

Article

Numerical Simulation of the Entrance Length in a Laminar Pipe Flow at Low Reynolds Numbers

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Abstract: According to Prandtl's boundary layer theory, the entrance length refers to the axial distance required for a flow to transition from its initial entry condition to a fully developed flow where the velocity profile stabilizes downstream. However, this theory remains applicable only under the assumption of $Re \gg 1$, while its validity diminishes under low-Reynolds-number conditions. This study utilizes OpenFOAM based on the finite volume method to numerically examine Newtonian and viscoelastic fluids in a laminar circular pipe flow. The objective is to determine the range of Reynolds numbers for which the differential equations from within the Prandtl boundary layer theory are strictly valid. Additionally, the study explores the effects of Reynolds numbers (Re) ranging from 50 to 100, solvent viscosity ratio (β) fixed at 0.3 and 0.7, and Weissenberg numbers (Wi) ranging from 0.2 to 5 on the entrance length and friction factor for the Oldroyd-B model. The results indicate the presence of a lower Reynolds number that impedes the attainment of the outcomes predicted by the Prandtl boundary layer theory for the entrance length. The inertia effect, the increase in solvent viscosity contribution, and the elastic effect exhibit a linear relationship with the entrance length and friction factor.

Keywords: Reynolds number; laminar pipe flow; Oldroyd-B model; entrance length

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1. Introduction

Prandtl's boundary layer theory provided the foundation for subsequent research in this field, with numerous scholars employing it as a foundational principle in their studies, particularly in investigating the entrance length. Prandtl pointed out that fluid flows with a low viscosity can be divided into a boundary layer near solid surfaces and interfaces, patched onto a nearly inviscid outer layer. In a pipe flow, when the boundary layer expands to the centerline of the conduit, the velocity profile ceases to vary along the axial direction, at which point the flow is termed a fully developed flow. The entrance length (L_e) is defined as the distance required for fluid flow to transition from pipe entry flow to a fully developed flow. For laminar flow, Prandtl derived the relationship between entrance length (L_e), Reynolds number (Re), and pipe diameter (D): $L_e/D = 0.06 Re$.

Determining the entrance length in a pipe is crucial in fluid mechanics and engineering applications. The sufficient straight pipe length must be ensured before using the measuring devices to avoid inaccuracies. Accurate estimation of entrance length is critical for minimizing energy losses in industrial fluid systems. By analyzing entrance effects, engineers can design more reliable, efficient, and cost-effective fluid systems.

Viscoelastic fluids [1] are complex fluids that simultaneously exhibit both viscous dissipation and elastic effects. Stress relaxation occurs over a characteristic relaxation time (λ), quantified by the Weissenberg number (Wi). Viscoelastic fluids exert a profound influence on modern engineering due to their unique rheological properties. The complex behavior of viscoelastic fluids presents significant prediction challenges, leaving many viscoelastic fluids phenomena still undiscovered.

The fluid properties of viscoelastic fluids also significantly influence the entrance length in laminar flow. For the study of entrance length in Newtonian and viscoelastic fluids, numerous researchers have carried out extensive investigations through mathematical theoretical analysis, numerical simulation, and experiments. Langhaar [2] solved the Navier–Stokes equations for laminar flow in a pipe. This solution was obtained by means of a linear approximation method, which determined the relationship between the entrance length and the Reynolds number using the velocity distribution curve. Campbell and Slattery [3] contemplated the effect of viscous dissipation within the boundary layer in the entrance region flow theory. Sparrow et al. [4] employed a theoretical analysis and numerical methods to derive an expression for the velocity distribution in the entrance region of the laminar flow in a pipe. They also discussed the factors affecting the entrance length. Friedmann et al. [5] conducted a study of the entrance length of a homogeneous viscous liquid in a pipe laminar flow within the range of low to medium Reynolds numbers. The flow characteristics, including the velocity distribution of the fluid, were obtained through the implementation of the numerical finite difference method and subsequently analyzed for working conditions in which the Reynolds number approaches zero.

Na and Yoo [6] numerically employed the SIMPLER algorithm in a non-uniform staggered grid system to investigate the developing flow of Oldroyd-B model in the planar entrance region. Alves et al. [7] investigated the flow characteristics of a PTT (Phan-Thien–Tanner) linear viscoelastic fluid in a two-dimensional 4:1 contraction flow, defining the entrance length as an axial location where the axial normal stress drops to 1 or 5% of its maximum value. Durst et al. [8] presented a relationship between the entrance length of a laminar flow pipe and the Reynolds number. They defined the entrance length as the axial location where the centerline velocity reaches 99% of its fully developed value. Poole and Ridley et al. [9] provided the results of a numerical investigation of the pipe flow of inelastic non-Newtonian fluids. They proposed a correlation between the power law exponent ($0.4 < n < 1.5$) and the entrance length segment in the range $0 < Re < 1000$. Yapici et al. [10] employed a finite-volume method to numerically simulate the steady-state flow of Oldroyd-B fluids and PTT-linear fluids in a two-dimensional rectangular channel. The effects of the Weissenberg number (Wi), computational mesh, inlet boundary conditions, and Reynolds number (Re) on the entrance length were investigated. Bertoco et al. [11] investigated the development length in pressure-driven viscoelastic fluid flows between parallel plates, modeled by the gPTT (generalized Phan-Thien–Tanner) constitutive equation. They used the finite-difference method to analyze the effects of model parameters. Mahdavi et al. [12] investigated the fluid properties of nanofluids in the entrance length of laminar flat plate flow. Numerical simulations revealed that the boundary layer immediately started forming at the beginning of the entrance region and merged long before the fully developed section. Furthermore, the location of the merging point is determined only by the Reynolds number. Lavrov [13] investigated the entrance length of Bingham fluid for a laminar flow between parallel flat plates at $0 < Bi < 1000$ by means of the momentum integral solution, the Batra–Kandasamy method, and the Gupta method.

