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Optimal Zero-Defect Solution for Multiple Inspection Items in Incoming Quality Control

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Abstract: This paper addresses the issues related to inaccurate inspections and high costs in incoming quality control. Incoming quality control refers to the initial inspection process that verifies whether externally provided products, materials, or services comply with specified quality requirements. Traditional methods inspect each item in sequence for a given part and terminate the inspection upon detecting a non-conforming item before proceeding to the next part. To reduce inspection times, we propose a novel approach termed 'selection of minimal inspection items', which formulates the selection of inspection items for a batch of parts as decision variables. This approach ensures that all non-conforming parts are detected while minimizing the total number of inspection items. We identify all the inspection items in the initial batch that cover all the non-conforming parts, then develop a set-covering approach to select the minimum inspection items that cover all non-conforming parts. Subsequently, the next batch of parts is inspected using the selected inspection items to identify as many non-conforming parts as possible. Compared to traditional inspection techniques, this approach demonstrates greater cost-effectiveness. Furthermore, we conduct experiments under scenarios with varying numbers of parts and inspection items across different batches to achieve zero-defect inspection, which ensures all non-conforming parts are identified and eliminated through systematic quality control procedures. Algorithms and programs are developed to implement the reported approach. The experimental results show that the proposed approach significantly reduces inspection times while maintaining high quality.

Keywords: incoming quality control; minimum inspection items; set covering; zero-defect

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1. Introduction and Background

Traditional incoming quality control (IQC) faces significant challenges in balancing inspection accuracy with cost efficiency, as achieving high detection rates often incurs prohibitive inspection costs. IQC spans nearly every stage of the product lifecycle, each characterized by distinct quality requirements. Inspection methods and analysis methods at each stage are different. The work in [1] proposes a definition of two phases for traditional IQC: quality planning and inspection execution. During the quality planning phase, engineers systematically correlate product design specifications with process parameters in order to establish comprehensive inspection criteria. In the subsequent execution phase, these predefined criteria are implemented to prevent non-conforming (NC) parts from entering the production lines.

To address the critical challenges of high inspection costs and low detection accuracy in the IQC process, researchers have explored various methods to achieve significant improvements. In [2], systematic workflow redesigns in furniture manufacturing are used to improve defect detection efficiency through streamlined protocols, demonstrating measurable time savings without compromising accuracy. CNN-based frameworks also achieve superior accuracy in experimental validations, outperforming traditional methods through adaptive feature extraction [3]. Automation solutions [4] optimize cost-effectiveness in high-volume production by dynamically adjusting resource allocation to demand fluctuations.

Despite these advancements, critical challenges persist in balancing accuracy with operational efficiency. Current sampling-based inspection methods, though preserving production rhythm, carry non-negligible risks of NC part propagation. Theoretical frameworks addressing this dilemma have emerged. Recent years have seen that some new methods introduce mathematical programming models and multi-criteria verification systems [5,6], yet practical implementations frequently encounter scalability constraints. To address these scalability constraints, we systematically classify and evaluate prevailing inspection methodologies, beginning with the quality management definition and applications.

Full Inspection: Full inspection requires comprehensive examination of every part, regardless of production batch size. Its advantage lies in its ability to identify quality issues in each part, providing the strictest quality control. However, it is time-consuming and costly, making it suitable for small-batch inspections or high-priced parts. For example, the inspection of battery packs by electric vehicle manufacturers falls into this category.

Recent studies have shown that automated inspections for full inspection enhances efficiency [7]. They use industrial cameras for rapid inspection to obtain the inspection data, yet interoperability challenges persist across heterogeneous production environments [8]. Current research focuses on cost-optimization frameworks, where machine learning algorithms dynamically adjust inspection parameters based on real-time quality metrics. One breakthrough model [9] demonstrates high cost reduction while maintaining high detection accuracy through adaptive sampling strategies.

Sampling Inspection: Sampling inspection involves probabilistic selection of representative parts from production batches, where quality assessment extrapolates sample results to the entire batch parts. This method demonstrates two principal advantages, enhancing resource efficiency for high-volume manufacturing systems and operating feasibility under time-sensitive production schedules. The methodology inherently carries zero-defect risks of defective part propagation through sampling gaps, with detection accuracy directly correlated to statistical selection protocols. Schilling and Neubauer [10] propose the application of acceptance sampling methods and the latest techniques, discussing their advantages and limitations. Bose et al. [11] propose an online sampling inspection model to address the economic production lot-sizing problem with imperfect quality and inspection issues. This approach significantly reduces inventory holding costs and increases the expected profit compared to full inspection and offline sampling. It offers an advantage, especially in high inspection cost scenarios, making it more efficient than traditional methods. However, for large-scale quality inspections, accuracy is relatively lower compared to 100% inspection.

