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On the Unboundedness of the Number of Natural Solutions for a Parameter-Dependent System of Equations

Dostonjon Numonjonovich Barotov

Department of Mathematics and Data Analysis, Faculty of Information Technology and Big Data Analysis, Financial University Under the Government of the Russian Federation, 4-th Veshnyakovsky Passage, 4, 109456 Moscow, Russia; dnbarotov@fa.ru

Abstract

In this paper, we consider a system of 10 equations from the standpoint of the number of its natural solutions, containing a non-negative integer parameter n and describing the magic state of the corresponding special table of numbers. As a result of the study, it is constructively proven that, for each natural number m, there exist natural numbers n_m and s_m such that, for a non-negative integer parameter n equal to n_m, this system has at least 2^m solutions, and all ten coordinates of each of these solutions are s_m-digit natural numbers, with the first, ninth, and tenth coordinates in decimal notation being represented only by the digits 0, 8, and 9, and the d-th coordinate, $d \in \{2, 3, \ldots, 8\}$, being represented only by a single digit, equal to $(d-1)$. This result, which constructively confirms the unboundedness of the number of solutions of this system depending on a non-negative integer parameter n, strengthens some recently published results.

Keywords: unboundedness of the number of solutions of the system; system of equations in natural numbers; algorithm; magic table; mathematical puzzle; the number of occurrences of the digit d in a natural number n; non-algebraic Diophantine equation in natural numbers

MSC: 11D45; 11Z05; 91A68; 11A67; 11A63

1. Introduction

Recently, in [1,2], a system of 10 equations in natural numbers was considered, containing a parameter—a non-negative integer n, of the form

$$\begin{cases} x_0 = 1 + \text{count}(0, n) + \text{count}(0, x_0) + \text{count}(0, x_1) + \cdots + \text{count}(0, x_9) \\ x_1 = 1 + \text{count}(1, n) + \text{count}(1, x_0) + \text{count}(1, x_1) + \cdots + \text{count}(1, x_9) \\ \ldots \\ x_9 = 1 + \text{count}(9, n) + \text{count}(9, x_0) + \text{count}(9, x_1) + \cdots + \text{count}(9, x_9) \end{cases}, \quad (1)$$

where $\text{count}(d, k)$ is the number of occurrences of the digit $d \in \{0, 1, \ldots, 9\}$ in the decimal representation of the number $k \in \{0, 1, 2, \ldots\}$. For a given n, this System (1) is related to the puzzle of filling a table containing n of the form

n	0	1	2	3	4	5	6	7	8	9

(2)

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with natural numbers such that the $(d+1)$-th of them is equal to the total number of occurrences of the digit $d \in \{0,1,\ldots,9\}$ in the entire table. In other words, if $(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)$ is a solution of System (1), then the table of the form

n	0	1	2	3	4	5	6	7	8	9
	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9

(3)

filled with this vector has a magic state, namely, for any digit $d \in \{0,1,\ldots,9\}$ indicated in the first row of Table (3), the number x_d indicated in the second row of Table (3) and standing below it is equal to the number of occurrences of the digit d in the entire Table (3). This problem, given its connection with Tables (2) and (3), can serve, along with other interesting (in the author's opinion) works, in particular those presented in [3–10], as a kind of bridge between the world of professional mathematics and the general public. It is also worth noting that, in our opinion, this System (1) without the parameter n is quite similar in meaning to the problems presented in [11,12], and the presence of the parameter n makes it non-trivial and, given its connections with Table (3), even "attractive". Regarding solvability and the number of distinct natural solutions that System (1) can have, the paper [1] contains a result confirming that System (1) may have no solution for some value of the parameter n. It also contains results confirming that System (1) can have at least one solution, at least two solutions, and at least ten solutions for some values of the parameter n. The paper [2] contains a result that covers the previous results and confirms that System (1) can have at least twenty solutions for some value of the parameter n. However, these papers do not establish either the uniform boundedness of the number of natural solutions of System (1), which depends on the parameter n, for all non-negative integers n, or the unboundedness of the number of natural solutions of System (1), which depends on the parameter n.