Chen [14] used the momentum integral method to analyze laminar Newtonian flow development in the entrance regions of circular pipes and parallel-plate channels at low Reynolds numbers. Everts and Meyer [15] experimentally investigated the hydrodynamic

and thermal entrance lengths for laminar Newtonian flow in smooth horizontal tubes under constant heat flux conditions.

However, Prandtl's boundary layer theory is highly effective under the assumption of a high Reynolds number ($Re \gg 1$). The theory is invalid under low-Reynolds-number conditions. It indicates that the effect of the Reynolds number governing the entrance length is only valid within a certain operational range. Previous studies on the entrance length have predominantly focused on CFD simulations of confined channel flow, while research on the characteristics of viscoelastic fluids in circular pipes remains scarce. This study investigates the entrance length of viscoelastic fluids in a laminar pipe flow using OpenFOAM [16] along with the RheoTool [17] package. The Oldroyd-B model, as a simple viscoelastic model, achieves mathematical tractability while maintaining physical objectivity. Numerical simulations of incompressible Newtonian and Oldroyd-B fluids in a laminar pipe flow are realized using the LCR (Log-Conformation Representation) method [18,19] to obtain the flow characteristics of the fluids. Numerous studies have demonstrated the reliability and precision of the LCR method in numerical calculations [20–23]. This study is conducted on the low Reynolds numbers within the framework of the boundary layer theory for Newtonian fluids, focusing on the factors that affect entrance lengths and friction factors in the Oldroyd-B model. Much consideration is given to the effects of Reynolds numbers (Re) ranging from 50 to 100, a solvent viscosity ratio (β) ranging from 0.3 to 0.7, and Weissenberg numbers (Wi) ranging from 0.2 to 5 on the entrance length and friction factor in the Oldroyd-B model. While simulating high Weissenberg number ($Wi \gg 1$) conditions in three-dimensional models presents significant computational challenges, this study provides the entrance length characteristics of viscoelastic fluids under elevated Weissenberg number conditions.

Combined with the above analysis, the main research content of this paper is as follows. Section 2 describes the mathematical model and numerical methods employed to solve the Oldroyd-B model equations. Section 3 presents and discusses the results of the numerical simulations in the Newtonian and Oldroyd-B fluids, and Section 4 provides conclusions and directions for future work on the entrance length.

2. Governing Equations and Methods

2.1. Governing Equations

For a viscoelastic fluid flow, the governing equations consist of the incompressible continuity equation and the momentum equation:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \boldsymbol{\tau} \quad (2)$$

where $\mathbf{u}$ is the velocity vector, ρ is the density, p is the pressure, $\boldsymbol{\tau}$ is the total stress tensor, which can be decomposed into the solvent ($\boldsymbol{\tau}_s$) and polymer ($\boldsymbol{\tau}_p$) stress tensors, as follows:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_s + \boldsymbol{\tau}_p \quad (3)$$

In Equation (3), $\boldsymbol{\tau}_s$ is given by the Newtonian law $\boldsymbol{\tau}_s = \eta_s [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$, where η_s is the solvent viscosity.

The constitutive equation for the Oldroyd-B model [24,25] is of the form

$$\boldsymbol{\tau}_p + \lambda \overset{\nabla}{\boldsymbol{\tau}}_p = \eta_p [\nabla \mathbf{u} + (\nabla \mathbf{u})^T] \quad (4)$$

where η_p is the polymer viscosity at the zero-shear rate, λ is the relaxation time. $\overset{\nabla}{\tau}_p$ denotes the upper converted time derivative of τ_p , which is defined as

$$\overset{\nabla}{\tau}_p = \frac{\partial \tau_p}{\partial t} + \mathbf{u} \cdot \nabla \tau_p - \tau_p \cdot \nabla \mathbf{u} - (\nabla \mathbf{u})^T \cdot \tau_p \quad (5)$$

The constitutive equation is formulated using the conformation tensor $\mathbf{C}$. The polymer stress and the conformation tensor are related to each other by

$$\tau_p = \frac{\eta_p}{\lambda} (\mathbf{C} - \mathbf{I}) \quad (6)$$

The conformation tensor $\mathbf{C}$ can be diagonalized as

$$\mathbf{C} = \mathbf{R} \mathbf{\Lambda} \mathbf{R}^T \quad (7)$$

where $\mathbf{R}$ is the eigenvector matrix fulfilling $\mathbf{R} \mathbf{R}^T = \mathbf{R}^T \mathbf{R} = \mathbf{I}$ and $\mathbf{\Lambda}$ is the diagonal matrix containing the eigenvalues of $\mathbf{C}$.

