

Article

Mathematical Modeling of Ride-Hailing Matching Considering Uncertain User and Driver Preferences: Interval-Valued Fuzzy Approach

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Abstract: This study introduces a fuzzy interval-valued approach using multi-objective linear programming to optimize passenger–driver matching in ride-hailing systems, addressing uncertainties in waiting times and fare preferences. The model aims to minimize total waiting times, balance job allocations among drivers, and reduce deviations in fare expectations between passengers and drivers. The proposed framework effectively manages operational uncertainties by incorporating interval-valued fuzzy parameters, including traffic variability and fluctuating demand patterns. Numerical experiments using real-world data demonstrate that the interval-valued fuzzy model significantly outperforms deterministic methods in reducing average waiting times, achieving higher request fulfillment rates, and ensuring a more equitable distribution of assignments among drivers. The results highlight the model’s robustness and adaptability, particularly under high uncertainty scenarios, and its ability to maintain service reliability and user satisfaction. While computational complexity remains a limitation, integrating the model with AI and IoT technologies offers promising avenues for scalability and real-time applications. These findings contribute to advancing optimization frameworks in ride-hailing systems, emphasizing uncertainty management’s importance in enhancing operational efficiency and fairness.

Keywords: interval-valued fuzzy programming; multi-objective programming; ride-hailing matching; uncertain parameters

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1. Introduction

Ride-hailing services have emerged as a crucial solution in modern urban transportation due to their ability to address several challenges associated with traditional transport systems. These services offer flexible and convenient mobility options, particularly in

urban areas where public transportation may be inadequate or inefficient [1,2]. Ride-hailing services play a pivotal role in multimodal commuting by providing essential first-mile and last-mile connectivity, thereby encouraging a shift from private vehicle use to mass public transportation, which can lead to a reduction in greenhouse gas emissions [3]. Despite concerns about increased traffic congestion and competition with public transit, ride-hailing services have been shown to improve transit availability in underserved areas and offer a safer and more efficient alternative to traditional taxis [2,4]. Additionally, the potential integration of autonomous electric vehicles within ride-hailing systems promises further reductions in carbon emissions and transportation costs, aligning with innovative city initiatives aimed at sustainable urban development [5,6]. However, the impact of ride-hailing on vehicle kilometers traveled and its environmental implications remain mixed, necessitating further research to optimize these services for sustainable urban mobility [7]. Overall, ride-hailing services are an important component of modern urban transportation, offering innovative solutions to longstanding mobility challenges while also presenting new opportunities for enhancing urban transit systems [6].

Recent research on ride-hailing matching and assignment has explored various innovative approaches to optimize the efficiency and effectiveness of these systems. One prominent method involves integrating Internet of Things (IoT) sensors with reinforcement learning (RL) to dynamically assign passengers to drivers, considering factors such as journey duration, driver proximity, and passenger preferences. This approach has demonstrated improved passenger matching efficiency, reduced wait times, and journey durations, outperforming traditional heuristic-based systems [8]. Another study proposes a one-stage end-to-end RL-based order-dispatching approach that utilizes a two-layer Markov decision process framework to address micro-view order-dispatching challenges, significantly enhancing matching efficiency and user experience in real-world benchmarks [9]. Additionally, a piecewise linear method has been applied to a non-linear programming model to optimize ride-hailing assignments by considering the service level, driver workload, and fuel consumption, resulting in reduced passenger waiting times, balanced driver workloads, and decreased fuel consumption [10]. Addressing uncertainty in travel times, an interval-valued fuzzy multi-objective linear programming approach has been developed, which efficiently handles ride-hailing matching challenges by reformulating constraints to manage uncertainty parameters [11]. Furthermore, a data-driven approach imposes pick-up time targets to maximize the probability of meeting these targets, thereby reducing trip cancellations and enhancing platform profitability [12]. The integration of vehicle assignment and rebalancing using fuzzy linear programming has also been explored to improve system performance under demand uncertainty [13]. In the context of coexisting ride-hailing and ride-sharing services, an optimization strategy that considers pricing fairness and user choices has been shown to increase the service ratio and profit without compromising service quality [14]. Heuristic methods for ride-pooling assignments have been proposed to balance system throughput and customer experience, highlighting the trade-offs between computational complexity and service quality [15]. Collectively, these studies underscore the potential of advanced computational techniques, including machine learning and optimization algorithms, to enhance ride-hailing matching and assignment processes, addressing challenges such as efficiency, sustainability, and user satisfaction [16].

Matching passengers and drivers in ride-hailing services under the uncertainty of travel times and price preferences presents several challenges. One significant issue is the inherent uncertainty in travel times, which complicates the optimization of matching algorithms. This uncertainty can be addressed using interval-valued fuzzy multi-objective linear programming, which reformulates constraints to manage these uncertainties effectively, thereby simplifying the solution process [11]. Additionally, drivers often have

heterogeneous costs and may behave strategically to maximize their profits, which can lead to inefficient matching and reduced social welfare. An incentive-compatible mechanism, such as a second-pricing algorithm, can encourage truthful cost reporting by drivers, thus improving the efficiency of the matching process and maximizing social welfare [17].

Furthermore, the stochastic nature of real-time ride-sourcing markets requires balancing multiple objectives, such as platform revenue, driver service scores, and pick-up distances, without pre-specifying the weights for each metric. This can be achieved through a multi-objective stochastic optimization approach that adapts to real-time performance gaps [18]. Another challenge is the spatially unbalanced demand for ride-hailing services, which can be mitigated by employing vehicular crowd-sensing strategies to improve spatial-temporal coverage while considering drivers' incomes [19]. Moreover, fairness in matching under uncertainty is crucial, as algorithmic decisions must respect individual preferences and be fair concerning merits, which are often uncertain. A fairness-aware framework can ensure utility-maximizing distributions over allocations while being robust to parameter perturbations [20]. Lastly, integrating pricing and matching decisions while considering future impacts can enhance sustainability and efficiency in ride-pooling services, leading to higher revenues and reduced vehicle usage [21]. These challenges highlight the complexity of designing practical matching algorithms in ride-hailing services, necessitating innovative approaches that consider uncertainty, strategic behavior, and fairness.

The uncertainty of the wait time and price parameters significantly impacts both efficiency and fairness in ride-hailing systems, as these factors influence the allocation of resources and the satisfaction of both drivers and passengers. Uncertain wait times, often caused by unpredictable traffic conditions, can lead to inefficiencies in driver-passenger matching and increased passenger dissatisfaction. Interval-valued fuzzy quadratic programming models have been proposed to better handle the uncertainty in waiting times, thereby improving the reliability of ride-hailing services and enhancing user welfare [22]. Similarly, the integration of vehicle assignment and rebalancing using fuzzy linear programming can mitigate the effects of uncertain demand and travel times, leading to more effective resource allocation and improved system performance [13]. On the pricing front, dynamic pricing policies that adapt to past demand randomness can optimize the distribution of servers and customers, thus enhancing overall efficiency [23]. However, these pricing strategies must also consider individual customer preferences for time and cost savings to prevent mismatches that could degrade service utility and provider profits [24]. Fairness, particularly in driver income, is another critical aspect affected by these uncertainties. Reinforcement learning frameworks have been developed to balance efficiency and fairness by optimizing driver incomes and ensuring an equitable distribution of earnings [25,26]. Overall, addressing the uncertainties in wait times and pricing through advanced optimization and machine learning techniques can significantly enhance both the efficiency and fairness of ride-hailing systems, ensuring sustainable and equitable urban mobility solutions.

Interval-valued fuzzy multi-objective linear programming was chosen as the main method in this study due to its ability to effectively handle uncertainty in multi-objective linear programming problems, particularly in contexts like ride-hailing, where travel times are unpredictable. This approach allows for the incorporation of interval-valued fuzzy numbers, which are adept at managing uncertain parameters by reformulating constraint coefficients, thus simplifying the linear programming formulations and enhancing solution efficiency [11]. The method's flexibility in dealing with imprecise data is further supported by its application in other studies, where fuzzy numbers and interval-based approaches have been used to convert imprecise objectives and constraints into crisp values, facilitating clearer decision-making processes [27]. Additionally, the interval-valued optimization

framework is recognized for its robustness in real-world scenarios where classical optimization assumptions do not hold, providing necessary optimality conditions that are crucial for solving complex, uncertain problems [28]. Collectively, these attributes make interval-valued fuzzy multi-objective linear programming a suitable choice for addressing the inherent uncertainties in ride-hailing and similar domains.