In this paper, we determine how many different natural solutions System (1) can have, i.e., we resolve the mutually exclusive questions raised above. Namely, we prove, and constructively at that, that for every natural number m there exist natural numbers n_m and s_m such that, for parameter n equal to n_m, this System (1) has at least 2^m solutions, and all ten coordinates of each of these solutions are s_m-digit natural numbers, with the first, ninth, and tenth coordinates in decimal notation representable only by the digits 0, 8, and 9, and the d-th coordinate, $d \in \{2,3,\ldots,8\}$, representable only by one digit, equal to $(d-1)$.

2. Used Notations

Definition 1. *We say that a natural number a, whose decimal representation is equal to $\overline{a_{t-1}a_{t-2}\ldots a_1a_0}$, where $\overline{a_{t-1}a_{t-2}\ldots a_1a_0} = a_0 \cdot 10^0 + a_1 \cdot 10^1 + \ldots + a_{t-1} \cdot 10^{t-1}$, can be written in decimal representation only with the digits $d_1, d_2, \ldots, d_k \in \{0,1,\ldots,9\}$ if the set of its digits satisfies the condition $\{a_{t-1}, a_{t-2}, \ldots, a_0\} \subseteq \{d_1, d_2, \ldots, d_k\}$.*

Let $\mathrm{count}(d,k)$, as indicated above, be the number of occurrences of the digit $d \in \{0,1,\ldots,9\}$ in the decimal representation of a non-negative integer k.

Let $concat(a,b) = \overline{a_{p-1}\ldots a_1a_0b_{q-1}\ldots b_1b_0}$ be a natural number obtained by concatenating natural numbers a and b, whose representations in the decimal system of calculation are equal to $\overline{a_{p-1}\ldots a_1a_0}$ and $\overline{b_{q-1}\ldots b_1b_0}$, respectively, where p and q are two positive integers.

3. Results

Recently, in [2], it was constructively proved that the number of different natural solutions of the equation $x = \mathrm{count}(0,x) + n$, representable in decimal notation using no more than three given digits 0, 8, and 9, and having the same number of digits, can grow without bound with increasing n. We begin our presentation by extending the

above-mentioned idea, based on the addition of a prefix and corresponding to the scalar equation $x = \text{count}(0, x) + n$, to a system of three equations, namely, the formulation and constructive proof of the following theorem.

Theorem 1. *For each natural number k, there exist natural numbers a_k and b_k such that, for a parameter n equal to a_k, the system of three equations of the form*

$$\begin{cases} x_0 = 1 + \text{count}(0, n) + \text{count}(0, x_0) + \text{count}(0, x_8) + \text{count}(0, x_9) \\ x_8 = 1 + \text{count}(8, n) + \text{count}(8, x_0) + \text{count}(8, x_8) + \text{count}(8, x_9) \\ x_9 = 1 + \text{count}(9, n) + \text{count}(9, x_0) + \text{count}(9, x_8) + \text{count}(9, x_9) \end{cases} \tag{4}$$

has at least 2^k solutions, all three coordinates of each of them being b_k-digit natural numbers such that in decimal notation they can only be represented by the digits 0, 8, and 9.

Proof. We present the proof by induction on k.

1. Let $k = 1$. Then $a_1 = \overline{\underbrace{99\ldots9}_{79 \text{ times}}\,\underbrace{88\ldots8}_{76 \text{ times}}\,\underbrace{00\ldots0}_{76 \text{ times}}}$ and $b_1 = 2$, since it is easy to see that the vector $(x_0, x_8, x_9) = (80, 80, 80)$, whose coordinates are b_1-digit natural numbers written in decimal notation using only the digits 0, 8, and 9, is a solution to System (4) for $n = a_1$.

2. Let $k = l$. Then suppose that there exist natural numbers a_l, b_l and a set of 2^l vectors of the form

$$(x_0(1), x_8(1), x_9(1)), \quad (x_0(2), x_8(2), x_9(2)), \quad \ldots, \quad \left(x_0(2^l), x_8(2^l), x_9(2^l)\right), \tag{5}$$

where $x_i(j)$ is a natural b_l-digit number written in the decimal notation using only the digits 0, 8, and 9 for all $i \in \{0, 8, 9\}$ and for all $j \in \{1, 2, \ldots, 2^l\}$, each of which is a solution of System (4) for $n = a_l$.