The equation for the conformation tensor in the Oldroyd-B model can be rewritten as

$$\frac{D\mathbf{C}}{Dt} = \mathbf{C}\mathbf{L} + \mathbf{L}^T\mathbf{C} + \frac{1}{\lambda}(\mathbf{I} - \mathbf{C}) \quad (8)$$

where $\mathbf{L} = \nabla \mathbf{u}$.

Defining $\tilde{\mathbf{\Omega}} = \mathbf{R}^T \frac{D\mathbf{R}}{Dt}$ and $\tilde{\mathbf{L}} = \mathbf{R}^T \mathbf{L} \mathbf{R}$, Equation (8) can be obtained as

$$\tilde{\mathbf{\Omega}} \mathbf{\Lambda} + \frac{D\mathbf{\Lambda}}{Dt} + \mathbf{\Lambda} \tilde{\mathbf{\Omega}}^T = \mathbf{\Lambda} \tilde{\mathbf{L}} + \tilde{\mathbf{L}}^T \mathbf{\Lambda} + \frac{1}{\lambda} \mathbf{I} - \frac{1}{\lambda} \mathbf{\Lambda} \quad (9)$$

The logarithm of $\mathbf{C}$ is defined as

$$\mathbf{S} = \log \mathbf{C} = \mathbf{R} \log \mathbf{\Lambda} \mathbf{R}^T \quad (10)$$

Equation (9) can be rewritten as

$$\frac{D\mathbf{S}}{Dt} = \mathbf{R} \left(\tilde{\mathbf{\Omega}} \log \mathbf{\Lambda} + \frac{D\mathbf{\Lambda}}{Dt} \mathbf{\Lambda}^{-1} + \log \mathbf{\Lambda} \tilde{\mathbf{\Omega}}^T \right) \mathbf{R}^T \quad (11)$$

The equation for the logarithmic conformation tensor in the Oldroyd-B model can be obtained:

$$\frac{D\mathbf{S}}{Dt} = \mathbf{\Omega} \mathbf{S} - \mathbf{S} \mathbf{\Omega} + 2\mathbf{B} + \frac{1}{\lambda} (e^{-\mathbf{S}} - \mathbf{I}) \quad (12)$$

where $\tilde{\mathbf{B}}_{ii} = \tilde{\mathbf{L}}_{ii}$, $\mathbf{B} = \mathbf{R} \tilde{\mathbf{B}} \mathbf{R}^T$, and $\mathbf{\Omega} = \mathbf{R} \tilde{\mathbf{\Omega}} \mathbf{R}^T$.

2.2. Geometry and Boundary Conditions

For axi-symmetric cases, the pipe is approximated by a wedge-shaped mesh with a small angle and a thickness of one cell, running along the centerline, straddling one of the coordinate planes, as shown in Figure 1. The axi-symmetric wedge planes must be specified as separate patches with a wedge shape. In this geometry, the x -coordinate represents the pipe axis, and the y -coordinate represents the pipe radius. The wedge boundary type is implemented using a 3D wedge-shaped mesh that mathematically represents axi-symmetric conditions, though it is not strictly a cylindrical-polar structured mesh in the traditional sense. A mesh diagram is shown in Figure 2.

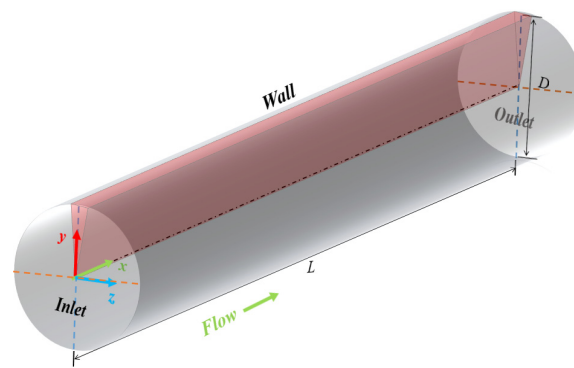


Figure 1. Geometry model diagram for pipe flow.

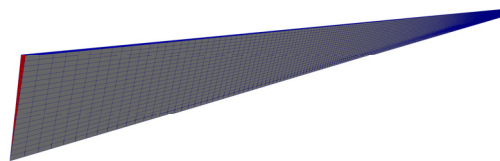


Figure 2. Mesh diagram.

The ratio of pipe length (L) to pipe diameter (D) is 50:1 to ensure fully developed flow conditions. x/D and y/D are the normalized x -coordinate and y -coordinate, respectively, and a uniform mesh is used in this geometry. The inlet is assigned a uniform velocity, the pipe wall is defined as a no-slip boundary condition, the center axis is treated as a symmetry condition, and the outlet is set at zero backpressure.