Several innovative methodologies and frameworks presented across the provided papers highlight the novelty in the mathematical modeling of ride-hailing matching with user and driver preferences under uncertainty, mainly through an interval-valued fuzzy approach. The use of interval-valued fuzzy numbers in multi-objective linear programming (MOLP) and quadratic programming models addresses the inherent uncertainties in ride-hailing, such as travel and waiting times, by reformulating the constraints to simplify solution processes and enhancing decision-making under uncertainty [11,22]. This approach is particularly effective in managing the unpredictability of traffic conditions and optimizing the allocation of drivers to passengers, thereby improving user welfare [22]. Additionally, integrating user preferences into ride-sharing models, as demonstrated by the preference-driven matching algorithm, underscores the importance of considering user satisfaction alongside efficiency. This algorithm captures user preferences to enhance the matching process, particularly in less-populated areas, and results in higher user satisfaction and increased demand [29].

Furthermore, the development of robust models, such as the matching-integrated vehicle rebalancing (MIVR) model, incorporates driver–customer matching into the vehicle rebalancing strategies to better handle demand uncertainty, thereby reducing customer wait times and improving service reliability [30]. The exploration of dynamic ride-pooling services, which predict matching probabilities and expected ride distances, further exemplifies the application of mathematical modeling to optimize ride-sharing under varying demand intensities [31]. These advancements collectively contribute to a more responsive and efficient ride-hailing system capable of addressing the complexities of urban mobility and enhancing the overall user experience through innovative mathematical approaches.

The research gap in this area lies in several aspects that have not been fully explored in the literature on efficient and fair ride-hailing systems, particularly in the context of uncertainty and variability in urban conditions. The identified gaps are as follows:

1. **Uncertainty in Waiting Times and Fare Preferences:** While various studies have attempted to optimize driver–passenger matching using mathematical models, many have not profoundly considered the uncertainty in waiting times and changing fare preferences. This research will address this gap using a fuzzy interval-valued approach to handle these uncertainties, which is more realistic in representing real-world conditions.
2. **Application of Multi-Objective Linear Programming in Ride-Hailing Systems:** Many studies focus on optimization within ride-hailing. However, few explore the application of multi-objective linear programming (MOLP) to handle uncertain parameters. MOLP has excellent potential to balance multiple operational objectives such as waiting times, demand fulfillment rates, and equitable job distribution. This gap presents an opportunity for further research to formulate solutions that accommodate multiple objectives simultaneously, in line with traffic dynamics and user preferences.
3. **Fairness and Efficiency in Job Distribution:** Previous studies have mainly focused on optimizing waiting times or demand fulfillment but have paid less attention to fairness in job distribution among drivers. In many cases, existing algorithms tend to give more jobs to higher-rated drivers, which can create imbalances. This gap is relevant to address by developing models that focus on efficiency and ensure a fairer distribution of jobs among all drivers.

4. **Real-Time Decision-Making in Ride-Hailing Platforms:** Although many ride-hailing platforms optimize their algorithms to handle dynamic traffic conditions and demand, most existing research has not effectively provided solutions for real-time decision-making that can directly respond to uncertainty in urban conditions. Research will fill this gap to provide a more responsive, fair, and efficient framework for making real-time decisions, considering the rapid changes in traffic and user preferences.
5. **Sustainability in Ride-Hailing Systems:** The balance between operational efficiency and sustainability in ride-hailing systems has not been thoroughly explored. While many studies focus on economic optimization and waiting times, the social and environmental impacts of unequal job distribution or inappropriate matching decisions are less addressed. This research will fill this gap by designing algorithms that are efficient and fair and support sustainability principles, such as reducing carbon emissions and ensuring a more equitable job distribution.
6. **Limitations in the Application of Fuzzy Models for Ride-Hailing:** Although fuzzy models have been applied in transportation systems in some contexts, there is limited research that develops fuzzy models that consider uncertainties in various ride-hailing variables, such as waiting times, fares, and passenger preferences. This research will contribute by developing a more holistic and applicable fuzzy model in the context of ride-hailing.

The urgency of this research lies in the growing demand for efficient and fair ride-hailing systems, especially in dynamic urban environments where uncertainty in waiting times, fare preferences, and traffic conditions can significantly impact service quality. As ride-hailing platforms continue to expand, optimizing driver–passenger matching and improving operational efficiency while ensuring fairness and sustainability become critical to enhancing customer experience and system sustainability. This study aims to address these challenges by developing innovative mathematical models that incorporate real-time decision-making, multi-objective optimization, and fuzzy logic to handle uncertainties, ensuring a more responsive, equitable, and efficient urban mobility system.

This study aims to:

1. Develop a mathematical model based on a fuzzy interval-valued approach to optimize the matching process in ride-hailing services, considering uncertainties in waiting times and fare preferences between passengers and drivers.
2. Enhance the operational efficiency of ride-hailing systems by reducing the average waiting times, increasing the request fulfillment rates, and achieving a fairer job distribution among drivers.
3. Explore the effectiveness of multi-objective linear programming to handle uncertain parameters, enabling solutions that adapt to dynamic traffic conditions and user preferences.
4. Provide a framework for ride-hailing platforms to support real-time decision-making that is more responsive, fair, and efficient under dynamic and uncertain conditions.
5. Contribute theoretically and practically to developing matching algorithms that prioritize efficiency, fairness, and sustainability, fostering modern urban mobility.

2. Preliminaries

2.1. Interval-Valued Fuzzy

This section provides a review of the foundational concepts related to level- (w^L, w^U) interval-valued trapezoidal fuzzy numbers, which are extensively utilized throughout this paper. The definitions, theorems, and operations related to fuzzy numbers will provide the mathematical structure for the optimization problem addressed in the subsequent chapters.

We first need to define w -level trapezoidal fuzzy numbers to understand fuzzy numbers. These numbers are characterized by their membership function, which describes the

degree of membership of an element to the fuzzy set. The concept of trapezoidal fuzzy numbers is commonly used because of its ability to represent uncertain or vague data effectively. The parameter w represents the degree of fuzziness, which affects the shape of the fuzzy number.

Definition 1 ([32]). A w -level trapezoidal fuzzy number, represented as $\tilde{A} = (a_1, a_2, a_3, a_4; w)$, is a fuzzy set defined on $\mathbb{R}$, where $0 < w \leq 1$. Its membership function is given as:

$$\mu_{\tilde{A}}(x) = \begin{cases} w \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ w, & a_2 \leq x \leq a_3, \\ w \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Let $F_{TN}(w)$ denote the set of all w -level trapezoidal fuzzy numbers, where $\tilde{A} = (a_1, a_2, a_3, a_4; w)$ satisfies $a_1 \leq a_2 \leq a_3 \leq a_4$, and $0 \leq w \leq 1$. Formally,

$$F_{TN}(w) = \left\{ \tilde{A} = (a_1, a_2, a_3, a_4; w), a_1 \leq a_2 \leq a_3 \leq a_4 \right\}, 0 < w \leq 1. \quad (2)$$

In order to extend the concept of fuzzy numbers, we introduce interval-valued fuzzy numbers. These numbers represent a range of possible values, with the lower and upper bounds determined by two fuzzy numbers. The interval-valued fuzzy number is a pair of trapezoidal fuzzy numbers, each with different membership levels.

Definition 2 ([33]). An interval-valued fuzzy set on $\mathbb{R}$, denoted as a level- (w^L, w^U) interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$, is defined as:

$$\tilde{\tilde{A}} = [\tilde{A}^L, \tilde{A}^U] = \left\langle (a_1^L, a_2^L, a_3^L, a_4^L; w^L), (a_1^U, a_2^U, a_3^U, a_4^U; w^U) \right\rangle,$$

where $\tilde{A}^L \in F_{TN}(w^L)$ is the lower trapezoidal fuzzy number and $\tilde{A}^U \in F_{TN}(w^U)$ is the upper trapezoidal fuzzy number. Their membership functions are:

For $\tilde{A}^L$:

$$\mu_{\tilde{A}^L}(x) = \begin{cases} w^L \frac{x-a_1^L}{a_2^L-a_1^L}, & a_1^L \leq x \leq a_2^L, \\ w^L, & a_2^L \leq x \leq a_3^L, \\ w^L \frac{a_4^L-x}{a_4^L-a_3^L}, & a_3^L \leq x \leq a_4^L, \\ 0, & \text{otherwise,} \end{cases} \quad (3)$$

For $\tilde{A}^U$:

$$\mu_{\tilde{A}^U}(x) = \begin{cases} w^U \frac{x-a_1^U}{a_2^U-a_1^U}, & a_1^U \leq x \leq a_2^U, \\ w^U, & a_2^U \leq x \leq a_3^U, \\ w^U \frac{a_4^U-x}{a_4^U-a_3^U}, & a_3^U \leq x \leq a_4^U, \\ 0, & \text{otherwise,} \end{cases} \quad (4)$$

Here, the conditions $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L, a_1^U \leq a_2^U \leq a_3^U \leq a_4^U, 0 < w^L \leq w^U \leq 1, a_1^U \leq a_1^L$ and $a_4^L \leq a_4^U$ hold. Additionally, it must satisfy that $\tilde{A}^L \subseteq \tilde{A}^U$.

