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Bidirectional Feedback Mechanism in Group Decision-Making: A Quantum Probability Theory Model Based on Interference Effects

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Abstract: Feedback in group decision-making (GDM) is an effective procedure for eliminating preference inconsistencies among experts. As the core of GDM, feedback controls the progress and cost of the process. However, the current feedback model seldom considers interference effects caused by the interaction among experts. In addition, the stubbornness of experts to change preferences through interaction is different. This study proposes a bidirectional feedback model that considers interference effects. The model integrating quantum probability theory (QPT) into a feedback mechanism has greater flexibility and is more conducive to revealing modern cognitive psychology. First, experts were classified into concordant and stubborn discordant groups according to their personality parameters. Bidirectional feedback was proposed for a stubborn discordant group to improve the efficiency of feedback process and reduce the consensus-reaching cost. QPT was then used to describe the probability of experts modifying their preferences during the game process. Combining the interference value determined by the quantum probability with the feedback mechanism, a bidirectional feedback model driven by a minimum feedback control parameter is proposed to ensure that a certain consensus level can be achieved with minimal adjustment. The proposed feedback mechanism considers interference effects produced by experts in the interaction and can capture the feelings of conflict and compromise.

Keywords: group decision-making; quantum probability theory; bidirectional feedback model; interference effects

MSC: 28-08

check for updates

Academic Editor: David Carf

Received: 18 December 2024
Revised: 18 January 2025
Accepted: 21 January 2025
Published: 24 January 2025

Citation: Cai, M.; Heng, Y. Bidirectional Feedback Mechanism in Group Decision-Making: A Quantum Probability Theory Model Based on Interference Effects. *Mathematics* **2025**, *13*, 379. https://doi.org/10.3390/math13030379

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1. Introduction

GDM theory arose with the development of welfare economics in Western countries. It is an important topic in the field of decision science and has wide realistic applications. It refers to a group of experts expressing their opinions on a limited number of alternatives, aggregating individual preferences into a collective preference, and then selecting the best solution [1,2]. The core of GDM is the "group", which requires the wisdom of the masses to make the best decision [3].

GDM is divided into two main procedures: the consensus-reaching process (CRP) and the selection of acceptable alternatives [4,5]. The CRP is the forerunner of selection and is responsible for the input and fusion of multiple opinion sources. With the development of

the information age, new social networks have emerged, which also require modification of traditional GDM. The interaction among experts may lead to division or polarization of the consensus. In a realistic decision-making environment, inconsistencies in consensus are inevitable. Therefore, a feedback mechanism is needed to generate an opinion adjustment model for experts to improve the consensus level of the group. Previous studies realized traditional feedback using the linear weighting method of the self-set feedback parameters. This mechanism does not maintain the balance between individual personality and group consensus and can increase the consensus cost and result in unnecessary waste of resources [6–8].

Defects in the feedback mechanism have been continuously addressed, and a relatively mature feedback mechanism has been gradually formed. According to the groups that needs to adjust their opinions, the existing feedback mechanisms are divided into unidirectional and bidirectional feedback mechanisms: (1) unidirectional feedback mechanism. In a unidirectional feedback mechanism, only the experts whose consensus level is less than the threshold value need to modify their opinions; concordant experts maintain their initial opinions by default. Ref. [9] proposed a feedback mechanism with structural hole spanners that include a two-stage consensus-reaching model considering the DMs' willingness. Ref. [10] developed a consensus-reaching process to improve consensus on the risk of human error. (2) Bidirectional feedback mechanism. In a bidirectional feedback mechanism [11], the adjustment of opinions is regarded as a negotiation process. The behavior-driven bidirectional feedback model proposed by [6] makes consensus reach the expected level through the maximum consensus-driven model, and then activates the maximum harmony-driven model to overcome "conflict behavior" and optimize both "group consensus" and "individual harmony". Ref. [12] proposed a bidirectional feedback mechanism in the social network context that comprehensively considers the trust level and the consensus level. Although significant achievements have been made in existing research on bidirectional feedback mechanisms, there remain several issues that necessitate further in-depth exploration. The existing bidirectional feedback mechanism ignores the interference effects (this refers to a phenomenon that occurs when a quantum system exists simultaneously in multiple possible states) of discordant and concordant groups when adjusting preferences. Quantum probability theory (QPT) [13] can address this issue.

Quantum probability theory is derived from a combination of "probability theory" and "quantum mechanics". It has been found that human behavior will violate the deterministic principle and total probability theorem in classical probability when the decision-making path is unknown [14]. In classical probability, people's belief is a jump in a certain state, and people's cognition and decision-making behavior are inherently uncertain, making them easily affected by the outside world [15]. Quantum decision theory states that, in a state of uncertainty, the first question will affect the answer to subsequent questions [16–18]. For example, a person may plan to buy a car, either an Audi or a Volkswagen. If he only asks about his preferences, he will definitely choose an Audi. However, if he first asks his wife's preferences (Volkswagen), and then asks about his own preferences, he might not choose an Audi so definitely. The QPT comprehensively considers subjectivity and objectivity in the decision-making process and models beliefs through the wave function (a mathematical function that encapsulates all possible information about a quantum system, providing a probabilistic description of the system's state). When multiple beliefs appear in people's minds simultaneously, this superposition state causes waves in different directions to collide with each other, resulting in interference effects. Recently, QPT has been applied in several fields [19,20]. Ref. [21] replaced classical probability with quantum probability amplitude, and the proposed quantum-like Bayesian network can make a more accurate prediction of decision-making behavior. Refs. [22,23] combined belief entropy with

quantum-like Bayes but with different emphases. Ref. [22] mainly used belief entropy to measure interference effects and explained the physical explanation of this method, while ref. [23] focused on the accuracy of the prediction of multiple-attribute decision-making (MAMD) results. Compared with classical probability, quantum logic has greater flexibility and randomness and has more advantages in describing people's uncertain belief state, which is more conducive to explaining people's judgments and decisions.

Based on the above analysis, we find that there are several research problems that need to be further addressed.

(1) The current bidirectional feedback mechanism overlooks the impact of personality differences on expert behavior and decision preference adjustment. It fails to distinguish between personality types, limiting the effectiveness of tailored feedback strategies.
(2) Most current methods for determining the interference term are based on prediction-oriented approaches, which rely heavily on the subjective judgment of experts. Therefore, one of the key issues this paper seeks to address is how to determine the value of the interference term from a more objective perspective.
(3) Previous bidirectional feedback consensus mechanisms ignored the interference of one decision-making subject's answer with another's, resulting in an absence of the subjectivity and uncertainty content of the decision-maker being portrayed in the bidirectional feedback mechanism.

Motivated by the challenges to fill the above research gaps and inspired by QPT and consensus decision-making theory, this study constructs a bidirectional feedback mechanism in group decision-making based on quantum probability theory. The main innovations include the following:

(1) Divide experts into three categories. Owing to the influence of experts' personalities on the feedback mechanism, the personality parameter is proposed to divide experts into three categories—stubborn, inconsistent, and harmonious experts, with different types of experts using different feedback methods (unidirectional or bidirectional).
(2) Quantify interference effects in the feedback mechanism. We construct the game process of a stubborn discordant group and a concordant group in feedback and use the powerful tool of QPT to calculate interference effects between them.
(3) Combine interference effects with a bidirectional feedback model to form opinions for adjusting preferences. The interference term is combined with the feedback model to propose different bidirectional feedback models to determine whether the stubborn discordant group modifies preferences.

This proposed bidirectional feedback model, incorporating quantum probability theory to address interference effects in group decision-making, opens several avenues for future research. The model not only enhances the understanding of decision-making dynamics among experts with different personality types but also offers a framework for exploring the complexities of collective decision processes in a variety of domains. In artificial intelligence, the model could inspire new approaches to adaptive learning systems where decision agents adjust their preferences based on dynamic interactions. In psychology, it may provide insights into how individuals' cognitive biases and preferences evolve in group settings, potentially informing interventions to reduce decision-making errors. Furthermore, in the collaborative economy, the model could be applied to optimize consensus-building and preference adjustment among diverse stakeholders, contributing to more efficient and equitable decision-making processes. Overall, the integration of quantum probability into decision science opens exciting possibilities for modeling human behavior in uncertain, interactive environments, thus setting the stage for future interdisciplinary research.

This study proposes a bidirectional feedback model driven by minimum feedback parameters by combining quantum probability and feedback processes for stubborn decision experts. Compared with previous feedback mechanisms, this model considers interference effects of stubborn discordant and concordant groups in the game process. The rest of this study is arranged as follows. In Section 2, the key procedures and related symbols of CRP are briefly introduced. In Section 3, the game process of stubborn discordant and concordant groups in feedback is established, and interference effects between them are quantified by quantum probability. In Section 4, the bidirectional feedback model considering interference effects is presented. In Section 5, an example is given to demonstrate the application and validity of our model. Comparative analysis is also given to show the benefit of our method. Finally, Section 6 gives the conclusions and future research directions.

2. Preliminary

This section briefly introduces the main steps and symbols of the CRP and some basic concepts of quantum probability.

2.1. The Consensus-Reaching Process

Let $\left\{DM^1, DM^2, \cdots, DM^m\right\}$ be a set of m decision-making experts, $X = \{x_1, x_2, \cdots, x_n\}$ be a set of n alternatives, and $C = \{c_1, c_2, \cdots, c_u\}$ be a set of u criteria. In GDM, the basic steps to reach consensus mainly include the following steps, as shown in Figure 1:

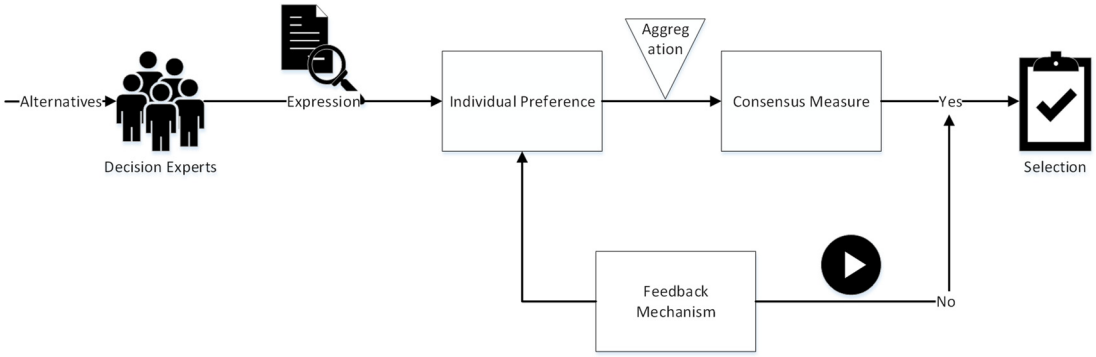


Figure 1. The consensus-reaching process.