Statistical Process Control (SPC): SPC continuously monitors production quality by collecting and analyzing real-time data, enabling immediate process adjustments. This approach effectively prevents defects while stabilizing manufacturing operations through systematic feedback mechanisms. Its implementation, however, demands substantial data infrastructure and precise sensor networks for reliable data acquisition.

The research in [12,13] confirms its particular effectiveness in highly automated, continuous production environments where rapid data throughput is achievable. Recent

innovations [14,15] develop predictive models to anticipate quality deviations during mass production phases, though current systems remain limited in dynamically adapting control parameters to real-time process variations. The methodology's reliance on fixed equipment configurations creates implementation barriers for flexible production lines. Emerging solutions focus on hybrid architectures that integrate traditional SPC frameworks with adaptive machine learning modules, seeking to balance stability with responsiveness.

Six Sigma: Six Sigma employs statistical analysis to minimize defects and enhance quality performance through systematic data evaluation. This methodology proves particularly effective in complex manufacturing environments where precise process control is critical, though its implementation demands specialized expertise and advanced analytical tools [16].

Recent advancements [17,18] integrate fuzzy inference systems to map parameter–quality relationships, generating visual capability profiles that benchmark against Six Sigma standards. Such frameworks enable proactive quality forecasting during system design phases, allowing manufacturers to optimize parameters before production initiation.

A persistent challenge emerges when inspection planning lacks alignment with execution data, creating disconnects between quality design and operational reality. Current research focuses on closing this loop through adaptive learning systems that dynamically correlate planning assumptions with real-world inspection outcomes. Comprehensive reviews [19] systematically analyze synergies between Six Sigma, lean manufacturing, and sustainability principles, identifying the critical integration barriers while proposing unified implementation frameworks for cross-disciplinary quality management.

Next, we explore the applications of IQC across different industries. Jin et al. [20] propose the application of lithium-ion battery quality management methods in the electronics field and discuss the advantages they bring to ensuring comprehensive project management of battery quality. As industrial products become increasingly complex, the demands for performance, safety, and environmental standards are also rising. For instance, in the case of electric vehicles, the power battery is a critical part, with its quality directly impacting vehicle performance and safety. The battery itself is a complex system, and its quality is influenced by multiple factors, such as the voltage, current, and capacity of individual battery cells, as well as the charging and discharging efficiency of the battery pack. The work in [21] proposes a data-driven methodology that utilizes multivariate process capability indices to identify cause–effect relationships in lithium-ion battery production and demonstrates significant reductions in production ramp-up time through enhanced quality assurance protocols. It makes the IQC of parts for electric vehicle power batteries a highly demanding process involving multiple inspection items and complex inspection methods. Current incoming inspection methods for parts often fail to meet these comprehensive requirements under the constraints of low cost and high efficiency. To address these challenges, we aim to explore an optimized inspection solution for achieving zero defects across multiple inspection items in IQC, with a specific focus on reducing inspection costs.

Building on these foundations, we propose an optimization method based on set-covering theory to minimize inspection costs. It addresses the issue of multiple inspection items in IQC, as well as the high costs and time-consuming nature of the process. Set covering is a combinatorial optimization technique used to identify a minimal set that covers all elements. Specifically, it involves selecting a subset of sets from a given family such that the union of these selected sets contains all elements while minimizing the number of selected subsets. Several studies have introduced approaches for processing quality data. Next, we provide an overview of these approaches and their practical applications.

Recent studies advance quality analytics through innovative data integration strategies. The work in [22,23] for electronics manufacturing combines association rule mining with a plan–do–check–act cycle to establish closed-loop quality improvement systems.

Pharmaceutical research [24] demonstrates machine learning models predicting critical quality attributes using early production data, achieving seamless integration with legacy systems. Emerging methodologies leverage real-time manufacturing data [25] to dynamically optimize process parameters, while metrology frameworks [26] strengthen data reliability through traceability protocols. The field's evolution from analytical models to data-driven paradigms [27,28] underscores the growing importance of mining complex production datasets for actionable insights. This synthesis informs our development of adaptive inspection models that assimilate historical data patterns with real-time process feedback, particularly focusing on scalable solutions for multi-variable quality systems.

The existing IQC methodologies, including full inspection, sampling protocols, and SPC frameworks, have some critical limitations. Current approaches do not reconcile the dual imperatives of zero-defect assurance and cost-effective implementation. Traditional full inspection guarantees defect elimination but incurs unsustainable expenses, whereas sampling and SPC methods optimize costs at the risk of undetected NC parts. Emerging techniques leveraging automation and machine learning improve scalability but lack adaptability to complex, multi-variable systems like lithium-ion battery production, where stringent safety standards demand 100% defect eradication under tight budget constraints. This unresolved challenge highlights a critical gap in scalable, low-cost IQC solutions for high-stakes industries, particularly in rapidly evolving sectors such as electric vehicle manufacturing.

