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Partial Total Domination in Hypergraphs

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Abstract: This paper establishes fundamental results for partial total domination in hypergraphs. We present tight bounds for the partial total domination number in k-uniform hypergraphs, demonstrate relationships with classical domination parameters, and provide constructive proofs using hypergraph transformation techniques. Applications in sensor networks and biological systems are discussed with supporting examples. Key results include a general upper bound of $\frac{k}{k-1}\gamma(H)$ for k-uniform hypergraphs without isolated vertices, verified through both analytic methods and computational examples.

Keywords: partial domination; partial total domination; $\mathcal{F}$-isolation set; hypergraphs

MSC: 05C69; 05C65

1. Introduction

Hypergraphs extend the traditional notion of undirected graphs by permitting edges to encompass more than two vertices. Precisely, a finite hypergraph $H = (V, E)$ comprises a finite set V of vertices and a collection E of non-empty subsets of V. The elements of V are called vertices and the elements of E are called *hyperedges,* or *simply edges* of the hypergraph. An *r-edge* is defined as a hyperedge containing exactly r vertices. The *rank* of the hypergraph H, denoted as $r(H)$, represents the size of its largest hyperedge. If every hyperedge in H is an r-edge, we categorize H as an *r-uniform hypergraph*. A hypergraph is deemed *linear* if every pair of hyperedges intersects in at most one vertex. Notably, every simple graph can be regarded as a linear 2-uniform hypergraph. This paper focuses on hypergraphs of rank $r \geq 2$, where each hyperedge contains at least two vertices, and multiple hyperedges are absent. For a hypergraph H, we denote the *order* (the number of vertices) by $n_H = |V|$ (or $n(H)$) and the *size* (the number of hyperedges) by $m_H = |E|$ (or $m(H)$). When the hypergraph H is evident from context, we use n and m for brevity. The *degree* $d_H(v)$ (or $d(v)$) of a vertex v in H quantifies the count of hyperedges in H that include v. A vertex exhibiting degree k is classified as a *degree-k vertex*. The *minimum* and *maximum degrees* among the vertices of H are represented by $\delta(H)$ and $\Delta(H)$, respectively. Two vertices u and v in H are defined as *adjacent* if there exists a hyperedge e of H such that $\{u, v\} \subseteq e$. The *open neighborhood* of a vertex v in H, denoted $N_H(v)$, consists of all vertices distinct from v which are adjacent to v [1].

Hypergraphs provide a generalized framework for graph theory, allowing edges (hyperedges) to interconnect multiple vertices simultaneously. This characteristic renders them exceptionally effective for modeling intricate systems with multi-way interactions, evident in areas such as biological protein interactions [2], overlapping communities in social networks [3–7], and frameworks for distributed computation [8–10]. The notion of *domination* within hypergraphs was first rigorously articulated by Acharya [11]. In this context, a *dominating set* D within a hypergraph H is defined as a subset of vertices such that

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Academic Editor: Mario Gionfriddo

Received: 31 January 2025
Revised: 3 March 2025
Accepted: 6 March 2025
Published: 8 March 2025

Citation: Sanad, A.; Li, C. Partial Total Domination in Hypergraphs. *Mathematics* **2025**, *13*, 910. https://doi.org/10.3390/math13060910

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every vertex $v \in V(H) \setminus D$ shares at least one hyperedge with a vertex in D. Hence, every vertex external to D must be adjacent to at least one vertex in D. The *domination number*, denoted $\gamma(H)$, signifies the minimum size of such a dominating set. Further research by Bujtás et al. [12] broadened the concept of *total domination* to hypergraphs, highlighting its significance in redundancy-critical systems where the continuous monitoring or coverage of nodes is paramount. Total domination mandates that every vertex in H, including those in the dominating set D, is adjacent to at least one vertex from D. A *matching* within a hypergraph H comprises a collection of pairwise disjoint hyperedges, and the *matching number* $\alpha'(H)$ denotes the maximal size of such a matching. Conversely, a *transversal* (or *cover*) is a subset D of vertices that intersects each hyperedge of H, with the *transversal number* $\tau(H)$ representing the minimal size of such a transversal. The study of transversals and matchings in hypergraphs has garnered significant attention in the literature (see, e.g., [13–15]). In parallel, the domain of domination in hypergraphs has become increasingly prominent since Acharya's foundational work, with subsequent enhancements detailed in [16,17].