(a) Preference representation

The preference value of expert DM^k for criterion c_j of alternative x_i is expressed by matrix $O^k = (o_{ij}^k)_{n\times u}$, where $o_{ij}^k \in [0,1]$. The greater the value of o_{ij}^k is, the higher the preference degree of DM^k for x_i on this criterion.

(b) Aggregation

After collecting the preference opinions of each expert on the alternative set, the individual preferences are aggregated into collective preferences through an aggregation operator f_{λ_k}:

$$o_{ij}^c = f_{\lambda_k}(o_{ij}^1, o_{ij}^2, \cdots, o_{ij}^m) \tag{1}$$

where $\lambda_k \in [0,1]$ and $\sum_{k=1}^m \lambda_k = 1$. In CRP, $\lambda_k = \{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ is the weight vector of experts. At present, the commonly used aggregation operators include the weighted average (WA) operator, ordered weighted average (OWA) operator, and importance induced ordered weighted average (I-IOWA) operator [24–26].

(c) Consensus measure

The consensus measure is a function to measure the similarity between experts' preferences. At present, the consensus measure is generally based on the distance and similarity measure, which mainly includes the consensus level measure of distance from collective preferences and the consensus level measure of pairwise distance between experts' preferences. In order to improve the consistency of decision-making opinions, the method of measuring the consensus level is constructed at the three levels of criteria, alternatives, and experts:

Level 1: The level of consensus at the criterion level. The level of consensus of expert DM^k on criterion c_j of alternative x_i is CE_{ij}^k:

$$CE_{ij}^{\mathrm{k}} = 1 - d(o_{ij}^k, o_{ij}^c) \tag{2}$$

Level 2: The level of consensus at the alternative level. The consensus level of expert DM^k on alternative x_i is CA_i^k:

$$CA_i^{\mathrm{k}} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^k \tag{3}$$

Level 3: The level of consensus at the expert level. The consensus level of expert DM^k is CI^k:

$$CI^k = \frac{1}{n}\sum_{i=1}^{n} CA_i^k \tag{4}$$

(d) Feedback mechanism

In GDM, it is often difficult to reach a "hard consensus", but it is feasible to consider reaching a "soft consensus". At this point, it is necessary to set a consensus threshold $\beta \in (0.5,\ 1)$ and then enter the recognition process.

First, identify experts with an insufficient consensus level at Level 3, and the identified experts form the set ECH:

$$ECH = \left\{k | CI^k < \beta\right\} \tag{5}$$

Then, after identifying the discordant experts, identify the alternatives with insufficient levels of consensus among these experts at Level 2, and the identified alternatives form the set ACH:

$$ACH = \left\{(k,i) | k \in ECH \wedge CA_i^k < \beta\right\} \tag{6}$$

Finally, among the identified alternatives, the criteria for an insufficient consensus level at Level 1 are identified, and the identified criteria form the set APS:

$$APS = \left\{(k,i,j) | (k,i) \in ACH \wedge CE_{ij}^k < \beta\right\} \tag{7}$$

To improve the level of group consensus, generally, the identified experts who do not contribute enough to the consensus level will adjust their preferences to be close to the collective preference so as to improve the consensus level:

$$o_{ij}^{k+1} = (1-\sigma)o_{ij}^k + \sigma o_{ij}^c \tag{8}$$

where $\sigma \in [0,\ 1]$ is the feedback control parameter of the preference adjustment degree.

2.2. Quantum Probability Theory in Decision-Making

New research suggests that the human mind does not, in nature, conform to traditional probability theory but works in a quantum-like way [27,28]. Quantum approaches allow humans to face uncertainty when making important decisions. Even if they have only limited spiritual resources, they still have to face complex problems. In quantum probability

theory, all fundamental events are represented by subspaces in Hilbert spaces. Events represented as subspaces of Hilbert spaces are shown in Figure 2.

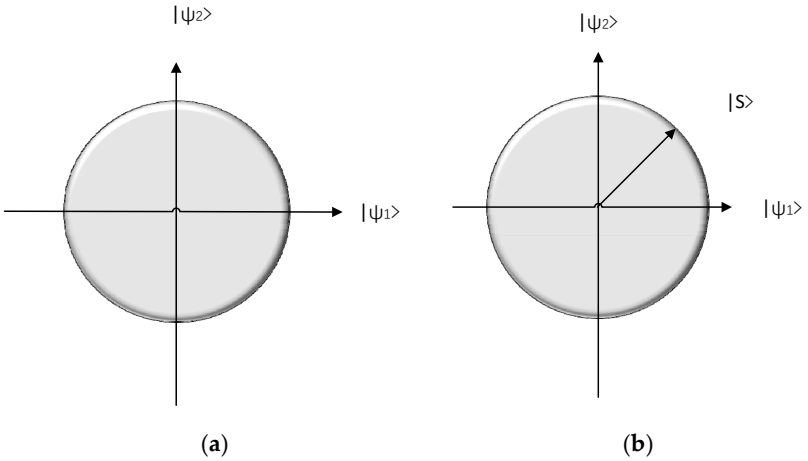


Figure 2. (**a**) Two-dimensional Hilbert space. (**b**) An event in Hilbert space.

Definition 1 ([29]). *Hilbert space is the abstraction of geometric finitely dimensional Euclidean space. Suppose H is a real linear space. If any two vectors x and y in H correspond to a real number, denoted as (x, y), the following conditions are met:*

(1) *For any two vectors x, y in H, $(x, y) = (y, x)$;*
(2) *For any three vectors x, y, z in H and real number α, β, there are $(\alpha x + \beta y, z) = \alpha(x, y) + \beta(y, z)$;*
(3) *For all vectors x in H, $(x, x) \geq 0$, and $(x, x) = 0$ is a necessary and sufficient condition for $x = 0$. Then, (x, y) is called an inner product on H, and H is called inner product space.*

In QPT, according to the superposition principle that embodies all conflict, ambiguity, and uncertainty factors of states, all possible event states form a Hilbert complex space H_+, where H_+ is the tensor product of each Hilbert subspace, i.e., $H_+ = H_1 \bigotimes H_2 \bigotimes \cdots \bigotimes H_n (H_1, \cdots, H_n \in H_+)$. Each state of the event corresponds to a vector in Hilbert space, called a state vector. Denoting the state vector as $|S\rangle$, a vector in a Hilbert space can be represented by a linear combination of orthonormal bases on that space.

Definition 2 ([30]). *A quantum state is a set of quantum representations used to represent the superposition of events. Any quantum state in the decision-making can be expanded as:*

$$|S\rangle = \sum_{n=0}^{\infty} \tau_n |\psi_n\rangle \tag{9}$$

where $\tau_n = \langle \psi_n || S \rangle$ is the wave function of $|\psi_n\rangle$, i.e., probability amplitude [31].

According to Born's rule, the probability of any event is expressed as the square of the probability amplitude [32]:

$$P_r(\psi_n) = ||\tau_n|\psi_n\rangle||^2 = ||\tau_n||^2 = |\varphi_{\psi_n} e^{i\theta_{\psi_n}}|^2 \tag{10}$$

where $P_r(\psi_n) = |\varphi_{\psi_n} e^{i\theta_{\psi_n}}|^2 = \varphi_{\psi_n} e^{i\theta_{\psi_n}} * \left(\varphi_{\psi_n} e^{i\theta_{\psi_n}}\right) * = \varphi_{\psi_n} e^{i\theta_{\psi_n}} * \varphi_{\psi_n} e^{-i\theta_{\psi_n}} = \varphi_{\psi_n}^2$, and the sum of squares of the probability amplitude must be equal to 1, i.e., $\sum_n p_r(\psi_n) = 1$.

Therefore, interference effects in a decision-making process can be derived from the probability formula of the union of N mutually exclusive events. Take two events as examples:

$$\begin{aligned} Pr(A \cup B) &= |e^{i\theta_A}\varphi_A + e^{i\theta_B}\varphi_B|^2 = (e^{i\theta_A}\varphi_A + e^{i\theta_B}\varphi_B)(e^{i\theta_A}\varphi_A + e^{i\theta_B}\varphi_B)* \\ &= e^{i\theta_A}\varphi_A e^{-i\theta_A}\varphi_A + e^{i\theta_B}\varphi_B e^{-i\theta_B}\varphi_B + e^{i\theta_A}\varphi_A e^{-i\theta_B}\varphi_B + e^{i\theta_B}\varphi_B e^{-i\theta_A}\varphi_A \\ &= |\varphi_A|^2 + |\varphi_B|^2 + |\varphi_A||\varphi_B|e^{i(\theta_A-\theta_B)} + |\varphi_A||\varphi_B|e^{i(\theta_B-\theta_A)} \\ &= |\varphi_A|^2 + |\varphi_B|^2 + 2|\varphi_A||\varphi_B|\cos(\theta_A - \theta_B) \end{aligned} \tag{11}$$

It is worth noting that quantum probability and classical probability have different rules for the probability distribution of events: the probability distribution of quantum events follows Born's rule, whereas the probability distribution of events in the classical case follows the Kolmogorov axiom.

3. Interference Effects in the Bidirectional Feedback Process

3.1. Problem Description

Psychological research shows that people are not independent individuals, and their behavior is easily influenced by the outside world to different degrees. Experts' personalities often have a significant impact on the modification process [6,33]. People's personalities are determined by a variety of factors, such as their environment, contact groups, and education level, as well as changes in the face of different problems. In this study, the personality characteristic of an expert is defined as a personality parameter, which reflects the positive degree of experts in modifying their preferences closer to the collective preferences. The personality parameter η of the experts was set to $[0,\ 1]$. Experts were divided into three types according to the values of the personality parameters, as shown in Figure 3.

(1) When $\eta \in [0, 0.5]$, experts are easygoing;
(2) When $\eta \in [0.5, 0.8]$, experts are rational;
(3) When $\eta \in [0.8, 1]$, experts are stubborn.

