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BER-Constrained Power Allocation for Uplink NOMA Systems with One-Bit ADCs

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Abstract

This study investigates bit error rate (BER)-constrained power allocation for uplink non-orthogonal multiple access (NOMA) systems in which a base station employs one-bit analog-to-digital converters. Although one-bit quantization significantly reduces hardware costs and receiver power consumption, it also introduces severe nonlinear distortions that degrade detection performance. To address this challenge, a pairwise error probability expression is first derived for the one-bit quantized uplink NOMA model, from which an analytical upper bound on the BER is obtained. Based on this characterization, a fairness-driven max–min power allocation strategy is formulated to minimize the BER of the worst-performing user. A closed-form solution for the optimal power allocation is obtained under binary phase-shift keying (BPSK) signaling. Simulation results verify the tightness of the analytical BER bound and demonstrate that the proposed power allocation scheme provides noticeable BER improvements that compensate for the performance degradation caused by one-bit quantization.

Keywords: non-orthogonal multiple access; joint maximum likelihood; bit error rate; one-bit ADCs; power allocation

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1. Introduction

The rapid growth of wireless devices and data-intensive applications has led to a substantial increase in traffic demand, which has exposed the limitations of traditional multiple-access techniques. Although orthogonal multiple access offers a simple and reliable framework by allocating disjoint time–frequency resources to individual users to prevent mutual interference, it inherently limits spectral efficiency because only one user can occupy a resource element at any given time. To address this inefficiency, non-orthogonal multiple access (NOMA) has been introduced as a more flexible strategy that enables multiple users to simultaneously share the same radio resources [1–3]. By employing superposition coding at the transmitter and multiuser detection at the receiver, NOMA significantly enhances spectral efficiency and operates closer to the capacity boundary of the multiple-access channel.

Among the various NOMA architectures, power-domain NOMA (PD-NOMA) has attracted considerable attention because of its straightforward implementation and compatibility with existing wireless infrastructures [4–6]. In PD-NOMA, users are multiplexed by allocating different transmit power levels depending on their channel qualities, and the receiver employs successive interference cancellation (SIC) to decode the superimposed

signals [6]. Power allocation plays a pivotal role in balancing user fairness and enhancing overall system performance because the assigned power levels determine the relative strength of each user's signal. Accordingly, several studies have explored joint optimization frameworks that integrate user clustering, beamforming, and power allocation to improve NOMA performance [7–9]. However, many of these approaches are formulated based on capacity or signal-to-interference-plus-noise ratio metrics, which often limits their applicability in practical deployments.

Although power allocation for NOMA has been studied extensively, relatively few works have examined the problem from the perspective of the bit-error-rate (BER). Most previous studies on power allocation that have considered BER also assumed SIC detection. Although SIC is attractive due to its low complexity, its performance can degrade significantly under strong interference or when received user powers are insufficiently separated. Therefore, joint maximum-likelihood (JML) detection has been explored as an optimal alternative for uplink NOMA, and its BER behavior has been characterized using pairwise error probability (PEP) analysis [10,11]. A prior BER study for two-user NOMA was reported in [10]; however, its effect on power allocation was shown to be minimal.

In parallel with these developments, low-resolution analog-to-digital converters (ADCs) have emerged as an effective approach to reduce hardware complexity and power consumption in high-speed wireless receivers [12–14]. Among these architectures, one-bit ADCs enable substantial savings in terms of circuit costs and energy usage by quantizing each received sample using a single bit. Despite these advantages, the extreme nonlinear distortion caused by one-bit quantization introduces considerable challenges in terms of performance characterization and the design of receivers. Consequently, prior research on NOMA with one-bit ADCs has concentrated primarily on achievable rate analysis, detector design, and information-theoretic performance limits [15–18]. For example, ref. [17] investigated a relay-assisted power allocation framework, whereas ref. [18] analyzed the capacity region of a two-user system equipped with one-bit ADCs and derived the corresponding optimal power allocation from an information-theoretic viewpoint. However, despite the critical importance of BER as a practical reliability metric, power allocation for NOMA systems based on the BER with one-bit ADCs has not been explored in the relevant literature.