2.3. Numerical Method

The laminar pipe flow is simulated using OpenFOAM, along with RheoTool package. For non-Newtonian fluids, the polymer stresses undergo a substantial increase, resulting in significant numerical instability during simulations. This study proposes the utilization of the LCR method to enhance the numerical stability of the simulations. In a recent study, Mohanty et al. [26] used the OpenFOAM-based LCR method to investigate the influence of the solvent viscosity ratio and Deborah number on the creeping flow of an Oldroyd-B viscoelastic fluid past a channel-confined cylinder. The solution in this study differs from that of Mohanty's study mainly in the discretization scheme used for the governing equations and the choice of solver. To enhance the convergence of iterations, a steady solution is achieved by obtaining a long-term solution to an unsteady problem, thereby retaining the time derivatives in the governing equations. The temporal derivative terms are discretized using the implicit Euler scheme. The convective terms in the momentum and constitutive equations are discretized with the CUBISTA (Convergent and Universally Bounded Interpolation Scheme for Treatment of Advection) [27] scheme for discretization, and the coupling of pressure and velocity is performed using the SIMPLE algorithm. Numerical relaxation of the governing equations is mainly used for the field relaxation by introducing relaxation factors to control variable updates. The numerical simulation focuses on field relaxation techniques specifically applied to pressure fields, velocity fields, stress fields, and tensors, and the relaxation factor for these fields is uniformly set to 1.

The pressure and velocity fields are solved using the smooth solver along with the GaussSeidel smoother which is an iterative solver but also as a widely used smoother. Its local error smoothing property makes it effective in eliminating high-frequency errors, particularly in elliptic partial differential equations. It is imperative to ensure that residuals vary smoothly during the solution process in order to enhance numerical stability. The

residual dimensionless value of the pressure and velocity fields is 10^{-7} . The stress field and conformation tensors are solved using the PBiCG (Preconditioned Bi-conjugate Gradient) solver along with the DILU (Diagonal-based Incomplete LU) preconditioner. This solver is utilized for the resolution of asymmetric coefficient matrices, yielding residual dimensionless values of 10^{-6} .

The dimensionless parameters are defined as follows:

$$Re = \frac{\rho V D}{\eta_0} \quad (13)$$

where V is the average velocity of the fluid, and $\eta_0 = \eta_s + \eta_p$ is the solvent and polymer viscosities at the zero-shear rate.

$$Wi = \frac{\lambda V}{D} \quad (14)$$

$$\beta = \frac{\eta_s}{\eta_0} \quad (15)$$

$$h_f = \lambda_f \frac{L}{D} \frac{V^2}{2g} = \frac{\Delta p_f}{\rho g} \quad (16)$$

where h_f is the friction head loss, λ_f is the friction factor, g is the gravitational acceleration which is typically taken to be 9.8 m/s^2 , and Δp_f is the pressure drop due to fluid resistance.

3. Results

3.1. Grid Independence Verification

To ensure the stability of the computational grid and the accuracy of the present results, mesh independency is demonstrated. Numerical computations are performed using five grid resolutions, with the mesh characteristics tabulated in Table 1. The numerical results for the entrance length of the Newtonian fluid at $Re = 100$ are verified using the dimensionless entrance length $L_e/D = 0.0575 Re$, as defined by Langhaar [2].

Table 1. Characteristics of meshes for pipe flow.

Mesh	Number of Nodes	$\Delta x/D$	$\Delta y/D$
M1	100×5	0.5	0.1
M2	200×10	0.25	0.05
M3	350×15	0.1429	0.0333
M4	500×20	0.1	0.025
M5	700×30	0.0714	0.0167

As shown in Figure 3, when $\Delta x/D \leq 0.25$, the entrance length of the Newtonian fluid is consistent with Langhaar's definitions [2] and remains essentially unchanged numerically. The use of 500×20 grid as the final computational grid is predicated on the necessity to ensure the stability of the results and calculation speed.

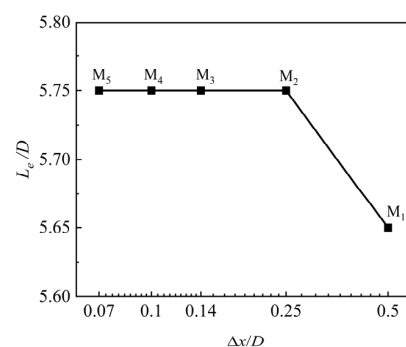


Figure 3. Entrance length for Newtonian fluid at $Re = 100$ with different $\Delta x/D$.

3.2. Accuracy Verification

To validate the LCR method, the results were compared with the entrance lengths of an Oldroyd-B fluid in a two-dimensional rectangular channel at $Re = 0.001$ and $\beta = 0.1$ [10]. The fundamental principles underpinning entrance length studies remain consistent, irrespective of the dimensionality of the geometric model employed. The dimensionality of the geometric model does not engender any difference in their numerical methods. In order to simplify the computational difficulty of reliability validation, the computational model is validated in comparison with that of Yapici et al. [10]. Dimensionless entrance lengths are, respectively, compared at $Wi = 0.1$ – 0.5 , and the results are shown in Figure 4. The maximum deviation is 2.89%, while the minimum deviation is 0.21%. These findings indicate that both the computational model and process can be considered reliable.

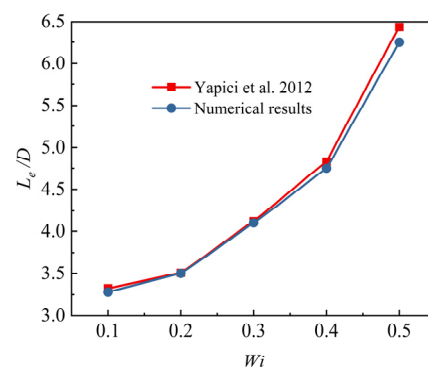


Figure 4. Comparison of L_e/D in the numerical results for the Oldroyd-B fluid and the results of Yapici et al. [10].

