v}_i = W_i + M_i u_i. \quad (3)$$

Design the following fixed-time disturbance observer:

$$\dot{\delta}_i = \hat{W}_i + M_i u_i, \quad (4)$$

$$\hat{W}_i = k_{1i} \text{sig}(\Pi_i)^p + k_{2i} \text{sig}(\Pi_i)^q + k_{3i} \text{sgn}(\Pi_i), \quad (5)$$

where $\hat{W}_i$ is the estimate of the lumped disturbance W_i , δ_i is an auxiliary variable, $\Pi_i = v_i - \delta_i$, $k_{1i}, k_{2i}, k_{3i} \in \mathbb{R}^{n \times n}$ are positive definite diagonal matrices with $\lambda_{\min}(k_{3i}) > \Omega_i$, $0 < p < 1$, and $q > 1$.

Since $\|W_i\| < \Omega_i < \lambda_{\min}(k_{3i})$, we have:

$$\begin{aligned} \Pi_i^T (W_i - k_{3i} \text{sgn}(\Pi_i)) &\leq \Pi_i^T W_i - \lambda_{\min}(k_{3i}) |\Pi_i| \\ &\leq |\Pi_i| \|W_i\| - \lambda_{\min}(k_{3i}) |\Pi_i| \\ &\leq |\Pi_i| (\|W_i\| - \lambda_{\min}(k_{3i})) \\ &\leq 0. \end{aligned} \quad (6)$$

Let $\tilde{W}_i = \hat{W}_i - W_i$. Subtracting Equation (3) from Equation (4) yields $\dot{\tilde{W}}_i = \dot{\delta}_i - \dot{v}_i = -\dot{\Pi}_i$. Choose the following Lyapunov function:

$$V_{doi} = \frac{1}{2} \Pi_i^T \Pi_i. \quad (7)$$

Differentiating Equation (7) and using Equations (3)–(6), we obtain:

$$\begin{aligned} \dot{V}_{doi} &= \Pi_i^T (\dot{v}_i - \dot{\delta}_i) \\ &= \Pi_i^T (W_i - k_{1i} \text{sig}(\Pi_i)^p - k_{2i} \text{sig}(\Pi_i)^q - k_{3i} \text{sgn}(\Pi_i)) \\ &\leq -\lambda_{\min}(k_{1i}) |\Pi_i|^T |\Pi_i|^p - \lambda_{\min}(k_{2i}) |\Pi_i|^T |\Pi_i|^q \\ &\quad + \Pi_i^T (W_i - k_{3i} \text{sgn}(\Pi_i)) \\ &\leq -\lambda_{\min}(k_{1i}) |\Pi_i|^T |\Pi_i|^p - \lambda_{\min}(k_{2i}) |\Pi_i|^T |\Pi_i|^q \\ &\leq -\lambda_{\min}(k_{1i}) 2^{\frac{p+1}{2}} V_{doi}^{\frac{p+1}{2}} - \lambda_{\min}(k_{2i}) n^{\frac{1-n}{2}} 2^{\frac{q+1}{2}} V_{doi}^{\frac{q+1}{2}} \\ &\leq -\mu_{1i} V_{doi}^{\frac{p+1}{2}} - \mu_{2i} V_{doi}^{\frac{q+1}{2}}, \end{aligned} \quad (8)$$

where $\mu_{1i} = \lambda_{\min}(k_{1i}) 2^{\frac{p+1}{2}}$ and $\mu_{2i} = \lambda_{\min}(k_{2i}) n^{\frac{1-n}{2}} 2^{\frac{q+1}{2}}$.

According to Lemma 1, Π_i converges to zero within a fixed settling time satisfying:

$$T_{doi} \leq \frac{1}{\mu_{1i}(1-p)} + \frac{1}{\mu_{2i}(q-1)}. \quad (9)$$

Remark 3. Compared with the finite-time disturbance observer $\hat{W}_i = k_{1i} \text{sig}(\Pi_i)^p + k_{3i} \text{sgn}(\Pi_i)$ [28], the fixed-time disturbance observer (5) incorporates an additional term $k_{2i} \text{sig}(\Pi_i)^q$. This term significantly enhances convergence performance when the state is far from the equilibrium point (i.e., when Π_i is large), essentially providing a "super-accelerator" for the system in the large-error region. Combined with the original "accelerator" $k_{1i} \text{sig}(\Pi_i)^p$ that operates in the small-error region, this design maintains extremely high convergence rates regardless of whether the system is far from or close to the equilibrium. It is precisely this synergistic effect of "balancing both near and far" that breaks the dependence of convergence time on initial conditions, thereby achieving superior fixed-time stability.

When $t > T_{doi}$, we have $\Pi_i \equiv 0$ and $\dot{\Pi}_i \equiv 0$. Furthermore, when $t > T_{doi}$, $\tilde{W}_i = 0$. This demonstrates that the designed fixed-time disturbance observer can achieve estimation of the i -th agent's lumped disturbance within a fixed settling time.

3.2. Distributed Event-Triggered Fixed-Time Formation Control Law Design

This section designs a distributed event-triggered fixed-time formation control law for the i -th follower agent based on the fixed-time disturbance observer and dynamic surface control technique. The control design comprises the following two steps.

Step 1: Define the formation error $e_{1i} \in \mathbb{R}^n$ of the i -th agent as:

$$e_{1i} = \sum_{j=1}^N a_{ij}(x_i - \sigma_{i,1} - x_j + \sigma_{j,1}) + b_i(x_i - \sigma_{i,1} - x_0), \quad (10)$$

where $\sigma_{i,1} \in \mathbb{R}^n$ represents the desired relative position between the i -th agent and the virtual leader, which is time-varying with $\dot{\sigma}_{i,1} = \sigma_{i,2}$.

Establish the event-triggered communication mechanism for the j -th agent:

$$x_j^c(t) = x_j(t_k^j), \quad \dot{x}_j^c(t) = \dot{x}_j(t_k^j), \quad t \in [t_k^j, t_{k+1}^j), \quad (11)$$

$$t_{k+1}^j = \inf \left\{ t > t_k^j \mid \|e_{xj}(t)\| \geq \varepsilon_{1j} \vee \|e_{\dot{x}j}(t)\| \geq \varepsilon_{2j} \right\}, \quad (12)$$

where $e_{xj}(t) = x_j(t) - x_j^c(t)$, $e_{\dot{x}j}(t) = \dot{x}_j(t) - \dot{x}_j^c(t)$; ε_{1j} and ε_{2j} are positive design constants; t_k^j denotes the k -th ($k = 1, 2, \dots, n$) triggering instant of the j -th agent, at which the j -th agent transmits $x_j(t_k^j)$ and $\dot{x}_j(t_k^j)$ to its neighbors via the communication network. The ultimate formation error bound linearly depends on the triggering thresholds ε_{1j} and ε_{2j} . A smaller threshold improves precision but increases the communication frequency. This trade-off allows designers to balance control accuracy and communication resource consumption according to application requirements.

Remark 4. The event-triggered communication mechanism revolutionizes traditional periodic communication into on-demand communication by introducing state-based triggering conditions. Specifically, each agent triggers new data transmission only when the error between the current local state or velocity measurement and the last broadcast value exceeds a preset threshold. This design significantly reduces redundant data transmission in the network, greatly decreases communication bandwidth occupancy and agent energy consumption while strictly guaranteeing cooperative control performance, achieving an optimal balance between control efficiency and resource conservation. In the proposed distributed architecture, each agent j measures its own velocity $\dot{x}_j(t)$ locally using onboard sensors (e.g., IMU, GPS, or optical flow sensors). At triggering instants t_k^j , agent j transmits both its position $x_j(t_k^j)$ and velocity $\dot{x}_j(t_k^j)$ to neighboring agents. Between triggering instants, neighbors use the last received velocity information $\dot{x}_j^c(t) = \dot{x}_j(t_k^j)$ for control computation. This approach is practically feasible since (1) each agent has access to its own velocity through local measurements; (2) transmitting velocity alongside position adds minimal overhead as both are typically included in standard state messages; and (3) the event-triggering mechanism ensures that velocity information is only transmitted when necessary, maintaining communication efficiency. The assumption that agents can measure their own velocities is standard in formation control literature and aligns with practical implementations where inertial measurement units are commonly available.

