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Coordinated Optimization of Train Timetables and Maintenance Windows for Plateau Railways

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Abstract

The conventional sequential planning of train timetables and maintenance windows cannot fully capture their interactions, particularly on plateau railways where infrastructure maintenance requirements are more demanding. This study develops a coordinated optimization model that jointly determines train arrival and departure times and the timing and duration of segmented rectangular maintenance windows. The model simultaneously minimizes total train travel time and maximizes maintenance-window duration while considering train departure and arrival time windows, section running times, dwell times, train headways, maintenance-window timing requirements, and safety separation between train operations and maintenance activities. A linear weighted-sum method is used to scalarize the bi-objective model, and a Benders-decomposition-based solution method is developed by separating discrete operating-sequence decisions from continuous timing decisions. The method incorporates optimality, feasibility, and conflict cuts to iteratively coordinate the master problem and subproblem. A case study of a certain plateau railway in China, involving 22 passenger trains and 25 section-based maintenance windows, demonstrates the applicability of the proposed approach. Sensitivity analysis reveals a clear trade-off between train operating efficiency and maintenance provision. With a weighting coefficient of $\lambda = 0.8$, the algorithm obtains a representative coordinated solution with a total train travel time of 8488.37 min and a maintenance-window duration of 200.64 min, illustrating the trade-off between transport efficiency and maintenance requirements. The proposed framework provides a practical modeling and solution approach for coordinated train timetabling and maintenance-window planning on plateau railways.

Keywords: plateau railway; train timetable; maintenance window; coordinated optimization; Benders decomposition

MSC: 90C11; 90C29; 90B06

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1. Introduction

The high-quality development of railways has placed higher demands on train operation organization, infrastructure condition maintenance, and operational safety. The Outline for Advancing Railway Development in Building China into a Transport Power in the New Era emphasizes the backbone role of railways in the integrated transport system

and calls for continuous improvements in transport service quality and safety. Against this background, the coordinated compilation of train timetables and maintenance windows has become a key issue in the transport organization of plateau railways.

Train timetables mainly specify train sequences, arrival and departure times, and the allocation of line capacity, whereas maintenance windows reserve the necessary time for infrastructure inspection, maintenance, and fault repair. Although they serve different purposes, both require the use of limited temporal–spatial resources of the railway line. Therefore, strong interactions and constraints exist between them during the planning process [1,2]. For plateau railways, the complex natural environment imposes additional pressure on infrastructure maintenance and further complicates the coordination between train timetables and maintenance windows. To address this issue, researchers have conducted extensive studies from the perspectives of research problems, mathematical models, and solution algorithms. Cordeau et al. [3] systematically reviewed models for train timetable optimization, classified them according to different types of optimization problems, and analyzed their characteristics in railway network and single-line scenarios. Focusing on passenger train timetable optimization, Caimi et al. [4] discussed the major factors that need to be considered in timetable optimization and summarized typical approaches for improving timetable robustness, including stochastic optimization and light robustness models. They also examined the application of these models in practical railway operations. Regarding the development of train timetable optimization models and algorithms, Ghoseiri et al. [5] formulated a bi-objective train timetable optimization model to minimize train travel time and operational energy consumption. A constraint transformation method was used to solve the model and obtain the corresponding Pareto frontier. From the perspective of improving railway capacity utilization, Murali et al. [6] developed an integer programming model for train timetable optimization and employed a genetic algorithm to solve it. Zhang et al. [7] investigated an integrated train stop planning and timetabling problem. A time–space network was adopted to represent train and passenger movements, and an integer linear programming model was formulated to jointly optimize train timetables and stopping plans while considering uncertainty in passenger demand across the railway network. The model was decomposed into two subproblems and solved using a variable neighborhood search algorithm. To make effective use of residual transport capacity, Qi et al. [8] proposed a train formation scheme for mixed passenger and freight transportation on high-speed railways and jointly optimized train stopping plans and timetables. A quadratic programming model was developed to accommodate time-dependent passenger and freight demands. Maintenance planning mainly addresses the scheduling of maintenance tasks within limited maintenance windows [9,10]. Existing studies have largely focused on maintenance cost control, maintenance window duration, and resource utilization efficiency, with particular attention to the rational allocation of maintenance window resources while satisfying maintenance requirements and safety constraints. Budai et al. [11] developed an optimization model that minimizes maintenance costs by grouping maintenance activities and carrying them out in a coordinated manner. A greedy algorithm was employed to reduce the impact of maintenance windows on train delays on single-track railways. Oudshoorn et al. [12] studied large-scale railway maintenance planning and formulated an optimization model with the objective of minimizing maintenance costs. Evolutionary algorithms, greedy heuristics, and a hybrid algorithm were employed for solution, and the results showed that the hybrid algorithm produced higher-quality maintenance plans. Unexpected disruptions during maintenance may cause substantial delays to scheduled trains and significantly reduce the service quality experienced by passengers. Unlike most studies that focus on deterministic maintenance planning, Ji et al. [13] developed a two-stage stochastic programming approach for train scheduling under uncertain maintenance plans. As train

frequencies continue to increase, the demand for maintenance windows for infrastructure inspection and maintenance becomes increasingly significant. Consequently, train timetables and maintenance windows can no longer be effectively planned through a simple sequential process [14]. Regarding the coordinated optimization of train timetables and maintenance windows, researchers have conducted extensive studies focusing primarily on mathematical model formulation and solution algorithm design. Zhang et al. [15] developed an integrated optimization model for train timetabling and maintenance planning based on a hierarchical time–space network, with the objectives of minimizing total passenger travel time and maintenance costs. The model incorporates practical constraints related to train headways, station capacity, maintenance duration, and maintenance costs. A solution algorithm combining constraint transformation and Lagrangian relaxation was designed to solve the model. Luan et al. [16] modeled maintenance windows as virtual trains and extended the conventional train timetabling model to achieve coordinated optimization of train timetables and maintenance windows. Yang et al. [17] introduced a distributionally robust optimization framework for the coordinated optimization of train timetables and maintenance windows. An integrated model with two types of binary variables and cross-resolution consistency constraints was formulated to robustly capture the joint decision-making process. The model was further reformulated as a multi-commodity network flow problem, and a decomposition heuristic based on the alternating direction method of multipliers was developed. Numerical experiments demonstrated the effectiveness and convergence of the proposed approach under uncertainty. Considering uncertainty in disruption duration, Yang et al. [18] proposed a maintenance adjustment strategy for major disruptions to address the rescheduling of train timetables and maintenance windows. A rolling-horizon framework was adopted to meet real-time rescheduling requirements, and a distributionally robust optimization model was developed. Han et al. [19] proposed an integrated optimization approach in which train stopping decisions were incorporated into the timetabling process and maintenance windows were represented as "virtual trains" scheduled together with actual trains. A mixed-integer linear programming model was formulated by considering container demand allocation during service, the avoidance of timetable conflicts among trains, and the minimization of disruptions caused by maintenance activities. To reduce the impact of maintenance on train operations, Ye et al. [20] proposed an integer linear programming model that jointly optimizes daily train timetables, maintenance window settings, and maintenance schedules. The maintenance duration and activities within each daily maintenance window were determined according to dynamic maintenance requirements and daily train traffic. A Lagrangian-relaxation-based heuristic algorithm was developed to solve the model efficiently.

Existing studies have mainly developed single-objective or multi-objective models with objectives such as minimizing train travel time and maintenance window costs or maximizing maintenance window duration, while establishing a solid basis for constraint formulation and algorithm design. Constraints related to train departure times, section running times, station dwell times, and train operating sequences provide useful references for developing a coordinated optimization model of train timetables and maintenance windows for plateau railways. However, under special operating conditions such as those of plateau railways, train operations are more strongly affected by line conditions and environmental factors, while maintenance requirements are also more demanding. Existing models and methods therefore remain insufficient to fully capture these operational characteristics. To address this gap, this study investigates the coordinated optimization of train timetables and maintenance windows for plateau railways, with particular emphasis on the interactions between train operations and maintenance activities under special operating conditions. However, the coordination of train timetables and maintenance windows under the opera-

tional conditions of plateau railways requires further consideration of the specific running and maintenance requirements of the railway. In particular, section-level running times, train headways and safety separations, and maintenance-window requirements directly affect the feasible coordination between train operations and maintenance activities. Based on these considerations, a coordinated optimization model for train timetables and maintenance windows is developed for plateau railway operations, and a solution method based on Benders decomposition is designed according to the structure of the model. The applicability of the proposed model and algorithm is examined using an anonymized plateau railway in China as a case study, with the aim of providing theoretical and methodological support for the coordinated optimization of train timetables and maintenance windows on plateau railways. The main contribution lies in integrating the plateau railway operating characteristics with the coordinated optimization of train timetables and maintenance windows, rather than in the use of Benders decomposition or the case application alone.

2. Problem Description

Plateau railways operate under harsh climatic conditions and complex geological environments, resulting in more demanding infrastructure service conditions, greater maintenance requirements, and higher maintenance planning complexity than conventional railways. Maintenance windows provide essential time periods for the inspection and maintenance of railway infrastructure, power supply systems, and communication and signaling equipment. In contrast, train timetables determine train operating sequences, arrival and departure times, stopping patterns, and train movements over railway sections. Both train operations and maintenance activities rely on the utilization of limited line capacity, resulting in significant interactions and mutual constraints during the planning process. Accordingly, this study investigates the coordinated optimization of train timetables and maintenance windows for plateau railways. Specifically, given the railway line configuration, station layout, train stopping patterns, and relevant technical operating standards, the arrival and departure times of each train at each station are jointly determined together with the start time, end time, and duration of each maintenance window, so as to coordinate train operations with maintenance window arrangements.

In railway transport planning, train timetables and maintenance windows are generally prepared using two basic approaches: sequential planning and integrated planning. Sequential planning treats train timetabling and maintenance window scheduling as two relatively independent stages carried out in sequence. Specifically, the train timetable may be prepared first and the maintenance windows arranged afterward, or the maintenance windows may be determined first and the train timetable subsequently adjusted. In contrast, integrated planning incorporates both decisions into a unified framework and optimizes them simultaneously. These two planning approaches differ mainly in their decision sequence and degree of coordination, leading to different mechanisms for arranging train operations and maintenance windows.

Taking the sequential planning approach in which the train timetable is prepared first and the maintenance window plan is arranged afterward as an example, as shown in Figure 1, once the operating times of up-direction trains i_2 and i_3 have been fixed and are no longer adjustable, the subsequent maintenance window can only be scheduled within the available time gaps between the predetermined train paths. To avoid conflicts between maintenance activities and train operations, the maintenance window is therefore often constrained to the interval between two consecutive trains. This may limit the available maintenance duration, making it difficult to fully satisfy the requirements of comprehensive maintenance activities and potentially creating safety risks.

If train timetabling and maintenance window scheduling are incorporated into a unified framework for coordinated optimization, both train operating times and the start and end times of maintenance windows can be treated as decision variables and determined jointly. By appropriately adjusting the schedules of selected trains, additional temporal flexibility can be created for maintenance-window placement while retaining feasible train operations. As shown in Figure 2, by moderately extending the dwell time of train i_1 at station k_2, train i_2 can overtake train i_1 at station k_2, allowing the departure time of train i_2 to be shifted earlier. Similarly, by adjusting the schedules of trains i_3 and i_4, train i_3 can overtake train i_4 at station k_2, thereby allowing the departure time of train i_3 to be shifted later. In this schematic example, the coordinated adjustment creates a longer feasible maintenance interval while maintaining feasible train operations.

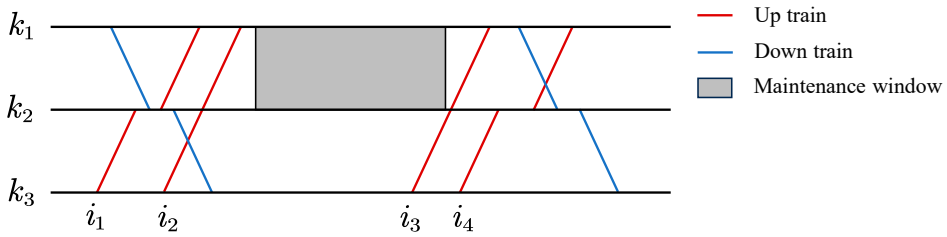


Figure 1. Schematic illustration of the sequential planning of train timetables and maintenance windows.

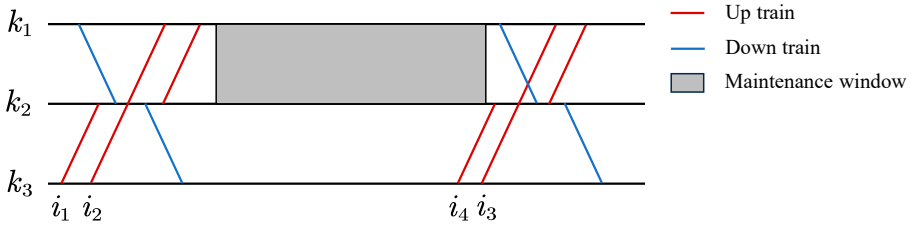


Figure 2. Schematic illustration of the integrated planning of train timetables and maintenance windows.

The schematic comparison illustrates that integrated planning provides additional decision-making flexibility by allowing train operating times and maintenance-window arrangements to be adjusted simultaneously. Figures 1 and 2 are intended to illustrate the difference in coordination mechanisms rather than to establish numerical superiority over sequential planning.

