

Article

A Spreadsheet Environment for Force, Torque and Strength of Materials Modeling: Bridging Analytical Mathematics and Engineering Practice

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Abstract

This paper presents and validates a unified spreadsheet-based framework for engineering mechanics education and preliminary design. Three modules are integrated within a single openly available workbook: multi-point resultant force and moment computation; axial normal stress with stress concentration effects for three geometric configurations (plate with hole, shoulder plate, stepped shaft); and beam deflection for simply supported and cantilever configurations under point loads. All governing equations are implemented as explicit closed-form expressions validated against analytical reference solutions for six independent cases; relative errors fall below 10^{-10} in all cases. Three worked exercises demonstrate the practical scope of the framework. A biomechanical multi-point force system yields joint moments of -6880 , $-33,421$, and $-58,241$ N·mm at the wrist, elbow, and shoulder, respectively. A tensile shoulder plate with $K_t \approx 1.85$ produces $\sigma_{\max} = 232$ MPa against $\sigma_y = 200$ MPa, identifying a design failure; a parametric redesign with fillet radius $r = 10$ mm reduces K_t to approximately 1.59 and $\sigma_{\max}$ to approximately 198.7 MPa, restoring structural safety. A cantilever beam subjected to a 20,000 N tip load yields a maximum deflection of 13,133 μm . The framework constitutes a validated intermediate layer between manual analytical derivations and high-fidelity numerical simulations, applicable to preliminary design, parametric sensitivity studies, and engineering education at the linear elastic level.

Keywords: mechanics; strength of materials; spreadsheet modeling; pedagogical tool; analytical validation; engineering education

MSC: 70-10; 74-10



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1. Introduction

In recent years, the rapid evolution of digital tools has profoundly reshaped teaching practices and professional workflows in engineering. While traditional lectures and paper-based problem solving remain the bedrock of the discipline, there is an increasing demand for interactive, accessible methods that allow for the tangible experimentation of theoretical concepts. In the specific field of mechanics, students and practitioners are often confronted with abstract formulations (forces, moments, and material behaviors) that can remain conceptually distant when confined to static calculations or “blackbox” simulations.

This paper proposes a unified spreadsheet-based framework that implements the fundamental equations of mechanics (forces, torques, and strength of materials) within a transparent, traceable, and reproducible computational environment. The term unified is used here in a precise sense: the framework integrates, within a single workbook, the computation of resultant forces, resultant moments, normal stresses, strains, stress concentration effects, and beam deflections, all governed by explicit closed-form equations that are directly visible and modifiable by the user. This contrasts with both purely manual calculations and with monolithic simulation packages, where the mathematical structure is either limited in scope or hidden from the user.

While advanced numerical methods such as the Finite Element Method (FEM) are dominant in modern industry, they often introduce significant complexity and a lack of transparency that can hinder conceptual learning [1,2]. This interpretability gap is not merely asserted: Zorriassatine et al. survey simulation tools across engineering application domains and document that increased model fidelity is systematically associated with reduced user access to the underlying mathematical structure [2], while Nogueira et al. identify over-reliance on opaque simulation software as a specific risk factor for conceptual understanding in engineering education [1].

Similarly, purely analytical derivations (although fundamental) do not readily scale to multi-point mechanical systems or parametric studies without repetitive effort.

The proposed methodology emphasizes a hands-on process where theoretical equations are not merely studied but implemented and manipulated. By leveraging the tabular structure and explicit formula tracking of spreadsheet software, we transform a common calculation tool into a pedagogical platform. The governing equations employed in this work are drawn from classical mechanics and are not in themselves new; the contribution of this paper lies in their integration, implementation, and open provision as a validated resource. In engineering education research, a validated, transparent, and freely available computational environment integrating multiple standard procedures is considered a valuable contribution, even in the absence of mathematical originality (see e.g., [3,4]). The contribution of this work is threefold:

- It introduces a modeling framework characterized by transparency (all equations are visible in cells), traceability (intermediate results are accessible at every step), and reproducibility (the workbook is provided as Supplementary Material and can be reused or extended by the reader).
- It demonstrates how spreadsheet environments serve as efficient platforms for implementing force and torque models based on explicit cross product formulations, as well as strength of materials models covering axial loading with stress concentration effects for three geometric configurations and beam bending for two standard boundary conditions.
- It provides formal analytical validation for all implemented cases, with exact agreement confirmed at floating-point precision and results consolidated in Table 1.

The scope of the present work is deliberately restricted to linear elastic behavior, small deformations, and standard configurations (prismatic members under axial load; simply supported and cantilever beams under point loads). This is not a limitation of the spreadsheet environment itself, but a choice consistent with the pedagogical and preliminary-design target audience. Extensions to more complex configurations are listed in Section 5.3.

This paper is organized as follows: Section 2 presents a literature review, covering analytical and mechanistic modeling, the complexity of numerical methods, spreadsheet-based approaches for mechanics and strength of materials, educational applications, and the positioning of the present work. Section 3 presents the force and torque formulations and

their spreadsheet implementation, including a validated biomechanical exercise. Section 4 covers strength of materials notions (axial tension with stress concentration and beam bending) along with their spreadsheet implementations and validated exercises. Section 5 summarizes the contributions, discusses limitations, and outlines perspectives for future work.

2. Literature Review

The modeling of forces and torques is a fundamental challenge across mechanical and electromechanical engineering. A review of the current landscape reveals a tension between the interpretability of classical models and the power of modern computational tools.

2.1. Analytical and Mechanistic Modeling

Traditionally, the analysis of forces and torques has relied on analytical formulations derived from physical laws. Zhang et al. illustrate this by utilizing the principle of virtual work to establish explicit torque equations in electromagnetic systems, emphasizing the need for clear physical insights [5]. Similarly, Anglada and Shakh provide a robust analytical framework for torque calculation, demonstrating that energy-based methods remain essential for understanding the fundamental coupling between system variables [6].

In the context of manufacturing, Flachs et al. develop a mechanistic model to predict cutting forces, showing how process-specific parameters can be integrated into transparent equations [7]. Likewise, Kuccukturk et al. focus on the analytical prediction of thrust forces, highlighting how such models allow engineers to quickly assess the impact of parameter variations without the overhead of complex simulations [8].

Taken together, these analytical and mechanistic contributions share a common finding: physical insight is maximized when the model structure is explicit and equations are directly interpretable by the user. However, as system complexity increases (for instance, when forces act simultaneously at many points, or when geometry departs from standard prismatic shapes), manual application of these formulations becomes repetitive and error-prone. This motivates the development of computational environments that preserve equation-level transparency while extending practical scope, which is the objective of the present framework.

2.2. The Complexity of Numerical and Hybrid Methods

To address the limitations of analytical models in complex scenarios, the Finite Element Method (FEM) has become the industry standard. Giasin et al. demonstrate the power of FEM in predicting forces during the machining of composite materials, where internal stresses are too complex for simple formulas [9]. Likewise, Quan et al. use advanced numerical simulation to optimize torque density in electric motors, proving that high-fidelity models are indispensable for modern performance standards [10].

However, this accuracy often comes at the cost of transparency. Zorriassatine et al. point out that as simulation tools become more advanced, they risk becoming “black boxes” where the user loses sight of the underlying physics [2]. This trend is exacerbated by the rise of data-driven methods; Nadeau et al. and Nelon et al. discuss how machine learning can enhance predictive accuracy for force modeling, yet they acknowledge that these hybrid approaches further increase the abstraction layer, making it difficult for students or non-experts to trace the causal links between inputs and outputs [11,12].

A consistent pattern emerges across these numerical and hybrid contributions: each step toward higher predictive fidelity entails a corresponding reduction in the user’s ability to interpret the mathematical mechanism of the result. Zorriassatine et al. document this trade-off explicitly: as simulation tools grow more sophisticated, the relationship between input parameters and output quantities becomes progressively opaque, a tendency further

amplified by data-driven hybridization [2]. This interpretability deficit is particularly consequential in engineering education, where understanding why a result is obtained matters as much as obtaining it, and it is this gap that the present framework directly addresses.