Differentiating Equation (9) and using Equation (1) yields:

$$\dot{e}_{1i} = (d_i + b_i)(P_i v_i + Q_i) - \sum_{j=1}^N a_{ij} \dot{x}_j - b_i \dot{x}_0 - (d_i + b_i) \sigma_{i,2} + \sum_{j=1}^N a_{ij} \sigma_{j,2}. \quad (13)$$

Design the virtual control law:

$$\alpha_i(t) = P_i^{-1} \left[\frac{1}{d_i + b_i} \begin{pmatrix} -k_{4i}\check{e}_{1i}(t) - k_{5i}\text{sig}(\check{e}_{1i}(t))^p - k_{6i}\text{sig}(\check{e}_{1i}(t))^q \\ -\Gamma_i \text{sgn}(\check{e}_{1i}(t)) + \sum_{j=1}^N a_{ij}\dot{x}_j + b_i\dot{x}_0 \\ + (d_i + b_i)\sigma_{i,2} - \sum_{j=1}^N a_{ij}\sigma_{j,2} \end{pmatrix} - Q_i \right], \quad (14)$$

$$\check{e}_{1i} = \sum_{j=1}^N a_{ij}(x_i - \sigma_{i,1} - x_j^c + \sigma_{j,1}) + b_i(x_i - \sigma_{i,1} - x_0), \quad t \in [t_k^j, t_{k+1}^j), \quad (15)$$

where $k_{4i}, k_{5i}, k_{6i}, \Gamma_i \in \mathbb{R}^{n \times n}$ are positive definite diagonal matrices.

Remark 5. In practical implementation, to effectively suppress chattering, the discontinuous sgn function in Equations (5), (13), and (18) can be replaced by the continuous and differentiable hyperbolic tangent function $\tanh(z/\delta)$, where $\delta > 0$ is a design parameter. This approximation completely eliminates chattering while preserving sign consistency. The parameter δ controls the steepness of the function near the origin: a smaller δ yields a more accurate approximation of sgn , leading to a smaller steady-state error but potentially higher sensitivity in numerical computation; a larger δ results in smoother system behavior but a correspondingly larger steady-state error. The property of fixed-time convergence is preserved qualitatively, but the bound of the steady-state error becomes dependent on δ .

Design the nonlinear filter:

$$T_{fi}\dot{\alpha}_{fi}(t) = -\tilde{\alpha}_{fi} - \text{sig}(\tilde{\alpha}_{fi})^p - \text{sig}(\tilde{\alpha}_{fi})^q - a_i \text{sgn}(\tilde{\alpha}_{fi}), \quad \alpha_{fi}(0) = \alpha_i(0), \quad (16)$$

where $\tilde{\alpha}_{fi} = \alpha_{fi}(t) - \alpha_i(t)$, $\alpha_{fi}(t) \in \mathbb{R}^n$ is the filtered vector, T_{fi} and a_i are positive design constants. Since the derivative of $\alpha_i(t)$ is unavailable, we pass $\alpha_i(t)$ through the above filter to obtain the differential term $\dot{\alpha}_{fi}(t)$ as a substitute for the derivative of $\alpha_i(t)$, thus resolving the issue of the unavailable derivative.

Step 2: Define the error vector $e_{2i} \in \mathbb{R}^n$ as:

$$e_{2i} = v_i - \alpha_{fi}. \quad (17)$$

Differentiating Equation (16) and using Equation (3) yields:

$$\dot{e}_{2i} = W_i + M_i u_i - \dot{\alpha}_{fi}. \quad (18)$$

Design the distributed event-triggered fixed-time formation control law:

$$u_i = M_i^{-1} \left(-k_{7i}e_{2i}(t) - k_{8i}\text{sig}(e_{2i}(t))^p - k_{9i}\text{sig}(e_{2i}(t))^q - \hat{W}_i + \dot{\alpha}_{fi} \right), \quad (19)$$

where $k_{7i}, k_{8i}, k_{9i} \in \mathbb{R}^{n \times n}$ are positive definite diagonal matrices.

Choose the following Lyapunov function:

$$V_i = \frac{1}{2} \left(e_{1i}^T e_{1i} + e_{2i}^T e_{2i} + \tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi} \right). \quad (20)$$

Differentiating Equation (19) yields:

$$\dot{V}_i = e_{1i}^T \dot{e}_{1i} + e_{2i}^T \dot{e}_{2i} + \tilde{\alpha}_{fi}^T \dot{\tilde{\alpha}}_{fi}. \quad (21)$$

Using Equations (12)–(16) and Young's inequality, we obtain:

$$\begin{aligned} e_{1i}^T \dot{e}_{1i} \leq & -\lambda_{\min}(k_{4i})e_{1i}^T e_{1i} - \lambda_{\min}(k_{5i})e_{1i}^T \text{sig}(\check{e}_{1i})^p - \lambda_{\min}(k_{6i})e_{1i}^T \text{sig}(\check{e}_{1i})^q \\ & - e_{1i}^T \Gamma_i \text{sgn}(\check{e}_{1i}(t)) + \frac{1}{2} \left(\lambda_{\min}(k_{4i}) \sum_{j=1}^N a_{ij} \varepsilon_{1j} + \sum_{j=1}^N a_{ij} \varepsilon_{2j} \right) I_P^T I_P \\ & + \left[l_i(d_i + b_i) + \frac{1}{2} \left(\lambda_{\min}(k_{4i}) \sum_{j=1}^N a_{ij} \varepsilon_{1j} + \sum_{j=1}^N a_{ij} \varepsilon_{2j} \right) \right] e_{1i}^T e_{1i} \\ & + \frac{l_i(d_i + b_i)}{2} (e_{2i}^T e_{2i} + \tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi}), \end{aligned} \quad (22)$$

where $I_P = [1, 1, \dots, 1]^T \in \mathbb{R}^n$, $\lambda_{\min}(P_i) = l_i$. Subtracting Equation (12) from Equation (14) and rearranging yields:

$$\check{e}_{1i}(t) = e_{1i}(t) + \sum_{j=1}^N a_{ij} e_{xj}(t). \quad (23)$$

Considering Equation (22), when the corresponding components of e_{1i} and $\check{e}_{1i}$ have opposite signs, assuming the first component of e_{1i} satisfies $e_{1i,1} < 0$ and the first component of $\check{e}_{1i}$ satisfies $\check{e}_{1i,1} > 0$, from Equations (14) and (22) we obtain $e_{1i,1}(t) + \sum_{j=1}^N a_{ij} \varepsilon_{1j} \geq 0$, hence $-\sum_{j=1}^N a_{ij} \varepsilon_{1j} \leq e_{1i,1} < 0$. Further induction yields $\|e_{1i}\| \leq \sqrt{n} \left| \sum_{j=1}^N a_{ij} \varepsilon_{1j} \right|$, i.e., when the corresponding components of e_{1i} and $\check{e}_{1i}$ have opposite signs, e_{1i} converges to the set $\{e_{1i} \mid \|e_{1i}\| \leq \sqrt{n} \left| \sum_{j=1}^N a_{ij} \varepsilon_{1j} \right|\}$.

In Equation (22), when the corresponding components of e_{1i} and $\check{e}_{1i}$ have the same sign, $\text{sgn}(\check{e}_{1i}(t)) = \text{sgn}(e_{1i}(t))$. Selecting the positive definite design matrix such that:

$$\lambda_{\min}(k_{4i}) \geq l_i(d_i + b_i) + \frac{1}{2} \left(\lambda_{\min}(k_{4i}) \sum_{j=1}^N a_{ij} \varepsilon_{1j} + \sum_{j=1}^N a_{ij} \varepsilon_{2j} \right), \quad (24)$$

Equation (21) can be written as:

$$\begin{aligned} e_{1i}^T \dot{e}_{1i} \leq & -\lambda_{\min}(k_{5i})e_{1i}^T \text{sig}(\check{e}_{1i})^p - \lambda_{\min}(k_{6i})e_{1i}^T \text{sig}(\check{e}_{1i})^q - \lambda_{\min}(\Gamma_i) \|e_{1i}\|^T \\ & + \frac{l_i(d_i + b_i)}{2} (e_{2i}^T e_{2i} + \tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi}) + \frac{1}{2} \left(\lambda_{\min}(k_{4i}) \sum_{j=1}^N a_{ij} \varepsilon_{1j} + \sum_{j=1}^N a_{ij} \varepsilon_{2j} \right) I_P^T I_P. \end{aligned} \quad (25)$$