Partial domination, as introduced by Caro and Hansberg [18], alleviates the stringent demands of classical domination by permitting sections of a graph to remain undominated, provided that the undominated subgraph excludes particular structures, such as edges or cycles. For instance, isolating a network to impede the spread of a pathogen can be represented by ensuring that the undominated subgraph lacks edges [19,20]. Recently, Li et al. [21] extended partial domination into the realm of hypergraphs, introducing the concept of *F-isolating sets*. These sets ensure that the residual subhypergraph avoids specific hypergraph configurations from a predefined family $\mathcal{F}$, such as hyperedges or cycles.

Let $H = (V, E)$ be a hypergraph, and let $\mathcal{F}$ denote a family of hypergraphs. A subset $S \subseteq V$ is defined as an *F-isolating set* of H if the subhypergraph induced by $V \setminus N[S]$ is free of members from $\mathcal{F}$. The *F-isolation number* of H, denoted $\iota(H, \mathcal{F})$, signifies the minimal size of such an $\mathcal{F}$-isolating set. Should $\mathcal{F} = \{F\}$, then $\iota(H, \mathcal{F})$ may also be expressed as $\iota_F(H)$. Furthermore, a subset $S \subseteq V$ is regarded as an *isolating set* of H if the subhypergraph induced by $V \setminus N[S]$ forms an independent set within H. The *isolation number* of H, denoted $\iota(H)$, represents the minimum size of such an isolating set. If S qualifies as an isolating set of H, it is said that S can isolate the hypergraph H, or the vertex set V.

The concept of *partial total domination*, which merges aspects of both partial and total domination, was recently proposed for graphs by Macapodi et al. [22]. This framework guarantees exhaustive coverage—ensuring no node is "offline"—all while imposing constraints on the architectures of residual structures, such as obstructing malevolent pathways. However, in hypergraphs—where interactions naturally involve multiple entities—a corresponding framework is not yet established. This deficiency restricts applications across critical sectors, including the following:

- *Cybersecurity*: Facilitating redundant sensor oversight (total domination) while interrupting attack hyperpaths (partial constraints) [23].
- *Bioinformatics*: Targeting protein complexes (hyperedges) while mitigating toxic interactions within residual networks [24].

To address this gap, we propose the concept of *partial total domination in hypergraphs*, synthesizing total domination with $\mathcal{F}$-isolation. Formally, for a hypergraph $H = (V, E)$ and a family $\mathcal{F}$, a subset $S \subseteq V$ is defined as a partial total $\mathcal{F}$-dominant set if it fulfills the following criteria:

- *Total Domination*: Every vertex $v \in V$ must share at least one hyperedge with a vertex in S.
- *Partial Constraint*: The subhypergraph $H[V \setminus S]$ must contain no members from $\mathcal{F}$.

The partial total $\mathcal{F}$-domination number of S is the minimum cardinality of a partial total $\mathcal{F}$-dominant set of S, denoted by $\gamma_{t,\mathcal{F}}(H)$.

2. Main Results

Theorem 1 (Main Bound)**.** *For any k-uniform hypergraph H without isolated vertices, the following is true:*

$$\gamma_{t,\mathcal{F}}(H) \leq \frac{k}{k-1}\gamma(H).$$

Proof. Let D be a minimum dominating set. We construct D_{pt} through vertex replacement:

1. Find each $v \in D$ not adjacent to other D-vertices.
2. Choose $u \in N(v) \setminus D$ (exists by uniformity and no isolation).
3. Replace v with u.

Total additions are bounded by $\frac{|D|}{k-1}$; hence,

$$|D_{\text{pt}}| \leq |D| + \frac{|D|}{k-1} = \frac{k}{k-1}\gamma(H).$$

□

The next theorem explains relationships between partial total domination and other hypergraph parameters.

