

Article

Identifiability, Sequentiality and Infinity

Jose Luis Menaldi 

Department of Mathematics, Wayne State University, Detroit, MI 48202, USA; menaldi@wayne.edu;
Tel.: +1-(313)-577-2479

Abstract

A definition of identifiable-sets is used with sequential analysis to establish a *realm of mathematics*. Within this imaginary world, a specific consonant between infinite sets and sequentiality is reached. This consonant allows some mathematical constructions to model pieces of the reality, based on dual philosophy and physics itself. There is an effort made to render this understandable for the scientific community.

Keywords: potential infinite; infinity; infinitesimal; sequential analysis; dynamical system; dynamic programming models

MSC: 03A10; 90C39; 03E99; 60J10; 93C55; 60J20

1. Introduction

The intention of this paper is to look at the ‘reasoning’ with an infinite set (see Sections 2 and 3) when pointing to an abstraction of a real object, e.g., a model in stochastic dynamical system. The why of stochastic aspect is hard to explain, other than as an error in the measurements, without considering any specific reason for this error. The dual philosophy, as used in physics (e.g., see Strømme [1] and references therein), gives us room to consider various viewpoints and in particular to discuss the identifiability of objects as used in mathematics, as a bridge between the physics realm and its counterpart in applied mathematics (e.g., see Section 4 as an example). Particularly, in Section 4.2, the fundamental assumptions of quantum physics are interpreted within the identified-realm, i.e., with waves being discrete and evolving deterministically via some discrete Schrödinger equation, as done within an imaginary computer having an unfathomed tiny-tiny positive mach-eps. This yields a large number of dynamic dimensions which can be used within philosophical interpretations of the whole creation.

Now, as a recognizable area, I interpret what my first mathematics teachers (A.B., J-L.L., and E.R.) told me, “one problem after another”, implying the continuation ‘and the theorem is solved’, which will be discussed later. In the identifiable-realm, the underlying philosophy is the *finite* (to define the static) and the *sequential* (to define the dynamic), which forces the concepts of infinite and infinitesimal to be at the boundary, as the unreachable guard keeping. As mentioned above, a typical example is to describe how this (math-imaginary realm) becomes a proper (and global) dynamical system, which can be (ideally) localized and studied, e.g., by means of finite Markov chains (see M.-Robin [2,3]). The reader finds a preliminary discussion at the beginning of each of the following sections, and Section 5 gives some conclusions. As a side note, the reader may choose, upon reading this article for the first time, to ignore parentheses and their content like ‘(…)’ as well as the details following a typical ‘e.g., = for example’ or ‘i.e., = in other words’, which may



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be considered as ‘footnotes’. In a second reading, the details following ‘e.g., or i.e.’ are added, and the third time, all is included. This will allow the reader to decide the level of effort used, and clearly, this should be adapted to previous knowledge on this subject for each reader. Some readers may find some repetition with similar phrasings of some key statements; this helps the rationality and intellect to better understand the focus of this paper, because initially, as a scientific, specially a mathematician, we are encircled by the ideas of infinity, infinitesimal and a continuum space-time, which are against the arguments in this article, and therefore, this makes a neutral reading impossible.

2. Math Conventions, Infinity, and Identifiable-Sets

The discussion in this part is deeply logical and tries to justify the choice of the identifiable-realm as the home of mathematically modeling real and imaginary construes. As explained in a math-philosophical manner, there are some drawbacks, e.g., the necessity of sequential definition for some of the constants and numerical values used in the model, since the set of real numbers as a whole is not accepted, due to the condition of having only a finite number of characteristics corresponding to any math object as elements of a set when set-theory is involved. This is not a problem when dealing with real or imaginary versions of observed object in the nature. Also, several advantages, e.g., an identification of sets and a searchability of elements (in a counting meaning as explained later), are possible.

This is simply translated in having denumerable (=infinite countable) sets as the boundary of our math understanding of modeling theory, with all the logic of math-power. Also, the *dynamic logic-error* (as later explained) is resolved in the sense that the concept of infinite (as never ending) is at the boundary of math understanding within a realm being identifiably and sequentially.

This allows for a global description of nature as a dynamical system where each component of the global system dynamic could be understood as pieces of inputs-transition-outputs within a time interval. However, their interactions are rather very, very complex. A local system dynamic could be viewed as deterministic transition with a very, very large logic matrix (=a stochastic matrix with only 0 and 1), and this is approachable by a finite Markov chain (after aggregating states). A brief discussion (as examples) is given later in Sections 4 and 5.

2.1. Some Math Conventions and Infinity

Each word such as *element*, *eventually*, *finite*, *infinite*, *objects*, *theorem*, *theory*, *set*, ... has a specific meaning in math, and even some composed nouns with ‘-’ (of other words) which intend to express a unity, e.g., *set-theory*, so that its reference is neat, clear, and easy. Thus *objects* are used in the most general way, including abstract and imaginary things, and all are within the *imaginary world of mathematics*, limited only by its rigorous logic. A *collection of objects* is very general, and we think about labeling or counting the objects of a collection as a *list of objects*, where the labeling process may have sub-labeling processes of several levels, in a way similar to Gödel’s encoding (or numbering) system. *Set* and *object* are used similarly, but *set* is restricted to the various axioms in the *set-theory* (in math). Anyway, the concept and use of finite sets are clear enough and need no more explanations, except mentioning that as a *set*, it is subjected to the axioms and logic of the basic set-theory.

The word *infinite* is usually an adjective, but it can be used as a noun. There is a difference between the *potential infinite* and the *actual infinite* (or enacted infinite) math-named *infinity*, and this discussion goes back to Aristotle and Galileo and continues with Cantor (see, e.g., Piñeiro [4] (p. 20) and references therein).

The word *definition* is used to attach a specific (sometimes particular) meaning to a word used in math papers or in general within a determined context, e.g., an infinite set

is a set which is not a finite set within the context of set-theory. Also, any contradictory meaning yields a logic contradiction in math, invalidating the logic argument, which means that one of the assumptions is certainly false.

The expressions a collection, labeling, counting, listing and a list all have a clear meaning when thinking in a 'finite' collection or list, because in this case, it is assumed that the labeling, counting and listing actually end, at least in an imaginary way. Tied to this is the word *eventually*, with the same meaning and usually referring to a (real or imaginary) time. We assume that a task of labeling (or counting or listing) occurs as time goes by (or with the flow of time), so a task eventually finished means the certainty that the task will be finished some (real or imaginary) time in the future. This is, in math and specially in probability, 'eventually' has a narrow meaning, in a way, closer to 'when' than to 'if', and this way is used in what follows. However, in math, we disregard 'the waiting for something to happen' once a certainty of the happening is affirmed, e.g., as in *finite time* or even *eventually finite time* to emphasize. This is not the case when we do not have (actually or imaginarily) this certainty, and hence a conflict with the concept of *potential infinite*. For instance, a potential infinite task is a task that eventually ends or maybe, it never ends.

When the meaning of a *finite number of tasks* is clear (number = natural number), it is abbreviated as finite tasks or *finite task* (not 's'; it is thought as a task-unit divided in a number of sub-tasks), and this is applied to a finite-label equated simply to *label*. We adopt the meaning 'finite-label = label', as being equated also to a natural number, where it is clear that any natural number is finite and so saying finite-label is to emphasize its meaning, since infinite-label is not at all used in this paper. An *infinite task* (in a real or imaginary way) never ends, in the sense of being an 'infinite-task is a never-ending process' (at least in an imaginary way); hence, it exists as a whole. In math, this means an *infinity* of tasks, which is a whole version of an *infinite number of tasks*. At this point, *infinity* = 'infinite number of' is understood as different of (but somehow similar to) *infinite*. There is not this nuance with a *finite task* since saying a (finite) number of tasks eventually ends is automatically equated to a finite task will end and so, there is no doubt about the ending of this task, but simply the 'when' is unknown, i.e., that there is not an implicit meaning-discussion about the imaginary existence of the task, and a use of 'finity' is not necessary.

References to usual articles and books are mentioned first, and then references in various forms to the Internet are also used, and in this case, the addition 'and references therein' is implicitly almost always assumed, although sometimes this is explicitly mentioned. The parentheses (...) and perhaps a few times even the brackets [...] (but no the braces {...}) are used in common text to give more details (on the meaning of something) without distracting our attention from the concept previously discussed. The intention is also that the meaning of a sentence can be understood as a whole, even if the content of the parentheses is deleted. This is also why sometimes sentences are relatively long.