3. Based on 1 and 2, it suffices to prove that each of the 2^{l+1} vectors of the form

$$(concat(p, x_0(1)), concat(p, x_8(1)), concat(p, x_9(1))),$$

$$(concat(p, x_0(2)), concat(p, x_8(2)), concat(p, x_9(2))), \quad \ldots,$$
$$\left(concat(p, x_0(2^l)), concat(p, x_8(2^l)), concat(p, x_9(2^l))\right),$$

$$(concat(p+1, x_0(1)), concat(p, x_8(1)), concat(p-1, x_9(1))),$$
$$(concat(p+1, x_0(2)), concat(p, x_8(2)), concat(p-1, x_9(2))), \quad \ldots,$$

$$\left(concat(p+1, x_0(2^l)), concat(p, x_8(2^l)), concat(p-1, x_9(2^l))\right), \tag{6}$$

where

$$p = \overline{8\,\underbrace{99\ldots9}_{10^{b_l} \text{ times}}} = 9 \cdot 10^{10^{b_l}} - 1, \tag{7}$$

is a solution of System (4) for

$$a_{l+1} = \overline{\underbrace{99\ldots9}_{r_9 \text{ times}}\,\underbrace{88\ldots8}_{r_8 \text{ times}}\,\underbrace{00\ldots0}_{r_0 \text{ times}}},$$

where

$$r_i = p \cdot 10^{b_l} - 3 \cdot \text{count}(i, p) + \text{count}(i, a_l),$$

and all three coordinates of each of which are a natural b_{l+1}-digit number, where $b_{l+1} = \left(10^{b_l} + 1 + b_l\right)$, written in decimal notation using only the digits 0, 8, and 9. Taking into account the equalities

$$p+1 = \overline{9\underbrace{00\ldots0}_{10^{b_l}\text{ times}}}, \quad p-1 = \overline{8\underbrace{99\ldots9}_{(10^{b_l}-1)\text{ times}}8},$$

which follow from (7), and (5), we obtain that all three coordinates of each of the vectors given in (6) are a natural b_{l+1}-digit number, written in decimal notation using only the digits 0, 8 and 9. It remains to show that each of the vectors given in (6) is a solution to System (4) for $n = a_{l+1}$. Indeed, taking into account (5) and the equality $\text{count}(d, concat(A, B)) = \text{count}(d, A) + \text{count}(d, B)$, where $d \in \{0, 1, ..., 9\}$, $A, B \in \mathbb{N}$, we obtain

$$(a) \quad concat(p, x_i(j)) - 1 - \text{count}(i, concat(p, x_0(j))) - \text{count}(i, concat(p, x_8(j))) -$$
$$- \text{count}(i, concat(p, x_9(j))) = p \cdot 10^{b_l} + x_i(j) - 1 - \text{count}(i, p) - \text{count}(i, x_0(j)) -$$
$$- \text{count}(i, p) - \text{count}(i, x_8(j)) - \text{count}(i, p) - \text{count}(i, x_9(j)) = p \cdot 10^{b_l} - 3 \cdot \text{count}(i, p) +$$
$$+ x_i(j) - 1 - \text{count}(i, x_0(j)) - \text{count}(i, x_8(j)) - \text{count}(i, x_9(j)) =$$
$$= p \cdot 10^{b_l} - 3 \cdot \text{count}(i, p) + \text{count}(i, a_l) \quad \forall i \in \{0, 8, 9\} \quad \text{and} \quad \forall j \in \{1, 2, \ldots, 2^l\};$$