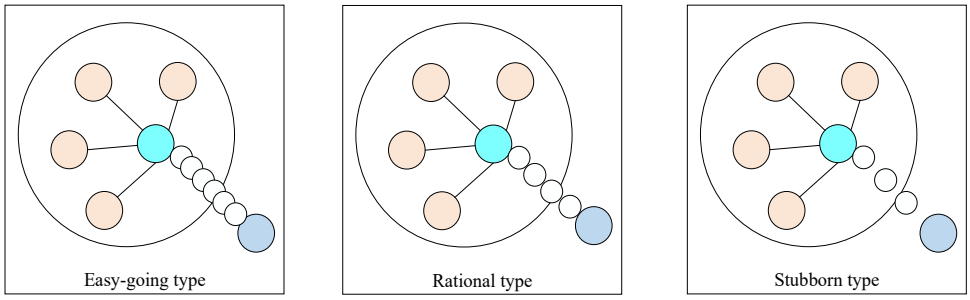


Figure 3. Three types of decision-making experts. The grey circle represents the expert who will modify his/her preference closer to the collective preference. The pink circles in a big circle form a group which the grey circle needs to close to. The blue circle represents the collective preferecene of the group. Three types of experts exhibit different behaviors when modifying their preferences. The white circle is the path for experts to modify preferences closer to group opinions, reflecting various difficulty for them to modify their preferences.

Easygoing experts probably make constant compromises for the interests of the collective group and blindly change their preferences in the feedback process to get close to collective preferences. Rational experts tend to consider both collective and individual interests and are willing to change their preferences to improve the level of consensus to a certain degree, whereas stubborn experts find it difficult to change their minds. In CRP, it costs more to make stubborn experts change their minds compared with easygoing experts.

Stubborn discordant experts are often unwilling to adjust their preferences to improve their level of consensus. To improve the subjective initiative of such experts in the feedback process, a compromise among concordant experts is needed. Firstly, we define the stubborn discordant and concordant groups as follows:

(1) The stubborn discordant experts are expressed as $SM = \left\{ DM^k | CI^k < \beta \right\}$, where $k \in 1, 2, \cdots, l(\leq m)$. The collective preferences of SM are expressed as $O^{SM} = \left(o_{ij}^{SM}\right)_{n \times u}$, where $o_{ij}^{SM} \in [0, 1]$ and $o_{ij}^{SM} = f_{\lambda k}(o_{ij}^1, o_{ij}^2, \cdots, o_{ij}^l)$.

(2) The concordant experts are expressed as $CM = \left\{ DM^k | CI^k \geq \beta \right\}$, where $k \in l + 1, \cdots, m$. The collective preferences of CM are expressed as $O^{CM} = \left(o_{ij}^{CM}\right)_{n \times u}$, where $o_{ij}^{CM} \in [0, 1]$ and $o_{ij}^{SM} = f_{\lambda_k}(o_{ij}^{l+1}, o_{ij}^{l+2}, \cdots, o_{ij}^m)$.

In the traditional unidirectional feedback process, only discordant groups make corresponding compromises while ignoring their "self-esteem". The unidirectional feedback mechanism cannot be applied to experts with high "self-esteem"; hence, bidirectional feedback was developed. With the continuous penetration of social psychology into GDM theory, a series of effects, such as interference and order among people, not only improves the information items not considered in GDM but also provides new inspiration for improving the bidirectional feedback mechanism. Through subjective human test reports, it can be found that one person's answer often affects another person's answer regarding the same event. Therefore, starting from interference effects in the bidirectional feedback process, this study analyses the causes, size, and results of interference effects, and optimizes the feedback link in GDM. We explored the following three issues in depth:

(1) Which experts in the discordant group need bidirectional feedback?

In *SM*, experts are unwilling to approach collective preferences in order to improve the level of consensus. The *CM* needs to make concessions and modify preferences. At this time, the *SM* will also actively approach the preferences of the *CM*. Both parties will constantly modify their preferences until they meet the consensus level of the group. In the process of modifying preferences, the degree to which one party modifies preferences will have interference effects on the other party. Therefore, stubborn discordant experts are selected for bidirectional feedback to improve the efficiency of the feedback process.

(2) How can the interference effects in the bidirectional feedback process be quantified?

The flexibility of QPT makes it advantageous for describing human decision-making behavior under uncertain circumstances. Therefore, the QPT can be used to quantify interference effects of concordant and discordant groups in the bidirectional feedback process.

(3) What impact will the generated interference item bring to the whole feedback result?

The magnitude of the interference affects the number of stages, adjustment intention, and degree of the feedback process. Therefore, designing reasonable bidirectional feedback models to reflect whether to consider the impact of "interference" on the overall result is required.

3.2. Classical vs. Quantum Probability of Modifying Preferences During Feedback Process

3.2.1. Classical Probability of Modifying Preferences Without Considering Interference Effects

Classical probability theory states that an event is always in a certain state, and the change in events is a transformation from a certain state to another certain state. The probability establishes a relationship between each event in the sample space and the real number through mapping. The game between the concordant and stubborn discordant groups in the decision-making process is shown in Figure 4. In the entire process, the

SM first selects whether to modify preferences in the first stage. Whether the *SM* choose to modify their preferences is often related to personality parameters. The larger the parameters, the less likely they are to modify their preferences. On this basis, the *CM* gives the probability of choosing to modify preferences. The two groups alternately choose whether to modify their preferences or not.

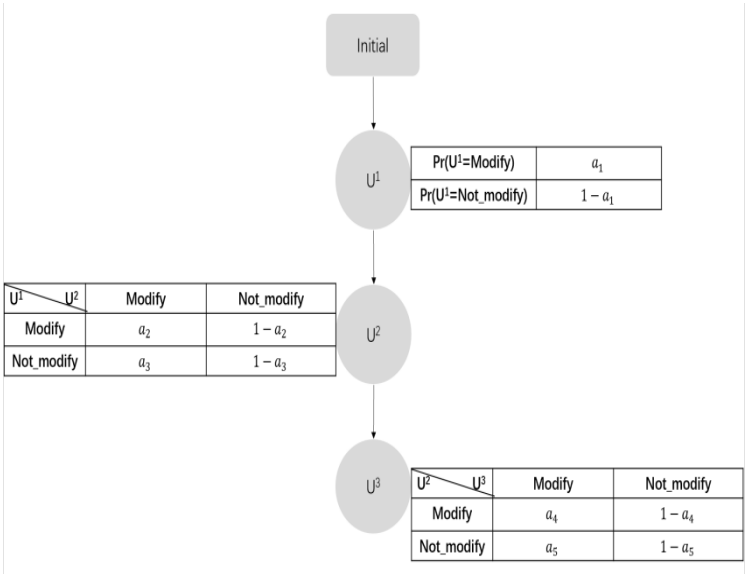


Figure 4. Classical Bayesian network of a game between *SM* and *DM* groups in the feedback process.

The aforementioned game process between the concordant and stubborn discordant groups can also be regarded as a Markov chain that describes the state sequence of experts' willingness to modify preferences, and each willing state depends on the previous finite states. Each willing state is extracted from a discrete state space (finite or infinite) and follows the Markov properties. Through the known probability distribution of modifying preferences, the conditional probabilities can be expressed by Equations (12)–(15), where U^q represents the stage q, and a_{2q-1} and a_{2q-2} correspond to the classical probability of *Modify* and *Not_modify* in stage q. According to the total probability calculation law, the following results can be derived:

$$P_r(U^3 = Modify|U^1 = Modify) = \frac{a_2a_4 + a_5(1-a_2)}{a_1[(a_2(1-a_4) + (1-a_2)(1-a_5) + a_2a_4 + a_5(1-a_2)]} \tag{12}$$

$$P_r(U^3 = Not_modify|U^1 = Modify) = \frac{a_2(1-a_4) + (1-a_2)(1-a_5)}{a_1[(a_2(1-a_4) + (1-a_2)(1-a_5) + a_2a_4 + a_5(1-a_2)]} \tag{13}$$

In the same way, we can obtain the probabilities of the *SM* choosing not to modify in U^1:

$$P_r(U^3 = Modify|U^1 = Not_modify) = \frac{a_3a_4 + a_5(1-a_3)}{(1-a_1)[(a_3(1-a_4) + (1-a_3)(1-a_5) + a_3a_4 + a_5(1-a_3)]} \tag{14}$$

$$P_r(U^3 = Not_modify|U^1 = Not_modify) = \frac{a_3(1-a_4) + (1-a_3)(1-a_5)}{(1-a_1)[(a_3(1-a_4) + (1-a_3)(1-a_5) + a_3a_4 + a_5(1-a_3)]} \tag{15}$$

As early as 1992 [34] observed experimentally that human actual decision-making data violated the classical probability theorem. Therefore, the traditional probability model cannot accurately simulate the decision-making behavior. In a real decision-making process, one person's choice often interferes with another person's choice, as shown in Figure 5. Therefore, in the next section, we introduce quantum probability considering interference

effects, which can explain the more practical decision-making behavior of the SM and CM in the feedback process.

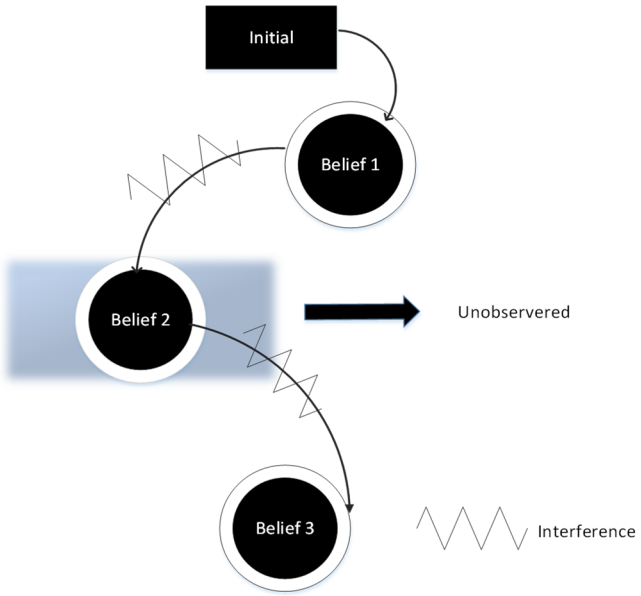


Figure 5. The quantum interference effects when the state of belief 2 is unobserved.