The set of all (w^L, w^U) -level trapezoidal fuzzy numbers are represented as:

$$F_{IVTN}(w^L, w^U) = \{ \tilde{A} = [\tilde{A}^L, \tilde{A}^U] = \langle (a_1^L, a_2^L, a_3^L, a_4^L; w^L), (a_1^U, a_2^U, a_3^U, a_4^U; w^U) \rangle : \tilde{A} \in F_{TN}(w^L), \tilde{A}^U \in F_{TN}(w^U), a_1^U \leq a_1^L, a_4^L \leq a_4^U, 0 < w^L \leq w^U \leq 1 \}.$$

Once we have defined interval-valued fuzzy numbers, it is essential to understand how to operate on these numbers. Specifically, we will look at extended addition and scalar multiplication, which allow us to combine and scale interval-valued fuzzy numbers meaningfully.

Definition 3 [34]. For $\tilde{A}, \tilde{B} \in F_{IVTN}(w^L, w^U)$ and a non-negative scalar $k \geq 0$, the operations of extended addition and scalar multiplication are defined as:

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= \left[\tilde{A}^L + \tilde{B}^L, \tilde{A}^U + \tilde{B}^U \right] \\ &= \langle (a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; w^L), (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; w^U) \rangle \\ k\tilde{A} &= \begin{cases} \langle (ka_1^L, ka_2^L, ka_3^L, ka_4^L; w^L), (ka_1^U, ka_2^U, ka_3^U, ka_4^U; w^U) \rangle, k > 0, \\ \langle (ka_1^L, ka_2^L, ka_3^L, ka_4^L; w^L), (ka_1^U, ka_2^U, ka_3^U, ka_4^U; w^U) \rangle, k > 0, \\ \langle (0, 0, 0, 0; w^L), (0, 0, 0, 0; w^U) \rangle = \tilde{0}, k = 0. \end{cases} \end{aligned}$$

Sometimes, we want to measure the “distance” from a number to zero. In this case, the signed distance from any number r to zero is just the number itself. This simple formula helps us to understand how far a number is from zero on the number line.

Definition 4 ([32]). For $r \in \mathbb{R}$, the signed distance from r to 0 is defined as $d(r, 0) = r$.

We often need to look at a part of the fuzzy set at a specific level in fuzzy logic. This is called the α -cut. It is like taking a slice of the fuzzy number at a certain “fuzziness” level to see which part of the fuzzy set it belongs to.

Definition 5 ([35]). The α -cut set of $\tilde{A} \in F_{IVTN}(w^L, w^U)$, denoted as $\tilde{A}(\alpha)$, is defined as (see Figure 1):

$$\tilde{A}(\alpha) = \left[\tilde{A}^L(\alpha), \tilde{A}^U(\alpha) \right] = \begin{cases} \left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha) \right] \cup \left[\tilde{A}_r^L(\alpha), \tilde{A}_r^U(\alpha) \right], 0 \leq \alpha \leq w^L, \\ \left[\tilde{A}_l^U(\alpha), \tilde{A}_r^U(\alpha) \right], w^L \leq \alpha \leq w^U, \end{cases}$$

where

$$\begin{aligned} \tilde{A}_l^L(\alpha) &= a_1^L + (a_2^L - a_1^L) \frac{\alpha}{w^L}, \tilde{A}_r^L(\alpha) = a_4^L + (a_4^L - a_3^L) \frac{\alpha}{w^L}, \\ \tilde{A}_l^U(\alpha) &= a_1^U + (a_2^U - a_1^U) \frac{\alpha}{w^U}, \tilde{A}_r^U(\alpha) = a_4^U + (a_4^U - a_3^U) \frac{\alpha}{w^U}. \end{aligned}$$

When we want to compare how far two interval-valued fuzzy numbers are from a reference point (like the starting point on a graph), we can use a special formula to calculate the signed distance. This helps us rank the fuzzy numbers by how far they are from the reference point.

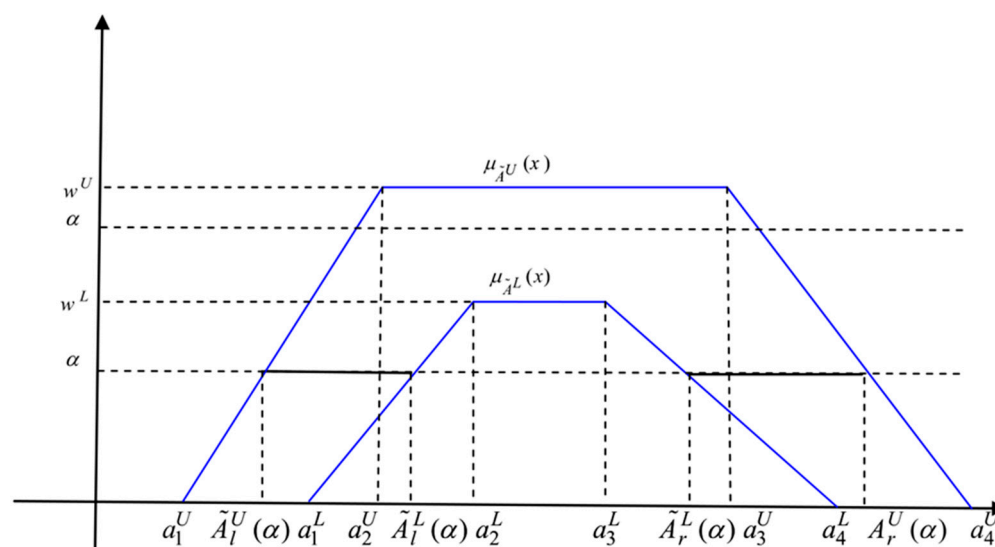


Figure 1. An α -cut of level- (w^L, w^U) interval-valued trapezoidal fuzzy number $\tilde{A}$ [11].

Theorem 1 ([35]). For $\tilde{A} \in F_{IVTN}(w^L, w^U)$, the signed distance of $\tilde{A}$ from O_1 (the y-axis) is calculated using the following expressions:

1. If $\tilde{A}^L = \tilde{A}^U = \tilde{A}$, the signed distance is:

$$d(\tilde{A}, O_1) = \frac{1}{4}[a_1 + a_2 + a_3 + a_4], \quad (5)$$

2. If $0 < w^L = w^U \leq 1$, the signed distance is:

$$d(\tilde{A}, O_1) = \frac{1}{8}[a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U], \quad (6)$$

3. If $0 < w^L < w^U \leq 1$, the signed distance is:

$$d(\tilde{A}, O_1) = \frac{1}{8}[a_1^L + a_2^L + a_3^L + a_4^L + 4a_1^U + 2a_2^U + 2a_3^U + 3(a_2^U + a_3^U - a_1^U - a_4^U)\frac{w^L}{w^U}]. \quad (7)$$

Theorem 1 provides a systematic approach for comparing level- (w^L, w^U) interval-valued trapezoidal fuzzy numbers using signed distance ranking.

Finally, we define how to rank interval-valued fuzzy numbers based on their distance from the reference point. This ranking tells us which fuzzy number is closer or farther from the reference point, helping us to understand their relative positions. If one number is closer, it is ranked higher; if they are at the same distance, they are considered equal.

Definition 6 ([35]). Let $\tilde{A}, \tilde{B} \in F_{IVTN}(w^L, w^U)$. The ranking of level- (w^L, w^U) interval-valued trapezoidal fuzzy numbers in $F_{IVTN}(w^L, w^U)$ is based on their signed distance $d(\cdot, O_1)$, and is defined as follows:

$$\tilde{A} \preceq \tilde{B} \text{ if and only if } d(\tilde{A}, O_1) \leq d(\tilde{B}, O_1), \quad (8)$$

$$\tilde{\tilde{A}} > \tilde{\tilde{B}} \text{ if and only if } d\left(\tilde{\tilde{A}}, O_1\right) > d\left(\tilde{\tilde{B}}, O_1\right), \quad (9)$$

$$\tilde{\tilde{A}} \approx \tilde{\tilde{B}} \text{ if and only if } d\left(\tilde{\tilde{A}}, O_1\right) = d\left(\tilde{\tilde{B}}, O_1\right). \quad (10)$$

It is important to note that the signed distance $(., O_1)$ acts as a linear ranking function. For any $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in F_{IVTN}(w^L, w^U)$ and $k \in \mathbb{R}$, the following property holds:

$$d\left(k\tilde{\tilde{A}} \oplus \tilde{\tilde{B}}, O_1\right) = kd\left(\tilde{\tilde{A}}, O_1\right) + kd\left(\tilde{\tilde{B}}, O_1\right).$$

Furthermore, the structure $(F_{IVTN}(w^L, w^U), \approx, <)$ adheres to the law of trichotomy, as described in [32]. This ensures that for any two fuzzy numbers, we have exactly one of the following: $\tilde{\tilde{A}} < \tilde{\tilde{B}}$, or $\tilde{\tilde{A}} \approx \tilde{\tilde{B}}$, or $\tilde{\tilde{B}} < \tilde{\tilde{A}}$.