Substituting Equation (18) into (17) and using $\tilde{W}_i = \hat{W}_i - W_i$, we obtain:

$$\begin{aligned} e_{2i}^T \dot{e}_{2i} \leq & -\lambda_{\min}(k_{7i})e_{2i}^T e_{2i} - \lambda_{\min}(k_{8i})(e_{2i}^T e_{2i})^{\frac{p+1}{2}} \\ & - \lambda_{\min}(k_{9i})(e_{2i}^T e_{2i})^{\frac{q+1}{2}} + \frac{1}{2} (e_{2i}^T e_{2i} + \tilde{W}_i^T \tilde{W}_i). \end{aligned} \quad (26)$$

From Equation (15), we have:

$$\tilde{\alpha}_{fi}^T \dot{\tilde{\alpha}}_{fi} \leq -\frac{1}{T_{fi}} \tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi} - \frac{1}{T_{fi}} (\tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi})^{\frac{p+1}{2}} - \frac{1}{T_{fi}} (\tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi})^{\frac{q+1}{2}} - \|\tilde{\alpha}_{fi}\| \left(\frac{a_i}{T_{fi}} - \|\dot{\alpha}_i\| \right). \quad (27)$$

Substituting Equations (24)–(26) into (20) and using Young's inequality, we obtain:

$$\begin{aligned}
 \dot{V}_i \leq & -\lambda_{\min}(k_{5i})e_{1i}^T \text{sig}(\check{\epsilon}_{1i})^p - \lambda_{\min}(k_{6i})e_{1i}^T \text{sig}(\check{\epsilon}_{1i})^q \\
 & - \lambda_{\min}(k_{8i})(e_{2i}^T e_{2i})^{\frac{p+1}{2}} - \lambda_{\min}(k_{9i})(e_{2i}^T e_{2i})^{\frac{q+1}{2}} \\
 & - \frac{1}{T_{fi}}(\tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi})^{\frac{p+1}{2}} - \frac{1}{T_{fi}}(\tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi})^{\frac{q+1}{2}} - \lambda_{\min}(\Gamma_i)\|e_{1i}\|^T \\
 & - \left(\lambda_{\min}(k_{7i}) - \frac{l_i(d_i + b_i)}{2} - \frac{1}{2} \right) e_{2i}^T e_{2i} \\
 & - \left(\frac{1}{T_{fi}} - \frac{l_i(d_i + b_i)}{2} \right) \tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi} \\
 & - \|\tilde{\alpha}_{fi}\| \left(\frac{a_i}{T_{fi}} - \|\dot{\alpha}_i\| \right) + \frac{1}{2} \tilde{W}_i^T \tilde{W}_i \\
 & + \frac{1}{2} \left(\lambda_{\min}(k_{4i}) \sum_{j=1}^N a_{ij} \epsilon_{1j} + \sum_{j=1}^N a_{ij} \epsilon_{2j} \right) I_P^T I_P.
 \end{aligned} \tag{28}$$

By appropriately selecting the positive definite design parameter matrix k_{7i} , positive design parameters T_{fi} and a_i such that:

$$\begin{aligned}
 \lambda_{\min}(k_{7i}) - \frac{l_i(d_i + b_i)}{2} - \frac{1}{2} &> 0, \\
 \frac{1}{T_{fi}} - \frac{l_i(d_i + b_i)}{2} &> 0, \\
 \frac{a_i}{T_{fi}} - \|\dot{\alpha}_i\| &> 0,
 \end{aligned} \tag{29}$$

we have:

$$\begin{aligned}
 \dot{V}_i \leq & -\lambda_{\min}(k_{5i})(e_{1i}^T e_{1i})^{\frac{p+1}{2}} - \lambda_{\min}(k_{6i})(e_{1i}^T e_{1i})^{\frac{q+1}{2}} \\
 & - \lambda_{\min}(k_{8i})(e_{2i}^T e_{2i})^{\frac{p+1}{2}} - \lambda_{\min}(k_{9i})(e_{2i}^T e_{2i})^{\frac{q+1}{2}} \\
 & - \frac{1}{T_{fi}}(\tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi})^{\frac{p+1}{2}} - \frac{1}{T_{fi}}(\tilde{\alpha}_{fi}^T \tilde{\alpha}_{fi})^{\frac{q+1}{2}} \\
 & - \|e_{1i}\|^T \left(\lambda_{\min}(\Gamma_i) - \lambda_{\min}(k_{5i}) \left| \sum_{j=1}^N a_{ij} \epsilon_{1j} \right|^p - \lambda_{\min}(k_{6i}) \left| \sum_{j=1}^N a_{ij} \epsilon_{1j} \right|^q \right) \\
 & + \frac{1}{2} \left(\lambda_{\min}(k_{4i}) \sum_{j=1}^N a_{ij} \epsilon_{1j} + \sum_{j=1}^N a_{ij} \epsilon_{2j} \right) I_P^T I_P \\
 & + \frac{1}{2} \tilde{W}_i^T \tilde{W}_i.
 \end{aligned} \tag{30}$$

By appropriately selecting the positive definite design parameter matrices Γ_i , k_{5i} , k_{6i} such that:

$$\lambda_{\min}(\Gamma_i) - \lambda_{\min}(k_{5i}) \left| \sum_{j=1}^N a_{ij} \epsilon_{1j} \right|^p - \lambda_{\min}(k_{6i}) \left| \sum_{j=1}^N a_{ij} \epsilon_{1j} \right|^q > 0, \tag{31}$$

and using Lemma 3, Equation (28) can be written as:

$$\dot{V}_i \leq -\mu_{3i} V_i^{\frac{p+1}{2}} - \mu_{4i} V_i^{\frac{q+1}{2}} + \psi + \frac{1}{2} \tilde{W}_i^T \tilde{W}_i, \tag{32}$$

where

$$\begin{aligned}\mu_{3i} &= 2^{\frac{p+1}{2}} \min \left\{ \lambda_{\min}(k_{5i}), \lambda_{\min}(k_{8i}), \frac{1}{T_{fi}} \right\}, \\ \mu_{4i} &= n^{\frac{1-q}{2}} 2^{\frac{q+1}{2}} \min \left\{ \lambda_{\min}(k_{6i}), \lambda_{\min}(k_{9i}), \frac{1}{T_{fi}} \right\}, \\ \psi &= \frac{1}{2} \left(\lambda_{\min}(k_{4i}) \sum_{j=1}^N a_{ij} \varepsilon_{1j} + \sum_{j=1}^N a_{ij} \varepsilon_{2j} \right) I_P^T I_P.\end{aligned}$$

Considering that $\tilde{W}_i = 0$ when $t > T_{doi}$, we have:

$$\dot{V}_i \leq -\mu_{3i} V_i^{\frac{p+1}{2}} - \mu_{4i} V_i^{\frac{q+1}{2}} + \psi. \quad (33)$$

4. Main Result

Theorem 1. Consider a distributed formation control system composed of one virtual leader and N follower agents described by Equations (1) and (2). Under Assumptions 1–3, by designing the fixed-time disturbance observer (4) and (5), virtual control law (13), event-triggered communication mechanism (10) and (11), fixed-time filter (15), and distributed event-triggered fixed-time formation control law (18), the system achieves formation control while ensuring that the formation error converges to a tight set centered at the origin within a fixed settling time.

Proof. Select the Lyapunov function:

$$V = \sum_{i=1}^N (V_i + V_{doi}). \quad (34)$$

Differentiating Equation (31) and using Equations (8) and (30), we obtain the following:

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^N \left(-\mu_{1i} V_{doi}^{\frac{p+1}{2}} - \mu_{2i} V_{doi}^{\frac{q+1}{2}} - \mu_{3i} V_i^{\frac{p+1}{2}} - \mu_{4i} V_i^{\frac{q+1}{2}} + \psi \right) \\ &\leq -\mu_{V1} V^{\frac{p+1}{2}} - \mu_{V2} V^{\frac{q+1}{2}} + \Psi,\end{aligned} \quad (35)$$

where

$$\begin{aligned}\mu_{V1} &= \min \{ \min \{ \mu_{11}, \mu_{31} \}, \dots, \min \{ \mu_{1N}, \mu_{3N} \} \}, \\ \mu_{V2} &= N^{\frac{1-q}{2}} 2^{\frac{1-q}{2}} \min \{ \min \{ \mu_{21}, \mu_{41} \}, \dots, \min \{ \mu_{2N}, \mu_{4N} \} \}, \\ \Psi &= \sum_{i=1}^N \psi.\end{aligned}$$

According to Lemma 2, the system exhibits practical fixed-time stability. Taking $\varepsilon = \frac{1}{2}$, the convergence time satisfies: $T \leq T_{\max} = \frac{2}{\mu_{V1}(1-p)} + \frac{2}{\mu_{V2}(q-1)}$, and $\left\{ \lim_{t \rightarrow T} x \mid V(x) \leq \min \left\{ \left(\frac{2\Psi}{\mu_{V1}} \right)^{\frac{1}{p}}, \left(\frac{2\Psi}{\mu_{V2}} \right)^{\frac{1}{q}} \right\} \right\}$.