3.2.2. Quantum Probability Considering Interference Effects

Ref. [35] defined quantum probability theory in Hilbert space so that it does not have to abide by many rules in Boolean logic. As shown in Figure 6, the first stage U^1 can be expressed in Hilbert subspace as Equation (16):

$$U^1 = e^{i\theta_{Modify}}\sqrt{a_1}Modify + e^{i\theta_{Not_Modifyy}}\sqrt{1-a_1}Not_Modify \tag{16}$$

where $Modify$ and Not_modify are a pair of orthogonal basis vectors representing Hilbert subspace.

$$Modify = \psi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$Not_modify = \psi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

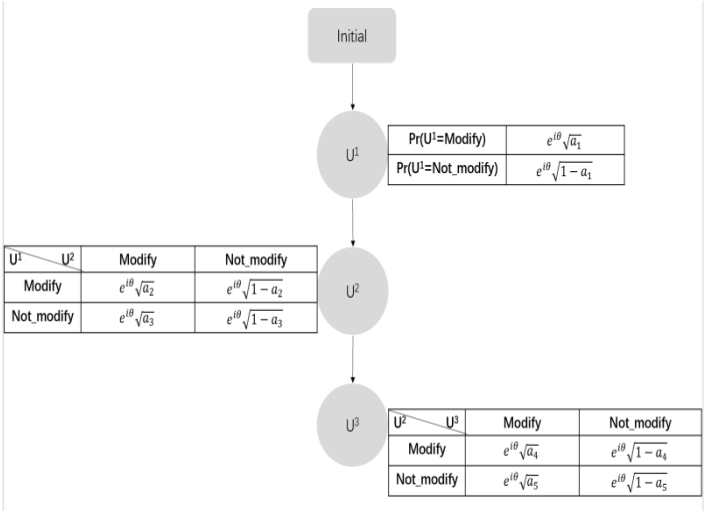


Figure 6. Quantum Bayesian network of a game between SM and DM groups in the feedback process.

Through Equation (10), we can convert the classical probability in Figure 4 into the amplitude in the quantum probability:

$$P_r(\psi_0) = |e^{i\theta_{\psi_0}}\varphi_{\psi_0}|^2 \rightarrow \varphi_{\psi_0} = e^{i\theta_{\psi_0}}\sqrt{P_r(\psi_0)} \tag{17}$$

In the feedback mechanism, the composite quantum state in the game process can be expressed as:

$$\begin{aligned}\varepsilon_c &= \varphi_{000}e^{i\theta_{000}}U_0^1U_0^2U_0^3 + \varphi_{001}e^{i\theta_{001}}U_0^1U_0^2U_1^3 + \cdots + \varphi_{111}e^{i\theta_{111}}U_1^1U_1^2U_1^3\\ &= \sqrt{a_1}\sqrt{a_2}\sqrt{a_4}e^{i\theta_{000}}U_0^1U_0^2U_0^3 + \sqrt{a_1}\sqrt{a_2}\sqrt{1-a_4}e^{i\theta_{001}}U_0^1U_0^2U_1^3 + \sqrt{a_1}\sqrt{1-a_2}\sqrt{a_5}e^{i\theta_{010}}U_0^1U_1^2U_0^3\\ &+\sqrt{1-a_1}\sqrt{a_3}\sqrt{a_4}e^{i\theta_{100}}U_1^1U_0^2U_0^3 + \sqrt{1-a_1}\sqrt{1-a_3}\sqrt{a_5}e^{i\theta_{110}}U_1^1U_1^2U_0^3 + \sqrt{1-a_1}\sqrt{a_3}\sqrt{1-a_4}e^{i\theta_{101}}U_1^1U_0^2U_1^3\\ &+\sqrt{a_1}\sqrt{1-a_2}\sqrt{1-a_5}e^{i\theta_{011}}U_0^1U_1^2U_1^3 + \sqrt{1-a_1}\sqrt{1-a_3}\sqrt{1-a_5}e^{i\theta_{111}}U_1^1U_1^2U_1^3\end{aligned} \tag{18}$$

where 0 is an abbreviation for ψ_0 and 1 is an abbreviation for ψ_1. U_0^q represents the situation where modification is selected in stage q, and U_1^q represents the situation where modification is not selected in stage q. $\varphi_{000}e^{i\theta_{000}}$ represents the probability amplitude that modification is selected in the first three stages, $\varphi_{001}e^{i\theta_{001}}$ represents the probability amplitude that modification is selected in the first two stages but modification is not selected in the third stage, $\varphi_{111}e^{i\theta_{111}}$ represents the probability amplitude that modification is not selected in the first three stages, and so on. It should be noted that the sum of squares of all probability amplitudes is equal to 1, that is: $\sum_{lmn}|\varphi_{U_l^1}\varphi_{U_m^1}\varphi_{U_n^1}|^2 = 1$.

In the bidirectional feedback process, interference effects among groups originates from the ripples generated by the superposition of different quantum states. When interference effects are considered, the classical probability is transformed into a quantum probability. Compared to the calculation results of classical probability, as shown in Equations (12)–(15), the following calculation results of quantum probability have one more term, which indicates interference effects among experts in the decision-making process.:

$$\begin{aligned}P_r(U^3 &= Modify|U^1 = Modify) = \alpha_1|\varphi_{000}e^{i\theta_{000}} + \varphi_{010}e^{i\theta_{010}}|^2\\ &= \alpha_1|\sqrt{a_1a_2a_4} + \sqrt{a_1(1-a_2)a_5}|^2\\ &= \alpha_1[a_1a_2a_4 + a_1(1-a_2)a_5 + 2(\sqrt{a_1a_2a_4}\sqrt{a_1(1-a_2)a_5}\cos\theta_1)]\end{aligned} \tag{19}$$

where $Interference = 2(\sqrt{a_1a_2a_4}\sqrt{a_1)1-a_2)a_4}cos\theta_1)$.

$$\begin{aligned}P_r(U^3 &= Not_modify|U^1 = Modify) = \alpha_1|\varphi_{001}e^{i\theta_{001}} + \varphi_{011}e^{i\theta_{011}}|^2\\ &= \alpha_1|\sqrt{a_1a_2(1-a_4)} + \sqrt{a_1(1-a_2)(1-a_5)}|^2\\ &= \alpha_1[a_1a_2(1-a_4) + a_1(1-a_2)(1-a_5) + 2(\sqrt{a_1a_2(1-a_4)}\sqrt{a_1(1-a_2)(1-a_5)}\cos\theta_1)]\end{aligned} \tag{20}$$

where α_1 is the normalized factor.

$$\begin{aligned}\alpha_1 &= 1/\Big\{[a_1a_2a_4 + a_1(1-a_2)a_5 + 2(\sqrt{a_1a_2a_4}\sqrt{a_1(1-a_2)a_5}\cos\theta_1)]\\ &+[a_1a_2(1-a_4) + a_1(1-a_2)(1-a_5) + 2(\sqrt{a_1a_2(1-a_4)}\sqrt{a_1(1-a_2)(1-a_5)}\cos\theta_1)]\Big\}\end{aligned} \tag{21}$$

In the same way, we can obtain the probabilities of the SM choosing not to modify in U^1:

$$\begin{aligned}\alpha_1 &= 1/\Big\{[a_1a_2a_4 + a_1(1-a_2)a_5 + 2(\sqrt{a_1a_2a_4}\sqrt{a_1(1-a_2)a_5}\cos\theta_1)]\\ &+[a_1a_2(1-a_4) + a_1(1-a_2)(1-a_5) + 2(\sqrt{a_1a_2(1-a_4)}\sqrt{a_1(1-a_2)(1-a_5)}\cos\theta_1)]\Big\}\end{aligned} \tag{22}$$

$$\begin{aligned} P_r(U^3 &= Not_modify|U^1 = Not_modify) = \alpha_2|\varphi_{101}e^{i\theta_{101}} + \varphi_{111}e^{i\theta_{111}}|^2 \\ &= \alpha_2|\sqrt{(1-a_1)a_3(1-a_4)} + \sqrt{(1-a_1)(1-a_3)(1-a_5)}|^2 \\ &= \alpha_2[(1-a_1)a_3(1-a_4) + (1-a_1)(1-a_3)(1-a_5) + 2(\sqrt{(1-a_1)a_3(1-a_4)}\sqrt{(1-a_1)(1-a_3)(1-a_5)}\cos\theta_2)] \end{aligned} \tag{23}$$

where α_2 is the normalized factor.

$$\begin{aligned} \alpha_2 &= 1/\Big\{[(1-a_1)a_3a_4 + (1-a_1)(1-a_3)a_5 + 2(\sqrt{(1-a_1)a_3a_4}\sqrt{(1-a_1)(1-a_3)a_5}\cos\theta_2)] \\ &+[(1-a_1)a_3(1-a_4) + (1-a_1)(1-a_3)(1-a_5) + 2(\sqrt{(1-a_1)a_3(1-a_4)}\sqrt{(1-a_1)(1-a_3)(1-a_5)}\cos\theta_2)]\Big\} \end{aligned} \tag{24}$$

θ is the key variable of the quantum probability. The value of θ determines the positive or negative value of the interference term, which reflects whether the interference effects caused by the interaction between experts have a positive impact.

- When $\theta = \pi/2$, quantum probability collapses into classical probability;
- When $\theta \in \left[0, \frac{\pi}{2}\right]$, the interference effect is positive;
- When $\theta \in \left[\frac{\pi}{2}, \pi\right]$, the interference effect is negative.

In this study, $cos\theta$ is the cause of the interference term formed by the interaction between the concordant and stubborn discordant groups. Because the concordant and stubborn discordant groups are not independent of each other, their interaction will affect the willingness of the other to modify their preferences. Therefore, compared with the classical probability, assuming that groups are independent, $cos\theta$ in the quantum probability of modifying preferences is a variable that measures the deviation of the original probability caused by interference effects between the two groups. Similarly, it can also reflect the influence of interference effects on the stride of adjusting preferences. $cos\theta$ is not a fixed value that reflects the uncertainty of the impact of the interference effects between the two groups on the results. Under different decision circumstances, different values are assigned to the probabilities shown in Figure 4. According to Equations (19)–(24), interference effects change with different θ. Therefore, we need to determine the value of θ that maximizes the expert's willingness to modify their preferences in the last stage of the decision-making process.