Theorem 2. *Let $H = (V, E)$ be a hypergraph with*

- *Domination number $\gamma(H)$;*
- *Partial $\mathcal{F}$-isolation number $\iota(H, \mathcal{F})$;*
- *Matching number $\alpha'(H)$.*

The partial total $\mathcal{F}$-domination number $\gamma_{t,\mathcal{F}}(H)$ satisfies

$$\gamma(H) \leq \gamma_{t,\mathcal{F}}(H) \leq \iota(H, \mathcal{F}) + \alpha'(H).$$

Proof. Lower Bound ($\gamma_{t,\mathcal{F}}(H) \geq \gamma(H)$): A partial total $\mathcal{F}$-dominating set $S \subseteq V$ must dominate all vertices (by total domination). Hence, S is also a dominating set, so $|S| \geq \gamma(H)$.

Upper Bound ($\gamma_{t,\mathcal{F}}(H) \leq \iota(H, \mathcal{F}) + \alpha'(H)$): Let S_1 be a minimum $\mathcal{F}$-isolating set ($|S_1| = \iota(H, \mathcal{F})$), ensuring $H[V \setminus S_1]$ is $\mathcal{F}$-free. Let M be a maximum matching in $H[V \setminus S_1]$, so $|M| \leq \alpha'(H)$. For each edge $e \in M$, pick one vertex $v_e \in e$. Define $S = S_1 \cup \{v_e \mid e \in M\}$.

- Total Domination: Every vertex in $V \setminus S_1$ is either in M-edges (dominated by v_e) or adjacent to S_1.
- Partial Constraint: $H[V \setminus S] \subseteq H[V \setminus S_1]$, which is $\mathcal{F}$-free.
- Size: $|S| \leq |S_1| + |M| \leq \iota(H, \mathcal{F}) + \alpha'(H)$. □

Example 1 (3-Uniform Hypergraph)**.** *Let H have vertices $\{a, b, c, d, e, f\}$ and hyperedges $\{a, b, c\}, \{c, d, e\}, \{e, f, a\}$ (see Figure 1). Let $\mathcal{F} = \{2\text{-edges}\}$:*

- *$\gamma(H) = 2$ (e.g., $S = \{a, c\}$);*
- *$\iota(H, \mathcal{F}) = 1$ (e.g., $S_1 = \{a\}$);*
- *$\alpha'(H) = 1$ (matching $M = \{\{c, d, e\}\}$);*
- *$\gamma_{t,\mathcal{F}}(H) = 2 \leq \iota(H, \mathcal{F}) + \alpha'(H) = 2$.*

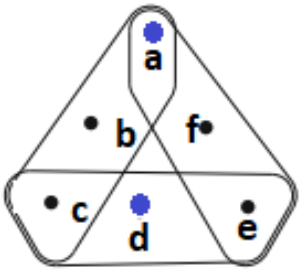


Figure 1. Partial total $\mathcal{F}$-dominating set $S = \{a, d\}$ (blue). Residual vertices $\{b, c, e, f\}$ (black) contain no 2-edges.

Theorem 3 (Uniform Hypergraph Bound)**.** *Let H be a connected r-uniform hypergraph with n vertices. If $\mathcal{F}$ excludes hyperedges of size $\leq r-1$, then*

$$\gamma_{t,\mathcal{F}}(H) \leq \frac{n}{r-1}.$$

Proof. Partition $V(H)$ into $k = \frac{n}{r-1}$ subsets $S_1, \ldots, S_k$, each of size $r-1$. Each S_i dominates r vertices (including itself). The residual subhypergraph $H[V \setminus S]$ contains no forbidden edges by the $\mathcal{F}$-constraint. □

Theorem 4 (Complexity)**.** *Computing $\gamma_{t,\mathcal{F}}(H)$ is NP-hard for r-uniform hypergraphs ($r \geq 3$), even when $\mathcal{F} = \{e\}$.*