Further considering Equation (22), when some corresponding components of e_{1i} and $\check{e}_{1i}$ have the same sign while others have opposite signs, assuming the first component of e_{1i} satisfies $e_{1i,1} < 0$ and the first component of $\check{e}_{1i}$ satisfies $\check{e}_{1i,1} > 0$, while the remaining components have the same sign, we can conclude from the above discussion that e_{1i} converges to the union of $\left\{ e_{1i} \mid \|e_{1i}\| \leq \sqrt{n} \left| \sum_{j=1}^N a_{ij} \varepsilon_{1j} \right| \right\}$ and a tight set centered at the origin, i.e., $\left\{ e_{1i} \mid \|e_{1i}\| \leq \sqrt{n} \left| \sum_{j=1}^N a_{ij} \varepsilon_{1j} \right| \right\}$.

In summary, e_{1i} converges to the set $\left\{e_{1i} \mid \|e_{1i}\| \leq \sqrt{n} \left| \sum_{j=1}^N a_{ij} \varepsilon_{1j} \right| \right\}$ within a fixed settling time satisfying:

$$T \leq T_{\max} = \frac{2}{\mu_{V1}(1-p)} + \frac{2}{\mu_{V2}(q-1)}. \quad (36)$$

□

Theorem 2. For the event-triggered communication mechanism defined by Equations (11) and (12), there exists a positive lower bound $\tau > 0$ for the inter-event times, i.e., $t_{k+1}^j - t_k^j \geq \tau$ for all $k \geq 0$ and $j = 1, \dots, N$, which guarantees the exclusion of Zeno behavior.

Proof. Consider the measurement errors $e_{xj}(t) = x_j(t) - x_j^c(t)$ and $e_{\dot{x}j}(t) = \dot{x}_j(t) - \dot{x}_j^c(t)$. Between consecutive triggering instants t_k^j and t_{k+1}^j , the control inputs remain constant, and the system dynamics are Lipschitz continuous according to Assumptions 2 and 3. Therefore, there exist positive constants $L_x, L_v > 0$ such that:

$$\|\dot{x}_j(t)\| \leq L_x, \quad \|\ddot{x}_j(t)\| \leq L_v, \quad \forall t \geq 0. \quad (37)$$

The time derivatives of the measurement errors satisfy:

$$\frac{d\|e_{xj}\|}{dt} = \frac{e_{xj}^T \dot{e}_{xj}}{\|e_{xj}\|} \leq \|\dot{x}_j(t) - \dot{x}_j^c(t)\| \leq 2L_x, \quad (38)$$

$$\frac{d\|e_{\dot{x}j}\|}{dt} = \frac{e_{\dot{x}j}^T \dot{e}_{\dot{x}j}}{\|e_{\dot{x}j}\|} \leq \|\ddot{x}_j(t) - \ddot{x}_j^c(t)\| \leq 2L_v. \quad (39)$$

Since the errors are reset to zero at each triggering instant t_k^j , the time needed for $\|e_{xj}(t)\|$ to grow from 0 to the threshold ε_{1j} is at least $\frac{\varepsilon_{1j}}{2L_x}$. Similarly, the time needed for $\|e_{\dot{x}j}(t)\|$ to reach ε_{2j} is at least $\frac{\varepsilon_{2j}}{2L_v}$. Therefore, the inter-event time is bounded below by:

$$\tau = \min \left\{ \frac{\varepsilon_{1j}}{2L_x}, \frac{\varepsilon_{2j}}{2L_v} \right\} > 0.$$

This positive lower bound guarantees that an infinite number of triggering events cannot accumulate in finite time, thus excluding Zeno behavior. □

5. Numerical Simulation

5.1. Three-Dimensional Simulation

This section considers a third-order nonlinear multi-agent system composed of one leader and four followers in three-dimensional space. The dynamic equations of follower $i \in \{1, 2, 3, 4\}$ are described by:

$$\begin{cases} \dot{x}_i = P_i v_i + Q_i, \\ \dot{v}_i = M_i u_i + f_i(\cdot) + d_i. \end{cases} \quad (40)$$

$$\text{where } P_i = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, Q_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, M_i = \frac{1}{m_i} I_3 = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

$$f_i(\cdot) = g_i - C_i v_i = \begin{bmatrix} -0.1v_x \\ -0.1v_y \\ -9.8 - 0.2v_z \end{bmatrix}, \text{ with } g_i = \begin{bmatrix} 0 \\ 0 \\ -9.8 \end{bmatrix} \text{ representing gravitational accelera-}$$

tion, $C_i = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$ denoting the air resistance coefficient proportional to velocity, and $d_i = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} = \begin{bmatrix} 0.1 \cdot \text{randn}() \\ 0.1 \cdot \text{randn}() \\ 0.2 \cdot \text{randn}() \end{bmatrix}$, where $\text{randn}()$ represents Gaussian white noise with zero mean and unit variance.

The leader's trajectory is given by:

$$\begin{cases} x = \left(\frac{3}{200}t\right) \cos\left(\frac{\pi}{40}t\right) \\ y = \left(\frac{3}{200}t\right) \sin\left(\frac{\pi}{40}t\right) \\ z = 0.1t \end{cases} \quad (41)$$

A random topology graph is selected based on graph theory, as shown in Figure 1.

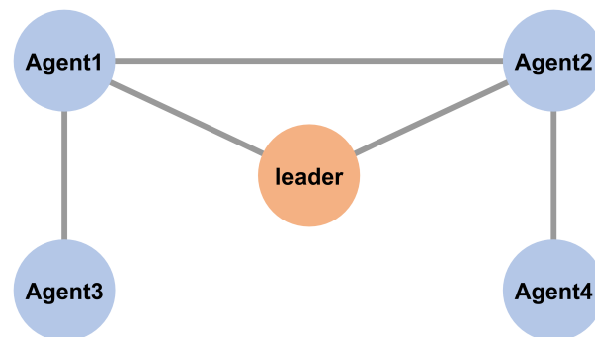


Figure 1. Topology graph of the MAS.

The corresponding adjacency matrix of Figure 1 is $A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. The communication matrix between followers and the leader is $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

The initial conditions are as follows.

- Followers: The initial positions of the follower agents are set as $x_1 = [1, 1, 0]$, $x_2 = [1, -1, 0]$, $x_3 = [-1, 1, 0]$, and $x_4 = [-1, -1, 0]$, and all velocity components of all followers are zero.
- Leader: Position and velocity at origin.

The time-varying formation deviations of each follower are shown in Table 2. Based on the control theory, the parameters are set as listed in Table 3. Furthermore, the maximum convergence time is specified as $T_{\max} = 2$ s, with parameters $p = \frac{1}{3}$ and $q = 3$. The simulation results are demonstrated in the following figures.

Remark 6. First select design parameters $l_i = 1$, $\varepsilon_{1i} = 0.01$ and $\varepsilon_{2i} = 0.01$, then choose design parameter matrices k_{4i} , k_{5i} , k_{6i} and Γ_i to satisfy: (24), $\lambda_{\min}(k_{5i}) \geq 0$, $\lambda_{\min}(k_{6i}) \geq 0$, (31). Select k_{7i} , k_{8i} , and k_{9i} to satisfy: $\lambda_{\min}(k_{7i}) - \frac{l_i(d_i+b_i)}{2} - \frac{1}{2} > 0$, $\lambda_{\min}(k_{8i}) \geq 0$, $\lambda_{\min}(k_{9i}) \geq 0$, thus ensuring the stability of the closed-loop formation control system. Furthermore, through repeated simulations with different parameter values, we observe that selecting larger values for k_{4i} , k_{5i} , k_{6i} ,

k_{7i} , k_{8i} and k_{9i} accelerates convergence speed, but larger Γ_i may cause control signal chattering. In practice, achieving desirable control performance requires iterative adjustment of all parameters.

Table 2. The time-varying formation deviations of each follower.