3. Coordinated Optimization Model

Plateau railways operate under complex conditions, with substantial demands for infrastructure maintenance and strong interactions between train operations and maintenance window scheduling. The traditional sequential planning approach cannot fully capture the coupling between train timetables and maintenance windows, which may result in reduced train operating efficiency or insufficient time for maintenance activities. To address this issue, a coordinated optimization model for train timetables and maintenance windows on plateau railways is developed. The model jointly considers train schedule adjustment and maintenance window allocation to better balance train operating efficiency and maintenance requirements.

3.1. Model Assumptions

(1) Only passenger train operations are considered, while the effects of freight trains on train timetabling and maintenance window scheduling are excluded. This study focuses on the coordination between passenger train timetables and maintenance windows, while freight train operations are beyond the scope of the present study.

(2) The maintenance windows considered are section-based maintenance windows at the train timetable design stage, while interstation maintenance windows are not considered.
(3) The planning horizon is set to $[0, 1440]$ min, corresponding to a study period of 1 day.
(4) Segmented rectangular maintenance windows are adopted as the maintenance window form, and maintenance windows are scheduled within the daytime period of 09:00–19:00 to focus on their coordination with passenger train operations.
(5) Railway sections are assumed to serve as the basis for dividing maintenance windows into segments, with each section corresponding to one maintenance window segment. To maintain a consistent planning standard at the timetable compilation stage, all maintenance window segments are assigned a common duration, while their timings are optimized separately for the corresponding railway sections.
(6) A double-track railway is considered. Up-direction and down-direction trains operate independently on separate tracks. Maintenance activities are arranged on the affected track, and train operations on the same track are required to satisfy the prescribed safety separation from the maintenance window.
(7) The number of trains of each type, train routing plans, and stopping patterns are assumed to be known in advance. The study therefore focuses only on train timetable optimization.
(8) Rolling stock circulation and utilization constraints are not considered, as the present study focuses on the coordination of train timetables and maintenance windows at the infrastructure and timetable planning level.

3.2. Notation

The notation used in the model and the corresponding definitions are summarized in Table 1.

Table 1. Description of model notation.

Type	Symbol	Description
Set	I	Set of all trains i, $I = I' \cup I''$
	I'	Set of EMU trains i', $I' \subseteq I$
	I''	Set of conventional-speed passenger trains i'', $I'' \subseteq I$
	K	Set of stations k
	Ω	Set of maintenance windows ω
Parameter	m	Number of stations
	M	A sufficiently large positive constant
	$\alpha_{i,1}^{\min}$	Earliest departure time of train i at the origin station
	$\alpha_{i,1}^{\max}$	Latest departure time of train i at the origin station
	$\alpha_{i,m}^{\min}$	Earliest arrival time of train i at the destination station
	$\alpha_{i,m}^{\max}$	Latest arrival time of train i at the destination station
	$T_{i,k}^{\min}$	Minimum dwell time of train i at station k
	$S_{i,k}$	Stop indicator for train i at station k: 1 if train i stops at station k, and 0 otherwise
	$T_{k,k+1}^{\min}$	Minimum running time between stations k and $k+1$
	$T_{i,k}^{ac}$	Additional acceleration time of train i at station k
	$T_{i,k}^{de}$	Additional deceleration time of train i at station k
	T^{aa}	Minimum arrival headway between adjacent trains at a station
	T^{dd}	Minimum departure headway between adjacent trains at a station
	$T_{dd}^{\min}$	Minimum departure headway between adjacent trains at the origin station
	$\alpha_{\omega,start}^{\min}$	Earliest start time of maintenance window ω
	$\alpha_{\omega,start}^{\max}$	Latest start time of maintenance window ω
	$\alpha_{\omega,end}^{\min}$	Earliest end time of maintenance window ω
	$\alpha_{\omega,end}^{\max}$	Latest end time of maintenance window ω
	$T_{\Omega}^{\min}$	Minimum duration of maintenance window ω
	$T_{\Omega}^{\max}$	Maximum duration of maintenance window ω
	T_{Ω}^{safe}	Safety interval between a train and a maintenance-window boundary

Table 1. *Cont.*

Type	Symbol	Description
Variable	$t^a_{i,k}$	Arrival time of train i at station$k$
	$t^d_{i,k}$	Departure time of train i at station k
	t^{start}_{ω}	Start time of maintenance window ω
	t^{end}_{ω}	End time of maintenance window ω
	T_{Ω}	Maintenance window duration, $T_{\Omega} = t^{end}_{\omega} - t^{start}_{\omega}$
	$q^d_{i,j,k}$	Departure-order variable at station k: 1 if train i departs before train j, and 0 otherwise
	$q^a_{i,j,k}$	Arrival-order variable at station k: 1 if train i arrives before train j, and 0 otherwise
	$q_{i,\omega,k}$	1 if train i passes through section $(k-1, k)$ before maintenance window ω starts, and 0 otherwise
	$q_{\omega,i,k-1}$	1 if train i passes through section $(k-1, k)$ after maintenance window ω ends, and 0 otherwise

3.3. Objective Functions

Minimizing train travel time is adopted as one of the optimization objectives in this study. The aim is to reduce the total travel time of each train from its origin to its destination as much as possible while satisfying train operation constraints and maintenance window requirements. As defined in Equation (1), the travel time of train i is the difference between its arrival time at the destination station and its departure time from the origin station. Minimizing train travel times can effectively improve operational efficiency, reduce passengers' in-vehicle time, and enhance transport service quality. It can also contribute to a more compact organization of train operations and improve the overall coordination of the train timetable.

$$\min Z_1 = \sum_{i \in I} \left(t^a_{i,m} - t^d_{i,1} \right) \tag{1}$$

To ensure the effective implementation of infrastructure maintenance activities, maximizing the duration of maintenance windows is adopted as the other optimization objective, as expressed in Equation (2). Subject to train timetabling constraints and maintenance window requirements, the start time, end time, and duration of each maintenance window are optimized to provide sufficiently long and continuous periods for maintenance activities. This objective helps improve the rationality of maintenance window allocation and the ability to accommodate maintenance operations, thereby achieving better coordination between train operations and infrastructure maintenance requirements.

$$\max\ Z_2 = T_{\Omega} \tag{2}$$

3.4. Constraints

(1) Train departure and arrival time window constraints

The natural environment along plateau railways is complex, with operations affected by high altitude, low temperatures, large day–night temperature variations, and changing weather conditions. Therefore, the determination of train departure and arrival times needs to account for operational safety, passenger travel demand, and connections with external transport services. To ensure that the train timetable satisfies the requirements of passenger transport organization on plateau railways, time window constraints are imposed on train departure and arrival times so that the actual departure and arrival times of each train remain within their allowable ranges. Equations (3) and (4) define the time window constraints for train departure times and terminal arrival times, respectively.

$$\alpha^{\min}_{i,1} \le t^d_{i,1} \le \alpha^{\max}_{i,1},\ \ \forall i \in I \tag{3}$$

$$\alpha^{\min}_{i_m} \le t^a_{i_m} \le \alpha^{\max}_{i_m},\quad \forall i \in I \tag{4}$$

(2) Train dwell time constraints

Train dwell time at intermediate stations is essential for passenger boarding and alighting, station operations, and safe train operation. Considering the technical characteristics and operating patterns of different train types, the proposed model imposes differentiated dwell time constraints on EMU trains and conventional-speed passenger trains. EMU trains mainly provide fast and direct passenger services, with high boarding and alighting efficiency and relatively simple station operations. To maintain their operating speed and overall timetable efficiency, their dwell times are fixed at the prescribed values. In contrast, under the complex operating conditions of plateau railways, conventional-speed passenger trains must not only satisfy basic passenger boarding and alighting requirements but also retain sufficient scheduling flexibility to accommodate maintenance windows and train crossings. Therefore, their dwell times can be adjusted according to the actual requirements of timetable construction, provided that they are no shorter than the prescribed minimum dwell times. Specifically, Equations (5) and (6) represent the dwell time constraints for EMU trains and conventional-speed passenger trains, respectively.

$$t^{d}_{i',k} - t^{a}_{i',k} = T^{\min}_{i',k} S_{i',k}, \quad \forall i' \in I', \ \forall k \in K \tag{5}$$

$$T^{\min}_{i'',k} S_{i'',k} \le t^{d}_{i'',k} - t^{a}_{i'',k} \le M S_{i'',k}, \quad \forall i'' \in I'', \ \forall k \in K \tag{6}$$

(3) Train section running time constraints

Train section running time is a fundamental constraint in train timetabling and is directly related to the safety and feasibility of train operations. Owing to factors such as long and steep gradients, complex track alignments, and high-altitude conditions, train running times on plateau railway sections must comply with the relevant technical standards. Therefore, the actual running time of a train between two adjacent stations is required to be no shorter than the minimum section running time specified for the corresponding train type. Additional acceleration and deceleration times are also considered according to the train stopping pattern. Equation (7) represents the train section running-time constraint.

$$t^{a}_{i,k+1} - t^{d}_{i,k} \ge T^{\min}_{k,k+1} + S_{i,k+1} T^{de}_{i,k+1} + S_{i,k} T^{ac}_{i,k}, \ \forall i \in I \quad \forall k, k+1 \in K \tag{7}$$

(4) Train safety headway constraints

Train safety separation mainly include the departure–arrival and arrival–departure intervals between trains operating in the same direction, as well as the departure and arrival headways between same-direction trains at the same station. Considering the relatively low density of passenger train services and the relatively large train headways on plateau railways at present, this study focuses on the arrival and departure headways between trains traveling in the same direction at the same station.

To ensure the safe operation of successive trains and maintain adequate station receiving and dispatching capacity, two consecutive trains arriving at the same station must satisfy the minimum arrival headway requirement. According to the operating patterns of the leading and following trains at the station, the minimum arrival headway can be classified into three types: the pass–stop arrival headway, where the leading train passes through the station and the following train stops; the stop–pass arrival headway, where the leading train stops and the following train passes through; and the stop–stop arrival headway, where both trains stop at the station, as shown in Figure 3.

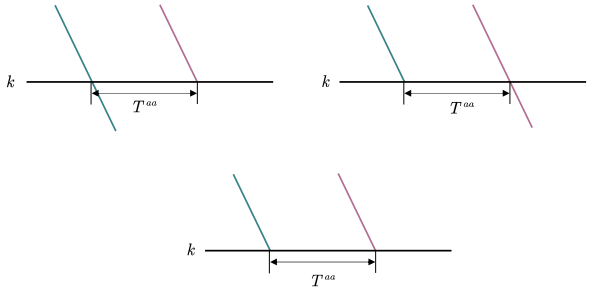


Figure 3. Schematic illustration of train arrival headways.

Corresponding to the arrival headway constraints, the minimum departure headway between two consecutive trains departing from or passing through the same station must also be satisfied to ensure the safe operation of successive trains and maintain sufficient section capacity. According to the operating patterns of the leading and following trains at the station, the minimum departure headway can be classified into four types: the pass–pass departure headway, where both trains pass through the station; the stop–stop departure headway, where both trains stop at the station; the stop–pass departure headway, where the leading train stops and the following train passes through; and the pass–stop departure headway, where the leading train passes through and the following train stops, as shown in Figure 4.

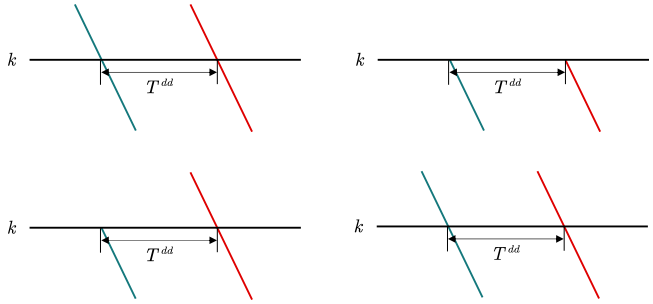


Figure 4. Schematic illustration of train departure headways.