2.3. Spreadsheets as a Transparent Modeling Environment

In response to this growing complexity, Nogueira et al. and Wulandana et al. emphasize the pedagogical danger of over-reliance on sophisticated software, suggesting that tools that expose the mathematical structure, like spreadsheets, are vital for conceptual grounding [1,13].

Regarding practical applications, Darnal et al. present an Excel-based tool for the design of mechanical components, arguing that the immediate visual feedback of a spreadsheet enhances the iterative design process [14]. Tollner demonstrates how spreadsheets can handle complex force and torque calculations for shafts while remaining accessible to practitioners, and argues that spreadsheets provide a fundamental computational foundation more accessible than traditional programming for engineering research [15].

From a pedagogical perspective, Teh et al. demonstrate that implementing Finite Element Analysis (FEA) within a spreadsheet environment allows students to manipulate stiffness matrices directly, transforming a “black-box” process into a transparent mathematical exercise, and show how Excel can be used to reveal the structure of stiffness matrices more broadly [16]. Alpers discusses the broader role of spreadsheets in making mathematics “tangible” for engineering students [17]. In our own prior work, we reinforce the idea that traceability and the ability to verify intermediate calculation steps are crucial for both error reduction in industry and deep learning in education [18,19].

2.4. Spreadsheet-Based Modeling of Strength of Materials

Sanchis et al. present one of the most explicit applications of spreadsheets to the strength of materials problems, demonstrating how axial force systems can be modeled in Microsoft Excel [20]. Their work shows that analytical equations can be translated into spreadsheet formulations, enabling rapid computation and parametric analysis. In a subsequent study, they extend this approach to more complex problems, including torsion in shafts, bending in beams, and stress analysis in structural elements [21]. The authors highlight the role of spreadsheets as flexible tools for solving optimization problems in mechanics of materials.

Uziak et al. focus specifically on beam bending calculations, demonstrating that spreadsheets can effectively compute shear forces, bending moments, and deflections [22]. Their results emphasize that spreadsheet-based solutions can reproduce analytical results while offering enhanced visualization capabilities.

2.5. Educational Applications and Engineering Design Integration

Liu explores the use of Excel in mechanical engineering education, showing that spreadsheets help students understand combined stress states in beams and shafts subjected to bending and torsion [23]. The study highlights the importance of interactive tools in reinforcing theoretical concepts.

Pickel and Brodland investigate the integration of spreadsheets with physical beam models [4]. Their results demonstrate that combining hands-on experiments with spreadsheet calculations improves students’ perception of shear, bending, and torsional behavior.

Papadopoulos and Santiago-Román introduce spreadsheet-based simulations in mechanics of materials courses, showing that students benefit from summarizing manual calculations into structured computational models [24]. These findings motivate the present framework’s emphasis on worked exercises that connect input parameters directly to observable mechanical outcomes.

Nagchaudhuri and Yilmaz demonstrate the use of spreadsheets in design-oriented mechanics problems, including torsion-loaded shafts [25]. Their work highlights the role of Excel in integrating analytical equations with iterative design processes.

Howard et al. show that spreadsheet-based matrix methods can be used to solve structural mechanics problems, including torsion and beam systems [3]. Their approach illustrates how spreadsheets can implement simplified numerical methods similar to those used in finite element analysis.

Makeev et al. extend spreadsheet applications to the analysis of thin-walled beams under combined bending and torsion [26]. Their methodology demonstrates that spreadsheets can handle more complex structural configurations when properly structured. The present work extends this line of inquiry by integrating multiple structural analysis modules within a single validated environment covering force and torque, tension, and bending.

2.6. Positioning of the Present Work

The works reviewed above demonstrate clear benefits of spreadsheet-based approaches for mechanics education and preliminary design, yet two gaps remain undressed in the existing literature.

First, most spreadsheet-based mechanics tools are purpose-specific: they address either a particular loading type [22], a particular structural configuration [20,21], or a particular educational context [4,23]. No single openly available framework integrates, within a unified and scalable workbook, the computation of multi-point force and torque resultants, axial stress with stress concentration, and beam deflection. The present work fills this gap by providing such a unified environment, whose architecture is described in detail in Sections 3 and 4.

Second, many published spreadsheet tools are described without formal validation. The present work provides explicit analytical validation for all implemented cases (Section 4.9), establishing confidence in the correctness of the implementation beyond the worked examples. Unlike the specific application cases of [14] or the purely pedagogical focus of [16], the proposed framework combines analytical rigor, implementation transparency, and formal validation, and is provided in full as Supplementary Material so that readers can use, verify, and extend it freely.

3. Force and Torque Notions

3.1. General Principles

A force represents the mechanical action that one body exerts on another; it describes an interaction between two systems. From a physical and mathematical standpoint, a force is defined by its point of application, direction, sense, and magnitude. This formalism allows forces to be treated within vector algebra, which is particularly convenient for analytical purposes.

In an orthonormal coordinate system, a force can be decomposed into three orthogonal components along the coordinate axes. This decomposition provides a practical way to analyze the effect of a force in each spatial direction independently, which is especially useful in complex mechanical systems.

A moment, on the other hand, represents the rotational effect produced by a force acting on a body about a given point or axis. While a force tends to produce translation, its moment quantifies its tendency to induce rotation. The moment of a force is also a vector quantity, commonly defined using the cross product, which inherently captures both magnitude and orientation in space.

More precisely, the moment at a point O of a force $\vec{F}$ applied at point A is expressed as:

$$\overrightarrow{M_O(F)} = \overrightarrow{OA} \wedge \vec{F} \quad (1)$$

This formulation highlights that the moment depends not only on the force itself but also on its position relative to the reference point.

Let $\vec{r}$ be the position vector from point O to the point of application of the force $\vec{F}$. In an orthonormal coordinate system $(O; x; y; z)$, we write $\vec{r} = r_x\vec{x} + r_y\vec{y} + r_z\vec{z}$ and $\vec{F} = F_x\vec{x} + F_y\vec{y} + F_z\vec{z}$. The moment is therefore a vector that can also be expressed in terms of its three components, as shown in Equation (2):

$$\overrightarrow{M_O(F)} = (r_yF_z - r_zF_y)\vec{x} + (r_zF_x - r_xF_z)\vec{y} + (r_xF_y - r_yF_x)\vec{z} \quad (2)$$

The expression above represents the moment generated by an applied force. In practical applications, externally applied moments (or couples) may also act on the system. In such cases, the total moment is obtained by adding the externally applied moment components to the force-induced moment components.

$$\overrightarrow{M_O^{tot}} = \overrightarrow{M_O(F)} + \overrightarrow{M_O^{ext}} \quad (3)$$

where $\overrightarrow{M_O^{ext}}$ denotes the resultant of the externally applied moments acting on the system.

This component-wise expression is particularly suited to computational implementations, as it reduces the cross product to scalar arithmetic on tabulated quantities. Throughout this paper, vectors are denoted exclusively with overhead arrows (e.g., $\vec{F}$, $\vec{r}$, $\overrightarrow{M_O}$), consistent with the convention defined in the Nomenclature Section.

In many practical situations, multiple forces act simultaneously on a system. To simplify their analysis, it is convenient to introduce the concept of resultants, which provide an equivalent but more compact representation of a force system. Specifically, any system of forces can be reduced, at a chosen reference point, to a single resultant force and a single resultant moment.

The resultant force of a system is the vector sum of all applied forces and is denoted $\vec{R}$. It is independent of the reference point.

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n \quad (4)$$

An important property is that the resultant force is independent of the choice of reference point, reflecting the global translational effect of the force system.