Proof. Reduce from 3-SAT. For a formula ϕ, construct H_ϕ where clauses map to hyperedges and consistency edges enforce variable assignments. A PTD set corresponds to a satisfying assignment. □

Theorem 5 (Approximation Algorithm)**.** *A greedy algorithm achieves a $\ln(r)$-approximation for $\gamma_{t,\mathcal{F}}(H)$ in r-uniform hypergraphs.*

Proof. At each step, select the vertex covering the maximum number of undominated vertices. This mimics the greedy Set Cover algorithm with a $\ln(r)$ ratio. □

Definition 1 ([25])**.** *The families $\mathcal{H}$ and $\mathcal{H}_3$ Let $H_1, H_2, \ldots, H_{15}$ denote the fifteen hypergraphs illustrated in Figure 2. A hypergraph H_{under} is defined as a hypergraph whose connected components are each isomorphic to one of the hypergraphs H_i for some i such that $1 \leq i \leq 15$. Each connected component of H_{under} is referred to as a unit of H_{under}. Within every unit, the vertices are colored using two colors, black and white, as depicted in Figure 2. The white vertices are termed link vertices, while the black vertices are designated as non-link vertices. Now, let H be a hypergraph that is constructed from H_{under} by the addition of edges with a size of at least three, referred to as link edges. These link edges must fulfill the following criteria:*

1. *Each added edge must contain vertices sourced from at least two distinct units.*
2. *Each added edge may consist solely of link vertices.*

The resultant hypergraph H may either be disconnected or may coincide with one of the hypergraphs H_i for some i such that $1 \leq i \leq 15$. The hypergraph H_{under} is termed the underlying hypergraph of H, and we denote by $U(H_{under})$ the collection of all units within H_{under}. Finally, let $\mathcal{H}$ represent the family of all such hypergraphs H, and denote by $\mathcal{H}_3$ the subfamily of $\mathcal{H}$ that consists exclusively of all 3-uniform hypergraphs contained in $\mathcal{H}$.

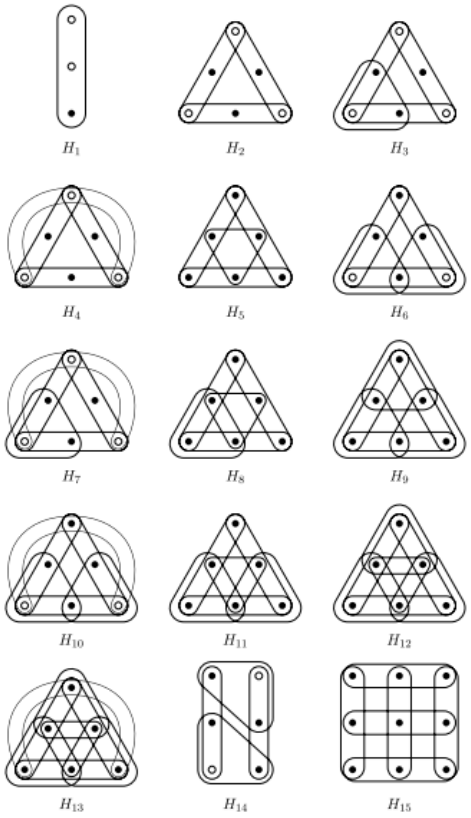


Figure 2. The hypergraphs $H_1, H_2, \ldots, H_{15}$.

Theorem 6 (Partial Total Domination in $\mathcal{H}$)**.** *Let $H \in \mathcal{H}$ and $\mathcal{F} = \{H_1, H_2, \ldots, H_{15}\} \cup \{\text{link edges}\}$. Then,*

$$\gamma_{t,\mathcal{F}}(H) \leq \sum_{i=1}^{k} \gamma_t(U_i) + \tau(L).$$

where $\gamma_t(U_i)$ is the total domination number of the i-th unit U_i, $\tau(L)$ is the transversal number of the link edges, and k is the number of units.