2.2. Interval-Valued Fuzzy Multi-Objective Linear Programming

Fuzzy linear programming (FLP), introduced by [36] and based on the fuzzy set theory pioneered by [37], addresses uncertainty in linear programming problems. FLP is categorized into three types, depending on the nature of uncertainty in the model [38]:

1. Fuzzy variables linear programming;
2. Fuzzy parameters linear programming;
3. Fuzzy variables and parameters linear programming.

In multi-objective linear programming with fuzzy parameters, uncertainty is introduced into both the objective functions and the constraints. Such a problem can be expressed as:

$$\begin{aligned} \min & \left(\sum_{i=1}^m \sum_{j=1}^n c_{ij1} x_{ij}, \sum_{i=1}^m \sum_{j=1}^n c_{ij2} x_{ij}, \sum_{i=1}^m \sum_{j=1}^n c_{ij3} x_{ij}, \dots, \sum_{i=1}^m \sum_{j=1}^n c_{ijk} x_{ij} \right), \\ \text{s.t.} & \sum_{i=1}^m \sum_{j=1}^n a_{ij} x_{ij} \geq b_l, \forall l, \\ & x_{ij} \geq 0, \forall i, j. \end{aligned} \quad (11)$$

Here, f_k represents the k -th objective function, a_{ij} denotes the constraint coefficients, b_l denotes the right-hand side of the constraints, and x_{ij} represents the decision variables. Let $f_k = \sum_{i=1}^m \sum_{j=1}^n c_{ijk} x_{ij}$, where c_{ijk} represents the parameters of the objective function. Hence, this problem is multi-objective linear programming.

In this section, we discuss the solution of interval-valued fuzzy multi-objective linear programming problems based on the approach proposed by [11]. A multi-objective optimization problem refers to an optimization challenge encompassing multiple objective functions. Mathematically, it can be written as:

$$\begin{aligned} \min & (f_1(x), f_2(x), f_3(x), \dots, f_o(x)), \\ \text{s.t.} & \sum_{i=1}^m \sum_{j=1}^n a_{ij} x_{ij} \geq b_l, \forall l, \\ & x_{ij} \geq 0, \forall i, j. \end{aligned} \quad (12)$$

Here, $\tilde{c}_{ijk}$ are the fuzzy parameters in the objective function, $\tilde{a}_{ij}$ are the fuzzy parameters in the constraints, $\tilde{b}_l$ represents the right-hand fuzzy parameters, and x_{ij} are the crisp decision variables.

The weighted sum method is often used to solve multi-objective problems. This method aggregates multiple objective functions into a single scalar composite function by assigning weights to each objective. The formulation is as follows:

$$\begin{aligned} \min & \sum_{k=1}^o \sum_{i=1}^m \sum_{j=1}^n w_k \tilde{c}_{ijk} x_{ij}, \\ \text{s.t.} & \sum_{i=1}^m \sum_{j=1}^n \tilde{a}_{ij} x_{ij} \geq \tilde{b}_l, \forall l, \\ & x_{ij} \geq 0, \forall i, j. \end{aligned} \quad (13)$$

Here, w_k is the weight assigned to the k -th objective function.

The membership value of $\tilde{c}_{ijk}$ may not always equal 1 during the optimization process. Instead, it is allowed to vary within the interval $[\lambda, 1]$, where $0 < \lambda < 1$. A level- $(\lambda, 1)$ interval-valued fuzzy number for $\tilde{c}_{ijk}$ is defined as (see Figure 2):

$$\tilde{c}_{ijk} = \left\langle \left(c_{ijk} - \delta_{ijk1}, c_{ijk} - \delta_{ijk2}, c_{ijk} + \delta_{ijk5}, c_{ijk} + \delta_{ijk6}; \lambda \right), \left(c_{ijk} - \delta_{ijk1}, c_{ijk} - \delta_{ijk2}, c_{ijk} + \delta_{ijk7}, c_{ijk} + \delta_{ijk8}; 1 \right) \right\rangle, \quad (14)$$

where $0 < \delta_{ijk1} < \delta_{ijk2} < \delta_{ijk3} < \delta_{ijk4} < c_{ijk}$, $0 < \delta_{ijk5} < \delta_{ijk6} < \delta_{ijk7} < \delta_{ijk8}$, $\forall k$.

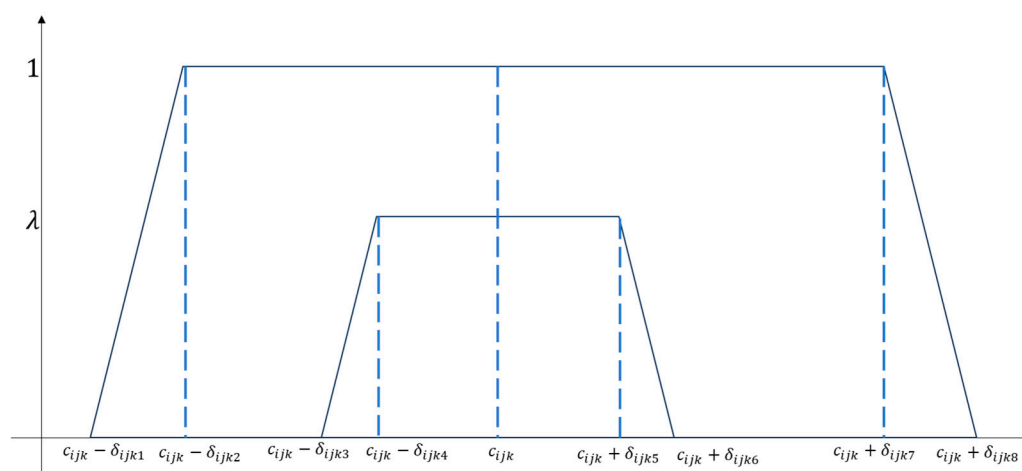


Figure 2. Level- $(\lambda, 1)$ interval-valued fuzzy numbers $\tilde{c}_{ijk}$ [11].

Similarly, $\tilde{a}_{ij}$ and $\tilde{b}_l$ are also defined as level- $(\lambda, 1)$ interval-valued fuzzy numbers:

$$\tilde{a}_{ij} = \left\langle \left(a_{ij} - \epsilon_{ij3}, a_{ij} - \epsilon_{ij4}, a_{ij} + \epsilon_{ij5}, a_{ij} + \epsilon_{ij6}; \lambda \right), \left(a_{ij} - \epsilon_{ij1}, a_{ij} - \epsilon_{ij2}, a_{ij} + \epsilon_{ij7}, a_{ij} + \epsilon_{ij8}; 1 \right) \right\rangle, \quad (15)$$

where $0 < \epsilon_{ij1} < \epsilon_{ij2} < \epsilon_{ij3} < \epsilon_{ij4} < a_{ij}$, $0 < \epsilon_{ij5} < \epsilon_{ij6} < \epsilon_{ij7} < \epsilon_{ij8}$, $\forall i, j$,

$$\tilde{b}_l = \left\langle \left(b_l - \omega_{l3}, b_l - \omega_{l4}, b_l + \omega_{l5}, b_l + \omega_{l6}; \lambda \right), \left(b_l - \omega_{l1}, b_l - \omega_{l2}, b_l + \omega_{l7}, b_l + \omega_{l8}; 1 \right) \right\rangle, \quad (16)$$

where $0 < \omega_{l1} < \omega_{l2} < \omega_{l3} < \omega_{l4} < b_l$, $0 < \omega_{l5} < \omega_{l6} < \omega_{l7} < \omega_{l8}$, $\forall l$.