In formulating the train safety headway constraints, the arrival and departure headways under different train operating conditions are parameterized in a unified manner to maintain model consistency and improve computational efficiency. Specifically, a standard arrival safety headway is used to represent all arrival headways, while a standard departure safety headway is used for all departure headways. Equations (8)–(11) define the train arrival and departure headway constraints, respectively.

$$-T^{aa} + M(1 - q^a_{j,i,k}) \geq t^a_{j,k} - t^a_{i,k} \geq T^{aa} - M(1 - q^a_{i,j,k}) \quad \forall i,j \in I, i \neq j \quad \forall k \in K \tag{8}$$

$$q^a_{i,j,k} + q^a_{j,i,k} = 1, \quad \forall i,j \in I, i \neq j, \ \forall k \in K \tag{9}$$

$$-T^{dd} + M(1 - q^d_{j,i,k}) \geq t^d_{j,k} - t^d_{i,k} \geq T^{dd} - M(1 - q^d_{i,j,k}) \quad \forall i,j \in I, i \neq j \quad \forall k \in K \tag{10}$$

$$q^d_{i,j,k} + q^d_{j,i,k} = 1, \quad \forall i,j \in I, i \neq j \quad \forall k \in K \tag{11}$$

To improve the regularity of train departures and the quality of passenger services at the origin station, the departure headway between adjacent trains is constrained. Equation (12) defines the minimum departure headway between adjacent trains departing from the same origin station. Specifically, the departure time of the following train must not be earlier than the departure time of the preceding train plus the minimum departure headway. This constraint prevents train departures from being excessively concentrated within a short period

and promotes a more even distribution of train services over the study period. Consequently, it provides passengers with a wider range of departure time choices and improves both the service level of the train timetable and the overall efficiency of transport organization.

$$-T_{dd}^{min} + M(1 - q_{i,j,1}^{d}) \geq t_{j,1}^{d} - t_{i,1}^{d} \geq T_{dd}^{min} - M(1 - q_{i,j,1}^{d}), \forall i, j \in I, i \neq j \tag{12}$$

(5) Train operation sequence consistency constraint

To ensure the orderly operation of trains traveling in the same direction within a railway section, a train operation sequence consistency constraint is introduced. For any two trains operating in the same direction, once their departure order at the origin station of a section has been determined, this order must remain unchanged throughout the section. In other words, a following train is not allowed to overtake the preceding train within the section. Equation (13) defines the train operation sequence consistency constraint between two adjacent stations, which ensures the continuity of train movements and the feasibility of the resulting train timetable.

$$q_{i,j,k}^{d} = q_{i,j,k+1}^{a}, \quad \forall i, j \in I, i \neq j \quad \forall k, k+1 \in K \tag{13}$$

(6) Fixed operating sequence constraint for EMU trains

EMU trains generally operate at the same or similar speed levels, and therefore there is limited need to arrange overtaking among EMU trains during train timetable construction. To simplify train operation organization and reduce the computational complexity of the model, a fixed operating sequence constraint is imposed on EMU trains. For any two EMU trains traveling in the same direction, once their departure order at the origin station of the line has been determined, this order must remain unchanged at all subsequent stations and sections. In other words, a later-departing EMU train is not allowed to overtake an earlier-departing EMU train. Equation (14) defines this constraint.

$$q_{i',j',k}^{a} = q_{i,j,k}^{d}, \quad \forall i', j' \in I', i' \neq j' \quad \forall k \in K \tag{14}$$

(7) Maintenance-window timing constraints

To ensure that maintenance window scheduling complies with the operational requirements of plateau railway maintenance activities, maintenance-window timing constraints are introduced. Owing to the complex climatic conditions along plateau railways, maintenance activities are particularly susceptible to extreme weather. Therefore, the allowable maintenance window period should be determined by considering local climatic conditions, operational safety requirements, and maintenance organization conditions. In the model, both the start and end times of each maintenance window are required to fall within predefined allowable time ranges to ensure that the resulting maintenance plan is practically feasible. Equations (15) and (16) define the maintenance-window timing constraints.

$$\alpha_{w,\text{start}}^{\min} \leq t_{w}^{\text{start}} \leq \alpha_{w,\text{start}}^{\max}, \forall w \in \Omega \tag{15}$$

$$\alpha_{w,end}^{\min} \leq t_{w}^{end} \leq \alpha_{w,end}^{\max}, \quad \forall \omega \in \Omega \tag{16}$$

(8) Maintenance window duration constraint

Considering the requirements of both maintenance activities and train operations on plateau railways, the duration of each maintenance window should be maintained within a reasonable range. The maintenance window must be sufficiently long to satisfy the basic time requirements of maintenance activities, while an excessively long duration should be avoided to prevent excessive disruption to train operations. Therefore, a maintenance

window duration constraint is introduced, requiring the duration of each maintenance window to remain between the prescribed minimum and maximum allowable durations. Equation (17) defines this constraint.

$$T_{\Omega}^{\min} \le T_{\Omega} \le T_{\Omega}^{\max} \tag{17}$$

(9) Safety separation constraints between train operations and maintenance windows

To ensure the safe coordination between train operations and maintenance activities, safety separation constraints between train operations and maintenance windows are introduced. The times at which trains enter or leave the relevant railway sections must maintain the prescribed safety intervals from the start and end times of the corresponding maintenance windows, thereby preventing temporal conflicts between train movements and maintenance activities. Although the two tracks operate independently, the maintenance window occupies the affected track and therefore requires a safety separation from train movements on that track. These constraints coordinate the relationship between train operating times and maintenance window boundaries and ensure the feasibility of both the train timetable and maintenance window schedule. The corresponding constraints are given in Equations (18)–(20).

$$t_{i,k-1}^{d} \ge t_{\omega}^{end} + T_{\Omega}^{safe} + M(q_{\omega,i,k-1} - 1), \ \forall i \in I \quad \forall \omega \in \Omega \quad \forall k-1 \in K \tag{18}$$

$$t_{i,k}^{a} \le t_{\omega}^{start} - T_{\Omega}^{safe} + M(1 - q_{i,\omega,k}), \forall i \in I \quad \forall \omega \in \Omega \quad \forall k \in K \tag{19}$$

$$q_{\omega,i,k-1} + q_{i,\omega,k} = 1, \quad \forall i \in I \quad \forall \omega \in \Omega \quad \forall k,k-1 \in K \tag{20}$$

4. Algorithm Design

4.1. Analysis of the Solution Approach

Since the coordinated optimization model of train timetables and maintenance windows developed in this study involves both train operation and maintenance window scheduling objectives, it is formulated as a bi-objective optimization problem. Directly solving the model within a bi-objective framework would not only increase the computational complexity but also make the subsequent decomposition algorithm more difficult to develop and implement. Therefore, a linear weighted-sum method is first applied to scalarize the two objectives. The total train travel-time objective and the maintenance-window duration objective are combined into a single overall objective function, thereby transforming the original bi-objective model into a single-objective mixed-integer programming model. This transformation preserves the original optimization objectives while providing a suitable basis for the subsequent decomposition procedure.

The resulting single-objective model contains both binary variables describing train operating sequences and the relative precedence between trains and maintenance windows, and continuous variables representing train arrival and departure times and the start and end times of maintenance windows. The binary variables exhibit strong combinatorial characteristics and constitute the main source of computational difficulty, whereas the continuous variables mainly determine the detailed timing decisions under a given train operating sequence. Since these two types of variables exhibit a clear hierarchical structure and are closely coupled, Benders decomposition is employed to separate the discrete decisions from the continuous optimization problem, thereby reducing the overall computational burden.

In the decomposition procedure, the train arrival-order variables and departure-order variables at each station, together with the variables describing the precedence relationship between trains and maintenance windows, are assigned to the master problem. The master

problem determines the discrete decisions associated with train operation organization and introduces an auxiliary variable to represent the objective value of the subproblem. Given a solution to the master problem, a subproblem containing only continuous variables is then constructed to optimize train arrival and departure times, dwell times, section running times, and the start times, end times, and durations of maintenance windows, thereby determining the detailed temporal coordination between train operations and maintenance activities.

In summary, the original bi-objective model is first transformed using the linear weighted-sum method, after which Benders decomposition is applied according to the variable types and structural characteristics of the model to achieve a hierarchical solution of the coordinated train timetabling and maintenance window optimization problem. This decomposition exploits the discrete–continuous structure of the model and iteratively coordinates the master problem and subproblem to solve the resulting mixed-integer optimization problem.

4.2. Linear Weighted-Sum Transformation of the Bi-Objective Model

The coordinated optimization model of train timetables and maintenance windows developed in this study considers both train operating efficiency and the effectiveness of maintenance window scheduling, and is therefore formulated as a bi-objective optimization problem. Specifically, the total train operating cost Z_1 is a minimization objective, whose value is expected to be as small as possible, whereas the maintenance window duration Z_2 is a maximization objective, whose value is expected to be as large as possible. Since the bi-objective model cannot be directly incorporated into the subsequent decomposition procedure in a convenient manner, the linear weighted-sum method is adopted to scalarize the original model and transform it into a single-objective optimization model. The resulting weighted objective function is given in Equation (21).

$$\min Z = \lambda \frac{Z_1 - Z_1^{\min}}{Z_1^{\max} - Z_1^{\min}} - (1-\lambda) \frac{Z_2 - Z_2^{\min}}{Z_2^{\max} - Z_2^{\min}} \tag{21}$$

Although Z_1 and Z_2 are both time-related objectives, they differ considerably in numerical scale. Z_1 represents the total travel time of all trains and generally has a relatively large magnitude, whereas Z_2 represents only the duration of the maintenance window and is therefore much smaller in magnitude. If the two objectives are directly combined using a weighted sum, the objective with the larger numerical magnitude may dominate the overall objective function, thereby distorting the intended preference represented by the weighting coefficients. Therefore, the two objectives are first normalized before the linear weighted-sum method is applied. In Equation (21), $Z_1^{\min}$ and $Z_1^{\max}$ denote the lower and upper reference values used to normalize the total train travel time, respectively, while $Z_2^{\min}$ and $Z_2^{\max}$ denote the lower and upper bounds used to normalize the maintenance-window duration. These values are determined before the weight-sensitivity analysis. Specifically, $Z_1^{\min}$ is obtained by solving the single-objective endpoint problem that minimizes the total train travel time, whereas $Z_1^{\max}$ is taken as the total train travel time obtained from the opposite endpoint problem that maximizes the maintenance-window duration. For the maintenance objective, $Z_2^{\min}$ and $Z_2^{\max}$ are directly given by the prescribed maintenance-window duration bounds in Equation (46). For the case study, the resulting normalization values are $Z_1^{\min} = 8312.78$min, $Z_1^{\max} = 15266.12$min, $Z_2^{\min} = 180$min, $Z_2^{\max} = 300$min. The two endpoint solutions satisfy the prescribed 3% optimality-gap tolerance and are therefore used as fixed reference values for objective normalization. A min–max normalization method is adopted to map the original objective functions Z_1 and Z_2 onto a common dimensionless scale, thereby effectively eliminating the influence of differences in units and numerical magnitudes on the optimization results. On this basis, a weighting parameter $\lambda \in [0, 1]$ is introduced, where λ represents the relative importance of the train operation

objective and $1-\lambda$ represents the relative importance of the maintenance window objective. The value of λ is determined by jointly considering train operation requirements and maintenance requirements. A sensitivity analysis is further conducted to compare the solutions obtained under different weighting schemes, and the parameter value that provides an appropriate balance between train operating efficiency and maintenance window scheduling performance is selected. Since $Z_1^{\min}$, $Z_1^{\max}$, $Z_2^{\min}$, and $Z_2^{\max}$ are all constants, they do not affect the optimal solution during the optimization process. Therefore, Equation (21) can be equivalently transformed into the linear objective function given in Equation (22).

$$\min Z = \frac{\lambda}{Z_1^{\max} - Z_1^{\min}} Z_1 - \frac{(1-\lambda)}{Z_2^{\max} - Z_2^{\min}} Z_2 \tag{22}$$

4.3. Formulation of the Master Problem and Subproblem

Following the linear weighted-sum transformation, the original bi-objective model is converted into a single-objective mixed-integer programming model. The resulting model contains both binary variables describing train operating sequences and the precedence relationships between trains and maintenance windows, and continuous variables representing train arrival and departure times, maintenance window start and end times, and maintenance window durations. The former mainly determine the train operation plan and exhibit strong combinatorial characteristics, whereas the latter are used to determine the detailed timing decisions under a given train operation plan. Since these two types of variables exhibit a clear hierarchical structure and are closely coupled in the model, Benders decomposition is adopted to separate the discrete and continuous decisions. Specifically, the discrete decision variables are assigned to the master problem, while the continuous time variables are assigned to the subproblem, thereby enabling a hierarchical solution of the original model.

$$\text{MP: } \min\theta \tag{23}$$

$$s.t. \quad q^a_{i,j,k} + q^a_{j,i,k} = 1, \quad \forall i,j \in I, i \neq j, \ \forall k \in K \tag{24}$$

$$q^d_{i,j,k} + q^d_{j,i,k} = 1, \quad \forall i,j \in I, i \neq j \quad \forall k \in K \tag{25}$$

$$q^a_{i,j,k} + q^a_{j,h,k} - q^a_{i,h,k} \leq 1, \quad \forall i,j,h \in I, i \neq j, j \neq h, i \neq h \quad \forall k \in K \tag{26}$$

$$q^d_{i,j,k} + q^d_{j,h,k} - q^d_{i,h,k} \leq 1, \quad \forall i,j,h \in I, i \neq j, j \neq h, i \neq h \quad \forall k \in K \tag{27}$$

$$q^d_{i,j,k} = q^a_{i,j,k+1}, \quad \forall i,j \in I, i \neq j \quad \forall k, k+1 \in K \tag{28}$$

$$q^a_{i',j',k} = q^d_{i,j,k}, \quad \forall i',j' \in I', i' \neq j' \quad \forall k \in K \tag{29}$$

$$q_{\omega,i,k-1} + q_{i,\omega,k} = 1, \quad \forall i \in I \quad \forall \omega \in \Omega \quad \forall k, k-1 \in K \tag{30}$$

$$q^a_{i,j,k}, \ q^d_{i,j,k} \in \{0,1\}, \ \forall i,j \in I, i \neq j \ \forall k \in K \tag{31}$$

$$q_{\omega,i,k-1}, q_{i,\omega,k} \in \{0,1\}, \forall i \in I \forall \omega \in \Omega \forall k = 2, 3, \ldots, m \tag{32}$$

At the initial iteration, the master problem consists of Equations (23)–(32), without any Benders cuts generated from the subproblem. Since the auxiliary variable θ is otherwise unrestricted in the initial master problem, a valid lower bound is imposed to ensure boundedness. According to Equation (22), $Z_1 \geq Z_1^{\min}$ and $Z_2 \leq Z_2^{\max}$; therefore,

$$\theta \geq \frac{\lambda Z_1^{\min}}{Z_1^{\max} - Z_1^{\min}} - \frac{(1-\lambda) Z_2^{\max}}{Z_2^{\max} - Z_2^{\min}} \tag{33}$$

No optimality, feasibility, or conflict cuts are included in the initial master problem. These cuts are generated iteratively according to the solution status of the continuous subproblem.

