

Article

An Integrated Weighted Fuzzy N-Soft Set–CODAS Framework for Decision-Making in Circular Economy-Based Waste Management Supporting the Blue Economy: A Case Study of the Citarum River Basin, Indonesia

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Abstract

The Citarum River Basin (DAS Citarum) in Indonesia faces significant challenges in waste management, necessitating a circular economy-based approach to reduce land-based pollution, which is critical for achieving the sustainability goals of the blue economy in the basin. This study addresses the complexity and inherent uncertainty in decision-making processes related to this challenge by developing a novel hybrid model, namely the Weighted Fuzzy N-Soft Set combined with the COmbinative Distance-based Assessment (CODAS) method. The model synergistically integrates the weighted 10R strategies in the circular economy, obtained via the Analytical Hierarchy Process (AHP), the capability of Fuzzy N-Soft Sets to represent uncertainty granularly, and the robust ranking mechanism of CODAS. Applied to a case study covering 16 types of waste in the Citarum River Basin, the model effectively processes expert assessments that are ambiguous regarding the 10R criteria. The results indicate that single-use plastics, particularly plastic bags (HDPE), styrofoam, transparent plastic sheets (PP), and plastic cups (PP), are the top priorities for intervention, in line with the high AHP weights for upstream strategies such as Refuse (0.2664) and Rethink (0.2361). Comparative analysis with alternative models, namely Fuzzy N-Soft Set-CODAS, Weighted Fuzzy N-Soft Set with row-column sum ranking, and Weighted Fuzzy N-Soft Set-TOPSIS, confirms the superiority of the proposed hybrid model in producing ecologically rational priorities, free from purely economic value biases. Further sensitivity analysis shows that the model remains highly robust across various weighting scenarios. This study concludes that the WFN-SS-CODAS framework provides a rigorous, data-driven, and reliable decision support tool for translating circular economy principles into actionable waste management priorities, directly supporting the restoration and sustainability goals of the blue economy in river basins. The findings suggest that targeting the high-priority waste types identified by the model addresses the dominant fraction of riverine pollution, indicating the potential for significant waste volume reduction. This research was conducted to directly contribute to achieving multiple targets under SDG 6 (Clean Water



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and Sanitation), SDG 12 (Responsible Consumption and Production), and SDG 14 (Life Below Water).

Keywords: circular economy; blue economy; weighted fuzzy N-soft set; CODAS; multi-criteria decision making; waste management (SDG 12)

MSC: 03E72; 03E75; 90B50; 91B76

1. Introduction

Waste management in major river basins in developing countries is a multidimensional challenge that intertwines ecological, social, and economic complexities within a dynamic landscape [1]. In Indonesia, these challenges are most visibly reflected in the Citarum River Basin, the longest and one of the most critical waterways in West Java. Historically a lifeline for surrounding communities, the Citarum has undergone severe degradation driven by industrial expansion, rapid urbanization, and weak waste-management systems. Currently, various types of waste, ranging from household and textile waste to industrial residues and plastic debris that account for nearly 60% of total waste accumulation, pollute the river, degrading environmental quality and community well-being. Government intervention under the “Citarum Harum” program has introduced waste-collection technologies and multi-agency governance mechanisms; however, the majority of these approaches remain linear and end-of-pipe, failing to address the structural root causes.

In response to today’s global environmental challenges, transforming waste-management systems requires integrated approaches that transcend the limitations of conventional linear models. In this context, applying Circular Economy (CE) principles to waste management emerges as a strategic solution that can comprehensively link terrestrial and aquatic dynamics and directly support Blue Economy (BE) objectives. CE emphasizes resource conservation through the hierarchical 10R strategies, such as refuse, rethink, reduce, reuse, recover, aimed at retaining material value within the economic cycle for as long as possible [2]. Conversely, BE emphasizes the regenerative and innovative utilization of aquatic systems through zero-waste principles. Applying CE strategies to waste management thus provides a holistic framework that reduces environmental pressure on aquatic ecosystems, thereby advancing the goals of the blue economy.

Strategically, applying circular economy principles to waste management is not merely an operational concept but a critical prerequisite for achieving several Sustainable Development Goals (SDGs) that are also central to the blue economy. This synergy directly supports SDG 6 (Clean Water and Sanitation) by reducing solid-waste pollution entering water bodies; SDG 12 (Responsible Consumption and Production) through the adoption of the 10R hierarchy that promotes sustainable production and consumption patterns; and SDG 14 (Life Below Water) by protecting marine ecosystems from plastic pollution, much of which originates from land-based waste streams. Additionally, it indirectly contributes to SDG 8 (Decent Work and Economic Growth) through the creation of green jobs in the recycling and waste-management sectors, and to SDG 13 (Climate Action) by reducing greenhouse gas emissions associated with primary material production and improper waste handling.

Although there is an increasing body of literature discussing CE and BE separately, studies that apply advanced CE decision-making tools to prioritize waste management actions in river basins, a key intervention for achieving BE outcomes, especially in developing countries, remain very limited. A significant challenge in this case lies in the complexity of decision-making. Waste-management systems involve multiple stakeholders, government, industry, local communities, and recycling actors, each holding heterogeneous priorities,

preferences, and risk perceptions [3]. Moreover, environmental data are often ambiguous, subjective, incomplete, and inconsistent across sources [4]. Therefore, decision-making for waste-management strategies requires mathematical tools capable of capturing uncertainty, representing linguistic assessments, and integrating diverse weights and parameters within a structured, systematic analytical framework.

Moreover, there is a significant implementation gap between global SDG policy narratives and the technical and operational realities on the ground. The link between circular economy actions and blue economy outcomes in river basins remains at a normative level, lacking an operational evaluation framework to determine which interventions are most prioritized, feasible, and capable of delivering the most significant impact across multiple SDGs simultaneously. For instance, policies that overemphasize recycling (downstream circular economy) without strengthening refuse and reduce strategies (upstream circular economy) risk diverting attention from the root causes of waste and may neglect the regenerative principles of the blue economy, resulting in waste still ending up in aquatic environments. Therefore, a systematic, measurable, and analysis-based approach is needed to evaluate and prioritize waste management strategies that consistently align with the objectives of the circular and blue economies.

Soft Set theory, introduced by Molodtsov [5], provides a non-probabilistic framework for dealing with conceptual and parameter-based uncertainty. While originally binary, the framework has evolved to accommodate broader applications. For example, Maji et al. [6] extended soft-set operations into more comprehensive logical representations, while Pei and Miao [7] strengthened their relationship with information systems. Several other studies across various domains have also enriched the development of this concept [8–12]. As the need to capture variations in rating levels increases, Fatimah et al. [13] introduced the N-Soft Set framework, which supports a multi-level rating system rather than a binary choice. However, N-Soft Sets still operate in a discrete environment, failing to account for linguistic vagueness. Korkmaz et al. [14] addressed this gap by developed the Weighted Fuzzy N-Soft Set, a hybrid model that integrates fuzzy membership, discrete grading, and parameter weights, an essential aspect of multi-criteria problems [15,16]. This significantly improves the representation of stakeholder preferences, particularly when they are inconsistent, ambiguous, or multidimensional. However, although Weighted Fuzzy N-Soft Sets excel at representing uncertainty, they do not inherently provide a ranking mechanism; consequently, they cannot model inputs in a way that translates into prioritized decisions.

Conversely, in the realm of multi-criteria decision-making (MCDM), the Combinative Distance-based Assessment (CODAS) method developed by Ghorabae et al. [17] is a robust and stable ranking approach. CODAS uses a combination of Euclidean and Hamming distances relative to the negative-ideal point to differentiate alternatives even when performance differences are slight. This advantage has encouraged many researchers to extend CODAS to more complex uncertainty environments. For example, Noor et al. [18] developed a version of CODAS for probabilistic dual hesitant fuzzy sets, demonstrating its ability to handle multi-level hesitation. Similarly, Garg et al. [19] extended CODAS to complex intuitionistic fuzzy group decision-making, further enhancing its adaptability to multi-stakeholder social contexts. However, it should be critically noted that the majority of CODAS developments remain separate from the soft set framework and have not considered the granular parameterized ratings characteristic of N-Soft Sets. In other words, CODAS excels at ranking but lacks the rich mechanisms for representing uncertainty found in fuzzy soft sets.

Considering the developments in these two research streams, soft sets and CODAS, there is a clear structural gap: to date, no hybrid framework exists that combines the

uncertainty representation capability of Weighted Fuzzy N-Soft Sets with the robust ranking power of CODAS. This methodological gap is particularly critical, given that no previous studies have integrated these two approaches in the context of circular economy–blue economy (CE–BE) or waste management in the Citarum River Basin. However, river-based waste management systems require methods that can capture stakeholders’ preference uncertainties while simultaneously providing clear and actionable priority rankings.

Based on this literature gap, the present study aims to formulate a hybrid Weighted Fuzzy N-Soft Set-CODAS model to support circular economy-based decision-making for waste management in the Citarum River Basin, thereby addressing a core challenge for its blue economy potential. The model is designed to handle linguistic uncertainty, accommodate multi-level parameters, provide measurable weights for criteria, and simultaneously offer a robust mechanism for ranking alternatives. Therefore, the proposed approach not only contributes methodologically to the applied mathematics literature, particularly in soft sets and MCDM, but also offers significant practical contributions for environmental policy and governance. Furthermore, this study has the potential to strengthen the transformation of the Citarum waste management system by providing waste-handling priorities grounded in CE principles and by mapping 16 waste types against the hierarchical 10R strategies. In this way, the study offers a concrete, data-driven step toward achieving cleaner rivers, more resilient community economies, and more sustainable waste management systems.

The structure of this paper includes: abstract, introduction, theoretical foundations of the fuzzy N-soft set, materials and methods, results and discussion, and conclusion.

2. Preliminaries

2.1. Circular Economy

The global economy has long operated under a linear “take, make, dispose” model, which has resulted in resource depletion, increased waste, and ecological degradation, despite supporting growth. The circular economy offers an alternative framework emphasizing resource efficiency and the regeneration of natural systems through sustainable design, production, and consumption practices [20]. Its core aim is to maintain materials and products in use for as long as possible by prioritizing repair, reuse, refurbishment, and recycling [21].

A key feature of this framework is its restorative and regenerative design approach. Products are intentionally created to be durable, repairable, or recyclable, contrasting with linear systems that often overlook sustainability at the design stage [22]. Frameworks have expanded from the initial 3Rs to the broader “10R” hierarchy, such as refuse, rethink, and remanufacture, which applies across consumer, industrial, policy context [22]. The framework focuses on reducing waste and pollution, extending product lifecycles, and supporting the renewal of natural systems.

Circular practices are increasingly implemented across manufacturing, construction, and energy sectors, including material reuse, modular product design, and the conversion of organic waste into bioenergy. Literature documents more than 45 circular strategies with numerous practical applications [20]. However, widespread adoption is hindered by regulatory gaps, limited incentives, high initial costs, technological constraints, and slow shifts in consumer behavior. Despite these barriers, potential benefits, such as cost savings, new business models, and enhanced brand value, continue to encourage the transition toward circular economic systems [23].

2.2. Blue Economy

The blue economy is an economic framework that promotes the sustainable use of ocean and marine resources to improve livelihoods, support economic growth, and

maintain healthy ecosystems. While it is part of the broader green economy, it specifically focuses on water-based systems. Its aim is to generate economic benefits while reducing environmental pressures and restoring ecological balance, thereby creating a sustainable cycle [24]. This approach emphasizes restorative practices, moving beyond older, often harmful extractive industries. It gained global attention after the 2012 UN Conference on Sustainable Development, which advocated for its adoption [25]. The framework spans multiple sectors, including fisheries, renewable ocean energy, tourism, biotechnology, and shipping.

Its core principles are sustainability, innovation, and circularity. Sustainability requires that all activities remain within the regenerative capacity of marine ecosystems, guiding practices such as fishing quotas, eco-friendly aquaculture, and climate-resilient coastal planning. Innovation drives the development of new technologies and business models that enable efficient resource use and reduced waste, such as offshore renewable energy of marine-derived pharmaceuticals. The circularity principle ensures that waste is minimized by repurposing by-products. For example, using shellfish shells in construction materials or as fertilizers [26].

The blue economy holds significant economic potential, with projections indicating that the ocean economy could double to over USD 3 trillion by 2030. Key growth areas include coastal tourism and marine renewable energy [27]. Beyond economic value, it addresses critical issues such as food security through sustainable aquaculture and contributes to climate change mitigation by protecting “blue carbon” ecosystems, such as mangroves and seagrass beds, that store substantial amounts of carbon [28].

2.3. Soft Set

Let U be a universal set and E a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U , and let $A, B \subseteq E$. A soft set is defined as follows.

Definition 1 ([5]). A pair (F, A) is called a soft set over U if F is a mapping from A to the collection of subsets of U , as shown in Equation (1).

$$F : A \rightarrow \mathcal{P}(U). \quad (1)$$

Mathematically, the soft set (F, A) can be expressed as:

$$(F, A) = \{(e, F(e)) | F(e) \in \mathcal{P}(U), e \in A\}. \quad (2)$$

Next, the definition of the soft subset operation is given as follows.

Definition 2 ([7]). Let (F, A) and (H, B) be soft sets over U . The set (F, A) is called a soft subset of (H, B) , denoted by $(F, A) \tilde{\subseteq} (H, B)$, if

- (i) $A \subseteq B$, and
- (ii) $\forall e \in A$ holds $F(e) \subseteq H(e)$

(F, A) is called a soft superset of (H, B) precisely when (H, B) is a soft subset of (F, A) . This relationship is represented by $(F, A) \tilde{\supseteq} (H, B)$.

In addition, the AND and OR operations on soft sets are defined as follows.

Definition 3 ([6]). Let (F, A) and (H, B) be soft sets over U . The AND operation between (F, A) and (H, B) , denoted by $(F, A) \wedge (H, B)$, is defined as $(K, A \times B)$ where for every $(a, b) \in A \times B$, holds $K(a, b) = F(a) \cap H(b)$.

Definition 4 ([6]). Let (F, A) and (H, B) be soft sets over U . The OR operation between (F, A) and (H, B) , denoted by $(F, A) \vee (H, B)$, is defined as $(K, A \times B)$ where for every $(a, b) \in A \times B$, holds $K(a, b) = F(a) \cup H(b)$.

2.4. N-Soft Set

Soft-set theory has evolved significantly since its introduction, leading to the development of N-Soft Sets, a framework that accommodates both binary and non-binary evaluations for decision-making. The concept was introduced by Fatimah et al. [13]. The definition of N-soft sets is given as follows, noting that each basic concept still refers to the soft set theory described earlier.

Definition 5 ([13]). Let $G = \{0, 1, \dots, N-1\}$, with $N \in \mathbb{N}$ and $N \geq 2$. A triple (F, A, N) is called an N-soft set over U if:

$$F : A \rightarrow 2^{U \times G}, \quad (3)$$

and for each $e \in A$, there exists a unique pair (u, g_e) , with $u \in U$ and $g_e \in G$, such that $(u, g_e) \in F(e)$. Furthermore, (u, g_e) can be written as in Equation (4).

$$F(e)(u) = g_e. \quad (4)$$

Suppose $U = \{u_1, u_2, u_3, \dots, u_m\}$ and $A = \{e_1, e_2, e_3, \dots, e_n\}$ are finite sets. Then N-soft set (F, A, N) can be represented in the following table [13].

In Table 1, g_{mn} means $F(u_m)(e_n)$, for every m and n .

Table 1. Tabular Representation of N-Soft Set (F, A, N) .

(F, A, N)	e_1	e_2	e_3	$\dots$	e_n
u_1	g_{11}	g_{12}	g_{13}	$\dots$	g_{1n}
u_2	g_{21}	g_{22}	g_{23}	$\dots$	g_{2n}
u_3	g_{31}	g_{32}	g_{33}	$\dots$	g_{3n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	g_{m1}	g_{m2}	g_{m3}	$\dots$	g_{mn}

Example 1. Suppose the owner of a coffee shop wants to introduce a new beverage to the menu. There are five different coffee recipes, denoted by $\{u_1, u_2, u_3, u_4, u_5\}$. The owner faces difficulties in deciding which recipe to select and therefore chooses to conduct a taste test with a group of loyal customers $A = \{e_1, e_2, e_3\}$ to assist in the decision-making process. For the taste test, each customer is required to assign a rating to every recipe. The shop owner adopts a scoring system ranging from 0 to 4, where 0 represents the lowest score and 4 represents the highest. Customers are asked to taste each recipe and assign a score from 0 to 4 based on flavor, appearance, and overall appeal. Accordingly, a 5-soft set $(F, A, 5)$ can be defined as follows:

$$F(e_1) = \{(u_1, 2), (u_2, 1), (u_3, 4), (u_4, 2), (u_5, 3)\},$$

$$F(e_2) = \{(u_1, 3), (u_2, 1), (u_3, 2), (u_4, 2), (u_5, 3)\},$$

$$F(e_3) = \{(u_1, 2), (u_2, 2), (u_3, 3), (u_4, 4), (u_5, 4)\}.$$

The tabular representation of $(F, A, 5)$ is given in Table 2 [13].

Table 2. Tabular Representation of 5-Soft Set $(F, A, 5)$ in Example 1.

$(F, A, 5)$	e_1	e_2	e_3
u_1	2	3	2
u_2	1	1	2
u_3	4	2	3
u_4	2	2	4
u_5	3	3	4

2.5. Fuzzy N-Soft Set

Research on N-soft sets continues to evolve, encouraging scholars to expand the theory through various combinations with other analytical approaches [29–33]. One such development is the fuzzy N-soft set, an extension of the traditional N-soft set in which the evaluation of each object with respect to a parameter is expressed using fuzzy values. This framework integrates the strengths of both N-soft sets and fuzzy sets in representing uncertainty. Furthermore, the fuzzy N-soft set is denoted as an (F, N) -soft set [14].