We are devoted to the inspection efficiency optimization problems in IQC processes modeled with set-covering theory. We develop an optimized set covering framework that integrates integer linear programming to achieve large inspection time reduction while maintaining the zero-defect objective.

The detailed structure of the paper is as follows: Section 2 presents the problem formulation and set-covering approach, including the mathematical modeling framework and experimental setup. Section 3 analyzes the computational results across multiple production batches, validating the scalability and robustness of the approach. Section 4 concludes with methodological implications and outlines future research directions in adaptive quality control systems.

2. A Set-Covering Approach to Select Minimal Inspection Items

This section presents the traditional inspection method and its associated challenges. Then, we apply a set-covering approach to resolve the problem and provide an experimental example.

2.1. Traditional Inspection Method

According to the existing IQC inspection method, an inspector randomly and non-repeatedly selects an individual part from a single batch of parts for inspection. Each part has multiple inspection items. If no NC items are found in any of the inspection items for the part, the part is conforming; otherwise, it is NC. During inspection, for the first selected part in the batch, if the first NC inspection item is detected, the part is NC, and the inspection of that part is halted. Inspection proceeds to the next part. A part is conforming if all of its inspection items are conforming. The inspector randomly selects parts from the batch without replacement until the inspection of all parts is completed. NC parts are returned to the supplier while conforming parts are accepted.

Let n be the number of parts in a batch, and m be the number of inspection items for each part. Table 1 shows the worst-case scenario for the number of inspection times conducted by a company during IQC with the traditional inspection method, aiming to achieve zero defects.

Table 1. Worst-case scenario of inspection for IQC.

n \ *m*	40	45	50	55	60
500	20,000	22,500	25,000	27,500	30,000
550	22,000	24,750	27,500	30,250	33,000
600	24,000	27,000	30,000	33,000	36,000
650	26,000	29,250	32,500	35,750	39,000
700	28,000	31,500	35,000	38,500	42,000

Based on the above data, we generate a 3D plane graph to visualize the proportional growth of inspection times relative to two variables: the number of inspection items and the number of parts, shown in Figure 1. The total inspection times are obtained by setting specific ranges for the number of parts and the number of inspection items. It is evident that the number of inspections is linearly related to both the number of inspection items and the number of parts.

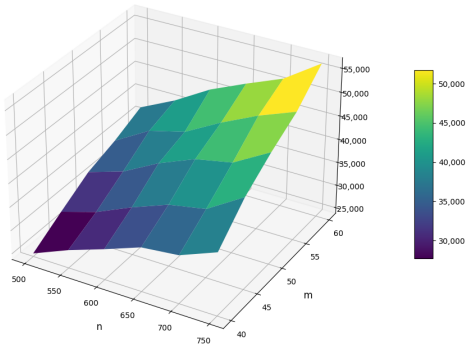


Figure 1. The inspection count linear graph.

The traditional inspection method ensures zero defects, but the large number of inspection times results in high inspection costs, which are unacceptable.

2.2. Selection of Minimal Inspection Items

A set-covering approach is applied to minimize the number of selected items and reduce the number of inspections for a single batch. We initially conduct the inspection for a single batch of parts. By applying the traditional inspection method, we select all NC inspection items to test all parts. In a situation where the quality of parts is relatively stable, the selected inspection items are used to inspect the next batch, maximizing the achievement of the zero-defect objective. This approach aims to eliminate errors caused by manual inspection conducted by inspectors and reduce the number of inspection items, thereby minimizing the number of inspections.

Due to the requirements of the minimal inspection item method, let P be the set of parts, S be the set of inspection items, and P' be the set of NC parts selected from P, $P' \subseteq P$. Let S' denote the set of inspection items selected from S, $S' \subseteq S$. Then, the relationship between an NC part and the inspection item s_j can be represented by an $n' \times m$ matrix A, where

$$A_{ij} = \begin{cases} 1, & p_i \text{ for } s_j \text{ is NC} \\ 0, & p_i \text{ for } s_j \text{ is conforming} \end{cases} \quad (1)$$

In Equation (1), A_{ij} indicates the element in the ith row and jth column of A. Specifically, $A_{ij} = 1$ means that part p_i is NC for inspection item s_j, and $A_{ij} = 0$ indicates that part

p_i is conforming for inspection item s_j. We define an indicator function $F : P \times S \to \{0,1\}$ to indicate whether p_i is NC on inspection item s_j. When $F(p_i, s_j) = 1$, it indicates that part p_i is NC on inspection item s_j, while when $F(p_i, s_j) = 0$, it indicates that part p_i is conforming on inspection item s_j. The example is shown in Table 2.