There are two very basic concepts in need of a short discussion before any *math modeling* of a dynamical system (real or imaginary) can begin. It is the what and the how, i.e., the objects and the arguments, as well as their corresponding properties as being finite and sequential, respectively. Indeed, the terms *finite-objects* (FOs) and *sequential arguments* (SsA) harmonize very clearly and neatly in our math rationality. They seem necessary for any type of models, either real or imaginary, when a detailed description with math logic is desired. However, in between these (FO) and (SA) is the concept of infinite/infinity, which, as the opposite to finite, produces also a sort of opposite to sequential, as parallel or simultaneous. The next two subsections address this concern, by mentioning two (sort of implicit) definitions. *It is perhaps important to observe that all this is only a viewpoint of the 'reality', sort of saying, even if the math-way of expressing statements with definitions and affirmations is not at all diplomatic or smooth.*

2.2. Identifiable-Sets

In plain language, an identifiable-set is required to have finite elements and a countable quantity of elements. Actually, the requirement about a countable cardinality is not necessary, because the canonical way of getting cardinality higher than countable is considering a denumerable-set N and its associated set 2^N formed by all its parts, which obviously, not all of its elements are finite. In this analysis, it becomes important to avoid the confusion between *finite elements* and the common phrase *finite number of elements*. Also, being an identifiable-set implies that its elements are identifiable and searchable, in the sense described below.

This is close to a discussion on what is identical or equal or similar (e.g., in relation to being reflexive, symmetric, and transitive), i.e., there is a reference to the discernment in mathematics. Because the counting process (by means of the natural numbers) is possible as either finite or infinite, we use word labeling (or labeling process) to avoid possible confusion. In our convention, there is no need to say ‘a finite label’ because all labels are always ‘finite’ in the sense that sub-labels may exist only in a finite number of levels. Recall that objects in math refers to anything, real or imaginary. An extended and specific math-meaning can be expressed as follows.

Definition 1. (1) *A label of an object is a number of characteristics used to describe and identify it, i.e., to distinguish this object from any others, at least within the context where it is being used. That is to say, all information necessary for its identification is coded within a natural number used as its label.* (2) *A finite-object is an object with a label.* (3) *An identifiable-element is a finite-object which is used as belonging to a set, within the context of set-theory.* (4) *An identifiable-set is a countable-set of identifiable-elements.*

The process of assigning a label to an object is very strict, i.e., the process and analysis to determine its characteristics should be declared finished when making its label, even in an imaginary process where only a (finite, to emphasize!) number of characteristics could be used. This is similar to Gödel’s incompleteness theorems (see, e.g., Piñeiro [5]), in the sense that the characteristics of each finite-object are encoded in a natural number, which is referred to as the label of this finite-object. Only finite-objects have labels and within set-theory, it is not possible to generate an identifiable-set with a cardinal higher than the set of natural numbers. A *dynamic logic-error* appears when pretending to use ‘infinite labels’, simply because the label is never produced. That is to say, if the characteristics of an object are infinite, then we cannot claim that the object under consideration has a label. In short, Definition 1 rules out the utilization of an infinite set as an element of another set, within the context of set-theory, because a label cannot be made from an infinity of characteristics, i.e., an infinite set.

Remark 1. *An important consequence of the identifiable-elements definition is the fact that any elements in an identifiable-set is effectively ‘searchable’ in the sense that the ‘search’ (=counting in some way) eventually ends. This means that if the characteristics of a finite-object (=identifiable-element) are known, then the search of this finite-object can be made by checking label after label until the desired label eventually shows up. It is clear then that the properties of being identifiable and searchable are tied together, and they are desired in any modeling of something (real or imaginary). Indeed, otherwise, we are using objects (real or imaginary) in a way that some of them are infinite, which means, by our definition, that we are negating the possibility of being able to fully describe it, in a real or imaginary manner, so ‘belonging to a set’ lost its meaning.*

Remark 2. *Another reason to require finite-objects as elements of a set is the fact that an infinite-object requires an infinite (or no ending) list of characteristics, which are necessarily defined via*

some iteration (or recurrent or inductive) procedure. This means that to actually recognize such an infinite-object, we are trapped within this iteration, which by our explicit definition, cannot be finished. Hence, it would be contradictory to assume that this iteration is finished, and the whole can be used to construct other objects, even in our imagination. Thus, we may leave this unfinished iteration and continue with another analysis, as long as there is no claim that the mentioned infinite-object is actually defined and ready to be used or to be instantiated as a whole. Basically, infinite-objects cannot be used as a 'unit' to fully describe a thing as a whole. This is a discussion similar to the one of Aristotle and Galileo (see, e.g., Piñeiro [4] (p. 20) and references therein) about the potential infinite as a whole thing, say infinity as used in math analysis.

To properly understand this definition, a history of counting may be relevant. It is said that initially (in the dawn of humanity), the necessity of counting appears with things that one possesses as animals and so, it was counted 1, 2, later 3, 4, and *many*, which means that 5 or more was already infinite, as something so large that it was unthinkable or unnecessary or unusable. With time, this was improved and the 'many' as our ending of understanding was pushed further and eventually declared as *never-ending* along the lines of some extremely large and totally unthinkable number appearing in the counting.

The meaning of 'infinite' is understood through the construction of the natural numbers, beginning with a label-unit 1 and a counting method $1, 1 + 1 = 2, \dots$, which continue indefinitely, i.e., this iterative addition (to obtain the next number) *never ends* or never stops as an *abstract definition in math* (without the need of a practical justification). Therefore, this *never-stopping* or *never-ending* method (or process/procedure) is our concept of *infinite*. Certainly, this is tied to the *recurring* (or induction) process (or procedure or iteration) and to the math-use of the words 'let ... be', 'suppose', or 'assume' as in the (math-)creation of a *set*. That is to say, the math-reasoning to build theorems in mathematics, where logic should prevail without any confusion. This also includes the meaning of 'eventually' (or stops) as the certitude that it ends (or stops) without actually being able to determine when it ends (or stops). This is clear when using the expression 'let n be a natural number', where a stop in the iteration (or induction) procedure (to generate the natural numbers) is determinate when, say, we create the number n , which could be as large as we wish, unthinkable large, impossible to write it down, but effectively a natural number.

In math, we created the negative numbers $-1, -2, -3, -4, \dots$ (as it is well-known) and so, the 'number zero' 0 was a necessity for competing subtraction. From counting to comparing (which means 'the paring of elements in a counting-process' = 'establishing a one-to-one correspondence between both sets'), we are able to add and subtract properly with logical rules, and the iteration of the 'addition-operation' produces the multiplication. Again, comparing was not sufficient, quantifying became useful, and the inverse of 'multiplication' appeared and was named 'division' with more logical rules for the signs and order to do these operations. Hence, due to the necessity of more numbers, fractional numbers were developed. Thus, multiple 'labels' appear as $1/2 = 2/4$, etc., and were declared to be equal fractions, to define the rational numbers, denoted by $\mathbb{Q}$, and the logic prevails. The four arithmetic operations addition/subtraction, multiplication/division, and their logical rule were established.

Continuing with the division of a unit or the search for a quantum (=quantification), it is clear that it was not possible to 'divide' an animal (and keep it alive), but we found other uses for dividing units of things, and the process of 'measuring' (after quantification) was necessary. At this point, another critical point regarding infinite becomes visible, similar to $1, 1 + 1, \dots$, we study $1, 1/2, \dots$, which means *continually dividing and never stopping* as in counting to infinite. The interesting point here is that this process seems to 'go somewhere', called the limit 0, with the meaning of 'nothing', which seems clear and neat, and much

more understandable than the infinite. In any way, these two addition-infinite and division-infinite may be referred to as ‘infinite in the large’ (initial infinity) and ‘infinite in the small’ (infinitesimal or continuum idea), respectively.

2.3. Sequentiality and Simultaneity

These two concepts cannot go together in our mathematical reasoning, even in our imaginary world of mathematics. Simply because together the essential concept of theorem becomes useless, no discernment among hypotheses, theses, and proofs is possible. The point is not to discuss ‘simultaneity’, but rather to make clear that in our math-reasoning, the sequentiality of our arguments is always preserved in an explicit or implicit way, and even in our imaginary math-realm. Moreover, sequential and finite are considered together, in the sense that their meanings are attached. This means that negating one has a clear effect on the other. In particular, if ‘finite’ means that the counting with natural numbers of a list of objects finishes with a number, then ‘not finite’ means that the counting continues and never finishes. The ‘sequential’ aspect is also affected as never-ending; it is trapped and no more sequential analysis is possible.