$$(b) \quad concat(p+1, x_0(j)) - 1 - \text{count}(0, concat(p+1, x_0(j))) - \text{count}(0, concat(p, x_8(j))) -$$
$$- \text{count}(0, concat(p-1, x_9(j))) = (p+1) \cdot 10^{b_l} + x_0(j) - 1 - \text{count}(0, p+1) - \text{count}(0, x_0(j)) -$$
$$- \text{count}(0, p) - \text{count}(0, x_8(j)) - \text{count}(0, p-1) - \text{count}(0, x_9(j)) = p \cdot 10^{b_l} + 10^{b_l} -$$
$$- 10^{b_l} - 0 - 0 + x_0(j) - 1 - \text{count}(0, x_0(j)) - \text{count}(0, x_8(j)) - \text{count}(0, x_9(j)) =$$
$$= p \cdot 10^{b_l} + \text{count}(0, a_l) = \left(p \cdot 10^{b_l} - 3 \cdot \text{count}(i, p) + \text{count}(i, a_l)\right)\Big|_{i=0} \quad \forall j \in \{1, 2, \ldots, 2^l\};$$

$$(c) \quad concat(p, x_8(j)) - 1 - \text{count}(8, concat(p+1, x_0(j))) - \text{count}(8, concat(p, x_8(j))) -$$
$$- \text{count}(8, concat(p-1, x_9(j))) = p \cdot 10^{b_l} + x_8(j) - 1 - \text{count}(8, p+1) - \text{count}(8, x_0(j)) -$$
$$- \text{count}(8, p) - \text{count}(8, x_8(j)) - \text{count}(8, p-1) - \text{count}(8, x_9(j)) = p \cdot 10^{b_l} - 0 - 1 - 2 +$$
$$+ x_8(j) - 1 - \text{count}(8, x_0(j)) - \text{count}(8, x_8(j)) - \text{count}(8, x_9(j)) = p \cdot 10^{b_l} -$$
$$- 3 + \text{count}(8, a_l) = \left(p \cdot 10^{b_l} - 3 \cdot \text{count}(i, p) + \text{count}(i, a_l)\right)\Big|_{i=8} \quad \forall j \in \{1, 2, \ldots, 2^l\};$$

$$(d) \quad concat(p-1, x_9(j)) - 1 - \text{count}(9, concat(p+1, x_0(j))) - \text{count}(9, concat(p, x_8(j))) -$$
$$- \text{count}(9, concat(p-1, x_9(j))) = (p-1) \cdot 10^{b_l} + x_9(j) - 1 - \text{count}(9, p+1) - \text{count}(9, x_0(j)) -$$
$$- \text{count}(9, p) - \text{count}(9, x_8(j)) - \text{count}(9, p-1) - \text{count}(9, x_9(j)) = p \cdot 10^{b_l} - 10^{b_l} - 1 -$$
$$- 10^{b_l} - (10^{b_l} - 1) + x_9(j) - 1 - \text{count}(9, x_0(j)) - \text{count}(9, x_8(j)) - \text{count}(9, x_9(j)) = p \cdot 10^{b_l} -$$
$$- 3 \cdot 10^{b_l} + \text{count}(9, a_l) = \left(p \cdot 10^{b_l} - 3 \cdot \text{count}(i, p) + \text{count}(i, a_l)\right)\Big|_{i=9} \quad \forall j \in \{1, 2, \ldots, 2^l\}.$$

Theorem 1 is proven. □

The upper bound proved in Lemma 1 of [1] guarantees that, for a given/fixed n, the number of distinct natural solutions to System (1) is finite. The upper bound obtained from Lemma 1, proved in [1], grows without bound as n increases, yet, it does not guarantee that the number of distinct natural solutions to System (1) can grow indefinitely with n. Now, based on the proven Theorem 1, we formulate and prove the following main result of this paper, which guarantees the unboundedness of the number of distinct natural solutions of System (1).