Given a universal set U , a set of parameters E , and $A \subseteq E$. Let $[0, 1]^U$ denote the set of all fuzzy sets on U , where each $\mu \in [0, 1]^U$ is defined as:

$$\mu = \{\langle u, \mu_u \rangle | u \in U\}, \quad (5)$$

with $\mu_u = \mu(u)$ for each $u \in U$. Then, the fuzzy N-soft set can be defined as follows.

Definition 6 ([14]). A pair (μ, K) is called a fuzzy N-soft set if $K = (F, A, N)$ is an N-soft set over U , and μ is a function that maps each $e \in A$ to a fuzzy set in $[0, 1]^{F(e)}$. For every element $e \in A$, it holds that $\mu(e) \in [0, 1]^{F(e)}$.

In line with the definition of an N-soft set, for each $e \in A$ and $u \in U$, there exists a unique pair $(u, g_e) \in U$ with $g_e \in G$ such that $\langle (u, g_e), \mu_e(u) \rangle$. This can be denoted equivalently as $\mu_e(u) = \mu(e)(u, g_e)$.

Let a universal set $U = \{u_1, u_2, u_3, \dots, u_m\}$ and a parameter set $A = \{e_1, e_2, e_3, \dots, e_n\}$ are finite sets. Let $K = (F, A, N)$ be an N-soft set over U . Then, the fuzzy N-soft set (μ, K) can be represented in the following tabular form [14] (see Table 3).

Table 3. Tabular Representation of Fuzzy N-Soft Set (μ, K) .

(μ, K)	e_1	e_2	e_3	...	e_n
u_1	$\langle g_{11}, \mu_{11} \rangle$	$\langle g_{12}, \mu_{12} \rangle$	$\langle g_{13}, \mu_{13} \rangle$	...	$\langle g_{1n}, \mu_{1n} \rangle$
u_2	$\langle g_{21}, \mu_{21} \rangle$	$\langle g_{22}, \mu_{22} \rangle$	$\langle g_{23}, \mu_{23} \rangle$	...	$\langle g_{2n}, \mu_{2n} \rangle$
u_3	$\langle g_{31}, \mu_{31} \rangle$	$\langle g_{32}, \mu_{32} \rangle$	$\langle g_{33}, \mu_{33} \rangle$	...	$\langle g_{3n}, \mu_{3n} \rangle$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\langle g_{m1}, \mu_{m1} \rangle$	$\langle g_{m2}, \mu_{m2} \rangle$	$\langle g_{m3}, \mu_{m3} \rangle$	...	$\langle g_{mn}, \mu_{mn} \rangle$

Example 2. Suppose a company is opening a job vacancy for a data analyst position and is trying to determine which candidate should be hired. There are three applicants, denoted by $\{u_1, u_2, u_3\}$. The company has collected information on each applicant based on four criteria: e_1 representing programming and data-visualization skills, e_2 representing work experience, e_3 representing

communication skills, and e_4 representing educational background. The company evaluates the applicants' qualifications according to the following criteria.

$$0 \leq \mu(e, u) \leq 0.2 \text{ if } g_e = 0,$$

$$0.2 \leq \mu(e, u) \leq 0.4 \text{ if } g_e = 1,$$

$$0.4 \leq \mu(e, u) \leq 0.7 \text{ if } g_e = 2,$$

$$0.7 \leq \mu(e, u) \leq 1 \text{ if } g_e = 3.$$

Thus, a fuzzy 4-soft set can be defined as follows.

$$\mu_{e_1} = \{ \langle (u_1, 2), 0.65 \rangle, \langle (u_2, 1), 0.26 \rangle, \langle (u_3, 0), 0.18 \rangle \},$$

$$\mu_{e_2} = \{ \langle (u_1, 1), 0.32 \rangle, \langle (u_2, 3), 0.78 \rangle, \langle (u_3, 2), 0.45 \rangle \},$$

$$\mu_{e_3} = \{ \langle (u_1, 2), 0.65 \rangle, \langle (u_2, 2), 0.48 \rangle, \langle (u_3, 3), 0.81 \rangle \},$$

$$\mu_{e_4} = \{ \langle (u_1, 3), 0.79 \rangle, \langle (u_2, 2), 0.46 \rangle, \langle (u_3, 1), 0.34 \rangle \}.$$

The tabular representation of the fuzzy 4-soft set is provided in Table 4.

Table 4. Tabular Representation of Fuzzy 4-Soft Set $(F, A, 4)$ in Example 2.

(μ, K)	e_1	e_2	e_3	e_4
u_1	$\langle 2, 0.65 \rangle$	$\langle 1, 0.32 \rangle$	$\langle 2, 0.65 \rangle$	$\langle 3, 0.79 \rangle$
u_2	$\langle 1, 0.26 \rangle$	$\langle 3, 0.78 \rangle$	$\langle 2, 0.48 \rangle$	$\langle 2, 0.46 \rangle$
u_3	$\langle 0, 0.18 \rangle$	$\langle 2, 0.45 \rangle$	$\langle 3, 0.81 \rangle$	$\langle 1, 0.34 \rangle$

Definition 7 ([14]). Suppose $0 \leq g_1 \leq g_2 < N$. The fuzzy soft set corresponding to the N-soft set (F, A, N) and the interval $[g_1, g_2]$ is defined as follows.

$$\mu_{g_1}^{g_2}(e, u) = \begin{cases} \mu(e)(u, g_e), & \text{if } (u, g_e) \in F(e) \text{ and } g_e \in [g_1, g_2], \\ 0, & \text{others.} \end{cases} \quad (6)$$

Example 3. Reconsider the fuzzy 4-soft set in Table 4. The fuzzy soft set corresponding to the fuzzy 4-soft set and the interval $[2, 3]$ is given in Table 5 [14].

Table 5. Tabular Representation of Fuzzy 4-Soft Set $(F, A, 4)$ and interval $[2, 3]$.

(μ, K)	e_1	e_2	e_3	e_4
u_1	0.65	0	0.65	0.79
u_2	0	0.78	0.48	0.46
u_3	0	0.45	0.81	0

Definition 8 ([14]). Weighted fuzzy N-soft sets are denoted by (F, A, N, w) where $w : A \rightarrow [0, 1]$ and $w(e_i)$ represents the degree of importance given to parameter e_i .

The Fuzzy N-Soft Set concept has undergone various developments and has been widely applied to solve diverse problems involving uncertainty, particularly in decision-making contexts [34–36].

2.6. Analytical Hierarchy Process

The Analytic Hierarchy Process (AHP), developed by T. L. Saaty in the 1970s, was designed to address complex decision problems in which statistical data and information are limited. AHP is a multi-criteria decision-making model used to derive ratio scales from discrete or continuous pairwise comparisons based on actual measurements, preferences, or judgments of relative importance. It incorporates multiple criteria and alternatives, each evaluated against all relevant criteria [37]. The general algorithm of AHP is presented as follows.

- (1) Determine the pairwise comparison matrix, which is expressed as Equation (7).

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix}. \quad (7)$$

Matrix B consists of numerical scores that indicate the degree to which one element is considered more important than another, following Saaty's established 1–9 scale. The scale also includes reciprocal values and permits intermediate scores between 1.1 and 1.9 for finer distinctions. A score of 1 indicates equal importance. Scores of 2, 3, and 4 represent slight, moderate, and moderately strong preferences, respectively. A score of 5 denotes a strong preference supported by judgment and experience. A score of 6 reflects a preference slightly above strong, whereas 7 indicates demonstrated and extreme importance in practice. Scores of 8 and 9 correspond to very high and the highest levels of dominance, respectively.

To maintain logical consistency in the comparisons, if element i is judged more important than element j by a given score, then the comparison of j to i must be assigned the reciprocal of that score. This ensures internal balance within the matrix. For cases in which two elements are nearly equal in importance, intermediate values (1.1–1.9) provide subtle distinctions without altering the intended meaning of the scale.

Based on the first axiom of reciprocity, Equation (8) is given as follows.

$$b_{ij} = \frac{1}{b_{ji}}. \quad (8)$$

- (2) Determine the normalization matrix of B , which is expressed as $C = [c_{ij}]_{n \times n}$. The elements of matrix C are calculated by Equation (9).

$$c_{ij} = \frac{b_{ij}}{\sum_{i=1}^n b_{ij}}. \quad (9)$$

- (3) The weight calculation, namely the eigenvector $w = [w_i]_{n \times 1}$ of the normalized matrix C , is performed by computing the arithmetic mean of each row of the matrix, as expressed in Equation (10).

$$w_i = \frac{\sum_{j=1}^n c_{ij}}{n}. \quad (10)$$

- (4) Obtaining the value of λ_{max} summing the columns of C and multiplying the resulting vector by the vector w , shown by Equation (11).

$$\left(\sum_{j=1}^n b_{ij} \right) w_j = \lambda_{max} w_i. \quad (11)$$

(5) Perform a ratio consistency test using Equation (12).

$$CR = \frac{CI}{RI}, \quad (12)$$

where the Consistency Index (CI) is computed as:

$$CI = \frac{\lambda_{max} - n}{n - 1}. \quad (13)$$

The Random Index (RI), as defined by Saaty [38], is a reference value that depends on the order (n) of the pairwise comparison matrix. It is used in the computation of the Consistency Ratio (CR) to evaluate the reliability of the judgment matrix. The corresponding RI values are as follows:

- $n = 1 - 2$: RI = 0 (inconsistency not applicable);
- $n = 3$: RI = 0.58;
- $n = 4$: RI = 0.90;
- $n = 5$: RI = 1.12;
- $n = 6$: RI = 1.24;
- $n = 7$: RI = 1.32;
- $n = 8$: RI = 1.41;
- $n = 9$: RI = 1.45;
- $n = 10$: RI = 1.49.

A pairwise comparison matrix is considered acceptably consistent when the resulting CR is below 0.10. If the CR is equal to or exceeds 0.10, the associated judgments must be revised to enhance consistency.

2.7. Fuzzy N-Soft Set Decision-Making Based on CODAS

CODAS was introduced by Ghorabae et al. [17] as an assessment and ranking method for Multi-Criteria Decision-Making (MCDM) problems. The fundamental principle of CODAS is to measure the distance of each alternative from the negative-ideal point (anti-ideal point) using two distance metrics: the Euclidean distance as the primary measure and the Taxicab or Hamming distance (l_1) as the secondary or supporting measure. Alternatives with larger distances are considered more desirable because they lie farther from the anti-ideal condition. The method was developed to provide a comparison mechanism that is sensitive to the magnitude of performance differences among alternatives, while also offering a secondary discrimination procedure when the primary distance is insufficient to differentiate them.

The intuition behind CODAS is that, after normalizing and weighting the performance of alternatives across all criteria, the anti-ideal value or worst value for each criterion is determined. For each alternative i , the weighted distance from this anti-ideal vector, is computed. The primary distance (l_2 -norm) captures the overall quadratic deviation from the anti-ideal. In contrast, the secondary distance (l_1 -norm) is used to resolve minor differences that cannot be distinguished solely through the l_2 metric. These distance measures are then aggregated into a composite score, which is used to generate the final ranking of alternatives. The use of these two norms provides CODAS with robustness in situations where a single metric may not be sufficiently informative.

The general CODAS algorithm extended to the Fuzzy Soft Set environment follows Peng and Garg [39]. However, in this study, the algorithm is modified to fit the Weighted Fuzzy N-Soft Set model:

- (1) Normalization of the weighted fuzzy N-soft set values by determining the cost and benefit parameters based on Equation (14).

$$\widetilde{(\mu, K)}(u_i) = \begin{cases} (\mu, K)(u_i), & e_j \text{ is benefit,} \\ 1 - (\mu, K)(u_i), & e_j \text{ is cost.} \end{cases} \quad (14)$$

- (2) Calculating the score function s_{ij} of $\widetilde{(\mu, K)}(u_i)$, $(i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n)$ with Equation (15).

$$s_{ij} = \frac{\exp(\widetilde{(\mu, K)}(u_i) - 1)}{1 + \widetilde{(\mu, K)}(u_i)}. \quad (15)$$

- (3) Forming a weighted normalized decision matrix $R = (r_{ij})_{m \times n}$. To reinforce the influence of criterion importance in the ranking phase, the score function values s_{ij} are weighted again using the same AHP-derived weights w_j , with elements expressed as weighted normalized performance values in Equation (16).

$$r_{ij} = w_j s_{ij}, \quad (16)$$

with w_j representing the weight of the j parameters and fulfilling the conditions $0 < w_j < 1$ and $\sum_{j=1}^n w_j = 1$.

- (4) Determining the negative ideal solution based on Equations (17) and (18).

$$N = (n_j)_{1 \times n}, \quad (17)$$

$$n_j = \min_{i=1,2,\dots,m} \{r_{ij}\}. \quad (18)$$

- (5) Computing the Euclidean distance vector $Eu = (Eu_i)_{1 \times m}$ and the Hamming distance vector $Ha = (Ha_i)_{1 \times m}$ of each alternative from the negative-ideal solution, as expressed in Equations (19) and (20).

$$Eu_i = \sqrt{\sum_{j=1}^n (r_{ij} - n_j)^2}, \quad (19)$$

$$Ha_i = \sum_{j=1}^n |r_{ij} - n_j|. \quad (20)$$

- (6) Constructing the relative assessment matrix based on the Euclidean and Hamming distances, as shown in Equations (21) and (22).

$$RA = (pr_{ik})_{m \times m}, \quad (21)$$

$$pr_{ik} = (Eu_i - Eu_k) + (\Phi(Eu_i - Eu_k) \times (Ha_i - Ha_k)), \quad (22)$$

where $k = 1, 2, 3, \dots, m$, and Φ denotes the threshold function used to detect the equivalence of the Euclidean distances between two alternatives, as expressed in Equation (23).

$$\Phi(Eu_i - Eu_k) = \begin{cases} 1, & \text{if } |Eu_i - Eu_k| \geq \Phi, \\ 0, & \text{if } |Eu_i - Eu_k| < \Phi. \end{cases} \quad (23)$$

In this function, Φ is a threshold parameter that can be selected by the decision-maker. It is recommended that the threshold be set to a value very close to zero, typically $0.01 \leq \Phi \leq 0.05$. If the difference in Euclidean distances between two alternatives exceeds Φ , then the alternatives are further compared using the Hamming distance.

- (7) Determining the assessment score of each alternative based on Equation (24).

$$RA_i = \sum_{k=1}^m pr_{ik}. \quad (24)$$

The alternative that has the highest RA_i score is the optimal choice for the decision-making problem under consideration.

A thorough understanding of the concepts and theoretical foundations discussed in this section serves as a critical basis for developing the subsequent research approach. The principles and formulations presented here guide the design of the method, the selection of data, and the analytical procedures described in the following section. Accordingly, the following section provides a detailed explanation of the materials used and the methodology applied to operationalize the conceptual framework in this study.

2.8. Weighted Fuzzy N-Soft Set–CODAS Model

This study employs an integrated model combining the Weighted Fuzzy N-Soft Set and the CODAS method to model the circular economy-based decision-making process for waste management in the Citarum River Basin, an approach critical for mitigating land-based pollution and supporting blue economy goals. This approach is selected for its ability to handle data uncertainty and ambiguity, provide more granular assessments compared to conventional models, and generate more accurate priority rankings for waste management. The methodological steps implemented follow the algorithm developed by Korkmaz et al. [14], which allows for the simultaneous consideration of fuzzy membership degrees and discrete grades assigned to each alternative across the defined parameters. The ranking of other options is carried out using the CODAS-based procedure introduced by Peng and Garg [39].

Before presenting the specific steps of the proposed framework, the hybrid Weighted Fuzzy N-Soft Set–CODAS model is formally defined.

Definition 9. Let $U = \{u_1, u_2, \dots, u_n\}$ be the universal set, $A = \{e_1, e_2, \dots, e_m\}$ be the set of parameters, and $L = (F, A, N)$ denote an N-soft set over U , where $F(e_j) = \{(u_i, g_{ij}) | u_i \in U\}$, $g_{ij} \in \{N \in \mathbb{N} | 0, 1, 2, \dots, N-1\}$, and $\mu_{e_j}(u_i) \in [0, 1]$, for all i, j . Suppose the weight vector $w_{AHP} = (w_1, w_2, \dots, w_m)$ satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^m w_j = 1$. Then the Weighted Fuzzy N-Soft Set associated with L and w_{AHP} is defined as the family of mappings:

$$(\mu, L)_{w_{AHP}} = \left\{ (\mu, L)_{w(e_j)} \mid e_j \in A \right\}, \quad (25)$$

where $(\mu, L)_{w(e_j)} : U \rightarrow [0, 1]$ and $(\mu, L)_{w(e_j)}(u_i) = w_j \mu_{e_j}(u_i)$, $i = 1, 2, 3, \dots, n$. The Weighted Fuzzy N-Soft Set may also be denoted by:

$$(F, \mu, A, N, w) = \left\{ \langle e_j, \left\{ \langle u_i, g_{e_j}, \mu_{e_j}(u_i) \rangle \mid u_i \in U \right\}, w_j \rangle \mid e_j \in A \right\}. \quad (26)$$

Definition 10. Suppose each parameter $e_j \in A$ has been classified as either a benefit or a cost. For each $e_j \in A$ and each $u_i \in U$, the following transformation is defined:

$$\widetilde{(\mu, L)}_{w(e_j)}(u_i) = \begin{cases} (\mu, L)_{w(e_j)}(u_i), & \text{if } e_j \text{ is benefit,} \\ 1 - (\mu, L)_{w(e_j)}(u_i), & \text{if } e_j \text{ is cost.} \end{cases} \quad (27)$$

For all criteria $e_j \in A$, whether benefit or cost, the value $\widetilde{(\mu, L)}_{w(e_j)}(u_i)$ can be uniformly interpreted as “the larger, the better” within the interval $[0, 1]$.