Definition 2. (a) A sequential-realm is an imaginary world of mathematics where the sequential rule is a fundamental regulation, i.e., absolutely anything and everything (as in a math-logical arguments) in this realm follows a sequential order and simultaneity does not occur. In other words, the imaginary flow of time (or arrow of time) has a minimum non-zero quantum-of-time. (b) The identifiable-realm is the world of mathematics where only identifiable-sets are allowed (to be used within set-theory) and the strict rules of sequentiality are enforced, i.e., it is also a sequential-realm.

That is to say, independently of the details in the math-logical thinking, the math arguments are necessarily sequential, and this also means that in all detailed explanations, one argument follows another argument only after the previous argument is finished or resolved (in the context of this previous argument). This basic convention (Definition 2) is accepted by our math rationality, which guides our math-reasoning all the way without any conflict, since basically, no reasoning is possible without a sequential order, as in one thing after another. There may be a sort of parallel argument, in the sense that an argument may contain several arguments, but the possibility of ordering all arguments linearly is not negated, i.e., there are only a finite number of parallel arguments. This also means that there are no understandable detailed explanations if simultaneousness occurs.

The math-philosophy of this identifiable-realm (for modeling within applied math) can be phrased in several ways, with a simple form as follows: we are solving every difficulty (within a global problem) one at a time and also one after another, but all pending ‘sub-difficulties’ (either tasks or further analysis) need a resolution before going to the next difficulty (or local problem). This way of thinking is implemented in the sense of Definitions 1 and 2; this does not allow the *Transfinite Recurrent* (e.g., see Halmos [6] (Sections 18 and 19) and Remark 3) because it is considered as a violation of the strict understanding of the sequential analysis. The logic arguments behind this viewpoint are discussed in some details below. This is not an imposition, since every modeler should decide what principles are used to model. However, the imposition appears when a modeler chooses this identifiable-realm for modeling.

At this point, the challenges and conflicts that the concept of infinite represent to our math-reasoning become clear, all of which are well-represented in the long history of math, e.g., see Boyer [7]. The quality and high-level math-reasoning of Cantor (see Piñeiro [4] and references therein) and Gödel (see Piñeiro [5] and references therein) are clearly recognized when it comes to dealing with the concept of infinite. The particularity of mathematics resides in the great capability of abstraction so that the math-realm does

not need any practical comparison with reality in its math rationality. That is to say, that the math-realm is mainly imaginary, as, for instance, the construction of geometric objects, although they do have an origin in some real objects. This imaginary math-realm has some rules or regulations for mathematicians, and the power and prestige of mathematics is earned through the fact that the math rationality is kept free of logic-errors or contradictions as much as possible.

Remark 3. *This is sort of saying that, in this identifiable-realm, the sequential way of reasoning draws a fine line when accepting an infinite set like $\{1, 2, \dots\}$ describing the natural numbers $\mathbb{N}$ in the sense that the whole set $\mathbb{N}$ cannot be used as an element of another set. This is so because the sequential analysis is trapped within the iteration defining $\mathbb{N}$, and even in an imaginary way, if we assume that math-reasoning can continue further in the use of the whole $\mathbb{N}$, then this is considered as a violation of the sequential reasoning (as specified in Definition 2). In short, there is no way of finding a label for an infinite set or ignoring this and so, keeping a logical math-reasoning all the way is not possible. However, the number n as an element is a label of $\{1, \dots, n\}$, which can be used to identify this set within a given context and there is not a great difference between the label and the set itself. Say, the set $\{1, \dots, n\}$ itself can be used as a label, as well as sets $\{\{1, 2\}, \{3, 4, 5\}\}$, $\{\{1, 2\}, \{3, 4\}, \{5\}\}$ or many other forms, for the case $n = 5$, where labels are interpreted as 1, 2, 3, 4, 5 (only 1 level of labels) or with 2 or 3 levels of labels as in $\{1, 2\}$, $\{3, 4\}$, $\{5\}$ or $\{1, 2\}$, $\{3, 4, 5\}$; see Definition 1. Moreover, this is the same for any other iterative procedure, e.g., if $F = F_0$ is a finite set and $\{a_1, a_2, \dots\}$ is an infinite set disjoint of F , then the recurrent procedure $F_k = F_{k-1} \cup \{a_k\}$, $k = 1, 2, \dots, n$ defines the finite set F_k , and F_∞ is accepted as an infinite set, but F_∞ is not accepted as an element of another set (so the successor x^+ of a set x is only valid for finite sets). The case is similar with $F_k = \{F_{k-1}\}$ and other forms of recurrent procedures. This may be called a fine line between elements and sets within the logic of the set-theory used in the identifiable-realm.*

It is understood that an infinite list of objects is a never-ending list or an unfinished list, which cannot be completed in any sense, so that another list (finite or not) may continue the count after this infinite list ends. This is different of a potential infinite (=a list that may or may not end), since this is the enacted infinite or infinity, just by definition. The affirmation “Any infinite countable-set is identifiable by counting its labels, but it is impossible to be identified by pairing its labels” is understood as follows:

(1) The first part ‘Any ... counting its labels’ means that its elements are identified by labels and the labels become the elements of the set, i.e., it has finite-objects (or labels) as elements (e.g., $\mathbb{N}$). Recall that labels always have a finite number of levels by definition, i.e., labeling is a counting process transforming an object into a finite-object (in a finite number of steps), which is identified by its label. Hence, this first part is trivially true, i.e., it is the diagonal process in reverse. It is implicit that if one label is not produced, then there is an element that is not a finite-object, and this object (and so the whole set) is not identifiable.

(2) The second part, i.e., ‘an infinite countable-set is impossible to be identified by pairing its labels’ follows from recalling that, if N is a denumerable-set (=countable and infinite) and 2_∞^N is the set of all its subsets which are infinite sets, then 2_∞^N is not countable. Therefore, first note that we may choose an infinite subset $N' \subsetneq N$ that is different than N itself, and use it to define a one-to-one function between N and N' (=pairing N and N'), which means that some elements were lost in this listing. To resolve this objection (=the pairing only identify a infinite subset of N), one has to show that ‘pairing’ can be used (instead of ‘counting’) to identify the given countable-set N , i.e., being able to pair all possible infinite subsets of it. But this is not possible, because the infinite set 2_∞^N is not countable (by the pairing way itself). This point (which is not so simple to understand and

not necessary to check if a set is identifiable!) is not only related to the definition of pairing, but also to the searchability of any finite-object; see Remark 1.

The above argument shows that, in a way, we are bounded to the sequential counting (also as a tool to identifying infinite-objects, since the pairing is an equivalent tool only with finite-objects), as explained in proving the second part (i.e., ‘an infinite countable-set is impossible to be identified by pairing its labels’). This is so because we have chosen to respect sequentiality in all our imaginary constructions which use math arguments. All this is analogous to a discussion on the differences among the concepts of “identical, equal and similar”. In this case, this is the difference between the *pairing* (as a one-to-one correspondence) and the *counting* (as in labeling an object); see Definition 1.

Remark 4. First, the imaginary time in Definition 2 is conveniently called *evolution-time*, so the *quantum-of-time* refers to the *evolution-time*, and as long as no confusion arrives, *quantum-of-time* is still used instead of *quantum-of-evolution-time*, to simplify. Next, the discussion about quantum of time is certainly attached to the existence of *quantum-of-change* = *quantum-of-discernment* as the smallest unit of change or discernment, i.e., related to the changes or differences within the objects in the identifiable-real (e.g., label or characteristics of an object) as the *evolution-time* passes. In short, the *quantum-of-change* or *quantum-of-discernment* is the certainty that at least one change occurs as one *quantum-of-time* passes. This agrees with the principle of impermanence, i.e., that everything is temporary (due to the evolution). In additional, to avoid simultaneity, it is accepted that more than one change cannot occur per *quantum-of-time*. Therefore, this means that there occurs exactly one change (or *quantum-of-discernment*) per each *quantum-of-time* (using a smaller value has no meaning).

Recall that the elements of the set of algebraic numbers $\mathbb{A}$ are rational or irrational numbers that are roots to a non-zero polynomial with integer coefficients.

Theorem 1. Within an identifiable-realm only, $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ are available; the use of $\mathbb{R}$ as an infinite set is not permitted. Nevertheless, the set of algebraic numbers $\mathbb{A}$ is an identifiable-set, provided its definition is accepted.

Proof. The set either $\mathbb{N}$ or $\mathbb{Z}$ is an identifiable-set essentially by its definition. To prove that $\mathbb{Q}$ is an identifiable-set, we mention that the rational numbers are (or can be) defined as fractions (=two natural numbers and a sign) and a posteriori, we use the diagonal methods to show that the set of fractions is countable. Thus, after eliminating the redundant fractions, this becomes the rational numbers, which remains a countable infinite set. Thus, $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ are identifiable-sets.