The master problem (MP) of the coordinated optimization model for train timetables and maintenance windows is formulated in Equations (23)–(32). The MP is primarily composed of discrete decision variables, including the arrival-order variables and departure-order variables of trains at each station, as well as variables describing the relative precedence between trains and maintenance windows. In addition, an auxiliary variable θ is introduced into the MP to represent an estimate of the optimal objective value of the subproblem under a given set of discrete decisions. Equations (26) and (27) define the transitivity constraints, which ensure the logical consistency of train arrival and departure orders at the same station. Specifically, if train i arrives at (or departs from) a station before train j, and train j arrives at (or departs from) the same station before train h, then train i must also arrive at (or depart from) the station before train h. By imposing these transitivity constraints, the precedence relationships represented by the train arrival-order and departure-order variables are guaranteed to satisfy strict logical consistency, thereby preventing cyclic orderings or conflicting precedence relationships. This ensures the feasibility and rationality of the train operating sequence determined by the MP.

Given the discrete decisions obtained from the MP, a subproblem (SP) containing only continuous variables is then constructed. The SP mainly includes the arrival and departure times of trains at each station, the start and end times of maintenance windows, and their durations. The SP is formulated in Equations (34)–(49).

$$SP: \min\left[\lambda \frac{\sum_{i\in I}(t_i^a - t_i^d)}{Z_1^{\max} - Z_1^{\min}} - (1-\lambda)\frac{T_\Omega}{Z_2^{\max} - Z_2^{\min}}\right] \tag{34}$$

$$s.t. \quad \alpha_{i,1}^{\min} \le t_{i,1}^d \le \alpha_{i,1}^{\max}, \quad \forall i \in I \tag{35}$$

$$\alpha_{i,m}^{\min} \le t_{i,m}^a \le \alpha_{i,m}^{\max}, \quad \forall i \in I \tag{36}$$

$$t_{i',k}^d - t_{i',k}^a = T_{i',k}^{\min} S_{i',k}, \quad \forall i' \in I' \quad \forall k \in K \tag{37}$$

$$T_{i'',k}^{\min} S_{i'',k} \le t_{i'',k}^d - t_{i'',k}^a \le M S_{i'',k}, \quad \forall i'' \in I'' \quad \forall k \in K \tag{38}$$

$$t_{i,k+1}^a - t_{i,k}^d \ge T_{k,k+1}^{\min} + S_{i,k+1} T_{i,k+1}^{de} + S_{i,k} T_{i,k}^{ac}, \quad \forall i \in I \quad \forall k, k+1 \in K \tag{39}$$

$$-T^{aa} + M\left(1 - \overline{q}_{j,i,k}^a\right) \ge t_{j,k}^a - t_{i,k}^a \ge T^{aa} - M\left(1 - \overline{q}_{i,j,k}^a\right) \quad \forall i,j \in I, i \ne j \quad \forall k \in K \tag{40}$$

$$-T^{dd} + M\left(1 - \overline{q}_{j,i,k}^d\right) \ge t_{j,k}^d - t_{i,k}^d \ge T^{dd} - M\left(1 - \overline{q}_{i,j,k}^d\right) \quad \forall i,j \in I, i \ne j \quad \forall k \in K \tag{41}$$

$$-T_{dd}^{\min} + M(1 - \overline{q}_{i,j,1}^d) \ge t_{j,1}^d - t_{i,1}^d \ge T_{dd}^{\min} - M(1 - \overline{q}_{i,j,1}^d), \forall i,j \in I, i \ne j \tag{42}$$

$$\alpha_{\omega,\text{start}}^{\min} \le t_\omega^{\text{start}} \le \alpha_{\omega,\text{start}}^{\max}, \quad \forall \omega \in \Omega \tag{43}$$

$$\alpha_{w,\text{end}}^{\min} \le t_w^{\text{end}} \le \alpha_{w,\text{end}}^{\max}, \forall w \in \Omega \tag{44}$$

$$t_\omega^{\text{end}} - t_\omega^{\text{start}} = T_\Omega, \quad \forall \omega \in \Omega \tag{45}$$

$$T_\Omega^{\min} \le T_\Omega \le T_\Omega^{\max} \tag{46}$$

$$t_{i,k-1}^d \ge t_\omega^{\text{end}} + T_\Omega^{\text{safe}} + M(\overline{q}_{\omega,i,k-1} - 1), \quad \forall i \in I \quad \forall \omega \in \Omega \quad \forall k-1 \in K \tag{47}$$

$$t_{i,k}^a \le t_\omega^{\text{start}} - T_\Omega^{\text{safe}} + M(1 - \overline{q}_{i,\omega,k}), \forall i \in I \quad \forall \omega \in \Omega \quad \forall k \in K \tag{48}$$

$$t_{i,k}^a, t_{i,k}^d, T_\Omega, t_\omega^{\text{start}}, t_\omega^{\text{end}} \ge 0, \quad \forall i \in I \quad \forall k \in K \quad \forall \omega \in \Omega \tag{49}$$

To construct the dual formulation of the continuous subproblem, each constraint in the primal subproblem (Equations (34)–(49)) is systematically associated with a dual variable. Table 2 summarizes the comprehensive mapping between the primal subproblem constraints and the corresponding dual variables, along with their standard mathematical

forms and sign restrictions. Based on this mapping, the dual model of the subproblem (Dual-SP) is formulated in Equations (50)–(67).

$$\begin{aligned}
&\text{DualSP}: \max \sum_{i\in I}(\alpha_{i,1}^{\min}u_i' - \alpha_{i,1}^{\max}u_i'' + \alpha_{i,m}^{\min}v_i' - \alpha_{i,m}^{\max}v_i'')+\\
&\sum_{i'\in I'}\sum_{k\in K} T_{i',k}^{\min}S_{i',k}\pi_{i',k} + \sum_{i''\in I''}\sum_{k\in K}(T_{i'',k}^{\min}S_{i'',k}\sigma_{i'',k}' - MS_{i'',k}\sigma_{i'',k}'')+\\
&\sum_{i\in I}\sum_{k=1}^{m-1}\left(T_{k,k+1}^{\min} + S_{i,k+1}T_{i,k+1}^{de} + S_{i,k}T_{i,k}^{ac}\right)\rho_{i,k}+\\
&\sum_{i,j\in I,i\neq j}\sum_{k\in K}[T^{aa} - M(1-\overline{q}_{i,j,k}^{a})](\mu_{i,j,k}' + \mu_{i,j,k}'')+\\
&\sum_{i,j\in I,i\neq j}\sum_{k\in K}[T^{dd} - M(1-\overline{q}_{i,j,k}^{d})](\phi_{i,j,k}' + \phi_{i,j,k}'')+\\
&\sum_{i,j\in I,i\neq j}[T_{dd}^{\min} - M(1-\overline{q}_{i,j,1}^{d})](\psi_{i,j}' + \psi_{i,j}'')+\\
&\sum_{\omega\in\Omega}(\alpha_{\omega,start}^{\min}s_\omega' - \alpha_{\omega,start}^{\max}s_\omega'' + \alpha_{\omega,end}^{\min}e_\omega' - \alpha_{\omega,end}^{\max}e_\omega'')+\\
&\sum_{i\in I}\sum_{\omega\in\Omega}\sum_{k=2}^{m}[T_{\Omega}^{safe} + M(\overline{q}_{\omega,i,k-1}-1)]\gamma_{\omega,i,k-1}+\\
&\sum_{i\in I}\sum_{\omega\in\Omega}\sum_{k=2}^{m}[T_{\Omega}^{safe} + M(\overline{q}_{i,\omega,k}-1)]\gamma_{i.\omega,k} + T_{\Omega}^{\min}\tau' + T_{\Omega}^{\max}\tau''
\end{aligned} \tag{50}$$

$$\begin{aligned}
s.t.\quad &\mathbf{1}[k=m](v_i' - v_i'') - \pi_{i,k} + \mathbf{1}[k\geq 2](\rho_{i,k-1} - \sum_{\omega\in\Omega}\gamma_{i,\omega,k})+\\
&\sum_{j\in I,i\neq j}(-\mu_{i,j,k}' + \mu_{i,j,k}'' + \mu_{j,i,k}' - \mu_{j,i,k}'') \leq \mathbf{1}[k=m]\frac{\lambda}{\overline{Z}_1}, \forall i\in I'\ \forall k\in K
\end{aligned} \tag{51}$$

$$\begin{aligned}
&\mathbf{1}[k=m](v_i' - v_i'') - \sigma_{i,k}' + \sigma_{i,k}'' + \mathbf{1}[k\geq 2](\rho_{i,k-1} - \sum_{\omega\in\Omega}\gamma_{i,\omega,k})+\\
&\sum_{j\in I,i\neq j}(-\mu_{i,j,k}' + \mu_{i,j,k}'' + \mu_{j,i,k}' - \mu_{j,i,k}'') \leq \mathbf{1}[k=m]\frac{\lambda}{\overline{Z}_1}, \forall i\in I''\ \forall k\in K
\end{aligned} \tag{52}$$

$$\begin{aligned}
&\mathbf{1}[k=1](u_i' - u_i'') + \pi_{i,k} - \mathbf{1}[k\leq m-1](\rho_{i,k} - \sum_{\omega\in\Omega}\gamma_{\omega,i,k})+\\
&\sum_{j\in I,i\neq j}(-\phi_{i,j,k}' + \phi_{i,j,k}'' + \phi_{j,i,k}' - \phi_{j,i,k}'')+\\
&\mathbf{1}[k=1]\sum_{j\in I,i\neq j}(-\psi_{i,j,k}' + \psi_{i,j,k}'' + \psi_{j,i,k}' - \psi_{j,i,k}'') \leq -\mathbf{1}[k=1]\frac{\lambda}{\overline{Z}_1}, \forall i\in I''\ \forall k\in K
\end{aligned} \tag{53}$$

$$\begin{aligned}
&\mathbf{1}[k=1](u_i' - u_i'') + \sigma_{i,k}' - \sigma_{i,k}'' - \mathbf{1}[k\leq m-1](\rho_{i,k} - \sum_{\omega\in\Omega}\gamma_{\omega,i,k})+\\
&\sum_{j\in I,i\neq j}(-\phi_{i,j,k}' + \phi_{i,j,k}'' + \phi_{j,i,k}' - \phi_{j,i,k}'')+\\
&\mathbf{1}[k=1]\sum_{j\in I,i\neq j}(-\psi_{i,j,k}' + \psi_{i,j,k}'' + \psi_{j,i,k}' - \psi_{j,i,k}'') \leq -\mathbf{1}[k=1]\frac{\lambda}{\overline{Z}_1}, \forall i\in I''\ \forall k\in K
\end{aligned} \tag{54}$$

$$s_\omega' - s_\omega'' - \xi_\omega + \sum_{i\in I}\sum_{k=2}^{m}\gamma_{i,\omega,k} \leq 0, \forall\omega\in\Omega \tag{55}$$

$$e_\omega' - e_\omega'' + \xi_\omega - \sum_{i\in I}\sum_{k=2}^{m}\gamma_{i,\omega,k-1} \leq 0, \forall\omega\in\Omega \tag{56}$$

$$\tau' - \tau'' - \sum_{\omega\in\Omega}\xi_\omega \leq -\frac{1-\lambda}{\overline{Z}_2} \tag{57}$$

$$u_i', u_i'', v_i', v_i'' \geq 0, \forall i\in I \tag{58}$$

$$\phi_{i,j}', \phi_{i,j}'', \psi_{i,j}', \psi_{i,j}'' \geq 0, \forall i,j\in I, i\neq j \tag{59}$$

$$\rho_{i,k} \geq 0, \forall i\in I\ \forall k\in K \tag{60}$$

$$\mu_{i,j,k}', \mu_{i,j,k}'' \geq 0, \forall i,j\in I, i\neq j\ \forall k\in K \tag{61}$$

$$\pi_{i',k} \in R, \forall i'\in I'\ \forall k\in K \tag{62}$$

$$\sigma_{i'',k}', \sigma_{i'',k}'' \geq 0, \forall i''\in I''\ \forall k\in K \tag{63}$$

$$s_\omega', s_\omega'', e_\omega', e'' \geq 0, \forall\omega\in\Omega \tag{64}$$

$$\xi_\omega \in R, \forall \omega \in \Omega \tag{65}$$

$$\tau', \tau'' \geq 0 \tag{66}$$

$$\gamma_{\omega,i,k-1}, \gamma_{i,\omega,k} \geq 0, \forall \omega \in \Omega \ \forall i \in I \ \forall k, k-1 \in K \tag{67}$$

Table 2. Systematic mapping between primal subproblem constraints and dual variables.