	Agent 1	Agent 2	Agent 3	Agent 4
σ_x	$0.5 \cos(\frac{\pi}{25}t)$	$-0.5 \sin(\frac{\pi}{25}t)$	$-0.5 \cos(\frac{\pi}{25}t)$	$0.5 \sin(\frac{\pi}{25}t)$
σ_y	$0.5 \sin(\frac{\pi}{25}t)$	$0.5 \cos(\frac{\pi}{25}t)$	$-0.5 \sin(\frac{\pi}{25}t)$	$-0.5 \cos(\frac{\pi}{25}t)$
σ_z	0	0	0	0
σ_{v_x}	$-\frac{\pi}{50}t \sin(\frac{\pi}{25}t)$	$-\frac{\pi}{50}t \cos(\frac{\pi}{25}t)$	$\frac{\pi}{50}t \sin(\frac{\pi}{25}t)$	$\frac{\pi}{50}t \cos(\frac{\pi}{25}t)$
σ_{v_y}	$\frac{\pi}{50}t \cos(\frac{\pi}{25}t)$	$-\frac{\pi}{50}t \sin(\frac{\pi}{25}t)$	$-\frac{\pi}{50}t \cos(\frac{\pi}{25}t)$	$\frac{\pi}{50}t \sin(\frac{\pi}{25}t)$
σ_{v_z}	0	0	0	0

Table 3. The parameters of each follower.

	Agent 1	Agent 2	Agent 3	Agent 4
k_1	d (10, 10, 10)	d (10, 10, 10)	d (10, 10, 10)	d (10, 10, 10)
k_2	d (10, 10, 10)	d (10, 10, 10)	d (10, 10, 10)	d (10, 10, 10)
k_3	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)
k_4	d (40, 40, 40)	d (40, 40, 40)	d (40, 40, 40)	d (40, 40, 40)
k_5	d (2.5, 2.5, 2.5)	d (2.5, 2.5, 2.5)	d (2.5, 2.5, 2.5)	d (2.5, 2.5, 2.5)
k_6	d (2.5, 2.5, 2.5)	d (2.5, 2.5, 2.5)	d (2.5, 2.5, 2.5)	d (2.5, 2.5, 2.5)
k_7	d (10, 10, 10)	d (10, 10, 10)	d (10, 10, 10)	d (10, 10, 10)
k_8	d (0.5, 0.5, 0.5)	d (0.5, 0.5, 0.5)	d (0.5, 0.5, 0.5)	d (0.5, 0.5, 0.5)
k_9	d (0.5, 0.5, 0.5)	d (0.5, 0.5, 0.5)	d (0.5, 0.5, 0.5)	d (0.5, 0.5, 0.5)
Γ	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)
T_f	d (0.2, 0.2, 0.2)	d (0.2, 0.2, 0.2)	d (0.2, 0.2, 0.2)	d (0.2, 0.2, 0.2)
a	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)	d (0.1, 0.1, 0.1)
ε_1	0.01	0.01	0.01	0.01
ε_2	0.01	0.01	0.01	0.01

Note: The notation $d(x, x, x)$ denotes $\text{diag}(x, x, x)$.

Figure 2 illustrates the complete motion trajectories and the formation evolution process of the multi-agent system. It specifically presents: (1) the movement paths of all agents; (2) the formation states at three key instants (initial, intermediate, and final); and (3) a schematic comparison between the initial formation (red dashed line) and the final formation (blue solid line). The figure visually confirms that the agents successfully achieve a stable transition from the initial to the target formation, with the followers maintaining consistent relative positions with the leader. The trajectories are smooth and free of abrupt changes, validating the effectiveness of the control algorithm and the coordination stability of the system.

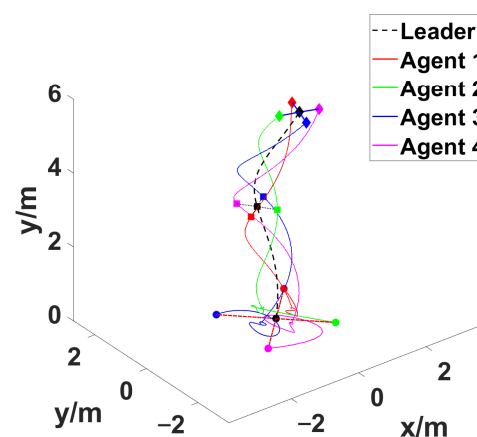


Figure 2. Trajectory graph of the MAS.

Figure 3 shows the position tracking errors of all agents, demonstrating that each follower effectively tracks its desired signal and achieves the time-varying formation objective. Figures 4–6 present the lumped disturbance estimation errors, the actual control input signals, and the event-triggering instants and intervals within the first 20 s, respectively. These figures collectively demonstrate that the proposed control scheme successfully achieves all control objectives while guaranteeing the fixed-time stability of the system.

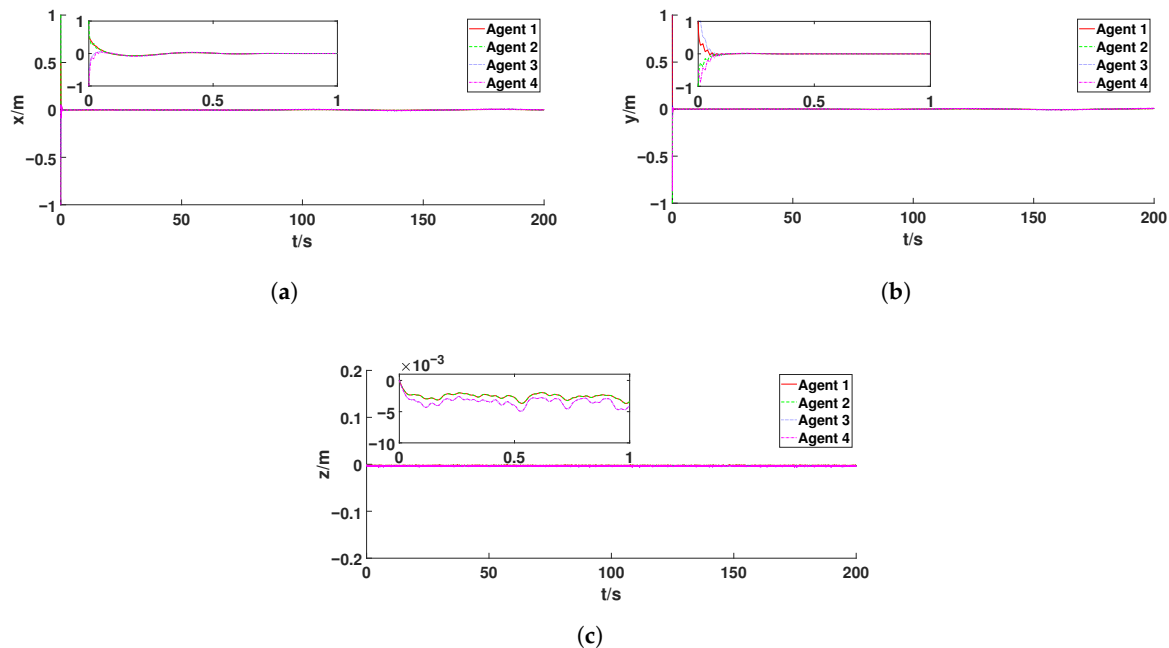


Figure 3. (a) State errors of the followers on the x-axis. (b) State errors of the followers on the y-axis. (c) State errors of the followers on the z-axis.