Definition 11. Based on the transformation values in Definition 10, for each alternative–criterion pair (u_i, e_j) with $\widetilde{(\mu, L)}_{w(e_j)}(u_i) \in [0, 1]$, the score function is defined as in Equation (28).

$$Sc_{ij} = \frac{\exp\left(\widetilde{(\mu, L)}_{w(e_j)}(u_i) - 1\right)}{1 + \widetilde{(\mu, L)}_{w(e_j)}(u_i)}, \quad i = 1, \dots, n; j = 1, \dots, m. \quad (28)$$

The steps taken are as follows.

Step 1: Determination of criteria weights using AHP.

- (1) The decision-maker conducts pairwise comparisons of all parameters in A using Saaty’s fundamental scale (1–9) [37] to construct matrix A according to Equation (7), where reciprocal elements follow Equation (8).
- (2) The matrix is normalized using Equation (9) to form $C = [c_{ij}]_{n \times n}$, and the priority weight vector $w = [w_{ij}]_{n \times 1}$ is obtained using Equation (10).
- (3) The Consistency Ratio (CR), given in Equation (12), is computed through the maximum eigenvalue λ_{max} (Equation (11)), the Consistency Index CI (Equation (13)), and the Random Index (RI). If $CR \geq 0.10$, the comparisons are deemed inconsistent, and the evaluator must revise the pairwise judgments in matrix A ; the computations are repeated until the consistency requirement is satisfied. If $CR < 0.10$, the comparisons are considered consistent, and the process proceeds to the next step.

Step 2: Evaluation of alternatives using the Weighted Fuzzy N-Soft Set.

- (4) The decision-maker evaluates each alternative $u_i \in U$ with respect to each parameter $e_j \in A$.
- (5) Alternatives (waste objects) are denoted by $U = \{u_1, u_2, u_3, \dots, u_m\}$, and criteria by $A = \{e_1, e_2, e_3, \dots, e_n\}$. The assessments are expressed through fuzzy values μ_{ij} and N-soft set values $g_{ij} \in G = \{0, 1, 2, \dots, N-1\}$, $N \geq 2 \in \mathbb{N}$, with $i = 1, 2, 3, \dots, m$ and $j = 1, 2, \dots, n$. According to Definition 6, these evaluations are represented in Table 3 as the Fuzzy N-Soft Set (μ, L) .
- (6) The interval grade $[g_1, g_2] \in G$ is selected to determine $\mu_{g_1}^{g_2}(e_j, u_i) \in (\mu, L)$.
- (7) The weighted fuzzy N-soft set value for $[g_1, g_2] \in G$ is computed using Equation (29):

$$\widetilde{(\mu, L)}_{w(e_j)}(u_i) = \mu_{g_1}^{g_2}(e_j, u_i) \times w_i, \quad (29)$$

with w_j is the corresponding priority weight.

- (8) The classification of each alternative (waste object) into the appropriate 10R criterion is obtained from the highest value of $\widetilde{(\mu, L)}_{w(e_j)}(u_i)$ for each $u_i \in U$.

Step 3: Ranking of waste-management priorities using the modified CODAS method.

- (9) The weighted normalized decision matrix $R = (r_{ij})_{m \times n}$ is obtained by normalizing the weighted values $\widetilde{(\mu, L)}_{w(e_j)}(u_i)$ in Equation (14), computing the score function Sc_{ij} (Equation (15)), and determining the elements r_{ij} using Equation (16).
- (10) The negative-ideal solution is computed using Equations (17) and (18).
- (11) The relative assessment matrix is constructed based on the Euclidean distance Eu_i (Equation (19)) and the Hamming distance Ha_i (Equation (20)). The elements of the relative assessment matrix RA are obtained using Equation (21), determined through Equation (22), and subject to the threshold condition in Equation (23).
- (12) The final assessment score for each alternative (waste object) is calculated using Equation (23). The priority order of waste-handling alternatives is determined from the ranking of the RA_i scores. A higher RA_i score indicates that the corresponding waste object should receive higher priority in waste management.

To enhance clarity and provide an integrated overview of the proposed decision-making framework, the above computational steps are visually summarized in the flowchart presented in Figure 1.

Justification of Two-Stage Weighting Procedure

In the proposed WFN-SS-CODAS framework, criterion weights are intentionally applied at two stages:

- (1) First weighting: During the construction of the Weighted Fuzzy N-Soft Set as expressed in Equation (29), where the fuzzy membership degree $\mu_{g_1}^{g_2}(e_j, u_i)$ is multiplied by the AHP-derived weights w_j . This integrates the strategic importance of each 10R criterion directly into the aggregated performance value of each alternative.
- (2) Second weighting: During the formation of the CODAS weighted normalized decision matrix as expressed in Equation (16), where the score function values Sc_{ij} are multiplied again by the same weight w_j . This ensures that the criterion importance is proportionally amplified in the final distance-based ranking mechanism.

This two-stage weighting is a deliberate design choice to enhance the discriminative power of the model. In waste-management prioritization, alternatives (waste types) often exhibit closely clustered performance profiles across multiple criteria. A single application of weights may be insufficient to generate clear, actionable rankings. The second weighting amplifies the influence of high-priority upstream strategies (e.g., Refuse, Rethink), enabling the CODAS algorithm to produce more differentiated final scores (RA_i) and a more stable, ecologically rational priority order. This approach is justified by the comparative analysis in Section 3.2, which demonstrates that the double-weighted WFN-SS-CODAS model yields superior differentiation and alignment with field data (volume, ecological impact) compared to single-weighted. The procedure does not introduce arbitrary bias but systematically reinforces the decision-maker's strategic preferences, making the model particularly suitable for complex environmental decision-making where subtle distinctions between alternatives are critical.

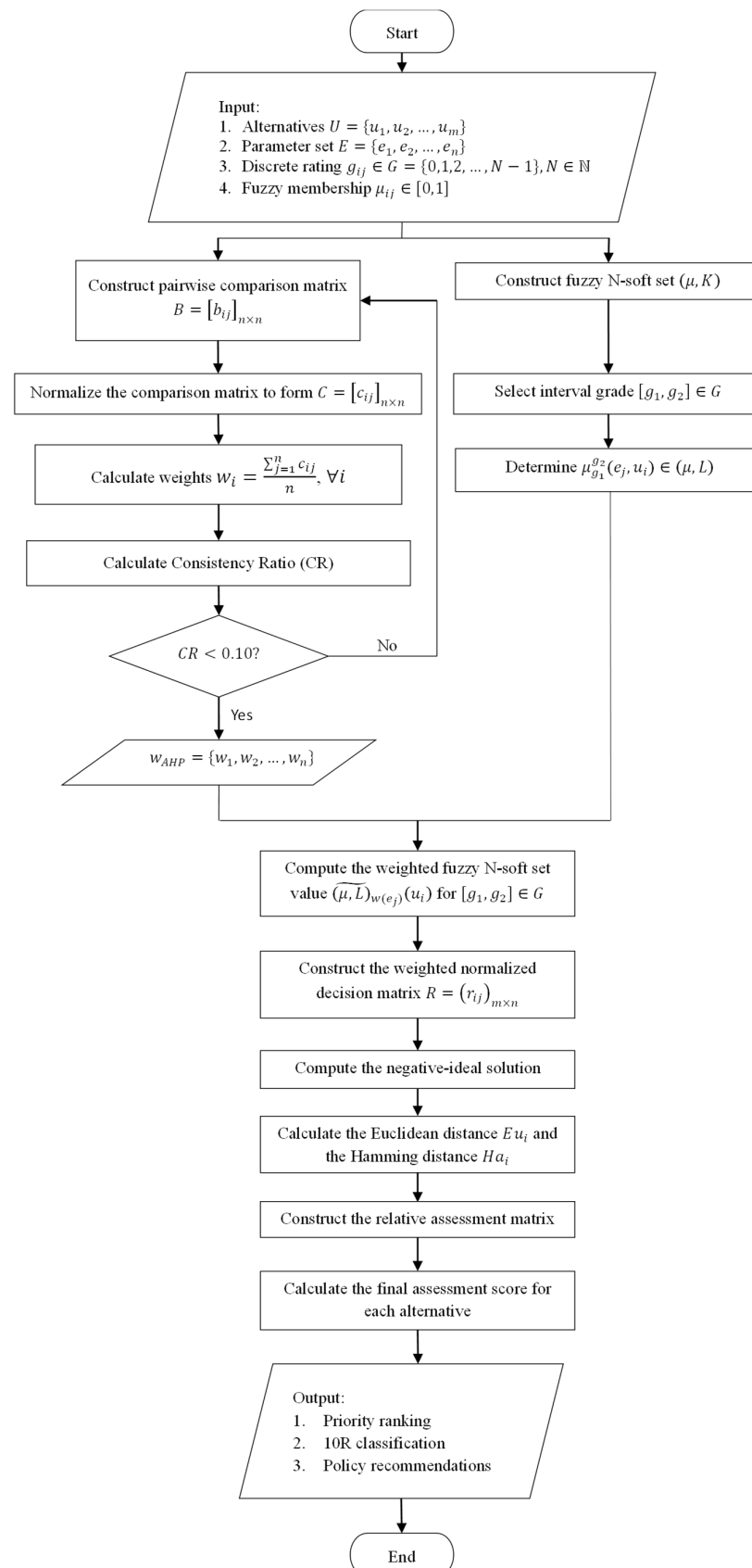


Figure 1. Flowchart of the proposed integrated weighted fuzzy N-soft set-CODAS framework.

3. Results

3.1. Application of the Weighted Fuzzy N-Soft Set–CODAS Algorithm

This study applies the proposed algorithm to a case study of waste management in the Citarum River, West Java. As a foundational demonstration of the WFN-SS–CODAS framework, this case study employs a single-expert input approach. All assessments for the AHP pairwise comparison and the Fuzzy N-Soft Set evaluations were provided by a single key expert, the founder of the Bening Saguling Foundation (BSF). This expert was selected for their deep, integrative, and operational knowledge of waste management across technical, community, and environmental dimensions in the Citarum Basin. Using a single consistent source ensures clarity and traceability in this proof-of-concept application, allowing the novel hybrid model to be demonstrated without the additional complexity of aggregating multiple conflicting judgments.

Primary data were collected through in-depth interviews with the management of the Citarum River, represented by the founder of the Bening Saguling Foundation (BSF), a non-profit organization focused on social, educational, and environmental initiatives, with a particular emphasis on river conservation and community empowerment, including local waste-picker families. The foundation was established in 2014 in response to growing concerns about the high volume of waste polluting the Saguling Reservoir and the Citarum River. The interview with the BSF founder was conducted to explore current waste-management practices, the challenges encountered, and perspectives on integrating circular and blue economy principles. These qualitative insights provided a deeper understanding of stakeholder preferences and key decision criteria. For example, BSF emphasized the importance of community empowerment and upstream waste-prevention strategies through waste banks, educational programs, and innovations in plastic recycling. The interviews also identified several evaluation parameters, such as the volume of waste collected, the economic value of recycled materials, and socio-economic impacts on riverbank communities, which were later formalized as criteria in the decision model.

In addition to interviews, field observations were conducted at the trash boom (floating waste barrier) located in the Saguling segment of the Citarum River. These observations recorded the types of waste accumulated at the trash boom and their relative quantities. The results indicated that organic waste, such as wood fragments, leaves, and water hyacinth, along with various types of plastics, dominated the river's waste composition. This aligns with previous reports showing that approximately 62.8% of Citarum River waste consists of organic materials, followed by macroplastics, textiles, rubber, metals, and paper. Data from BSF confirmed these findings, noting that single-use plastic packaging, styrofoam, and plastic bags constitute a significant fraction. Once collected using the trash boom and conveyor system, the waste was sorted by material category, and the composition data were documented for further analysis. These waste categories were then mapped to the 10R principles of the circular economy framework. Each waste type was linked to a relevant 10R strategy. For example, low-value plastics were aligned with Recycle and Repurpose pathways, such as melting into recycled plastic boards or furniture; organic waste was associated with Recover options, such as composting or biogas; and Refuse/Reduce strategies were implemented through community education and waste-prevention programs, such as waste banks. This categorization ensured that each waste type and its corresponding intervention strategy were systematically grouped within the appropriate 10R category (Refuse, Rethink, Reduce, Reuse, Repair, Refurbish, Remanufacture, Repurpose, Recycle, Recover). Structuring the data using the 10R framework ensured that the decision design aligned with circular-economy principles, while the riverine and community context was incorporated through socio-environmental indicators identified during the interviews.

Building on field observations and the analysis of the Citarum Repair Report, the waste types polluting the Citarum River were systematically identified and categorized. This inventory establishes the set of decision alternatives for the multi-criteria analysis, as summarized in Table 6.

Table 6. Types of Waste in the Citarum River.

Category	Material	Waste Item
Organic	Organic	Water Hyacinth Wood
Non-Organic	Rubber	Rubber
	HDPE	Plastic Bag Plastic Bottle Clear Plastic Sheet
	PP	Plastic Cup
	Glass	Glass Bottle
	PET	PET Bottle
	LDPE	Gallon Cap
Residual	Mixed Plastic	Multi-Layer Pack (MLP) Used Beverage Carton (UBC)
	Polystyrene	Styrofoam
	Textile	Bag Clothing
Hazardous Waste (B3)	B3	B3

Source: <https://lookerstudio.google.com> (accessed on 27 September 2025).

The evaluation data presented in this table were obtained through an in-depth interview with the founder of a BSF, and the corresponding data source link is provided below the table for transparency. Although the referenced source lists more than sixteen waste objects, the founder indicated that the sixteen waste types included in Table 6 represent those most frequently encountered in the Citarum River Basin, and it was suggested by the founder that this be factored into the data processing. Accordingly, these 16 waste objects were selected for evaluation based on the expert's operational experience and field observations.

The classification of the 16 different waste types is defined as the universe of objects, denoted by U , namely:

$$U = \{u_1, u_2, u_3, \dots, u_{16}\}, \quad (30)$$

where for each $u_i \in U$, $1, 2, 3, \dots, 16$, the elements correspond, respectively, to water hyacinth (u_1), wood (u_2), rubber (u_3), plastic bag (u_4), plastic bottle (u_5), clear plastic sheet (u_6), plastic cup (u_7), glass bottle (u_8), PET bottle (u_9), gallon cap (u_{10}), multi-layer pack (u_{11}), used beverage carton (u_{12}), styrofoam (u_{13}), bag (u_{14}), clothing (u_{15}), and B3 (u_{16}). To evaluate these objects with respect to Circular Economy principles, the 10R framework is adopted as the set A , where $A \subseteq E$ denotes the set of evaluation criteria; thus, the set A can be defined as follows:

$$A = \{e_1, e_2, e_3, \dots, e_{10}\}, \quad (31)$$

where for each $e_j \in A$, $j = 1, 2, 3, \dots, 10$, the elements correspond, respectively, to Refuse (e_1), Rethink (e_2), Reduce (e_3), Reuse (e_4), Repair (e_5), Refurbish (e_6), Remanufacture (e_7), Repurpose (e_8), Recycle (e_9), and Recover (e_{10}). Furthermore, the evaluation of parameters is conducted using the rating scale $R = \{0, 1, 2, 3\}$, where each value represents a specific assessment level, described as follows:

- A score of 0 indicates "very difficult",
- A score of 1 indicates "difficult",

- A score of 2 indicates “easy”, and
- A score of 3 indicates “very easy”.

The basis for assessment used is presented in Table 7 below.

Table 7. Assessment Basis for Parameter Evaluation.

Category	Assessment Basis	Score
Very Difficult	Very challenging to implement; requires substantial cost, technology, and effort.	0
Difficult	Applicable but faces constraints such as high costs or limited infrastructure.	1
Easy	Relatively easy to implement due to adequate technological, market, or policy support.	2
Very Easy	Very easy to implement, with low cost, high benefits, and immediate positive impact.	3

Based on the rating scale defined in Table 7, the fuzzy membership intervals for each score are assigned according to their relevance and contribution to the evaluation objectives, as presented in Table 8, with reference to the intervals proposed by Korkmaz et al. [14]. The integration of discrete ratings with continuous membership degrees represents a core innovation of the Fuzzy N-Soft Set framework, enabling the model to capture the uncertainty inherent in expert assessments.

Table 8. Membership Degree Interval for Each Category.

Score	Membership Degree Interval
0	$0.0 \leq \mu_e(u) \leq 0.15$
1	$0.15 < \mu_e(u) \leq 0.40$
2	$0.40 < \mu_e(u) \leq 0.85$
3	$0.85 < \mu_e(u) \leq 1.0$

Step 1: Determination of the weights.

The relative importance of the 10R criteria is determined using the Analytical Hierarchy Process (AHP). The founder of the Bening Saguling Foundation, serving as the expert evaluator, conducted pairwise comparisons of all criteria using Saaty’s fundamental scale (1–9). This process produced the pairwise comparison matrix and the derived priority weights, as presented in Table 9.

Table 9. Pairwise Comparison Matrix of The Parameters.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	Weights
e_1	1	1	3	5	5	6	6	6	7	8	0.2664
e_2	1	1	3	4	4	5	5	5	6	7	0.2361
e_3	0.3333	0.3333	1	3	3	4	4	4	5	6	0.1462
e_4	0.2	0.25	0.3333	1	2	3	3	3	4	5	0.0956
e_5	0.2	0.25	0.3333	0.5	1	2	2	2	3	4	0.0686
e_6	0.1667	0.2	0.25	0.3333	0.5	1	2	2	3	3	0.0546
e_7	0.1667	0.2	0.25	0.3333	0.5	0.5	1	2	2	3	0.0453
e_8	0.1667	0.2	0.25	0.3333	0.5	0.5	0.5	1	2	3	0.0394
e_9	0.1429	0.1667	0.2	0.25	0.3333	0.3333	0.5	0.5	1	2	0.0276
e_{10}	0.125	0.1429	0.1667	0.2	0.25	0.3333	0.3333	0.3333	0.5	1	0.0201
Sum	3.5012	3.7429	8.7833	14.95	17.0833	22.6667	24.3333	25.8333	33.5	42	1

To illustrate the calculation of scores in the pairwise comparison matrix in Table 9, consider the following example.