Considering Definitions 3 and 6 and Theorem 1, the optimization problem can be reformulated as:

$$\begin{aligned}
 & \min \sum_{k=1}^o w_k t_k, \\
 & \text{subject to :} \\
 & \frac{1}{8} \left[12 \sum_{i=1}^m \sum_{j=1}^n a_{ij} x_{ij} - 12b_l + \sum_{i=1}^m \sum_{j=1}^n (-4\epsilon_{ij1} - 2\epsilon_{ij2} - \epsilon_{ij3} - \epsilon_{ij4} + \epsilon_{ij5} + \epsilon_{ij6} + 2\epsilon_{ij7}) x_{ij} \right. \\
 & \quad + 4\omega_{l1} + 2\omega_{l2} + \omega_{l3} + \omega_{l4} - \omega_{l5} - \omega_{l6} - 2\omega_{l7} \\
 & \quad \left. + 3\lambda (\sum_{i=1}^m \sum_{j=1}^n (\epsilon_{ij1} - \epsilon_{ij2} + \epsilon_{ij7} - \epsilon_{ij8}) x_{ij} - \omega_{l1} + \omega_{l2} - \omega_{l7} + \omega_{l8}) \right] \geq 0, \forall l, \\
 & \frac{1}{8} \left[12 \sum_{i=1}^m \sum_{j=1}^n c_{ijk} x_{ij} + \sum_{i=1}^m \sum_{j=1}^n (-4\delta_{ijk1} - 2\delta_{ijk2} - \delta_{ijk3} - \delta_{ijk4} + \delta_{ijk5} + \delta_{ijk6} + 2\delta_{ijk7}) x_{ij} \right. \\
 & \quad \left. + 3\lambda (\sum_{i=1}^m \sum_{j=1}^n (\delta_{ijk1} - \delta_{ijk2} + \delta_{ijk7} - \delta_{ijk8}) x_{ij}) \right] \leq t_k, \forall k, \\
 & t_k, x_{ij} \geq 0, \forall i, j, k.
 \end{aligned} \tag{17}$$

This formulation captures the essence of interval-valued fuzzy multi-objective linear programming and provides a structured approach to handle uncertainty in both the objectives and constraints.

3. Ride-Hailing Matching with Interval-Valued Fuzzy Travel Time and Passenger–Driver Preferences

In the ride-hailing ecosystem, platforms face a complex optimization problem in which the main goal is to match passenger requests with available vehicles efficiently and fairly. However, this problem is complicated by several key factors. First, the uncertainty in waiting time is caused by unpredictable traffic conditions, varying driving speeds, and dynamic weather conditions. The waiting time becomes a variable that is difficult to determine deterministically, requiring a fuzzy-based approach to handle this uncertainty.

Second, differing price preferences between passengers and drivers add to the challenge of the matching process. Passengers often expect lower fares aligned with their budgets, while drivers seek compensation that reflects their time and effort. This misalignment, mainly when the fare preferences are expressed as uncertain intervals, requires a mechanism to balance fairness and efficiency in matching.

Third, limited vehicle resources in certain areas or during specific times may result in unfulfilled requests, reducing user trust in the platform. Platforms must optimize vehicle distribution to maximize the number of fulfilled requests and minimize the total waiting time.

Based on these problems, an optimization model is needed to consider operational efficiency and fairness in matching fares. By integrating a fuzzy-based approach to handle parameter uncertainty, this model aims to minimize the total waiting time, the number of unfulfilled requests, and the deviation in fare preferences between passengers and drivers. This solution is expected to significantly contribute to overcoming modern ride-hailing systems' practical challenges.

3.1. Limitations of the Model

This section highlights the key limitations of the proposed model to provide context for its implementation and help define the boundaries within which it can be effectively applied.

1. **Uncertainty in Waiting Time:** One of the inherent limitations of this model is the handling of uncertainty in the waiting time, which is affected by factors like traffic conditions, varying driving speeds, and weather. These conditions make the waiting time challenging to determine deterministically. Consequently, waiting times are considered fuzzy parameters that account for this inherent uncertainty.
2. **Limited Vehicle Availability:** This model is constrained by the assumption that each vehicle can only serve one request at a time. This limitation directly impacts

the availability of vehicles and restricts the number of requests that can be fulfilled, mainly when vehicle resources are limited in specific regions or periods.

3. **Unfulfilled Requests:** In many instances, requests cannot be fulfilled due to geographic limitations or the unavailability of vehicles. As a result, this model includes a mechanism to account for unfulfilled requests, but such instances can negatively affect a platform's reputation and user trust.
4. **Mismatch in Fare Expectations:** A significant challenge is the mismatch in fare expectations between passengers and drivers. Passengers generally expect lower fares that fit their budgets, while drivers seek adequate compensation reflecting their time and effort. This discrepancy, mainly when expressed as interval-based uncertain preferences, adds complexity to the fare-matching mechanism.
5. **Fairness in Fare Alignment:** This model also aims to manage the deviations in fare expectations. These deviations, represented as separate variables for underachievement and overachievement, reflect discrepancies between the agreed-upon fare and the expected fare for both parties. While it aims to minimize such deviations, the model acknowledges that a perfect match may not always be possible.
6. **Operational Efficiency:** Vehicles can serve only one request at a time, ensuring that the allocation process remains efficient and conflict-free. This constraint is necessary to maintain operational efficiency, although it reduces the flexibility in meeting multiple simultaneous passenger requests.

3.2. Assumptions of the Model

The following assumptions underlie the development of the proposed model, establishing conditions and boundaries for its application.

1. **Fuzzy Representation of Waiting Time:** It is assumed that the waiting time between passenger requests and vehicle arrivals is fuzzy, reflecting real-world waiting time's variable and unpredictable nature. This assumption is central to modeling uncertainty and handling the stochastic aspects of waiting time.
2. **Fare Preferences:** This model assumes that passengers have a budgetary constraint that drives their fare expectations, and drivers expect a fare that compensates them for their time and effort. Fare preferences are represented as intervals to accommodate the inherent uncertainty and potential variations in individual expectations.
3. **Limited Vehicle Resources:** This model assumes limited vehicle availability, especially during peak hours or in higher-demand areas. This limitation constrains the system's ability to fulfill all requests, making resource optimization crucial to system performance.
4. **Single Request Per Vehicle:** Each vehicle is assumed to be able to serve one request at a time, preventing conflicts in vehicle allocation. This assumption simplifies the allocation process while ensuring that each vehicle is assigned optimally, contributing to operational efficiency.
5. **Dynamic Matching Process:** This model assumes that the demand for rides and vehicle availability fluctuate dynamically based on external factors such as the time of day, location, and weather. Consequently, the matching process is adaptable as vehicles adjust to varying conditions and requests.
6. **Implementation of Big M Method:** The Big M method is utilized within the optimization process to handle decision variables effectively. This approach ensures that complex constraints are met, such as ensuring a request is fulfilled or unfulfilled, without introducing conflicts or contradictions.
7. **Operational Efficiency and Feasibility:** This model assumes that operational efficiency can be maximized by assigning one vehicle to one request at a time. This

ensures the system remains efficient and can optimize the number of fulfilled requests while minimizing the waiting time.

3.3. Notation List

Set

I : Set of passenger ride requests.

J : Set of available vehicles.

Parameters

$\tilde{t}_{ij}$

$\tilde{t}_{ij}$: Fuzzy waiting time between request i and vehicle j .

$\tilde{c}_{ij}^p$

$\tilde{c}_{ij}^p$: Fare expected by the passenger for request i , served by vehicle j .

$\tilde{c}_{ij}^d$

$\tilde{c}_{ij}^d$: Fare expected by the driver for request i , served by vehicle j .

Decision Variables

x_{ij} : Binary variable, 1 if vehicle j is assigned to request i , 0 otherwise.

y_i : Binary variable, 1 if request i is unfulfilled, 0 otherwise.

d_{ij}^- : Deviation variable for underachievement in fare alignment for request i and vehicle j .

d_{ij}^+ : Deviation variable for overachievement in fare alignment for request i and vehicle j .

The waiting time is one of the leading indicators of user experience in ride-hailing services. Passengers expect vehicles to arrive as quickly as possible after they make a request. However, waiting times can vary due to the vehicle's distance from the passenger, traffic conditions, and the driver's speed. To reflect the uncertainty in waiting time, we use the fuzzy parameter $\tilde{t}_{ij}$, which represents the waiting time between request i and vehicle j . The model aims to minimize the total waiting time for all pairs, expressed as:

$$\min \sum_{i \in I} \sum_{j \in J} \tilde{t}_{ij} x_{ij}, \quad (18)$$

Not all requests can be fulfilled, especially when vehicles are limited or passengers are in hard-to-reach locations. Unfulfilled requests harm the platform's reputation and reduce user trust. The model uses the variable y_i , which equals 1 if request i is not fulfilled. By minimizing y_i , the platform ensures more requests are fulfilled, increasing service reliability:

$$\min \sum_{i \in I} y_i, \quad (19)$$

Passengers have fare expectations aligned with their budgets, while drivers expect fares that reflect their time and effort. The mismatch between these preferences can lead to dissatisfaction or even failed matches. The deviation between the passenger fare $\tilde{c}_{ij}^p$ and the driver fare $\tilde{c}_{ij}^d$ is represented by two variables:

- d_{ij}^- : Underachievement in fare alignment.
- d_{ij}^+ : Overachievement in fare alignment.