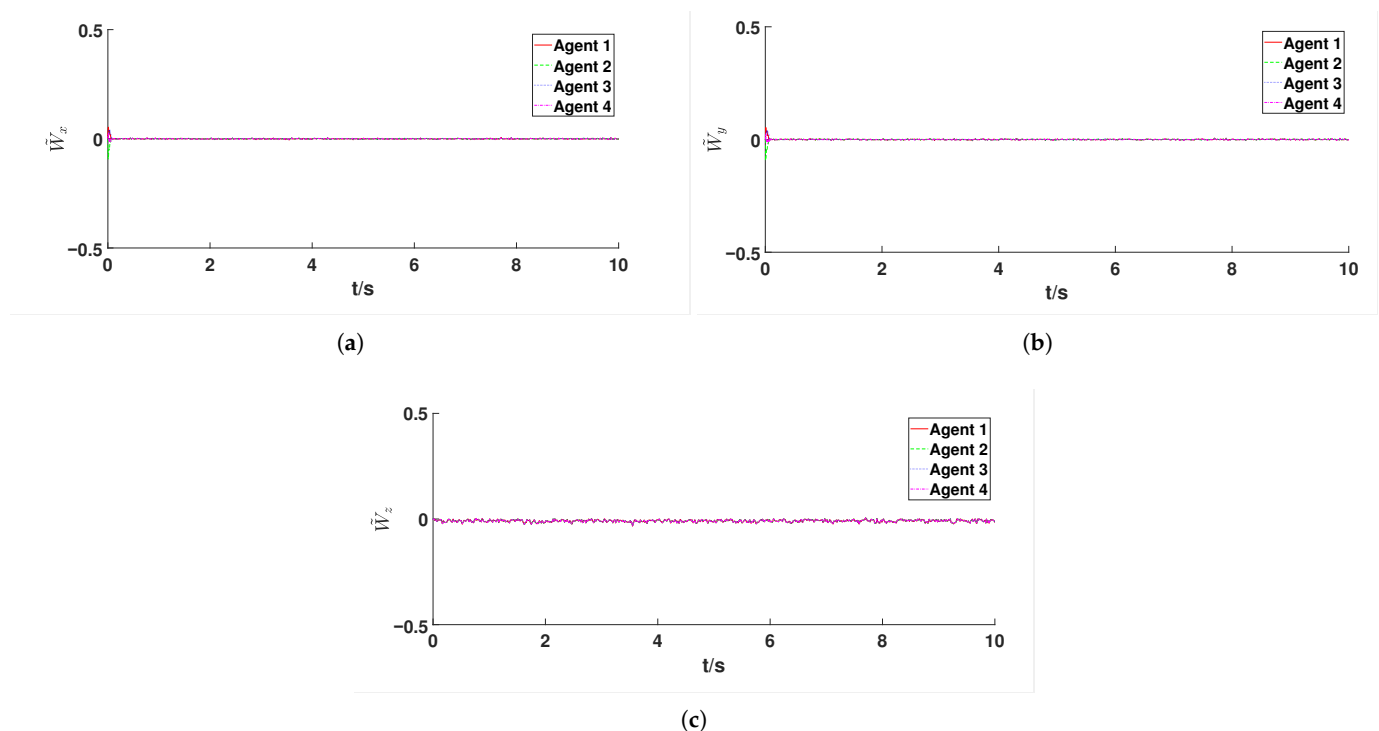


Figure 4. (a) Lumped disturbance errors of the followers on the x-axis. (b) Lumped disturbance errors of the followers on the y-axis. (c) Lumped disturbance errors of the followers on the z-axis.

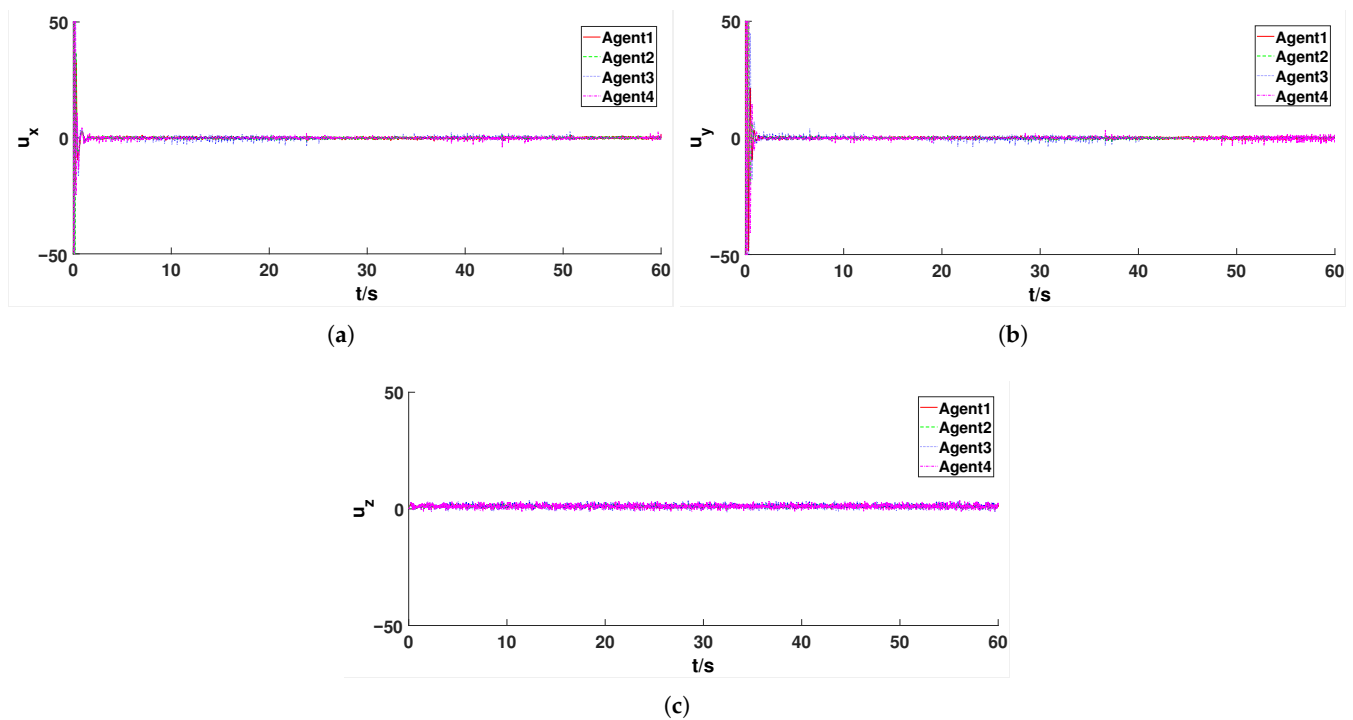


Figure 5. (a) The control input of the followers on the x-axis. (b) The control input of the followers on the y-axis. (c) The control input of the followers on the z-axis.

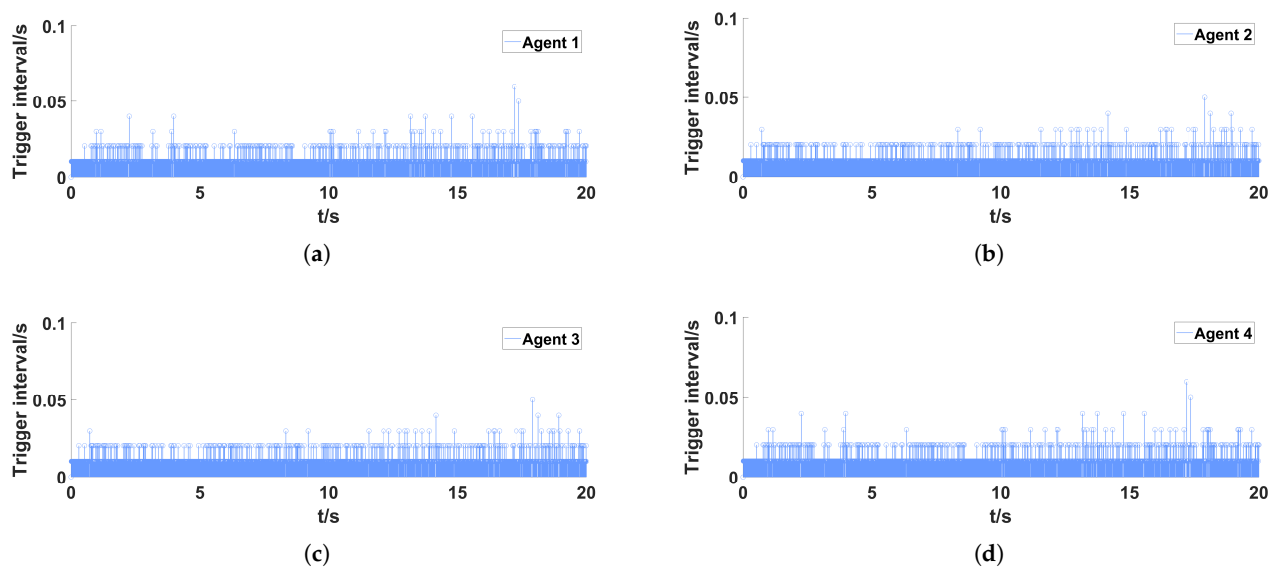


Figure 6. (a) The event-triggering time and triggering interval of the follower agent 1 within the first 20 s. (b) The event-triggering time and triggering interval of the follower agent 2 within the first 20 s. (c) The event-triggering time and triggering interval of the follower agent 3 within the first 20 s. (d) The event-triggering time and triggering interval of the follower agent 4 within the first 20 s.

