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Two TOPSIS-Based Approaches for Multi-Choice Rough Bi-Level Multi-Objective Nonlinear Programming Problems

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Abstract: The multi-choice rough bi-level multi-objective nonlinear programming problem (MR-BLMNPP) has noticeably risen in various real applications. In the current model, the objective functions have a multi-choice parameter, and the constraints are represented as a rough set. In the first phase, Newton divided differences (NDDs) are utilized to formulate a polynomial of the objective functions. Then, based on the rough set theory, the model is converted into an Upper Approximation Model (UAM) and Lower Approximation Model (LAM). In the second phase, two Technique of Order Preferences by Similarity to Ideal Solution (TOPSIS)-based models are presented to solve the MR-BLMNPP. A TOPSIS-based fuzzy max–min and fuzzy goal programming (FGP) model are applied to tackle the conflict between the modified bi-objective distance functions. An algorithm for solving MR-BLNPP is also presented. The applicability and efficiency of the two TOPSIS-based models suggested in this study are presented through an algorithm and a numerical illustration. Finally, the study presents a bi-level production planning model (BL-PPM) as an illustrative application.

Keywords: bi-level optimization; multi-objective programming; multi-choice programming; rough set; TOPSIS

MSC: 90-06; 90C29; 90C70; 90C90

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Academic Editor: Jun He

Received: 17 February 2025
Revised: 29 March 2025
Accepted: 2 April 2025
Published: 9 April 2025

Citation: El Sayed, M.A.; Farahat, F.A.; Elsisy, M.A.; Alsabaan, M.; Ibrahem, M.I.; Elwahsh, H. Two TOPSIS-Based Approaches for Multi-Choice Rough Bi-Level Multi-Objective Nonlinear Programming Problems. *Mathematics* **2025**, *13*, 1242. https://doi.org/10.3390/math13081242

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1. Introduction

The main mathematical optimization issue for expressing decentralized decision models in a hierarchical organization with several interacting decision-makers (DMs) is the bi-level programming problem (BL-PP). Due to its multiple applications in many significant sectors, including engineering, science, finance, management, banking, economics, agriculture, and so on, BL-PP is extensively researched and the subject of a great deal of interest in the literature [1–4].

Particularly with the development of economic integration and in the age of big data, BL-PP has recently emerged in the decentralized department era and has grown extremely complex [5,6]. A technique for handling a bi-level multi-objective optimization problem (BL-MOOP) was presented by Ranarahu et al. [7]. Baky et al. created the FGP technique to solve fuzzy BL-MOOP [6]. Youness et al. demonstrated BL-MOOP using fuzzy integers [8]. Using concepts from interval programming, Ren created a technique to address the totally fuzzy BL-MOOP [9].

Recently, Chen and Chen focused on multi-level optimization problems (ML-OPs) [3]. FGP models were suggested by Pramanik and Roy to solve ML-OPs [10]. Lachhwani addressed an ML-OP solution using the FGP methodology [4]. Osman et al. demonstrated an interactive method for rational ML-OPs under fuzziness [11]. Osman et al. proposed parametric ideas of rational fuzzy ML-OP [12]. Sharma et al. created a chance-constrained optimization method for a BL-MOOP [13]. Additionally, the approach was used to solve the production planning issue. Masood et al. employed a BL-MOOP to allocate the available water at Pakistan's Taunsa Barrage to all competitive water-related areas in the best possible way [14].

In certain optimization models, when there are several data points for a parameter, multi-choice programming (MCP) is used. The introduction of MCP refers to Healy's [15] study of a particular mixed-integer layout scenario. There are several options for a border in MCP, but only one should be used to maximize the objective function [16–19]. In mixed binary programming or MCP, each binary variable creates several independent options from which one variable must be chosen. The applications of MCP approaches have grown in importance in a variety of fields, including finance, manufacturing, transportation, military reasoning, innovation, and more. The supply and demand parameters should be multi-choice due to variations in an item's market price [11,20–22]. When we convert an MCP problem into a regular numerical programming problem, we extend the following important issues: selecting boundaries for binary codes, limiting binary codes with additional restrictions, and a greater number of possibilities for a specific variable [17]. We use various mathematical approach assumptions, particularly the interpolating polynomial strategy for a multiple-choice parameter, to get over these difficulties. To construct a mixed-integer nonlinear programming issue, consistent interpolating polynomials are used instead of multi-choice parameters [17,23].

Pawlak [24,25] presented rough set theory (RST), a novel mathematical technique for flawed data analysis that addresses uncertainty. Several industries, including healthcare, engineering, decision support, and environmentalism, have identified uses for it [10,26,27]. Unlike fuzzy sets, it conveys ambiguity by using a limit district of a set rather than a membership function. Abou-Elnaga et al. suggested a new approach for constructing the Pareto frontier of rough BL-MOOP [28]. Saad et al. established a treatment for a Rough Interval ML-OP [29]. A rough ML-MOOP has been demonstrated by Emam et al. [30].

The premise behind TOPSIS, a well-known multiple-criteria decision-making (MCDM) technique, is that the alternative that is chosen should be the one that is far from the negative ideal solution (NIS) and the one that is closest to the positive ideal solution (PIS) [2,31,32]. TOPSIS converts the non-commensurable and contradicting m-objectives into bi-objective, commensurate, and frequently conflicting functions. It was initially created to address an MCDM problem by Lai et al. [33]. The TOPSIS idea was expanded by Chen [34] to address multi-person MCDM in a fuzzy domain. For BL-MOOPs, Baky and Abo-Sinna expanded the TOPSIS methodology [2]. Elsayed et al. introduced an adapted TOPSIS approach for BL-MOOPs with ambiguous integers [35]. The TOPSIS technique for fuzzy rough BL-MOOPs was demonstrated by Elsisy and Elsayed [26]. An intuitionistic fuzzy parameter fuzzy BL-MOOPs based on the TOPSIS technique has been suggested by Singh

et al. [36]. A new bi-level TOPSIS-based neutrosophic programming technique for land allocation to medium farm holders was introduced by Angammal and Hannah [37]. For the production planning problem, Kamal et al. employed an FGP algorithm with multi-choice BL-PP [38].

In many real-world situations, it is sometimes impossible to obtain unambiguous data from mathematically modeled data [3,6,30,35,39]. This ambiguity can manifest as a multi-choice question, a rough sense question, or both [6,27,40]. Owing to the ambiguous structure of the issues, it is typically challenging to precisely analyze the raw data in practice, even though the majority of BL-MOOP studies have concentrated on deterministic and fuzzy methods [7,8,10–12]. Consequently, we provide and design a new MR-BLMNPP, which has never been discussed elsewhere before.

In this paper, we introduce MR-BMNPP. In the proposed model, the objective function coefficients are of the multi-choice type, and the set of constraints is modeled as a rough set. Firstly, we avoid the nature of the multi-choice parameters using the NDD interpolating method. Secondly, to tackle the roughness of the constraints, the model is converted into a UAM and LAM. Then, we present two TOPSIS-based models to solve such problems as we first solve the UAM if the obtained solution belongs to the lower approximation set end; otherwise, we must solve the LAM. A TOPSIS-based fuzzy max–min and FGP model was applied to tackle the conflict between the modified bi-objective distance functions. The applicability and efficiency of the two TOPSIS-based models suggested in this study were presented through an algorithm and a numerical illustration. Finally, a BL-PPM is exhibited.

This paper is organized as follows: Following the introduction, Section 2 presents some notions and preliminaries. In Section 3, the mathematical formulation of the MR-BLMNPP was introduced. Section 4 incorporates the two TOPSIS-based models. An algorithm of the two TOPSIS-based models is explained in Section 5. A numerical example, the application of the BLPPM, and comparisons are shown in Section 6. A discussion on the uncertainty is presented in Section 7. Finally, some conclusions are presented in Section 8.

2. Notations

The main ideas of the MCP and RST are introduced in this section [17,41–44]. A linear MCP problem's numerical model is shown as [17,23]

$$\max Z = \sum_{j=1}^{n} c_j x_j \tag{1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq \left\{ b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, \ldots, b_i^{(k_i)} \right\}, \quad i = 1, 2, 3, \ldots, m, \tag{2}$$

$$x_j \geq 0, \quad j = 1, 2, 3, \ldots, n. \tag{3}$$

The right-hand side of Equation (2) has a set of k_i number of goals, out of which only one objective is to be chosen to maximize the objective function.

Definition 1. *Let Θ be the universal set, $\mathcal{R}$ be the equivalence relation on Θ, $[\theta]_{\mathcal{R}}$ be the set of equivalence class of $\mathcal{R}$, and Ω be a non-empty subset of Θ. The upper and lower approximations of the set $\boldsymbol{\Omega}$ are defined as*

$$\overline{\mathcal{R}\boldsymbol{\Omega}} = \{\theta \in \Theta : [\theta]_{\mathcal{R}} \cap \boldsymbol{\Omega} \neq \emptyset\}$$
$$\underline{\mathcal{R}\boldsymbol{\Omega}} = \{\theta \in \Theta : [\theta]_{\mathcal{R}} \subseteq \boldsymbol{\Omega}\}$$
$$\aleph\boldsymbol{\Omega} = \overline{\mathcal{R}\boldsymbol{\Omega}} - \underline{\mathcal{R}\boldsymbol{\Omega}}$$

If $\aleph\boldsymbol{\Omega} \neq \emptyset$, then set $\boldsymbol{\Omega}$ is called a rough set [24,25].

Definition 2. *The collection of all sets with the same upper and lower approximations is called a rough set, denoted by* $\left(\overline{\mathcal{R}\Omega}, \underline{\mathcal{R}\Omega}\right)$ [24].

3. Mathematical Formulation

The MR-BLMNPP can be modeled as follows, where the decision variables X_1 and X_2 are controlled by the leader and the follower [45–47]:

[Leader]

$$\max_{X_1} f_{1j}(X) = \sum_{k=1}^{r_1} \left\{ c_k^{1j(1)}, c_k^{1j(2)}, \ldots, c_k^{1j(v)} \right\} \prod_{l=1}^{n} x_l^{\beta_{1jkl}},\ j = 1, 2, \ldots, Q_1 \tag{4}$$

where X_2 solves

[Follower]

$$\max_{X_2} f_{2j}(X) = \sum_{k=1}^{r_2} \left\{ c_k^{2j(1)}, c_k^{2j(2)}, \ldots, c_k^{2j(v)} \right\} \prod_{l=1}^{n} x_l^{\beta_{2jkl}},\ j = 1, 2, \ldots, Q_2 \tag{5}$$

subject to

$$x \in S(X)$$

where $S(X)$ is a rough attainable region, and $\underline{S}(X) \subseteq S(X) \subseteq \overline{S}(X)$, $\underline{S}(X)$ and $\overline{S}(X)$ are the lower and upper approximation sets for $S(X)$.

$$\underline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} g_i(X) \leq 0, & i = 1, 2 \ldots, r_1 \\ g_i(X) = 0, & i = r_1 + 1, \ldots, r_2 \\ g_i(X) \geq 0, & i = r_2 + 1, \ldots, m_1 \end{array} \right\} \neq \emptyset \tag{6}$$

$$\overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} G_i(X) \leq 0, & i = 1, 2 \ldots, r_1 \\ G_i(X) = 0, & i = r_1 + 1, \ldots, r_2 \\ G_i(X) \geq 0, & i = r_2 + 1, \ldots, m_2 \end{array} \right\} \neq \emptyset \tag{7}$$

Also, $x_1 = (x_{11}, x_{12}, \ldots, x_{1n_1}) \in R^{n_1}, x_2 = (x_{21}, x_{22}, \ldots, x_{2n_2}) \in R^{n_2}, X = (x_1, x_2) \in R^n$, and x_l represents any variable of x_1 and x_2, $n = n_1 + n_2$. $f_{ij}(\mathrm{X})$, $i = 1, 2, j = 1, 2, \ldots, Q_i$ is a nonlinear polynomial function with multi-choice coefficients, $c_k^{1j(1)}, c_k^{1j(2)}, \ldots, c_k^{1j(q)}$, $c_k^{2j(1)}, c_k^{2j(2)}, \ldots, c_k^{2j(v)} \in R; \beta_{ijkl} \in R$.