When stubborn discordant and concordant groups enter the feedback mechanism, a multi-stage game process may occur. We extrapolate the quantum probabilities between the stubborn discordant and concordant groups from two stages to multi-stages, as shown in Figure 7, and Equations (25)–(28) are obtained as follows:

$$\begin{aligned} P_r(U^N &= Modify|U^1 = Modify) = \alpha_1 a_1 \\ &* \left(\underbrace{a_2a_4a_6\cdots a_{2N-4}a_{2N-2} + \cdots + (1-a_2)(1-a_5)(1-a_7)\cdots(1-a_{2N-3})a_{2N-1}}_{2^{N-2}} \right) + 2Interference \end{aligned} \tag{25}$$

$$\begin{aligned} P_r(U^N &= Not_modify|U^1 = Modify) = \alpha_1 a_1 \\ &* \left(\underbrace{a_2a_4a_6\cdots a_{2N-4}(1-a_{2N-2}) + \cdots + (1-a_2)(1-a_5)(1-a_7)\cdots(1-a_{2N-3})(1-a_{2N-1})}_{2^{N-2}} \right) + 2Interference \end{aligned} \tag{26}$$

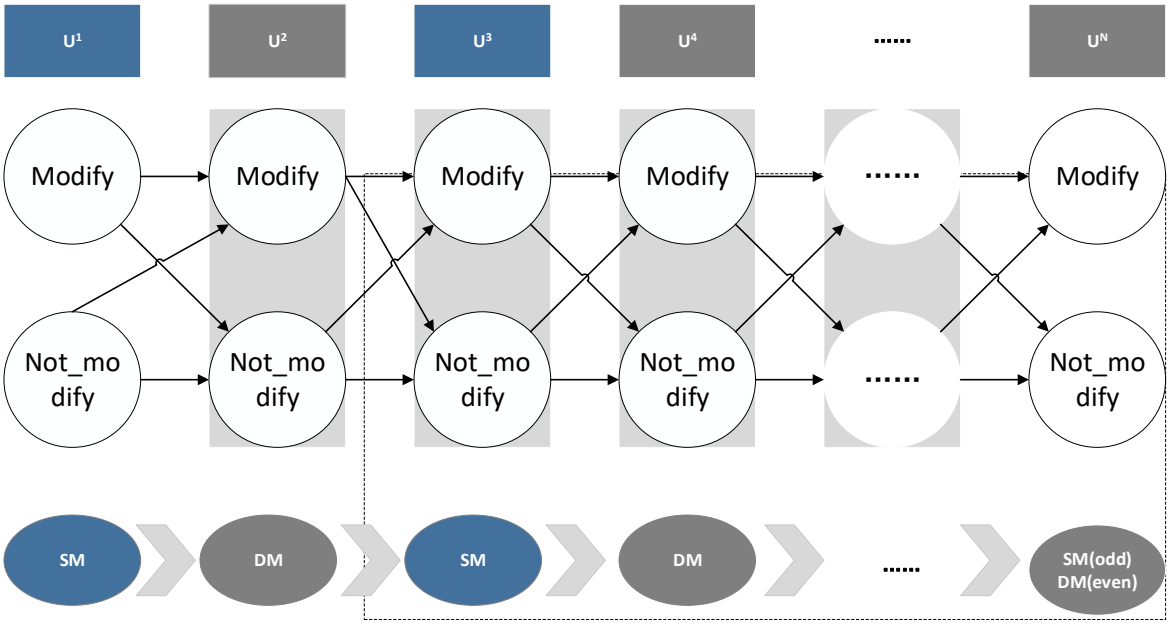


Figure 7. Interference effects of the SM and DM groups in multi-stage games.

In the same way, we can obtain the probabilities of the SM choosing not to modify in U^1:

$$P_r(U^N = Not_modify|U^1 = Modify) = \alpha_1 a_1 * \left(\underbrace{a_2 a_4 a_6 \cdots a_{2N-4}(1-a_{2N-2}) + \cdots + (1-a_2)(1-a_5)(1-a_7)\cdots(1-a_{2N-3})(1-a_{2N-1})}_{2^{N-2}} \right) + 2Interference \quad (27)$$

$$P_r(U^N = Not_modify|U^1 = Not_modify) = \alpha_2(1-a_1) * \left(\underbrace{a_3 a_4 a_6 \cdots a_{2N-4}(1-a_{2N-2}) + \cdots + (1-a_3)(1-a_5)(1-a_7)\cdots(1-a_{2N-3})(1-a_{2N-1})}_{2^{N-2}} \right) + 2Interference \quad (28)$$

By comparison, we can find that the calculation results of classical probability and quantum probability are inconsistent, showing that classical probability cannot describe human decision-making behavior under uncertainty. The $cos\theta$ obtained in this section will be used as the interference indicator of the degree of adjustment of the experts' preferences in the modeling of the bidirectional feedback model.

4. Bidirectional Feedback Mechanism Considering Interference Effects

4.1. GDM Framework with Bidirectional Feedback Mechanism

The traditional GDM process generally includes CRP and selection, whereas the GDM framework in this study is divided into nine steps:

Step 1. Input the preferences of each expert for the criteria of different alternatives.

Step 2. Aggregate individual preferences into collective preferences by an aggregation operator.

Step 3. Calculate the consensus level of the criteria, alternatives, and experts.

Step 4. Select feedback mechanisms.

Step 4.1. Identify experts whose consensus level is less than the threshold, and alternatives and criteria that are below the threshold among these experts.

Step 4.2. Classify the identified experts according to personality parameters.

Step 4.3. Select the feedback mechanism (bidirectional or unidirectional) that these experts must follow. Stubborn experts enter bidirectional feedback, whereas others by default enter the traditional unidirectional feedback.

Step 5. Calculate the interference values between the SM and CM groups.

Step 6. Generate preference adjustment opinions according to the proposed bidirectional feedback model.

Step 7. Update the consensus levels of experts after adjusting their preferences and check whether they are less than the threshold value. If they are less than the threshold value, the feedback mechanism is entered again; if they are not less than the threshold, then the feedback mechanism is ended.

Step 8. Aggregate the preferences of all experts whose consensus level is not less than the threshold into collective preferences.

Step 9. Calculate the sum of the scores of each alternative on each criterion through the collective preference matrix. The alternatives are sorted according to the scores, and the best alternative is selected.

The aforementioned steps, except for step 6, have been described in the previous section; the bidirectional feedback model will be described in detail in the next section.

4.2. Bidirectional Feedback Model

In the previous section, the quantum probabilities considering the interference effects of the SM and CM groups in the game process are calculated. Through calculations, the selection of SM in U^1 produces two different results, and different results correspond to different interference values. Therefore, according to the differences between these two situations, this study considers the feedback mechanism in two aspects: (1) the first-stage SM accepts the modification and (2) the first-stage SM rejects the modification.

In the feedback mechanism, the control parameter σ of the preference adjustment degree is the key index used by experts to modify the stride, which determines the progress of the feedback process. The larger the σ, the smaller is the proportion of original opinions retained by the experts. Considering the improvement in the consensus level, the cost of the preference adjustment cannot be ignored. Therefore, obtaining the maximum consensus level under minimum preference change is the core content of the feedback mechanism. However, previous studies on the bidirectional feedback mechanism did not consider interference effects between the two groups. To solve this problem, we propose models for SM acceptance and rejection of modification in U^1. Interference effects considered in the proposed model are quantified in Section 3.2 and reflected in σ; that is, after one group provides adjustment opinions first, the opinions of the other party will also be affected and changed. The parameters and variables used in the model are listed in Table 1.

(1) First-stage SM accepts the modification

When the SM accepts the modification in the first stage, the phase angle of the corresponding amplitude is θ_1. Changing θ_1 produces different interference terms. In the bidirectional feedback mechanism of this study, our goal is to find the maximum value of willingness to adjust preferences for the last stage by manually adjusting θ_1; that is, when $\theta_1 = \theta_1'$, $max\{P_r(U^3 = Modify|U^1 = Modify)\}$. The bidirectional feedback model is driven by the minimum feedback control parameter (i.e., the minimum adjustment step), and Models (29) and (30) are proposed to ensure that the final consensus levels of the experts are greater than the threshold.

Table 1. Variables and parameters in the bidirectional feedback model.

Parameters	Description
β	Consensus threshold
ε^k	Minimum feedback parameter value satisfying consensus level for stubborn discordant expert DM^k
$CI^{k^{(q)}}$	Consensus level of stubborn discordant expert DM^{k} at stage "q"
$CI^{h^{(q)}}$	Consensus level of concordant expert DM^h at stage "q"
$CA_i^{k^{(q)}}$	Consensus level of stubborn discordant expert DM^k for alternative x_i at stage "q"
$CA_i^{h^{(q)}}$	Consensus level of concordant expert DM^h for alternative x_i at stage "q"
$CE_{ij}^{k^{(q)}}$	Consensus level of stubborn discordant expert DM^k on criterion u_j of x_i at stage "q"
$CE_{ij}^{h^{(q)}}$	Consensus level of concordant expert DM^h on criterion u_j of x_i at stage "q"
$\cos\theta$	Interference term
Decision Variables	Description
$\sigma^{k^{(q)}}$	Feedback parameter for stubborn discordant expert DM^k at stage "q" of the feedback process
$\gamma^{h^{(q)}}$	Feedback parameter for concordant expert DM^h at stage "q" of the feedback process
$o_{ij}^{k^{(q)}}$	Original preferences for stubborn discordant expert DM^k at stage "q" of the feedback process
$o_{ij}^{h^{(q)}}$	Original preferences for concordant expert DM^h at stage "q" of the feedback process
$o_{ij}^{c^{(q)}}$	Original collective preference for experts at stage "q" of the feedback process
$oo_{ij}^{k^{(q)}}$	Adjusted preferences for stubborn discordant expert DM^k at stage "q" of the feedback process
$oo_{ij}^{h^{(q)}}$	Adjusted preferences for concordant expert DM^h at stage "q" of the feedback process
$oo_{ij}^{c^{(q)}}$	Adjusted collective preference for experts at stage "q" of the feedback process