Table 2. Matrix A.

p_i	s_1	s_2	s_3	s_4
p_1	0	1	0	1
p_2	1	0	0	0
p_3	0	0	1	0
p_4	1	1	0	0
p_5	0	0	0	1

Each element in S' corresponds to at least one NC part, where

$$P' = \{p_i \mid \exists s_j \in S, F(p_i, s_j) = 1\} \tag{2}$$

Item s_j is said to be a critical inspection item of p_i if $F(p_i, s_j) = 1$. Formally, the set of critical inspection items of p_i is

$$S_{c_{p_i}} = \{s_j \in S \mid F(p_i, s_j) = 1\} \tag{3}$$

Equation (3) means that $S_{c_{p_i}}$ represents the subset of inspection items S, and part p_i is NC. Each element in P' corresponds to at least one NC item, where

$$S' = \{s_j \mid \exists p_i \in P', F(p_i, s_j) = 1\} \tag{4}$$

An NC part p_i is said to be a critical one of s_i if $F(p_i, s_j) = 1$. Formally, the set of critical parts of s_j is

$$P_{d_{s_j}} = \{p_i \in P' \mid F(p_i, s_j) = 1\} \tag{5}$$

To minimize the number of selected inspection items, we present a binary variable z_j for each inspection. The variable z_j represents whether or not an inspection item has been selected. In particular, $z_j = 1$ indicates that s_j is selected, and $z_j = 0$ signifies that s_j is not selected. In this way, we obtain a candidate set of inspection items S'. For each $s_j \in S'$, we have

$$\sum_{j=1}^{m} A_{ij} \cdot z_j \geq 1 \tag{6}$$

According to Equation (6), there exists $j \in \{1, 2, \ldots, m\}$ such that $A_{ij} \cdot z_j = 1$. The fact that $A_{ij} \cdot z_j = 1$ means that $A_{ij} = 1$, and $z_j = 1$; that is to say, s_j is selected. In summary, Equation (6) ensures that, for each NC part, at least one of its critical inspection items is selected to determine that it is NC. Then, an objective function is given to select the fewest inspection items, as shown below:

$$\min z = \sum_{j=1}^{m} z_j \tag{7}$$

By combining Equations (6) and (7), an ILPP is created to minimize the number of inspection items to test all NC parts, represented as the set-covering problem of inspection item (SEPII).

SEPII:

$$\min z = \sum_{j=1}^{m} z_j$$

$$\begin{cases} \sum_{j=1}^{m} A_{ij} \cdot z_j \geq 1, & i \in \{1, 2, \ldots, n\} \\ z_j \in \{0, 1\}, & j \in \{1, 2, \ldots, m\} \end{cases} \tag{8}$$

Theorem 1. *SEPII selects the minimal number of inspection items to inspect all NC parts in P.*

Proof of Theorem 1. For each part $p_i \in P$, Equation (6) ensures that there exists $j \in \{1, 2, \ldots, k\}$ such that $A_{ij} \cdot z_j = 1$, suggesting $A_{ij} = 1$ and $z_j = 1$. By $A_{ij} = 1$, part p_i for the inspection item s_j is NC. According to $z_j = 1$, s_j is selected. In summary, at least one inspection item is selected for the NC part p_i. By considering all NC parts in P, Equation (6) ensures that each NC part has at least one selected inspection item. Finally, the objective function minimizes the number of inspection items to be selected. □

Next, we use the selected inspection items to inspect the next batch of parts with the same quantity.

2.3. Synthesis of the Minimal Inspection Item Method

This section proposes a minimal inspection item method for the SEPII problem. It optimizes the selection of inspection items to minimize the inspection times while ensuring the accurate identification of all NC parts. The synthesis process is outlined as follows.

During the inspection process, we have w batches of parts, each of the same quantity. Assuming the quality of the parts remains relatively stable, priority is given to analyze the first batch to identify the number of NC parts and their corresponding inspection items. These identified inspection items are then used to inspect the remaining batches of parts to detect the NC parts.