This model aims to minimize the total deviation, maintaining a fare balance as follows:

$$\min \sum_{i \in I} \sum_{j \in J} d_{ij}^- + d_{ij}^+, \quad (20)$$

The platform must ensure that the proposed fares are acceptable for passengers and drivers. Fare deviation must be measured and managed to keep the matching results fair.

Fare deviation is regulated through the relationship between passenger fares, driver fares, and deviation variables as follows:

$$\tilde{c}_{ij}^p x_{ij} + d_{ij}^- - d_{ij}^+ = \tilde{c}_{ij}^d x_{ij}, \forall i \in I, \forall j \in J \quad (21)$$

Each vehicle can only serve one request at a time. This prevents allocation conflicts and ensures operational efficiency. The model limits the number of requests that can be assigned to vehicle j as follows:

$$\sum_{i \in I} x_{ij} \leq 1, \forall j \in J, \quad (22)$$

Each request must be processed by matching it with one vehicle or marking it as unfulfilled. This constraint ensures that every request i has a clear outcome as follows:

$$\sum_{j \in J} x_{ij} = 1 - y_i, \forall i \in I, \quad (23)$$

These constraints ensure that decision variables remain within their appropriate domains as follows:

$$x_{ij}, y_i \in \{0, 1\}, \forall i \in I, \forall j \in J. \quad (24)$$

$$d_{ij}^-, d_{ij}^+ \geq 0, \forall i \in I, \forall j \in J.$$

Using the 1, M , M weighted multi-objective function and with similar steps as described before, we have ride-hailing matching with an uncertain travel time in the fuzzy sense as follows:

$$\begin{aligned} & \min \frac{1}{8} \left[12 \sum_{i \in I} \sum_{j \in J} t_{ij} x_{ij} + \sum_{i \in I} \sum_{j \in J} \left(-4\delta_{ij1} - 2\delta_{ij2} - \delta_{ij3} - \delta_{ij4} + \delta_{ij5} + \delta_{ij6} + 2\delta_{ij7} \right) x_{ij} \right. \\ & \quad \left. + 3\lambda \sum_{i \in I} \sum_{j \in J} \left(\delta_{ij1} - \delta_{ij2} + \delta_{ij7} - \delta_{ij8} \right) x_{ij} \right] + M \sum_{i \in I} y_i + M \sum_{i \in I} \sum_{j \in J} \left(d_{ij}^- + d_{ij}^+ \right), \\ & \text{subject to :} \\ & \frac{1}{8} \left(12c_{ij}^p - 4\omega_{ijk1}^p - 2\omega_{ijk2}^p - \omega_{ijk3}^p - \omega_{ijk4}^p + \omega_{ijk5}^p + \omega_{ijk6}^p + 2\omega_{ijk7}^p + 3\lambda(\omega_{ijk1}^p - \omega_{ijk2}^p \right. \\ & \quad \left. + \omega_{ijk7}^p - \omega_{ijk8}^p) \right) x_{ij} + d_{ij}^- - d_{ij}^+ = \frac{1}{8} \left(12c_{ij}^d - 4\omega_{ijk1}^d - 2\omega_{ijk2}^d - \omega_{ijk3}^d - \omega_{ijk4}^d \right. \\ & \quad \left. + \omega_{ijk5}^d + \omega_{ijk6}^d + 2\omega_{ijk7}^d + 3\lambda(\omega_{ijk1}^d - \omega_{ijk2}^d + \omega_{ijk7}^d - \omega_{ijk8}^d) \right) x_{ij}, \forall i \in I, \forall j \in J, \\ & \sum_{i \in I} x_{ij} \leq 1, \forall j \in J, \\ & \sum_{j \in J} x_{ij} = 1 - y_i, \forall i \in I, \\ & x_{ij}, y_i \in \{0, 1\}, \forall i \in I, \forall j \in J. \\ & d_{ij}^-, d_{ij}^+ \geq 0, \forall i \in I, \forall j \in J. \end{aligned} \quad (25)$$

where M is a big number (big M).

The algorithm for this method can be seen in Algorithm 1.

Algorithm 1. Fuzzy-Based Vehicle Assignment for Ride-Hailing

1: Input:

- 2: Taxi trip dataset (Manhattan, January 15, 2024).
- 3: Number of vehicles (1000), Number of requests.
- 4: Fuzzy travel time estimates and fare preferences.
- 5: Maximum waiting time = 5 minutes.

6: Step 1: Data Preparation

- Load dataset of requests with timestamps, pickup, and drop-off locations.
 - Calculate travel times between requests using a traffic simulation (e.g., using OSMnx or similar tools).
-

Algorithm 1. *Cont.*

- Generate fuzzy travel times and fuzzy fare expectations for both passengers and drivers (as fuzzy trapezoidal numbers).
 - 7: **Step 2: Fuzzy Waiting Time and Fare Calculation**
 - Travel time are modeled as fuzzy trapezoidal numbers based on vehicle location and congestion level.
 - Fare preferences are modeled as fuzzy trapezoidal numbers.
 - 8: **Step 3: Optimization Using proposed model.**
 - 9: **Output:** List of assignments, total waiting time for all fulfilled requests, and deviations from expected fares.
-

4. Numerical Experiments

4.1. Case Study

To evaluate the performance of the proposed model, we conducted experiments using a publicly available dataset of taxi trips in Manhattan (<https://www.nyc.gov/site/tlc/about/tlc-trip-record-data.page>, accessed on 15 April 2024). This dataset includes detailed trip information such as pick-up and drop-off locations and request timestamps. For this study, we utilized data from the ride requests recorded at noon on 15 January 2024, focusing exclusively on trips within Manhattan. Travel distances and times between locations were estimated using the OSMnx library, as described in [39].

The experimental setup was designed to mimic a real-world ride-hailing scenario. A total of 1000 drivers were randomly distributed across various locations in Manhattan, and the maximum allowable waiting time was set to 5 min. Each driver could only accept a new request after completing their current trip. The total number of vehicles remained constant throughout the simulation, with no additions or removals. Drivers were allowed to continuously pick up and drop off passengers based on the optimization decisions made by the model. Requests were grouped into 0.5 min time-window batches, and unfulfilled requests were carried over to the next batch for reconsideration. Any request that exceeded the 5 min waiting time limit was removed from the queue. The distribution of the number of requests during the simulation is illustrated in Figure 3.

Travel times were treated as uncertain due to the dynamic nature of traffic conditions. As OSMnx does not provide explicit error margins in its travel time estimates, we generated multiple travel time scenarios to account for this uncertainty. These scenarios included:

1. Travel times are based on free-flow conditions (OSMnx-calculated values).
2. Travel times at congestion levels of 50%, 60%, 70%, 80%, and 90%.
3. Travel times are derived from constant vehicle speeds of 20, 30, and 40 km/h over the shortest distance.

The average of these nine scenarios was used to estimate the travel times, and these travel times, stored in seconds, directly influenced the waiting times, which were modeled as interval-valued fuzzy numbers to capture the uncertainty. An example of the fuzzy representation for waiting time uncertainty is shown in Figure 4, where ϵ represents the residual error from travel time forecasting.

In addition to uncertainty regarding the waiting time, the figure also accounted for uncertainty in user and driver fare preferences. Fares were represented using interval-valued fuzzy trapezoidal numbers, with the fare preferences randomly generated within a range of 1.00 USD to 2.00 USD per kilometer. This approach ensured that variability in fare expectations from both passengers and drivers was adequately captured without compromising the quality of the numerical experiments.

The key parameters for the case study are summarized in Table 1 to provide clarity regarding the simulation settings.

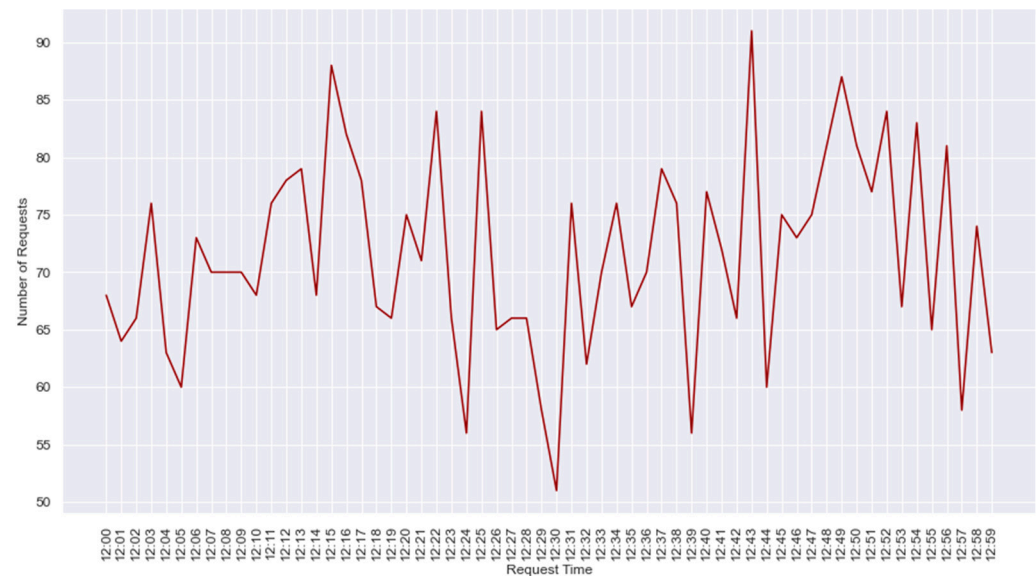


Figure 3. Number of requests [11].