- The comparison between e_1 (refuse) and e_3 (reduce) was assigned a value of 3, indicating that the founder of the Bening Saguling Foundation considers refuse to be more important than reduce.
- Meanwhile, for the opposite comparison, e_3 against e_1 , the value obtained is $1/3$, indicating that reduce is considered less important than refuse.
- The values on the diagonal elements of the matrix represent the comparison of a parameter with itself, which consistently has a value of 1 because they are considered to have the same level of importance.

The final weights are then derived by normalizing and aggregating the entire comparison matrix, yielding the priority distribution shown in the Weights column.

Through the consistency validation process, the values obtained were $\lambda_{max} = 10.8283$, a Consistency Index of $CI = 0.0920$, and a Consistency Ratio of $CR = 0.0618$. Since $CR = 0.0618 < 0.10$, the level of inconsistency is acceptable. The weight distribution places strong emphasis on upstream strategies, with Refuse (0.2664), Rethink (0.2361), and Reduce (0.1462) collectively accounting for nearly 65% of the total decision weight, aligning with the Circular Economy's prevention-oriented ethos.

Step 2: Evaluation of Alternatives Using the Weighted Fuzzy N-Soft Set.

The founder of the Bening Saguling Foundation, acting as the expert evaluator, assessed the membership degrees of all criteria within the interval $[0, 1]$. The evaluation of the sixteen waste types with respect to the ten criteria was carried out using the 4-soft set framework $(F, A, 4)$, in which each waste-criterion pair was assigned a discrete rating g_{ij} ranging from 0 to 3. The 4-soft set $(F, A, 4)$ is defined following the formulation presented in Equation (32).

$$(F, A, 4) = \left\{ (e_j, F(e_j)) \mid j = 1, 2, 3, \dots, 10, F(e_j) \in 2^{U \times R} \right\}, \quad (32)$$

where

$$F(e_1) = \{(u_1, 0), (u_2, 0), (u_3, 0), (u_4, 3), (u_5, 2), (u_6, 2), (u_7, 3), (u_8, 1), (u_9, 2), (u_{10}, 2), (u_{11}, 2), (u_{12}, 2), (u_{13}, 3), (u_{14}, 0), (u_{15}, 0), (u_{16}, 0)\}$$

$$F(e_2) = \{(u_1, 0), (u_2, 0), (u_3, 0), (u_4, 3), (u_5, 3), (u_6, 3), (u_7, 3), (u_8, 1), (u_9, 3), (u_{10}, 2), (u_{11}, 2), (u_{12}, 2), (u_{13}, 3), (u_{14}, 0), (u_{15}, 0), (u_{16}, 0)\}$$

$$F(e_3) = \{(u_1, 0), (u_2, 0), (u_3, 0), (u_4, 3), (u_5, 3), (u_6, 2), (u_7, 3), (u_8, 2), (u_9, 3), (u_{10}, 3), (u_{11}, 2), (u_{12}, 2), (u_{13}, 3), (u_{14}, 0), (u_{15}, 3), (u_{16}, 1)\}$$

$$F(e_4) = \{(u_1, 2), (u_2, 3), (u_3, 1), (u_4, 1), (u_5, 2), (u_6, 0), (u_7, 1), (u_8, 3), (u_9, 3), (u_{10}, 2), (u_{11}, 0), (u_{12}, 0), (u_{13}, 0), (u_{14}, 3), (u_{15}, 3), (u_{16}, 0)\}$$

$$F(e_5) = \{(u_1, 0), (u_2, 2), (u_3, 2), (u_4, 0), (u_5, 0), (u_6, 0), (u_7, 0), (u_8, 0), (u_9, 0), (u_{10}, 0), (u_{11}, 0), (u_{12}, 0), (u_{13}, 0), (u_{14}, 3), (u_{15}, 3), (u_{16}, 0)\}$$

$$F(e_6) = \{(u_1, 0), (u_2, 3), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 0), (u_7, 0), (u_8, 0), (u_9, 0), (u_{10}, 0), (u_{11}, 0), (u_{12}, 0), (u_{13}, 0), (u_{14}, 2), (u_{15}, 2), (u_{16}, 3)\}$$

$$F(e_7) = \{(u_1, 0), (u_2, 1), (u_3, 2), (u_4, 0), (u_5, 0), (u_6, 0), (u_7, 0), (u_8, 0), (u_9, 0), (u_{10}, 0), (u_{11}, 0), (u_{12}, 0), (u_{13}, 0), (u_{14}, 0), (u_{15}, 0), (u_{16}, 0)\}$$

$$F(e_8) = \{(u_1, 3), (u_2, 3), (u_3, 2), (u_4, 2), (u_5, 2), (u_6, 0), (u_7, 2), (u_8, 2), (u_9, 3), (u_{10}, 2), (u_{11}, 1), (u_{12}, 2), (u_{13}, 1), (u_{14}, 2), (u_{15}, 3), (u_{16}, 0)\}$$

$$F(e_9) = \{(u_1, 3), (u_2, 3), (u_3, 2), (u_4, 2), (u_5, 2), (u_6, 2), (u_7, 2), (u_8, 3), (u_9, 3), (u_{10}, 2), (u_{11}, 0), (u_{12}, 1), (u_{13}, 1), (u_{14}, 2), (u_{15}, 2), (u_{16}, 0)\}$$

and

$$F(e_{10}) = \{(u_1, 3), (u_2, 3), (u_3, 2), (u_4, 2), (u_5, 2), (u_6, 3), (u_7, 2), (u_8, 0), (u_9, 2), (u_{10}, 2), (u_{11}, 2), (u_{12}, 2), (u_{13}, 3), (u_{14}, 2), (u_{15}, 2), (u_{16}, 0)\}$$

The discrete rankings defined through Equation (32) produce the initial evaluation matrix presented in Table 10.

Table 10. Tabular Representation of 4-Soft Set $(F, A, 4)$.

$(F, A, 4)$	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
u_1	0	0	0	2	0	0	0	3	3	3
u_2	0	0	0	3	2	3	1	3	3	3
u_3	0	0	0	1	2	0	2	2	2	2
u_4	3	3	3	1	0	0	0	2	2	2
u_5	2	3	3	2	0	0	0	2	2	2
u_6	2	3	2	0	0	0	0	0	2	3
u_7	3	3	3	1	0	0	0	2	2	2
u_8	1	1	2	3	0	0	0	2	3	0
u_9	2	3	3	3	0	0	0	3	3	2
u_{10}	2	2	3	2	0	0	0	2	2	2
u_{11}	2	2	2	0	0	0	0	1	0	2
u_{12}	2	2	2	0	0	0	0	2	1	2
u_{13}	3	3	3	0	0	0	0	1	1	3
u_{14}	0	0	2	3	3	2	0	2	2	2
u_{15}	0	3	2	3	3	2	0	3	2	2
u_{16}	0	1	0	0	0	0	0	0	0	0

The discrete evaluation presented in Table 10 offers an initial overview. For example, single-use plastics such as plastic bags (u_4) and Styrofoam (u_{13}) receive high scores (3) for the heavily weighted criteria Refuse, Rethink, and Reduce, indicating strong potential for upstream prevention strategies. Conversely, hazardous waste (u_{16}) consistently receives low scores, highlighting the difficulty of applying standard Circular Economy principles. In practice, however, the implementation of the 10R strategies often exists along a continuum, influenced by technical feasibility, economic efficiency, infrastructure availability, and social factors. By integrating fuzzy concepts into the N-soft set framework, the membership degree of each alternative with respect to the 10R criteria can be assessed more flexibly and contextually. This enables the study to produce a more realistic and comprehensive analysis of the waste-management potential mapped within the 10R framework.

To incorporate the nuance of expert judgment, the discrete scores in Table 10 are fuzzified. Each score g_{ij} is paired with a specific fuzzy membership degree μ_{ij} assigned by the expert evaluator, resulting in the Fuzzy N-Soft Set (μ, K) , where $K = (F, A, 4)$, defined as follows:

$$\begin{aligned} \mu_{e_1} = \{ & \langle (u_1, 0), 0.05 \rangle, \langle (u_2, 0), 0.10 \rangle, \langle (u_3, 0), 0.10 \rangle, \langle (u_4, 3), 0.98 \rangle, \langle (u_5, 2), 0.75 \rangle, \\ & \langle (u_6, 2), 0.80 \rangle, \langle (u_7, 3), 0.92 \rangle, \langle (u_8, 1), 0.30 \rangle, \langle (u_9, 2), 0.65 \rangle, \langle (u_{10}, 2), 0.55 \rangle, \\ & \langle (u_{11}, 2), 0.50 \rangle, \langle (u_{12}, 2), 0.45 \rangle, \langle (u_{13}, 3), 0.97 \rangle, \langle (u_{14}, 0), 0.10 \rangle, \langle (u_{15}, 0), 0.15 \rangle, \\ & \langle (u_{16}, 0), 0.10 \rangle, \end{aligned}$$

$$\mu_{e_2} = \{ \langle (u_1, 0), 0.05 \rangle, \langle (u_2, 0), 0.10 \rangle, \langle (u_3, 0), 0.10 \rangle, \langle (u_4, 3), 0.97 \rangle, \langle (u_5, 3), 0.95 \rangle, \\ \langle (u_6, 3), 0.97 \rangle, \langle (u_7, 3), 0.95 \rangle, \langle (u_8, 1), 0.30 \rangle, \langle (u_9, 3), 0.98 \rangle, \langle (u_{10}, 2), 0.55 \rangle, \\ \langle (u_{11}, 2), 0.50 \rangle, \langle (u_{12}, 2), 0.45 \rangle, \langle (u_{13}, 3), 0.97 \rangle, \langle (u_{14}, 0), 0.10 \rangle, \langle (u_{15}, 3), 0.95 \rangle, \\ \langle (u_{16}, 1), 0.35 \rangle, \}$$

$$\mu_{e_3} = \{ \langle (u_1, 0), 0.05 \rangle, \langle (u_2, 0), 0.10 \rangle, \langle (u_3, 0), 0.10 \rangle, \langle (u_4, 3), 0.95 \rangle, \langle (u_5, 3), 0.90 \rangle, \\ \langle (u_6, 2), 0.80 \rangle, \langle (u_7, 3), 0.94 \rangle, \langle (u_8, 2), 0.50 \rangle, \langle (u_9, 3), 0.95 \rangle, \langle (u_{10}, 3), 0.88 \rangle, \\ \langle (u_{11}, 2), 0.55 \rangle, \langle (u_{12}, 2), 0.45 \rangle, \langle (u_{13}, 3), 0.94 \rangle, \langle (u_{14}, 2), 0.75 \rangle, \langle (u_{15}, 2), 0.65 \rangle, \\ \langle (u_{16}, 0), 0.10 \rangle, \}$$

$$\mu_{e_4} = \{ \langle (u_1, 2), 0.55 \rangle, \langle (u_2, 3), 0.98 \rangle, \langle (u_3, 1), 0.35 \rangle, \langle (u_4, 1), 0.20 \rangle, \langle (u_5, 2), 0.80 \rangle, \\ \langle (u_6, 0), 0.10 \rangle, \langle (u_7, 1), 0.30 \rangle, \langle (u_8, 3), 0.98 \rangle, \langle (u_9, 3), 0.94 \rangle, \langle (u_{10}, 2), 0.45 \rangle, \\ \langle (u_{11}, 0), 0.05 \rangle, \langle (u_{12}, 0), 0.05 \rangle, \langle (u_{13}, 0), 0.10 \rangle, \langle (u_{14}, 3), 0.98 \rangle, \langle (u_{15}, 3), 0.96 \rangle, \\ \langle (u_{16}, 0), 0.00 \rangle, \}$$

$$\mu_{e_5} = \{ \langle (u_1, 0), 0.05 \rangle, \langle (u_2, 2), 0.75 \rangle, \langle (u_3, 2), 0.70 \rangle, \langle (u_4, 0), 0.10 \rangle, \langle (u_5, 0), 0.10 \rangle, \\ \langle (u_6, 0), 0.10 \rangle, \langle (u_7, 0), 0.10 \rangle, \langle (u_8, 0), 0.00 \rangle, \langle (u_9, 0), 0.10 \rangle, \langle (u_{10}, 0), 0.05 \rangle, \\ \langle (u_{11}, 0), 0.05 \rangle, \langle (u_{12}, 0), 0.05 \rangle, \langle (u_{13}, 0), 0.10 \rangle, \langle (u_{14}, 3), 0.98 \rangle, \langle (u_{15}, 3), 0.99 \rangle, \\ \langle (u_{16}, 0), 0.00 \rangle, \}$$

$$\mu_{e_6} = \{ \langle (u_1, 0), 0.05 \rangle, \langle (u_2, 3), 0.98 \rangle, \langle (u_3, 0), 0.10 \rangle, \langle (u_4, 0), 0.10 \rangle, \langle (u_5, 0), 0.10 \rangle, \\ \langle (u_6, 0), 0.10 \rangle, \langle (u_7, 0), 0.10 \rangle, \langle (u_8, 0), 0.05 \rangle, \langle (u_9, 0), 0.10 \rangle, \langle (u_{10}, 0), 0.05 \rangle, \\ \langle (u_{11}, 0), 0.05 \rangle, \langle (u_{12}, 0), 0.05 \rangle, \langle (u_{13}, 0), 0.10 \rangle, \langle (u_{14}, 2), 0.75 \rangle, \langle (u_{15}, 2), 0.80 \rangle, \\ \langle (u_{16}, 0), 0.00 \rangle, \}$$

$$\mu_{e_7} = \{ \langle (u_1, 0), 0.05 \rangle, \langle (u_2, 1), 0.30 \rangle, \langle (u_3, 2), 0.75 \rangle, \langle (u_4, 0), 0.10 \rangle, \langle (u_5, 0), 0.10 \rangle, \\ \langle (u_6, 0), 0.10 \rangle, \langle (u_7, 0), 0.10 \rangle, \langle (u_8, 0), 0.10 \rangle, \langle (u_9, 0), 0.10 \rangle, \langle (u_{10}, 0), 0.05 \rangle, \\ \langle (u_{11}, 0), 0.05 \rangle, \langle (u_{12}, 0), 0.05 \rangle, \langle (u_{13}, 0), 0.10 \rangle, \langle (u_{14}, 0), 0.15 \rangle, \langle (u_{15}, 0), 0.15 \rangle, \\ \langle (u_{16}, 0), 0.00 \rangle, \}$$

$$\mu_{e_8} = \{ \langle (u_1, 3), 0.90 \rangle, \langle (u_2, 3), 0.98 \rangle, \langle (u_3, 2), 0.70 \rangle, \langle (u_4, 2), 0.80 \rangle, \langle (u_5, 2), 0.75 \rangle, \\ \langle (u_6, 0), 0.10 \rangle, \langle (u_7, 2), 0.55 \rangle, \langle (u_8, 2), 0.75 \rangle, \langle (u_9, 3), 0.98 \rangle, \langle (u_{10}, 2), 0.45 \rangle, \\ \langle (u_{11}, 1), 0.20 \rangle, \langle (u_{12}, 2), 0.40 \rangle, \langle (u_{13}, 1), 0.35 \rangle, \langle (u_{14}, 2), 0.85 \rangle, \langle (u_{15}, 3), 0.96 \rangle, \\ \langle (u_{16}, 0), 0.00 \rangle, \}$$

$$\mu_{e_9} = \{ \langle (u_1, 3), 0.90 \rangle, \langle (u_2, 3), 0.88 \rangle, \langle (u_3, 2), 0.75 \rangle, \langle (u_4, 2), 0.70 \rangle, \langle (u_5, 2), 0.80 \rangle, \\ \langle (u_6, 2), 0.60 \rangle, \langle (u_7, 2), 0.55 \rangle, \langle (u_8, 3), 0.99 \rangle, \langle (u_9, 3), 0.99 \rangle, \langle (u_{10}, 2), 0.78 \rangle, \\ \langle (u_{11}, 0), 0.05 \rangle, \langle (u_{12}, 1), 0.35 \rangle, \langle (u_{13}, 1), 0.20 \rangle, \langle (u_{14}, 2), 0.50 \rangle, \langle (u_{15}, 2), 0.45 \rangle, \\ \langle (u_{16}, 0), 0.00 \rangle, \}$$

and

$$\mu_{e_{10}} = \{ \langle (u_1, 3), 0.95 \rangle, \langle (u_2, 3), 0.99 \rangle, \langle (u_3, 2), 0.80 \rangle, \langle (u_4, 2), 0.70 \rangle, \langle (u_5, 2), 0.65 \rangle, \\ \langle (u_6, 3), 0.90 \rangle, \langle (u_7, 2), 0.50 \rangle, \langle (u_8, 0), 0.00 \rangle, \langle (u_9, 2), 0.45 \rangle, \langle (u_{10}, 2), 0.55 \rangle, \\ \langle (u_{11}, 2), 0.60 \rangle, \langle (u_{12}, 2), 0.75 \rangle, \langle (u_{13}, 3), 0.99 \rangle, \langle (u_{14}, 2), 0.55 \rangle, \langle (u_{15}, 2), 0.50 \rangle, \\ \langle (u_{16}, 0), 0.00 \rangle, \}$$

Based on the definition of Fuzzy N-Soft Set (μ, K) with $K = (F, A, 4)$, the tabular representation is shown in Table 11. Weight components are added to each criterion without aggregation in Fuzzy 4-Soft Set.

Table 11. Tabular Representation of Weighted Fuzzy 4-Soft Set (μ, K, w) .