Let $B = \{B_1, B_2, \ldots, B_w\}$ be the set of w batches of parts, where each batch contains n parts, i.e., $B_k = \{p_{k_1}, p_{k_2}, \ldots, p_{k_n}\}$, for $k \in \{1, 2, \ldots, w\}$. We first inspect all parts in the first batch B_1.

$$P' = \{p_{1i} \in B_1 \mid \exists s_j \in S, F(p_{1i}, s_j) = 1\} \tag{9}$$

With Equation (9), we obtain P', and it is the set of NC parts in B_1. Then, we identify the critical inspection items S'_1.

$$S'_1 = \{s_j \in S \mid \exists p_{1i} \in P', F(p_{1i}, s_j) = 1\} \tag{10}$$

With Equation (10), we obtain S'_1, which is the critical inspection item set that covers all the NC parts in B_1. Then, we use S'_1 to inspect the remaining batches; for each batch B_k, identify the set of NC parts:

$$P'_k = \{p_{ki} \in B_k \mid \exists s_j \in S', F(p_{ki}, s_j) = 1\} \tag{11}$$

This method minimizes the number of inspection items and ensures efficient use of the most critical inspection items to detect NC parts in all batches, reducing the number of inspections required while maintaining accuracy.

2.4. An Instance of the Approach

We have the IQC data of parts from a lithium battery module for a certain automobile. The dataset comprises 20 batches of parts, each containing 500 parts. Each batch includes 53 inspection items, and the sequence of these inspection items is consistent across all batches. We give the part details as shown in Table 3.

Table 3. Part details.

Material Code	Supplier Batch No.	Material Name	Delivery Quantity
3TCCM0010014	46LY211010F1	Nickel Cobalt Manganese Oxide (NCM), CK-63, 100,120	500

The inspection items' details are shown in Table 4.

Table 4. Inspection items details.

Inspection Item	Inspection Item Name	Specification
1	Appearance	Black-gray powder, stable under normal temperature and pressure
2	D10	$\geq$1.2 μm
3	D50	3.0–4.5 μm
4	D90	5.0–9.0 μm
5	D99	$\leq$15.0 μm
6	Specific Surface Area	0.7–1.0 mm^2/g
7	Tapped Density	$\geq$2.0 g/cm^3
8	Moisture Content	$\leq$0.05%
9	Magnetic Substances	<100 ppb
10	PH	$\leq$11.80
11	Residual Total Lithium	$\leq$500 ppm
12	Lithium Content (Li)	6.5–8.0%
13	Nickel Content (Ni)	34.0–38.0%
14	Cobalt Content (Co)	5.0–7.0%
15	Manganese Content (Mn)	15.5–18.5%
16	Iron Content (Fe)	$\leq$0.005%
17	Chromium Content (Cr)	$\leq$0.002%
18	Sodium Content (Na)	$\leq$0.030%
19	Calcium Content (Ca)	$\leq$0.015%
20	Magnesium Content (Mg)	$\leq$0.025%
21	Copper Content (Cu)	$\leq$0.0015%
22	Zinc Content (Zn)	$\leq$0.005%
23	Titanium Content (Ti)	0.06–0.12%
24	Potassium Content (K)	$\leq$0.02%
25	Zirconium Content (Zr)	0.22–0.30%
26	Aluminum Content (Al)	0.07–0.13%
27	Sulfur Impurity Content	$\leq$0.135%
28	Initial Discharge Specific Capacity	175–183 mAh/g
29	Initial Efficiency	85–90%
30	Magnetic Substances $\geq$ 200 μm	$\leq$2 particles
31	Magnetic Substances 100 < D $\leq$ 200 μm	$\leq$20 particles
32	Magnetic Substances 50 < D $\leq$ 100 μm	$\leq$50 particles
33	Oxygen Content	$\leq$2.0%
34	Trace Silicon Content	$\leq$0.002%
35	Carbon Impurity Content	$\leq$0.001%
36	Chloride Content	$\leq$0.003%
37	Particle Size Distribution Deviation	$\leq$5%
38	Surface Oxide Content	$\leq$0.01%
39	Powder Flowability	$\geq$95%
40	Thermal Stability	$\geq$400 °C
41	Discharge Rate Performance	$\geq$80% (10C)
42	Electrical Conductivity	$\geq$20 μS/cm
43	Electrolyte Absorption Performance	$\geq$95%
44	Chemical Stability	$\geq$6 months under normal temperature
45	Dust Content	$\leq$50 ppm
46	Water Absorption Rate	$\leq$0.01%
47	Transportation Temperature Adaptability	−30 °C to +70 °C
48	Sealing Performance	$\leq$1 mbar/24 h

Table 4. *Cont.*

Inspection Item	Inspection Item Name	Specification
49	Anti-Vibration Performance	No physical deformation
50	Explosion Resistance	≥150 psi
51	Particle Size Distribution Uniformity	≤0.5 μm
52	High-Temperature Cycle Performance	≥500 cycles
53	Low-Temperature Cycle Performance	≥300 cycles

Each IQC inspection item corresponds to a specific verification standard for the raw materials of automotive lithium batteries. Next, we use the first batch as the sample for selecting inspection items.