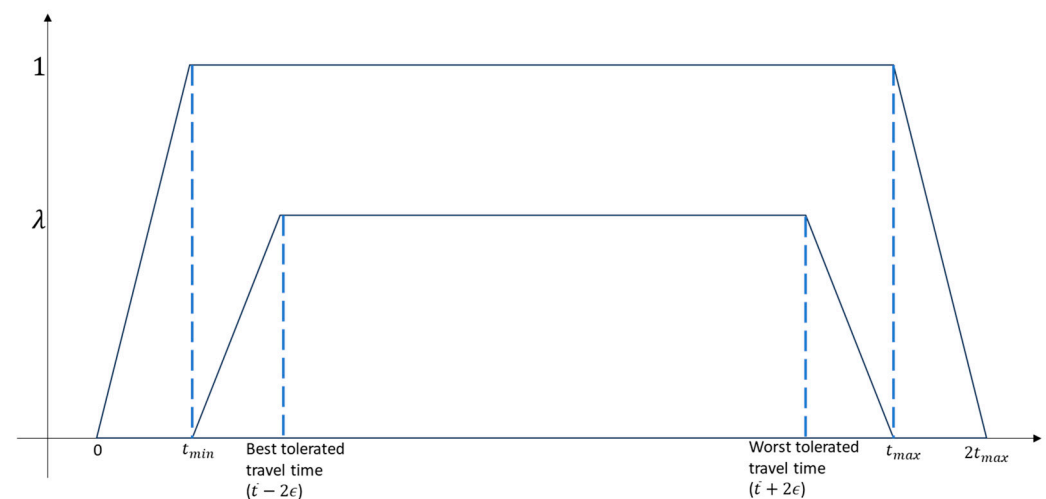


Figure 4. The interval-valued fuzzy trapezoidal number for waiting time uncertainty [11].

Table 1. Parameter settings for ride-hailing matching problem [11].

Parameter	Value	Parameter	Values
Number of passengers per request	1 request	λ	0.8
Number of vehicles	1000 vehicles	M	99,999
Maximum tolerable waiting time	5 min	ϵ	Forecasting errors
Time window for batching	0.5 min	$\bar{t}$	Forecasted waiting time
Number of batches	20 batch	t_{min}, t_{max}	Minimum and maximum waiting Times based on nine scenarios

The proposed method is designed to work effectively on a wide range of parameter sets, including those listed in Table 1. Thus, the process is effective for the tested parameters and can be applied reliably under a wide range of different conditions, making it flexible and reliable in dealing with a wide range of scenarios.

This experimental framework provides a robust basis for analyzing the performance of the proposed model under realistic conditions, capturing the inherent uncertainties in travel times and fare preferences to better reflect the dynamics of ride-hailing systems.

4.2. Experimental Results

In this section, we present the numerical simulation results, comparing the deterministic and interval-valued fuzzy linear programming models to evaluate their performance in addressing the uncertainties in ride-hailing systems. The waiting time, a critical indicator of user experience, is influenced by the inherent uncertainty in pick-up travel times, which vary due to factors such as traffic conditions and vehicle availability. For the deterministic model, the best-case travel time is used as a parameter for calculation; however, the resulting solutions may lose feasibility in real-world scenarios as they fail to account for dynamic conditions like traffic fluctuations. Additionally, the analysis incorporates uncertainty in user and driver fare preferences, as passengers typically prefer fares that align with their budgets. In contrast, drivers expect fares that fairly compensate for their time and effort. These conflicting preferences can impact the matching process if not managed effectively. The interval-valued fuzzy model addresses these challenges by introducing interval-valued parameters for both waiting time and fare preferences, creating a more robust and adaptable solution compared to the deterministic approach. This enables the model to accommodate variability better, ensuring fairness and feasibility in the allocation of vehicles to ride requests.

The average waiting time for the simulation results using the deterministic model is 1.94 min, while for the interval-valued fuzzy model, the average waiting time is slightly longer at 1.95 min. Although the difference is slight, the longest waiting time for the deterministic model is recorded at 14.77 min, whereas for the interval-valued fuzzy model, the longest is 15.14 min. Furthermore, the variance in waiting time for the deterministic model is 7.50, slightly lower than the variance for the interval-valued fuzzy model, which is 7.62. This indicates that although the difference in the average waiting time between the two models is insignificant, the interval-valued fuzzy model tends to exhibit a slightly higher level of uncertainty in the required waiting times. It can be seen in Figure 5.

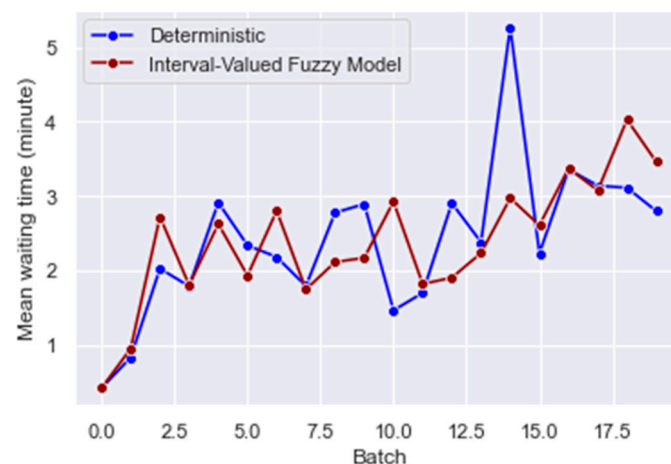


Figure 5. Mean waiting time [11].

Figure 5 illustrates a comparative analysis of the mean waiting times between the deterministic model and the interval-valued fuzzy model across multiple batches. The deterministic model demonstrates significant fluctuations, with noticeable peaks indicating longer waiting times in specific batches. Conversely, the interval-valued fuzzy model shows a more stable and consistent performance, maintaining lower mean waiting times in the

majority of cases. This outcome highlights the robustness of the fuzzy model in managing uncertainties inherent in ride-hailing systems, particularly under varying conditions such as traffic and vehicle availability. The reduced variability and lower mean waiting times achieved by the interval-valued fuzzy model underscore its potential to enhance user experience by minimizing delays and improving service reliability.

The total number of requests in the simulation results using the deterministic model is 823, while the interval-valued fuzzy model serves 829 requests. The average number of requests served per batch for the deterministic model is 41.15, while for the interval-valued fuzzy model, it is 41.45. Additionally, the variance in the number of requests served per batch for the deterministic model is 997.29, slightly higher than the 966.79 variances observed for the interval-valued fuzzy model. This indicates that although the deterministic model serves a slightly smaller total number of requests, it demonstrates a slightly higher variability in the number of requests served per batch compared to the interval-valued fuzzy model. It can be seen in Figure 6.

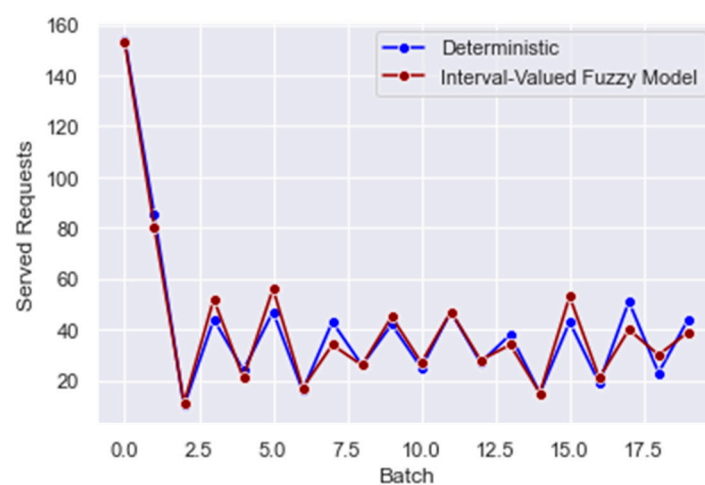


Figure 6. Service level [11].