$$\min\sigma^{k^{(q)}} \begin{cases} CI^{k(q)} \geq CI^{k(q-1)} \\ CI^{h^{(q)}} \geq \beta \\ CI^{k^{(q)}} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{k^{(q)}}, CI^{h^{(q)}} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{h^{(q)}} \\ CA_i^{k^{(q)}} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{k^{(q)}}, CA_i^{h^{(q)}} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{h^{(q)}} \\ CE_{ij}^{k^{(q)}} = 1 - d(oo_{ij}^{k^{(q)}}, oo_{ij}^{c^{(q)}}), CE_{ij}^{h'} = 1 - d(oo_{ij}^{h^{(k)}}, oo_{ij}^{c^{(q)}}) \\ oo_{ij}^{k^{(q)}} = [1 - \sigma^{k^{(q)}} * (cos\theta_1)^{q-1}]o_{ij}^{k^{(q)}} + \sigma^{k^{(q)}} * (cos\theta_1)^{q-1} o_{ij}^{c^{(q)}} \\ oo_{ij}^{h^{(q)}} = o_{ij}^{h^{(q)}} \\ oo_{ij}^{c^{(q)}} = f_\omega(oo_{ij}^{k^{(q)}}, oo_{ij}^{h^{(k)}}), k = 1, \cdots, l, h = l+1, \cdots, m \\ 0 \leq \sigma^{k^{(q)}} \leq \varepsilon^k, (k, i, j) \in APS, \#APS \neq 0 \end{cases} \tag{29}$$

$$\min\gamma^{h^{(q)}} \begin{cases} CI^{k(q)} \geq CI^{k(q-1)} \\ CI^{h^{(q)}} \geq \beta \\ CI^{h^{(q)}} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{h^{(q)}}, CI^{k^{(q)}} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{k^{(q)}} \\ CA_i^{h^{(q)}} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{h^{(q)}}, CA_i^{k^{(q)}} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{k^{(q)}} \\ CE_{ij}^{h'} = 1 - d(oo_{ij}^{h^{(k)}}, oo_{ij}^{c^{(q)}}), CE_{ij}^{k^{(q)}} = 1 - d(oo_{ij}^{k^{(q)}}, oo_{ij}^{c^{(q)}}) \\ oo_{ij}^{k^{(q)}} = o_{ij}^{k^{(q)}} \\ oo_{ij}^{h^{(q)}} = [1 - \gamma^{h^{(q)}} * (cos\theta_1)^{q-1}]o_{ij}^{h^{(q)}} + \gamma^{h^{(q)}} * (cos\theta_1)^{q-1} o_{ij}^{c^{(q)}} \\ oo_{ij}^{c^{(q)}} = f_\omega(oo_{ij}^{k^{(q)}}, oo_{ij}^{h^{(k)}}), k = 1, \cdots, l, h = l+1, \cdots, m \\ (i, j) \in APS, \#APS \neq 0 \end{cases} \tag{30}$$

When concordant experts adjust their preferences, they choose experts with the smallest personality parameters in the CM group to adjust their preferences.

(2) First-stage SM rejects the modification.

When the SM rejects the modification in the first stage, the phase angle of the corresponding amplitude is θ_2. Changing θ_2 also produces different interference terms. Similarly, we find the maximum value of the willingness to adjust preferences for the last stage by manually adjusting θ_2; that is, when $\theta_2 = \theta_2'$, $max\{P_r(U^3 = Modify | U^1 = Not_modify)\}$. The bidirectional feedback model is driven by the minimum feedback control parameter (i.e., the minimum adjustment step), and Models (31) and (32) are proposed to ensure that the final consensus levels of the experts are greater than the threshold.

$$\min \sigma^{k(q)} \begin{cases} CI^{k(q)} \geq CI^{k(q-1)} \\ CI^{h(q)} \geq \beta \\ CI^{k(q)} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{k(q)}, CI^{h(q)} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{h(q)} \\ CA_i^{k(q)} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{k(q)}, CA_i^{h(q)} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{h(q)} \\ CE_{ij}^{k(q)} = 1 - d(oo_{ij}^{k(q)}, oo_{ij}^{c(q)}), CE_{ij}^{h'} = 1 - d(oo_{ij}^{h(k)}, oo_{ij}^{c(q)}) \\ oo_{ij}^{k(q)} = [1 - \sigma^{k(q)} * (cos\theta_2)^{q-2}] o_{ij}^{k(q)} + \sigma^{k(q)} * (cos\theta_2)^{q-2} o_{ij}^{c(q)} \\ oo_{ij}^{h(q)} = o_{ij}^{h(q)} \\ oo_{ij}^{c(q)} = f_\omega(oo_{ij}^{k(q)}, oo_{ij}^{h(k)}), k = 1, \cdots, l, h = l+1, \cdots, m \\ 0 \leq \sigma^{k(q)} \leq \varepsilon^k, (k, i, j) \in APS, \#APS \neq 0 \end{cases} \tag{31}$$

$$\min \gamma^{h(q)} \begin{cases} CI^{k(q)} \geq CI^{k(q-1)} \\ CI^{h(q)} \geq \beta \\ CI^{h(q)} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{h(q)}, CI^{k(q)} = \frac{1}{n}\sum_{i=1}^{n} CA_i^{k(q)} \\ CA_i^{h(q)} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{h(q)}, CA_i^{k(q)} = \frac{1}{u}\sum_{j=1}^{u} CE_{ij}^{k(q)} \\ CE_{ij}^{h'} = 1 - d(oo_{ij}^{h(k)}, oo_{ij}^{c(q)}), CE_{ij}^{k(q)} = 1 - d(oo_{ij}^{k(q)}, oo_{ij}^{c(q)}) \\ oo_{ij}^{k(q)} = o_{ij}^{k(q)} \\ oo_{ij}^{h(q)} = [1 - \gamma^{h(q)} * (cos\theta_2)^{q-2}] o_{ij}^{h(q)} + \gamma^{h(q)} * (cos\theta_2)^{q-2} o_{ij}^{c(q)} \\ oo_{ij}^{c(q)} = f_\omega(oo_{ij}^{k(q)}, oo_{ij}^{h(k)}), k = 1, \cdots, l, h = l+1, \cdots, m \\ (i, j) \in APS, \#APS \neq 0 \end{cases} \tag{32}$$

The implementation steps of the proposed feedback mechanism are described in Figure 8.

The traditional feedback mechanism does not adequately differentiate between experts' individual differences, resulting in a uniform feedback approach for all experts during the opinion adjustment process. This practice may lead stubborn experts to adhere to their original views, hindering the smooth achievement of group consensus. To address this issue, the feedback mechanism proposed in this paper classifies experts by introducing personalized parameters and selects corresponding feedback strategies based on different types of experts. QPT is utilized to describe the probability of decision-makers revising their preferences, and the derived quantum probability can more accurately depict decision-

makers' decision-making behavior during the feedback process. This personalized feedback mechanism effectively avoids the interference of stubborn experts in the group consensus process, thereby enhancing decision-making efficiency and coordination in group decision-making, and increasing the flexibility and accuracy of the model. This method accelerates the formation of consensus and optimizes the group decision-making process by tailored adjustments to experts' feedback approaches. The method proposed in this paper can be applied to practical scenarios such as emergency decision-making, risk management, and medical decision-making. However, the proposed method still has some limitations in practical applications. This method relies on the setting of experts' personalized parameters, and different parameter choices may lead to result fluctuations. Furthermore, accurately identifying experts' personality types poses challenges.

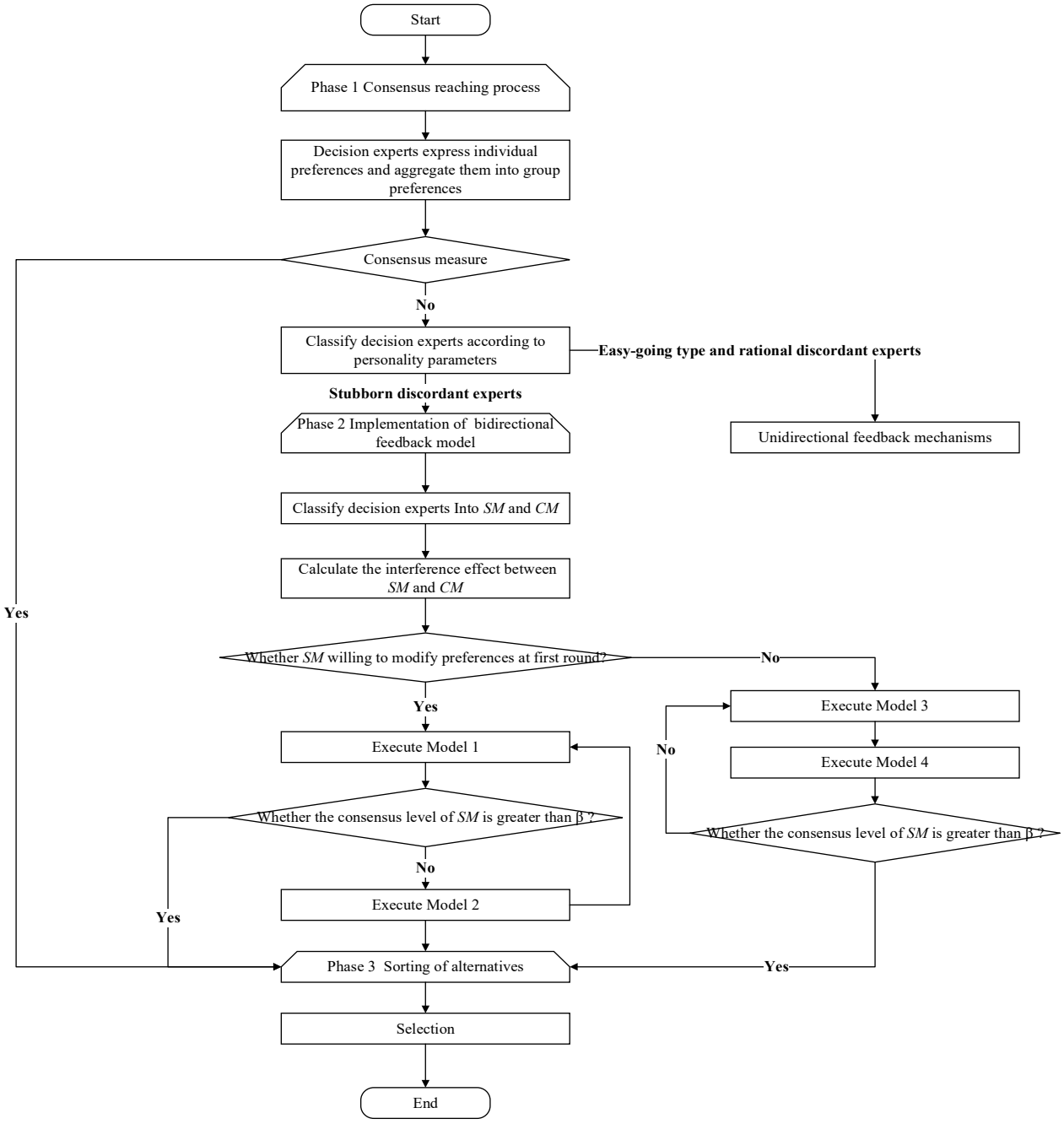


Figure 8. Steps of GDM involving the bidirectional feedback mechanism.