	$e_1, w_1 =$ 0.2664	$e_2, w_2 =$ 0.2361	$e_3, w_3 =$ 0.1462	$e_4, w_4 =$ 0.0956	$e_5, w_5 =$ 0.0686	$e_6, w_6 =$ 0.0546	$e_7, w_7 =$ 0.0453	$e_8, w_8 =$ 0.0394	$e_9, w_9 =$ 0.0276	$e_{10}, w_{10} =$ 0.0201
u_1	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 2, 0.55 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 3, 0.90 \rangle$	$\langle 3, 0.90 \rangle$	$\langle 3, 0.95 \rangle$
u_2	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 2, 0.75 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 1, 0.30 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 3, 0.88 \rangle$	$\langle 3, 0.99 \rangle$
u_3	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 1, 0.35 \rangle$	$\langle 2, 0.70 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 2, 0.75 \rangle$	$\langle 2, 0.70 \rangle$	$\langle 2, 0.75 \rangle$	$\langle 2, 0.80 \rangle$
u_4	$\langle 3, 0.98 \rangle$	$\langle 3, 0.97 \rangle$	$\langle 3, 0.95 \rangle$	$\langle 1, 0.20 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 2, 0.80 \rangle$	$\langle 2, 0.70 \rangle$	$\langle 2, 0.70 \rangle$
u_5	$\langle 2, 0.75 \rangle$	$\langle 3, 0.95 \rangle$	$\langle 3, 0.90 \rangle$	$\langle 2, 0.80 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 2, 0.75 \rangle$	$\langle 2, 0.80 \rangle$	$\langle 2, 0.65 \rangle$
u_6	$\langle 2, 0.80 \rangle$	$\langle 3, 0.97 \rangle$	$\langle 2, 0.80 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 2, 0.60 \rangle$	$\langle 3, 0.90 \rangle$
u_7	$\langle 3, 0.92 \rangle$	$\langle 3, 0.95 \rangle$	$\langle 3, 0.94 \rangle$	$\langle 1, 0.30 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 2, 0.55 \rangle$	$\langle 2, 0.55 \rangle$	$\langle 2, 0.50 \rangle$
u_8	$\langle 1, 0.30 \rangle$	$\langle 1, 0.30 \rangle$	$\langle 2, 0.50 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 0, 0.00 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 2, 0.75 \rangle$	$\langle 3, 0.99 \rangle$	$\langle 0, 0.00 \rangle$
u_9	$\langle 2, 0.65 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 3, 0.95 \rangle$	$\langle 3, 0.94 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 3, 0.99 \rangle$	$\langle 2, 0.45 \rangle$
u_{10}	$\langle 2, 0.55 \rangle$	$\langle 2, 0.55 \rangle$	$\langle 3, 0.88 \rangle$	$\langle 2, 0.45 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 2, 0.45 \rangle$	$\langle 2, 0.78 \rangle$	$\langle 2, 0.55 \rangle$
u_{11}	$\langle 2, 0.50 \rangle$	$\langle 2, 0.50 \rangle$	$\langle 2, 0.55 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 1, 0.20 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 2, 0.60 \rangle$
u_{12}	$\langle 2, 0.45 \rangle$	$\langle 2, 0.45 \rangle$	$\langle 2, 0.45 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 0, 0.05 \rangle$	$\langle 2, 0.40 \rangle$	$\langle 1, 0.35 \rangle$	$\langle 2, 0.75 \rangle$
u_{13}	$\langle 3, 0.97 \rangle$	$\langle 3, 0.97 \rangle$	$\langle 3, 0.94 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 1, 0.35 \rangle$	$\langle 1, 0.20 \rangle$	$\langle 3, 0.99 \rangle$
u_{14}	$\langle 0, 0.10 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 2, 0.75 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 3, 0.98 \rangle$	$\langle 2, 0.75 \rangle$	$\langle 0, 0.15 \rangle$	$\langle 2, 0.85 \rangle$	$\langle 2, 0.50 \rangle$	$\langle 2, 0.55 \rangle$
u_{15}	$\langle 0, 0.15 \rangle$	$\langle 3, 0.95 \rangle$	$\langle 2, 0.65 \rangle$	$\langle 3, 0.96 \rangle$	$\langle 3, 0.99 \rangle$	$\langle 2, 0.80 \rangle$	$\langle 0, 0.15 \rangle$	$\langle 3, 0.96 \rangle$	$\langle 2, 0.45 \rangle$	$\langle 2, 0.50 \rangle$
u_{16}	$\langle 0, 0.10 \rangle$	$\langle 1, 0.35 \rangle$	$\langle 0, 0.10 \rangle$	$\langle 0, 0.00 \rangle$	$\langle 0, 0.00 \rangle$	$\langle 0, 0.00 \rangle$	$\langle 0, 0.00 \rangle$	$\langle 0, 0.00 \rangle$	$\langle 0, 0.00 \rangle$	$\langle 0, 0.00 \rangle$

The weights determined through the AHP method in Table 9 were then aggregated with the Fuzzy 4-Soft Set to form the Weighted Fuzzy N-Soft Set (μ, K, w) . To focus the analysis on the most feasible strategies, the value interval $[2, 3]$ (“Easy” to “Very Easy”) was selected. This choice is grounded in the practical context of the Citarum River Basin, where implementing organizations (e.g., local government, NGOs like BSF) operate under significant technological and budgetary constraints. In this setting, strategies rated as “Difficult” or “Very Difficult” (scores 0–1) are often synonymous with high cost, complex technology, or low scalability, making them less actionable in the short to medium term. Focusing on the higher-feasibility interval $[2, 3]$ therefore aligns the model with the study’s goal of generating a pragmatic and immediately relevant priority list, identifying interventions where high strategic value per AHP weights, coincides with high implementability. The representation of this focused model is presented in Table 12.

Table 12. Tabular Representation of Weighted Fuzzy 4-Soft Set (μ, K, w) and $[2, 3]$.

$\mu_2^3(e)$	$e_1, w_1 =$ 0.2664	$e_2, w_2 =$ 0.2361	$e_3, w_3 =$ 0.1462	$e_4, w_4 =$ 0.0956	$e_5, w_5 =$ 0.0686	$e_6, w_6 =$ 0.0546	$e_7, w_7 =$ 0.0453	$e_8, w_8 =$ 0.0394	$e_9, w_9 =$ 0.0276	$e_{10}, w_{10} =$ 0.0201
u_1	0	0	0	0.55	0	0	0	0.90	0.90	0.95
u_2	0	0	0	0.98	0.75	0.98	0	0.98	0.88	0.99
u_3	0	0	0	0.94	0	0	0	0.95	0.99	0.98
u_4	0.98	0.97	0.95	0	0	0	0	0.80	0.70	0.70
u_5	0.75	0.95	0.90	0.80	0	0	0	0.75	0.80	0.65
u_6	0.80	0.97	0.80	0	0	0	0	0	0.60	0.90
u_7	0.92	0.95	0.94	0	0	0	0	0.55	0.55	0.50
u_8	0	0	0.50	0.98	0	0	0	0.75	0.99	0
u_9	0.65	0.98	0.95	0.94	0	0	0	0.98	0.99	0.45
u_{10}	0.55	0.55	0.88	0.45	0	0	0	0.45	0.78	0.55
u_{11}	0.50	0.50	0.55	0	0	0	0	0	0	0.60
u_{12}	0.45	0.45	0.45	0	0	0	0	0	0	0.75
u_{13}	0.97	0.97	0.94	0	0	0	0	0	0	0.99
u_{14}	0	0	0.75	0.98	0.98	0.75	0	0.85	0.50	0.55
u_{15}	0	0.95	0.65	0.96	0.99	0.80	0	0.96	0.45	0.50
u_{16}	0	0	0	0	0	0	0	0	0	0

As an interpretation of Table 12, for example, organic waste such as water hyacinth (u_1) and wood (u_2) previously received scores 0 for the parameters Refuse (e_1), Rethink

(e_2), and Reduce (e_3). By setting the score interval [2, 3] as the feasibility threshold, both types of waste receive a fuzzy value of 0 for these three parameters, indicating that these organic wastes are challenging to manage using these upstream strategies. Conversely, waste that receives a score of 2 or 3, such as plastic bags (u_4), which scored 3 with a membership value of 0.98 for e_1 (Refuse), is valued according to its degree of membership. This value indicates the potential for management through refusal or early-use prevention strategies. Meanwhile, hazardous waste (u_{16}) obtains a value of 0 across all parameters when the [2, 3] interval is applied, indicating that none of the 10R techniques are suitable for handling this type of waste. Therefore, B3 waste requires a special management mechanism outside the scope of this assessment framework.

Furthermore, the final weighted values for the [2, 3] interval are calculated as the product of the fuzzy membership degree and the criterion weight ($w_j \times \mu_2^3(e_j, u_i)$). The results of this calculation are presented in Table 13, which serves as the definitively prepared matrix for the subsequent CODAS ranking.

Table 13. Weighted Table of Fuzzy 4-Soft Set (μ , K , w) and [2, 3].

$\mu_2^3(e)$	$e_1, w_1 =$ 0.2664	$e_2, w_2 =$ 0.2361	$e_3, w_3 =$ 0.1462	$e_4, w_4 =$ 0.0956	$e_5, w_5 =$ 0.0686	$e_6, w_6 =$ 0.0546	$e_7, w_7 =$ 0.0453	$e_8, w_8 =$ 0.0394	$e_9, w_9 =$ 0.0276	$e_{10}, w_{10} =$ 0.0201
u_1	0	0	0	0.053	0	0	0	0.035	0.025	0.019
u_2	0	0	0	0.094	0.051	0.054	0	0.039	0.024	0.020
u_3	0	0	0	0	0.048	0	0.034	0.028	0.021	0.016
u_4	0.261	0.229	0.139	0	0	0	0	0.032	0.019	0.014
u_5	0.200	0.224	0.132	0.076	0	0	0	0.030	0.022	0.013
u_6	0.213	0.229	0.117	0	0	0	0	0	0.017	0.018
u_7	0.245	0.224	0.137	0	0	0	0	0.022	0.015	0.010
u_8	0	0	0.073	0.094	0	0	0	0.030	0.027	0
u_9	0.173	0.231	0.139	0.090	0	0	0	0.039	0.027	0.009
u_{10}	0.147	0.130	0.129	0.043	0	0	0	0.018	0.022	0.011
u_{11}	0.133	0.118	0.080	0	0	0	0	0	0	0.012
u_{12}	0.120	0.106	0.066	0	0	0	0	0	0	0.015
u_{13}	0.258	0.229	0.137	0	0	0	0	0	0	0.020
u_{14}	0	0	0.110	0.094	0.067	0.041	0	0.033	0.014	0.011
u_{15}	0	0.224	0.095	0.092	0.068	0.044	0	0.038	0.012	0.010
u_{16}	0	0	0	0	0	0	0	0	0	0

To illustrate the calculation process for the final values of each object under the 10R parameters, consider the following example.

- The object water hyacinth waste (u_1), which is evaluated under parameter e_4 (Reuse), receives a fuzzy membership degree of $\mu_2^3(e_4, u_1) = 0.55$, while the priority weight of e_4 , obtained through the AHP method, is $w_4 = 0.0956$. Then, the final value of u_1 under e_4 is calculated as:

$$w_4 \times \mu_2^3(e_4, u_1) = 0.0956 \times 0.55 = 0.053.$$

- Next, the same object (u_1) under parameter e_8 (Repurpose), receives a membership degree of $\mu_2^3(e_8, u_1) = 0.90$. The AHP weight for e_8 is $w_8 = 0.0394$, then, the final value of u_1 under e_8 is calculated as:

$$w_8 \times \mu_2^3(e_8, u_1) = 0.0394 \times 0.90 = 0.035.$$

This calculation procedure is applied consistently across all objects for each parameter within the 10R framework (from Refuse to Recover), ensuring that every object–

parameter combination produces a final value that reflects the combined contribution of its fuzzy membership degree and the parameter's importance weight.

Step 3: Ranking of Waste-Handling Priorities Using the Modified CODAS Method.

The CODAS method is applied to the weighted matrix presented in Table 13. The first step involves transforming the weighted values using a specific non-linear scoring function, $Sc_{ij} = \frac{\exp\left(\left(\widetilde{(\mu, L)}_{w(e_j)}(u_i) - 1\right)\right)}{1 + \left(\widetilde{(\mu, L)}_{w(e_j)}(u_i)\right)}$, accentuates the performance differences among objects.

The results of this transformation are provided in Table 14.

Table 14. Score Function.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
u_1	0.367879	0.367879	0.367879	0.368379	0.367879	0.367879	0.3678794	0.36809971	0.36799254	0.367945
u_2	0.367879	0.367879	0.367879	0.369413	0.368342	0.368398	0.3678794	0.368152247	0.36798374	0.3679521
u_3	0.367879	0.367879	0.367879	0.367879	0.36829	0.367879	0.3680874	0.368021041	0.367959449	0.367926
u_4	0.37874	0.376363	0.371149	0.367879	0.367879	0.367879	0.3678794	0.368063917	0.36794502	0.3679152
u_5	0.374441	0.376016	0.370839	0.368892	0.367879	0.367879	0.3678794	0.368041785	0.367967194	0.3679103
u_6	0.375275	0.376363	0.370224	0.367879	0.367879	0.367879	0.3678794	0.367879441	0.367932009	0.3679383
u_7	0.377519	0.376016	0.371059	0.367879	0.367879	0.367879	0.3678794	0.367967194	0.367920421	0.3678977
u_8	0.367879	0.367879	0.368816	0.369413	0.367879	0.367879	0.3678794	0.368041785	0.368011191	0.3678794
u_9	0.372855	0.376504	0.371149	0.369288	0.367879	0.367879	0.3678794	0.368152247	0.368011191	0.3678943
u_{10}	0.371521	0.370754	0.370711	0.36821	0.367879	0.367879	0.3678794	0.367938337	0.367967194	0.3679015
u_{11}	0.370883	0.370263	0.368999	0.367879	0.367879	0.367879	0.3678794	0.367879441	0.367879441	0.3679057
u_{12}	0.370342	0.369816	0.368648	0.367879	0.367879	0.367879	0.3678794	0.367879441	0.367879441	0.3679204
u_{13}	0.378506	0.376363	0.371059	0.367879	0.367879	0.367879	0.3678794	0.367879441	0.367879441	0.3679521
u_{14}	0.367879	0.367879	0.36996	0.369413	0.368671	0.368181	0.3678794	0.368075503	0.367915162	0.3679015
u_{15}	0.367879	0.376016	0.369445	0.36935	0.368694	0.368226	0.3678794	0.368138599	0.367905719	0.3678977
u_{16}	0.367879	0.367879	0.367879	0.367879	0.367879	0.367879	0.3678794	0.367879441	0.367879441	0.3678794

Consider the following calculation for Sc_{11} and Sc_{14} .

$$Sc_{11} = \frac{\exp(0 - 1)}{1 + 0} = \frac{0.367879}{1} = 0.367879,$$

$$Sc_{14} = \frac{\exp(0.053 - 1)}{1 + 0.053} = \frac{0.3879}{1.053} = 0.368379.$$

The matrix whose scores have been calculated from Table 14 is used to construct the final weighted normalized decision matrix $R = (r_{ij})$, where each element is $r_{ij} = w_j \times Sc_{ij}$. This matrix, presented in Table 15, synthesizes all previous evaluation, weighting, and transformation steps, and serves as the direct input for the CODAS distance computation.

Table 15. Weighted Normalization Matrix.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
u_1	0.098003	0.086856	0.053784	0.035217	0.025237	0.020086	0.0166649	0.014503129	0.010156594	0.0073957
u_2	0.098003	0.086856	0.053784	0.035316	0.025268	0.020115	0.0166649	0.014505199	0.010156351	0.0073958
u_3	0.098003	0.086856	0.053784	0.035169	0.025265	0.020086	0.0166744	0.014500029	0.010155681	0.0073953
u_4	0.100896	0.088859	0.054262	0.035169	0.025237	0.020086	0.0166649	0.014501718	0.010155283	0.0073951
u_5	0.099751	0.088777	0.054217	0.035266	0.025237	0.020086	0.0166649	0.014500846	0.010155895	0.007395
u_6	0.099973	0.088859	0.054127	0.035169	0.025237	0.020086	0.0166649	0.01449445	0.010154923	0.0073956
u_7	0.100571	0.088777	0.054249	0.035169	0.025237	0.020086	0.0166649	0.014497907	0.010154604	0.0073947
u_8	0.098003	0.086856	0.053921	0.035316	0.025237	0.020086	0.0166649	0.014500846	0.010157109	0.0073944
u_9	0.099329	0.088893	0.054262	0.035304	0.025237	0.020086	0.0166649	0.014505199	0.010157109	0.0073947
u_{10}	0.098973	0.087535	0.054198	0.035201	0.025237	0.020086	0.0166649	0.01449677	0.010155895	0.0073948
u_{11}	0.098803	0.087419	0.053948	0.035169	0.025237	0.020086	0.0166649	0.01449445	0.010153473	0.0073949
u_{12}	0.098659	0.087314	0.053896	0.035169	0.025237	0.020086	0.0166649	0.01449445	0.010153473	0.0073952
u_{13}	0.100834	0.088859	0.054249	0.035169	0.025237	0.020086	0.0166649	0.01449445	0.010153473	0.0073958
u_{14}	0.098003	0.086856	0.054088	0.035316	0.025291	0.020103	0.0166649	0.014502175	0.010154458	0.0073948
u_{15}	0.098003	0.088777	0.054013	0.03531	0.025292	0.020105	0.0166649	0.014504661	0.010154198	0.0073947
u_{16}	0.098003	0.086856	0.053784	0.035169	0.025237	0.020086	0.0166649	0.01449445	0.010153473	0.0073944

To illustrate the process of calculating the entries of the weighted normalization matrix in Table 15, consider the following example.