Dual Variable	Associated Primal Constraint	Sign Restriction
u'_i	Equation (35) Earliest departure time at origin	≥ 0
u''_i	Equation (35) Latest departure time at origin	≥ 0
v'_i	Equation (36) Earliest arrival time at destination	≥ 0
v''_i	Equation (36) Latest arrival time at destination	≥ 0
$\pi_{i',k}$	Equation (37) Fixed dwell time (EMU trains)	$\in R$(free)
$\sigma'_{i'',k}$	Equation (38) Minimum dwell time (Conv. trains)	≥ 0
$\sigma''_{i'',k}$	Equation (38) Maximum dwell buffer (Conv. trains)	≥ 0
$\rho_{i,k}$	Equation (39) Minimum section running time	≥ 0
$\mu'_{i,j,k}$	Equation (40) Minimum station arrival headway	≥ 0
$\mu''_{i,j,k}$	Equation (40) Station arrival headway (reverse)	≥ 0
$\phi'_{i,j,k}$	Equation (41) Minimum station departure headway	≥ 0
$\phi''_{i,j,k}$	Equation (41) Station departure headway (reverse)	≥ 0
$\psi'_{i,j}$	Equation (42) Minimum departure headway at origin	≥ 0
$\psi''_{i,j}$	Equation (42) Origin departure headway (reverse)	≥ 0
s'_ω	Equation (43) Earliest start time of window	≥ 0
s''_ω	Equation (43) Latest start time of window	≥ 0
e'_ω	Equation (44) Earliest end time of window	≥ 0
e''_ω	Equation (44) Latest end time of window	≥ 0
ξ_ω	Equation (45) Segmented window duration definition	$\in R$(free)
τ'	Equation (46) Global minimum window duration	≥ 0
τ''	Equation (46) Global maximum window duration	≥ 0
$\gamma_{\omega,i,k-1}$	Equation (47) Separation: Train passes after window	≥ 0
$\gamma_{i,\omega,k}$	Equation (48) Separation: Train passes before window	≥ 0

In the dual subproblem (Dual-SP), $\mathbf{1}[A]$ denotes the indicator function, which is defined in Equation (68) as

$$\mathbf{1}[A] = \begin{cases} 1, & \textit{if } A \textit{ holds}, \\ 0, & \textit{otherwise}. \end{cases} \tag{68}$$

4.4. Construction of Benders Cuts

After the master problem and subproblem have been separated, an information exchange mechanism must be established between them. Solving the master problem alone only determines the discrete decisions associated with the train operating sequence and the relative precedence between trains and maintenance windows, and therefore cannot directly yield the overall optimal solution of the original problem. The subproblem, in contrast, optimizes only the continuous timing variables under a given set of discrete decisions. Therefore, the solution information obtained from the subproblem must be converted into additional constraints for the master problem, which is then iteratively updated. According to the solution status of the subproblem, the resulting Benders cuts can be classified into optimality cuts and feasibility cuts. When the current subproblem is feasible and bounded, an optimality cut is generated from its optimal dual solution; when the current subproblem is infeasible, a feasibility cut is generated from an extreme ray of the dual problem. By successively adding these cuts, the feasible region of the master problem is progressively tightened, thereby gradually approaching the optimal solution of the original problem.

Let Λ denote the vector of dual variables associated with the subproblem. When the dual subproblem is feasible and bounded under the current master-problem solution, let $\overline{\Lambda}$

denote its optimal solution. According to the strong duality theorem, the optimality cut shown in Equation (69) can then be constructed.

$$\begin{aligned}
&\sum_{i\in I}\left(\alpha_{i,1}^{\min}\overline{u}_i' - \alpha_{i,1}^{\max}\overline{u}_i'' + \alpha_{i,m}^{\min}\overline{v}_i' - \alpha_{i,m}^{\max}\overline{v}_i''\right) + \sum_{i'\in I'}\sum_{k\in K} T_{i',k}^{\min} S_{i',k}\overline{\pi}_{i',k} \\
&+ \sum_{i''\in I''}\sum_{k\in K}\left(T_{i'',k}^{\min} S_{i'',k}\overline{\sigma}_{i'',k}' - M S_{i'',k}\overline{\sigma}_{i'',k}''\right) + \sum_{i\in I}\sum_{k=1}^{m-1}\left(T_{k,k+1}^{\min} + S_{i,k+1}T_{i,k+1}^{de} + S_{i,k}T_{i,k}^{ac}\right)\overline{\rho}_{i,k} \\
&\quad + \sum_{i,j\in I, i\neq j}\sum_{k\in K}\left[T^{aa} - M\left(1 - q_{i,j,k}^{a}\right)\right]\left(\overline{\mu}_{i,j,k}' + \overline{\mu}_{i,j,k}''\right) \\
&\quad + \sum_{i,j\in I, i\neq j}\sum_{k\in K}\left[T^{dd} - M\left(1 - q_{i,j,k}^{d}\right)\right]\left(\overline{\phi}_{i,j,k}' + \overline{\phi}_{i,j,k}^{'}\right) \\
&\quad + \sum_{i,j\in I, i\neq j}\left[T_{dd}^{\min} - M\left(1 - q_{i,j,1}^{d}\right)\right]\left(\overline{\psi}_{i,j}' + \overline{\psi}_{i,j}''\right) \\
&+ \sum_{\omega\in\Omega}\left(\alpha_{\omega,start}^{\min}\overline{s}_{\omega}' - \alpha_{\omega,start}^{\max}\overline{s}_{\omega}'' + \alpha_{\omega,end}^{\min}\overline{e}_{\omega}' - \alpha_{\omega,end}^{\max}\overline{e}_{\omega}''\right) + T_{\Omega}^{\min}\overline{\tau}' - T_{\Omega}^{\max}\overline{\tau}'' \\
&\quad + \sum_{i\in I}\sum_{\omega\in\Omega}\sum_{k=2}^{m}\left[T_{\Omega}^{safe} + M\left(q_{\omega,i,k-1} - 1\right)\right]\overline{\gamma}_{\omega,i,k-1} \\
&\quad + \sum_{i\in I}\sum_{\omega\in\Omega}\sum_{k=2}^{m}\left[T_{\Omega}^{safe} + M\left(q_{i,\omega,k} - 1\right)\right]\overline{\gamma}_{i,\omega,k} \le \theta
\end{aligned} \tag{69}$$

When the continuous subproblem defined by Equations (34)–(49) is infeasible under the current master solution, a Farkas infeasibility certificate is obtained from an extreme ray of the corresponding dual cone. Let $\widehat{\Lambda}^{n}$ denote such an extreme ray at iteration n. The feasibility cut is obtained by substituting $\widehat{\Lambda}^{n}$ for the dual variables in the dual objective function in Equation (50). Because any master solution that admits a feasible continuous timetable must satisfy the corresponding Farkas inequality, the resulting expression must be nonpositive. This gives the feasibility cut in Equation (70).

When the dual subproblem is unbounded under the current master-problem solution, let its extreme ray be denoted by $\widehat{\Lambda}$. According to Farkas' lemma, the feasibility cut given in Equation (70) can then be constructed.

$$\begin{aligned}
&\sum_{i\in I}\left(\alpha_{i,1}^{\min}\hat{u}_i' - \alpha_{i,1}^{\max}\hat{u}_i'' + \alpha_{i,m}^{\min}\hat{v}_i' - \alpha_{i,m}^{\max}\hat{v}_i''\right) + \sum_{i'\in I'}\sum_{k\in K} T_{i',k}^{\min} S_{i',k}\hat{\pi}_{i',k} \\
&+ \sum_{i''\in I''}\sum_{k\in K}\left(T_{i'',k}^{\min} S_{i'',k}\hat{\sigma}_{i'',k}' - M S_{i'',k}\hat{\sigma}_{i'',k}''\right) + \sum_{i\in I}\sum_{k=1}^{m-1}\left(T_{k,k+1}^{\min} + S_{i,k+1}T_{i,k+1}^{de} + S_{i,k}T_{i,k}^{ac}\right)\hat{\rho}_{i,k} \\
&\quad + \sum_{i,j\in I, i\neq j}\sum_{k\in K}\left[T^{aa} - M\left(1 - q_{i,j,k}^{a}\right)\right]\left(\hat{\mu}_{i,j,k}' + \hat{\mu}_{i,j,k}^{'}\right) \\
&\quad + \sum_{i,j\in I, i\neq j}\sum_{k\in K}\left[T^{dd} - M\left(1 - q_{i,j,k}^{d}\right)\right]\left(\hat{\phi}_{i,j,k}' + \hat{\phi}_{i,j,k}''\right) \\
&\quad + \sum_{i,j\in I, i\neq j}\left[T_{dd}^{\min} - M\left(1 - q_{i,j,1}^{d}\right)\right]\left(\hat{\psi}_{i,j}' + \hat{\psi}_{i,j}''\right) \\
&+ \sum_{\omega\in\Omega}\left(\alpha_{\omega,start}^{\min}\hat{s}_{\omega}' - \alpha_{\omega,start}^{\max}\hat{s}_{\omega}'' + \alpha_{\omega,end}^{\min}\hat{e}_{\omega}' - \alpha_{\omega,end}^{\max}\hat{e}_{\omega}''\right) + T_{\Omega}^{\min}\hat{\tau}' - T_{\Omega}^{\max}\hat{\tau}'' \\
&\quad + \sum_{i\in I}\sum_{\omega\in\Omega}\sum_{k=2}^{m}\left[T_{\Omega}^{safe} + M\left(q_{\omega,i,k-1} - 1\right)\right]\hat{\gamma}_{\omega,i,k-1} \\
&\quad + \sum_{i\in I}\sum_{\omega\in\Omega}\sum_{k=2}^{m}\left[T_{\Omega}^{safe} + M\left(q_{i,\omega,k} - 1\right)\right]\hat{\gamma}_{i,\omega,k} \le 0
\end{aligned} \tag{70}$$

Thus, Equations (69) and (70) have the same coefficient structure: Equation (69) uses an optimal dual solution and is bounded above by θ, whereas Equation (70) uses an extreme ray and is bounded above by zero.

In addition, to improve the algorithm's ability to exclude master-problem solutions that lead to infeasible subproblems, this study introduces conflict cuts based on conflict analysis in addition to the standard feasibility cuts. When the subproblem becomes infeasible for a given set of discrete decisions from the master problem, an irreducible inconsistent subsystem (IIS) can be further extracted, in addition to constructing a feasibility cut from a dual extreme ray, to identify the key constraints responsible for the subproblem infeasibility. An IIS is a minimal infeasible subsystem consisting of a subset of constraints and variable bounds from an infeasible model. The subsystem itself remains infeasible, whereas removing any one of its constraints or variable bounds restores feasibility. Based on the key constraints identified by the IIS, the corresponding binary variables in the master problem on which these constraints depend can be traced to form a set of conflict variables. A conflict cut can then be constructed to prevent the master problem from repeatedly generating the same or similar infeasible discrete decisions in subsequent iterations.

In addition to the standard feasibility cut, an IIS-based conflict cut is introduced to more directly identify the discrete decisions responsible for an infeasible timing subproblem. When the subproblem is infeasible at iteration n, an irreducible inconsistent subsystem $\mathcal{R}^n$ is extracted from Equations (35)–(49). The IIS may contain both constraints that are independent of the master decisions and constraints whose right-hand sides depend on the binary sequence variables. In the present model, only Equations (40)–(42) and Equations (47)–(48) contain master-problem binary variables. Therefore, the conflict-variable set C^n is constructed by collecting the binary variable associated with each of these constraints that appears in $\mathcal{R}^n$. Constraints in the IIS that do not contain binary variables are retained in the IIS but do not contribute variables to C^n.

Let q_c^n denote the value of conflict variable q_c in the current master solution. To prevent the same IIS-inducing combination of sequence decisions from being generated again, the following conflict cut is added to the master problem:

$$\sum_{c=C^n, q_c^n=1} (1 - q_c) + \sum_{c \in C^n, q_c^n=0} q_c \geq 1 \tag{71}$$

Equation (71) requires at least one conflict variable in C^n to change its value relative to the current infeasible master solution. Therefore, the same binary assignment on all IIS-related sequence variables cannot be generated again.

Proposition 1. *The conflict cut in Equation (71) is valid; that is, it does not eliminate any feasible solution of the original problem.*

Proof. Suppose that a feasible master solution q^* violates Equation (71). Since all variables in C^n are binary, this implies $q_c^* = q_c^n$ for every $c \in C^n$. For each IIS constraint originating from Equations (40)–(42) or Equations (47)–(48), the associated Big-M term therefore remains identical to that under the current infeasible master solution. All other constraints in the IIS are independent of the master binary variables and hence also remain unchanged. Consequently, the same subsystem $\mathcal{R}^n$ remains infeasible under q^*, contradicting the assumed feasibility of q^*. Therefore, every feasible solution satisfies Equation (71), and the conflict cut cannot exclude an optimal feasible solution.

4.5. Algorithm Solution Procedure

After the model decomposition and the construction of the different types of Benders cuts are completed, the overall solution procedure of the algorithm is described in a unified manner. The proposed solution method is based on the iterative solution of the master problem and subproblem, in which the master problem is progressively strengthened by adding optimality cuts, feasibility cuts, and conflict cuts according to the solution status of the subproblem. During the iterative process, the global upper and lower bounds are updated to evaluate the quality of the incumbent solution and monitor the convergence behavior of the algorithm.