Theorem 2. *For every natural number m, there exist natural numbers n_m and s_m such that, for parameter n equal to n_m, System (1) has at least 2^m different solutions, all ten coordinates of each of which are s_m-digit natural numbers, and the first, ninth, and tenth coordinates in decimal notation can be represented using only the digits 0, 8, and 9, and the d-th coordinate, $d \in \{2,3,\ldots,8\}$, can be represented using only one digit, equal to $(d-1)$.*

Proof. According to Theorem 1, we have that for a given arbitrary natural number m there exist natural numbers a_m, $b_m \geq 2$ and 2^m distinct three-dimensional vectors of the form

$$(x_0(1), x_8(1), x_9(1)), \quad (x_0(2), x_8(2), x_9(2)), \quad \ldots, \quad (x_0(2^m), x_8(2^m), x_9(2^m)) \tag{8}$$

such that, firstly, $x_i(j)$ is a natural s_m-digit number, where $s_m = b_m$, written in decimal notation using only the digits 0, 8 and 9 for all $i \in \{0,8,9\}$ and for all $j \in \{1,2,\ldots,2^m\}$, and secondly, each of the vectors is a solution of System (4) for $n = a_m$. We argue that each of the 2^m distinct ten-dimensional vectors of the form

$$(x_0(1), x_1(1), \ldots, x_9(1)), \quad (x_0(2), x_1(2), \ldots, x_9(2)), \quad \ldots, \quad (x_0(2^m), x_1(2^m), \ldots, x_9(2^m)), \tag{9}$$

where

$$x_d(j) = \underbrace{\overline{dd\ldots d}}_{s_m \text{ times}} \quad \forall j \in \{1,2,\ldots,2^m\}, \quad d \in \{1,2,3,4,5,6,7\},$$

obtained by a slight modification from the vectors given in (8), is a solution of System (1) for

$$n = n_m = \underbrace{\overline{999\ldots9}}_{r_9 \text{ times}}\,\underbrace{\overline{888\ldots8}}_{r_8 \text{ times}}\,\underbrace{\overline{777\ldots7}}_{r_7 \text{ times}}\ldots\underbrace{\overline{000\ldots0}}_{r_0 \text{ times}},$$

where

$$r_t = \begin{cases} \text{count}(t, a_m), & \text{if} \quad t \in \{0,8,9\} \\ \frac{t}{9} \cdot (10^{s_m} - 1) - s_m - 1, & \text{if} \quad t \in \{1,2,\ldots,7\} \end{cases}.$$

Indeed, in view of (8) and (9), we have

$$x_d(j) - 1 - \sum_{i=0}^{9} \text{count}(d, x_i(j)) = x_d(j) - 1 - \sum_{i \in \{1,2,3,4,5,6,7\}} \text{count}(d, x_i(j)) -$$

$$- \sum_{i \in \{0,8,9\}} \text{count}(d, x_i(j)) = x_d(j) - 1 - 0 - \sum_{i \in \{0,8,9\}} \text{count}(d, x_i(j)) = x_d(j) - 1 -$$

$$- \sum_{i \in \{0,8,9\}} \text{count}(d, x_i(j)) = \text{count}(d, a_m) \quad \forall j \in \{1,2,\ldots,2^m\} \quad \text{and} \quad \forall d \in \{0,8,9\},$$

$$x_d(j) - 1 - \sum_{i=0}^{9} \text{count}(d, x_i(j)) = x_d(j) - 1 - \text{count}(d, x_d(j)) - \sum_{\substack{i=0 \\ i \neq d}}^{9} \text{count}(d, x_i(j)) =$$

$$= x_d(j) - 1 - s_m - 0 = \frac{d}{9} \cdot (10^{s_m} - 1) - s_m - 1 \quad \forall j \in \{1,2,\ldots,2^m\} \quad \text{and} \quad \forall d \in \{1,2,\ldots,7\}.$$

Theorem 2 is proven. □

For clarity, we will show an illustrative numerical example for small m.

(0) The starting vector $(x_0(1), x_1(1), \ldots, x_9(1)) = (80, 11, 22, 33, 44, 55, 66, 77, 80, 80)$ will be the solution of System (1) for

$$n = \overline{\underbrace{999\ldots9}_{r_9(0)\text{ times}}\ \underbrace{888\ldots8}_{r_8(0)\text{ times}}\ \underbrace{777\ldots7}_{r_7(0)\text{ times}}\ldots\underbrace{000\ldots0}_{r_0(0)\text{ times}}},$$

where $(r_0(0), r_1(0), \ldots, r_9(0)) = (76, 8, 19, 30, 41, 52, 63, 74, 76, 79)$.