3.3. Lower Reynolds Number

Prandtl's boundary layer theory states that the viscous properties of fluids exert an influence on the flow within a very small thickness, denoted by δ , near the surface of the object. In the event that the characteristic dimension in the direction of fluid flow is denoted by l , the boundary layer thickness is denoted as $\frac{\delta}{l} \sim \frac{1}{\sqrt{Re_l}}$. This theory applies to high-Reynolds-number flows where inertia dominates viscosity. Specifically, Re_l should satisfy $Re_l \gg 1$ and $\delta \ll l$ must hold. The hydrodynamic entrance length (L_e) demonstrates a linear proportionality to the Reynolds number (Re), as expressed by the relation $L_e/D = C \cdot Re$, where C is the dimensionless development coefficient. However, under low-Reynolds-number conditions, the aforementioned theoretical relationship becomes invalid. Therefore, numerical simulations identify the range of Reynolds numbers that satisfy $\delta/l \ll 1$, ensuring the boundary layer differential equations' validity.

The laminar flow characteristics of Newtonian fluid were examined in a pipe with different Reynolds numbers. The simulation results for a dimensionless entrance length were compared with $L_e/D = 0.0575 Re$, as defined by Langhaar [2]. Figure 5 reflects the effect of Re on the dimensionless entrance length of Newtonian fluid. Both simulations and theory show a linear growth of entrance length with increasing Re . For $Re \geq 40$, the simulated and theoretical results agree closely. Below $Re = 40$, the results increasingly deviate from the theory as Re decreases. The deviation grows systematically with decreasing Re , consistently exceeding theoretical predictions. We identified a critical Reynolds number ($Re_L = 40$) below which Prandtl's boundary layer conditions fail. This lower boundary, represented by $Re_L = 40$, signifies the point at which the condition of strict satisfaction is no longer met. Consequently, Prandtl boundary layer differential equations cannot be strictly established.

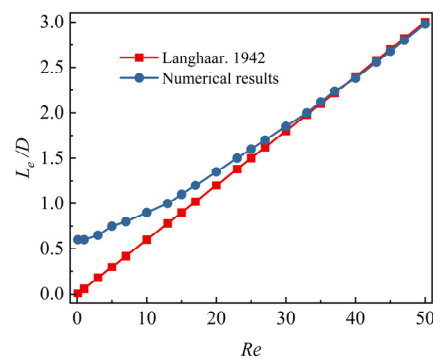


Figure 5. Comparison of L_e/D in the Newtonian fluid between numerical results and Langhaar's results [2].

3.4. Influence of Dimensionless Entrance Length and Friction Factor

3.4.1. Reynolds Number

The Reynolds number directly affects laminar flow fields through inertial forces. We quantify inertia's effect on entrance length and friction factor across Reynolds numbers. In Figure 6a–c, the dimensionless centerline velocities of the pipe are represented for various Reynolds numbers in the Newtonian and Oldroyd-B fluids with $\beta = 0.5$. The centerline velocities increase with x/D for both fluids, with Oldroyd-B showing more pronounced development. Newtonian fluids reach the expected maximum velocity (twice the average velocity), matching the analytical solutions. The Oldroyd-B fluids show a stronger velocity development before reaching a fully developed flow.

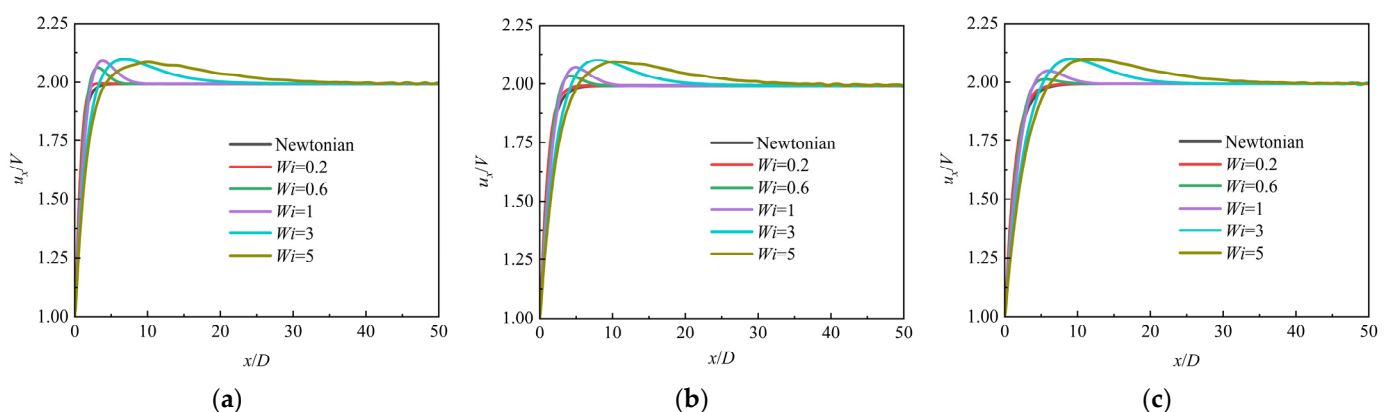


Figure 6. The dimensionless centerline velocities of pipes in Newtonian and Oldroyd-B fluids with $\beta = 0.5$. (a) $Re = 50$; (b) $Re = 75$; (c) $Re = 100$.