Modeling Objective Function via NDD Interpolation

For the multi-choice parameters, interpolating polynomials are created while taking certain integral values for the node points into account [39,43]. Each node corresponds to exactly one multi-choice parameter value. If a parameter has v alternatives, then only v nodes are needed. At these nodes, the produced interpolating polynomial exactly crosses the affine function once. Using the NDD approach, we replace a multi-choice parameter with an interpolating polynomial [39]. Suppose that $0, 1, 2, \ldots, v-1$ for v number of nodes where $c_k^{1j(1)}$, $c_k^{1j(2)}$, $\ldots$, $c_k^{1j(v)}$ is the associated functional values of the v distinct nodes' interpolating polynomial, as shown in Table 1. Therefore, a polynomial $P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)$ of degree $(v-1)$ will be used to interpolate the given data [39,41]:

$$P_{v-1}(\gamma) = \left(c_k^{1j(v)}\right)^{\gamma+1},\ \gamma = 0, 1, 2, \ldots, v-1;\ k = 1, 2 \ldots, r_1;\ j = 1, 2, \ldots, Q_1, \tag{8}$$

Table 1. Data points.

$\gamma = \omega^{\gamma}_{c_k^{1j(v)}}$	0	1	2	...	$v-1$
$f\left(\omega^{\gamma}_{c_k^{1j(v)}}\right) = c_k^{1j(v)}$	$c_k^{1j(1)}$	$c_k^{1j(2)}$	$c_k^{1j(3)}$	...	$c_k^{1j(v)}$

We use the form found in Table 2 to establish the NDD. The associated formula is utilized to ascertain the NDDs [39,41]:

$$\begin{aligned} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right) = \; & f\left[\omega^{0}_{c_k^{1j(v)}}\right] + \left(\omega_{c_k^{1j}}\right) f\left[\omega^{0}_{c_k^{1j(1)}}, \omega^{1}_{c_k^{1j(2)}}\right] \\ & + \left(\omega_{c_k^{1j}}\right)\left(\omega_{c_k^{1j}} - \omega^{1}_{c_k^{1j(v)}}\right) f\left[\omega^{0}_{c_k^{1j(v)}}, \omega^{1}_{c_k^{1j(v)}}, \omega^{2}_{c_k^{1j(v)}}\right] + \ldots \\ & + \left(\omega_{c_k^{1j}}\right)\left(\omega_{c_k^{1j}} - \omega^{1}_{c_k^{1j(v)}}\right) \ldots \left(\omega_{c_k^{1j}} - \omega^{v-1}_{c_{ij}^{(q)}}\right) f\left[\omega^{0}_{c_k^{1j(v)}}, \omega^{1}_{c_k^{1j(v)}}, \ldots, \omega^{v-1}_{c_k^{1j(v)}}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right) = \; & c_k^{1j(1)} + \left(\omega_{c_k^{1j}}\right)\left(c_k^{1j(2)} - c_k^{1j(1)}\right) \\ & + \left(\omega_{c_k^{1j}}\right)\left(\omega_{c_k^{1j}} - 1\right)\left(\frac{c_k^{1j(3)} - 2c_k^{1j(2)} + c_k^{1j(1)}}{\omega^{2}_{c_k^{1j}} - \omega^{0}_{c_k^{1j}}}\right) + \ldots \\ & + \left(\sum_{\gamma=1}^{v} \frac{c_k^{1j(\gamma)}}{\prod_{\gamma \neq l+1, l=0}^{v-1}\left(\omega^{\gamma-1}_{c_k^{1j}} - \omega^{l}_{c_k^{1j}}\right)}\right) \end{aligned} \tag{10}$$

where $f\left[\omega^{0}_{c_k^{1j(1)}}, \omega^{1}_{c_k^{1j(2)}}\right] = c_k^{1j(2)} - c_k^{1j(1)}$ and $f\left[\omega^{0}_{c_k^{1j(1)}}, \omega^{1}_{c_k^{1j(2)}}, \omega^{2}_{c_k^{1j(3)}}\right] = \left(\frac{c_k^{1j(3)} - 2c_k^{1j(2)} + c_k^{1j(1)}}{\omega^{2}_{c_k^{1j}} - \omega^{0}_{c_k^{1j}}}\right)$,

also, $P_{u-1}\left(\vartheta^{\gamma}_{c_k^{2j(v)}}\right)$ may be expressed as

$$\begin{aligned} P_{u-1}\left(\vartheta^{\gamma}_{c_k^{2j(v)}}\right) = \; & c_k^{2j(1)} + \left(\vartheta_{c_k^{2j}}\right)\left(c_k^{2j(2)} - c_k^{2j(1)}\right) \\ & + \left(\vartheta_{c_k^{2j}}\right)\left(\vartheta_{c_k^{2j}} - 1\right)\left(\frac{c_k^{2j(3)} - 2c_k^{2j(2)} + c_k^{2j(1)}}{\vartheta^{2}_{c_k^{2j}} - \vartheta^{0}_{c_k^{2j}}}\right) + \ldots \\ & + \left(\sum_{\gamma=1}^{v} \frac{c_k^{2j(\gamma)}}{\prod_{\gamma \neq l+1, l=0}^{v-1}\left(\vartheta^{\gamma-1}_{c_k^{2j}} - \vartheta^{l}_{c_k^{2j}}\right)}\right) \end{aligned} \tag{11}$$

Table 2. Divided difference table.

$\omega^{\gamma}_{c_{ij}^{v(q)}}$	$f\left(\omega^{\gamma}_{c_{ij}^{v(q)}}\right)$	First DD	Second DD	...	$(v-1)^{th}$ order DD
0	$c_k^{1j(1)}$				
		$f\left[\omega^{0}_{c_k^{1j(1)}}, \omega^{1}_{c_k^{1j(2)}}\right]$			
1	$c_k^{1j(2)}$		$f\left[\omega^{0}_{c_{ij}^{1(q)}}, \omega^{1}_{c_{ij}^{2(q)}}, \omega^{2}_{c_{ij}^{3(q)}}\right]$		
		$f\left[\omega^{1}_{c_k^{1j(2)}}, \omega^{2}_{c_k^{1j(3)}}\right]$			
2	$c_k^{1j(3)}$		$f\left[\omega^{1}_{c_k^{1j(1)}}, \omega^{2}_{c_k^{1j(2)}}, \omega^{3}_{c_k^{1j(3)}}\right]$		
		$f\left[\omega^{2}_{c_k^{1j(3)}}, \omega^{3}_{c_k^{1j(4)}}\right]$			
3	$c_k^{1j(4)}$				

Table 2. *Cont.*

...	...		$f\left[\omega^{(v-3)}_{c^{(v-2)(q)}_{ij}}, \omega^{(v-2)}_{c^{(v-1)(q)}_{ij}}, \omega^{(v-1)}_{c^{v(q)}_{ij}}\right]$
$v-2$	$c_k^{1j(v-2)}$	$f\left[\omega^{(v-3)}_{c_k^{1j(v-2)}}, \omega^{(v-2)}_{c_k^{1j(v-1)}}, \omega^{(v-1)}_{c_k^{1j(v)}}\right]$	
$v-1$	$c_k^{1j(v-1)}$	$f\left[\omega^{v-2}_{c_k^{1j(v-1)}}, \omega^{v-1}_{c_k^{1j(v)}}\right]$	

Similarly, the interpolating polynomials that were put up for the leader and follower objective function coefficients were expressed as $P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)$, $P_{u-1}\left(\vartheta^{\gamma}_{c_k^{2j(v)}}\right)$, and the multi-choice parameters in the subsequent mathematical model were subrogated by their interpolating polynomial. The rough set of constraints for the MR-BLMNPP evolves into a UAM and LAM [6,20,47]:

UAM:

$$\underbrace{\max}_{x_1} f_{1j}(X,\omega) = \sum_{k=1}^{r_1} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}, \quad j = 1,2,\ldots,Q_1 \tag{12}$$

where x_2 solves

$$\underbrace{\max}_{x_2} f_{2j}(X,\vartheta) = \sum_{k=1}^{r_2} P_{u-1}\left(\vartheta^{\gamma}_{c_k^{2j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{2jkl}}, \quad j = 1,2,\ldots,Q_2 \tag{13}$$

subject to

$$x \in \overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} G_i(X) \le 0, & i = 1,2\ldots,r_1 \\ G_i(X) = 0, & i = r_1+1,\ldots,r_2 \\ G_i(X) \ge 0, & i = r_2+1,\ldots,m_2 \end{array} \right\} \neq \emptyset \tag{14}$$

LAM :

$$\underbrace{\max}_{x_1} f_{1j}(X,\omega) = \sum_{k=1}^{r_1} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}, \quad j = 1,2,\ldots,Q_1 \tag{15}$$

where x_2 solves

$$\underbrace{\max}_{x_2} f_{2j}(X,\vartheta) = \sum_{k=1}^{r_2} P_{u-1}\left(\vartheta^{\gamma}_{c_k^{2j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{2jkl}}, \quad j = 1,2,\ldots,Q_2 \tag{16}$$

subject to

$$x \in \underline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} g_i(X) \le 0, & i = 1,2\ldots,r_1 \\ g_i(X) = 0, & i = r_1+1,\ldots,r_2 \\ g_i(X) \ge 0, & i = r_2+1,\ldots,m_1 \end{array} \right\} \neq \emptyset \tag{17}$$

Definition 3. *For any feasible $x_1\left(x_1 \in \overline{S}(X)\right)$ given by the leader if x_2 $\left(x_2 \in \overline{S}(X)\right)$ is the Pareto optimal solution of the BL-MNPP, then (x_1, x_2) is a feasible solution for the UAM.*

Definition 4. *A point x^** is a *Pareto optimal solution for the UAM if no other feasible solution $x \in \overline{S}(X)$ exists, such that $f_{1j}(X^*,\omega) \le f_{1j}(X,\omega)$ for at least one objective function $f_{1j}(X,\omega)$.*

Definition 5. *x^* is called the surely Pareto optimal solution iff x^* is the Pareto optimal solution of the MR-BLMNPP for the UAM and $x^* \in \underline{S}(X)$. Otherwise, this solution is called a possibly Pareto optimal solution.*

4. TOPSIS-Based Models for MR-BLMNPP

The two TOPSIS-based models for solving the MR-BLMNPP are presented in this section after calculating the UAM and LAM using rough set approximation and NDD interpolation [28,31,39,46]. Prior to solving the LAM, we solve the UAM using two TOPSIS models. If the result is surely Pareto optimal, we stop.