5. Numerical Example and Comparative Analysis

5.1. Numerical Example

Under the influence of COVID-19, many large-scale events have been affected. At present, a company has undertaken an outdoor activity, so it is necessary to select a site for this activity. Through pre-evaluation, the company screened out five sites $\{x_1, x_2, x_3, x_4, x_5\}$. In order to select the best site for the event, the company convened five experts $\{DM^1, DM^2, DM^3, DM^4, DM^5\}$ to evaluate sites according to six criteria: u_1: airtightness, u_2: rent, u_3: site size, u_4: capacity, u_5: transportation, and u_6: equipment. Five preference matrices of experts were obtained. (According to the survey, the personality parameters of the five experts are $\eta = (0.4,\ 0.92,\ 0,\ 0,\ 0)$):

$$O^1 = \begin{pmatrix} 0.6 & 0.5 & 0.5 & 0.4 & 0.7 & 0.3 \\ 0.1 & 0.9 & 0.2 & 0.8 & 0.8 & 0.7 \\ 0.8 & 0.9 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.1 & 0.4 & 0.3 & 0.7 & 0.2 & 0.3 \\ 0.7 & 0.5 & 0.5 & 0.4 & 0.6 & 0.5 \end{pmatrix} \quad O^2 = \begin{pmatrix} 0.8 & 1 & 0 & 0.4 & 0.2 & 0.7 \\ 0.7 & 0.6 & 0.6 & 0.2 & 0.3 & 0.4 \\ 0.2 & 0.5 & 0.9 & 0.5 & 0.5 & 0.6 \\ 0.4 & 0.9 & 0.7 & 0.2 & 0.5 & 0.5 \\ 0 & 0.8 & 0.9 & 0.8 & 0.2 & 0.1 \end{pmatrix}$$

$$O^3 = \begin{pmatrix} 0.5 & 0.5 & 0.3 & 0.3 & 0.5 & 0.3 \\ 0.5 & 0.5 & 0.7 & 0.2 & 0.2 & 0.5 \\ 0.9 & 0 & 0.5 & 0.7 & 0.2 & 0.3 \\ 1 & 0.4 & 0.1 & 0.9 & 0.7 & 0.7 \\ 0.3 & 1 & 0.1 & 0.8 & 0 & 0.9 \end{pmatrix} \quad O^4 = \begin{pmatrix} 0.2 & 1 & 0.8 & 0 & 0.3 & 0 \\ 0.6 & 0.6 & 0.7 & 0.2 & 0.3 & 0.5 \\ 0.9 & 0.2 & 0.2 & 0.2 & 0.9 & 1 \\ 1 & 0.1 & 0.1 & 0.9 & 0.3 & 0 \\ 0.3 & 0.6 & 0.5 & 0.5 & 0.4 & 0.6 \end{pmatrix}$$

$$O^5 = \begin{pmatrix} 0.5 & 0.6 & 0.5 & 0.3 & 0.5 & 0.4 \\ 0.2 & 0 & 0.1 & 0.7 & 0.7 & 0.8 \\ 0.3 & 0.5 & 0.7 & 0.6 & 0.5 & 0.6 \\ 0.4 & 0.8 & 0.7 & 0.3 & 0.4 & 0.5 \\ 0.6 & 0.5 & 0.6 & 0.5 & 0.5 & 0.6 \end{pmatrix}$$

Step 1: The process of preference aggregation and the consensus measure:

(1) Each expert is given the same weight, and the collective preference matrix is obtained through the aggregation operator (WA operator):

$$O^c = \begin{pmatrix} 0.52 & 0.72 & 0.42 & 0.28 & 0.44 & 0.34 \\ 0.42 & 0.52 & 0.46 & 0.42 & 0.46 & 0.58 \\ 0.62 & 0.42 & 0.5 & 0.44 & 0.46 & 0.54 \\ 0.42 & 0.52 & 0.38 & 0.6 & 0.42 & 0.4 \\ 0.46 & 0.68 & 0.52 & 0.6 & 0.34 & 0.54 \end{pmatrix}$$

(2) The consensus level of the criteria, alternatives, and experts can be obtained through Equations (2)–(4):

$$CE^1 = \begin{pmatrix} 0.92 & 0.78 & 0.92 & 0.88 & 0.74 & 0.94 \\ 0.68 & 0.62 & 0.74 & 0.62 & 0.66 & 0.88 \\ 0.82 & 0.52 & 0.7 & 0.76 & 0.74 & 0.66 \\ 0.68 & 0.88 & 0.92 & 0.9 & 0.78 & 0.9 \\ 0.76 & 0.82 & 0.98 & 0.8 & 0.74 & 0.96 \end{pmatrix} \quad CE^2 = \begin{pmatrix} 0.72 & 0.72 & 0.58 & 0.86 & 0.76 & 0.64 \\ 0.72 & 0.92 & 0.86 & 0.78 & 0.84 & 0.82 \\ 0.58 & 0.92 & 0.6 & 0.94 & 0.96 & 0.94 \\ 0.98 & 0.62 & 0.68 & 0.6 & 0.92 & 0.9 \\ 0.54 & 0.88 & 0.62 & 0.8 & 0.86 & 0.56 \end{pmatrix}$$

$$CE^3 = \begin{pmatrix} 0.98 & 0.78 & 0.88 & 0.98 & 0.94 & 0.96 \\ 0.92 & 0.98 & 0.76 & 0.78 & 0.74 & 0.92 \\ 0.72 & 0.58 & 1 & 0.74 & 0.74 & 0.76 \\ 0.42 & 0.88 & 0.72 & 0.7 & 0.72 & 0.7 \\ 0.84 & 0.68 & 0.58 & 0.9 & 0.66 & 0.64 \end{pmatrix} \quad CE^4 = \begin{pmatrix} 0.68 & 0.72 & 0.72 & 0.72 & 0.86 & 0.66 \\ 0.82 & 0.92 & 0.76 & 0.78 & 0.84 & 0.92 \\ 0.72 & 0.78 & 0.7 & 0.76 & 0.56 & 0.54 \\ 0.78 & 0.58 & 0.72 & 0.7 & 0.88 & 0.6 \\ 0.76 & 0.92 & 0.98 & 0.9 & 0.94 & 0.94 \end{pmatrix}$$

$$CE^5 = \begin{pmatrix} 0.98 & 0.88 & 0.92 & 0.98 & 0.94 & 0.94 \\ 0.78 & 0.48 & 0.64 & 0.72 & 0.76 & 0.78 \\ 0.68 & 0.92 & 0.8 & 0.84 & 0.96 & 0.94 \\ 0.98 & 0.72 & 0.68 & 0.7 & 0.98 & 0.9 \\ 0.86 & 0.82 & 0.92 & 0.9 & 0.84 & 0.94 \end{pmatrix}$$

$$CA^1 = (0.86, 0.7, 0.7, 0.84, 0.84), CA^2 = (0.71, 0.82, 0.82, 0.78, 0.71), CA^3 = (0.92, 0.85, 0.76, 0.69, 0.72)$$

$$CA^4 = (0.73, 0.84, 0.68, 0.71, 0.91), CA^5 = (0.94, 0.69, 0.86, 0.83, 0.88)$$

$$CI^1 = 0.79, CI^2 = 0.77, CI^3 = 0.79, CI^4 = 0.77, CI^5 = 0.84$$

(3) In this GDM problem, the consensus threshold is set as 0.8, and the following is obtained through Equations (5)–(7):

$$\begin{aligned} APS = \{&(1,2,1),(1,2,2),(1,2,3),(1,2,4),(1,2,5),(1,3,2),(1,3,3),(1,3,4),(1,3,5),(1,3,6),(2,1,1),(2,1,2),\\ &(2,1,3),(2,1,5),(2,1,6),(2,4,2),(2,4,3),(2,4,4),(2,5,1),(2,5,3),(2,5,6),(3,3,1),(3,3,2),(3,3,4),\\ &(3,3,5),(3,3,6),(3,4,1),(3,4,3),(3,4,4),(3,4,5),(3,4,6),(3,5,2),(3,5,3),(3,5,5),(3,5,6),(4,1,1),\\ &,(4,1,2),(4,1,3),(4,1,4),(4,1,6),(4,3,1),(4,3,2),(4,3,3),(4,3,4),(4,3,5),(4,3,6),(4,4,1),(4,4,2),\\ &(4,4,3),(4,4,4),(4,4,6)\} \end{aligned}$$

Step 2: Enter the bidirectional mechanism proposed in this study:

(1) Identify elements of a bidirectional feedback mechanism:

Set the personality parameters $\eta_2 = 0.94 > 0.85$ of expert DM^2, and we have:

$$SM = \{DM^2\}$$
$$CM = \{DM^5\}$$
$$CM = \{DM^5\} \quad APS^1 = \{(2,1,1),(2,1,2),(2,1,3),(2,1,5),(2,1,6),(2,4,2),(2,4,3),(2,4,4),(2,5,1),(2,5,3),(2,5,6)\}$$

(2) Calculate the interference effects between the SM and CM groups. The calculation results of quantum probability as shown in Table 2 can be obtained through Figure 9.

Table 2. The calculation results of the interference effects between the *SM* and *CM* groups.