We analyzed the inertial effects on entrance lengths for Newtonian and Oldroyd-B fluids at $Re = 50, 75$, and 100 . Figure 7 reveals that the dimensionless entrance length in Oldroyd-B model is found to be particularly influenced by Re . The entrance lengths in Oldroyd-B model grow with Re , consistent with Yapici et al. [10]. The Oldroyd-B model shows significantly longer entrance lengths than the Newtonian fluids. The dependence of entrance length on Re intensifies with higher Weissenberg numbers (Wi). L_e/D in the Oldroyd-B model exhibits minimal Re sensitivity at $Wi < 1$, but a strong dependence at $Wi \geq 1$.

Oldroyd-B fluid's properties affect both entrance length and friction factor λ_f . Figure 8 reflects λ_f for Newtonian and Oldroyd-B fluids at different Re values with $\beta = 0.5$. A close examination reveals that the slopes of curves for the Newtonian and Oldroyd-B fluids are essentially the same. λ_f in the Oldroyd-B model decreases more sharply with Re and

remains lower than Newtonian values. At $Wi = 0.2$, λ_f nearly matches the Newtonian behavior, converging as elasticity decreases.

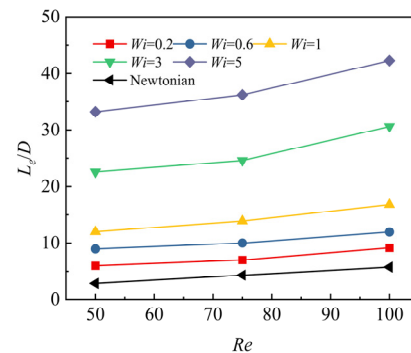


Figure 7. Variation in L_e/D in Newtonian and Oldroyd-B fluids at different Re values with $\beta = 0.5$.

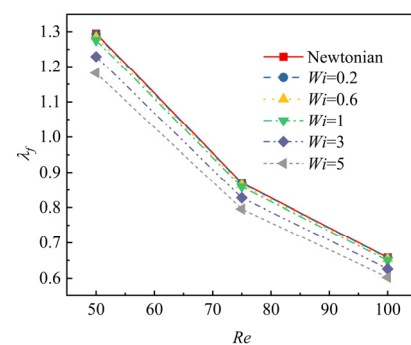


Figure 8. Variation in λ_f in Newtonian and Oldroyd-B fluids at different Re values with $\beta = 0.5$.

3.4.2. Solvent Viscosity Ratio

The solvent viscosity ratio β has been shown to reflect the contribution of a polymer to solution viscosity. It reveals an inverse relationship between β and the solute's viscous contribution in the viscoelastic fluid. Figure 9 shows that decreasing β (increasing solute viscosity) augments L_e/D in Oldroyd-B fluids. The slope of the curve increases gradually as the contribution of solute viscosity increases. L_e/D exhibits a smoother curve and greater resistance to variation with β when $Wi < 1$. Conversely, under conditions of high Wi ($Wi \geq 1$), the results are more susceptible to β , and the greater the Wi , the more pronounced the increase in L_e/D .

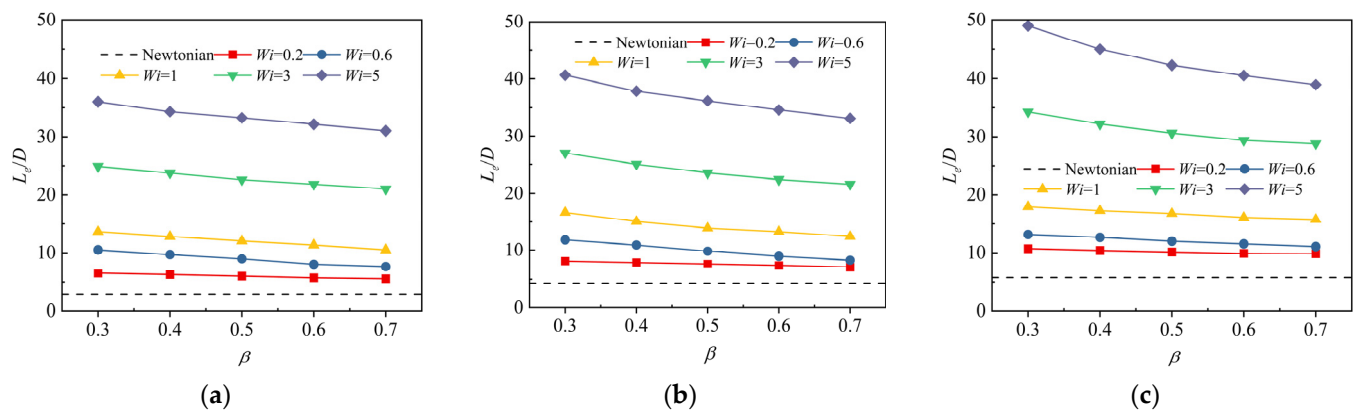


Figure 9. Variation in L_e/D in Newtonian and Oldroyd-B fluids at different β . (a) $Re = 50$; (b) $Re = 75$; (c) $Re = 100$.