4.1. The TOPSIS Model for the Leader Problem of the UAM

The TOPSIS approach is used to handle the following leader issue [6,31,46]:

$$\underbrace{\max}_{x_1} f_{1j}(X,\omega) = \sum_{k=1}^{r_1} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}, \quad j = 1,2,\ldots,Q_1 \tag{18}$$

subject to

$$x \in \overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} G_i(X) \le 0, & i = 1,2\ldots,r_1 \\ G_i(X) = 0, & i = r_1+1,\ldots,r_2 \\ G_i(X) \ge 0, & i = r_2+1,\ldots,m_2 \end{array} \right\} \neq \emptyset \tag{19}$$

The bi-objective distance equations for the leader issue are developed in accordance with Lai et al. [33]:

$$\min d_P^{PIS^F}(X) = \left\{ \sum_{j=1}^{Q_1} \delta_{1j}^p \left[\frac{f_{1j}^* - \sum_{k=1}^{r_1} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}}{f_{1j}^* - f_{1j}^-} \right]^p \right\}^{\frac{1}{p}} \tag{20}$$

$$\max d_P^{NIS^F}(X) = \left\{ \sum_{j=1}^{Q_1} \delta_{1j}^p \left[\frac{\sum_{k=1}^{r_1} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}} - f_{1j}^-}{f_{1j}^* - f_{1j}^-} \right]^p \right\}^{\frac{1}{p}} \tag{21}$$

subject to

$$x \in \overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} G_i(X) \le 0, & i = 1,2\ldots,r_1 \\ G_i(X) = 0, & i = r_1+1,\ldots,r_2 \\ G_i(X) \ge 0, & i = r_2+1,\ldots,m_2 \end{array} \right\} \neq \emptyset \tag{22}$$

where $f_{1j}^* = \max\limits_{x\in\overline{S}(x)} \sum_{k=1}^{r_1} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}$ is the unique PIS, $f_{1j}^- = \min\limits_{x\in\overline{S}(x)} \sum_{k=1}^{r_1} P_{v-1}\left(\omega^{\gamma}_{c_k^{1j(v)}}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}$ is the unique NIS, and δ_{1j} is the goals' respective weights. We ensure that the membership functions $\mu_F^{PIS}(X)$ and $\mu_F^{NIS}(X)$ of Model (20)–(22) is linear across $\left(d_p^F\right)^*$ and $\left(d_p^F\right)^-$, which are

$$\left(d_P^{PIS^F}\right)^* = \min_{x\in\overline{S}(\boldsymbol{x})} d_P^{PIS^F}(X) \quad \& \quad \left(d_P^{PIS^F}\right)^- = \max_{x\in\overline{S}(\boldsymbol{x})} d_P^{PIS^F}(X) \tag{23}$$

$$\left(d_P^{NIS^k}\right)^* = \max_{x\in\overline{S}(\boldsymbol{x})} d_P^{NIS^k}(X) \quad \& \quad \left(d_P^{NIS^F}\right)^- = \min_{x\in\overline{S}(\boldsymbol{x})} d_P^{NIS^F}(X) \tag{24}$$

Thus, $\mu_F^{PIS}(X) \equiv \mu_{d_P^{PIS^F}}(X)$ and $\mu_F^{NIS}(X) \equiv \mu_{d_P^{NIS^F}}(X)$ are obtainable as [2,31,32]

$$\mu_F^{PIS}(X) = \begin{cases} 1 & \text{if } d_P^{PIS^F}(X) < \left(d_P^{PIS^F}\right)^* \\ \frac{\left(d_P^{PIS^F}\right)^- - d_P^{PIS^F}(x)}{\left(d_P^{PIS^F}\right)^- - \left(d_P^{PIS^F}\right)^*} & \text{if } \left(d_P^{PIS^F}\right)^* \le d_P^{PIS^F}(X) \le \left(d_P^{PIS^F}\right)^- \\ 0 & \text{if } \left(d_P^{PIS^F}\right)^- < d_P^{PIS^F}(X) \end{cases} \tag{25}$$

$$\mu_F^{NIS}(X) = \begin{cases} 1 & \text{if } d_P^{NIS^F}(X) > \left(d_P^{NIS^F}\right)^* \\ \frac{d_P^{NIS^F}(x) - \left(d_P^{NIS^F}\right)^-}{\left(d_P^{NIS^F}\right)^* - \left(d_P^{NIS^F}\right)^-} & \text{if } \left(d_P^{NIS^F}\right)^- \le d_P^{NIS^F}(X) \le \left(d_P^{NIS^F}\right)^* \\ 0 & \text{if } d_P^{NIS^F}(X) < \left(d_P^{NIS^F}\right)^- \end{cases} \tag{26}$$

The first model used is the fuzzy max–min decision model [2,31,46]. Thus, Model (20)–(22) is equivalent to the later Tchebycheff edition.

model (*I*):

$$\max \lambda \tag{27}$$

subject to

$$(1-\lambda)\left(d_P^{PIS^F}\right)^- - \left\{\sum_{j=1}^{Q_1} \delta_{1j}^p \left[\frac{f_{1j}^* - \sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}}{f_{1j}^* - f_{1j}^-}\right]^p\right\}^{\frac{1}{p}} \ge -\lambda\left(d_P^{PIS^F}\right)^* \tag{28}$$

$$\left\{\sum_{j=1}^{Q_1} \delta_{1j}^p \left[\frac{\sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}} - f_{1j}^-}{f_{1j}^* - f_{1j}^-}\right]^p\right\}^{\frac{1}{p}} + (\lambda-1)\left(d_P^{NIS^F}\right)^- \ge \lambda\left(d_P^{NIS^F}\right)^* \tag{29}$$

$$x \in \overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} G_i(X) \le 0, & i = 1, 2 \ldots, r_1 \\ G_i(X) = 0, & i = r_1 + 1, \ldots, r_2 \\ G_i(X) \ge 0, & i = r_2 + 1, \ldots, m_2 \end{array} \right\} \ne \emptyset \tag{30}$$

Here is how the additional model, which relies on the FGP method [6,47], resolves the conflicting bi-objective distance functions in the leader issue:

model (*II*):

$$\min Z = D_F^{PIS^+} + D_F^{NIS^-}$$

subject to

$$\frac{\left(d_P^{PIS^F}\right)^- - \left\{\sum_{j=1}^{Q_1} \delta_{1j}^p \left[\frac{f_{1j}^* - \sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}}{f_{1j}^* - f_{1j}^-}\right]^p\right\}^{\frac{1}{p}}}{\left(d_P^{PIS^F}\right)^- - \left(d_P^{PIS^F}\right)^*} + D_F^{PIS^-} - D_F^{PIS^+} = 1, \tag{31}$$

$$\frac{\left\{\sum_{j=1}^{Q_1} \delta_{1j}^p \left[\frac{\sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}} - f_{1j}^-}{f_{1j}^* - f_{1j}^-}\right]^p\right\}^{\frac{1}{p}} - \left(d_P^{NIS^F}\right)^-}{\left(d_P^{NIS^F}\right)^* - \left(d_P^{NIS^F}\right)^-} + D_F^{NIS^-} - D_F^{NIS^+} = 1, \tag{32}$$

$$x \in \overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} G_i(X) \le 0, & i = 1,2\ldots,r_1 \\ G_i(X) = 0, & i = r_1+1,\ldots,r_2 \\ G_i(X) \ge 0, & i = r_2+1,\ldots,m_2 \end{array} \right\} \neq \emptyset \tag{33}$$

The fundamental tenet of the BLOP is that the leader makes decisions and then solicits the follower's optimal course of action [5,6,30,31]. To avoid extensive computational efforts, we use the models presented in [2,6]. To demonstrate the effectiveness of the suggested fuzzy max–min model and FGP model, that is, the UAM for the MR-BLMNPP has a workable solution even in the absence of any leniency for the leader choice variables, we have used the leader's decision variables as restricting constraints.

4.2. TOPSIS Models for UAM of MR-BLMNPP

By modifying the distance families $d_P^{PIS^B}$ and $d_P^{NIS^B}$, the two suggested TOPSIS-based models can be applied to obtain a solution to MR-BLMNPP.

$$d_P^{PIS^B}(x) = \left\{ \sum_{i=1}^{2} \left(\sum_{j=1}^{Q_i} \delta_{ij}^p \left[\frac{f_{ij}^* - \sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{1jkl}}}{f_{ij}^* - f_{ij}^-} + \frac{f_{ij}^* - \sum_{k=1}^{r_2} P_{u-1}\left(\vartheta_{c_k^{2j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{2jkl}}}{f_{ij}^* - f_{ij}^-} \right]^p \right) \right\}^{\frac{1}{p}} \tag{34}$$

$$d_P^{NIS^B}(x) = \left\{ \sum_{i=1}^{2} \left(\sum_{j=1}^{Q_i} \delta_{ij}^p \left[\frac{\sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{1jkl}} - f_{ij}^-}{f_{ij}^* - f_{ij}^-} + \frac{\sum_{k=1}^{r_2} P_{u-1}\left(\vartheta_{c_k^{2j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{2jkl}} - f_{ij}^-}{f_{ij}^* - f_{ij}^-} \right]^p \right) \right\}^{\frac{1}{p}} \tag{35}$$

where $\delta_{ij}, i = 1,2, j = 1,2,\ldots,Q_i$ are the objectives' respective weights. And also, $f_{ij}^* = \max\limits_{x \in \overline{S}(x)} f_{ij}(X)$, $f_{ij}^- = \min\limits_{x \in \overline{S}(x)} f_{ij}(X), i = 1,2, j = 1,2,\ldots,Q_i$, correspondingly. Next, the MR-BLMNPP, as expressed in Equations (12)–(14), was translated into the following distance functions [2,46]:

$$\min d_P^{PIS^B}(x) = \left\{ \sum_{i=1}^{2} \left(\sum_{j=1}^{Q_i} \delta_{ij}^p \left[\begin{array}{l} \frac{f_{ij}^* - \sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{1jkl}}}{f_{ij}^* - f_{ij}^-} \\ + \frac{f_{ij}^* - \sum_{k=1}^{r_2} P_{u-1}\left(\vartheta_{c_k^{2j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{2jkl}}}{f_{ij}^* - f_{ij}^-} \end{array} \right]^p \right) \right\}^{\frac{1}{p}} \tag{36}$$

$$\max d_P^{NIS^B}(x) = \left\{ \sum_{i=1}^{2} \left(\sum_{j=1}^{Q_i} \delta_{ij}^p \left[\begin{array}{l} \frac{\sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{1jkl}} - f_{ij}^-}{f_{ij}^* - f_{ij}^-} \\ + \frac{\sum_{k=1}^{r_2} P_{u-1}\left(\vartheta_{c_k^{2j(v)}}^{\gamma}\right) \prod_{l=1}^{n} x_l^{\beta_{2jkl}} - f_{ij}^-}{f_{ij}^* - f_{ij}^-} \end{array} \right]^p \right) \right\}^{\frac{1}{p}} \tag{37}$$

subject to

$$x \in \overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{lll} G_i(X) \le 0, & i = 1,2\ldots,r_1 & \\ G_i(X) = 0, & i = r_1+1,\ldots,r_2, & x_1 = x_1^* \\ G_i(X) \ge 0, & i = r_2+1,\ldots,m_2 & \end{array} \right\} \neq \emptyset \tag{38}$$