Figure 6 compares the performance of the deterministic model and the interval-valued fuzzy model in terms of the number of served requests across multiple batches. Initially, both models exhibit a sharp drop in the number of served requests, likely reflecting a transition or adjustment phase in the system. As the batches progress, the interval-valued fuzzy model consistently performs slightly better or equally as well as the deterministic model in serving requests, particularly in scenarios where variability and uncertainty are more pronounced.

The fuzzy model's ability to account for uncertainties in factors such as waiting times and fare preferences allows it to allocate vehicles more effectively, leading to a higher or comparable number of fulfilled ride requests. The deterministic model, on the other hand, shows more fluctuations and slightly less stability in performance. Overall, this suggests that the interval-valued fuzzy model provides a more reliable approach for managing dynamic ride-hailing environments, ensuring a higher level of service reliability despite uncertainties.

The total number of jobs obtained by driver-partners for the simulation results using the deterministic model is 823, while the number of jobs obtained using the interval-valued fuzzy model is 829. The average number of jobs obtained per batch for the deterministic model is 0.823, while for the interval-valued fuzzy model, it is 0.829. Additionally, the variance in the number of jobs obtained per batch for the deterministic model is 0.584, slightly higher than the 0.544 variance observed for the interval-valued fuzzy model. Although the deterministic model results in a slightly smaller total number of jobs, it

exhibits a slightly higher variability in the number of jobs obtained per batch compared to the interval-valued fuzzy model. It can be seen in Figure 7.

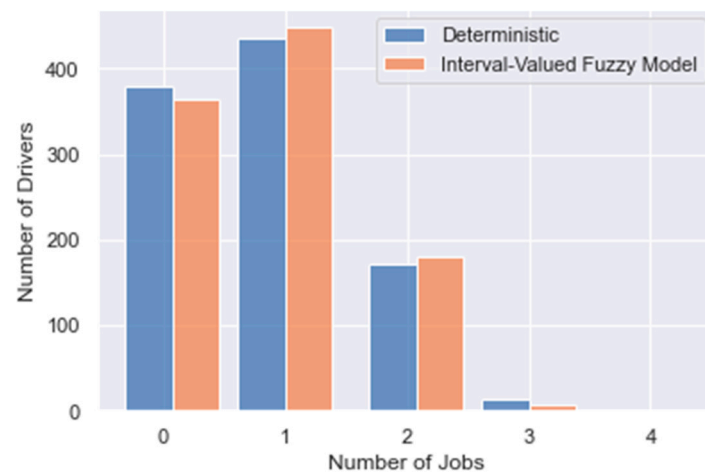


Figure 7. Distribution of the number of driver jobs [11].

Figure 7 compares the distribution of the number of jobs completed by drivers using the deterministic model and the interval-valued fuzzy model. The majority of drivers in both models are assigned precisely one job, indicating an effort to distribute the workload evenly among the available drivers. However, the interval-valued fuzzy model shows a slightly higher count of drivers assigned one job compared to the deterministic model, reflecting its ability to optimize assignments under uncertain conditions.

For drivers with no assigned jobs, the deterministic model has a marginally higher count, suggesting a less efficient utilization of driver resources. Conversely, for drivers completing two jobs, the deterministic model slightly outperforms the fuzzy model. Very few drivers are assigned three or more jobs in either model, which aligns with the goal of balancing assignments to prevent overburdening individual drivers.

Overall, the interval-valued fuzzy model demonstrates improved fairness and utilization in job distribution while effectively managing uncertainties in the system. This balanced approach ensures both driver satisfaction and operational efficiency.

The numerical results demonstrate that the interval-valued fuzzy model consistently outperforms the deterministic model in managing uncertainties in ride-hailing systems. In terms of the mean waiting time, the fuzzy model delivers more stable and lower results, showcasing its efficiency in addressing uncertainties such as traffic conditions and vehicle proximity to passengers. Additionally, the number of served requests is more consistent with the fuzzy model, reflecting its superior ability to maximize vehicle utilization and reliably meet user demand. Regarding driver job distribution, the fuzzy model achieves a more balanced allocation of jobs, with fewer drivers left without assignments, enhancing fairness and operational efficiency. Overall, these findings highlight that the interval-valued fuzzy model is a more robust and adaptive approach compared to the deterministic model, particularly in handling the complexities and dynamic uncertainties inherent in ride-hailing systems.

5. Discussions

This study introduces a fuzzy interval-valued model to address uncertainties in waiting times and pricing preferences in ride-hailing systems. The proposed approach offers significant advantages over traditional deterministic models, particularly in managing dynamic and unpredictable factors. The numerical results demonstrate that the fuzzy model achieves superior operational efficiency, reflected in reduced and more stable aver-

age waiting times across various scenarios. This improvement enhances user experience by minimizing delays and increasing the reliability of ride-hailing services.

Another key advantage of the fuzzy model lies in its ability to balance request fulfillment rates and fairness in driver assignment. By effectively minimizing the mismatch between passenger and driver price expectations, the model ensures more equitable outcomes for both parties. Additionally, the model distributes jobs more evenly among drivers, reducing idle times and overburdening, which contributes to a more sustainable and fair operational environment.

The global relevance of this approach is evident in its adaptability to dynamic urban ecosystems, where traffic variability and diverse user preferences are common challenges. The fuzzy model's integration into ride-hailing platforms could advance matching algorithms, enabling real-time, adaptive decision-making. With further enhancements through AI and IoT technologies, the model could improve scalability and responsiveness, making it a valuable tool for modern transportation systems. Furthermore, the findings offer insights for policymakers to design equitable pricing regulations that account for uncertainty and promote fairness among stakeholders.

Despite its advantages, the model requires higher computational resources compared to deterministic approaches, presenting challenges for large-scale implementation. Future research should focus on optimizing the computational efficiency of the fuzzy model to enable broader adoption. Additionally, expanding the scope of the model to incorporate non-monetary factors, such as comfort and safety, would provide a more comprehensive framework for decision-making. Validation in diverse regional contexts with varying cultural, economic, and traffic conditions is also essential to assess the model's generalizability.

The interval-valued fuzzy (IVF) approach in linear programming has several limitations that must be considered. Computationally, this approach is more complex than conventional linear programming because each variable is represented as an interval, requiring more operations. Additionally, determining accurate intervals depends on sufficient data or expert opinion, which can affect objectivity. The IVF solution, being interval-based, can also introduce ambiguity in interpretation and decrease confidence if the uncertainty is too broad. IVF may not always produce optimal solutions because it tends to offer a conservative, compromise solution. Therefore, while it is helpful in highly uncertain situations, this approach requires solid data and appropriate algorithms to generate effective solutions.

In this study, we observe that the fuzzy approach has been utilized in several previous works. For example, studies such as [13,22,40] have applied fuzzy models to address travel time uncertainty; however, they have not incorporated the uncertainty of fare preferences, which is a crucial factor in ride-hailing systems. Similarly, the study in [11] developed a fuzzy interval-valued matching model but did not consider fare preference uncertainty. In contrast, the model proposed in this research brings novelty by effectively incorporating fare preference uncertainty, providing a more comprehensive and reliable solution for matching drivers and passengers in dynamic conditions. This contribution adds significant value to the existing body of knowledge by addressing a gap that previous studies overlooked.

In conclusion, the fuzzy interval-valued model significantly contributes to addressing the complexities of modern ride-hailing systems. By offering robust and adaptable solutions to uncertainty, the model sets a strong foundation for enhancing efficiency, fairness, and user satisfaction while also opening avenues for further research and practical applications.

6. Conclusions

This study addresses the challenges of uncertainty in ride-hailing systems by employing a fuzzy interval-valued approach to optimize the matching process between passengers and drivers. The proposed model effectively incorporates uncertainties into waiting times

and pricing preferences, which are critical factors influencing user satisfaction and operational efficiency. The numerical simulations demonstrate that the fuzzy model outperforms traditional deterministic approaches by achieving lower and more stable waiting times, higher request fulfillment rates, and more equitable job distribution among drivers. The findings highlight the model's robustness in managing dynamic conditions, such as traffic variability and mismatched pricing expectations, offering a practical and adaptive solution for modern ride-hailing platforms. Furthermore, the integration of fuzzy interval-valued parameters allows for a more realistic representation of real-world uncertainties, enhancing the relevance and applicability of the model to urban transportation ecosystems globally. While this study establishes a strong foundation for improving ride-hailing operations, certain limitations remain. The computational demands of the fuzzy model may hinder its scalability, and additional factors, such as safety and environmental considerations, could be incorporated to broaden its scope. Future research should explore these areas and validate the model's performance across diverse regional settings. In summary, the fuzzy interval-valued approach provides a significant contribution to the optimization of ride-hailing systems by addressing inherent uncertainties. This research not only advances theoretical understanding but also offers practical insights for enhancing the fairness, efficiency, and reliability of ride-hailing services, with potential implications for broader applications in dynamic and uncertain operational contexts.

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