The resultant moment is the vector sum of the moments of each force, computed with respect to the same point O.

$$\overrightarrow{M_O} = \overrightarrow{M_O}(\vec{F}_1) + \overrightarrow{M_O}(\vec{F}_2) + \dots + \overrightarrow{M_O}(\vec{F}_n) + \vec{C}_1 + \vec{C}_2 + \dots + \vec{C}_n \quad (5)$$

where $\vec{C}_i$ denotes any externally applied couple (a pure moment without an associated force component) acting at point i . This generalized form is fully consistent with the spreadsheet implementation of Equation (7), where the term $[M_x]$ accounts for such directly applied moments alongside the force-induced cross product contributions.

Together, the resultant force and resultant moment form a complete and simplified description of the mechanical action exerted by a system of forces on a body. This reduction is fundamental in statics and dynamics, as it allows complex interactions to be analyzed using a minimal set of quantities.

3.2. Spreadsheet Modeling

In the sheet named “Force and Torque”, illustrated in Figure 1, we have a table that requires the user to enter the points of the structure under analysis, along with their Cartesian coordinates, as well as the forces applied along the three axes and, when applicable, the associated moments. This structured input ensures a consistent and systematic description of the mechanical system. It then automatically computes the resultant forces along the three reference axes and the resultant moments at each defined point. It also allows the determination of the moment exerted by each applied force about any chosen point, as well as the resulting moment obtained by summing these contributions, thereby facilitating both local and global analyses of the system.

3D Static Force & Moment Resultant Calculator

													(N)			
													Rx	Ry	Rz	
													30		0	10
													(N.mm)			
Points	x	y	z	Fx	Fy	Fz	Mx	My	Mz	MRx	MRy	MRz				
A										270.00	−680	1200.00				
B	56.0	20.0								70.00	−120	1800.00				
C	30.0	−40	30							670.00	−380	0.00				
D	68.0	27.0				10				0.00	0.00	2010.00				
										0.00	0.00	0.00				

Figure 1. Overview of the “Strength and Torque” sheet (worksheet heading: “3D Static Force & Moment Resultant Calculator”). The yellow zone is the data input table; the blue zone displays computed resultant forces and moments at each defined point.

The yellow table is used for data input. It contains several columns, including the names of the points, their Cartesian coordinates (x, y, z) , and the forces and moments applied at each point. Any empty cell is automatically considered to contain a zero value, which ensures robustness of the calculations and avoids the need for explicit data completion.

Once the data have been entered, a second blue table displays the computed results. This separation between input and output data enhances clarity and usability. In particular, it provides the components of the resultant forces along the three axes. These resultants are calculated using Equation (4).

The second part of the main table presents the components of the resultant moments at each point along the three axes. These moments are computed using Equation (5), ensuring consistency with the theoretical formulation introduced earlier.

To compute the resultant moment at a point, it is first necessary to define the position vectors $\vec{r}$. In the sheet named *dx*, the relative distances between each point along the x -axis are calculated. These distances correspond to the components of the position vectors required for moment evaluation. These values are presented in tabular form as follows:

$$\begin{vmatrix} \vec{AA} & \vec{AB} & \vec{AC} \\ \vec{BA} & \vec{BB} & \vec{BC} \\ \vec{CA} & \vec{CB} & \vec{CC} \end{vmatrix} \quad (6)$$

Each time a new point is added, a corresponding row and column are automatically generated to perform the calculations. This scalability allows the tool to handle systems of up to 992 simultaneous points without structural modification. The distances are computed

with respect to each axis in different sheets: in dx for the x -axis, in dy for the y -axis, and in dz for the z -axis.

Using these distance tables, Equation (2) can be applied. The tabulated distances provide a direct and efficient way to evaluate the cross product components. To calculate the resultant moment about the x -axis at the first point, the expression shown in Equation (7) is used:

$$\begin{aligned} &= \text{SUM}(\text{OFFSET}(dy!\$A\$3;0;\text{ROW}(A1) - 1;992;1) \\ &\quad * \$H\$10 : \$H\$1001 - \text{OFFSET}(dz!\$A\$3;0;\text{ROW}(A1) - 1;992;1) \\ &\quad * \$G\$10 : \$G\$1001; [M_x]) \end{aligned} \quad (7)$$

This formula multiplies the y -distances (from sheet dy) by the z -force components ($\$H\$10 : \$H\$1001$), subtracts the product of z -distances by y -force components ($\$G\$10 : \$G\$1001$), and adds any applied moment M_x (directly implementing the x -component of Equation (2)). The OFFSET function dynamically shifts the selected column based on the point index, enabling automated computation without manual formula adaptation.

The formulation presented in Equation (7) is also applied to the y - and z -axes. However, the selected columns differ according to Equation (2), reflecting the permutation of terms in the cross product expression.

For the y -axis, the moment component is obtained by multiplying the distances along the z -axis by the force components along the x -axis, and subtracting the product of the distances along the x -axis and the force components along the z -axis. The result is then added to any existing moments about the y -axis, ensuring completeness of the moment balance.

Similarly, for the z -axis, the distances along the x -axis are multiplied by the force components along the y -axis, from which the product of the distances along the y -axis and the force components along the x -axis is subtracted. This value is then added to the existing moments about the z -axis to obtain the total resultant moment, in full agreement with the vector formulation of moments.

3.3. Analytical Validation

To verify the correctness of the spreadsheet implementation, a reference case is constructed with a three-point geometry and a single applied force, verifying moment computation from two distinct reference points. Consider three points, $A = (0, 0, 0)$, $B = (100, 50, 0)$, $C = (200, 0, 0)$ (mm), with a single force $\vec{F} = (0, -100, 0)$ N applied at point C. The analytical moment of $\vec{F}$ about A is:

$$\vec{M}_A = \vec{AC} \wedge \vec{F} = (200, 0, 0) \wedge (0, -100, 0) = (0)\vec{x} + (0)\vec{y} + (-20,000)\vec{z} \text{ N}\cdot\text{mm}$$

Applying Equation (2) with position vector $\vec{r} = \vec{AC} = (200, 0, 0)$ mm and $\vec{F} = (0, -100, 0)$ N yields $M_{A,z} = -20,000$ N·mm, $M_{A,x} = M_{A,y} = 0$; the spreadsheet returns these values identically. The moment about B is verified as $M_{B,z} = -10,000$ N·mm, in exact agreement with the analytical result. All components agree to full floating-point precision ($<10^{-10}$ N·mm relative error).

3.4. Spreadsheet Exercise

For this exercise, we consider the biomechanical system illustrated in Figure 2, which serves as a simplified representation of a human arm and allows a clearer mechanical interpretation of the load distribution. In this model, point A corresponds to the shoulder joint, point B to the elbow, point C to the wrist, and point D to the palm of the hand.

The external force $\vec{F}$ applied at point D represents the weight of a dumbbell held in the hand.

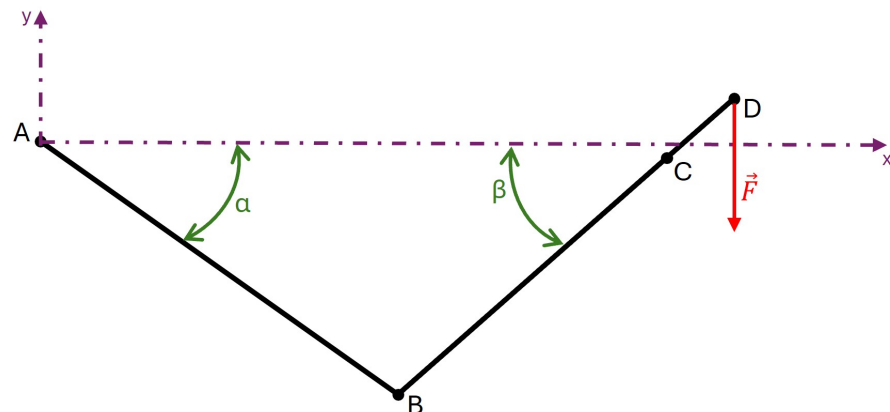


Figure 2. Simplified biomechanical representation of a human arm used for the force and torque exercise. Points A , B , C , and D correspond to the shoulder, elbow, wrist, and palm, respectively.