- In Table 14, the value of the Sc_{11} scoring function for the u_1 object at the e_1 parameter is $Sc_{11} = 0.367879$. With the e_1 parameter weight of $w_1 = 0.2664$, the weighted value r_{11} is calculated as follows:

$$r_{11} = w_1 \times Sc_{11} = 0.2664 \times 0.367879 = 0.098003.$$

In general, for every $Sc_{ij} = 1$, the value of r_{ij} for object u_i on parameter e_j will be equal to the weight of parameter e_j .

- The value of the Sc_{14} scoring function for the u_1 object at the e_4 parameter is $Sc_{14} = 0.368376$. With the e_4 parameter weight of $w_4 = 0.0956$, the weighted value r_{14} is calculated as follows:

$$r_{14} = w_4 \times Sc_{14} = 0.0956 \times 0.368376 = 0.035217.$$

The CODAS algorithm requires identifying a benchmark for the worst performance. The Negative Ideal Solution (NIS), defined as the minimum value for each criterion across all alternatives, is determined from Table 15 and presented in Table 16.

Table 16. Negative Ideal Solution.

Parameter	Value
e_1	0.098003
e_2	0.086856
e_3	0.053784
e_4	0.035169
e_5	0.025237
e_6	0.020086
e_7	0.016665
e_8	0.014494
e_9	0.010153
e_{10}	0.007394

Note that the minimum value of parameter e_1 across all alternatives, as shown in Table 15, is 0.098003 (observed in $u_1, u_2, u_3, u_8, u_{14}, u_{15}$, and u_{16}). Therefore, the Negative Ideal Solution for parameter e_1 is 0.098003. Mathematically, this can be written as:

$$n_1 = \min_{i=1,2,\dots,16} \{r_{i1}\} = 0.098003.$$

The same calculation also applies to determining the negative ideal solution using other criteria.

Based on the matrix in Table 15 and the NIS in Table 16, the Euclidean (Eu_i) and Hamming (Ha_i) distances of each trash object through the NIS values are calculated. The results, presented in Table 17, quantify how far each alternative is from the worst-case scenario, with larger distances indicating better overall performance.

Table 17. Euclidean and Hamming Distance.

Alternative	Euclidean Distance	Hamming Distance
u_1	0.0000486493	0.0000608671
u_2	0.000153	0.000222
u_3	0.0000303371	0.000046336
u_4	0.003551	0.005384
u_5	0.002635	0.004208
u_6	0.00283	0.004319
u_7	0.00324	0.004959
u_8	0.000201	0.000293
u_9	0.00248	0.003989
u_{10}	0.001255	0.002099
u_{11}	0.000992	0.001527
u_{12}	0.000807	0.001226
u_{13}	0.003499	0.0053
u_{14}	0.000342	0.000531
u_{15}	0.001941	0.002376
u_{16}	0	0

Consider the following calculation for u_1 .

$$\begin{aligned}
 Eu_1 &= \sqrt{\sum_{j=1}^{10} (r_{1j} - n_j)^2} \\
 &= \sqrt{(r_{11} - n_1)^2 + (r_{12} - n_2)^2 + \dots + (r_{1,10} - n_{10})^2} \\
 &= \sqrt{(0.098003 - 0.098003)^2 + (0.086856 - 0.086856)^2 + \dots + (0.0074 - 0.007394)^2} \\
 &= \sqrt{0.00000000227996 + 0.0000000000753179 + 0.00000000000974395 + 0.00000000000173748} \\
 &= \sqrt{0.00000000236676} \\
 &= 0.0000486493 \\
 Ha_1 &= \sum_{j=1}^{10} |r_{1j} - n_j| \\
 &= |r_{11} - n_1| + |r_{12} - n_2| + |r_{13} - n_3| + \dots + |r_{1,10} - n_{10}| \\
 &= |0.098003 - 0.098003| + |0.086856 - 0.086856| + \dots + |0.0074 - 0.007394| \\
 &= 0.0000477489 + 0.00000867859 + 0.00000312153 + 0.00000131813 \\
 &= 0.0000608671
 \end{aligned}$$

The core comparative mechanism of CODAS is summarized in the relative assessment matrix (RA). This matrix is constructed by performing pairwise comparisons of the distances in Table 17, using both Euclidean and Hamming distances with a threshold function. A threshold value of $\Phi = 0.02$ is specified based on the verification provided by Ghorabae et al. [17], which confirmed that this threshold yields consistent results. The resulting matrices using the threshold $\Phi = 0.02$, presented in Table 18a,b, show the relative performance P_{rik} of each object i compared with every other object k .

Table 18. (a) Matrix P_{rik} ; (b) Matrix P_{rij} .

(a)								
	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
u_1	0	−0.000104	0.0000183	−0.0035	−0.00259	−0.00278	−0.00319	−0.00015
u_2	0.000104	0	0.000123	−0.0034	−0.00248	−0.00268	−0.00309	−0.000048
u_3	−0.0000183	−0.00012	0	−0.00352	−0.0026	−0.0028	−0.00321	−0.00017
u_4	0.003503	0.003398	0.003521	0	0.000916	0.000721	0.000311	0.003351
u_5	0.002586	0.002482	0.002604	−0.00092	0	−0.0002	−0.00061	0.002434
u_6	0.002782	0.002677	0.0028	−0.00072	0.000196	0	−0.00041	0.00263
u_7	0.003192	0.003087	0.00321	−0.00031	0.000606	0.00041	0	0.00304
u_8	0.000152	−0.000048	0.00017	−0.00335	−0.00243	−0.00263	−0.00304	0
u_9	0.002431	0.002327	0.00245	−0.00107	−0.00015	−0.00035	−0.00076	0.002279
u_{10}	0.001206	0.001102	0.001224	−0.0023	−0.00138	−0.00158	−0.00199	0.001054
u_{11}	0.000943	0.000839	0.000961	−0.00256	−0.00164	−0.00184	−0.00225	0.000791
u_{12}	0.000759	0.000654	0.000777	−0.00274	−0.00183	−0.00202	−0.00243	0.000607
u_{13}	0.00345	0.003346	0.003469	−0.000052	0.000864	0.000669	0.000258	0.003298
u_{14}	0.000294	0.000189	0.000312	−0.00321	−0.00229	−0.00249	−0.0029	0.000142
u_{15}	0.001892	0.001788	0.00191	−0.00161	−0.00069	−0.00089	−0.0013	0.00174
u_{16}	−0.000049	−0.00015	−0.00003	−0.00355	−0.00263	−0.00283	−0.00324	−0.0002
(b)								
	u_9	u_{10}	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}
u_1	−0.00243	−0.00121	−0.00094	−0.00076	−0.00345	−0.00029	−0.00189	0.000049
u_2	−0.00233	−0.0011	−0.00084	−0.00065	−0.00335	−0.00019	−0.00179	0.000153
u_3	−0.00245	−0.00122	−0.00096	−0.00078	−0.00347	−0.00031	−0.00191	0.00003
u_4	0.001071	0.002297	0.002559	0.002744	0.000052	0.003209	0.001611	0.003551
u_5	0.000155	0.00138	0.001643	0.001827	−0.00086	0.002292	0.000694	0.002635
u_6	0.00035	0.001576	0.001839	0.002023	−0.00067	0.002488	0.00089	0.00283
u_7	0.00076	0.001986	0.002249	0.002433	−0.00026	0.002898	0.0013	0.00324
u_8	−0.00228	−0.00105	−0.00079	−0.00061	−0.0033	−0.00014	−0.00174	0.000201
u_9	0	0.001225	0.001488	0.001673	−0.00102	0.002137	0.000539	0.00248
u_{10}	−0.00123	0	0.000263	0.000447	−0.00224	0.000912	−0.00069	0.001255
u_{11}	−0.00149	−0.00026	0	0.000184	−0.00251	0.000649	−0.00095	0.000992
u_{12}	−0.00167	−0.00045	−0.00018	0	−0.00269	0.000465	−0.00113	0.000807
u_{13}	0.001019	0.002244	0.002507	0.002691	0	0.003156	0.001558	0.003499
u_{14}	−0.00214	−0.00091	−0.00065	−0.00046	−0.00316	0	−0.0016	0.000342
u_{15}	−0.00054	0.000686	0.000949	0.001133	−0.00156	0.001598	0	0.001941
u_{16}	−0.00248	−0.00125	−0.00099	−0.00081	−0.0035	−0.00034	−0.00194	0

Consider the following calculation for pr_{12} of Table 18a with $\Phi = 0.02$ as stated before. First, we need to check for the binary function of $\Phi(Eu_i - Eu_k)$.

$$\begin{aligned}
 |Eu_1 - Eu_2| &= |0.0000486493 - 0.000153| \\
 &= |-0.0001043507| \\
 &= 0.0001043507.
 \end{aligned}$$

We have the binary function of $\Phi(Eu_i - Eu_k)$:

$$\Phi(Eu_1 - Eu_2) = \begin{cases} 1, & \text{if } |-0.0001043507| \geq 0.02, \\ 0, & \text{if } |-0.0001043507| < 0.02. \end{cases}$$

Therefore, the calculation for pr_{12} as expressed below.

$$\begin{aligned}
 pr_{12} &= (Eu_1 - Eu_2) + (\Phi(Eu_1 - Eu_2) \times (Ha_1 - Ha_2)) \\
 &= (0.0000486493 - 0.000153) + (\Phi(-0.0001043507) \\
 &\quad \times (0.0000608671 - 0.000222)) \\
 &= (-0.0001043507) + (0 \times (-0.0001611329)) \\
 &= (-0.0001043507) + 0
 \end{aligned}$$

The final assessment score (RA_i) for each waste object is calculated by summing all elements in the corresponding row of the relative assessment matrices from Table 18a,b ($RA_i = \sum_{k=1}^{16} pr_{ik}$). These scores and the resulting final rankings are presented in Table 19.

Table 19. Alternative Scores and Rankings.

Alternative	Score	Rank
u_1	−0.02323	14
u_2	−0.02156	13
u_3	−0.02352	15
u_4	0.032815	1
u_5	0.018152	5
u_6	0.021280	4
u_7	0.027841	3
u_8	−0.02079	12
u_9	0.015674	6
u_{10}	−0.00393	8
u_{11}	−0.00814	9
u_{12}	−0.01109	10
u_{13}	0.031977	2
u_{14}	−0.01853	11
u_{15}	0.007044	7
u_{16}	−0.02401	16

To illustrate the score calculation for each object, the following is an example calculation for object u_1 :

$$\begin{aligned}
 RA_1 &= \sum_{k=1}^{16} pr_{1k} \\
 &= 0 - 0.0001043 + 0.0000183 - 0.0035 - 0.00259 - 0.00278 - 0.00319 - 0.00015 \\
 &\quad - 0.00243 - 0.00121 - 0.00094 - 0.00076 - 0.00345 - 0.00029 - 0.00189 + \\
 &\quad 0.000049 \\
 &= -0.02323.
 \end{aligned}$$

The rankings in Table 19 provide a clear and mathematically derived priority order for waste-management interventions. To facilitate interpretation, the final CODAS scores are further grouped into three priority classes, namely high-, medium-, and low-priority waste. This classification is derived from the distribution of the assessment scores and their relative distances from the negative-ideal solution. Waste types with substantially larger positive scores are classified as high priority, reflecting strong alignment with the highly weighted upstream circular economy strategies, particularly Refuse, Rethink, and Reduce. Medium-priority waste exhibits moderate assessment scores, indicating partial alignment with upstream strategies and stronger feasibility under midstream strategies

such as Reuse, Repurpose, or Recycle. Low-priority waste corresponds to alternatives with small or negative assessment scores, suggesting limited effectiveness within the evaluated 10R framework under the current weighting structure.

As a complement to the tabular representation in Table 19, a graph was created to visualize the final appraisal scores (RA_i) for each waste object. This visualization, presented in Figure 2, enables a more intuitive and rapid understanding of the priority hierarchy generated by the hybrid Weighted Fuzzy N-Soft Set and CODAS model.

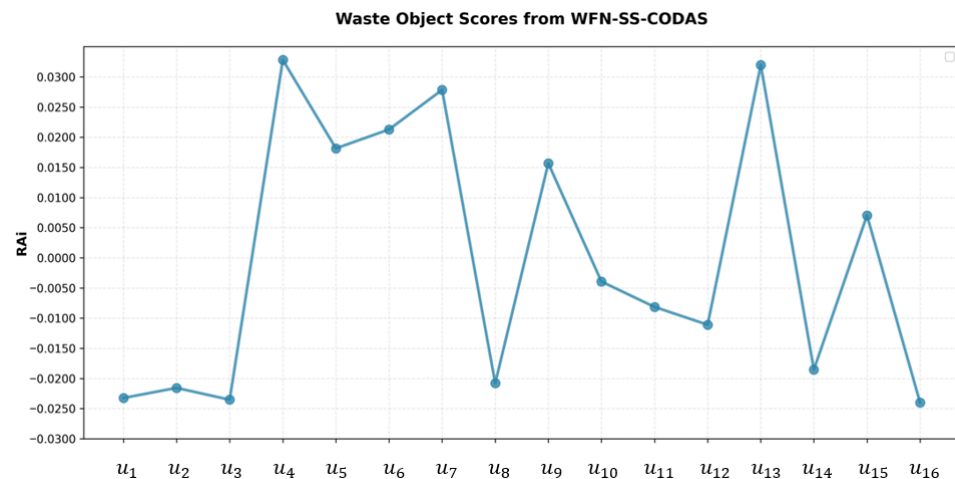


Figure 2. Final Scores of waste object derived from Weighted Fuzzy N-Soft Set–CODAS model.

Based on the graph in Figure 2, a pronounced disparity in CODAS scores across the evaluated waste types is clearly observable. To facilitate interpretation, the results are qualitatively organized into three broad priority tiers according to score magnitude and relative ranking: high-priority waste (Ranks 1–4), medium-priority waste (Ranks 5–7), and low-priority waste (Ranks 8–16). This tiering reflects the natural clustering exhibited by the CODAS scores and supports a structured discussion of intervention priorities.

- High-priority waste: Waste objects u_4 (Plastic Bags), u_{13} (Styrofoam), u_6 (Clear Plastic Sheets), and u_7 (Plastic Glasses) appear as the tallest points on the graph, reflecting their higher RA_i scores compared with other objects. This indicates that these objects consistently exhibit superior performance relative to nearly all others in the relative assessment matrix. Their elevated positions visually represent their status as the most critical targets for immediate waste-management intervention.
- Medium-priority waste: Objects such as u_5 (Plastic Bottles), u_9 (PET Bottles), u_{14} (Bags), and u_{15} (Clothes) exhibit moderate positive CODAS scores. In Figure 2, this group occupies an intermediate range, reflecting partial alignment with the highly weighted upstream strategies while showing stronger feasibility under reuse-, repurpose-, or recycle-oriented approaches. Their positions correspond to Ranks 5–7 in Table 19. Waste object u_{14} is classified as medium-priority waste because, although it shows relatively high feasibility under reuse- and repurpose-oriented strategies, its performance under the most heavily weighted upstream criteria, that is Refuse, Rethink, and Reduce, is comparatively weaker. As a result, its overall CODAS score positions it between the high-priority single-use plastics and the low-priority waste categories.
- Low-priority waste: Most organic, complex residual, and hazardous (B3) waste objects, such as u_1 (Water Hyacinth), u_3 (Rubber), u_{11} (Multi-Layer Pack), u_{12} (Used Beverage Cartons), and u_{16} (Hazardous Waste). These objects are represented by very low or negative CODAS scores and are positioned below the zero axis in Figure 2. In particular, hazardous waste (B3) attains the lowest assessment score, highlighting the

inherent difficulty of effectively applying the 10R strategies to this category. Waste objects u_2 , u_8 , and u_{10} are included in the CODAS evaluation but fail within the low-priority category. Their relatively small assessment scores indicate limited distances from the negative-ideal solution, implying lower immediate impact or feasibility for circular economy-based intervention under the current weighting scheme.

This graph not only replicates the information in Table 19 but also strengthens the interpretation of the results. The considerable vertical distance between the highest and lowest points underscores the discriminative power of the CODAS method in distinguishing objects with highly divergent performance. Thus, Figure 2 serves as a visual tool that conveys to stakeholders and policymakers the key message that resources and interventions should primarily be directed toward the waste objects represented by the highest points, namely, single-use plastic waste that can be prevented at the upstream stage.

3.2. Comparative Analysis

To rigorously evaluate the effectiveness, robustness, and discriminative capacity of the proposed hybrid Weighted Fuzzy N-Soft Set–CODAS (WFN-SS–CODAS) model, a comparative analysis was conducted against two other methodological frameworks:

- (1) Fuzzy N-Soft Set–CODAS (FN-SS–CODAS): This model applies the CODAS ranking to an unweighted fuzzy N-soft set based on Peng and Garg [39], meaning no AHP weighting is incorporated at the fuzzy stage.
- (2) Weighted Fuzzy N-Soft Set with Row-Sum Ranking (WFN-SS–R): This model integrates AHP weights into the fuzzy N-soft set but uses a simple row-sum aggregation for ranking, following Korkmaz et al. [14], representing a single-weighting approach.
- (3) Weighted Fuzzy N-Soft Set–TOPSIS (WFN-SS–TOPSIS) model: This model applies the TOPSIS ranking to the weighted fuzzy N-soft set, based on Zulfarnain et al. [40].

In contrast, the proposed WFN-SS–CODAS model employs a two-stage weighting strategy that is weights are applied first during the construction of the weighted fuzzy N-soft set, and again during the CODAS normalization phase. This design intentionally amplifies the influence of high-importance criteria (e.g., Refuse, Rethink) to enhance differentiation among alternatives, a critical need in waste prioritization where performance scores are often closely clustered. The same dataset, consisting of evaluations of 16 waste objects against the 10R criteria, was used across all models to ensure a fair and consistent comparison.