The proposed algorithm terminates when either the prescribed optimality-gap tolerance is satisfied or the maximum number of iterations is reached. If the algorithm terminates because the optimality gap satisfies the specified tolerance, the incumbent solution is regarded as a solution within the prescribed optimality accuracy. Otherwise, if the maximum iteration limit is reached before the optimality-gap criterion is satisfied, the algorithm returns the best feasible solution obtained, together with the final upper bound, lower bound, and remaining optimality gap. In the latter case, no claim of convergence to the prescribed optimality tolerance is made.

To clearly present the execution logic of the algorithm, the detailed solution steps and the overall solution procedure are summarized in Table 3 and Figure 5, respectively.

Table 3. Solution procedure of the Benders decomposition algorithm.

Solution Procedure of the Benders Decomposition Algorithm
Step 1: Problem decomposition and initialization
Decompose the original problem into the master problem (MP) and subproblem (SP). Initialize the global upper bound as $UB^* = +\infty$ and the global lower bound as $LB^* = -\infty$. Set the current iteration number to $n = 0$, and specify the maximum number of iterations $maxn$ and the optimality-gap threshold $minGap$
Step 2: Solve the master problem (MP)
Solve the current MP. Its constraints include those associated with the discrete decision variables in the original problem, as well as the optimality cuts, feasibility cuts, and conflict cuts generated by the subproblem and added to the MP in previous iterations. Obtain the current master-problem solution $\overline{q}^n$ and its objective value θ. Update the global lower bound: $LB^* = max\{\theta, LB^*\}$
Step 3: Solve the dual subproblem (Dual-SP) and add cuts to the MP
Substitute the master-problem solution obtained in Step 2, $\left(q^{a,n}_{i,j,k}, q^{d,n}_{i,j,k}, q^{n}_{\omega,i,k-1}, q^{n}_{i,\omega,k}\right)$, into the continuous subproblem (SP) defined by Equations (34)–(49), and solve the resulting linear programming problem. If the SP is feasible and bounded:Solve the corresponding dual subproblem (Dual-SP) and obtain its optimal dual solution $\overline{\Lambda}^n$ and objective value. By strong duality, the SP objective value provides the objective value of a feasible solution to the original problem. Update the global upper bound as $UB = \min\{UB, Z^n_{SP}\}$, and add the optimality cut in Equation (68) to the MP. Otherwise, if the SP is infeasible: Obtain an extreme ray $\widehat{\Lambda}^n$ of the dual problem as a Farkas infeasibility certificate and add the feasibility cut in Equation (70) to the MP. Meanwhile, extract an IIS $\mathcal{R}^n$ from the infeasible SP. For the IIS constraints originating from Equations (40)–(42) and Equations (47) and (48), identify the corresponding master binary variables $\left(q^{a}_{i,j,k}, q^{d}_{i,j,k}, q_{\omega,i,k-1}, q_{i,\omega,k}\right)$, construct the conflict-variable set C^n, and add the conflict cut in Equation (71) to the MP.
Step 4: Check the termination criteria
Check the gap between the global upper and lower bounds and the number of iterations. If either termination criterion is satisfied, terminate the algorithm and output the solution corresponding to the global upper bound. Otherwise, update the iteration number as $n = n + 1$ and return to Step 2

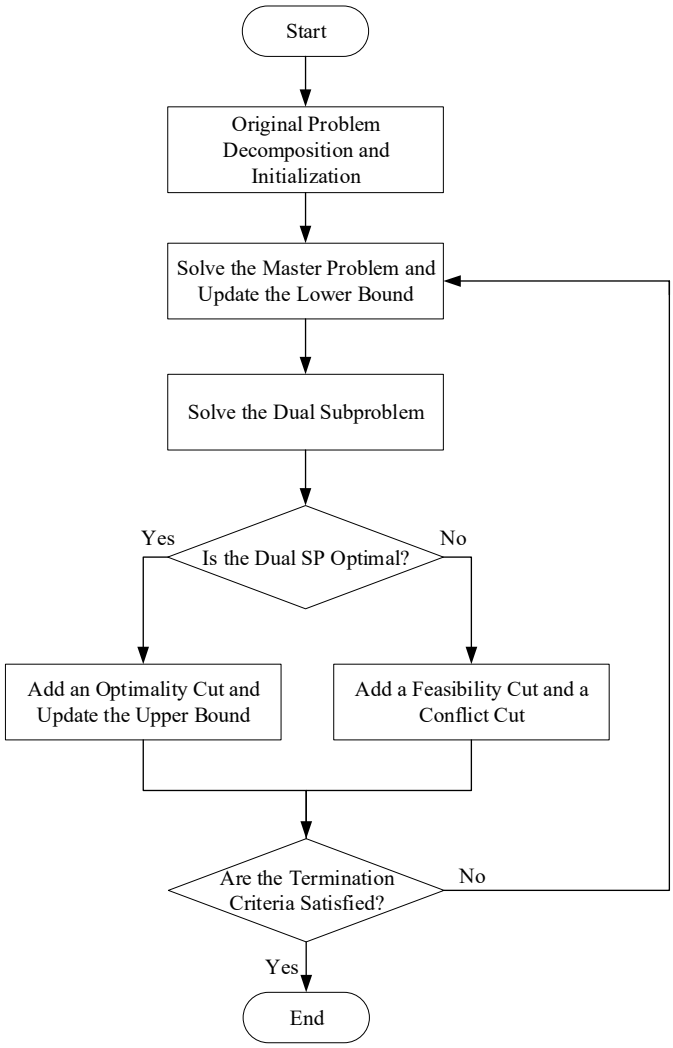


Figure 5. Flowchart of the Benders decomposition algorithm.

4.6. Validation of the Benders Decomposition Algorithm

To verify the correctness of the solution procedure of the proposed Benders decomposition algorithm and the satisfiability of the model constraints, a small-scale coordinated optimization case is designed for validation. This small-scale numerical experiment aims to demonstrate the iterative coordination mechanism between the master problem and subproblem, analyze the convergence process of the global upper and lower bounds, and confirm whether the resulting timetable and maintenance-window plan satisfy operational constraints, maintenance requirements, and safety separation.

The test line consists of six stations (A, B, C, D, E, and F), with section distances of 97 km, 48 km, 47 km, 23 km, and 33 km in the up direction. The allowable operating period is set to [06:00, 24:00], during which a total of 10 passenger trains (both EMU trains and conventional-speed passenger trains) are scheduled. Up-direction trains D2, D4, D6, D8, and K2 depart from Station A and terminate at Station F; down-direction trains D1, D3, D5, D7, and K1 depart from Station F and terminate at Station A. All trains stop at intermediate stations B, C, D, and E, with fixed dwell times of 2 min for EMU trains and minimum dwell times of 3 min for conventional passenger trains. The departure and arrival headways at all stations, as well as the minimum departure headway at origin stations, are set to 5 min. The additional acceleration and deceleration times are 2 min and 3 min, respectively. Segmented maintenance windows are arranged on sections between stations A and F, with an allowable maintenance period of [09:00, 19:00], duration limits of [180, 240] min, and a safety interval

of 10 min between train paths and maintenance windows. The minimum section running times are shown in Table 4.

Table 4. Minimum section running times of the small-scale plateau railway case.

Section	Distance (km)	Minimum Section Running Time (min)			
		EMU		Conventional	
		Down	Up	Down	Up
A–B	97	38	38	57	57
B–C	48	19	19	28	28
C–D	47	19	19	28	28
D–E	23	9	9	14	14
E–F	33	13	13	19	19

Since this instance primarily serves to illustrate the algorithmic convergence and validate constraint compliance, the two normalized objectives are directly scalarized using the linear weighted-sum method without further sensitivity analysis on the weighting parameter. The algorithm was coded in MATLAB R2023a and solved using the Gurobi Optimizer 13.0.0.

The computational results indicate that the total train travel time is 1406 min and the maintenance-window duration is 240 min. The algorithm converged after 114 iterations with a total computation time of 5.42 s. The final upper and lower bounds converged to the same objective value, resulting in an optimality gap (Gap) of 0%, indicating convergence for this small-scale instance. The evolution of the upper and lower bounds during the iterations is illustrated in Figure 6. As shown in Figure 6, the gap between the upper and lower bounds progressively decreases as cuts are added during the iterative process, providing numerical evidence of convergence for this small-scale instance. The final coordinated train timetable and maintenance-window plan are shown in Figure 7.

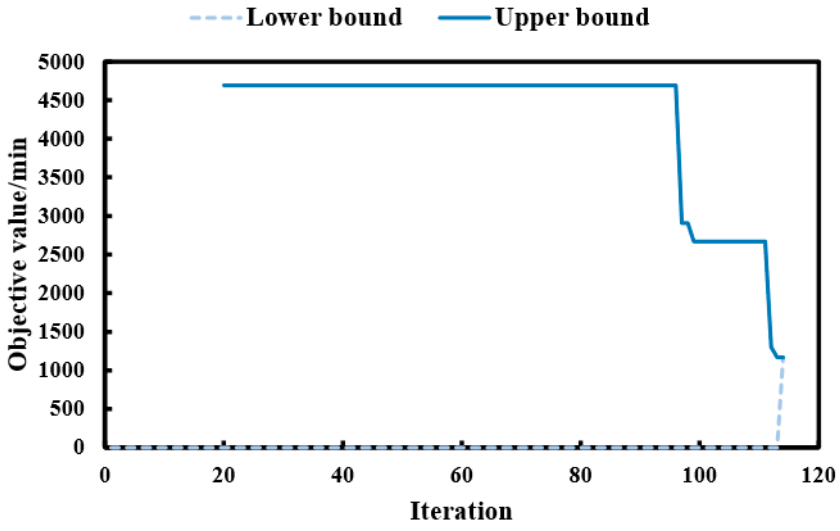


Figure 6. Convergence of upper and lower bounds in the small-scale instance.

As shown in Figure 7, the train timetable and maintenance windows are well coordinated across time and space. Train paths are continuously scheduled across adjacent sections, the train operation sequence complies with all order constraints, and train paths completely avoid the maintenance-window regions without temporal–spatial conflicts. All constraints regarding section running times, station dwell times, headways, and safety separations between train movements and maintenance activities are strictly satisfied. Mean-

while, the maintenance windows are properly deployed within the allowable time range and achieve the required continuous duration. These results confirm that, for this small-scale instance, the algorithm reaches a zero-gap solution and the resulting timetable satisfies the modeled operational and maintenance constraints.

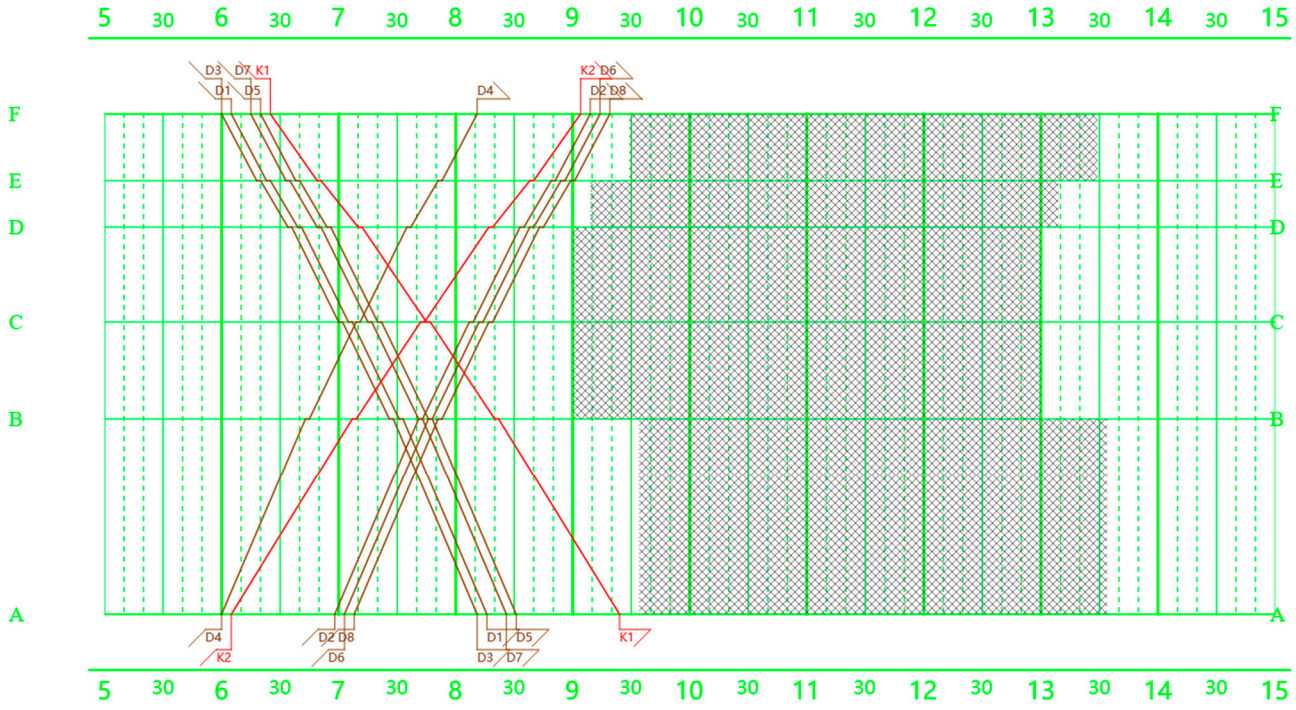


Figure 7. Coordinated train timetable and maintenance windows of the small-scale instance.