Let E be the estimated number of inspections in the worst-case scenario, and R be the actual number of inspection times required according to the traditional inspection method or the minimal inspection item method. In IQC inspection, the NC rate of parts is also used to assess the effectiveness of the inspection method. Let μ be the NC rate of parts. Then, we have

$$\mu = \frac{n'}{n} \tag{12}$$

Let P^r be the NC parts selected by the traditional inspection method. When we employ the traditional inspection method, $P' = P^r$. Let α be the accuracy rate used to measure the inspection method. Then, we have

$$\alpha = \frac{|P'|}{|P^r|} \tag{13}$$

For the first batch, $\alpha = 1$. The relationship between an inspection item s_j and its corresponding NC parts can be represented by an $n' \times m'$ matrix A' with

$$A'_{ij} = \begin{cases} 1, & \text{if } p_i \in P_{d_{s_j}} \\ 0, & \text{if } p_i \notin P_{d_{s_j}} \end{cases}$$

We identify the selected inspection items from the first batch that can ensure all the NC parts are detected, as shown below.

$$S' = \{s_4, s_6, s_7, s_9, s_{12}, s_{16}, s_{20}, s_{21}, s_{27}, s_{30}, s_{37}, s_{39}, s_{41}, s_{44}, s_{46}, s_{47}, s_{50}, s_{52}\}$$

$$P' = \{P_{d_{s_4}}, P_{d_{s_6}}, P_{d_{s_7}}, P_{d_{s_9}}, P_{d_{s_{12}}}, P_{d_{s_{16}}}, P_{d_{s_{20}}}, P_{d_{s_{21}}}, P_{d_{s_{27}}}, P_{d_{s_{30}}}, P_{d_{s_{37}}}, P_{d_{s_{39}}}, P_{d_{s_{41}}}, P_{d_{s_{44}}}, P_{d_{s_{46}}}, P_{d_{s_{47}}}, P_{d_{s_{50}}}, P_{d_{s_{52}}}\}$$

$$\begin{aligned}
&P_{d_{s_4}} = \{p_{279}, p_{385}\}, && P_{d_{s_6}} = \{p_{496}\}, && P_{d_{s_7}} = \{p_4, p_{493}\}, \\
&P_{d_{s_9}} = \{p_{93}\}, && P_{d_{s_{12}}} = \{p_{25}\}, && P_{d_{s_{16}}} = \{p_{14}\}, \\
&P_{d_{s_{20}}} = \{p_{429}, p_{471}\}, && P_{d_{s_{21}}} = \{p_{196}, p_{499}\}, && P_{d_{s_{27}}} = \{p_{197}\}, \\
&P_{d_{s_{30}}} = \{p_{65}\}, && P_{d_{s_{37}}} = \{p_{228}, p_{485}\}, && P_{d_{s_{39}}} = \{p_{108}, p_{420}\}, \\
&P_{d_{s_{41}}} = \{p_{393}, p_{478}, p_{498}\}, && P_{d_{s_{44}}} = \{p_5, p_{428}\}, && P_{d_{s_{46}}} = \{p_{151}, p_{493}\}, \\
&P_{d_{s_{47}}} = \{p_{292}, p_{390}, p_{494}\}, && P_{d_{s_{50}}} = \{p_1\}, && P_{d_{s_{52}}} = \{p_{19}\}.
\end{aligned}$$

Then, we have the results shown in Table 5.

Table 5. Inspection results for the traditional inspection method.

E	R	$n - \|P'\|$	$\|P'\|$	μ	$\|P^r\|$	α
26,500	25,754	471	29	5.8%	29	100%

Next, we apply the minimal inspection item method to calculate for the next batch of data B_2. We use the inspection items from S' to inspect the next batch of parts, performing item-by-item sequential inspection on all parts, and then compile the relevant inspection results. Since the cost differences between different inspection items are negligible, the weight of each element in S' is set to 1 during the calculation. Then, we have the optimized results as shown in Table 6.

Table 6. Inspection results for the minimal inspection item method.

E	R	$n - \lvert P' \rvert$	$\lvert P' \rvert$	μ	$\lvert P^r \rvert$	α
26,500	17	468	32	6.40%	33	93.94%

We then compare the results of the two methods. The results indicate that using the minimal inspection item method significantly reduces inspection times, as we use the inspection items from the sample batch. However, the inspection accuracy is too low. Next, we will analyze the results to improve the accuracy of the inspection.

3. Experimental Results

In this section, the proposed approach is applied to a more complex example to improve the inspection accuracy. First, we analyze the change in inspection times and NC rates under the condition of the same number of inspection items but varying quantities of parts in different batches of parts. Then, we analyze the scenario with different batches of parts, where the number of parts remains the same, but the inspection items vary.