The FN-SS–CODAS model serves as the baseline benchmark. This model leverages the Fuzzy N-Soft Set structure to capture uncertainty and the granularity of expert assessments, but does not incorporate the relative importance of the 10R criteria through AHP-derived weights. In other words, it assumes that all 10R principles have equal strategic significance. On the other hand, the WFN-SS–R model integrates AHP weights into the Fuzzy N-Soft Set to form a weighted evaluation matrix. However, this model does not employ a more advanced ranking mechanism, such as CODAS, instead relying on simpler aggregation and ranking procedures, namely, row and column sums, which lack the discriminative power of distance-based evaluation. The WFN-SS–TOPSIS model also uses the same weighted evaluation matrix (Table 13) but replaces the simple ranking procedure with the TOPSIS method, allowing alternatives to be ranked based on their distances from the ideal and anti-ideal solutions while keeping the input data and weights unchanged. Finally, the proposed WFN-SS–CODAS model provides a comprehensive framework that synergistically combines weighted and uncertain data representation with CODAS's strong dual-distance ranking capabilities.

To link this theoretical comparison with practical realities and policy relevance, and to justify the two-stage weighting strategy, the ranking results are critically evaluated

against additional field data that reflect real-world urgency and feasibility. These data include estimates of daily waste, the potential economic value (if processed), and approximate processing costs, derived from the technological complexity of applying the 10R strategies to each waste type. These data were obtained from field observations and BSf reports, providing an important benchmark for assessing the practical rationality of each model's output.

Importantly, the two-stage weighting in WFN-SS–CODAS is designed to align the mathematical discrimination with ecological urgency that ensures that high-volume, low-value pollutants (e.g., plastic bags, styrofoam) are distinctly prioritized over materials with higher economic value but lower volumetric impact (e.g., PET bottles), thereby supporting preventive, upstream interventions consistent with circular and blue economy principles.

The results of the comparative analysis, including the final assessment scores (RA_i) and the corresponding ranks for each waste object, are systematically presented in Table 20.

Table 20. Comparison of Scores and Rankings of Trash Objects by Different Ranking Methods.

Objects	RA_i WFN-SS–CODAS	Rank	RA_i FN-SS–CODAS	Rank	RA_i WFN-SS–R	Rank	RA_i WFN-SS–TOPSIS	Rank
u_1	−23.23	14	−7.52	15	−5.59	15	1.14	15
u_2	−21.56	13	−4.25	10	−1.69	12	3.06	11
u_3	−23.52	15	−6.00	13	−5.20	14	4.54	4
u_4	32.82	1	9.21	1	9.02	3	4.78	1
u_5	18.15	5	6.36	5	9.10	2	4.51	5
u_6	21.28	4	5.90	6	6.42	6	4.42	6
u_7	27.84	3	7.90	3	7.96	4	4.65	3
u_8	−20.79	12	−5.24	12	−3.20	13	1.87	14
u_9	15.67	6	7.19	4	9.39	1	4.41	7
u_{10}	−3.93	8	−1.32	8	3.98	8	3.51	9
u_{11}	−8.14	9	−5.08	11	−0.10	10	3.01	12
u_{12}	−11.09	10	−6.29	14	−1.04	11	2.74	13
u_{13}	31.98	2	8.65	2	7.72	5	4.74	2
u_{14}	−18.53	11	−2.90	9	0.60	9	3.22	10
u_{15}	7.04	7	3.46	7	6.14	7	4.09	8
u_{16}	−24.01	16	−10.07	16	−9.02	16	0	16

Furthermore, a visual summary of the score distributions for all four models is shown in Figure 3.

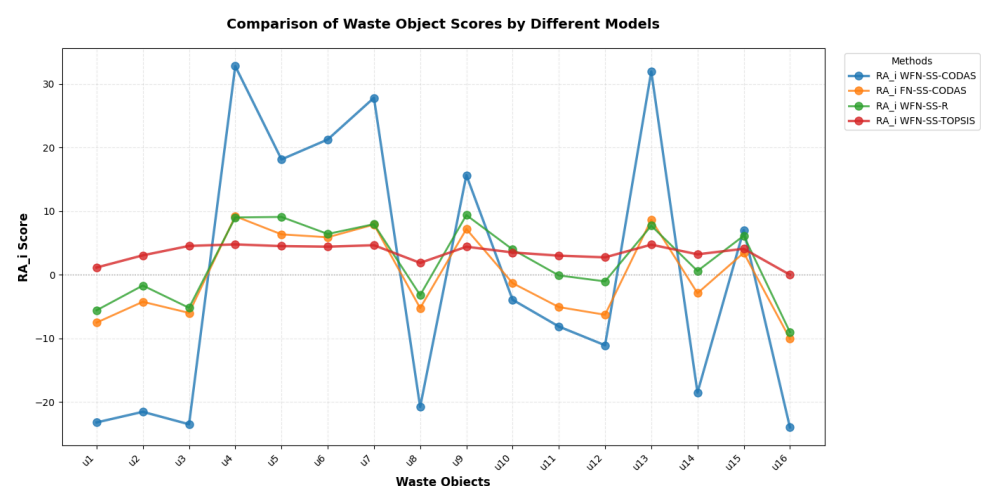


Figure 3. Comparison of Waste Object Scores by Different Models.

Initial observations from Table 20 and Figure 3 reveal variations in the range of scores. The crucial task is to determine which model's score distribution and rankings best align with the pragmatic demands of waste management in the Citarum context. To conduct

this assessment, the ranking results are presented alongside field data on waste volume, economic value, and processing costs, as summarized in Table 21.

Table 21. Supporting Field Data: Volume, Economic Value, and Estimated Processing Costs.

Notation	Object	Estimated Daily Volume (% of Total Waste)	Economic Value (Potential Revenue)	Estimated Processing Cost and Complexity
u_1	Water Hyacinth	Very High	Low (Compost/Fertilizer)	Very Low (simple composting); Medium–High (biogas)
u_2	Wood	High	Low–Medium (firewood/craft material)	Low (sorting, cutting); Medium (creative recycling)
u_3	Rubber	Medium	Very Low	High. Very difficult to recycle, limited market.
u_4	Plastic Bags (HDPE)	>20% (Very High)	Very Low	Low when collected, Very High for economically viable recycling. Refuse/Reduce strategies are most effective.
u_5	Plastic Bottles (HDPE)	Medium	Medium–High	Low–Medium. Recycling market is relatively strong. Recycle is feasible.
u_6	Clear Plastic Sheets (PP)	High	Low	Medium. Often contaminated, low market value.
u_7	Plastic Cups (PP)	High	Low–Medium	Medium. Has a recycling market but limited. Recycle possible with incentives.
u_8	Glass Bottles	Medium	Very Low	High. Heavy, high transport cost, breakage risk. Reuse is preferable to recycling.
u_9	PET Bottles	Medium	High (strong and stable recycling market)	Low–Medium. Easy to collect and sell. Recycling is highly economically viable.
u_{10}	Gallon Caps (LDPE)	Medium	Low	Medium. Often mixed with other materials; sorting is difficult.
u_{11}	Multi-Layer Pack (MLP)	Medium	Very Low	Very High. Recycling technology is complex and uneconomical at small scale.
u_{12}	Used Beverage Cartons (UBC)	Medium	Very Low	Very High. Recycling technology is complex and uneconomical at small scale.
u_{13}	Styrofoam (Polystyrene)	High	Very Low (almost zero)	Very High. Difficult to recycle, no market. Refuse is the only rational strategy.
u_{14}	Bags (Textile)	Low	Medium (for reuse/shopping bags)	Low for reuse; High for recycling into raw material.
u_{15}	Clothes (Textile)	Low	Medium–High (for wearable secondhand clothing)	Low for reuse; High for recycling (into rags/insulation).
u_{16}	Hazardous Waste (B3)	Low (but extremely dangerous)	None	Very High. Requires special handling, licensed facilities, and advanced technology.

In the context of river basin restoration, daily waste volume is the most critical indicator because it directly relates to pollution loads, ecosystem degradation, and disruptions to blue-economy activities such as fisheries, tourism, and water security. A model that fails to prioritize high-volume waste will produce decisions misaligned with ecological needs on the ground.

The WFN-SS-CODAS model demonstrates strong environmental coherence by consistently placing high- to very-high-volume waste, such as Plastic Bags (u_4), Styrofoam (u_{13}), Plastic Cups (u_7), and Clear Plastic Sheets (u_6), at the highest ranks. These four are major contributors to macroplastic pollution, easily fragment into microplastics, and severely damage aquatic biota. Their high priority emphasizes upstream prevention strategies (Refuse, Rethink, Reduce), and the CODAS mechanism strengthens the performance of objects that excel on these high-weight criteria. In contrast, the FN-SS-CODAS model, which does not use AHP weights, exhibits a different discriminative pattern due to the absence of strategic weighting. While it also successfully identifies u_4 and u_{13} as top priorities and ranks u_7 highly (3rd), its compressed score range and the altered relative order of

priorities, compared to the double-weighted model, indicate that treating all 10R criteria as equally important dilutes the strategic emphasis on the highest-weighted upstream strategies, potentially obscuring focus on the most critical threats as defined by the circular economy hierarchy.

The ranking produced by the WFN-SS-R model reveals a significant strategic risk. The placement of PET Bottles (u_9) in first place indicates a strong bias toward their “High” economic value despite their only “Medium” volume. This priority misaligns with the mission of river-ecosystem restoration. A premature focus on PET Bottles, although economically beneficial for communities, risks diverting resources away from more pressing issues: the massive accumulation of low-value, highly persistent plastics such as Plastic Bags and Styrofoam. Meanwhile, the WFN-SS-TOPSIS model yields similar top-tier prioritization of Plastic Bags (u_4) and Styrofoam (u_{13}) but displays distinct mathematical behavior in the compressed normalized score range (0–0.478). This model elevates Rubber (u_3) to rank 4, a position significantly higher than in CODAS variants, highlighting sensitivity to specific criterion profiles within its distance calculations. This elevation is inconsistent with field data and expert assessment, which classify rubber as a lower-priority material due to its medium volume, very low economic value, and high processing complexity. The WFN-SS-CODAS model assigns a more proportional position to PET Bottles, namely rank 6, indicating a balanced understanding between economic circularity potential and the urgent need to reduce high-volume macroplastic pollution.

All models assign a low rank to hazardous waste (B3) (u_{16}), which is environmentally reasonable because its volume is “Low” even though its hazard level is high. This allows the focus to remain on controlling high-volume waste that has direct impacts on the ecosystem. The cases of Multi-Layer Packaging (u_{11}) and Used Beverage Cartons (u_{12}) are more informative. The WFN-SS-CODAS model assigns both a lower priority, not because the 10R strategies are irrelevant, but because of technical and economic misalignment, very low economic value and “Very High” processing costs. This conclusion directly guides policy recommendations toward upstream mechanisms such as Extended Producer Responsibility (EPR), rather than downstream community-based interventions.

The low ranking of Water Hyacinth (u_1) and Wood (u_2), despite their “Very High” volumes, indicates limitations in the initial assessment of the Recovery strategy. If the fuzzy scores for low-cost processing options, such as composting or biogas, are too low, the model automatically lowers their final priority ranking.

The results of the analysis confirm that WFN-SS-CODAS is the most coherent, rational, and ecologically relevant model for integrating the Circular Economy and Blue Economy within the Citarum River context. The intentional two-stage weighting proves critical in enhancing score differentiation and stabilizing rankings under sensitivity tests (Section 3.3), without overfitting to economic biases. Compared with the other two models:

- (1) FN-SS-CODAS can provide a basic overview but lacks strategic depth due to the absence of criterion weighting.
- (2) WFN-SS-R introduces strategic weighting but employs an oversimplified ranking mechanism that fails to resolve closely scored alternatives, resulting in a bias toward economically valuable waste.
- (3) WFN-SS-TOPSIS offers mathematical robustness through its normalized distance approach, but its compressed scoring range reduces discriminative clarity for mid-tier alternatives, making it less optimal for nuanced policy differentiation.
- (4) WFN-SS-CODAS unify strategic weights (AHP), expert fuzzy assessments, and the enhanced discriminative power of double-weighted CODAs, generating a clear, actionable priority list aligned with ecological urgency, emphasizing prevention, and offering differentiated policy directions for each waste type.

Overall, this model not only excels mathematically but also delivers the most actionable policy roadmap for accelerating the recovery of the Citarum River. The two-stage weighting strategy, while increasing model sensitivity, is justified by its ability to produce discriminative, ecologically grounded priorities essential for effective waste-management intervention in complex riverine systems.

3.3. Sensitivity Analysis

A sensitivity analysis was conducted to evaluate the robustness and reliability of the WFN-SS-CODAS model to changes in the weights of the 10R criteria. The main objective is to ensure that the final ranking does not depend excessively on a particular set of weights. Instead, it reflects a stable priority structure under reasonable parameter variations. A robust model should maintain its ranking pattern even when moderate changes are made to the criterion weights. The analysis was performed by modifying the original AHP weight vector

$$w = (0.2664, 0.2361, 0.1462, 0.0956, 0.0686, 0.0546, 0.0453, 0.0394, 0.0276, 0.0201).$$

Moreover, recalculating the entire WFN-SS-CODAS procedure. These changes affect not only the CODAS weighting but also the weights used in the Fuzzy N-Soft Set, as both are structurally integrated. Three scenarios were designed to test the model's response to shifts in strategic emphasis:

- (1) Scenario 1 (equal weights): All 10R criteria were assigned equal weights, $w_j = 0.1$. This scenario assesses whether the original rankings are overly dependent on the AHP-derived weight structure.
- (2) Scenario 2 (upstream focus): The weights of Refuse, Rethink, and Reduce were increased by 20% from their original values. To maintain the total weight at 1, the weights of downstream strategies were reduced proportionally. This scenario evaluates the model's consistency when preventive strategies are emphasized.
- (3) Scenario 3 (downstream focus): The weights of Recycle and Recover were increased by 50%, with compensation taken from reductions in upstream strategies. This scenario simulates policy orientations that prioritize downstream solutions such as recycling and energy recovery.

Each weight vector was normalized, and new rankings for the 16 waste objects were generated. Table 22 presents the comparison of CODAS ranking changes across the three weighting scenarios.

A comparison is also presented in graphical form in Figure 4.

The model demonstrates high stability for both the highest- and lowest-priority waste objects across all scenarios. Hazardous Waste (u_{16}) consistently receives the lowest score, reaffirming its position as a low-priority object. This outcome is reasonable considering its hazardous nature and relatively small volume, making it more suitable for specialized handling mechanisms rather than mass-scale waste-reduction interventions. Meanwhile, Plastic Bags (u_4) and Styrofoam (u_{13}) maintain high scores in Scenarios 2 and 3, underscoring their environmental urgency given their significant volume and pollution levels. Under Scenario 1 (equal weighting), both still fall within the range of positive scores, although not dominant, indicating that weighting strategies play a crucial role in emphasizing the criticality of these objects.

Regarding shifts in strategic emphasis, the model exhibits logical responses to changes in policy focus. Under Scenario 2 (upstream emphasis), waste objects such as u_4 , u_{13} , u_7 , and u_5 achieve the highest scores because they align well with preventive strategies such as Refuse and Reduce. These findings reinforce the priorities established in the

baseline model and validate the importance of upstream-oriented approaches in achieving integrated circular-economy and blue-economy outcomes. Conversely, under Scenario 3 (downstream emphasis), plastic bags (u_4) retain the highest score (0.03167), followed by plastic bottles (u_5 , 0.01744) and PET bottles (u_9 , 0.01509). This reflects their continued high volume and ecological impact, even when policy emphasis shifts toward recycling. The strong performance of u_5 and u_9 aligns with their high recycling potential and economic value under this downstream-focused weighting.

Table 22. Comparison of Weight Vector Scenarios.

Objects	RA_i Scenario 1	RA_i Scenario 2	RA_i Scenario 3
u_1	0.03182	−0.03993	−0.02254
u_2	0.19637	−0.03907	−0.02104
u_3	0.09548	−0.03918	−0.02128
u_4	0.15037	0.05580	0.03167
u_5	0.16290	0.03109	0.01744
u_6	0.07788	0.03642	0.02049
u_7	0.08826	0.04753	0.02690
u_8	0.02601	−0.03636	−0.02029
u_9	0.20696	0.02700	0.01509
u_{10}	0.03459	−0.00632	−0.00401
u_{11}	−0.14846	−0.01320	−0.00790
u_{12}	−0.14753	−0.01827	−0.01078
u_{13}	0.09672	0.05455	0.03095
u_{14}	0.14817	−0.03197	−0.01817
u_{15}	0.20756	0.01224	0.00672
u_{16}	−1.22710	−0.04032	−0.02326

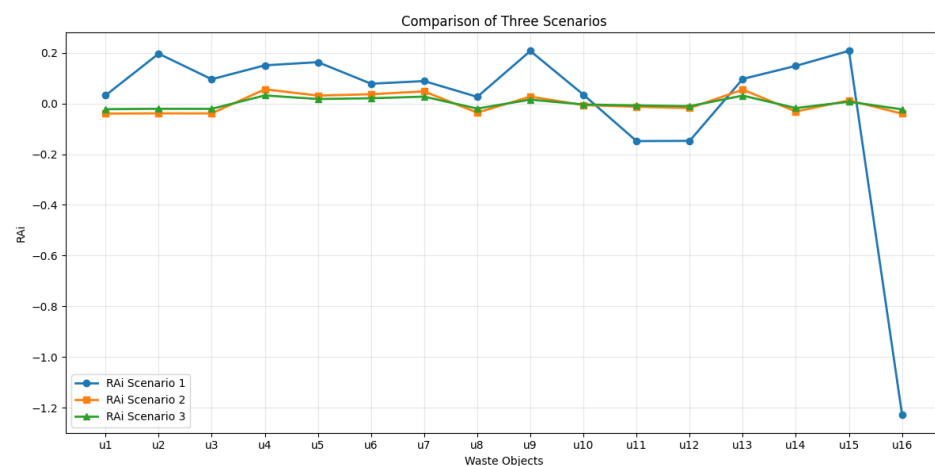


Figure 4. Comparison of Three Weight Vector Scenarios.