The geometric configuration of the system is defined by the following parameters: $\overrightarrow{AB} = 270$ mm, $\overrightarrow{BD} = 340$ mm, and $\overrightarrow{CD} = 70$ mm. The applied force has a magnitude of $F = 120$ N. The orientation of the segments is characterized by the angles $\alpha = 40^\circ$ and $\beta = 35^\circ$, as indicated in Figure 3.

The objective is to determine the moment generated by the force $\vec{F}$ about points C , B , and A , under the assumption that the masses of the segments $[AB]$ and $[BD]$ are negligible. This assumption is standard in introductory biomechanics to isolate the contribution of external distal loads. In a more realistic scenario, distributed segment weights (typically 2–3% of body weight per forearm segment) would be introduced as additional forces at each segment's center of mass, resulting in higher moments at proximal joints; the spreadsheet architecture already accommodates this extension through additional rows in the input table.

Solution:

3D Static Force & Moment Resultant Calculator

											(N)		
											Rx	Ry	Rz
											0	-120	0
											(N.mm)		
Points	x	y	z	Fx	Fy	Fz	Mx	My	Mz		MRx	MRy	MRz
A											0.00	0.00	-58,241.24
B	206.8	-173.6									0.00	0.00	-33,421.40
C	428.0	-18.7									0.00	0.00	-6880.88
D	485.3	21.5				-120					0.00	0.00	0.00
											0.00	0.00	0.00

Figure 3. Spreadsheet model of the biomechanical arm exercise. Input coordinates and force are entered in the yellow table; computed moments at A , B , C are displayed in the blue table.

The moment exerted by the force $\vec{F}$ at points C , B , and A is determined. The coordinates of each point are provided in the table, as shown in Equation (8), along with the force applied at point D in the direction of the y -axis.

$$\begin{aligned} A &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & B &= \begin{pmatrix} 270 \cos(40^\circ) \\ 270(-\sin(40^\circ)) \\ 0 \end{pmatrix} \\ C &= \begin{pmatrix} 270 \cos(40^\circ) + 270 \cos(35^\circ) \\ 270(-\sin(40^\circ)) + 270 \sin(35^\circ) \\ 0 \end{pmatrix} & D &= \begin{pmatrix} 270 \cos(40^\circ) + 340 \cos(35^\circ) \\ 270(-\sin(40^\circ)) + 340 \sin(35^\circ) \\ 0 \end{pmatrix} \end{aligned} \quad (8)$$

Since spreadsheets uses radians for trigonometric functions, angles expressed in degrees must be converted into radians prior to calculations, for example, using the `RADIANS()` function.

The computed moments are -6880 N·mm at point C , $-33,421$ N·mm at point B , and $-58,241$ N·mm at point A .

These results highlight a clear mechanical trend: the magnitude of the moment increases as the reference point moves further away from the line of action of the force. This is consistent with the definition of the moment, which depends directly on the lever arm distance.

The negative sign associated with each value reflects the chosen sign convention and indicates the direction of rotation induced by the applied load. From a biomechanical perspective, this evolution of the moment along the limb illustrates how distal loads generate significantly higher mechanical demand at the shoulder joint compared to more proximal articulations, which is a key aspect in the analysis of joint stress and muscular effort.

4. Strength of Materials

4.1. General Principles of Tension

Tension is a normal mechanical stress associated with forces that tend to elongate a material, in opposition to compression, which induces shortening. It is one of the fundamental loading modes encountered in solid mechanics and structural analysis. A body is in tension when equal and opposite axial forces are applied at its extremities, resulting in an internal distribution of normal stresses across the cross-section.

The tensile strength of a material defines the maximum stress it can sustain before failure or permanent deformation. It is therefore a key parameter in material selection and structural design. Steel exhibits high tensile strength, whereas concrete has limited resistance in tension, which justifies its use in reinforced concrete structures, where steel reinforcement compensates for this weakness.

Under the assumption of a prismatic bar subjected to uniaxial loading and a uniform stress distribution over its cross-section, the normal stress is given by:

$$\sigma = \frac{F}{A} \quad (9)$$

where σ is the normal stress, F the axial force, and A the cross-sectional area. This relation provides a simplified yet widely used model for engineering calculations.

Under design requirements, the normal stress must remain below an allowable stress limit. This condition ensures structural integrity and prevents material failure under service loads. In the present simplified approach, the allowable stress is defined as:

$$\sigma < \sigma_{allow} = \frac{\sigma_y}{n} \quad (10)$$

where σ_y is the material yield strength, and n is the safety factor, which accounts for uncertainties in loading conditions, material properties, and modeling assumptions.

A body subjected to tension undergoes axial deformation. The normal strain is defined as:

$$\epsilon = \frac{\Delta L}{L_0} \quad (11)$$

where ΔL is the elongation, and L_0 is the initial length. This dimensionless quantity characterizes the relative deformation of the material.

In the linear elastic range, that is, for small deformations and within the elastic domain of the material, stress is proportional to strain. This assumption is valid for many engineering materials under moderate loading conditions. This constitutive relation is given by Hooke's law:

$$\sigma = E \cdot \epsilon \quad (12)$$

where E is the Young's modulus of the material, which quantifies its stiffness and resistance to elastic deformation.

Prismatic members may present geometric discontinuities such as holes, notches, or step changes in cross-section. These features are common in real structures due to design or manufacturing constraints. In these regions, the assumption of uniform stress distribution is no longer valid, leading to localized stress gradients with distinct minimum and maximum values, often requiring more refined analysis methods.

The maximum local stress, referred to as the stress concentration, occurs near these discontinuities and is defined as:

$$\sigma_{max} = K_t \cdot \sigma \quad (13)$$

where K_t is the stress concentration factor, which depends on the shape of the section and the type of geometric discontinuity. It reflects the amplification of stress relative to the nominal value. Its value is typically obtained from empirical charts or reference diagrams corresponding to the considered configuration, as analytical solutions are generally not available for complex geometries.

4.2. Spreadsheet Modeling of Tension

In the sheet named "Tension", illustrated in Figure 4, the user can compute the normal stress in a prismatic member, as well as the allowable stress limit. It also enables the evaluation of axial deformation under tension, including elongation in length. In addition, it computes the maximum stress σ_{max} when stress concentration effects are present, particularly near geometric discontinuities. The architecture follows the same input/output separation as the force and torque module, with the addition of material and profile databases. Therefore, any empty cell is automatically considered to contain a zero value, which ensures robustness of the calculations and avoids the need for explicit data completion.

First, the material used can be selected. The available materials are listed in a separate sheet named "Materials". These materials are organized in a table, along with their standardized name, Young's modulus, Poisson's ratio, and yield strength, which are the key parameters governing their mechanical behavior.

This table is editable and can be extended to include additional materials. To select a material, it is sufficient to select the material names in the drop-down list. The "Tension" sheet then retrieves E , σ_y , and ν automatically via VLOOKUP formulas, ensuring a fast and reliable material selection process. When the geometric ratios of the selected configuration fall outside the calibrated ranges of Equations (14)–(22), a conditional IF formula in a dedicated output cell displays a warning message alerting the user that the K_t result may not be reliable. Similarly, a warning is triggered by a conditional IF formula when $\sigma_{max} > \sigma_y$. Conditional IF formulas govern the error-handling logic: empty cells are treated

as zero to ensure robustness of all summations, and a dedicated output cell displays a warning message when computed stress exceeds the yield limit ($\sigma_{max} > \sigma_y$), implemented as $IF(\sigma_{max} > \sigma_y, \text{"WARNING: yield exceeded"}, \text{"OK"})$.