5.2. Contrast Experiment

To validate the superiority of the proposed control law, we conduct simulation comparisons with the following distributed event-triggered finite-time formation control law:

$$u_{c,i} = M_i^{-1} \left(-k_{7i} e_{2i}(t) - k_{8i} \text{sig}(e_{2i}(t))^p - \hat{W}_i + \dot{\alpha}_{fi} \right), \quad (42)$$

$$T_{fi} \dot{\alpha}_{c,fi}(t) = -\tilde{\alpha}_{c,fi} - \text{sig}(\tilde{\alpha}_{c,fi})^p - a_i \text{sgn}(\tilde{\alpha}_{c,fi}), \quad \alpha_{c,fi}(0) = \alpha_{c,i}(0), \quad (43)$$

$$\alpha_{c,i}(t) = P_i^{-1} \left[\frac{1}{d_i + b_i} \left(-k_{4i}\check{e}_{1i}(t) - k_{5i}\text{sig}(\check{e}_{1i}(t))^p - \Gamma_i \text{sgn}(\check{e}_{1i}(t)) \right) + \sum_{j=1}^N a_{ij}\dot{x}_j + b_i\dot{x}_0 + (d_i + b_i)\sigma_{i,2} - \sum_{j=1}^N a_{ij}\sigma_{j,2} \right] - Q_i, \quad (44)$$

where all symbols and parameter values maintain the same meanings and settings as their counterparts in the proposed control law, with the subscript c denoting the comparative controller.

The simulation and comparison results, depicted by blue solid lines and red solid lines, respectively, in Figure 7, demonstrate that compared with the distributed event-triggered finite-time formation control law, the designed formation control law enables the formation errors of the four follower agents to converge more rapidly to a neighborhood centered at zero.

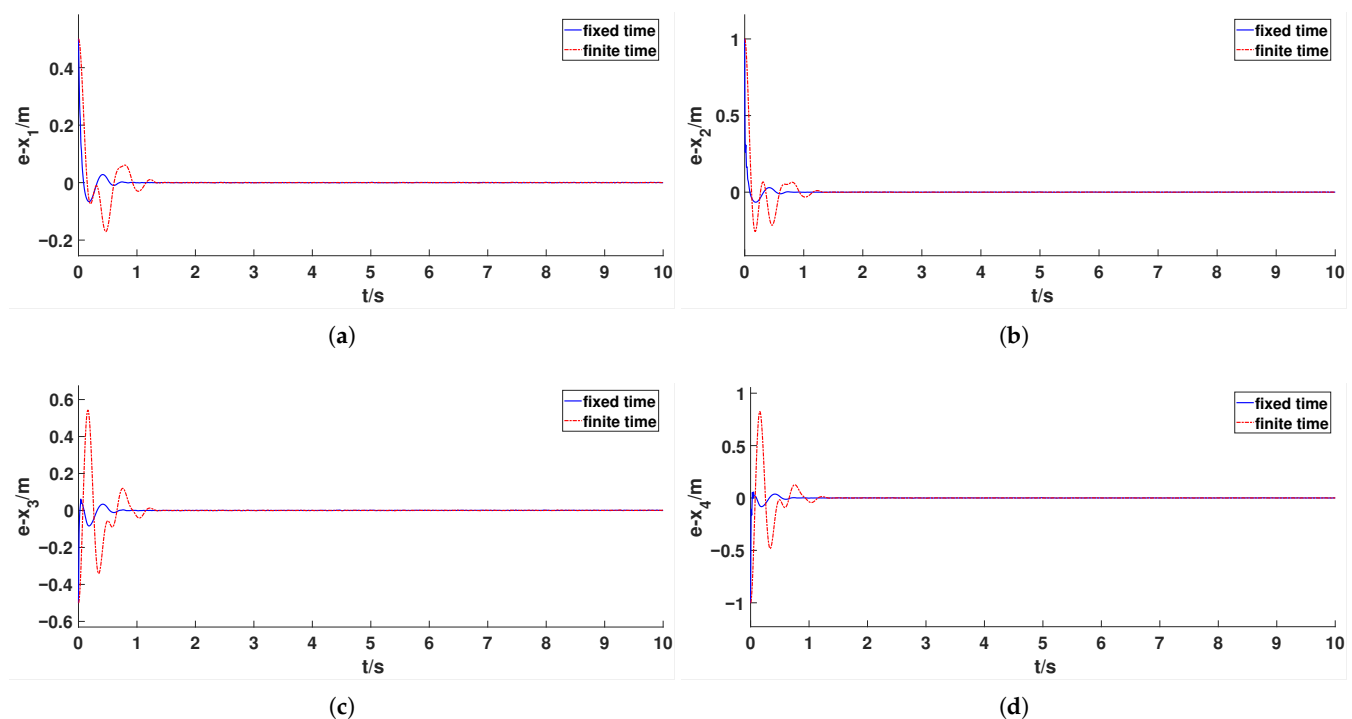


Figure 7. (a) The comparison results of the follower agent 1 in the two situations. (b) The comparison results of the follower agent 2 in the two situations. (c) The comparison results of the follower agent 3 in the two situations. (d) The comparison results of the follower agent 4 in the two situations.

Table 4 presents a comprehensive performance comparison between the proposed control scheme and the comparative method. The experimental results demonstrate that the proposed approach achieves significant improvements in several critical performance metrics. Firstly, in terms of convergence speed, the proposed method exhibits a 49.3% acceleration, reducing the convergence time from 1.262 s to 0.639 s. Secondly, regarding computational and communication efficiency, the proposed event-triggered mechanism yields a 93.4% extension in the average triggering interval and a 24.1% reduction in total triggering events, which substantially alleviates the computational burden and communication overhead. Although the proposed method shows slight increments in post-convergence tracking accuracy metrics, these marginal differences are within acceptable limits and do not compromise the overall control performance. In summary, the proposed control strategy effectively balances rapid convergence and resource efficiency, making it

particularly suitable for practical applications with limited computational resources and communication bandwidth.

Table 4. Performance comparison between proposed and comparative methods.

Performance Metric	Proposed Method	Comparative Method	Improvement
Convergence Time (s)	0.639	1.262	49.3% faster
Average Error (post-convergence)	0.000603	0.000569	–
Maximum Absolute Error (post-convergence)	0.002663	0.002427	–
Control Input Variance (post-convergence)	4.959149	4.852426	–
Average Triggering Interval (s)	0.0238	0.0123	93.4% longer
Total Triggering Count	1265	1667	24.1% reduction

6. Conclusions

This paper proposes a distributed event-triggered fixed-time time-varying formation control scheme for a class of nonlinear multi-agent systems with model uncertainties and unknown time-varying disturbances. By designing a fixed-time disturbance observer to accurately estimate lumped disturbances and integrating event-triggered mechanisms with dynamic surface control techniques, a distributed control law is constructed that ensures time-varying formation tracking while significantly reducing communication and computational resource consumption. Theoretical analysis demonstrates that the closed-loop system achieves practical fixed-time stability and effectively avoids Zeno behavior. Simulation results indicate that the proposed scheme outperforms traditional finite-time control methods in terms of convergence speed, control accuracy, and resource efficiency.

The methodological innovations of this work primarily lie in the systematic integration and comprehensive proof of existing techniques including fixed-time control, event-triggered mechanisms, and disturbance observers, rather than in radical breakthroughs in individual technical components. The principal contribution resides in providing a cohesive solution framework that addresses the complex problem of cooperative control under simultaneous challenges of time-varying formations, model uncertainties, and unknown disturbances. The advantages of the proposed approach are manifested in its guaranteed convergence within a predetermined time independent of initial conditions, substantial reduction in communication resource consumption, and effective suppression of compound disturbances. However, certain limitations should be acknowledged, including the relative complexity of parameter tuning and specific requirements on the continuity of system dynamics. This comprehensive design and rigorous proof offer a reference with both theoretical soundness and practical utility for cooperative control of multi-agent systems in complex scenarios.