Additionally, the friction factor is influenced by the solvent viscosity ratio. Figure 10 shows the law of variation in an Oldroyd-B fluid's λ_f for different β at $Re = 50, 75$, and 100 . λ_f increases with β , but remains nearly constant for $Wi = 0.2$, matching Newtonian behavior. At $Wi \geq 1$, the effect of solvent viscosity ratio intensifies with Wi , producing λ_f values substantially below Newtonian levels.

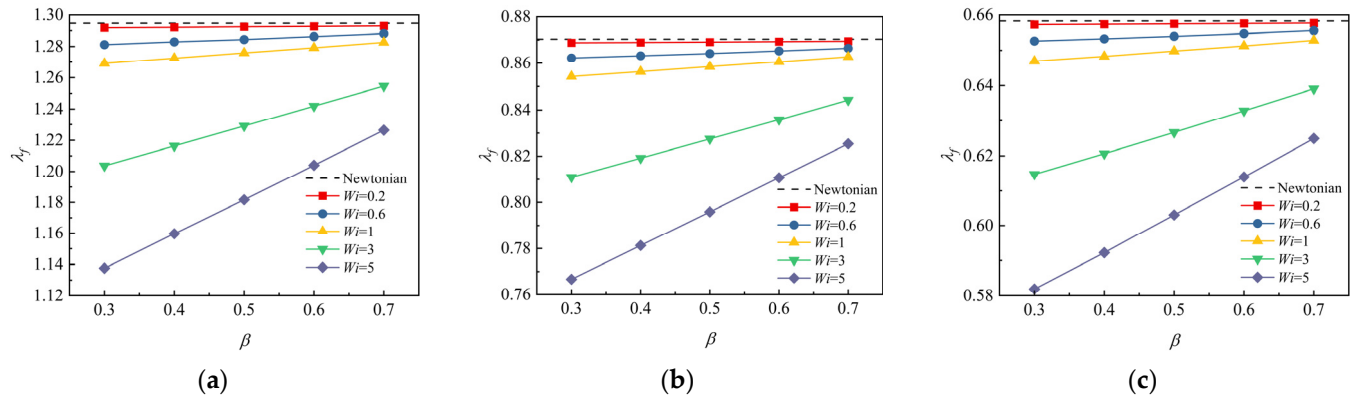


Figure 10. Variation in λ_f in Newtonian and Oldroyd-B fluids at different β . (a) $Re = 50$; (b) $Re = 75$; (c) $Re = 100$.

3.4.3. Weissenberg Numbers

The Weissenberg number is indicative of the extent to which the elastic properties of a fluid predominate during shear or tensile flow. The effect of varying Wi on the entrance length in Oldroyd-B fluid at $Re = 50, 75$, and 100 is demonstrated in Figure 11. As Wi increases, L_e/D in the Oldroyd-B fluid gradually increases. At $Wi < 1$, L_e/D changes minimally, especially at $Re = 50$, while $Re = 100$ shows dramatic increases. For $Wi \geq 1$, L_e/D rises sharply with Wi , particularly at $Re = 100$, while $Wi = 5$ produces extreme elongation.

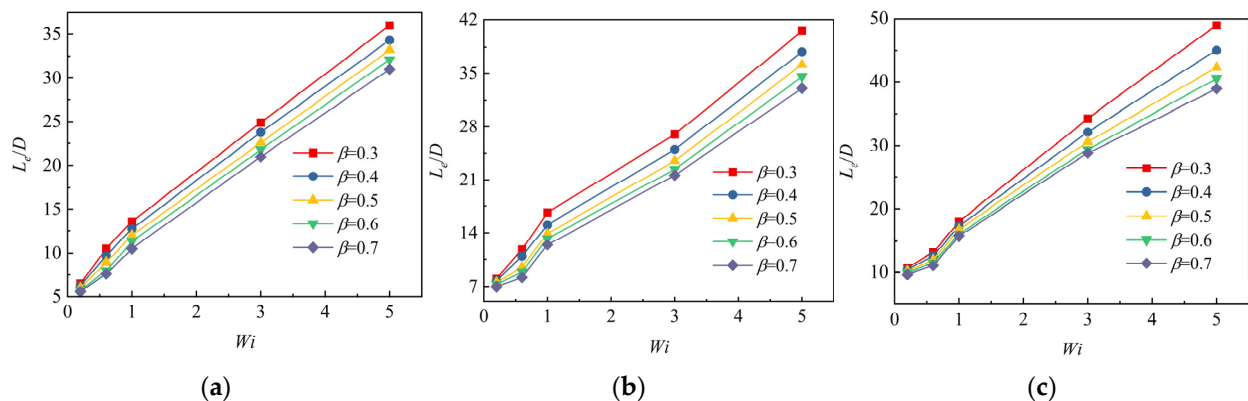


Figure 11. Variation of L_e/D in the Newtonian and Oldroyd-B fluid at different Wi . (a) $Re = 50$; (b) $Re = 75$; (c) $Re = 100$.

Figure 12 presents the friction factor curves in Oldroyd-B fluid across Wi values. At $Wi = 0.2$, β variations produce negligible changes in λ_f . Higher Wi values cause sharp λ_f reductions, with pronounced elastic effects dominating at elevated Wi . Strong elasticity and solute viscosity combine to dramatically reduce λ_f , demonstrating how viscoelastic properties govern pipe flow behavior.