Then, the membership functions $\mu_{d_P^{PIS^B}}(X)$ and $\mu_{d_P^{NIS^B}}(X)$ are built [2,31,46] as

$$\mu_{d_P^{PIS^B}}(X) = \begin{cases} 1 & \text{if} \quad d_P^{PIS^B}(X) < \left(d_P^{PIS^B}\right)^* \\ \frac{\left(d_P^{PIS^B}\right)^- - d_P^{PIS^B}(x)}{\left(d_P^{PIS^B}\right)^- - \left(d_P^{PIS^B}\right)^*} & \text{if } \left(d_P^{PIS^B}\right)^* \le d_P^{PIS^B}(X) \le \left(d_P^{PIS^B}\right)^- \\ 0 & \text{if } \left(d_P^{PIS^B}\right)^- < d_P^{PIS^B}(X) \end{cases} \tag{39}$$

$$\mu_{d_P^{NIS^B}}(X) = \begin{cases} 1 & \text{if } d_P^{NIS^B}(X) > \left(d_P^{NIS^B}\right)^* \\ \frac{d_P^{NIS^B}(x) - \left(d_P^{NIS^B}\right)^-}{\left(d_P^{NIS^B}\right)^* - \left(d_P^{NIS^B}\right)^-} & \text{if } \left(d_P^{NIS^B}\right)^- \le d_P^{NIS^B}(X) \le \left(d_P^{NIS^B}\right)^* \\ 0 & \text{if } d_P^{NIS^B}(X) < \left(d_P^{NIS^B}\right)^- \end{cases} \tag{40}$$

Accordingly, we may create the final fuzzy max–min model (I) and FGP model (II) as follows:

Model (*I*):

$$\max \tau \tag{41}$$

subject to

$$(1-\tau)\left(d_P^{PIS^B}\right)^- - \left\{\sum_{i=1}^{2}\left(\sum_{j=1}^{Q_i}\delta_{ij}^p\left[\frac{f_{ij}^* - \sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}}{f_{ij}^* - f_{ij}^-} + \frac{f_{ij}^* - \sum_{k=1}^{r_2} P_{u-1}\left(\vartheta_{c_k^{2j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{2jkl}}}{f_{ij}^* - f_{ij}^-}\right]^p\right)\right\}^{\frac{1}{p}} \ge -\tau\left(d_P^{PIS^B}\right)^* \tag{42}$$

$$\left\{\sum_{i=1}^{2}\left(\sum_{j=1}^{Q_i}\delta_{ij}^p\left[\frac{\sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}} - f_{ij}^-}{f_{ij}^* - f_{ij}^-} + \frac{\sum_{k=1}^{r_2} P_{u-1}\left(\vartheta_{c_k^{2j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{2jkl}} - f_{ij}^-}{f_{ij}^* - f_{ij}^-}\right]^p\right)\right\}^{\frac{1}{p}} + (\tau-1)\left(d_P^{NIS^B}\right)^- \ge \tau\left(d_P^{NIS^B}\right)^* \tag{43}$$

$$x \in \overline{S}(X) = \left\{ x \in R^n \middle| \begin{array}{ll} G_i(X) \le 0, & i = 1, 2 \ldots, r_1 \\ G_i(X) = 0, & i = r_1+1, \ldots, r_2, \\ G_i(X) \ge 0, & i = r_2+1, \ldots, m_2 \end{array} \right\} \ne \emptyset, \quad x_1 = x_1^*, \tag{44}$$

Drawing on the FGP technique, the second proposed model is as follows:

Model (*II*):

$$\min Z = D_B^{PIS^+} + D_B^{NIS^-} \tag{45}$$

subject to

$$\frac{\left(d_P^{PIS^B}\right)^- - \left\{\sum_{i=1}^{2}\left(\sum_{j=1}^{Q_i}\delta_{ij}^p\left[\frac{f_{ij}^* - \sum_{k=1}^{r_1} P_{v-1}\left(\omega_{c_k^{1j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{1jkl}}}{f_{ij}^* - f_{ij}^-} + \frac{f_{ij}^* - \sum_{k=1}^{r_2} P_{u-1}\left(\vartheta_{c_k^{2j(v)}}^{\gamma}\right)\prod_{l=1}^{n} x_l^{\beta_{2jkl}}}{f_{ij}^* - f_{ij}^-}\right]^p\right)\right\}^{\frac{1}{p}}}{\left(d_P^{PIS^B}\right)^- - \left(d_P^{PIS^B}\right)^*} + D_B^{PIS^-} - D_B^{PIS^+} = 1 \tag{46}$$

$$\frac{\left\{\sum_{i=1}^{2}\left(\sum_{j=1}^{Q_i}\delta_{ij}^{p}\left[\begin{array}{l}\frac{\sum_{k=1}^{r_1}P_{v-1}\left(\omega^{\gamma}_{c^{1j(v)}_{k}}\right)\prod_{l=1}^{n}x_l^{\beta_{1jkl}}-f_{ij}^{-}}{f_{ij}^{*}-f_{ij}^{-}}\\ +\frac{\sum_{k=1}^{r_2}P_{u-1}\left(\vartheta^{\gamma}_{c^{2j(v)}_{k}}\right)\prod_{l=1}^{n}x_l^{\beta_{2jkl}}-f_{ij}^{-}}{f_{ij}^{*}-f_{ij}^{-}}\end{array}\right]^{p}\right)\right\}^{\frac{1}{p}}-\left(d_P^{NIS^B}\right)^{-}}{\left(d_P^{NIS^B}\right)^{*}-\left(d_P^{NIS^B}\right)^{-}}+D_B^{NIS^-}-D_B^{NIS^+}=1 \quad (47)$$

$$x\in\overline{S}(X)=\left\{x\in R^n\left|\begin{array}{ll}G_i(X)\le 0, & i=1,2\ldots,r_1\\ G_i(X)=0, & i=r_1+1,\ldots,r_2,\\ G_i(X)\ge 0, & i=r_2+1,\ldots,m_2\end{array}\right.\right\}\neq\emptyset,\qquad \boldsymbol{x}_1=\boldsymbol{x}_1^{*}, \quad (48)$$

5. Algorithm of the Two TOPSIS-Based Models for the MR-BLMNPP

Following the above description of the MR-BLMNPP, the procedure for the two TOPSIS-based models that were suggested for this model is described below:

Step 1:	Formulate the MR-BLMNPP.
Step 2:	Apply the NDD interpolation to tackle the multi-choice uncertainty in the objective functions using Equations (10) and (11).
Step 3:	Construct the UAM Equations (12)–(14) and LAM Equations (15)–(17).
Step 4:	Calculate the individual minimum and maximum values for the objective functions.
Step 5:	Construct the PIS and NIS payoff tables of the leader problem in Equations (18) and (19).
Step 6:	Set up $d_2^{PIS^F}(x)$& $d_2^{NIS^F}(x)$ according to Equations (20) and (21).
Step 7:	Elicit the membership functions $\mu_{d_2^{PIS^F}}(x)$ and $\mu_{d_2^{NIS^F}}(x)$ using Equations (25) and (26).
Step 8:	If the leader accepts model (**I**) to solve the MR-BLMNPP, then go to Step 9; otherwise, if it decides to use model (**II**), go to Step 10.
Step 9:	Formulate and solve model (**I**) (Equations (27)–(30)) for the leader of the UAM, then go to Step 11.
Step 10:	Formulate and solve model (**II**) (Equations (31)–(33)) for the leader of the UAM, then go to Step 11.
Step 11:	Set $\boldsymbol{x}_{1l}=\boldsymbol{x}_{1l}^{*}=\left(x_{11}^{*},x_{12}^{*},\ldots,x_{1n_1}^{*}\right)$.
Step 12:	Construct the PIS and NIS payoff tables of the UAM of the MR-BLMNPP.
Step 13:	Set up $d_2^{PIS^B}(x)$ and $d_2^{NIS^B}(x)$ according to Equations (34) and (35), respectively.
Step 14:	Elicit $\mu_{d_2^{PIS^B}}(x)$ and $\mu_{d_2^{NIS^B}}(x)$ using Equations (39) and (40).
Step 15:	Formulate and solve model (**I**) (Equations (41)–(44)) for the UAM of the MR-BLMNPP, and go to Step 17.
Step 16:	Formulate and solve model (**II**) (Equations (45)–(48)) for the UAM of the MR-BLMNPP.
Step 17:	If the compromise solution belongs to $\underline{S}(\boldsymbol{x})$, then go to Step 18l otherwise, go to Step 19.
Step 18:	If the leader is satisfied with the solution, and a Pareto optimal solution is obtained, go to Step 21; otherwise, go to Step 20.
Step 19:	Solve the LAM Equations (15)–(17). If the leader is satisfied with the solution, then a possibly Pareto optimal solution is obtained; go to Step 21, or else go to Step 20.
Step 20:	Elicit another kind of $\mu_{d_2^{PIS^F}}(x)$ and $\mu_{d_2^{NIS^F}}(x)$, and go to Step 8.
Step 21:	End.