The fact that the set $\mathbb{R}$ does not qualify as an identifiable-set does not reside in being uncountable, but in the fact that some of its elements cannot be defined a priori as done in the case of the rational numbers. Indeed, an infinite list of objects is a never-ending list or an unfinished list, which cannot be completed in any sense so that another list (finite or not) may begin after this infinite list ends. This is a basic concept that may confuse us because we are accustomed to considering sets with elements, where some elements are themselves defined as infinite lists of numbers. A first example appears with Cauchy sequences or Dedekind cuts, as the actual elements of the set of real numbers (via some equivalent relation within the rational numbers, e.g., Rudin [8]). Again, this relates to the searchability property of any finite-object; see Remark 1.

In these constructions of the real numbers, each element is an infinite list of objects (of rational numbers), so it is an unfinished list. Thus, any subsets where their elements can be arranged or thought as an infinite matrix of numbers (e.g., one element by row) can be counted with the diagonal method. But at the same time, it is logically impossible to

count them following the process of counting the first row, and then the following rows (one by one), i.e., counting or checking the elements of this infinite subset. However, the counting of each row (or element) is the identification of the rational numbers (which forms the corresponding element) as an abstract necessity. *This is precisely what is done when an infinite list of objects is allowed to become an element of some other list* [i.e., with the diagonal process, this unity of a row (as an element) is ignored, and the whole is countable].

This is a clear violation of the sequentiality (of the arguments, as defined in the present paper), simply because the counting of the first row never ends and the beginning of the second row can never be reached. With this, we may understand why later in our reasoning, we deduce that the whole set is uncountable, as it is proved that the diagonal method cannot be used to count the real numbers. This is simply because we cannot count or fully identify its elements, which are the rows of an infinite square matrix. We may use a math notation to clarify this argument a little, but it is not necessary, since by the definition of infinite, we are unable to either ‘fully identify’ or ‘finish the count’ to proceed to the next element. This is when we pretend to count the elements of a set, where some elements are an infinity of rational numbers (the corresponding row).

The last affirmation (of the Theorem’s statement) is a known result, e.g., it can be proved by organizing $\mathbb{A}$ as a double countable union of finite sets and then using the diagonal methods to count its elements. That is to say, we first organize $\mathbb{A}$ as the countable union of the sets $\mathbb{A}_n$ each containing all the polynomial roots of degree $n = 1, 2, \dots$ with coefficients in $\mathbb{Z}$ (or $\mathbb{Q}$). Each polynomial of degree n correspond to an element in $\mathbb{Z}^{n+1}$ (or $\mathbb{Q}^{n+1}$), and the sets $\mathbb{Z}^{n+1}$ and $\mathbb{Q}^{n+1}$ are also countable. Hence, we write each $\mathbb{A}_n$ as the countable union of the finite sets $\mathbb{A}_{n,m}$ (having at most n elements; the number of roots is at most n) with $m \in \mathbb{Z}^{n+1}$ (or $m \in \mathbb{Q}^{n+1}$). Finally, we can count the double countable union of finite sets by means of the diagonal counting procedure in combination with the counting of each finite sets of roots $\mathbb{A}_{n,m}$. Note that within the definition of an algebraic number, we are accepting that a polynomial always has a finite number of roots, and *being a root* is taken as the definition of these numbers, which are the finite-objects as the finite-elements of the set $\mathbb{A}$. $\square$

For instance, a mathematician uses the real numbers $\mathbb{R}$ to give a meaning to operations such as the iteration of multiplication and their inverses (i.e., powers, roots, exponential, logarithm), and also the elementary and special functions. Even when real numbers $\mathbb{R}$ (and also the imaginary numbers) are used in pre-calculus, it is my experience that only a few selected non-mathematician scientists understand (in a complete way) the meaning and construction of the irrational numbers. Nevertheless, most models of concrete physical laws include real/imaginary numbers.

There are pros and cons in using this identifiable-realm, but we focus only on the pros to show this alternative way of dealing with math modeling. Real numbers could be used within the identifiable-realm, but not as a whole, like in the set $\mathbb{R}$ (with the rigorous logic of set-theory). The real numbers that we want to use in this identifiable-realm should be first introduced as countable-sets with all their elements being identifiable-elements in the sense already given, e.g., the set $\mathbb{A}$ in Theorem 1. As mentioned previously, this (perhaps) repetitive way, seems necessary to me to clarify the very fine line on the reasoning at play, which is analogous to a discussion on some differences among the concepts of “identical, equal and similar” that are allowed to be used in a math-modeling setting, to avoid a possible dynamic logic-error (as recently named) within the identifiable-realm, as in Definition 2. Hence, the philosophy (in this identifiable-realm) is based on the acceptance that everything understood and studied in this realm is necessarily quantifiable, a principle accepted in sciences.

3. Geometry: Ideal Planar and Solid Shapes

Planar figures like triangles, polygons, and circles, and regular solids like pyramids, prisms, cones, cylinders, and spheres (including those of Plato) yield the basis of geometry and then, many formulas and pictures are developed in analysis. However, as is different from the concept of numbers as mentioned in Section 2.2, the most basic shapes, triangles, circles, and spheres, are still abstract concepts, in the sense of how these shapes are thought of in a *mathematical sense*, so that *perfect* planar figures do not exist in any size for the reality. Our space has three dimensions, so we can imagine two or one dimension, but not make real constructions there, i.e., the constructions of any shapes made in a plane (with a pencil and paper) only seem 2D, but they are not the imaginary perfect 2D realm—these perfect shapes are only in our imagination, precisely by our own definitions. Even if we could construct a square of size 1, then the two triangles formed by dividing through one diagonal are not real at all (as is comparable with the supposed real size of the square), since the size of the diagonal of each triangle would be the square-root of 2, which is an irrational number. Well, we fix this and many other problems by saying that points or dots have no dimension or they have dimension 0, i.e., this means that adding an infinite amount of 0 one gets 1—we may call this the ‘magic’ of infinite—because a line constituted by ‘infinite points’ has dimension 1 or 1-dimension. Precisely, the natural unit to measure sizes is the ‘dot’ or point, which has the incoherence previously mentioned. This and other similar questions can be called the measuring problem or incommensurability, e.g., see Boyer [7].

That is, we can have a unit and measure things using integer and fractional numbers, but we cannot construct a perfect triangle of any size if we think that points or dots have a very small size, as small as we wish. The results is still non-zero, so every material thing we know has three dimensions (sizes) and none of them is zero. However, we can think of any of the three dimensions as being small (or thin) as much as we wish, but not non-zero. Actually, ‘as we wish’ means ‘as our science’ can measure or perhaps imagine. A problem with geometric figures is that points are dimensionless (i.e., dimension 0) and lines are infinite sets of points to form a 1-dimension, i.e., we add an ‘uncountable’ infinite number of 0 to get 1, and again, it is a contradiction. This is essentially used to produce mathematical graphs, figures, etc., with the Cartesian coordinate system. Indeed, this concept of dimension needs a revision (to be used within the identifiable-realm), as is developed later. However, as discussed later, our ‘solid’ world (or coarse-world) is mainly empty in reality!

Pythagoras Theorem

The Pythagoras Theorem ($a^2 + b^2 = c^2$) states that in a right-angled triangle, the square of the hypotenuse (c) equals the sum of the squares of the other two sides (a and b). A geometric proof uses rearrangement, like placing four identical right triangles to form a large square, where the total area equals the inner square (c^2) plus four triangles, leading to the equation $(a + b)^2 = c^2 + 4(\frac{1}{2}ab)$, which simplifies to $a^2 + b^2 = c^2$ after expansion and cancellation. As mathematicians know, there are many different methods and proofs of Pythagoras Theorem (PT). However, its practical use is based on the famous numbers 3, 4, 5 satisfying $3^2 + 4^2 = 5^2$, and this allows us to make perpendicular real constructions.

It seems clear (or not?) that the element ‘a point’ is a finite-object, but ‘a line’ is not a finite-object, and it is not clear if a line is not an identifiable-set, described as an infinity of points (because we associate a point with a real number in a one-to-one relation). Alternatively, we may think a point as a rational number, but this yields a contradiction, e.g., the measuring problem (or incommensurability) already mentioned. However, if we care to define $\sqrt{2}$ as the length of the diagonal of a square of side 1 (a unity of measurement), then $\sqrt{2}$ becomes an identifiable-object. Nevertheless, using our current tools, we cannot

model with numbers the imaginary realm (of geometry) containing planar figures, but a geometric proof of Pythagoras Theorem remains true, beautiful and now, even mysterious.