3.1. Minimal Inspection Item Method for Different Numbers of Parts

During IQC, various batches of parts are inspected simultaneously. Quality management personnel aim to perform IQC on different batches of parts with varying numbers of parts while minimizing inspection times. The selected inspection items for each batch of parts are the same set S' that is selected from the sample batch. Each batch contains a different number of parts, denoted by n_w. Let E_w be the estimated number of inspection times in the worst-case scenario, R_w be the actual number of inspection times required according to the minimal inspection item method, and μ_w be the NC rate of parts for each batch. We redefine the following equations:

$$\mu_w = \frac{|P'_w|}{n_w} \tag{14}$$

Let P^r_w be the actual NC part selected from each batch, and α_w be the accuracy rate used to measure the difference between the minimal inspection item method and the traditional inspection method. Then, we have

$$\alpha_w = \frac{|P'_w|}{P^r_w} \tag{15}$$

By using the minimal inspection item method separately on each batch, we obtain the results shown in Table 7.

Table 7. Inspection results of the minimal inspection item method for reducing batch size.

w	n_w	$\lvert S'_w \rvert$	$n_w - \lvert P'_w \rvert$	$\lvert P'_w \rvert$	μ_w	$\lvert P^r_w \rvert$	α_w
1	500	18	471	29	5.80%	29	100.00%
2	500	18	471	29	5.82%	31	93.94%
3	480	18	452	28	5.89%	30	94.25%
4	460	18	433	27	5.97%	29	94.70%
5	440	18	414	26	5.84%	27	95.25%

Table 7. *Cont.*

w	n_w	$\lvert S'_w \rvert$	$n_w - \lvert P'_w \rvert$	$\lvert P'_w \rvert$	μ_w	$\lvert P^r_w \rvert$	α_w
6	420	18	396	24	5.71%	25	95.95%
7	400	18	378	22	5.57%	23	96.84%
8	380	18	359	21	5.41%	21	97.95%
9	360	18	341	19	5.22%	19	98.92%
10	340	18	322	18	5.29%	18	100.00%
11	320	18	303	17	5.31%	17	100.00%
12	300	18	284	16	5.33%	16	100.00%
13	280	18	265	15	5.36%	15	100.00%
14	260	18	245	15	5.77%	15	100.00%
15	240	18	226	14	5.83%	14	100.00%
16	220	18	207	13	5.91%	13	100.00%
17	200	18	189	11	5.50%	11	100.00%
18	180	18	170	10	5.56%	10	100.00%
19	160	18	151	9	5.63%	9	100.00%
20	140	18	132	8	5.71%	8	100.00%
21	120	18	113	7	5.83%	7	100.00%

As seen in Table 7, the first batch of parts undergoes full inspection to determine the minimum number of inspection items. From the second batch onward, the inspection items selected from the first batch are used, while the number of parts gradually decreases. As the number of parts decreases, the NC rate of parts remains stable. When the number of parts is reduced to a certain level, the accuracy of the minimal inspection item method detection is the same as that of the traditional inspection method, and ultimately, it trends toward zero defects.

3.2. Minimal Inspection Item Method for Different Numbers of Inspection Items

When the number of inspection items varies while the number of parts remains constant, the inspection items for each batch of parts are the same set S_w, and the selected inspection items differ. S'_{total} represents the set of selected inspection item sets across different batches; then, we have

$$S'_{\text{total}} = \{S'_1, S'_2, \ldots, S'_w\}$$

The previous section has proven the accuracy of inspection for different quantities of parts in different batches for the same inspection items. As the number of inspection items increases, the minimal inspection item method is required to identify the minimum number of inspection items for each batch of parts. This section only covers inspection for one batch. We analyze the inspection results, as shown in Table 8.

As the number of inspection items increases, the inspection times also rise, while the NC rate slightly increases and accuracy improves. When the number of inspection items reaches a certain threshold, the accuracy of inspections using the minimal inspection item method begins to match that of the traditional inspection method, achieving zero-defect inspection.

Table 8. Inspection results of the the minimal inspection item method for the same batch size.

w	n_w	S_w	$\lvert S'_w \rvert$	$n_w - \lvert P'_w \rvert$	$\lvert P'_w \rvert$	μ_w	$\lvert P^r_w \rvert$	α_w
1	500	53	18	471	29	5.80%	33	87.88%
2	500	54	18	471	29	5.80%	34	85.29%
3	500	55	18	471	29	5.80%	35	88.23%
4	500	56	18	470	30	6.00%	34	88.23%