Several waste objects exhibit considerable score variation across scenarios, indicating model sensitivity to weight changes. For instance, Wood (u_2) has a relatively high score in Scenario 1 (0.19637), but drops sharply in Scenario 2 (−0.03907), illustrating that its priority level is heavily influenced by strategic context. Clothing (u_{15}) also undergoes ranking changes despite consistently receiving positive scores. This phenomenon underscores the importance of context-specific weighting, enabled through structured methods such as AHP, to avoid misinterpretation of priority levels. Objects such as Multi-Layer Pack (u_{11}) and Used Beverage Cartons (u_{12}) consistently receive negative scores across all scenarios, indicating their low compatibility with standard 10R strategies. The primary factors influencing this outcome include high processing costs and low economic value. This

consistency reinforces the model's ability to identify waste streams that are better addressed through upstream policy instruments, such as extended producer responsibility, rather than local operational solutions.

The sensitivity analysis results indicate that the WFN-SS-CODAS model possesses strong robustness and logical coherence. Although the scores and rankings of particular objects shift under different weight scenarios, the model's core recommendations remain stable. It consistently identifies large-volume and high-impact waste, such as plastic bags and Styrofoam, as top priorities for upstream preventive interventions, while classifying complex, low-value waste streams as low priorities. The model's ability to adjust its outputs in response to shifts in policy orientation, such as moving from prevention to recycling, without producing anomalies demonstrates its reliability as a decision-support tool. Consequently, policymakers can rely on the prioritization generated under the initial AHP weighting scheme while using sensitivity analysis to understand the implications of alternative policy emphases for integrated circular-economy and blue-economy planning in the Citarum River Basin.

4. Discussion

4.1. Comparative Positioning with Related Ranking Methods

The proposed Weighted Fuzzy N-Soft Set–CODAS (WFN-SS-CODAS) model demonstrates significant advancements over conventional Multi-Criteria Decision-Making (MCDM) approaches, particularly through its integration of granular uncertainty representation and a robust ranking mechanism. Traditional methods such as TOPSIS and VIKOR, which rely on distance measures from ideal solutions, tend to be less adaptable to heterogeneous, interval-scaled, or ambiguity-laden environmental datasets. In contrast, the Combinative Distance-based ASsessment (CODAS) algorithm employs two types of distance metrics, Euclidean and Hamming, which enhance discriminatory sensitivity, especially when alternatives exhibit minimal performance differences. This feature is crucial in waste-management prioritization, where subtle variations in material characteristics or circular-economy potential require more precise differentiation.

Compared with stand-alone Fuzzy Soft Set models, the WFN-SS-CODAS framework offers the advantage of simultaneously capturing quantitative and qualitative uncertainty. Intuitionistic Fuzzy Sets (IFS) can model degrees of membership and non-membership, but do not provide the multigranular assessment structure of N-Soft Sets. Similarly, Hesitant Fuzzy Sets (HFS) can accommodate multiple membership values but lack a parameterized weighting mechanism. The proposed model synthesizes these strengths by integrating AHP-derived weights into the Fuzzy N-Soft Set structure, thereby reducing the subjectivity typically associated with assigning criterion weights.

Empirical evaluation also shows that the WFN-SS-CODAS model outperforms two comparative approaches: the unweighted FN-SS-CODAS model and the WFN-SS-R model based on row-sum ranking. The resulting prioritization demonstrates that the model not only aligns with ecological urgency, e.g., emphasizing high-volume, low-economic-value plastics such as plastic bags and styrofoam, but also avoids economic bias that could shift attention toward high-value materials with limited volumetric impact. Thus, the model emerges as a more reliable and adaptable tool for supporting circular- and blue-economy planning, particularly in complex, data-scarce environmental contexts such as the Citarum River Basin.

4.2. Theoretical Advancements and Model Integration

The integration of the Analytical Hierarchy Process (AHP) into the Fuzzy N-Soft Set framework represents a significant theoretical contribution to the development of

soft-set literature. The extensions introduced by Fatimah et al. [13] enhanced the representational flexibility of soft sets. Still, they did not provide a systematic mechanism for parameter weighting. The study by Korkmaz et al. [14] introduced the concept of the Weighted Fuzzy N-Soft Set. Still, it did not integrate this structure with a ranking technique capable of supporting comprehensive decision-making. Therefore, the present research fills this methodological gap by combining the Weighted Fuzzy N-Soft Set with the CODAS algorithm, producing a complete decision-support framework—from uncertainty representation to object prioritization.

From a theoretical standpoint, this model extends the parameterization concept in soft sets, shifting from binary or purely discrete assessments to weighted evaluations aligned with AHP principles. Assigning weights to the 10R criteria ensures that strategic orientations, such as prioritizing upstream prevention (Refuse, Rethink) over downstream strategies (Recycle, Recover), are directly reflected in the evaluation process. This approach is substantially stronger than conventional weighting techniques such as row-sum ranking, which are prone to expert bias. Furthermore, the use of interval-based fuzzy membership values combined with discrete rating levels enables the model to capture both the evaluator's degree of confidence and the categorical performance of each object—a feature not offered by classical fuzzy sets nor by conventional N-soft sets.

4.3. Methodological Workflow and Rationale

The WFN-SS-CODAS framework is designed to transform qualitative inputs containing uncertainty into defensible quantitative rankings through a series of interrelated stages. The first stage, weight determination using AHP, ensures that the relative importance of each 10R criterion is systematically established through pairwise comparisons, with consistency verified ($CR < 0.10$). This step addresses a standard limitation in many fuzzy-based models, which assign weights either equally or arbitrarily. The second stage constructs the Fuzzy N-Soft Set based on expert evaluations, where each waste object is assessed using discrete ratings (0–3) and fuzzy membership degrees. This layered representation not only captures performance but also reflects the degree of ambiguity or confidence in the assessment. The focus on the interval $[2, 3]$, corresponding to the categories “Easy” to “Very Easy,” yields a more practically relevant representation, as it includes only strategies that are realistically implementable. The third stage applies the CODAS algorithm to generate the final ranking. The combination of Euclidean and Hamming distances ensures that even minor differences among waste types are effectively detected without introducing instability in the ranking. The threshold parameter Φ further regulates when the Hamming distance should be activated, preventing arbitrary tie-breaking.

This workflow is not only mathematically consistent but also adaptive to implementation contexts. This is reflected in the model output, which prioritizes plastic bags and styrofoam, two materials with high pollution impacts and limited recycling potential, consistent with prevention-oriented principles in the circular economy. Thus, the model demonstrates its ability to translate sustainability principles into concrete decision hierarchies.

4.4. Analysis of Computational Complexity

Implementing the WFN-SS-CODAS model entails a significant computational burden due to its multi-stage aggregation and ranking procedures [41]. The AHP requires solving an eigenvalue problem derived from an $m \times m$ pairwise comparison matrix, while the construction of the Fuzzy N-Soft Set involves processing $n \times m \times k$ data points. The CODAS algorithm further adds complexity through the creation of an $n \times n$ pairwise comparison matrix across waste types, which grows quadratically with the number of alternatives. Nevertheless, this level of complexity remains manageable for small- to

medium-scale environmental problems, such as the present study, which involves a limited number of waste objects and 10 criteria. The dominant complexity term, $O(n^2 \times m)$, is considered acceptable for practical decision-making contexts. However, applications at much larger scales may require optimization techniques, such as approximation algorithms or parallel computation. The model also incorporates several efficiency strategies, including the use of a narrowed fuzzy interval [2, 3] and an AHP consistency check that prevents recalculations stemming from inconsistent judgments. Thus, the required computational load is justified by the precision and reliability of the results produced.

The analysis of computational complexity indicates that the WFN-SS-CODAS model is a polynomial-time algorithm, consistent with modern MCDM methods. The AHP procedure operates with a complexity of $O(m^3)$, which remains efficient for a relatively small number of criteria. The construction of the Fuzzy N-Soft Set requires $O(n \times m \times k)$ operations. At the same time, the CODAS ranking phase entails the computation of Euclidean and Hamming distances for all alternative pairs, with a complexity of $O(n^2 \times m)$. Compared with other hybrid models such as fuzzy DEMATEL or fuzzy ANP, which often rely on iterative matrix computations, WFN-SS-CODAS offers efficiency advantages through one-time AHP weighting and a deterministic ranking mechanism. However, for large-scale applications such as waste management in metropolitan regions, the development of heuristic methods or distributed computing architectures represents a promising direction for future research.

4.5. Limitations and Future Research Directions

Despite offering substantial theoretical and practical contributions, the WFN-SS-CODAS model still has several limitations. First, the model's inputs in this case study rely on the assessments of a single, highly knowledgeable expert. While this ensured internal consistency and deep contextual relevance for this proof-of-concept application, it introduces risks of individual subjectivity and limits the representativeness of the diverse stakeholder landscape in the Citarum River Basin (e.g., government agencies, industrial operators, different community groups). This design choice is aligned with the scope of the Fuzzy N-Soft Set, which models graded uncertainty from a single decision-making source. A comprehensive multi-stakeholder assessment would require transitioning to a Hesitant Fuzzy N-Soft Set model, capable of handling sets of possible grades and membership values from multiple experts.

Second, its reliance on expert judgment in determining AHP weights inherently contains elements of subjectivity that cannot be entirely eliminated. Third, assigning discrete ratings and fuzzy membership degrees may impose cognitive burdens on evaluators. Fourth, the current model is not designed to dynamically update weights or assessments when new data becomes available, limiting its adaptability to static context. These limitations open avenues for future research. Future research could explore the following promising directions:

- (1) **Extension to Group Decision-Making:** Developing a Hesitant Fuzzy N-Soft Set-CODAS or Hesitant Fuzzy Multi-Granular N-Soft Set-CODAS framework. This would involve defining aggregation mechanisms for multiple experts' assessments (e.g., using the mode for discrete grades and the median or interval-valued means for fuzzy memberships and multi-granular for N-Soft Set), adapting the CODAS distance measures to compare hesitant sets, and incorporating inter-expert reliability metrics (e.g., Fleiss' Kappa for grade agreement). Such an extension would enhance the model's applicability to participatory, multi-stakeholder environmental governance without altering the core hybrid integration logic established in this study.

- (2) Machine-learning-based approaches could be explored to calibrate AHP weights using historical data or to estimate fuzzy membership degrees from partial inputs.
- (3) Dynamic framework extension: The model may be extended into a dynamic framework. For example, through time-series soft sets or adaptive weighting mechanisms, to accommodate evolving waste streams and policy priorities.
- (4) Cross-domain applications, such as in renewable energy planning, sustainable supply chain management, or healthcare resource allocation, would provide insight into the model's generalizability and the specific adaptations required.
- (5) Computational optimization: For large-scale applications (e.g., metropolitan waste management with hundreds of waste categories), developing heuristic methods or parallel computing architectures would improve the model's operational efficiency and scalability.

These developments would strengthen the role of fuzzy soft-set-based models in supporting environmental governance practices aligned with sustainable development objectives, building directly upon the foundational integration presented in this work.

4.6. Implications for Circular Economy-Based Waste Management Supporting the Blue Economy

The prioritization outputs generated by the WFN-SS-CODAS model carry significant implications for advancing circular-economy-based waste management, a foundational action for supporting the blue economy in the Citarum River Basin. The resulting rankings do not merely reflect mathematical calculations; they also provide a strategic framework that aligns waste-management interventions with the core principles of both paradigms. The placement of plastic bags (u_4), styrofoam (u_{13}), transparent plastic sheets (u_6), and plastic cups (u_7) as high-priority waste types underscore a critical finding: achieving effective river basin restoration and blue economy goals must begin with aggressive upstream prevention, guided by circular economy principles.

From a circular-economy perspective, prioritizing single-use plastics directly targets the entry point of the linear “take–make–dispose” model. The substantial weights assigned to Refuse (0.2664) and Rethink (0.2361) confirm that the most sustainable and economically sound strategy is to prevent these bulky, low-value materials from entering economic systems, and ultimately the river environment. This aligns with the foundational circular-economy principle of narrowing resource flows. From a blue-economy standpoint, upstream prevention of plastic waste is a prerequisite for achieving “zero waste” and maintaining the ecological integrity of aquatic systems. Plastics and styrofoam are among the most persistent and visually disruptive pollutants in water bodies, with a high likelihood of breaking down into microplastics that can infiltrate food webs. Accordingly, the model's output highlights the need for policy instruments, such as extended producer responsibility (EPR) and restrictions on specific single-use items, to advance circular and blue economy objectives simultaneously.

The ranking of other waste types further demonstrates the model's ability to articulate strategic differentiation. For example, PET bottles (u_9) receive a relatively high ranking due to their recycling value, aligning with circular economy principles that emphasize closing material loops through recycling. However, their position below upstream prevention targets reinforce the circular-economy hierarchy, in which prevention is prioritized over recycling. This also mitigates the common tendency to overprioritize materials with economic value, even when their ecological impacts are less severe than those of non-recyclable pollutants. From a blue-economy perspective, improving PET recycling can help reduce land-based pollution loads and support the cleanliness of water bodies. Conversely, the low rankings of complex composite materials such as Multi-Layer Packaging (u_{11}) and Used Beverage Cartons (u_{12}) indicate significant implementation barriers. Their extremely

low economic value and high technical processing complexity render them unsuitable for management through conventional circular-economy pathways, including informal-sector recycling. The model's identification of the need for alternative approaches, such as design-for-recycling regulations at the producer level, highlights relevant policy implications for using circular-economy levers to support blue-economy objectives.

The case of organic waste, such as water hyacinth (u_1) and wood (u_2), which also receive low priority despite their high volume, illustrates another critical dynamic. The low scores for Recover (e.g., composting or biogas) may reflect expert perceptions of technical and market limitations rather than theoretical circular potential. This indicates a gap between the circular economy's "biological nutrient" concept and its current feasibility for implementation in the local context. Bridging this gap requires developing value chains capable of effectively utilizing organic waste, generating environmental benefits while contributing to the blue economy by reducing organic loads in water bodies. The synthesis between ecological urgency and economic value is also evident in the consistent assignment of hazardous waste (u_{16}) to the lowest priority. Despite its low volume, its management is critical. It requires specialized mechanisms beyond circular- and blue-economy strategies for municipal solid waste. This highlights that hazardous waste necessitates its own regulatory and infrastructural protocols and should not be incorporated into a community-based circular strategy.

Overall, the WFN-SS-CODAS model demonstrates that successfully applying circular economy strategies to support blue economy goals in complex systems, such as the Citarum River Basin, requires differentiated approaches tailored to each waste type. These strategies include:

- (1) Prioritizing upstream prevention for high-volume, low-value, and highly persistent pollutants through reduction- and ban-oriented policies;
- (2) Leveraging market-based recycling for high-value materials such as PET and selected textiles to strengthen material circulation and reduce pollution loads;
- (3) Developing advanced policies and technologies for low-value complex materials such as MLPs that cannot be addressed through conventional mechanisms;
- (4) Promoting bioeconomy development for organic waste streams to unlock circular potential and reduce pressure on aquatic ecosystems.

By explicitly quantifying and ranking trade-offs, the WFN-SS-CODAS model translates circular economy principles into a practical priority framework for waste management, addressing key implementation needs for reducing land-based pollution in support of blue economy goals. The resulting hierarchy of interventions is concrete and actionable, ensuring that resource allocation yields optimal benefits for both economic systems and aquatic environments.

5. Conclusions

This study successfully developed and validated the Weighted Fuzzy N-Soft Set-CODAS (WFN-SS-CODAS) framework, designed to address the challenges of waste management prioritization under uncertainty, using circular economy principles to support blue economy goals in river basins. The conclusions of this research are strongly supported by several key findings. First, the proposed framework effectively integrates the analytical rigor of AHP with the uncertainty representation capabilities of the Fuzzy N-Soft Set. This integration is demonstrated through mathematically consistent, rational AHP-derived weights ($CR = 0.0618 < 0.10$), thereby overcoming the issue of arbitrary weighting, a long-standing major limitation in fuzzy soft-set approaches. The dominance of the Refuse (0.2664), Rethink (0.2361), and Reduce (0.1462) criteria underscores the centrality of prevention-based strategies aligned with circular and blue economy principles.

Second, the model yields decisions that are ecologically urgent while economically defensible. The comparative analysis shows that the model accurately identifies single-use plastics, such as plastic bags and styrofoam, as top priorities due to their high volume and environmental persistence. Meanwhile, high-value but low-volume materials, such as PET bottles, are placed in secondary-priority tiers. These findings contrast with the WFN-SS-R model, which tends to be biased toward purely economic value, and the FN-SS-CODAS model, which exhibits weaker discriminatory ability due to the absence of weighting. Third, the proposed framework demonstrates strong robustness and reliability. This is evidenced by sensitivity analyses showing that the model's core recommendations remain consistent across various weighting configurations, including equal-weighting, upstream-focused, and downstream-focused scenarios. The persistently low rankings of complex composite materials and hazardous waste across all scenarios further reinforce the model's ability to distinguish waste streams that can be practically managed from those requiring specialized handling.

The main theoretical contribution of this study lies in the fact it not only extends fuzzy-set applications but also successfully integrates the AHP as a foundational weighting mechanism within the Fuzzy N-Soft Set process, which is subsequently combined with the ranking strength of CODAS. From a practical perspective, the model has proven to be more rational, robust, and transparent than the comparative models, and is highly relevant for circular- and blue-economy-based waste management in river basin contexts. Future research directions include optimizing computational efficiency for large-scale urban waste applications, integrating the model with geographic information systems (GIS) for spatially driven prioritization, and extending its application to other domains characterized by uncertainty and multi-stakeholder involvement, such as renewable energy planning and sustainable supply chain management.

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