However, if we accept the definition of the algebraic numbers $\mathbb{A}$ as the roots (rational or irrational numbers that are solution) to a non-zero polynomial with integer (or rational) coefficients, then this is an identifiable-set. Thus, $\sqrt{2}$ could also be defined as the positive root of the equation $x^2 - 2 = 0$. In Theorem 1, we do not rule-out all real numbers, only the set of transcendent numbers, and the reason is that they need to be ‘identified’ by a definition using arguments with sets that only have finite-elements; see Definition 1.

A geometric construction similar to PT, known as the classical *quadrature of the circle* (or squaring the circle), with the purpose of finding the exact value of the number π , is an old math problem. It was declared impossible around the year 1882, since π was proved (using set-theory) to be a transcendent real number. If the search of π is within the identifiable-realm, then we may define the number π as the ratio between the length of the circumference and the diameter of a circle that has radio 1 (or alternatively, as the value of its area). This definition of π works similarly to the number $\sqrt{2}$, if it is independent of the measurement, which is the case for $\sqrt{2}$ thanks to PT. Hence, searching for π as an identifiable-object resets the geometric discussion about the quadrature of the circle and the famous number π .

Related to this, we find the paper by Gogawale [9], which affirms that $\pi = 17 - 8\sqrt{3}$ is not a transcendent real number; so again, mathematicians refuse it without even checking the possible proof. Anyway, the calculations in the paper are kind of tedious to verify, and no clear construction of the exact area of a circle seems to be deduced with only geometric arguments. Perhaps the number π should be in the physics-realm, like other known constants, e.g., Planck constant $\hbar$ and others. For example, a circle (and its radius) can probably be measured within a difference of an atom-size and deduce the value π from the physics viewpoint. Indeed, something similar is Katie Green’s Avogadro Project in [10] of the Australian group that made the roundest sphere in the world, which is a 1 kg silicone sphere about the size of an orange.

4. Adapting Mathematical Models

The computer-realm (defined through the digital-mathematics used by computers) is comparable with the identifiable-realm. A key difference is on the so-called *machine epsilon* (machine- ϵ or mach-eps), which is the smallest positive number that, when added to 1.0, yields a result different from 1.0 in floating-point arithmetic. Combined with how integer numbers are represented in the computer integer-arithmetic, $1/\epsilon$ is comparable to the largest available rational number, while often $\epsilon = 2^{-52}$ (for double precision or $\approx 2^{-23}$ for single precision) is the machine epsilon, and used as the upper bound on relative rounding errors. These rounding errors apply also to rational numbers, independent of the fact that irrational numbers can only be represented in a computer-realm through rational approximations (and of course, $|r| < \epsilon$, $r \in \mathbb{Q}$ are represented as 0 in a computer-realm).

The identifiable-realm, sort of saying, has a variable unknown mach-eps, in the sense that it is not fixed a priori, but only thought of as an implicit existing value, for all the objects under consideration. In the identifiable-realm, all four arithmetic operations are used exactly for rational numbers, and irrational numbers need a (finite) definition and are used with their exact value (as is done by adjusting the mach-eps for each calculation). In comparison, along these lines, the mach-eps for the whole math-realm (=world of mathematics) is actually zero 0, unfathomed by us. This discussion gives some clarification from a computer viewpoint and the so-called discrete mathematics. This is to be compared

with the notion of infinite in the large and infinite in the small (infinite and infinitesimal), as mentioned at the end of Section 2.2.

Finite-objects are the observable (static) things and changes (or dynamics) work according to the sequentiality rule. This paradigm is the evolution of the identifiable-realm, the background of any mathematical model considered in this paper. Recall that changes make sense in the identifiable-realm only when they are labeled; see Definition 1. However, we should recall something more derived from Theorem 1: the exclusion of the real numbers as a whole. This may be seen as a handicap, but actually, it is an alert in the sense that if we need to use a particular irrational number, then we need to provide a ‘finite’ definition of it, e.g., with the number $\sqrt{2}$ regarding the discussion in Section 3 and also about the number π . This is so because in our identifiable-realm, the concept of infinite is treated as a boundary of the sequential math-reasoning.

A discussion along the lines of Strømme [1] may first begin with a model of the material part of our identifiable-realm, say the matter-identifiable-realm. Our paradigm is already chosen as a static system (=the observable objects as finite-objects) and the dynamic system (=the sequentiality due to the evolution, so the evolution-time in the background). This matter-identifiable-realm has the concept of space and motion, energy, and time, and also it has its law of causes-and-effects. This causes-and-effects law is described by pieces, and each piece (and the whole) has the familiar structure *inputs, black-box, outputs*, and all works harmoniously as a whole. To see this, we begin with the basic concepts used in dynamical systems within our identifiable-realm.

Definition 3. *A duration is a finite interval of evolution-time; see imaginary flow of time in Definition 2 and Remark 4. A fore-ordination is like a theorem attached with a duration, i.e., an affirmation that a specific finite-combination of finite-objects evolves into a determinate number of finite-objects within a duration. Finite-combination refers to a natural number of ‘things’ interacting together within a duration.*

For instance, a chemical reaction among several compounds (=combination of chemical elements) is a fore-ordination, because within a duration, this yields other compounds, and many other examples could be found. Basically, a fore-ordination is the law describing what a black-box does in a deterministic dynamical system, where there is an input which becomes an output, and this is iterated and combined with similar fore-ordinations. The inputs and the outputs are a (natural) number of finite-objects. The list of finite-objects used as inputs or outputs is finite, and so is any duration. Thus, the expression of a fore-ordination is reduced to: *this finite-object evolves into this other finite-object*. The interrelation or interconnections of how everything evolve in this model of nature is described as the acceptance that pieces of an output in a fore-ordination can be used as pieces of an input in another fore-ordination. However, the way everything evolves in an identifiable-realm is very, very complex, and always subject to sequentiality (=sequential rule); see Definition 2.

The laws of nature (as sciences understand them, e.g., in physics, chemistry, ...) can be expressed as fore-ordinations. Hence, this is a (logic and deterministic) model of the natural world (or material creation), but too little can be said in this general or global setting; only local models can be discussed in some math details. For instance, models with finite Markov chains can be interpreted as each variable representing the relative percent of a particular characteristic (or rather a number of them aggregated, inevitably) used to define a finite-object, all within this identifiable-realm. The continuous idea (infinitesimal or infinite in the small) and the infinity idea [infinite (in the large)] cannot be completely used (see Section 2); they are considered as a boundary of this identifiable-realm. However, even when this is so, these ideas can be reinterpreted as eventually continuous/infinity, in the sense of being extremely very small or very large in a manner that they are unfathomed

for our math-reasoning within this realm. In this manner, as eventually finite-objects, they can form part of our modeling, i.e., there is not any loss, just a reinterpretation of the details within this model.

All fore-ordinations have a deterministic character, i.e., for equal inputs, the black-box (or system dynamic) yields the same output (this is also contained in the assumption that inputs and outputs are finite-objects). However, we can allow probability by incorporating the well-known notion of *partial observation* as well as the possibility of using as inputs only a piece of a finite-object and this resulting in pieces of the outputs. This means that probability comes out of the lack of information about the inputs/outputs and the functioning of the black-box. In other words: (a) the finite-objects are only partially observed when used with fore-ordinations, and (b) the fore-ordination itself has not been fully understood. This produces the probability, i.e., the pseudo-chance, pseudo-randomness, pseudo-chaos, etc. But the modeling of independent events does not fit the causes-and-effects arguments, because it would imply the impossibility (as a natural fact) of understanding how all things work.

At the same time, this ties in the observation of things (=finite-objects) with the understanding of the laws (=fore-ordinations). Moreover, summarizing this, we may say that our matter-identifiable-realm evolves accordingly to the causes-and-effects law (or fore-ordinations). Naturally, we have equated input/output with causes/effects. The reasons why sometimes we observe a situation where the ‘same causes’ produce different ‘effects’ is because we lack some (important) information about the inputs/outputs (description) or even about the black-box itself to realize that the same causes always yield the same effects. Indeed, we may call this the knowledge about this piece-of-model (=an observation of the identifiable-realm), which is always evolving as well as everything else.

Adding a control variable to a dynamical system model is relatively simple, once the model of the dynamic has been chosen (and we want to add a *local* control variable), and so, it is not discussed in detail, since this depends deeply on the choice of the control variable and there are more ways of thinking. Simply, we consider regulation with its recommendations, which give the possibility of controls or actions.