This scheme can be extended to cooperative control of similar nonlinear systems such as UAVs and UGVs, and future work will further investigate formation control problems in the presence of communication delays. For time-varying communication delays with a known upper bound $\tau_{\max}$, the core solution involves introducing a fixed-time state predictor to compensate for the lack of state information during the delay period. For packet drops following a Bernoulli model (with loss probability $p < 1$), performance is maintained by incorporating a compensation term related to p into the event-triggering condition. Under this extended framework, the upper bound of the system's convergence time is proven to be the sum of the fixed convergence time T under ideal conditions

and the maximum delay $\tau_{\max}$, thereby preserving the core characteristic of fixed-time convergence. At the level of theoretical proof, the stability analysis requires extending the original Lyapunov function into two forms: firstly, a Lyapunov–Krasovskii functional that includes an integral of states over the delay interval to handle the delay effect; secondly, a stochastic Lyapunov function (in expectation form) to address the randomness introduced by packet drops. Stability is proven by demonstrating that the expectation of this functional or stochastic function converges within a fixed time. Regarding the avoidance of Zeno behavior, the lower bound on the inter-event time needs to be recalculated. The new lower bound will be the positive lower bound h derived under ideal conditions minus the maximum delay $\tau_{\max}$. As long as the design ensures $h > \tau_{\max}$, a strictly positive time interval between any two actual triggers is guaranteed, thereby precluding Zeno behavior.

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References

1. Tang, J.; Duan, H.; Lao, S. Swarm intelligence algorithms for multiple unmanned aerial vehicles collaboration: A comprehensive review. *Artif. Intell. Rev.* **2023**, *56*, 4295–4327. [\[CrossRef\]](#)
2. Hamidi, H.; Kamankesh, A. An approach to intelligent traffic management system using a multi-agent system. *Int. J. Intell. Transp. Syst. Res.* **2018**, *16*, 112–124. [\[CrossRef\]](#)
3. Marco, K.; Goyal, S.; Kaur, R. Underwater wireless sensor network-based static and mobile node deployment using jellyfish optimization algorithm. *Int. J. Distrib. Sens. Netw.* **2025**, *2025*, 9964297. [\[CrossRef\]](#)
4. Qu, X.; Li, C.; Jiang, S.; Liu, G.; Zhang, R. Multi-agent reinforcement learning-based cooperative encirclement control of autonomous surface vehicles against multiple targets. *J. Mar. Sci. Eng.* **2025**, *13*, 1558. [\[CrossRef\]](#)
5. Olfati-Saber, R.; Fax, J.A.; Murray, R.M. Consensus and cooperation in networked multi-agent systems. *Proc. IEEE* **2007**, *95*, 215–233. [\[CrossRef\]](#)
6. Liu, Y.; Liu, J.; He, Z.; Li, Z.; Zhang, Q.; Ding, Z. A survey of multi-agent systems on distributed formation control. *Unmanned Syst.* **2024**, *12*, 913–926. [\[CrossRef\]](#)
7. Chen, Q.; Wang, Y.; Jin, Y.; Wang, T.; Nie, X.; Yan, T. A survey of an intelligent multi-agent formation control. *Appl. Sci.* **2023**, *13*, 5934. [\[CrossRef\]](#)
8. Farooq, A.; Xiang, Z.; Chang, W.J.; Aslam, M.S. Recent advancement in formation control of multi-agent systems: A review. *Comput. Mater. Contin.* **2025**, *83*, 3. [\[CrossRef\]](#)
9. Yan, C.; Zhang, W.; Guo, H.; Zhu, F.; Qian, Y. Distributed adaptive time-varying formation control for Lipschitz nonlinear multi-agent systems. *Trans. Inst. Meas. Control* **2022**, *44*, 272–285. [\[CrossRef\]](#)
10. Ren, Y.; Zhang, K.; Jiang, B.; Cheng, W.; Ding, Y. Distributed fault-tolerant time-varying formation control of heterogeneous multi-agent systems. *Int. J. Robust Nonlinear Control* **2022**, *32*, 2864–2882. [\[CrossRef\]](#)
11. Mohamed, A.; Marino, M.; Watkins, S.; Jaworski, J.; Jones, A. Gusts encountered by flying vehicles in proximity to buildings. *Drones* **2023**, *7*, 22. [\[CrossRef\]](#)
12. Guo, S.; Chen, M.; Pang, W. Path planning for autonomous underwater vehicles based on an improved artificial jellyfish search algorithm in multi-obstacle ocean current environment. *IEEE Access* **2023**, *11*, 31010–31023. [\[CrossRef\]](#)
13. Zhu, B.; Karimi, H.R.; Zhang, L.; Zhao, X. Neural network-based adaptive reinforcement learning for optimized backstepping tracking control of nonlinear systems with input delay. *Appl. Intell.* **2025**, *55*, 129. [\[CrossRef\]](#)
14. Li, J.; Du, J.; Chang, W.J. Robust time-varying formation control for underactuated autonomous underwater vehicles with disturbances under input saturation. *Ocean Eng.* **2019**, *179*, 180–188. [\[CrossRef\]](#)

15. Qu, X.R.; Liang, X.; Hou, Y.H. Fuzzy state observer-based cooperative path-following control of autonomous underwater vehicles with unknown dynamics and ocean disturbances. *Int. J. Fuzzy Syst.* **2021**, *23*, 1849–1859. [[CrossRef](#)]
16. Amato, F.; Ambrosino, R.; Ariola, M. *Finite-Time Stability and Control*; Springer: London, UK, 2014.
17. Yu, J.; Yu, S.; Li, J.; Yan, Y. Fixed-time stability theorem of stochastic nonlinear systems. *Int. J. Control* **2019**, *92*, 2194–2200. [[CrossRef](#)]
18. Yao, Q. Disturbance observer-based robust fixed-time integrated trajectory tracking control for space manipulator. *Robotica* **2022**, *40*, 3214–3232. [[CrossRef](#)]
19. Zhang, D.; Hu, J.; Cheng, J.; Wu, Z.G.; Yan, H. A novel disturbance observer based fixed-time sliding mode control for robotic manipulators with global fast convergence. *IEEE/CAA J. Autom. Sin.* **2024**, *11*, 661–672. [[CrossRef](#)]
20. Yan, F.; Chen, G. Distributed consensus and coordination control of networked multi-agent systems. In *Consensus and Synchronization in Complex Networks*; Springer: Berlin/Heidelberg, Germany, 2012; pp. 51–68.
21. Wang, J.; Du, J.; Li, J. Distributed finite-time velocity-free robust formation control of multiple underactuated AUVs under switching directed topologies. *Ocean Eng.* **2022**, *266*, 112967. [[CrossRef](#)]
22. Chen, Q.; Jin, Y.; Wang, T.; Wang, Y.; Yan, T.; Long, Y. UAV formation control under communication constraints based on distributed model predictive control. *IEEE Access* **2022**, *10*, 126494–126507. [[CrossRef](#)]
23. Li, J.; Du, J.; Li, Y.; Xu, G. Distributed robust prescribed performance 3-D time-varying formation control of underactuated AUVs under input saturations and communication delays. *IEEE J. Ocean. Eng.* **2023**, *48*, 649–662. [[CrossRef](#)]
24. Ong, P.; Cortés, J. Performance-barrier-based event-triggered control with applications to network systems. *IEEE Trans. Autom. Control* **2023**, *69*, 4230–4244. [[CrossRef](#)]
25. Kim, J.H.; Yoo, S.J. Distributed event-triggered adaptive output-feedback formation tracking of uncertain underactuated underwater vehicles in three-dimensional space. *Appl. Math. Comput.* **2022**, *424*, 127046. [[CrossRef](#)]
26. Wang, H.; Su, B. Event-triggered formation control of AUVs with fixed-time RBF disturbance observer. *Appl. Ocean Res.* **2021**, *112*, 102638. [[CrossRef](#)]
27. Zhao, W.; Xia, Y.; Zhai, D.H.; Cui, B. Adaptive event-triggered coordination control of unknown autonomous underwater vehicles under communication link faults. *Automatica* **2023**, *158*, 111277. [[CrossRef](#)]
28. Huang, D.; Huang, T.; Qin, N.; Li, Y.; Yang, Y. Finite-time control for a UAV system based on finite-time disturbance observer. *Aerosp. Sci. Technol.* **2022**, *129*, 107825. [[CrossRef](#)]
29. Guo, T.; Liu, Y.; Hu, H.X. Adaptive connectivity-preserving formation control with a dynamic event-triggering mechanism. *J. Frankl. Inst.* **2025**, *362*, 107331. [[CrossRef](#)]
30. Wang, F.; Lai, G. Fixed-time control design for nonlinear uncertain systems via adaptive method. *Syst. Control Lett.* **2020**, *140*, 104704. [[CrossRef](#)]
31. Liu, M.R.; Wu, L.B.; Wang, H.Q.; Guo, L.D.; Huang, S.J. Disturbance observer-based nonsingular fixed-time fuzzy adaptive event-triggered output feedback control of uncertain nonlinear systems. *Fuzzy Sets Syst.* **2025**, *498*, 109132. [[CrossRef](#)]

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