In this study, a BER-constrained power allocation framework is proposed for uplink NOMA systems with one-bit ADCs. The analysis begins by deriving a BER expression based on a PEP-oriented approach tailored to the one-bit quantized NOMA signal model. Using this expression, a fairness-driven max–min power allocation strategy is formulated to minimize the worst-user BER. The derived BER bound enables closed-form solutions for the binary phase-shift keying (BPSK) case, which yields an analytically tractable and computationally efficient power allocation rule. The results of a theoretical analysis and a numerical simulation demonstrate that the proposed power allocation achieved substantial improvements in terms of BER across different modulation formats and coded systems, while effectively compensating for the performance degradation caused by the severe nonlinearity of one-bit quantization.

The remainder of this study is organized as follows. Section 2 describes the uplink NOMA system with one-bit ADCs and introduces the JML detection process. In Section 3, a BER analysis is presented based on the derived PEP. Section 5 provides simulation results that verify the accuracy of the analysis and the benefits of the proposed power allocation schemes. Finally, Section 6 concludes by summarizing the findings and suggesting some possible directions for future research.

Notation

The notations $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and Hermitian (conjugate transpose) of a matrix, respectively. The operators $\mathbb{P}(\cdot)$ and $\mathbb{E}(\cdot)$ represent the probability and the expectation of an event, respectively, and $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ refer to the real and imaginary parts of the complex number, respectively. The function $\log(\cdot)$ represents a natural logarithm. The symbols $|\cdot|$ and $\|\cdot\|_0$ represent the absolute value and zero norm (i.e., the number of nonzero elements), respectively. The indicator function $\mathbb{1}\{\mathcal{X}\}$ equals 1 if an event $\mathcal{X}$ is true and 0 otherwise. The sign function $\text{Sign}(x)$ is defined as 1 for $x \geq 1$ and -1 otherwise. The Q-function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du$. Finally, $\mathbb{R}$ and $\mathbb{C}$ represent sets of real and complex numbers, respectively.

2. System Model

In this section, the signal model for uplink NOMA system using one-bit ADCs is considered. Subsequently, the joint ML detection rule is introduced for the uplink NOMA system.

2.1. Signal Model

The uplink NOMA system under consideration consists of a base station (BS) equipped with N antennas and two single-antenna users, denoted as the cell-center user (c) and the cell-edge user (e) (Figure 1). The cell-center user is assumed to experience a higher average channel gain, whereas the cell-edge user suffers from a lower average channel gain due to the increased path loss. Accordingly, the received baseband signal vector at the BS can be expressed as

$$\bar{\mathbf{r}} = \sqrt{P_c d_c^{-\alpha}} \bar{\mathbf{g}}_c \bar{x}_c + \sqrt{P_e d_e^{-\alpha}} \bar{\mathbf{g}}_e \bar{x}_e + \bar{\mathbf{n}}, \quad (1)$$

where P_u ($u \in \{c, e\}$) and d_u denote the transmit power and the distance from the BS of user u , respectively. The parameter α represents the path-loss exponent, and $\bar{\mathbf{g}}_u \in \mathbb{C}^{N \times 1}$ denotes the wireless channel vector from user u to the BS. Each element of $\bar{\mathbf{g}}_u$ is modeled as an independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance, i.e., $\bar{\mathbf{g}}_u \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$. The additive white Gaussian noise (AWGN) vector is denoted by $\bar{\mathbf{n}} \in \mathbb{C}^{N \times 1}$, whose elements are i.i.d. complex Gaussian random variables with zero mean and unit variance. The variable $\bar{x}_u \in \mathbb{C}$ denotes the modulation symbol transmitted by user u , satisfying $\mathbb{E}[|\bar{x}_u|^2] = 1$. For notational convenience, the effective channel vector from user u to the BS is defined as

$$\bar{\mathbf{h}}_u \triangleq \sqrt{P_u d_u^{-\alpha}} \bar{\mathbf{g}}_u \in \mathbb{C}^{N \times 1}. \quad (2)$$