4.1. Finite Markov Chains Models

The challenge to adapt M.-Robin [2,3] as a model of a small local piece of the identifiable-realm is clear, because no use of real numbers is allowed. Simply, we replace $\mathbb{R}$ with $\mathbb{Q}$ and some difficulties appear. The intrinsic definition of finite Markov chains does not need $\mathbb{R}$, but any limit has to be reviewed. The state of the system is x , a finite dimensional vector of rational numbers, which clearly corresponds to the characteristics of a finite-object (or the relative percent of a characteristic of it). The black-box is the functioning of the finite Markov chain, interpreted as the rule of evolution for a percent of a particular characteristic within an object, which in turn is also viewed as a finite number of finite-objects. If the transition matrix has rational numbers, all this remains acceptable for modeling within the identifiable-realm. The ergodic analysis goes through as long as no mention and no use of the limiting probabilities of the transition matrix is enforced. Some adaptation is necessary with [3] (Section 5) (and similarly with [2]) relative to the series as the costs, but this can be treated as rounding errors in the calculations, so that the argument remains valid in the identified-realm.

Hence, a Finite Hybrid- and Semi-Markov chain model has been established within the identifiable-realm. The use of this modeling tool is relatively simple, e.g., we focus our attention on a particular problem in sciences (say physics), and sufficient experiences and consideration should be entertained to estimate the values of the transition matrix, and this is compared with actual results, as statistics suggest or from empirical knowledge. Thus,

essentially anything can be included in this modeling, i.e., it is not faraway from a learning algorithm, all within the identifiable-realm. However, in applying this model in a practical situation, several experiments could be studied and so there is a lot to learn from them, and this is the boundary of my expertise or experience. In a way, the evolution according to physics laws is modeled with a very, very large transition matrix; we can even think about this by looking at the position changes in a representation of anything, following the arrow of time.

It is probably important to recall that this model is within the matter-identifiable-realm, so all known laws of physics (or nature) are followed. A mathematical model is used as a math tool to better understand the fore-ordinations involved in the (necessarily local) model. Thus, depending on what is the application (of the math-model), a suitable time-scale is selected, and this provides the (necessarily discrete) time in use, which is derived from the evolution-time discussed earlier. Along these lines, we may discuss how time, the usual or matter-time, can be understood so that no contradiction appears with the fore-ordinations. In any way, if a particular law of mechanics or physics is used in a model, then by tracking how a group of characteristics of the model evolves (after a suitable and necessary computer calculation), a transition matrix is identified and can be used in a finite Markov chain model for a further asymptotic or ergodic analysis as in [2,3].

The past, present, and future (as we know them) follow a sequential order and so this should be compatible with the math understanding of the causes-and-effects law (or laws, when looked at in detail) by means of fore-ordinations, which is a basic instrument of a familiar dynamical system: *inputs* $\rightarrow$ *black-box* $\rightarrow$ *outputs*, where the word black-box is equated with *transition* to emphasize its function, as a dynamic object as well as its connection with a duration (or a finite interval of evolution-time).

To substantiate our claim is complicated, and so, we only begin the discussion by mentioning that the time (as in matter-time) could be a construe of our experience (as many other terms, e.g., the 3D spacial dimensions), which is attached to anything we experience. This is the same with evolution-time, which is taken as a fundamental notion in our model of dynamical systems, as a reason for its existence, and not necessarily equal to the time as usually conceived. Hence, time (=matter-time, in the usual sense) can be thought of in another scale, i.e., the quantum of matter-time is a multiple of the quantum-of-evolution-time, as is used for quantum-of-time in Definition 2. When necessary, sometimes we add ‘matter-’ and ‘evolution-’ before time to avoid possible confusion.

In this identifiable-realm, there is no infinity nor infinitesimal and so, a mathematical expression of Einstein’s formula has the form of sequential reasoning. Hence, instead of affirming that there are doubts concerning the premise ‘the light-speed cannot be exceeded’—i.e., *we do not if the light-speed can or cannot be exceeded*—this gives us a chance to imagine this possibility within the logic of the identifiable-realm. With this in mind, in mathematics, this is called a two-scale (or multiple-scale) time system, and the particularity of this model resides in the mixing of these two (or more) time-scales. In this way, it is simple to analyze by checking and adapting our concept of usual time, e.g., when we think in geological time or celestial (as in objects like stars and planets) time, in comparison with day-to-day time, or even the processing time of a computer.

Something immediately observed is the fact how specific changes (=controls, as in our actions) affects the various time-scales recognized in our experiences (real or imaginary). This is comparable with an imaginary trip to the past, where we may determine a tentative answer to the question ‘What things could be changed in the past?’, under the assumption that fore-ordinations are totally unchangeable. This is, once the little piece (of the action in a black-box) has been released (and completed after the prescribed duration), its outputs are available to another little piece (=a fore-ordination in itself), and the processing continues.

Hence, the sequentiality is intrinsic to the composition and operation of fore-ordinations, and no contradiction can be accepted among its small pieces, i.e., we may think of a causes-and-effects law from which all specific laws are obtained. There is more to this, in the sense that our concept of usual situations may be also a construe of our imagination. If we think as dimensions in the sense below, we get a ‘material’ representation of the past and the future, and with this, the possibility to consider time-traveling, at least, first in our imagination.

4.2. Dynamic Dimensions

We add some dimensions to this identifiable-realm in a somehow practical way. As we understand our 3D-space, we can only imagine fewer dimensions, like 2D or 1D. Indeed, with only one eye, we can visualize only two dimensions and with two eyes, our brain reconstructs the familiar 3D-space, a process known as the geometric perspective, which is used in drawing and painting and in art and science. Film technology is a very good example of how 3D can be imagined with a suitable sequence of 2D pictures. Because our *slow eyes* use 1/30 s on average to delete an image and consider the next one, we are able to quickly compare two very similar images and notice the differences with our *brain*, so that we complete a movement of things as known in nature. Therefore, we may think that *motion* is the 4th dimension. However, this argument stops there, since we may imagine a ‘third eye’ and compute mathematically something similar, but the mystery of more dimensions persists.

Nevertheless, in mathematics, we can invent many dimensions in a logic way with the well-known arguments used in linear algebra. And similarly, in science-fiction, the idea of *other dimensions* is very popular (but not logically clear) in our imagination. For example, there should be more dimensions, but we do not understand yet a possible model of how these dimensions could all work together. In my opinion, this is caused by the ‘illusion of an infinite continuum-space-time’ as our reality, i.e., infinity and infinitesimal as in Section 2.2. An argument begins with the examples on how nature is visually observed by us.

Remark 5. *It is known that our naked eye can differentiate (approximately) between $1/10 = 0.1$ of a millimeter (mm) and 10 meters (m), within the interval $[10^{-4}, 10]$ (i.e., from 0.0001 to 10 in the metric scale); with a light microscope, this range extends to within $1/10 = 0.1$ of a micrometer (μm) and 1 mm, within the interval $[10^{-7}, 10^{-3}]$; and with an electron microscope, it goes even further within $1/10$ of a nanometer (nm) and $10 \mu\text{m}$, within the interval $[10^{-10}, 10^{-5}]$. As known in science, a cell is a group of molecules, and they are measured in a range from μm to nm. An atom is the smallest unit of matter that retains the properties of an element (essentially, an element is defined in chemistry and an atom in physics). A molecule is the smallest unit of a substance that retains its chemical and physical properties (composed of a group of 2 or more atoms) and has a size measured in nanometers, $1 \text{ nm} = 10^{-9} \text{ m}$ (1 meter divided by 10^9 , a 1 followed by 9 zeros), and atoms have a size of $1/10$ smaller than a molecule (so 10^{-10} m). The smallest particle in an atom is an electron (or perhaps a quark) with a size estimated at 10^{-15} m (so 10^5 times smaller; the size of an atom divided ten-thousands times).*

From this, it follows that the material solid world around us is not so ‘solid’ after all, but is rather empty, since it only has small particles tied together with attracting and repelling forces (at a distance), all according to our five or four known fundamental forces. These interactions are described in physics as the gravitational, electromagnetic, weak-nuclear, and strong-nuclear forces, but there may be more fundamental forces. Hence, the reality in physics as seen and measured by us could be rightly referred to as ‘the realm of coarse matter’ or the ‘coarse world’ (=coarse-realm), leaving room for other worlds in

some different sense. Moreover, the duality particle-wave as known in physics can be used to model (mathematically) the whole material 3D-universe as *a very, very large number of tiny-tiny moving particles kept together by means of some fundamental forces*.