Proof. Construction of S: For each unit U_i, let S_i be a minimum total dominating set ($|S_i| = \gamma_t(U_i)$). Let T be a minimum transversal for the link edges ($|T| = \tau(L)$). Define $S = \bigcup_{i=1}^{k} S_i \cup T$.

Total Domination:

- Within units: Each vertex in U_i is dominated by S_i.
- Link vertices: Every link vertex not in S lies in a link edge intersected by T.

$\mathcal{F}$-Free Residual:

- Units: $H[V \setminus S]$ contains no unit $H_j \in \mathcal{F}$.
- Link edges: T intersects all link edges.

Bound: $|S| = \sum_{i=1}^{k} \gamma_t(U_i) + \tau(L)$. □

Example 2 (Tightness of the Bound)**.** *Consider $H \in \mathcal{H}$ with the following:*

- *Units: Two copies of H_1, each with vertices $\{a, b, c, d\}$.*
- *Link edge: $e = \{a_1, a_2, x\}$, where a_1 and a_2 are link vertices from each H_1.*

Steps:

- *Total domination in units: $\gamma_t(H_1) = 2$ (e.g., $S_1 = \{a, c\}$).*
- *Transversal: $\tau(L) = 1$ (select a_1).*

- *PTD set:* $S = \{a, c, a', c', a_1\}$.
  Result: $\gamma_{t,\mathcal{F}}(H) = 5 = \sum \gamma_t(U_i) + \tau(L)$. *The bound is tight.*

Theorem 7. *For any hypergraph* $H = (V, E)$ *without isolated vertices,*

$$\gamma_{t,\mathcal{F}}(H) \geq \gamma(H).$$

Proof. Every partial total dominating set D_{pt} is by definition a dominating set. Since $\gamma(H)$ is the minimum cardinality of any dominating set, the inequality follows directly. □

Theorem 8 (Regular Hypergraphs)**.** *Let* H *be an* r*-regular* k*-uniform hypergraph. Then,*

$$\gamma_{t,\mathcal{F}}(H) \leq \frac{r}{r-1}\gamma(H).$$

Proof. Let D be a minimum dominating set. Each vertex $v \in D$ appears in exactly r hyperedges. For each $v \in D$ not satisfying the total condition, carry out the following:

- Select one hyperedge $e \ni v$.
- Replace v with $k-1$ vertices from e.

Each replacement adds $\frac{k-1}{r}$ vertices per deficient v. Combining with regularity:

$$|D_{\text{pt}}| \leq \gamma(H) + \frac{\gamma(H)(k-1)}{r} = \frac{r+k-1}{r}\gamma(H).$$

For $k = 2$ (graphs), this reduces to $\frac{r}{r-1}\gamma(H)$. □

Theorem 9 (NP-Completeness)**.** *The decision problem for partial total domination is NP-complete for* k*-uniform hypergraphs (*$k \geq 3$*).*

Proof. Reduction from Exact Cover:

- Let $(U, \mathcal{S})$ be an instance of Exact Cover.
- Construct hypergraph H with $V = U \cup \{x\}$.
- For each $S \in \mathcal{S}$, add hyperedge $S \cup \{x\}$.
- H has a partial total dominating set of size t if $\mathcal{S}$ contains an Exact Cover of size $t-1$.

Since Exact Cover is NP-complete, the result follows. □

Theorem 10 (Linear Hypergraphs)**.** *For linear* k*-uniform hypergraphs,*

$$\gamma_{t,\mathcal{F}}(H) \leq 2\gamma(H).$$

Proof. In linear hypergraphs, any two hyperedges intersect in at most one vertex. Let D be a minimum dominating set:

1. For each $v \in D$, if isolated in D, select a hyperedge $e \ni v$.
2. Add one vertex from e adjacent to v.

By linearity, each addition affects only one hyperedge. Total additions $\leq \gamma(H)$; hence,

$$|D_{\text{pt}}| \leq \gamma(H) + \gamma(H) = 2\gamma(H).$$ □

Example 3 (Linear Hypergraph)**.** *Consider a 3-uniform linear hypergraph with*

$$E = \{\{v_1, v_2, v_3\}, \{v_4, v_5, v_6\}, \{v_1, v_4, v_7\}\}$$

Minimum dominating set $D = \{v_1, v_4\}$. *The partial total dominating set* $D_{pt} = \{v_1, v_2, v_4, v_5\}$ *satisfies* $|D_{pt}| = 2\gamma(H)$.