6. Numerical Illustration

Consider the MR-BLMNPP below, in which the set of constraints is modeled as a rough environment and the objective function factors are multi-choice parameters:

[Leader]

$$\underbrace{\max}_{\boldsymbol{x_1}}\begin{pmatrix} f_{11}(X)=c_{11}^{1}x_1^{2}+c_{11}^{2}x_2^{3}+c_{11}^{3},\\ \underset{\sim}{f}_{12}(X)=c_{12}^{1}x_1^{2}+c_{12}^{2}x_1x_2\end{pmatrix},$$

where x_2 solves

[Follower]

$$\underbrace{\max}_{x_2}\begin{pmatrix}\widetilde{f}_{21}(X)=c_{21}^{1}x_1^2+c_{21}^{2}x_1x_2^2,\\ \widetilde{f}_{22}(X)=c_{22}^{1}x_1^3+c_{22}^{2}x_2^2\end{pmatrix},$$

subject to

$$x\in S(X)$$
$$\begin{Bmatrix} x_1^2+x_2^2\le 16,\\ x_1+x_2\le 5,\\ x_1,\ x_2\ge 0,\end{Bmatrix}\subseteq S(X)\subseteq\begin{Bmatrix} x_1^2+x_2^2\le 36,\\ x_1\le 5.5,\\ x_2\le 5.5,\\ x_1,\ x_2\ge 0,\end{Bmatrix}$$

where the multi-choice parameters are described as

$$c_{11}^{1}=\{15,18,20\};\qquad c_{11}^{2}=\{16,19,22,25\};\qquad c_{11}^{3}=\{15,16,18,19\};$$
$$c_{12}^{1}=\{16,19,22,25\};\qquad c_{12}^{2}=\{15,18,20\};\qquad c_{21}^{1}=\{25,27,28,30\};$$
$$c_{21}^{2}=\{16,19,22,25\};\qquad c_{22}^{1}=\{12,15,19,24\};\qquad c_{22}^{2}=\{8,10,13,17\}$$

Firstly, based on the proposed NDD formula, the MR-BLMNPP is converted into a BLMNPP with a rough environment as

[Leader]

$$\underbrace{\max}_{x_1}\begin{pmatrix}f_{11}(X,\omega)=\begin{pmatrix}[15+3\omega_1-0.5\omega_1(\omega_1-1)]x_1^2+[16+3\omega_2]x_2^3+\\ \left[15+\omega_3+0.5\omega_3(\omega_3-1)-\frac{1}{6}\omega_3(\omega_3-1)(\omega_3-2)\right]\end{pmatrix},\\ f_{12}(X,\omega)=\left([16+3\omega_2]x_1^2+[15+3\omega_1-0.5\omega_1(\omega_1-1)]x_1x_2\right)\end{pmatrix},$$

where x_2 solves

[Follower]

$$\underbrace{\max}_{x_2}\begin{pmatrix}f_{21}(X,\omega)=\begin{pmatrix}\left[25+2\omega_4-0.5\omega_4(\omega_4-1)+\frac{1}{6}\omega_4(\omega_4-1)(\omega_4-2)\right]x_1^2\\ +[16+3\omega_2]x_1x_2^2,\end{pmatrix}\\ f_{22}(X,\omega)=\begin{pmatrix}[12+3\omega_5+0.5\omega_5(\omega_5-1)]x_1^3\\ +[8+2\omega_6+0.5\omega_6(\omega_6-1)]x_2^2\end{pmatrix}\end{pmatrix}$$

subject to

$$x\in S(X);\qquad 0\le w_1\le 2;\qquad 0\le w_2\le 2;\qquad 0\le w_3\le 3;$$
$$0\le w_4\le 3;\qquad 0\le w_5\le 2;\qquad 0\le w_6\le 2;$$
$$\begin{Bmatrix} x_1^2+x_2^2\le 16,\\ x_1+x_2\le 5,\\ x_1,\ x_2\ge 0,\end{Bmatrix}\subseteq S(X)\subseteq\begin{Bmatrix} x_1^2+x_2^2\le 36,\\ x_1\le 5.5,\\ x_2\le 5.5,\\ x_1,\ x_2\ge 0,\end{Bmatrix}$$

Starting with the two TOPSIS-based models, the UAM of the rough BLMNPP is solved. Table 3 provides a summary of each value's maximum and minimum. For the leader problem, Tables 4 and 5 provide the PIS and NIS payout tables, respectively.

Table 3. Individual maximum and minimum values.

	$f_{11}(X,\omega)$	$f_{12}(X,\omega)$	$f_{21}(X,\omega)$	$f_{22}(X,\omega)$
$max\ f_{ij}(X,\omega)$	3794	929.27	2240.816	3235.875
$min\ f_{ij}(X,\omega)$	14.995	0	0	0

Table 4. PIS payoff table of the leader UAM of the rough BLMNPP.

	$f_{11}(X,\omega)$	$f_{12}(X,\omega)$	x_1	x_1	w_1	w_2	w_3
$max\ f_{11}(X,\omega)$	3794.255*	390.28	2.398	5.5	2	2	2.9434
$max\ f_{12}(X,\omega)$	747.95	929.27*	5.5	2.398	2	2	1.234

$$F^* = (f_{11}^*, f_{12}^*) = (3794.255; 929.27)$$

Table 5. NIS payoff table of the leader UAM of the rough BLMNPP.

	$f_{11}(X,\omega)$	$f_{12}(X,\omega)$	x_1	x_1	w_1	w_2	w_3
$min\ f_{11}(X,\omega)$	14.995*	0	0	0	0	1.157	0.0566
$min\ f_{12}(X,\omega)$	31.37	0*	0	0.839	1.175	1.155	1.234

$$F^- = \left(f_{11}^-, f_{12}^-\right) = (14.995,\ 0)$$

Assume that $\delta_{11} = \delta_{12} = 0.5$, then the equations of $d_p^{PIS^F}(X,\omega)$ and $d_P^{NIS^F}(X,\omega)$ when $p = 2$ are

$$d_2^{PIS^F}(X,\omega) = \left\{ \begin{array}{l} (0.5)^2 \left[\dfrac{3794.255 - \left(\begin{array}{l} [15 + 3\omega_1 - 0.5\omega_1(\omega_1 - 1)]x_1^2 + [16 + 3\omega_2]x_2^3 + \\ \left[15 + \omega_3 + 0.5\omega_3(\omega_3 - 1) - \frac{1}{6}\omega_3(\omega_3 - 1)(\omega_3 - 2)\right] \end{array} \right)}{3794.255 - 14.995} \right]^2 \\ + (0.5)^2 \left[\dfrac{929.27 - \left([16+3\omega_2]x_1^2 + [15+3\omega_1 - 0.5\omega_1(\omega_1-1)]x_1x_2\right)}{929.27 - 0} \right]^2 \end{array} \right\}^{\frac{1}{2}}$$

$$d_2^{NIS^F}(X,\omega) = \left\{ \begin{array}{l} (0.5)^2 \left[\dfrac{\left(\begin{array}{l} [15 + 3\omega_1 - 0.5\omega_1(\omega_1 - 1)]x_1^2 + [16 + 3\omega_2]x_2^3 + \\ \left[15 + \omega_3 + 0.5\omega_3(\omega_3 - 1) - \frac{1}{6}\omega_3(\omega_3 - 1)(\omega_3 - 2)\right] \end{array} \right) - 14.995}{3794.255 - 14.995} \right]^2 \\ + (0.5)^2 \left[\dfrac{\left([16+3\omega_2]x_1^2 + [15+3\omega_1 - 0.5\omega_1(\omega_1-1)]x_1x_2\right) - 0}{929.27 - 0} \right]^2 \end{array} \right\}^{\frac{1}{2}}$$

Thus, we compute $\max d_2^{PIS^F}(X,\omega) = 0.7071, \min d_2^{PIS^F}(X,\omega) = 0.2151$, and $\max d_2^{NIS^F}(X,\omega) = 0.5418, \min d_2^{NIS^F}(X,\omega) = 0$. Then, we have $d_2^{F^*} = (0.2151, 0.5418)$ and $d_2^{F^-} = (0.7071, 0)$; therefore, $\mu_{d_2^{PIS^F}}(X,\omega)$ *and* $\mu_{d_2^{NIS^F}}(X,\omega)$ can be obtained as

$$\mu_{d_2^{PIS^F}}(X,\omega) = \frac{0.7071 - d_2^{PIS^F}(X,\omega)}{0.7071 - 0.2151} = 1.4372 - 2.03 * d_2^{PIS^F}(X,\omega)$$

$$\mu_{d_2^{NIS^F}}(X,\omega) = \frac{d_2^{NIS^F}(X,\omega) - 0}{0.5418 - 0} = 1.8457 * d_2^{NIS^F}(X,\omega)$$

The proposed two TOPSIS-based models of the leader UAM are as follows:

Model (I): Fuzzy Max–Min Method	**Model (II): FGP Method**
$\max\ \vartheta$ subject to $1.4372 - 2.03 * d_2^{PIS^F}(X,\omega) \geq \vartheta,$ $1.8457 * d_2^{NIS^F}(X,\omega) \geq \vartheta,$ $x_1^2 + x_2^2 \leq 36,$ $x_1 \leq 5.5,$ $x_2 \leq 5.5,$ $0 \leq \omega_1 \leq 2,$ $0 \leq \omega_2 \leq 2,$ $0 \leq \omega_3 \leq 3,$ $x_1,\ x_2 \geq 0,$ $\vartheta \in [0,1], \quad \vartheta = 0.95014,$ $\vartheta(X^*, \omega^*) = (2.9439, 5.228, 2, 2, 2.943,).$	$\min Z = D_F^{PIS^+} + D_F^{NIS^-}$ subject to $1.4372 - 2.03 * d_2^{PIS^F}(X,\omega) + D_F^{PIS^-} - D_F^{PIS^+} = 1,$ $1.8457 * d_2^{NIS^F}(X,\omega) + D_F^{NIS^-} - D_F^{NIS^+} = 1,$ $x_1^2 + x_2^2 \leq 36,$ $x_1 \leq 5.5,$ $x_2 \leq 5.5,$ $0 \leq \omega_1 \leq 2,$ $0 \leq \omega_2 \leq 2,$ $0 \leq \omega_3 \leq 3,$ $x_1,\ x_2 \geq 0, \quad D_F^{PIS^-},\ D_F^{PIS^+},\ D_F^{NIS^-},\ D_F^{NIS^+} \geq 0,$ $Z = 0.0506, \quad (X^*, \omega^*) = (5.496, 2.4062, 2, 2, 2.943).$

Using Lingo 20, the solution of the leader UAM is obtained as $(X^*, \omega^*) = (2.9439, 5.228, 2, 2, 2.943)$ for model (I) and $(X^*, \omega^*) = (5.496, 2.4062, 2, 2, 2.943)$ for model (II).