5. Case Study

5.1. Case Description and Data Sources

To evaluate the applicability of the proposed model and algorithm in practical applications, an actual plateau railway in China is selected as the case study. To protect the confidentiality of the original case information, the railway name and station and section identifiers are anonymized in this manuscript. The case data are derived from the original operational and maintenance data of the selected railway. The operational data include section running times, station dwell times, train stopping patterns, and allowable arrival and departure time ranges, while the maintenance-related data include the section division and maintenance-window settings. These data are used to construct the train service and maintenance-window inputs for the proposed model. The plateau-specific operating conditions are reflected in these case-specific inputs, particularly the section running times, station dwell times, stopping patterns, and maintenance-window requirements. No additional altitude-related variable is introduced, as the present study focuses on timetable–maintenance-window coordination rather than on modeling the physical effects of the plateau environment itself. Based on the requirements of passenger train operations and maintenance window scheduling, the main parameters are specified as follows. The dwell time of EMU trains at each station is set to 2 min, while the minimum dwell time of conventional-speed passenger trains is set to 3 min. The additional acceleration and deceleration times for all trains are set to 2 min and 3 min, respectively. The safety-separation interval between train operations and maintenance windows is set to 10 min, and the minimum departure headway between adjacent trains at the origin station is set to 25 min, according to the operational and safety requirements considered for the studied plateau railway. To simplify the case study, the departure and arrival headways at

all stations are assumed to be identical and are both set to 3 min. The above operational parameters are specified based on the operational conditions and planning requirements of the selected case railway. They are treated as case-specific input parameters rather than universal regulatory limits.

For maintenance window scheduling, a segmented rectangular maintenance-window scheme is adopted, with the allowable time range for daytime maintenance windows set to [09:00, 19:00]. According to the section division of the plateau railway line, a total of 25 maintenance window sections are considered: Section A–B, Section B–C, Section C–D, Section D–E, Section E–F, Section F–G, Section G–H, Section H–I, Section I–J, Section J–K, Section K–L, Section L–M, Section M–N, Section N–O, Section O–P, Section P–Q, Section Q–R, Section R–S, Section S–T, Section T–U, Section U–V, Section V–W, Section W–X, Section X–Y, Section Y–Z. The minimum and maximum durations of each maintenance window section are set to 180 min and 300 min, respectively, according to the maintenance-window requirements considered for the studied plateau railway. The daytime maintenance period of [09:00, 19:00] and the maintenance-window duration range of 180–300 min are likewise adopted from the maintenance planning setting of the selected case railway. These values define the feasible maintenance scheduling range in the case study.

Regarding train service data, the allowable arrival and departure time range is set to [06:00, 24:00], within which a total of 22 passenger trains are scheduled to operate. The up-direction trains include D2, D4, D6, D8, D10, D12, D14, D16, D18, K2, and K4, all departing from Station Z and terminating at Station A. The down-direction trains include D1, D3, D5, D7, D9, D11, D13, D15, D17, K1, and K3, all departing from Station A and terminating at Station Z. The stopping pattern of each train is determined according to the train category, transport organization requirements, and operating conditions at stations along the line. The specific stopping patterns of the up-direction and down-direction trains are shown in Figures 8 and 9, respectively.

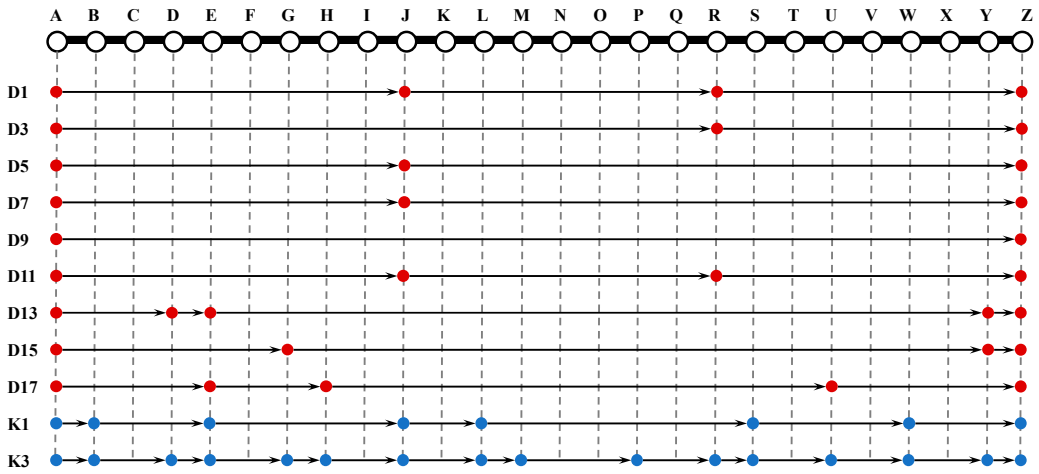


Figure 8. Stopping pattern of down-direction passenger trains on the certain plateau railway in China.

To improve the alignment between passenger train departure times and passenger travel demand, the departure time windows for different train categories are specified according to train type and transport organization requirements. Specifically, the departure time window for trains D1–D12 is set to [06:00, 12:00], that for trains D13–D18 is set to [12:00, 18:00], and that for trains K1–K4 is set to [06:00, 18:00].

This case study involves a total of 26 stations, 25 section-based maintenance windows, and 22 passenger trains, including 18 EMU trains and 4 conventional-speed passenger trains. The model contains both continuous variables, such as train arrival and departure times and the start and end times of maintenance windows, and binary variables, such as train

operating sequences and the precedence relationships between trains and maintenance windows. These binary decisions give the model strong combinatorial characteristics. Therefore, Benders decomposition is adopted to exploit the discrete–continuous structure of the model and improve computational tractability.

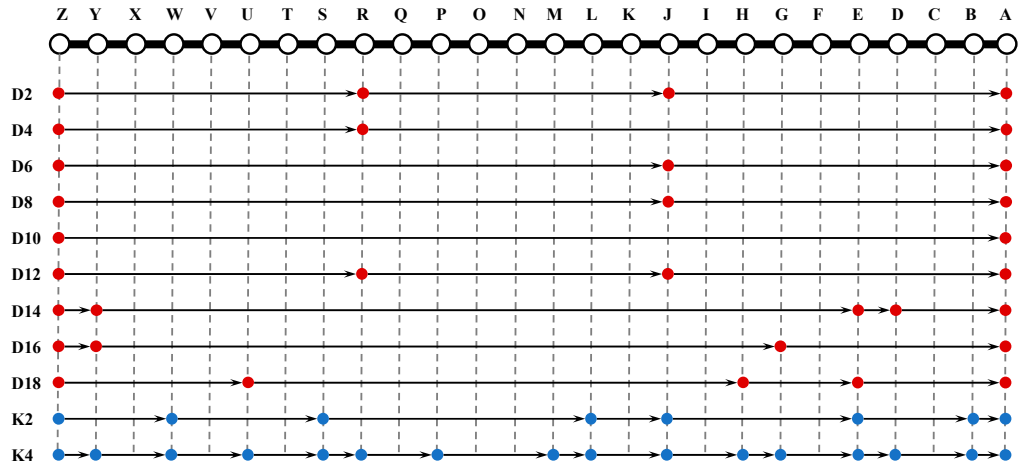


Figure 9. Stopping pattern of up-direction passenger trains on the certain plateau railway in China.

All computational experiments were conducted on a computer running Windows 11, equipped with an Intel Core i7 processor and 16 GB of RAM. The proposed algorithm was implemented in MATLAB R2023a, and both the master problem and subproblem were solved using the Gurobi Optimizer 13.0.0. The termination criteria were set as follows: the algorithm terminates when either the number of iterations reaches 8000 or the current optimality gap (Gap) is no greater than 3%.

5.2. Solution Results and Sensitivity Analysis

With the weighting coefficient λ set to 0.5, the model was solved using the Benders decomposition algorithm. After 8000 iterations, the algorithm obtained the current feasible coordinated solution. The results show that the optimized total train travel time is 9394.54 min, the maintenance window duration is 235.33 min, and the final optimality gap (Gap) is 27.86%. The coordinated optimization results of the train timetable and maintenance windows are shown in Figure 10.

When the weighting coefficient λ is set to 0.5, equal importance is assigned to the train operating efficiency objective and the maintenance window provision objective. The model must therefore balance the reduction in total train travel time against the extension of maintenance window duration. Because these two objectives are partially conflicting, extending the maintenance window reduces the available temporal–spatial capacity for scheduling train paths, whereas further reducing total train travel time limits the feasible time range available for maintenance windows. As a result, the interactions among train path adjustment, maintenance window timing, and safety headway constraints become stronger, which increases the computational difficulty of the model.

Meanwhile, the model contains a large number of binary decision variables, resulting in a large search space for the master problem. Within a limited number of iterations, the Benders decomposition algorithm is therefore unable to tighten the upper and lower bounds sufficiently rapidly, leading to a relatively large final optimality gap (Gap). Although the final Gap for this solution is 27.86%, the resulting timetable still satisfies the constraints related to train operations, maintenance window scheduling, and safety separation. The solution therefore demonstrates the model's ability to coordinate the temporal–spatial conflicts between train paths and maintenance windows. However, given the relatively

large Gap, these results are more appropriately used to demonstrate the feasibility and coordination capability of the proposed model, rather than as evidence of a solution with a high degree of optimality.

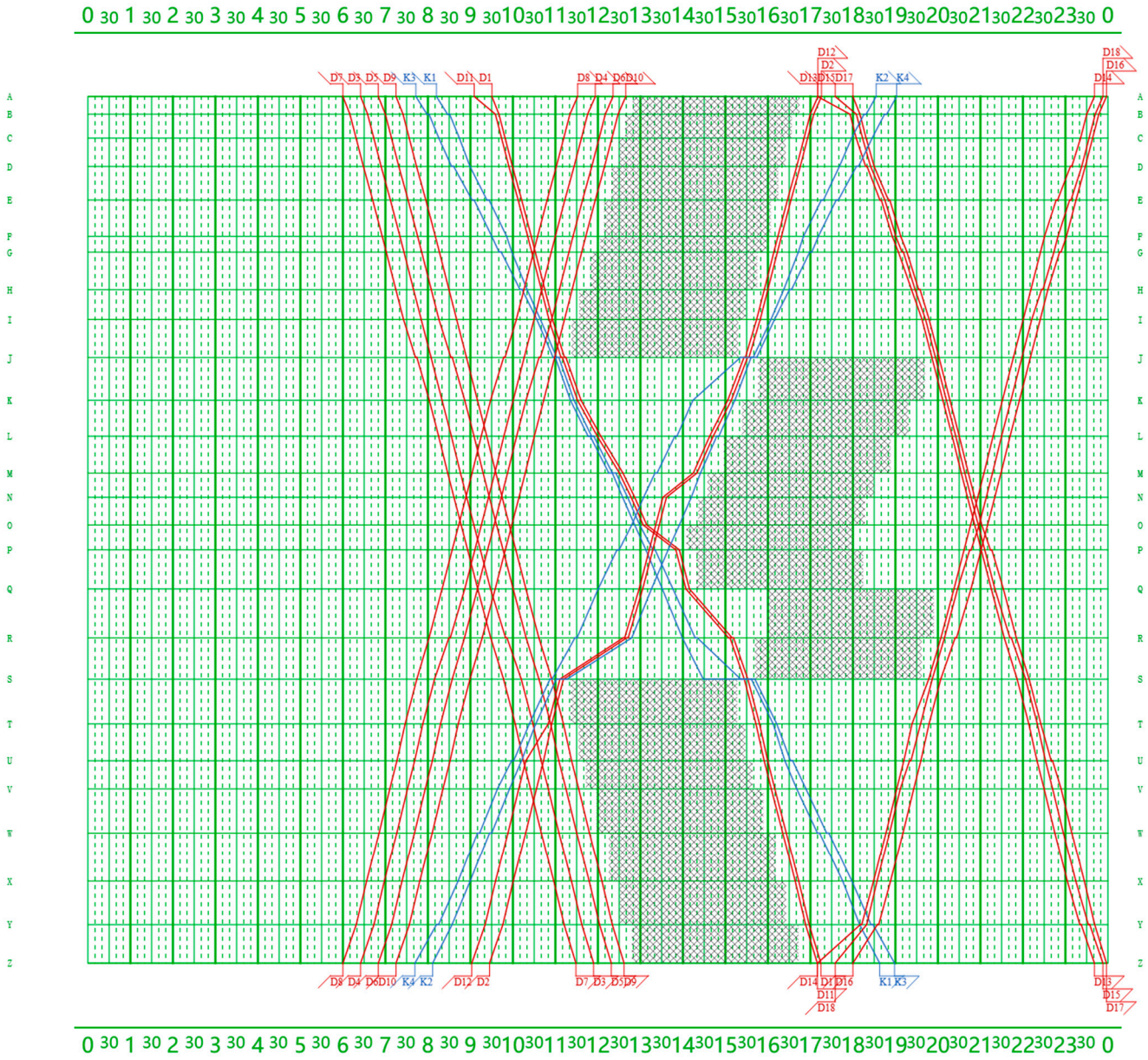


Figure 10. Coordinated optimization results of the train timetable and maintenance windows ($\lambda = 0.5$).