Table 8. *Cont.*

w	n_w	S_w	$\lvert S'_w \rvert$	$n_w - \lvert P'_w \rvert$	$\lvert P'_w \rvert$	μ_w	$\lvert P^r_w \rvert$	α_w
5	500	57	19	470	30	6.00%	36	83.33%
6	500	58	19	470	30	6.00%	34	88.23%
7	500	59	18	470	30	6.00%	33	90.91%
8	500	60	18	470	30	6.00%	33	90.91%
9	500	61	18	469	31	6.20%	35	88.57%
10	500	62	20	469	31	6.20%	32	96.88%
11	500	63	20	469	31	6.20%	32	96.88%
12	500	64	20	469	31	6.20%	33	93.94%
13	500	65	21	468	32	6.40%	33	96.97%
14	500	66	21	468	32	6.40%	33	96.97%
15	500	67	21	468	32	6.40%	33	96.97%
16	500	68	21	467	33	6.60%	34	97.06%
17	500	69	22	466	34	6.68%	34	100.00%
18	500	70	22	466	34	6.68%	34	100.00%
19	500	71	22	466	34	6.68%	34	100.00%
20	500	72	22	466	34	6.68%	34	100.00%

4. Conclusions

This paper proposes a set-covering approach for IQC inspection. First, for a given batch of parts, a full inspection is conducted to identify all NC parts. Then, a set-covering approach is applied to determine the minimal set of inspection items such that each NC part is identified by at least one inspection item. These selected inspection items are then used to test the next batch of parts with the same quantity, thereby minimizing inspection times for that batch. Experimental results presented in Tables 5 and 6 demonstrate that this method effectively reduces inspection times. Furthermore, to achieve zero-defect inspection, we conduct experiments by keeping the inspection items fixed while gradually reducing the number of parts. The results shown in Table 7 indicate that decreasing the number of parts to a certain threshold enables zero-defect inspection. We generate a graph as shown in Figure 2 to observe this rule.

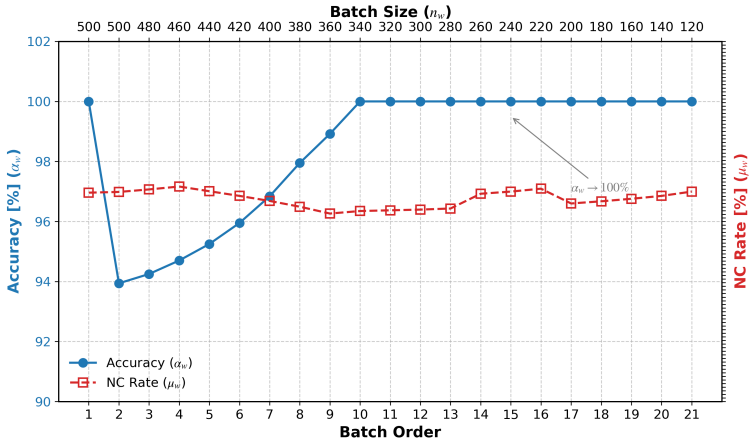


Figure 2. Accuracy improvement with reducing batch size under fixed inspection items.

Additionally, we examine the impact of varying the number of inspection items on batches with a fixed number of parts, as shown in Table 8. We generate a graph as shown in Figure 3.

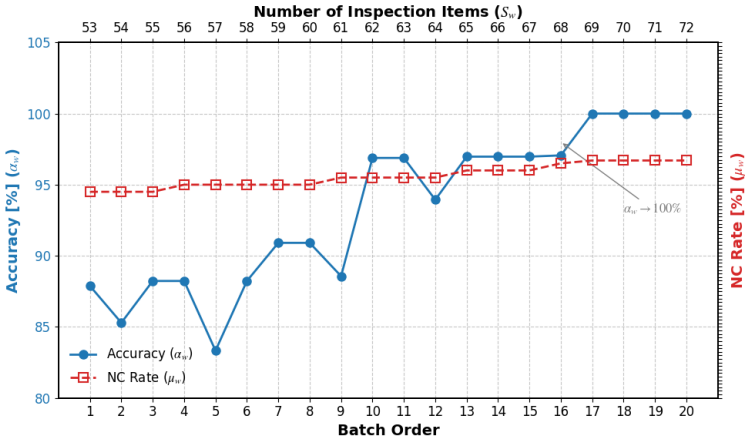


Figure 3. Accuracy improvement with increasing inspection items under fixed batch size.

The results show that as the number of inspection items increases to a certain level, the accuracy of the minimal inspection item method approaches that of the traditional inspection method. We do not consider factors such as the cost of each inspection item or the variability of inspection items for different parts.

In future work, we will focus on scaling up batch sizes while rigorously maintaining 100% detection accuracy and minimizing inspection items. A dynamic optimization strategy will be developed to enhance the set-covering method by systematically incorporating the inherent non-conformity probabilities of individual inspection items. This approach aims to maximize inspection efficiency through adaptive item selection, ensuring optimal coverage of critical defects across expanded production batches.

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