(1) Vectors

$$\left(concat(p, x_0(1)), \underbrace{111\ldots1}_{103\text{ times}}, \underbrace{222\ldots2}_{103\text{ times}}, \ldots, \underbrace{777\ldots7}_{103\text{ times}}, concat(p, x_8(1)), concat(p, x_9(1))\right)$$

and

$$\left(concat(p+1, x_0(1)), \underbrace{111\ldots1}_{103\text{ times}}, \underbrace{222\ldots2}_{103\text{ times}}, \ldots, \underbrace{777\ldots7}_{103\text{ times}}, concat(p, x_8(1)), concat(p-1, x_9(1))\right)$$

will be solutions of System (1) for

$$n = \overline{\underbrace{999\ldots9}_{r_9(1)\text{ times}}\ \underbrace{888\ldots8}_{r_8(1)\text{ times}}\ \underbrace{777\ldots7}_{r_7(1)\text{ times}}\ldots\underbrace{000\ldots0}_{r_0(1)\text{ times}}},$$

where $r_d(1) = \begin{cases} p \cdot 10^2 - 3 \cdot \text{count}(d, p) + r_d(0), \text{ if } d \in \{0, 8, 9\} \\ \frac{d}{9} \cdot \left(10^{101} - 1\right) \cdot 10^2 - 101 + r_d(0), \text{ if } d \in \{1, 2, ..., 7\} \end{cases}$, $p = 9 \cdot 10^{100} - 1$.

At the end of the illustration, we note that this process of doubling the number of solutions by increasing/expanding n, according to Theorems 1 and 2, we can repeat as many times as we like.

At the end of this paper we state a fact that follows directly from Theorem 2 and which is the answer to the problem, which is a modified version of the second problem given in [1].

Corollary 1. *For each natural number m there is a corresponding number N_m such that System (1) for $n = N_m$ will have at least m distinct solutions.*

4. Discussion

In view of the "tower-like" growth of the sequence $\{n_k\}_{k\in\mathbb{N}}$, for the k-th element of which the System (1), according to Theorem 2, has at least 2^k solutions, it is appropriate to note the following problem, similar to the problem posed for the equation $x + s(x) = n$ [13–17], as a problem for further research: find the minimum value of the parameter n for which System (1) has at least k different natural solutions. In addition, firstly, it is advisable to generalize the obtained results related to the decimal number system to other number systems, and, secondly, to look for applied or cryptographic problems, for example, from the field of finance or security, such as [18–22], and try to apply the applicable approaches and results presented in this article, in the mathematical description or in the asymptotic analysis of which. Thus, this study, on the one hand, fully and constructively resolves the question of the unbounded growth of the number of natural solutions of System (1), and on the other hand, opens up some directions for further research, in particular those mentioned above.

Also, for the sake of analogy and comparison, it should be noted that the problem considered in this work is somewhat similar in appeal and in name to the problem about a magic square [23–25]. However, the problem about a magic square is algebraic and linear, whereas the problem examined in this work is neither algebraic nor linear.

5. Conclusions

As a result of the study, it was constructively proven that, for every natural number k, there exist natural numbers a_k and b_k such that, for parameter n equal to a_k, System (4) has at least 2^k solutions. Moreover, all three coordinates of each of these solutions are b_k-digit natural numbers, which in decimal notation can only be represented by the digits 0, 8, and 9. It was also proven that, for every natural number m, there exist natural numbers n_m and s_m such that, for parameter n equal to n_m, System (1) has at least 2^m distinct solutions. Moreover, all ten coordinates of each of these solutions are s_m-digit natural numbers, with the first, ninth, and tenth coordinates in decimal notation represented only by the digits 0.8 and 9, and the d-th coordinate, $d \in \{2, 3, \ldots, 8\}$, can be represented by just one digit, equal to $(d-1)$. Thus, it was established that the number of natural solutions of System (1), depending on the integer parameter n, can grow without bound as n increases.

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