As shown in Figure 10, the train timetable and maintenance windows exhibit good coordination in their temporal–spatial distribution. The maintenance windows are arranged in the form of segmented rectangular windows, with the windows for different sections scheduled at staggered times within the prescribed period. This arrangement avoids the concentrated occupation of line capacity that would result from a simultaneous closure of the entire line and helps reduce the impact of maintenance windows on overall transport operations. The train paths generally avoid the maintenance window periods and sections, indicating that the model can effectively satisfy the conflict-avoidance and safety headway constraints between train operations and maintenance activities. Meanwhile, some trains avoid maintenance windows through adjustments to their departure times, operating

sequences, and dwell times. This allows sufficient maintenance window duration to be maintained while preserving the continuity and feasibility of train operations. At Station S, an EMU train overtakes a conventional-speed passenger train, indicating that the model can flexibly adjust train operating sequences according to the operational characteristics of different train categories, thereby further improving the rationality and coordination of the resulting timetable. Overall, the results show that the proposed model can generate a feasible coordinated timetable and maintenance-window plan under the considered case settings.

However, Figure 10 also shows that some train paths operating close to the maintenance window periods are noticeably affected by the maintenance arrangements. In several sections, train schedules and operating sequences are adjusted, and some trains need to modify their dwell times or operating times to avoid conflicts with maintenance windows. This indicates that, under the current weighting coefficient, although the model can achieve coordinated planning of train timetables and maintenance windows, a trade-off remains between train operating efficiency and maintenance window duration. Therefore, a sensitivity analysis of the weighting coefficient is necessary to examine how the total train travel time, maintenance window duration, and overall coordination performance vary under different weight settings. This analysis can support the selection of a more appropriate weighting coefficient, so that maintenance requirements are adequately satisfied while the impact of maintenance windows on train operations is minimized as much as possible. The corresponding total train travel time and maintenance window duration under different weighting coefficients are presented in Table 5.

Table 5. Comparison of total train travel time and maintenance-window duration under different weighting coefficients.

No	λ	Total Train Travel Time (min)	Maintenance Window Duration (min)	Gap	Termination Status
1	0	15,266.12	300.00	1.75%	Gap tolerance satisfied
2	0.1	13,399.13	297.69	3.41%	Iteration limit reached
3	0.2	13,092.31	290.37	4.63%	Iteration limit reached
4	0.3	11,495.27	274.28	10.37%	Iteration limit reached
5	0.4	9893.85	243.69	16.38%	Iteration limit reached
6	0.5	9394.54	235.33	27.86%	Iteration limit reached
7	0.6	8974.75	225.33	24.51%	Iteration limit reached
8	0.7	8902.48	217.46	15.62%	Iteration limit reached
9	0.8	8488.37	200.64	8.73%	Iteration limit reached
10	0.9	8403.66	192.48	4.28%	Iteration limit reached
11	1	8312.78	180.00	2.82%	Gap tolerance satisfied

The reported optimality gaps vary across weighting coefficients and are larger for several intermediate weight settings, indicating differences in computational difficulty among the scalarized instances. These gap values should not be interpreted as a direct quantitative measure of the conflict between the two objectives. Independently of the gap values, Table 5 shows a clear trade-off: increasing λ generally reduces total train travel time while shortening the maintenance-window duration.

As shown in Table 5 and Figure 11, as the weighting coefficient λ gradually increases, the model places greater emphasis on train operating efficiency. Accordingly, the total train travel time generally decreases, whereas the maintenance window duration shows a downward trend. When λ increases from 0 to 1, the total train travel time decreases from 15,266.12 min to 8312.78 min, indicating that increasing the weight assigned to the train operation objective can effectively reduce train travel time and improve timetable compactness. Meanwhile, the maintenance window duration decreases from 300.00 min to

180.00 min, demonstrating a clear trade-off between maintenance window duration and train operating efficiency. When λ is relatively small, the model gives greater priority to maintaining a sufficiently long maintenance window, and the resulting duration approaches its maximum allowable value. However, the train timetable is more strongly constrained by the maintenance window arrangement, resulting in a longer total train travel time. In contrast, when λ is relatively large, the model places greater emphasis on reducing total train travel time. Although train operating efficiency is improved, the maintenance window duration is progressively shortened, thereby reducing the time available for maintenance activities. Therefore, the weighting coefficient should not be selected by simply favoring one objective. Instead, the weighting coefficient should be selected according to the desired trade-off between train operating efficiency and maintenance-window availability.

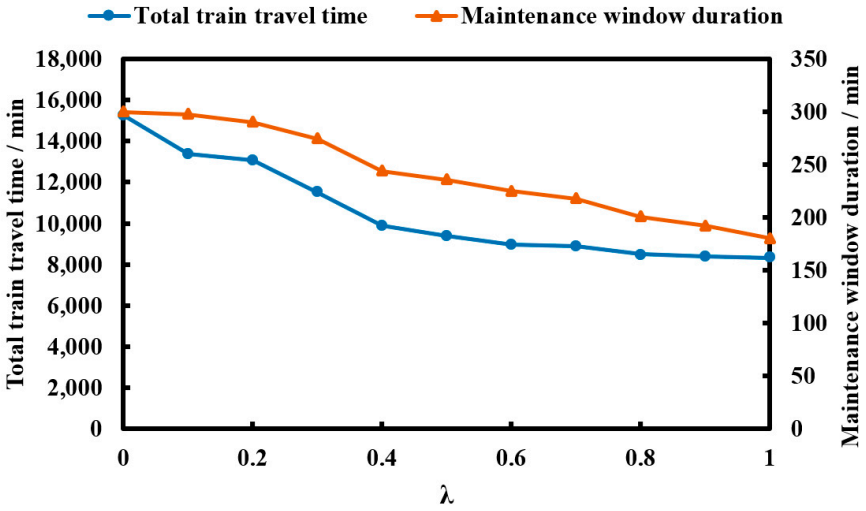


Figure 11. Effect of the weighting coefficient on the solution results.

A comprehensive comparison of the optimization results under different weighting coefficients shows that, as λ increases from 0 to 0.8, the total train travel time decreases markedly. This indicates that assigning a larger weight to the train operating efficiency objective makes the train paths more compact and mitigates, to some extent, the effects of train waiting, conflict avoidance, and dwell time adjustments near maintenance windows. Between $\lambda = 0.8$ and $\lambda = 0.9$, the total train travel time decreases from 8488.37 min to 8403.66 min, corresponding to a reduction of 84.71 min, while the maintenance-window duration decreases from 200.64 min to 192.48 min, corresponding to a reduction of 8.16 min. The $\lambda = 0.8$ solution is therefore selected as a representative feasible trade-off solution for further illustration. This selection is not intended to identify a formal Pareto knee point or a mathematically optimal compromise. Moreover, the solution obtained for $\lambda = 0.8$ has an optimality gap of 8.73% and does not satisfy the prescribed 3% optimality-gap tolerance within the computational limit. Therefore, it is not regarded as a certified optimal or near-optimal solution.

As shown in Figure 12, when the weighting coefficient is set to $\lambda = 0.8$, the model places greater emphasis on the train operating efficiency objective, and the resulting train timetable is overall more compact. Specifically, under this setting, the train paths are connected more smoothly before and after the maintenance windows, and the additional dwell time and running-time adjustments required to avoid maintenance windows are reduced. Through coordinated adjustments to departure times, operating sequences, and dwell times, some trains achieve a more continuous operating process, thereby reducing the impact of maintenance windows on total train travel time. The maintenance windows still adopt the form of segmented rectangular windows and are arranged in a staggered manner

along the line direction within the prescribed time period. The train paths are able to avoid the maintenance window regions while satisfying the safety headway requirements, and no intersections occur between train paths and maintenance window regions. These results indicate that the solution with $\lambda = 0.8$ satisfies the modeled train-operating, maintenance-window, and safety-separation constraints. Accordingly, the $\lambda = 0.8$ solution is used as a representative feasible example to illustrate the trade-off between train operating efficiency and maintenance-window availability.

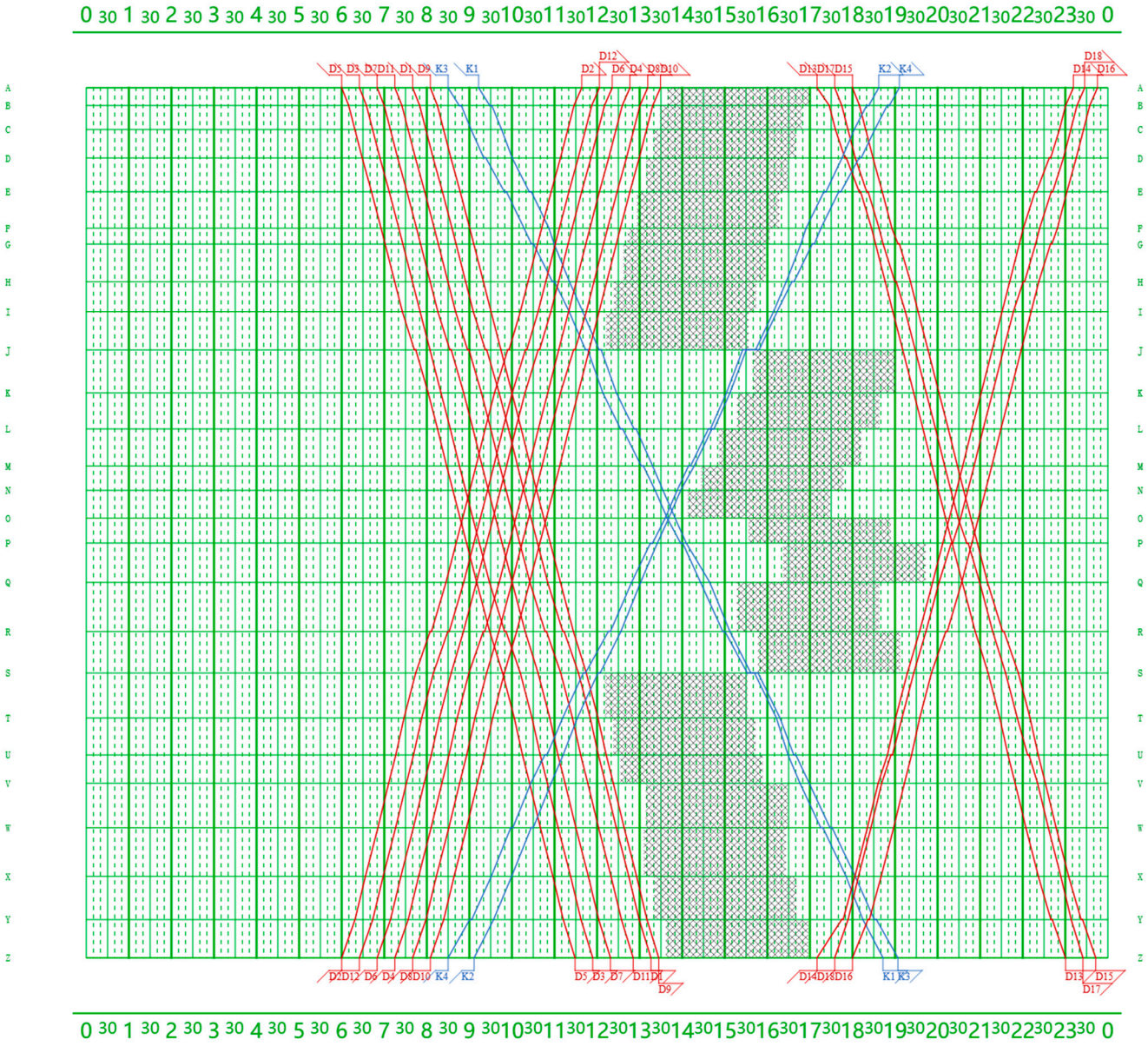


Figure 12. Coordinated optimization results of the train timetable and maintenance windows ($\lambda = 0.8$).

6. Conclusions

This study develops a coordinated optimization framework for train timetables and maintenance windows in plateau railway operations. The proposed model jointly considers train sequencing, train arrival and departure times, and segmented rectangular maintenance windows, while incorporating operational constraints and train–maintenance safety separa-

tion. The framework provides an integrated representation of the interaction between train operations and infrastructure maintenance under the considered plateau railway conditions.

To solve the resulting mixed discrete–continuous optimization problem, a Benders decomposition approach is developed by assigning the discrete sequence-related decisions to the master problem and the continuous time-related decisions to the subproblem. IIS-based conflict cuts are incorporated to identify infeasible combinations of discrete decisions and progressively exclude such combinations from the master problem. The proposed solution framework provides a systematic way to handle the coupling between train sequencing and maintenance-window scheduling without changing the original optimization model.

The case study based on an anonymized plateau railway demonstrates that the proposed framework can generate a feasible coordination scheme for passenger train operations and maintenance windows while satisfying the prescribed operational and safety requirements. The results further show that the coordination of train timetables and maintenance windows is important for avoiding temporal conflicts between train movements and maintenance activities and for maintaining the feasibility of both operating and maintenance plans. The computational results support the applicability of the proposed decomposition framework to the considered case; however, the obtained optimality gaps indicate that the solutions should not be interpreted as demonstrating guaranteed near-optimality.

From a practical perspective, the proposed framework can provide decision support for railway timetable compilation and maintenance-window planning in plateau railway operations. By jointly considering train movements and maintenance activities, planners can evaluate the temporal compatibility between passenger services and infrastructure maintenance before implementing the timetable and maintenance plan. The framework can also serve as a basis for further studies involving heterogeneous maintenance durations, freight train operations, rolling-stock circulation, and more detailed operational characteristics of plateau railways.

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