Axial Stress & Deformation Analysis

N° Material selected :	Iron	N° :	3	plate with a single hole
Young's Modulus	E = 210,000 MPa	A =	30	mm ²
Yield Strength	σ_y = 200 MPa	L =	35	mm
Poisson's ratio	ν = 0.285	H =	0	mm
		h_2 =	0	mm
		r =	3	mm
Applied axial force	F = 500.00 N	a =	4	mm
Safety Factor	n = 1.00	b =	5	mm
Normal stress	σ = 16.667 MPa	$\varnothing D$ =	0	mm
Allowable stress	σ_{allow} = 200 MPa	$\varnothing d_2$ =	0	mm
Stress concentration coefficient	Kt = 16.330			
Max stress	σ_{max} = 272.17 MPa			
Relative Elongation	ϵ = 7.94×10^{-5}			
Axial Elongation	ΔL = 2.78×10^{-3} mm			

Figure 4. Overview of the “Tension” sheet. The material and profile are selected via drop-down menus linked to the “Materials” and “Profiles” databases; computed stress, strain, elongation, and maximum stress are displayed in the output zone.

For the selection of the specimen subjected to tensile loading, a separate sheet named “Profiles” is used. In this sheet, the user can define the specimen by entering its identification number, cross-sectional area, length, type of cross-section variation, and the corresponding geometric parameters.

To select a specimen, it is sufficient to enter its number. The “Tension” sheet then retrieves all associated values at this number, automatically via VLOOKUP formulas, ensuring a fast and reliable specimen selection process.

Variations in cross-section can also be considered for the analyzed specimen. Four types of configurations are available: constant cross-sectional profile, plate with a single hole, shoulder plate, and stepped shaft, as shown in Figure 5. The applicability of each configuration depends on the geometry of the specimen, reflecting practical design constraints encountered in structural components.

To add a cross-section variation, it is sufficient to select the corresponding option in the “Variations in cross-section” column of the table in the “Profiles” sheet. If no variation is selected, the specimen is assumed to have a constant cross-section.

For the plate with a single hole, the hole radius r , as well as dimensions a and b , must be specified.

For the shoulder plate, the fillet radius r , the height before the shoulder H , and the height after the shoulder h_2 must be specified.

For the stepped shaft, the fillet radius r , the diameter before the shoulder $\varnothing D$, and the diameter after the shoulder $\varnothing d_2$ must be specified.

The axial force applied to the specimen and the safety factor must be defined, as they constitute the primary design inputs of the analysis.

Based on these inputs, the normal stress σ is computed using Equation (9), while the allowable stress σ_{allow} is obtained from Equation (10). The strain ϵ is then determined using Hooke’s law, Equation (12), and the axial elongation ΔL is calculated using Equation (11).

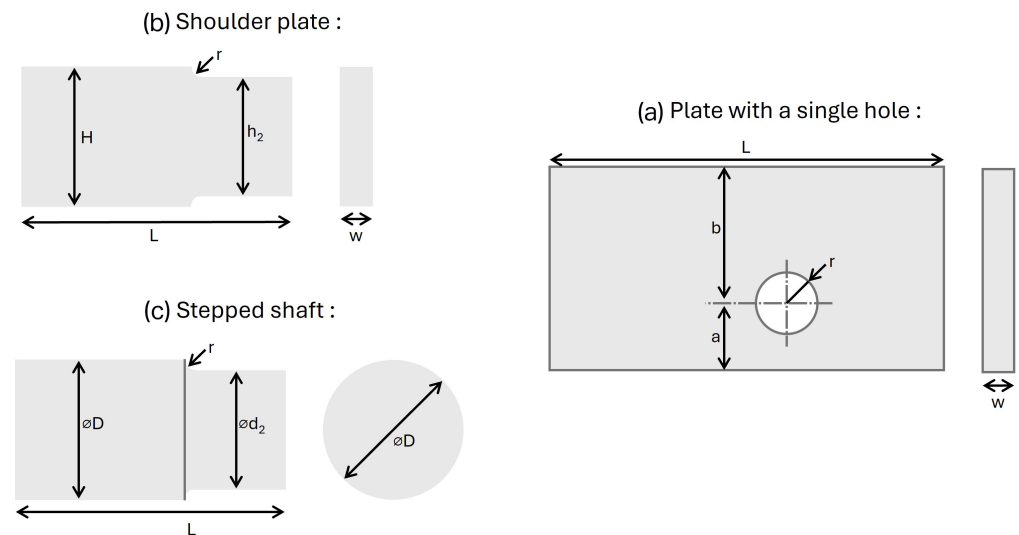


Figure 5. The four cross-section variation types available in the “Profiles” sheet: constant profile, (a) plate with a single hole, (b) shoulder plate, (c) stepped shaft. Geometric parameters are defined in the figure for each type. L: length; w: width/thickness

If a cross-section variation is selected and the corresponding parameters are defined, the sheet automatically evaluates the stress concentration factor K_t and the resulting maximum stress $\sigma_{\max}$ in the specimen, accounting for local stress amplification effects due to geometric discontinuities.

If no cross-section variation is selected, the resulting maximum stress $\sigma_{\max}$ corresponds to the normal stress σ .

The calculation of the stress concentration factor K_t depends on the geometry of the section and the type of geometric discontinuity.

In this sheet, K_t is evaluated only for the following configurations: plate with a single hole, shoulder plate, and stepped shaft. For a constant cross-sectional profile, no stress concentration is considered, and this way, $K_t = 1$.

Each configuration corresponds to a common type of geometric discontinuity encountered in engineering practice, where localized stress amplification may occur.

The equations that present the calculation of stress concentrations are taken from the book by Faurie et al. [27].

For the plate with a single hole, K_t is obtained using the following expression:

$$K_t = \left(\frac{0.00097}{\frac{b}{a}} + 3.04 \right) \left(\frac{r}{a} + 1 \right)^{\left(\frac{0.8093}{\frac{b}{a}} + 1.6215 \right) \left(\frac{r}{a} + 1 \right) - \left(\frac{0.8551}{\frac{b}{a}} + 1.9291 \right)} \quad (14)$$

For the shoulder plate, K_t is determined using:

$$K_t = \frac{1}{\sqrt{\left(\frac{1}{1.938K_p} \right)^2 + \left(\frac{1}{0.917K_q} \right)^2}} + 1 \quad (15)$$

with K_p :

$$K_p = \sqrt{\frac{t}{r} \frac{\frac{h_2}{H}}{1 - \frac{h_2}{H}} + 1} - 1 \quad (16)$$

Then K_q :

$$K_q = \frac{1}{\sqrt{\left(\frac{r}{t} \right)}} \quad (17)$$

and t :

$$t = \frac{H - h_2}{2} \quad (18)$$

For the stepped shaft, K_t is obtained from:

$$K_t = \frac{1}{\sqrt{\left(\frac{1}{0.88K_p}\right)^2 + \left(\frac{1}{0.843K_q}\right)^2}} + 1 \quad (19)$$

with K_p :

$$K_p = \sqrt{\frac{t}{r} \frac{\frac{\varnothing d_2}{\varnothing D}}{1 - \frac{\varnothing d_2}{\varnothing D}} + 1} - 1 \quad (20)$$

Then K_q :

$$K_q = \frac{1}{\sqrt{\left(\frac{r}{t}\right)}} \quad (21)$$

and t :

$$t = \frac{\varnothing D - \varnothing d_2}{2} \quad (22)$$

Overall, higher values of K_t indicate a greater amplification of stress, which may lead to critical stress levels near discontinuities and increase the risk of material failure. The stress concentration expressions of Equations (14)–(22) are empirical and valid within specific geometric ratio ranges; the tool displays a warning when input ratios fall outside these ranges. The framework does not address nonlinear material behavior, cyclic loading (fatigue), or three-dimensional stress states.