Assume that $\delta_{ij} = 0.25$, then the equations of $d_p^{PIS^B}(X, \omega)$ and $d_P^{NIS^B}(X, \omega)$ when $p = 2$ are

$$d_2^{PIS^B}(X,\omega) = \left\{ \begin{array}{l} (0.25)^2 \left[\dfrac{3794.255 - \left(\begin{array}{l} [15 + 3\omega_1 - 0.5\omega_1(\omega_1 - 1)]x_1^2 + [16 + 3\omega_2]x_2^3 + \\ [15 + \omega_3 + 0.5\omega_3(\omega_3 - 1) - \omega_3(\omega_3 - 1)(\omega_3 - 2)] \end{array} \right)}{3794.255 - 14.995} \right]^2 \\ +(0.25)^2 \left[\dfrac{929.27 - \left([16+3\omega_2]x_1^2 + [15+3\omega_1 - 0.5\omega_1(\omega_1 - 1)]x_1x_2 \right)}{929.27 - 0} \right]^2 \\ +(0.25)^2 \left[\dfrac{2240.618 - \left(\begin{array}{l} \left[25 + 2\omega_4 - 0.5\omega_4(\omega_4 - 1) + \frac{1}{6}\omega_4(\omega_4 - 1)(\omega_4 - 2)\right]x_1^2 \\ +[16 + 3\omega_2]x_1x_2^2, \end{array} \right)}{2240.618 - 0} \right]^2 \\ +(0.25)^2 \left[\dfrac{3235.875 - \left(\begin{array}{l} [12 + 3\omega_5 + 0.5\omega_5(\omega_5 - 1)]x_1^3 \\ +[8 + 2\omega_6 + 0.5\omega_6(\omega_6 - 1)]x_2^2 \end{array} \right)}{3235.875 - 0} \right]^2 \end{array} \right\}^{\frac{1}{2}}$$

$$d_2^{NIS^B}(X,\omega) = \left\{ \begin{array}{l} (0.25)^2 \left[\dfrac{\left(\begin{array}{l} [15 + 3\omega_1 - 0.5w_1(\omega_1 - 1)]x_1^2 + [16 + 3\omega_2]x_2^3 + \\ \left[15 + \omega_3 + 0.5\omega_3(\omega_3 - 1) - \frac{1}{6}\omega_3(\omega_3 - 1)(\omega_3 - 2)\right] \end{array} \right) - 14.995}{3794.255 - 14.995} \right]^2 \\ +(0.25)^2 \left[\dfrac{\left([16+3\omega_2]x_1^2 + [15+3\omega_1 - 0.5\omega_1(\omega_1 - 1)]x_1x_2 \right) - 0}{929.27 - 0} \right]^2 \\ +(0.25)^2 \left[\dfrac{\left(\begin{array}{l} \left[25 + 2\omega_4 - 0.5\omega_4(\omega_4 - 1) + \frac{1}{6}\omega_4(\omega_4 - 1)(\omega_4 - 2)\right]x_1^2 \\ +[16 + 3\omega_2]x_1x_2^2, \end{array} \right) - 0}{2240.618 - 0} \right]^2 \\ +(0.25)^2 \left[\dfrac{\left(\begin{array}{l} [12 + 3\omega_5 + 0.5\omega_5(\omega_5 - 1)]x_1^3 \\ +[8 + 2\omega_6 + 0.5\omega_6(\omega_6 - 1)]x_2^2 \end{array} \right) - 0}{3235.875 - 0} \right]^2 \end{array} \right\}^{\frac{1}{2}}$$

So, $\max d_2^{PIS^B}(X) = 0.5, \min d_2^{PIS^B}(X) = 0.1694$, and $\max d_2^{NIS^B}(X) = 0.4, \min d_2^{NIS^B}(X) = 0$. Thus, we have $d_2^{B^*} = (0.1694, 0.4)$ and $d_2^{B^-} = (0.5, 0)$, so $\mu_{d_2^{PIS^B}}(X)$ and $\mu_{d_2^{NIS^B}}(X)$ can be obtained as

$$\mu_{d_2^{PIS^B}}(X,\omega) = 1.5124 - 3.024 * d_2^{PIS^B}(X,\omega)$$

$$\mu_{d_2^{NIS^B}}(X,\omega) = 2.5 * d_2^{NIS^B}(X,\omega)$$

The proposed two TOPSIS-based models for MR-BLMNPP may be formulated as follows:

Model (I): Fuzzy Max–Min Method	**Model (II): FGP Method**
$\max\delta$	$\min Z = D_B^{PIS^+} + D_B^{NIS^-}$
subject to	subject to
$1.5124 - 3.024 * d_2^{PIS^B}(X,\omega) \geq \delta,$	$1.5124 - 3.024 * d_2^{PIS^B}(X,\omega) + D_B^{PIS^-} - D_B^{PIS^+} = 1,$
$2.5 * d_2^{NIS^B}(X,\omega) \geq \delta,$	$2.5 * d_2^{NIS^B}(X,\omega) + D_B^{NIS^-} - D_B^{NIS^+} = 1,$
$x_1^2 + x_2^2 \leq 36,$	$x_1^2 + x_2^2 \leq 36,$
$x_1 = 2.9433,$	$x_1 = 5.496,$
$x_2 \leq 5.5,$	$x_2 \leq 5.5,$
$0 \leq \omega_1 \leq 2,$	$0 \leq \omega_1 \leq 2,$
$0 \leq \omega_2 \leq 2,$	$0 \leq \omega_2 \leq 2,$
$0 \leq \omega_3 \leq 3,$	$0 \leq \omega_3 \leq 3,$
$0 \leq \omega_4 \leq 3,$	$0 \leq \omega_4 \leq 3,$
$0 \leq \omega_5 \leq 2,$	$0 \leq \omega_5 \leq 2,$
$0 \leq \omega_6 \leq 2,$	$0 \leq \omega_6 \leq 2,$
$x_2 \geq 0, \quad \delta \in [0,1]$	$D_B^{PIS^-},\ D_B^{PIS^+},\ D_B^{NIS^-},\ D_B^{NIS^+} \geq 0,$
$\delta = 0.8419,$	$Z = 0,$
$(X^*,\omega^*) = (2.9439, 5.228, 2, 2, 2.943, 3, 2, 2)$	$(X^*,\omega^*) = (5.496, 2.4062, 2, 2, 2.5422, 2.864, 2, 1.837)$

Using Lingo 20, the solution of the UAM of the MR-BLMNPP is obtained as $(X^*,\omega^*) = (2.9439, 5.228, 2, 2, 2.943, 3, 2, 2)$ for model (I) and $(X^*,\omega^*) = (5.496, 2.4062, 2, 2, 2.5422, 2.864, 2, 1.837)$ for model (II). The obtained solution of the two TOPSIS-based models for the UAM is $x^* \notin \underline{S}(x)$. So, we must solve the LAM. The results for the UAM and LAM are listed in Tables 6 and 7, respectively.

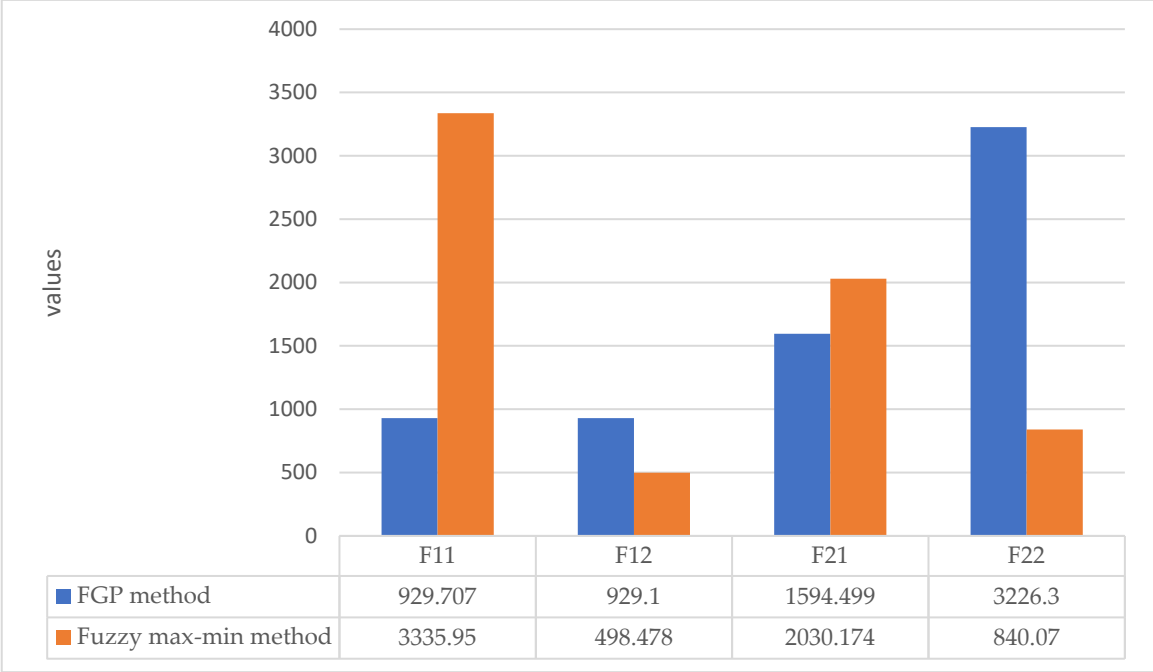
Table 6. Comparison between the TOPSIS-based FGP and fuzzy max–min method for the UAM.

FGP Method	**Fuzzy Max–Min Method**
$f_{11} = 929.707$	$f_{11} = 3335.95$
$f_{12} = 929.1$	$f_{12} = 498.478$
$f_{21} = 1594.499$	$f_{21} = 2030.174$
$f_{22} = 3226.3$	$f_{22} = 840.07$
$(X^*,\omega^*) =$	(X^*,ω^*)
$(5.496, 2.4062, 2, 2, 2.5422, 2.864, 2, 1.837)$	$= (2.9439, 5.228, 2, 2, 2.943, 3, 2, 2)$

Table 7. Comparison between the TOPSIS-based FGP and fuzzy max–min methods for the LAM.

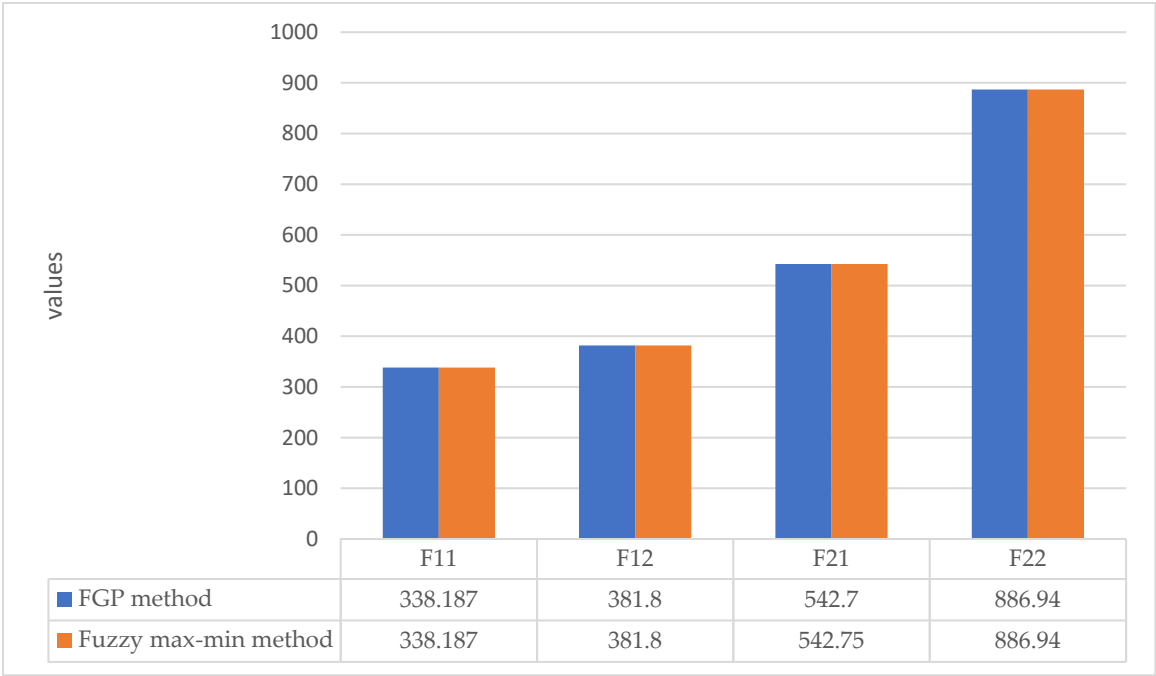
FGP Method	**Fuzzy Max–Min Method**
$f_{11} = 338.187$	$f_{11} = 338.187$
$f_{12} = 381.8$	$f_{12} = 381.8$
$f_{21} = 542.75$	$f_{21} = 542.75$
$f_{22} = 886.94$	$f_{22} = 886.94$
$(X^*,\omega^*) = (3.564, 1.436, 2, 2, 2.9433, 3, 2, 2)$	$(X^*,\omega^*) = (3.564, 1.436, 2, 2, 2.9433, 3, 2, 2)$

The outcomes of the TOPSIS-based fuzzy max–min and FGP methods for solving the LAM and UAM are shown in Tables 6 and 7, respectively. The FGP and fuzzy max–min methods are the same for the LAM but different for the UAM, according to the data that were acquired. The values of various objectives in both levels, using the TOPSIS-based FGP and fuzzy max–min methods for the UAM and LAM, are represented by Figures 1 and 2, respectively.