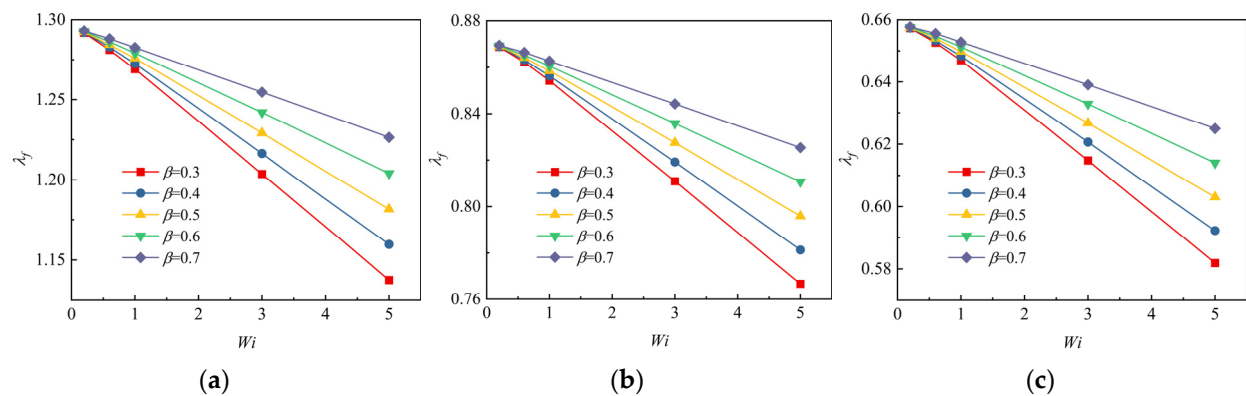


Figure 12. Variation in λ_f in the Newtonian and Oldroyd-B fluids at different Wi . (a) $Re = 50$; (b) $Re = 75$; (c) $Re = 100$.

4. Conclusions

This study employs OpenFOAM-based numerical simulations to systematically investigate the entrance length and friction factor in the Oldroyd-B model in a pipe flow. According to the Prandtl boundary layer theory, the condition for the establishment of boundary layer is $\delta/l \ll 1$. For Newtonian fluids, a lower limit Re_L ($Re_L = 40$) exists. Below this threshold $Re < 40$, the condition $\delta/l \ll 1$ is not strictly satisfied. This prevents the Prandtl's boundary layer differential equation from being strictly established.

The entrance length and friction factor in the Oldroyd-B model are both affected by its fluid inertia effect. As the Reynolds number increases, the entrance length of the fluid tends to increase, but the friction factor shows a sharp decrease. Under identical conditions, the Oldroyd-B model exhibits a longer entrance length and a smaller friction factor in comparison to the Newtonian fluid.

As the solvent viscosity ratio β increases, the solvent viscosity's contribution to the fluid increases, causing the entrance length in the Oldroyd-B model to decrease while its friction factor increases. The entrance length and friction factor are more affected by β in highly elastic ($Wi \geq 1$) conditions relative to low-elastic ($Wi < 1$) conditions.

The entrance length in the Oldroyd-B model is subject to an increase proportional to the rise in the Weissenberg number of the fluid. Concurrently, the friction factor undergoes a decrease. Under identical conditions, the results of the Weissenberg number approximate those of the solvent viscosity ratio β . The entrance length and friction factor vary to a lesser extent for $Wi < 1$, and the effect is more drastic for $Wi \geq 1$.

For fluid transport systems, heat exchange equipment, chemical reactors, and other engineering fields, the accurate calculation of entrance length directly impacts system performance, energy consumption, and operational safety. Although entrance length is a classic fluid mechanics problem, it remains highly relevant in emerging fields such as renewable energy and biomedical engineering.

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Nomenclature

Nomenclature		PBiCG	Preconditioned Bi-conjugate Gradient
		PTT	Phan-Thien–Tanner
		SIMPLE	Semi-Implicit Method for Pressure
			Linked Equations
		SIMPLER	Semi-Implicit Method for Pressure Linked
			Equations Revised
Bi	Bingham number	<i>Greek letters</i>	
C	Conformation tensor	β	Solvent viscosity ratio
D	Diameter	δ	Boundary layer thickness
g	Gravitational acceleration	η	Viscosity
h_f	Friction head loss	λ	Relaxation time
l	Characteristic length	λ_f	Friction factor
L	Length	ρ	Density
L_e	Entrance length	τ	Stress tensor
M	Mesh	Δ	Change symbol
n	Exponential	Λ	Diagonal matrix composed of eigenvalues
p	Pressure	Ω	Anti-symmetry matrix
R	Eigenvector matrix	<i>Superscripts</i>	
Re	Reynolds number	∇	Gradient
Re_L	Lower Reynolds number	T	Transposition
S	Logarithm of conformation tensor	<i>Subscripts</i>	
t	Time	f	Friction
u	Velocity vector	i	ith segment
V	Average velocity	l	Characteristic length
Wi	Weissenberg number	L	Low
<i>Abbreviations</i>		s	Solvent
	Convergent and Universally	p	Polymer
CUBISTA	Bounded Interpolation Scheme	0	Zero-shear rate
	for Treatment of Advection		
DILU	Diagonal-based Incomplete LU		
gPTT	Generalized Phan-Thien Tanner		
log	Logarithm		
LCR	Log-Conformation Representation		

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