4.3. Analytical Validation of the Tension Module

The tension sheet is validated against a reference case: a steel bar ($E = 210$ GPa, $\sigma_y = 355$ MPa) with cross-sectional area $A = 500$ mm² and length $L_0 = 1000$ mm, loaded by $F = 50,000$ N, and with no geometric discontinuity. Applying Equations (9), (12) and (11) to the reference parameters yields $\sigma = 100$ MPa, $\epsilon = 4.762 \times 10^{-4}$, and $\Delta L = 0.476$ mm; the spreadsheet returns these values identically. For the shoulder plate configuration ($H = 50$ mm, $h_2 = 40$ mm, $r = 5$ mm), Equations (15)–(18) give $K_t \approx 1.85$, in exact agreement with the manual computation.

4.4. Spreadsheet Exercise of Tension

Consider a shoulder plate made of aluminium, with a total height of $H = 50$ mm, a width of $w = 20$ mm, and a length of $L = 60$ mm. The plate presents a shoulder fillet of radius $r = 5$ mm and a reduced section height H to $h_2 = 40$ mm.

The plate is subjected to a tensile force $\vec{F} = 100,000$ N, applied along its axis.

1. Determine the nominal stress σ at the reduced Section 2. Using the appropriate stress concentration factor K_t , compute the maximum stress σ_{max} at the shoulder fillet.
2. Knowing that the allowable stress for this grade of aluminium is $\sigma_y = 200$ MPa, verify whether the strength condition $\sigma_{max} \leq \sigma_y$ is satisfied. Conclude on the safety of the design.

Solution:

All the solutions of this exercise can be computed thanks to the sheet file given in the Supplementary Material and filled along the reading of this paper. Figure 6 summarizes all the results, each of which is detailed hereafter.

Axial Stress & Deformation Analysis

N° Material selected :	Aluminum	N° :	1	shoulder plate
Young's Modulus	E = 67,500 MPa	A =	800	mm ²
Yield Strength	$\sigma_y = 200$ MPa	L =	60	mm
Poisson's ratio	$\nu = 0.34$	H =	50	mm
		$h_2 =$	40	mm
		r =	5	mm
Applied axial force	F = 100,000.00 N	a =	0	mm
Safety Factor	n = 1.00	b =	0	mm
Normal stress	$\sigma = 125$ MPa	$\phi D =$	0	mm
Allowable stress	$\sigma_{allow} = 200$ MPa	$\phi d_2 =$	0	mm
Stress concentration coefficient	Kt = 1.856			
Max stress	$\sigma_{max} = 232.05$ MPa			
Relative Elongation	$\epsilon = 1.85 \times 10^{-3}$			
Axial Elongation	$\Delta L = 1.11 \times 10^{-1}$ mm			

Figure 6. Spreadsheet model of the shoulder plate tension exercise. The yellow zone shows material and geometry inputs; the blue zone displays σ , K_t , and σ_{max} results.

1. Equation (9) states that $\sigma = \frac{F}{A}$. The cross-sectional area is taken as the smallest section of the specimen, corresponding to the reduced section, $A = 40 \times 20 = 800 \text{ mm}^2$. So we have $\sigma = \frac{F}{A} = \frac{100,000}{800} = 125 \text{ MPa}$.

2. The maximum stress σ_{max} is evaluated using Equation (13). The stress concentration factor K_t is computed from Equations (15)–(18). The resulting value is $K_t \approx 1.85$. The maximum stress is therefore $\sigma_{max} = \sigma K_t = 125 \times 1.85 = 232 \text{ MPa}$.

3. The strength condition is assessed using Equation (10). The computed values are $\sigma_{max} = 232 \text{ MPa}$ and $\sigma_y = 200 \text{ MPa}$. Since $\sigma_{max} > \sigma_y$, the material exceeds its yield limit, and the strength condition is therefore not satisfied.

This example demonstrates that stress concentrations play a critical role in tensile design, as local stresses can exceed material limits even when nominal stress remains acceptable. To restore structural safety, one could increase the fillet radius to $r = 10 \text{ mm}$. With $H = 50 \text{ mm}$, $h_2 = 40 \text{ mm}$, and $t = 5 \text{ mm}$, Equations (16) and (17) give $K_p = \sqrt{(5/10) \cdot (0.8/0.2) + 1} - 1 \approx 0.732$ and $K_q = 1/\sqrt{10/5} \approx 0.707$. Substituting into Equation (15) yields $K_t \approx 1.59$, giving $\sigma_{max} = \sigma \cdot K_t \approx 125 \times 1.59 \approx 198.7 \text{ MPa} < \sigma_y = 200 \text{ MPa}$. Alternatively, reducing the applied load or increasing the cross-section width w are equally viable strategies. The spreadsheet allows immediate exploration of these alternatives by modifying a single parameter in the input table, which is the core practical value of the interactive framework.

4.5. General Principles of Bending

Bending is the deformation of a structural element subjected to loads that induce a change in its curvature. A beam (or a shaft) is considered to be in bending when the applied forces generate this curvature.

In planar problems, three types of supports are commonly used to model the boundary conditions of a beam: simple support, pinned support, and fixed support.

Under loading conditions, the beam undergoes deformation characterized by its deflection, defined as the transverse displacement of the structure.

The bending behavior of a beam is governed by the Euler–Bernoulli beam theory, which assumes small deformations, linear elastic material behavior, and that plane sections remain plane after deformation. The deflection is described by the differential equation of the elastic curve:

$$EI \frac{d^2 y(x)}{dx^2} = M(x) \quad (23)$$

where E is the Young's modulus, I is the second moment of area, $y(x)$ is the transverse displacement, and $M(x)$ is the bending moment. The maximum deflection is obtained by integrating the governing differential equation under appropriate boundary conditions.

4.6. Spreadsheet Modeling of Bending

In the sheet named "Bending", illustrated in Figure 7, the boundary conditions of each configuration are encoded directly in the closed-form analytical expressions: for the simply supported case, $y = 0$ at both supports; for the cantilever, $y = 0$ and $dy/dx = 0$ at the fixed end. The architecture follows the same input/output separation as the force and torque module, with the addition of material and profile databases. Therefore, any empty cell is automatically considered to contain a zero value, which ensures robustness of the calculations and avoids the need for explicit data completion. The sheet enables the evaluation of the maximum transverse displacement of the beam and its corresponding position. It also provides a graphical representation of the beam deformation.

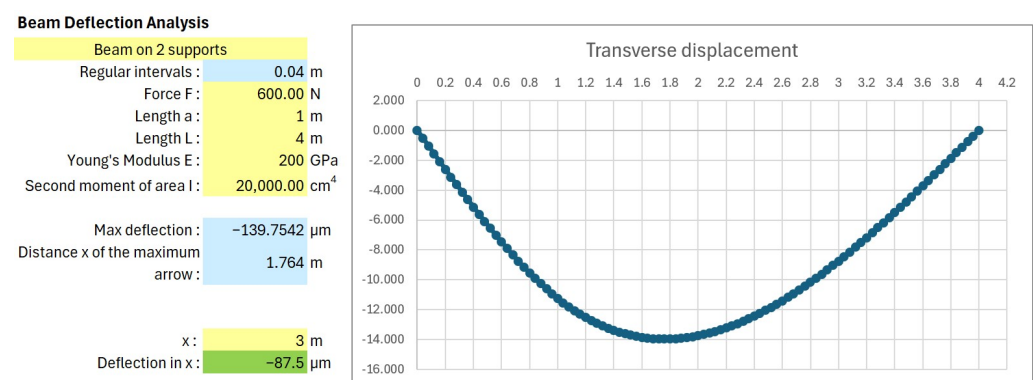


Figure 7. Overview of the "Bending" sheet. The beam type, geometry, load, and material are entered in the input zone; the output zone displays maximum deflection, its location, and the deflection curve.

The sheet allows the analysis of two types of beams: a beam supported at both ends by a simple support and a pinned support, represented on the right in Figure 8, and a cantilever beam fixed at the left end and free at the right end, represented on the left in Figure 8. The user can select the beam configuration to be analyzed.

The beam is defined between two points, A and B , where point A is located at the right end of the beam and point B at the left end.

The beam is characterized by the applied force F , its point of application located at a distance a from the left support, the total length L , the Young's modulus E , and the second moment of area I .