	F11	F12	F21	F22
FGP method	929.707	929.1	1594.499	3226.3
Fuzzy max-min method	3335.95	498.478	2030.174	840.07

Figure 1. TOPSIS-based FGP and fuzzy max–min method for the UAM.



	F11	F12	F21	F22
FGP method	338.187	381.8	542.7	886.94
Fuzzy max-min method	338.187	381.8	542.75	886.94

Figure 2. TOPSIS-based FGP and fuzzy max–min method for the LAM.

Bi-Level Production Planning Model

For the sake of simplicity and to clarify whether the two TOPSIS-based models that have been suggested can be used to solve problems in the actual world, here, we have examined six distinct machine types for the industrial production planning problem: a drill press, band saw, jig saw, lathe, grinder, and milling machine [38]. Three goods are to be produced using all of their capacities. Every machine type's current capacity is expressed in hours per week. Table 8 lists the total time that each machine is available for use as well as the amount of multi-choice time that each product requires.

Table 8. Portfolio of available capacities with multi-choice data.

Machine Type	Available Time	Machine Time		
		Product (1)	**Product (2)**	**Product (3)**
Milling machine (m)	$b_1 = 1400$	$a_{11} = 12$	$a_{12} = 17$	$a_{13} = 0$
Lathe (l)	$b_2 = 1000$	$a_{21} = 3$	$a_{22} = 9$	$a_{23} = 8$
Grinder (g)	$b_3 = 1750$	$a_{31} = 10$	$a_{32} = 13$	$a_{33} = 15$
Jig saw (s)	$b_4 = 1325$	$a_{41} = 6$	$a_{42} = 0$	$a_{43} = 16$
Drill press (d)	$b_5 = 900$	$a_{51} = 0$	$a_{52} = 12$	$a_{53} = 17$
Band saw (b)	$b_6 = 1075$	$a_{61} = 9.5$	$a_{62} = 9.5$	$a_{63} = 4$
Profit (P)		$c_{11} = \{40, 50, 60, 80\}$	$c_{12} = \{90, 100, 120\}$	$c_{13} = \{15, 18, 19, 20\}$
Product liability (L)		$c_{21} = \{0.5, 0.6, 0.8, 1\}$	$c_{22} = \{0.7, 0.8, 1\}$	$c_{23} = \{0.5, 0.6, 0.8, 1\}$
Quality (Q)		$c_{31} = \{80, 90, 100, 110\}$	$c_{32} = \{60, 70, 80\}$	$c_{33} = \{40, 50, 60, 80\}$
Workers' satisfaction (W)		$c_{41} = \{10, 20, 30\}$	$c_{42} = \{90, 100, 120\}$	$c_{43} = \{60, 70, 80\}$

Using the data given in Table 8 and applying the NDD interpolation, the model of the BL-PPM is obtained as follows:

Leader

$$\underbrace{\max}_{x_1, x_2} \begin{pmatrix} Z_{11} = \begin{pmatrix} \left[40 + 10\omega_1 + \frac{5}{3}\omega_1(\omega_1 - 1)(\omega_1 - 2)\right]x_1 + [90 + 10\omega_2 + 5\omega_2(\omega_2 - 1)]x_2 \\ + \left[15 + 3\omega_3 - \omega_3(\omega_3 - 1) + \frac{1}{3}\omega_3(\omega_3 - 1)(\omega_3 - 2)\right]x_3 \end{pmatrix} & \textbf{Profit} \\ Z_{12} = \begin{pmatrix} \left[0.5 + 0.1\omega_4 + \frac{0.1}{2}\omega_4(\omega_4 - 1) - \frac{0.1}{6}\omega_4(\omega_4 - 1)(\omega_4 - 2)\right]x_1 \\ + \left[0.7 + 0.1\omega_5 + \frac{0.1}{2}\omega_5(\omega_5 - 1)\right]x_2 + \left[0.5 + 0.1\omega_6 + \frac{0.2}{2}\omega_6(\omega_6 - 1)\right]x_3 \end{pmatrix} & \textbf{Product liability} \end{pmatrix}$$

Follower

$$\underbrace{\max}_{x_3} \begin{pmatrix} Z_{21} = \begin{pmatrix} [80 + 10\omega_7]x_1 + [60 + 10\omega_8]x_2 \\ + \left[40 + 10\omega_1 + \frac{5}{3}\omega_1(\omega_1 - 1)(\omega_1 - 2)\right]x_3 \end{pmatrix} & \textbf{Quality} \\ Z_{22} = \begin{pmatrix} [10 + 10\omega_9]x_1 + [90 + 10\omega_2 + 5\omega_2(\omega_2 - 1)]x_2 \\ + [60 + 10\omega_8]x_3 \end{pmatrix} & \textbf{Worker's satisfaction} \end{pmatrix}$$

subject to

$12x_1 + 17x_2 \leq 1400$ (***Milling Machine***)
$3x_1 + 9x_2 + 8x_3 \leq 1000$ (***Lathe***)
$10x_1 + 13x_2 + 15x_3 \leq 1750$ (***Grinder***)
$6x_1 + 16x_3 \leq 1325$ (***Jig saw***)
$12x_2 + 17x_3 \leq 900$ (***Drill press***)
$9.5x_1 + 9.5x_2 + 4x_3 \leq 1075$ (***Bandsaw***)
$0 \leq \omega_1 \leq 3, 0 \leq \omega_2 \leq 2, 0 \leq \omega_3 \leq 3, 0 \leq \omega_4 \leq 3, 0 \leq \omega_5 \leq 2,$
$0 \leq \omega_6 \leq 3, 0 \leq \omega_7 \leq 3, 0 \leq \omega_8 \leq 2, 0 \leq \omega_9 \leq 2, x_1, x_2, x_3 \geq 0$

The leader issue continues as follows, based on the two TOPSIS-based models that have been offered:

Model (I): Fuzzy Max–Min Method	**Model (II): FGP Method**
$\max \vartheta$ subject to $1.078 - 1.5246 * d_2^{PIS}(X,\omega) \geq \vartheta,$ $1.513 * d_2^{NIS}(X,\omega) \geq \vartheta,$ $12x_1 + 17x_2 \leq 1400$ $3x_1 + 9x_2 + 8x_3 \leq 1000$ $10x_1 + 13x_2 + 15x_3 \leq 1750$ $6x_1 + 16x_3 \leq 1325$ $12x_2 + 17x_3 \leq 900$ $9.5x_1 + 9.5x_2 + 4x_3 \leq 1075$ $x_1, x_2, x_3 \geq 0,$ $0 \leq \omega_1 \leq 3,$ $0 \leq \omega_2 \leq 2,$ $0 \leq \omega_3 \leq 3,$ $0 \leq \omega_4 \leq 3,$ $0 \leq \omega_5 \leq 2,$ $0 \leq \omega_6 \leq 3,$ $\delta \in [0,1],\ \vartheta = 0.995,$ $(X^*, \omega^*) = (71.118, 28.1, 33.11, 3, 2, 3, 3, 2, 3)$	$\min \quad Z = D_1^{PIS^+} + D_2^{NIS^-}$ subject to $1.078 - 1.5246 d_2^{PIS} + D_1^{PIS^-} - D_1^{PIS^+} = 1$ $1.513 d_2^{NIS} + D_2^{NIS^-} - D_2^{NIS^+} = 1$ $12x_1 + 17x_2 \leq 1400$ $3x_1 + 9x_2 + 8x_3 \leq 1000$ $10x_1 + 13x_2 + 15x_3 \leq 1750$ $6x_1 + 16x_3 \leq 1325$ $12x_2 + 17x_3 \leq 900$ $9.5x_1 + 9.5x_2 + 4x_3 \leq 1075$ $x_1, x_2, x_3, D_1^{PIS^-}, D_1^{PIS^+}, D_2^{NIS^-}, D_2^{NIS^+} \geq 0,$ $0 \leq \omega_1 \leq 3,$ $0 \leq \omega_2 \leq 2,$ $0 \leq \omega_3 \leq 3,$ $0 \leq \omega_4 \leq 3,$ $0 \leq \omega_5 \leq 2,$ $0 \leq \omega_6 \leq 3,$ $Z = 0,$ $(X^*, \omega^*) = (65.468, 36.14, 27.43, 3, 2, 3, 3, 2, 3)$

Using Lingo 20, the solution of the leader is $(X^*, \omega^*) = (71.118, 28.1, 33.11, 3, 2, 3, 3, 2, 3)$ for model (I) and $(X^*, \omega^*) = (65.468, 36.14, 27.43, 3, 2, 3, 3, 2, 3)$ for model (II). The BL-PPM is carried out using the last two suggested TOPSIS-based models as follows:

Model (I): Fuzzy Max–Min Method	**Model (II): FGP Method**
$\max \delta$ subject to $1.089 - 2.276 d_2^{PIS^B}(X,\omega) \geq \delta,$ $2.353 d_2^{NIS^B}(X,\omega) \geq \delta,$ $12x_1 + 17x_2 \leq 1400,$ $3x_1 + 9x_2 + 8x_3 \leq 1000,$ $10x_1 + 13x_2 + 15x_3 \leq 1750,$ $6x_1 + 16x_3 \leq 1325,$ $12x_2 + 17x_3 \leq 900,$ $9.5x_1 + 9.5x_2 + 4x_3 \leq 1075,$ $x_1 = 71.118,$ $x_2 = 28.1,$ $x_3 \geq 0,$ $0 \leq \omega_1 \leq 3,$ $0 \leq \omega_2 \leq 2,$ $0 \leq \omega_3 \leq 3,$ $0 \leq \omega_4 \leq 3,$ $0 \leq \omega_5 \leq 2,$ $0 \leq \omega_6 \leq 3,$ $0 \leq \omega_7 \leq 3,$ $0 \leq \omega_8 \leq 2,$ $0 \leq \omega_9 \leq 2,$ $\delta \in [0,1], \ \delta = 0.995,$ $(X^*, \omega^*) =$ $(71.118, 28.1, 33.11, 3, 2, 3, 3, 2, 3, 3, 2, 2)$	$\min \quad Z = D_1^{PIS^+} + D_2^{NIS^-}$ subject to $1.089 - 2.276 d_2^{PIS^B} + D_1^{PIS^-} - D_1^{PIS^+} = 1,$ $2.353 d_2^{NIS^B} + D_2^{NIS^-} - D_2^{NIS^+} = 1,$ $12x_1 + 17x_2 \leq 1400,$ $3x_1 + 9x_2 + 8x_3 \leq 1000,$ $10x_1 + 13x_2 + 15x_3 \leq 1750,$ $6x_1 + 16x_3 \leq 1325,$ $12x_2 + 17x_3 \leq 900,$ $9.5x_1 + 9.5x_2 + 4x_3 \leq 1075,$ $x_1 = 65.468,$ $x_2 = 36.14,$ $x_3, D_1^{PIS^-}, D_1^{PIS^+}, D_2^{NIS^-}, D_2^{NIS^+} \geq 0,$ $0 \leq \omega_1 \leq 3,$ $0 \leq \omega_2 \leq 2,$ $0 \leq \omega_3 \leq 3,$ $0 \leq \omega_4 \leq 3,$ $0 \leq \omega_5 \leq 2,$ $0 \leq \omega_6 \leq 3,$ $0 \leq \omega_7 \leq 3,$ $0 \leq \omega_8 \leq 2,$ $0 \leq \omega_9 \leq 2,$ $Z = 0, \quad (X^*, \omega^*) =$ $(65.468, 36.14, 27.43, 3, 2, 3, 3, 2, 3, 3, 2, 2)$

Using Lingo 20, the solution of the multi-choice BL-PPM is $(X^*, \omega^*) = (71.118, 28.1, 33.11, 3, 2, 3, 3, 2, 3, 3, 2, 2)$ for model (I) and $(X^*, \omega^*) = (65.468, 36.14, 27.43, 3, 2, 3, 3, 2, 3, 3, 2, 2)$ for model (II). The objective values are given in Table 9.

Table 9 presents a comparative analysis of the two proposed TOPSIS models for handling the multi-choice BL-PPM. The findings show that there is a reasonable degree of agreement between the objective values derived from the TOPSIS-based fuzzy max–min and FGP methods. The values of various objectives in both levels, using the TOPSIS-based FGP and Fuzzy max–min methods for the BL-PPM, are represented by Figure 3. A PC with a Core i5 CPU running at 2.8 GHz, 8 GB of RAM, and a 64-bit operating system is used to carry out the computations. The Lingo programming software, LINGO 20, computed numerical models.

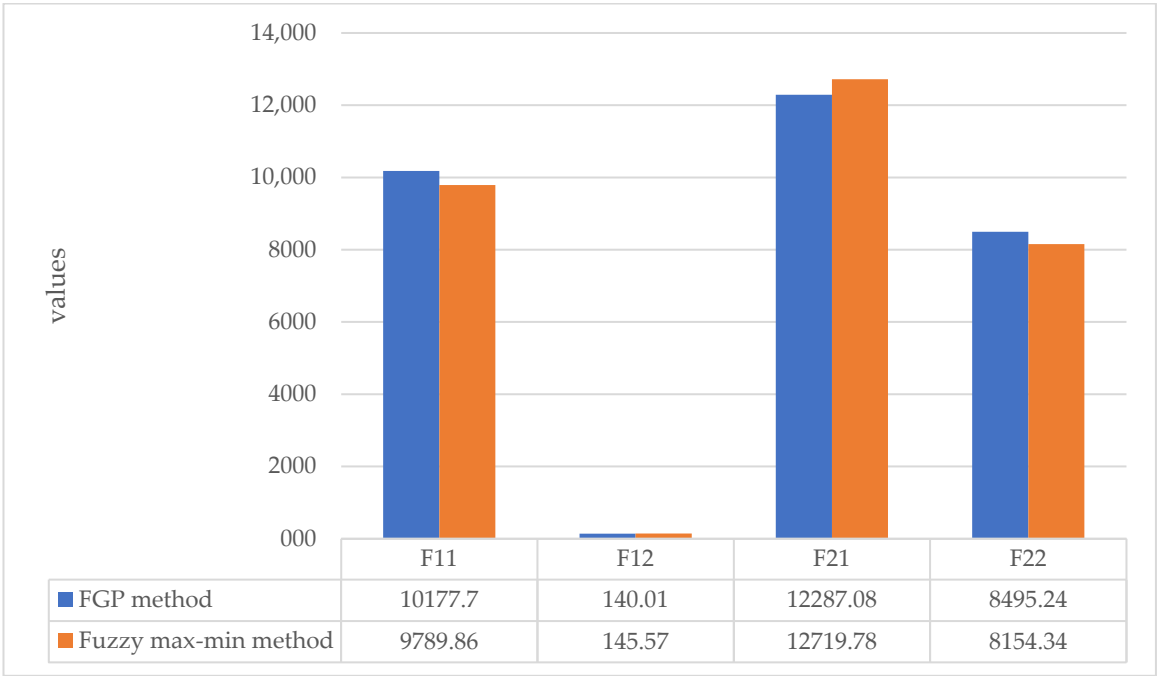


Figure 3. TOPSIS-based FGP and fuzzy max–min method for the BL-PPM.

Table 9. Comparison between TOPSIS-based fuzzy max–min and FGP methods for BL-PPM.

Fuzzy Max–Min Method	FGP Method
$Z_{11} = 9789.86$	$Z_{11} = 10,177.7$
$Z_{12} = 145.57$	$Z_{12} = 140.01$
$Z_{21} = 12,719.78$	$Z_{21} = 12,287.08$
$Z_{22} = 8154.34$	$Z_{22} = 8495.24$

7. Discussion

Both the TOPSIS-based FGP and fuzzy max–min methods can be used by the DM in the algorithm to solve the MR-BLMNPP. It is also possible to formulate the final models using other types of membership functions, such as parabolic and hyperbolic membership functions, but the model will be more complicated. To guarantee the suitability and computational effectiveness of the suggested models for resolving the BL-PPM, numerical data for the TOPSIS-based fuzzy max–min and FGP methods are displayed in Table 10. The elapsed runtime per second and the total number of solver iterations are displayed. The two models' total solver iterations are rather similar. It is evident that the fuzzy max–min model's elapsed runtime is shorter than that of the FGP model. Additionally included were

the model class, integer variables, nonlinear variables, and the total number of constraints. Numerous organizations involved in supply chain networks and logistic systems can benefit greatly from the study that has been conducted here. Any logistic operation where single-type uncertainty is insufficient to define specific parameters can benefit greatly from the suggested model's ability to handle two-fold uncertainty.

Newton's divided difference interpolation method is a widely used technique for constructing interpolating polynomials. Here are the advantages and disadvantages of this method:

Advantages

Flexibility: The method can easily accommodate new data points without needing to re-calculate the entire polynomial, making it efficient for incremental data.

Higher-Order Polynomials: It allows the construction of higher-order interpolating polynomials, which can provide better approximations for functions that are well behaved over the interval of interest.

Numerical Stability: The divided difference table helps in maintaining numerical stability, reducing the impact of round-off errors when computing polynomial coefficients.

Easy to Implement: The algorithm is straightforward to implement, making it accessible for educational purposes and practical applications.

Error Analysis: The method provides a clear way to analyze interpolation errors, which can be useful in assessing the quality of the approximation.

Disadvantages

Computational Complexity: For large datasets, the computation of divided differences can become cumbersome, leading to increased computational time and complexity.

Runge's Phenomenon: Like other polynomial interpolation methods, it can suffer from oscillations at the edges of the interpolation interval, especially with high-degree polynomials.

Global Approximation: The resulting polynomial is a global approximation; small changes in the data can significantly affect the entire polynomial.

Overfitting: Using a high-degree polynomial can lead to overfitting, where the polynomial fits the data points well but performs poorly on other values.

Limited to Polynomial Functions: The method assumes that the function can be well approximated by a polynomial, which may not hold true for all functions, particularly those with discontinuities or sharp changes.

Conclusion: The NDD interpolation method is a powerful tool for polynomial interpolation, particularly useful for its flexibility and numerical stability. However, users should be cautious of its limitations, especially regarding computational complexity and the potential for overfitting. When choosing an interpolation method, it is essential to consider the specific characteristics of the data and the desired accuracy of the approximation.

Table 10. Numerical data of the two TOPSIS-based models for production planning issues.

	Fuzzy Max–Min Method	**FGP Method**
Total solver iteration	6	7
Elapsed runtime seconds	0.27	2.32
Model class	NLP	NLP
Total variables	11	14
Nonlinear variables	10	10
Integer variables	0	0
Total constraints	27	27

8. Conclusions

This study formulated the multi-choice rough bi-level multi-objective nonlinear programming problem (MR-BLMNPP) and proposed a solution method using two TOPSIS-

based models. The approach begins by constructing a polynomial representation of the objective function using NDD interpolation. Subsequently, a UAM and LAM are developed using rough set theory. To solve the MR-BLMNPP, a novel strategy is introduced: If the solution to the UAM lies within the lower approximation set, it is accepted as the optimal solution. Otherwise, the LAM is solved. A numerical example is presented to demonstrate the effectiveness of the proposed methods. Additionally, a BL-PPM is formulated and solved using the two TOPSIS-based models.

One of the main advantages of the proposed TOPSIS-based models is their adaptability to a wide range of uncertain domains while requiring minimal computational resources to find the optimal solution. Specifically, the BL-PPM was solved using the TOPSIS-based fuzzy max–min method and FGP model, with runtimes of 0.27 and 2.32 s, respectively, demonstrating high efficiency. The models required a total of six and seven solver iterations, respectively, and each model contained 27 constraints.

Looking ahead, several promising research directions in the field of MR-BLMNPPs remain unexplored. Potential areas for future investigation include the following:

- Multi-choice rough multi-objective multi-item solid fractional transportation models.
- Fully multi-choice bi-level production planning models (BL-PPMs).
- Bi-level supply chain models with multi-choice parameters.

Author Contributions: Conceptualization, M.A.E.S., M.A.E. and F.A.F.; methodology, M.A.E.S.; software, M.A.; validation, M.I.I., H.E. and M.A.E.S.; formal analysis, M.A.E.S.; investigation, M.A.E.; resources, F.A.F.; data curation, H.E.; writing—original draft preparation, M.A.E.S.; writing—review and editing, M.I.I.; visualization, M.A.; supervision, M.A.E.S.; project administration, M.A.E.; funding acquisition, M.A. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by Researchers Supporting (project number RSPD2025R636), King Saud University, Riyadh, Saudi Arabia.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author(s).

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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