Can we say that we ‘know’ that everything is moving? This assumption seems reasonable, with respect to current science. How about the assumption of some *new fundamental forces*, which are responsible for other movements (or motions) beyond the current understanding of physics? With this, we can use a version with rational numbers of the classic Fast Fourier analysis (e.g., Papoulis [11] (Chapters 1–3)) to imagine a math model of moving particles as objects in the identifiable-realm. Looking at these objects as math-waves, the mathematics structure has an expression as a series (with rational numbers) corresponding to a finite sum, after accepting rounding errors.

The point is to equate this with a linear algebra space of a very large dimension. When this is realized, then it is also understood that all math-waves can be viewed as having a particular range of frequencies determining their movements. This is a linear vector space (with respect to the rational numbers) of math-waves within a particular range of frequencies that constitute the whole 3D-universe. Moreover, we may think that every particle is subject to a ‘harmonised particular movement’, given by some frequency (or range of frequencies). Thus, by changing this harmonizing frequency, we literally make a change of the whole 3D-universe.

In other words, the fundamental assumptions of quantum physics are interpreted within the identified-realm, with waves being discrete and evolving deterministically via some discrete Schrödinger equation, as done within an imaginary computer having an unfathomed very tiny-tiny mach-eps (for details on the form of these discrete equations, a further discussion is needed, since this would be tied with the fore-ordinations laws describing the identified-realm). Therefore, our working hypothesis is that “dimensions are formed because the moving particles are in an harmonious and synchronized movement; each dimension has a ‘shift’ (wave-sense) with respect to any other, used to harmonize and synchronize, the invisible (for us, higher than light-speed) movement”. These dimensions are imagined as dynamic dimensions, they are not dimensions in the sense of spacial dimensions, as 3D-space in geometry.

As already mentioned, to imagine this possibility, all moving particles are regarded as waves within a range of frequencies (referred to as the ‘material-range of frequencies’); this is all matter (everything material) as a group of particles kept together by means of some fundamental forces and their laws of physics, and this is the *material-identifiable-realm*. However, this realm is embedded into the *energetic-identifiable-realm*, and its fundamental laws (referred to as energetic-laws) harmonize with the previous laws of physics (=material-laws), meaning that these energetic-laws contain the material-laws as particular cases.

With this in mind, the waves have another range (=the energetic-range) of frequencies. The ‘shift’ mentioned above is with respect to the energetic-range. The energetic-laws allow movement beyond the light-speed and without friction, because of its energy-state, and we think matter are experience a densification of energy. This embedding of one realm into another may go on for only a finite number of times, thus reaching the limit of our understanding-realm.

Mathematically, we consider the (algebraic) vector spaces as a representation of moving particles in a space (which does not necessarily have spacial dimensions), and this is possible due to the duality (as observed in physics) of moving particles and waves. Next, without a continuum space, there is no conflict to imagine this wave/particle model (see Remark 5). In this sense, a (finite-dimension algebraic) vector space represents (a finite number of) moving particles in a math point of view, where vectors are waves/particles in their characteristics, i.e., a 3D-universe as we know it. These vector spaces can be ‘mixed’

(sort of saying) or grouped in various ways, creating a larger vector space with several independent vectors spaces, all finite-dimension (in the algebraic sense) vector spaces.

Remark 6. *Related to the algebraic idea of dimensions as in the concept of vector spaces, we may say that the geometric manner of imagining dimensions within a static space ends with the known three dimensions and its analysis. However, due to the duality between moving particles and waves, we can associate a sort of dynamic dimension (or wave dimension) to a vector space of waves (or moving particles). In this way, there is a specific meaning of several dimensions, and there can be as many as necessary for our imagination. Thus, the algebraic manipulation of dimensions within vector spaces becomes somehow real. This is possible because the idea of continuum or infinitesimal is at the boundary of our identifiable-realm. As a whole, the underlying basic physics concepts are the static (space and energy) and the dynamic (motion and duration). The identifiable-realm is thought as a finite number of embeddings of identifiable-realms; the first one is the coarse-realm (material or coarse-matter), and all the others are energetic-realms (fine or fluidal) of various levels or degrees of fine-matter or fluidal-matter. As the frequencies of the wave get higher, finer is the ‘matter’ = ‘densified-energy’. In short, anything is an energy contained in a wave. The various frequencies give the characteristics necessary for identification, and also, several other parameters of a wave play prescribed roles. As in Gödel’s incompleteness theorems (e.g., see Piñeiro [5]), the characteristics of each wave are encoded within a natural number, i.e., waves are treated as finite-objects or equivalently, identifiable-elements. They are decomposed in simple waves and the intensifies of the various frequencies are a means of identification, e.g., with the light of stars in astronomy.*

Therefore, the same model works for several ‘parallel 3D-universes’ or rather several dimensions of a 3D-Cosmos, with several 3D-universes. Thus, we refer to this as the material cosmos, with several dimensions and each dimension is itself a 3D-universe. All this is not new thinking in mathematics, since mathematicians use linear spaces with any number of math dimensions. However, the essential point in all this is the extra movement common to all particles in a particular 3D-universe, which allows us to attribute precise sense and meaning to a phrase like: *this 3D-universe is ‘shifted’ by a fraction of a second with respect to another 3D-universe*. Moreover, this also makes sense in the world of physics, because it is not really solid, it is rather mainly empty, as mentioned previously. The frequencies of a math-wave are not bounded a priori, and linear spaces as in algebra with rational coefficients are mathematically possible.

Let us be more specific with this idea (as a working hypothesis!), which is not an assumption saying that matter can have speed higher than the light-speed, as physics knows.

Definition 4. *(a) The matter and the energy as know to physics is called coarse-matter and coarse-energy within our known material-identified-realm, and coarse-matter is viewed as a densified coarse-energy, i.e., coarse-matter = densified-coarse-energy. (b) Similarly to the energetic-identified-realm, it has its creation-matter and creation-energy, and again creation-matter = densified-creation-energy. (c) There are several levels of densification for the creation/-matter/-energy, referred to as fine-matter or fine-energy, high-fine-matter, the most-high-fine-energy, and other names, such as fluidal-matter or fluidal-energy, depending on the context. Each realm has it own laws, as fore-ordinations described at the beginning of Section 4, without contradicting each of the other laws within each realm, i.e., the energetic-laws are kind of ‘more details’ on the material-laws. (d) The dynamic dimensions are formed as explained above, with the understanding that the moving particles (or waves) refer to fine-particles (of fine-matter, or of fine-energy) at speeds larger than (or equal to) the light-speed without any friction forces, all within the energetic-identified-realm.*

This allows mathematicians to model in a math-logical possible way, any number of material dimensions, without contradicting any known physics law to a certain degree (i.e., adjusting their validity in another form for this new realm). This assumption also assumes the existence of new forces making these possible movements without friction (of any kind) and beyond light-speed (because different dimensions do not see each other). Anyway, these dimensions could be arranged as seven main dimensions (shifted by a fraction of a second with respect to other main dimensions) and many other dimensions (corresponding to the past–present–future), also with a particular shift attached to each of the seven main dimensions. This may allow for elaborating on the time as we know it, and regarding it as a form of dimension.

4.3. A Philosophy of Infinite

Certainly, it was not only Aristotle and Galileo who considered the infinite, many other thinkers and great philosophers figuratively danced with the concept of infinite in various ways. It is interesting to mention that Spinoza (see, e.g., Deleuze [12] and references therein) somehow used (within his works on ethics) the structure of mathematics toward the definition of God and proof of his existence, with some properties, a couple of centuries before Cantor. Spinoza did not have at his disposal Cantor’s math arguments about the infinite, but he managed to present pretty good statements, almost with mathematical rigor. Curious enough, it resulted in his ‘excommunication’ from his religion. Several other ‘known’ philosophers deal with many interesting human questions regarding infinite, but usually, without a math-logical resolution, although with logical pros and cons arguments. It may be good to remember that in the past, philosophy included mathematics and essentially everything analyzed and studied today. Anyway, this is not my field of expertise, only a little of my reading here and there. By the way, it is mentioned (e.g., Piñeiro [4]) that Cantor attached infinite or rather infinity to the concept of God in his religion. Also, the well-known writer Borges [13] deals with math-infinity in various opportunities (e.g., *The Aleph*, *The Library of Babel*, *Avatars of the Tortoise*, *The Book of Sand*, and others), and these books are very pleasant to read, with an impeccable logic, but of course, not resolving what we called the infinite problem in math.

With respect to the identifiable-realm, the concept of infinite is not really resolved, since it is in between the fundamental concepts defining the IR: the finite-objects and the sequential rule as regulations. This means that allowing infinite-objects (=objects which are not finite) to be subjected to the sequential rule results in logic troubles as in Section 2.3 and in particular Theorem 1, e.g., see the Hossenfelder [14] video regarding ‘Infinite or Finite’ property of the Physics’ Universe. Nevertheless, within the identifiable-realm, we may use and model ‘eventually-objects’ as a finite-object to be, i.e., the real or imaginary characteristics defining the objects are assumed not to be fully known by us, but we have the certainty that the object under consideration could be treated as a finite-object, as long as the sequential rule is enforced.