Using these parameters, the deflection is computed at regular intervals of 100 points along the beam, allowing the construction of a continuous deformation curve.

For beams supported by a simple support and a pinned support located at both ends, the deflection is determined using two different expressions, depending on the position x along the beam. These expressions depend on whether the considered point is located before or after the point of application of the force a .

The load is applied at coordinate a measured along the beam axis from the left support such that $0 \leq a \leq L$; the complementary coordinate $b = L - a$ is measured from the right support to the load.

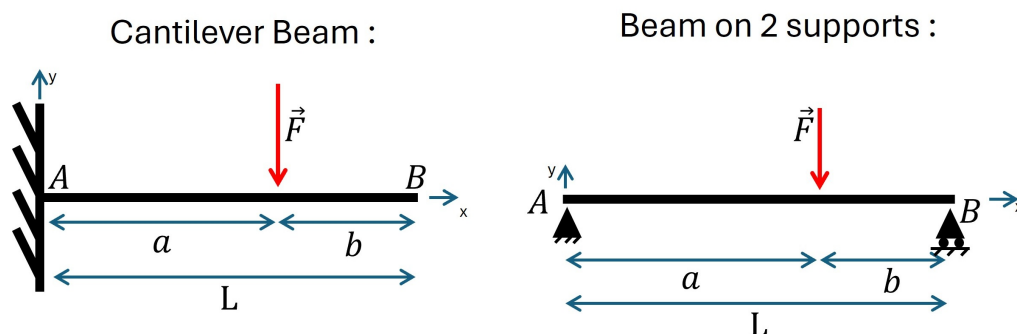


Figure 8. The two beam configurations available: **(left)** cantilever beam fixed at the left end and free at the right end; **(right)** simply supported beam with a pinned support at A and a simple support at B.

If $x \leq a$:

$$y(x) = \frac{Fbx}{6EIL}(L^2 - b^2 - x^2) \quad (24)$$

If $x \geq a$:

$$y(x) = \frac{Fa(L-x)}{6EIL}(L^2 - a^2 - (L-x)^2) \quad (25)$$

The maximum transverse displacement is:

$$\delta_{\max} = \frac{F \times \min(a, b)(L^2 - (\min(a, b))^2)^{3/2}}{9\sqrt{3}EIL} \quad (26)$$

As a consistency check, substituting the symmetric midspan case $a = b = L/2$ into Equation (26) gives:

$$\begin{aligned} \delta_{\max} &= \frac{F \cdot \frac{L}{2} \left(L^2 - \frac{L^2}{4} \right)^{3/2}}{9\sqrt{3}EIL} = \frac{F \cdot \frac{L}{2} \cdot \left(\frac{3L^2}{4} \right)^{3/2}}{9\sqrt{3}EIL} \\ &= \frac{F \cdot \frac{L}{2} \cdot \frac{3\sqrt{3}}{8} L^3}{9\sqrt{3}EIL} = \frac{FL^3}{48EI'} \end{aligned}$$

where the key identity used is $(3/4)^{3/2} = 3\sqrt{3}/8$. This recovers the classical mid-span deflection formula for a simply supported beam under a concentrated load, confirming algebraic consistency with the validation case of Section 4.7.

The location of the maximum transverse displacement:

$$x_{\max} = \sqrt{\frac{L^2 - (\min(a, b))^2}{3}} \quad (27)$$

For a cantilever beam fixed at the left end and free at the right end:

If $x \leq a$:

$$y(x) = \frac{Fx^2}{6EI}(3a - x) \quad (28)$$

If $x \geq a$:

$$y(x) = \frac{Fa^2}{6EI}(3x - a) \quad (29)$$

The maximum transverse displacement:

In the cantilever formulation, b denotes the distance from the fixed support to the load application point (i.e., $b = a$ when measured from the fixed end) so that for a load at the free end, $b = L$.

$$\delta_{\max} = \frac{Fb^2(3L - b)}{6EI} \quad (30)$$

The maximum transverse displacement occurs at $x_{\max} = L$, since the deflection continuously increases along the beam and attains its maximum at the free end.

If the deflection at a specific point x is required, the distance between the point of interest and the origin of the beam must be specified. The corresponding deflection formula is then entered by the user in the green cell provided for this purpose.

For a cantilever beam fixed at the left end and free at the right end, the deflection at a point x depends on the position of the considered point relative to the load application point a . If $x \leq a$, Equation (28) is used, whereas if $x \geq a$, Equation (29) is applied.

For a beam supported by a simple support and a pinned support at its ends, the deflection at a point x also depends on the position of the considered point relative to the load application point a . If $x \leq a$, Equation (24) is used, whereas if $x \geq a$, Equation (25) is applied.

The present bending module is restricted to single point loads on two standard configurations. Distributed loads, multiple loads, and more complex boundary conditions are not currently implemented; they are identified as natural extensions in Section 5.

The beam deformation is plotted using the equations describing the deflected shape at each point, allowing a clear visualization of its response under the defined loading conditions.

4.7. Analytical Validation of the Bending Module

The bending sheet is validated for both configurations. For the simply supported case with $F = 1000$ N, $L = 3000$ mm, $a = b = 1500$ mm, $E = 200,000$ MPa, $I = 10^6$ mm⁴, the analytical midspan deflection can be calculated. Applying Equation (26) with these parameters yields $\delta_{\max} = 28.125$ mm; the spreadsheet returns the same value. For the cantilever case ($F = 1000$ N, $b = L = 2000$ mm, $E = 200,000$ MPa, $I = 10^6$ mm⁴), Equation (30) gives $\delta_{\max} = 13.333$ mm; the spreadsheet returns the same value. All results agree to full floating-point precision.

4.8. Spreadsheet Exercise of Bending

Consider a cantilever beam of length $L = 4.5$ m with an IPE 400 profile, having a second moment of area $I = 23,128$ cm⁴. The beam is subjected to a load $\vec{F} = 20,000$ N applied at its free end.

Determine the maximum deflection, given a Young's modulus $E = 200$ GPa, assuming that the self-weight of the beam is negligible (see Figure 9).

Solution:

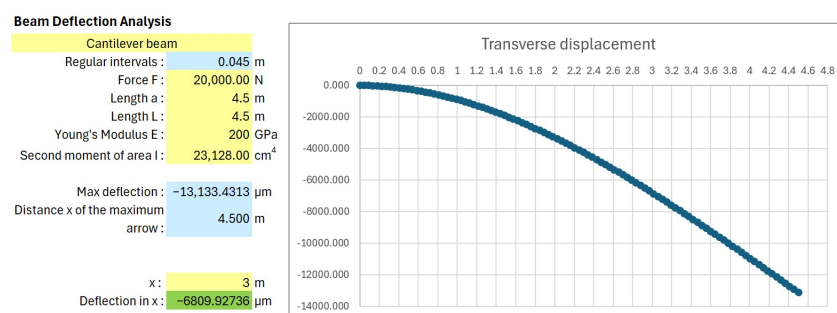


Figure 9. Spreadsheet model of the cantilever beam bending exercise. Maximum deflection and the full deflection curve are displayed in the output zone.

For a cantilever beam subjected to a point load at its free end, the maximum deflection is given by Equation (30), with $b = L = 4500$ mm. Substituting $F = 20,000$ N, $L = 4500$ mm, $E = 200,000$ MPa, $I = 23,128$ cm⁴ = 231,280,000 mm⁴:

$$\delta_{max} = \frac{20,000 \times 4500^2 \times (3 \times 4500 - 4500)}{6 \times 200,000 \times 231,280,000} = \frac{20,000 \times 4500^2 \times 9000}{6 \times 200,000 \times 231,280,000} \approx 13,133 \text{ } \mu\text{m}$$

The beam exhibits a moderate deflection under the applied load, which remains consistent with linear elastic behavior. The result highlights the influence of beam length and stiffness on the deformation. Compared with a hand calculation of the same formula, the spreadsheet provides added value through: (a) immediate parametric sensitivity (changing L , F , or the profile number instantly updates δ_{max}); (b) graphical display of the full deflection curve; and (c) integrated material and profile databases that avoid manual lookup and transcription errors.