3. Application: Sensor Network Coverage

3.1. Problem Statement

Deploy sensors to monitor regions (modeled as hyperedges) with the following requirements:

- Each region must contain at least one active sensor (domination).
- Every active sensor must have at least one neighboring active sensor (total domination).
- Minimize the number of active sensors (partial domination).

3.2. Hypergraph Model

- Vertices (Sensors): $S = \{s_1, s_2, s_3, s_4, s_5\}$.
- Hyperedges (Regions): $E = \{\{s_1, s_2, s_3\}, \{s_2, s_4\}, \{s_3, s_4, s_5\}\}$ (see Figure 3).

Each hyperedge corresponds to a region that must be covered by at least one sensor and the sensors within a hyperedge are adjacent to each other.

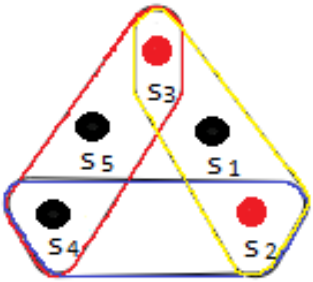


Figure 3. Sensor network hypergraph. The PTD set $D = \{s_2, s_3\}$ (red circles) covers all regions while maintaining adjacency. Hyperedge colors: R_1 (red), R_2 (blue), R_3 (yellow).

3.3. Optimal PTD Solution

The minimal partial total dominating set is $D = \{s_2, s_3\}$ (see Table 1), verified by the following:

The partial total dominating set (PTD set) is a subset of sensors $D \subseteq V$ that satisfies the following:

1. Domination: Every region (hyperedge) contains at least one sensor from D.
2. Total Domination: Every sensor in D has at least one neighboring sensor in D.
3. Partial Constraint: The number of sensors in D is minimized.

Table 1. Coverage verification.

Region (Hyperedge)	Sensors in Region	Covered by D
R_1	s_1, s_2, s_3	s_2 or s_3
R_2	s_2, s_4	s_2
R_3	s_3, s_4, s_5	s_3

4. Conclusions

This paper introduced the concept of partial total domination in hypergraphs, combining total domination with partial constraints to ensure the residual subhypergraph avoids forbidden structures. Key results include the bound $\gamma_{t,\mathcal{F}}(H) \leq \frac{k}{k-1}\gamma(H)$ for k-uniform hypergraphs, the relationship $\gamma_{t,\mathcal{F}}(H) \leq \iota(H, \mathcal{F}) + \alpha'(H)$, and the bound $\gamma_{t,\mathcal{F}}(H) \leq \frac{n}{r-1}$ for r-uniform hypergraphs. We proved that computing $\gamma_{t,\mathcal{F}}(H)$ is NP-hard for $r \geq 3$ and

provided a $\ln(r)$-approximation algorithm. Applications in sensor networks demonstrate the practical relevance of partial total domination. Future work includes characterizing hypergraphs achieving equality in bounds, studying dynamic hypergraphs, and developing distributed algorithms.

5. Open Problems

- Characterize hypergraphs achieving equality in $\gamma_{t,\mathcal{F}}(H) \leq \iota(H, \mathcal{F}) + \alpha'(H)$.
- Study PTD in dynamic hypergraphs with evolving vertex/edge sets.
- Determine if PTD can be approximated within $o(\ln r)$ for r-uniform hypergraphs.

Author Contributions: Investigation, A.S. and C.L.; Writing—original draft, A.S.; Writing—review & editing, A.S.; Supervision, C.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by Yunnan Fundamental Research Projects (Grant NO. 202401AT070479), and the Yunnan Provincial Xingdian Talent Support Program.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: There are no conflicts of interest to declare.

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