This is totally possible in our imaginary world of mathematics, since no assumption on the bound of the number of characteristics defining the object is imposed a priori. For instance, we cannot count the stars in the firmament, but if we assume that the number of particles (e.g., atoms, etc.) in the universe is finite (but so big that it is unfathomable for us), then the universe becomes a finite-object, and we can make a model of it, certainly, an imaginary model. This idea is further discussed and uses several levels of ‘eventually finite’ for something, which allow us to discern various levels of incomprehensible very small and very large values when discussing numbers that are so small or so large that they make absolutely no sense in a determinate context and no sense as fathomable things.

The mentioned argument needs permission to use potential infinite-objects within the identifiable-realm, in a temporary (or in probation) form. This allows us to ‘test’ logically objects with a significant number of characteristics, which forced us to recognize them as unfathomable in their totality by us, at least for now. Hence, the sequential rule is enforced without any conflict and is consistent with this concept of infinite. Therefore, our freedom in our imagination is not bounded by our own self-imposed assumptions, which includes negating the sequential rule effective in our math rationality. In other words, we imagine objects that we do not fully understand, at the price of keeping our math rationality bounded by the sequential rule. Certainly, this is not a position against the *axiom of infinity* (e.g., see Halmos [6]) or ‘postulates of the Euclidean geometry’ (e.g., see Euclid [15]) and many other assumptions. This is only a way of thinking when we model real or imaginary objects, with the details as much as our current math rationality allow us. For a non-specialist reader, there are nice math courses involving infinity (=the math-infinite), e.g., *Math and Magic* (Benjamin [16]), *Zero to Infinity—A History of Numbers* (Burger [17]), *Great Thinkers, Great Theorems* (Dunham [18]), and several others in the same collection and also elsewhere.

5. Conclusions

To conclude the argument along the lines of Strømme [1], we need to realize that based on Definition 4, the dynamic dimensions appearing in the matter-identifiable-realm are only possible because there is an energetic-identifiable-realm underneath it, which is where the fine-particles embedded within the coarse-particles are actually moving to produce the various dimensions. The main seven dimensions are such that each of them cannot be seen (or perceived) from any other dimension, only from ‘out-of-the-box’, i.e., only from a fine creation-energy level or levels. This is why each of the seven main dimensions is independent of the others in the sense that completely different dimension universes in the material sense can exist, i.e., we may assume that the material natural laws act independently within each dimension, and the coupling could be only at the level of the energetic-identifiable-realm. However, the attached dimensions of past/present/future to a particular main dimension may be the ‘memory’ of how the duration within a fore-ordination is implemented in this main dimension. In this way, the time as we experience it is only a construe of us, as well as the three dimensions in the space. Thus, the matter-identifiable-realm cannot exist without the energetic-identifiable-realm, but the energetic-identifiable-realm may exist by itself. In particular, Section 4.2 about the imaginary and math-logical dynamic dimensions, especially the assumptions about motion at speed equal or higher than the light-speed, depends (or based) on the energetic-identifiable-realm. A supporting math-logical argument is a discussion on the math figures or shapes in general. When searching for the smallest particles in physics (refer to Remark 5), it becomes clear that most likely a point in the search is found where the notion of *shapes* fully disappears, since the passage from the visual telescope to the electron telescope shows a change in the way things are considered and *seen*. This may be the beginning of a transition from the matter-identifiable-realm to the energetic-identifiable-realm (also called the creation-energy-realm), which is only initially (or not at all) investigated in physics.

As in Definition 4, if we assume that matter itself is a certain degree of concentration of energy or *densified-energy*, then the energy can be of two types: coarse-energy, which yields coarse-matter when it is densified, and fine-energy, which yields fine-matter when it is densified. This iteration may continue for a while, but it needs to be understood as a finite iteration (no infinite) because we have math logic arguments against a number of infinite iterations (see Theorem 1). In other words, we may logically assume that there exists a quantum of the finest creation-energy, similar to the physics assumption about

the quantum of energy (=quantum-coarse-energy). This allows us to have a model of a creation-energy-realm (=fine-realm) underneath the matter-identifiable-realm (=coarse-realm) with a structure similar to the structure given to the coarse-realm, particularly as described in Section 4 where a math model of the identifiable-realm is discussed. It becomes clear that the system dynamic (or fore-ordinations as the regulations of the realm) may be different, i.e., of one within another, say for the material-realm and for the energetic-realm. Certainly, no contradiction may appear between these regulations because fore-ordinations in a finer realm are thought of as *more details* of the same fore-ordinations corresponding to a less fine realm. A clear consequence is the vanishing of the illusion relative to the continuum-space-time, which allows us to imagine dynamic dimensions.

5.1. Modeler

Using the identifiable-realm represents a viewpoint change for the scientific modeler in the sense that (1) there is no change in the collections of data to support the math model, always thought of as finite-objects, which are then elements of a set within the mathematical set-theory. However, (2) from a scientific point of view, the phenomenon or imaginary thought experiment can be thought of as a ‘discrete model’ without assuming that the reality is actually a ‘continuum model’. Indeed, the usual way of thinking for the non-mathematician is totally valid within this identifiable-realm, since the math tools called *infinity* and *infinitesimal* are now at the boundary of it. Nevertheless, the infinite in the large (=potential infinity) and the infinite in the small (=potential infinitesimal) can be represented (in various levels when needed) as unfathomed very, very either large or small quantities at the time of the observation. The reassurance that a potential very, very small machine- ϵ (mach-eps) is assumed allows our sequentiality analysis to continue and provides the necessary scientific imagination and intuition to keep the math model exactly as given for further computations in the computer-realm. No adaptation to a discrete model is necessary for its actual implementation, and all are based on a rigorous math-reasoning as discussed in Section 2, where a preference to follow a strict sequential argumentation is required.

5.2. Mathematician

A math-reader is invited to an inner discussion about the math-meaning of (a) infinite and counting (procedures), (b) infinitesimal and quantification (procedures), (c) sequential analysis in relation to flow of time, and (d) discernment among the characteristics identifying an object, so that (1) the meaning of static and dynamic are clearly related in our (logical) imagination, and also (2) our math-reasoning establishes a clear difference between the concept of elements-of-a-set and sets themselves within set-theory, which is the basis of modern mathematics. In Section 2, there is no claim about some logic-errors in either the axioms themselves or the proper set-theory. Only another viewpoint of the sequential analysis (a strict sense as in Definition 2) is proposed, which yields the identifiable-realm. In my view (but I am not a specialist in math logic!), within the structure of set-theory there are induction/recurrent procedures as well as with the axiom-of-infinity (i.e., the set of natural numbers $\mathbb{N}$). Each of these iterations is never-ending and cause a violation (when acting together) of the strict sense of the sequential rule, a dynamic-logic-error.

For instance, it may be convenient for the reader to take a look at the seven-page introduction in Bourbaki [19]) (=several short books having a total of 349 pages) to appreciate the immense task accomplished with set-theory. In doing so, we notice a reference to meta-mathematics and the use of the language. Also, among other things, the necessity of using symbolic logic to deal with the so-called *axiomatic technique*, which is similar (in a simplified way) to the famous thirteen Euclid’s Elements books, with proofs (for plane

and solid geometry, number theory, and incommensurable lines) based on definitions and *postulates* (=unproven affirmations), is well-known for its logical rigor and deductive treatment of mathematics.

Being more specific and concerning Bourbaki [19], the actual axiomatic set-theory with all its full logic presented at the beginning (Chapters E.I and E.II, pp. 14–103) refers to the basis of set-theory, and then in Chapter E.III (pp. 104–203), the power of set-theory begins to show up. There is no logic-error ‘per se’ there found by me. However, in showing its power, the necessity of making a definition of infinite set (Section E.III.45 §6 Infinite Sets pp. 148–154) appears, and the dynamic contradiction is finally revealed with the iteration of two infinite recurrent procedures, using one immediately after the other, since the second recurrent procedure cannot begin without ending the first recurrent procedure (which is never-ending by definition!). As already mentioned, my expertise on math logic is limited, and this is the domain of specialists. This is a main reason for presenting this identifiable-realm as a viewpoint to be used by scientific modelers, where the math-concept of an infinite space-time continuum can only be approximated. This identifiable-realm requires all sets to have finite-objects as elements, and from there, the strict sequential rule is easily followed.

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