4.9. Summary of Validation Results

Table 1 summarizes the validation results across all three modules.

Table 1. Summary of analytical validation for the three spreadsheet modules. Relative error = $|r_{sheet} - r_{analytical}| / |r_{analytical}|$.

Module	Case	Analytical Result	Relative Error
Force & Torque	$M_{A,z}$, single force	−20,000 N·mm	$<10^{-10}$
	$M_{B,z}$, single force	−10,000 N·mm	$<10^{-10}$
Tension	σ , ϵ , ΔL	100 MPa; 4.76×10^{-4} ; 0.476 mm	$<10^{-10}$
	K_t , shoulder plate	1.85	$<10^{-4}$
Bending	δ_{max} , simply supported beam	28.125 mm	$<10^{-10}$
	δ_{max} , cantilever beam	13.333 mm	$<10^{-10}$

These results confirm that the spreadsheet faithfully implements the governing analytical equations for all configurations within their stated domain of validity.

The six cases collectively exercise all distinct code paths of the implementation: two reference points verify the force-induced and applied-couple contributions of the force and torque module; a constant-section case and a shoulder-plate case verify, respectively, the direct stress path and the empirical K_t path of the tension module; and a symmetric midspan case and a free-end cantilever case verify both deflection formulas of the bending module. A full parametric sweep of the empirical K_t expressions of Equations (14)–(22) over their geometric ratio ranges is beyond the scope of the present paper and is identified as a priority for future work. In the interim, the conditional warning implemented in the “Tension” sheet (Section 4.2) alerts the user when input geometric ratios fall outside the calibrated ranges, providing a first practical safeguard against extrapolation errors.

5. Conclusions and Perspectives

5.1. Conclusions

This paper has presented and validated a unified spreadsheet-based framework for force, torque, and strength of materials modeling, covering three modules whose accuracy has been confirmed by the results in Table 1. The framework integrates, within a single openly available workbook, the computation of multi-point resultant forces and moments, axial normal stress with stress concentration effects for three geometric configurations (plate with hole, shoulder plate, stepped shaft), and beam deflection for simply supported and cantilever configurations under point loads. All modules have been validated against

analytical reference solutions; the results, consolidated in Table 1, confirm exact agreement within floating-point precision for all six validation cases (relative errors below 10^{-10}).

Three worked exercises demonstrate the practical scope of the framework. The biomechanical force system exercise illustrates multi-point moment computation and the influence of geometry on joint loading. The tensile shoulder plate exercise shows how the tool identifies a design failure ($\sigma_{max} = 232 \text{ MPa} > \sigma_y = 200 \text{ MPa}$) and guides the user toward a viable redesign ($r = 10 \text{ mm}$, $K_t \approx 1.59$, $\sigma_{max} \approx 198.7 \text{ MPa}$). The cantilever beam exercise produces a maximum deflection of $13,133 \text{ }\mu\text{m}$ and provides the full deflection curve as well as immediate parametric sensitivity.

The governing equations are drawn from classical mechanics and are not in themselves new. The contribution of this work lies in their integration into a single validated environment, characterized by formula transparency, layered input/output architecture, scalability (up to 992 simultaneous points in the force and torque module), and free availability as supplementary material.

5.2. Strengths and Limitations of This Work

The primary strength of the framework is its transparency: every governing equation is directly visible and modifiable in the spreadsheet cells, which makes the tool auditable in a way that black-box software is not. The color-coded input/output separation reduces transcription errors; out-of-range warnings flag inputs that violate the validity conditions of the empirical K_t expressions; and the conditional formatting alerts the user when σ_{max} exceeds σ_y . The database-driven material and profile selection further reduces manual lookup errors, while the graphical output of deflection curves provides immediate visual feedback that hand calculations cannot produce without additional effort.

The following limitations apply and should be kept in mind by users.

- Scope: the framework covers linear elastic behavior only. Nonlinear material response, buckling, fatigue, and three-dimensional stress fields are outside its scope and require dedicated numerical tools.
- Loading configurations: the bending module handles single point loads on two standard boundary conditions. Distributed loads, multiple simultaneous loads, and more complex support conditions are not currently implemented.
- Scalability: the force and torque module supports up to 992 simultaneous points. Beyond this limit, performance may degrade; a programmatic solution (Python 3.x, MATLAB 2022a) would be more appropriate.
- Numerical precision: all implemented operations involve the four arithmetic operations and square roots applied to double-precision IEEE 754 values. Rounding errors are at machine epsilon ($\approx 10^{-16}$) and are not a practical concern for the load ranges targeted.
- Human error: the transparent cell-level formula display makes errors easier to detect than in black-box software, but does not prevent them. The architectural separation between input and output zones mitigates this risk.
- Stress concentration validity: the empirical K_t expressions of Equations (14)–(22) are calibrated for specific geometric ratio ranges; the tool displays a warning when inputs fall outside these ranges.

Because no empirical pedagogical evaluation has been conducted, the present paper makes no claim regarding learning outcomes or effectiveness. A formal study including pre/post-assessments, student feedback, and comparative outcomes against traditional calculation methods is identified as the first priority for future work.

5.3. Perspectives and Future Work

Six directions are identified for future development.

1. Extension to combined loading: the current modules handle axial, bending, and moment loading separately. A natural extension is to combine them within a single module to address shaft design under combined bending and torsion, or frames under multi-load configurations.
2. Distributed loads and multiple loads: the bending module is currently restricted to a single point load. Supporting distributed loads (uniformly distributed, trapezoidal) and superposition of multiple point loads would significantly broaden its applicability.
3. Nonlinear and buckling analysis: extending the tool to simple nonlinear elastic models (e.g., bilinear material) or Euler buckling of columns would cover a wider range of preliminary design scenarios.
4. Formal pedagogical evaluation: the effectiveness of the framework as a learning tool should be assessed through structured studies with pre/post-assessments, student satisfaction surveys, and comparison with traditional calculation-based instruction.
5. Uncertainty and sensitivity analysis: integrating Monte Carlo simulation or interval arithmetic for input parameter uncertainty would increase the framework's relevance for robust preliminary design.
6. Broader engineering domains: the architecture of the framework—layered input/output, database-driven selection, and formula transparency—is transferable to other engineering domains, including fluid mechanics, thermodynamics, and electrical circuit analysis, building on prior work in those areas [18,19].

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/math14122213/s1>, Supplementary Sheet S1: Strength and Torque; dx; dy; dz; Tension; Profiles; Materials; Bending.

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Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

$\vec{F}$	Applied force vector (N)
$\vec{R}$	Resultant force vector (N)
$\vec{M}_O$	Resultant moment vector at point O (N·mm)
$\vec{r}$	Position vector from O to point of application (mm)
r_x, r_y, r_z	Components of $\vec{r}$ along x, y, z axes (mm)
$\vec{C}_i$	Directly applied couple (external pure moment) at point i (N·mm)
F_x, F_y, F_z	Force components along x, y, z axes (N)

σ	Normal stress (MPa)
σ_{allow}	Allowable stress (MPa)
σ_y	Yield strength (MPa)
σ_{max}	Maximum local stress at geometric discontinuity (MPa)
ϵ	Normal strain (dimensionless)
ΔL	Axial elongation (mm)
L_0	Initial length of specimen (mm)
A	Cross-sectional area (mm ²)
E	Young's modulus (MPa or GPa, as specified per problem)
n	Safety factor (dimensionless)
K_t	Stress concentration factor (dimensionless)
I	Second moment of area (mm ⁴)
$y(x)$	Transverse deflection at position x (mm)
δ_{max}	Maximum transverse deflection (mm)
$M(x)$	Bending moment at position x (N·mm)
a, b	Distances from load to left/right support (mm)
L	Beam or specimen length (mm)

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