U^1	*Modify*	$\Pr(U^3{=}M \mid U^1{=}M)$	$\frac{0.2569+0.1475\cos\theta_1}{0.385+0.2546\cos\theta_1}$
		$\Pr(U^3{=}N \mid U^1{=}M)$	$\frac{0.128+0.1071\cos\theta_1}{0.385+0.2546\cos\theta_1}$
	Not_modify	$\Pr(U^3{=}M \mid U^1{=}N)$	$\frac{0.2873+0.2566\cos\theta_2}{0.65+0.4427\cos\theta_2}$
		$\Pr(U^3{=}N \mid U^1{=}N)$	$\frac{0.3627+0.1861\cos\theta_2}{0.65+0.4427\cos\theta_2}$

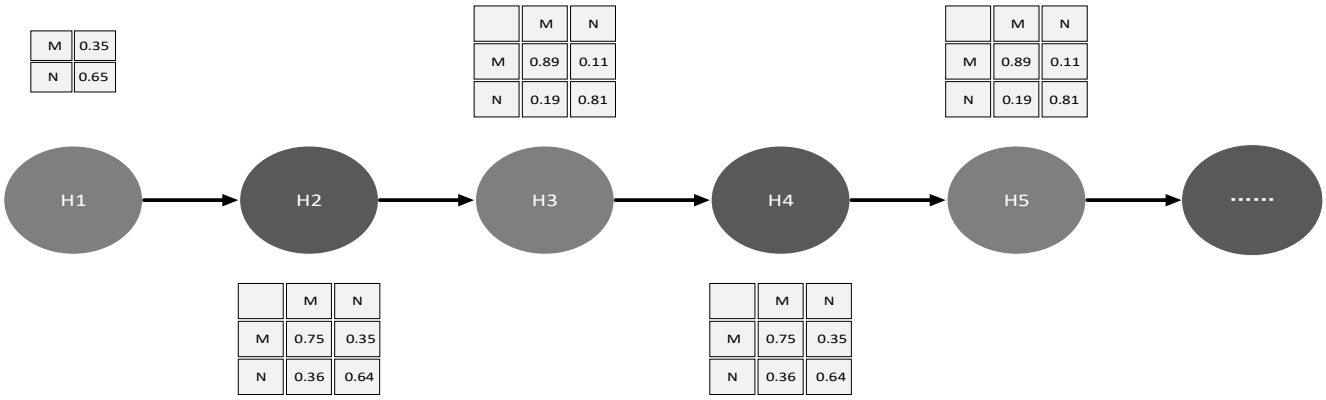
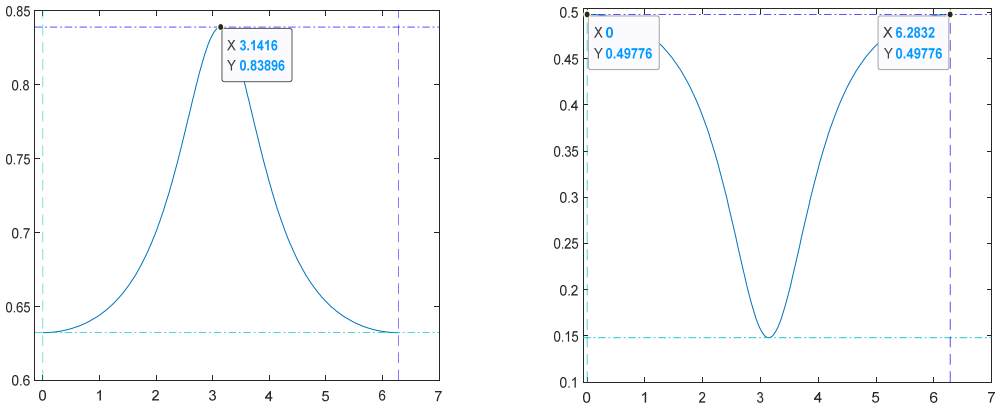


Figure 9. The known probability distribution of the *SM* and *CM* groups in the game process.

Figure 10 reflects how the angle of the phase angle in our method affects the quantum probability. x ranges from 0 to 2π. If $0 < x < \pi$, the probability of the SM modifying their preferences increases, while the probability of the SM not modifying their preferences decreases. If $\pi < x < 2\pi$, the probability of the SM modifying their preferences decreases, while the probability of the SM not modifying their preferences increases. A reverse variation tendency exists between these two stations. When $x = \pi$, the quantum probability has reached its extreme value and there is an absolutely negative or positive interference between the two groups. Overall, Figure 10 shows the wave transformation under different phase values. This also makes our decision results more abundant and specific.



(**a**) First-stage *SM* accepts the modification. (**b**) First-stage *SM* rejects the modification.

Figure 10. The various quantum probability values that can be achieved by varying the phase angle. (Note that y represents the quantum probability and x represents the angle of the phase angle).

(3) In this example, *SM* chooses to modify the preference in the first stage. Therefore, Models (29) and (30) are executed and the feedback results as shown in Table 3 are obtained:

Table 3. Some important indicators calculated in the bidirectional mechanism.

Feedback Stage	Feedback Subgroup	Feedback Issues	Feedback Parameter	Consensus Level	
				DM^2	DM^5
U^1	*SM*	(2,1,1), (2,1,2), (2,1,3), (2,1,5), (2,1,6), (2,4,2), (2,4,3), (2,4,4), (2,5,1), (2,5,3), (2,5,6)	$\sigma^{(1)} = 0.226$	0.794	0.84
U^2	*DM*	(2,1,1), (2,1,2), (2,1,3), (2,1,5), (2,1,6), (2,5,1), (2,5,3) (2,5,6)	$\gamma^{(2)} = 0.43$	0.78	0.82
U^3	*SM*	(2,1,1), (2,1,2), (2,1,3), (2,1,6), (2,5,1), (2,5,3), (2,5,6)	$\sigma^{(3)} = 0.22$	0.80	0.82

When the consensus level of all experts is greater than the threshold, the feedback mechanism is ended. The best alternative obtained through the selection process is x_2.

5.2. Comparative Analysis

To demonstrate the necessity of considering interference effects between the stubborn discordant and concordant groups in the bidirectional feedback mechanism, different feedback mechanisms in Table 4 are compared and discussed.

Table 4. Comparison of some indicators in different feedback mechanisms.

Feedback Mechanism	Feedback Stage	Sum of Feedback Parameters		Consensus Level		Loss (Feedback Parameters × Personality Parameters)
		SM	CM	SM	CM	
Traditional feedback	5	0.24	0.1	0.8	0.84	0.23
Unidirectional feedback	1	0.5	0	0.83	0.84	0.46
Bidirectional feedback without interference effects	2	0.23	0.05	0.80	0.84	0.21
Bidirectional feedback considering interference effects	3	0.446	0.43	0.80	0.82	0.41

(1) Necessity of the bidirectional feedback mechanism. First, experts have different attitudes towards preference adjustment. The more easygoing the expert is, the lower is the cost of persuading the expert to adjust his or her preferences. Therefore, we multiply the sum of the feedback parameters of experts and personality parameters as Loss to reflect the price paid in the process of preference adjustment. Static feedback parameters are used in traditional and unidirectional feedback mechanisms, which are subjectively selected according to the degree of consensus. However, random selection leads to an increase in feedback frequency or cost, which greatly reduces the flexibility of feedback mechanisms and group initiatives. As shown in Table 4, although the Loss of the traditional feedback is lower than that of the unidirectional feedback, the Feedback stage is the largest. In the unidirectional feedback mechanism, only the discordant group adjusted their preferences, and their attitude toward blindly pursuing consensus maximization was not conducive to cost control, resulting in the largest *Loss*. Moreover, in realistic GDM, it is often unrealistic for only one party to compromise.

(2) Interference effects in the bidirectional feedback mechanism. The bidirectional feedback mechanism successfully reflects that "feedback" is also a dynamic process of "negotiation". The feedback parameters used are calculated using the optimization model, and the cost is constrained on the premise of meeting the consensus level. Therefore, the determination of feedback parameters is more reasonable. However, there is a certain interference effect in the interaction among the groups. Therefore, simulating a more realistic feedback process is a deficiency of the previous bidirectional feedback mechanism. As shown in Table 4, after considering the interference effects between the concordant and stubborn discordant groups, the Feedback stage and Loss are increased compared with the bidirectional feedback mechanism without considering interference effects. This can be a disadvantage; however, it is more realistic because it is the result of simulating the interaction between the *SM* and *DM* in the process of real feedback bargaining. Interference effects between interactions can be positive or negative, and different interference values produce different feedback results.

Based on the above analysis, this paper makes several contributions that offer new perspectives compared to previous research methods and results.

First, it addresses the limitation of ignoring individual differences among experts in earlier studies by proposing a personalized feedback mechanism. While previous models assumed all experts were homogeneous and used uniform feedback methods, this paper categorizes experts into stubborn, inconsistent, and harmonious types, applying unidirectional or bidirectional feedback based on their personalities. This approach improves the efficiency and effectiveness of decision-making by better adapting to experts' personality differences.

Second, this paper quantifies the interference effect in the feedback mechanism, filling the gap left by previous studies, which often relied on subjective judgment. Using QPT, it constructs a game process between stubborn and consistent groups and provides an objective method for assessing interference effects, enhancing the scientific accuracy of the decision-making model.

Finally, the paper combines the interference effect with the bidirectional feedback model, addressing the previous model's failure to account for the interaction between decision-makers. By introducing a disturbance term, it forms a dynamic mechanism for adjusting decision preferences, making the group decision-making process more flexible and improving the model's applicability and accuracy.

6. Conclusions

In this study, an innovative bidirectional feedback model incorporating quantum probability theory (QPT) is proposed to address the interference effect between the unanimous group and the stubbornly opposing group in group decision-making (GDM). The model emphasizes the dynamic interactions among experts, in which the decision of one group affects the other, which plays a key role in overcoming the shortcomings of traditional feedback mechanisms in ignoring the interference effect. The model also classifies experts into three categories based on their personality parameters and personalizes the feedback method: especially for stubborn experts, two-way feedback helps to increase their initiative to modify their preferences.

Through the results, it is shown that the interference effect is not fixed but changes dynamically according to the group's willingness to modify their preferences and is quantified through the QPT. This approach not only provides us with a deeper understanding of how expert interactions affect the feedback process but also provides a more realistic framework for dynamic simulation of group decision-making. Numerical results from MATLAB2018b simulations support the validity of the model, showing that it is able not only to effectively deal with interference effects but also to shorten the time required to reach consensus.

The advantages of the model are that it is able to quantify and resolve interference effects, providing a flexible and adaptable decision-making framework that is suitable for complex decision-making environments. However, some limitations remain. The current methodology for determining the expert personality parameters needs further refinement, as the existing methodology is somewhat subjective. In addition, although the interference effect is dynamically changing, how to determine the most suitable interference values for different decision-making situations is still an open question. Future research should focus on optimizing the method of determining personality parameters and exploring the selection of interference values in different decision-making situations.

Author Contributions: Methodology, M.C.; data curation, Y.H.; writing—original draft, M.C.; visualization, Y.H.; project administration, M.C.; funding acquisition, M.C. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Humanities and Social Science Fund of Ministry of Education of China (23YJA630005) and the National Natural Science Foundation of China (NSFC) (71871121).

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

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