To enable symbol detection under one-bit quantization, the complex-valued received signal in (1) is reformulated in an equivalent real-valued representation $\mathbf{r} = [\text{Re}\{\bar{\mathbf{r}}\}, \text{Im}\{\bar{\mathbf{r}}\}]^T$ as

$$\mathbf{r} = \sqrt{P_c d_c^{-\alpha}} \mathbf{g}_c x_c + \sqrt{P_e d_e^{-\alpha}} \mathbf{g}_e x_e + \mathbf{n}, \quad (3)$$

where

$$\mathbf{g}_u = \begin{bmatrix} \text{Re}\{\bar{\mathbf{g}}_u\} & -\text{Im}\{\bar{\mathbf{g}}_u\} \\ \text{Im}\{\bar{\mathbf{g}}_u\} & \text{Re}\{\bar{\mathbf{g}}_u\} \end{bmatrix}, \quad \mathbf{x}_u = \begin{bmatrix} \text{Re}\{\bar{x}_u\} \\ \text{Im}\{\bar{x}_u\} \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} \text{Re}\{\bar{\mathbf{n}}\} \\ \text{Im}\{\bar{\mathbf{n}}\} \end{bmatrix}, \quad (4)$$

and the effective channel matrix is defined as $\mathbf{h}_u \triangleq \sqrt{P_u d_u^{-\alpha}} \mathbf{g}_u \in \mathbb{R}^{2N \times 2}$. The real-valued symbol vector $\mathbf{x}_u \in \mathcal{X}^2$ is drawn from the underlying modulation alphabet $\mathcal{X}$, which corresponds to a $\sqrt{M}$ -ary pulse-amplitude modulation (PAM) constellation.

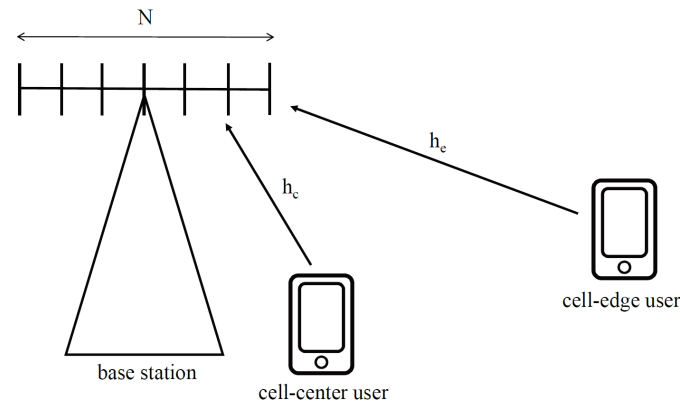


Figure 1. System model of uplink NOMA where the BS is equipped with multiple antennas and the two users each have a single antenna.

At the BS, each antenna output is passed through a pair of one-bit ADCs that separately quantize the real and imaginary components. This quantization imposes a hard-limiting operation by converting each entry of $\mathbf{r}$ into its sign. The resulting quantized observation corresponding to the i -th dimension is expressed as

$$q_i = \text{sign}(r_i) = \text{sign}(h_{c,i}^T \mathbf{x}_c + h_{e,i}^T \mathbf{x}_e + n_i), \quad (5)$$

where $i \in \mathcal{I} = \{1, \dots, 2N\}$ and $h_{u,i}^T$ denotes the i -th row of the matrix $\mathbf{h}_u$.

2.2. JML Detection

The base station performs symbol vector detection based on the quantized observation vector $\mathbf{q} = [q_1, \dots, q_{2N}]^T$, which is obtained from the hard-limiting operation described in (5). Given that each quantized element q_i depends on the transmitted symbols of both users and additive noise, the joint conditional probability of the quantized observation can be expressed as

$$\mathbb{P}\{\mathbf{q} | x_c, x_e\} = \prod_{i=1}^{2N} \mathbb{P}\{q_i | c_i(x_c, x_e)\}, \quad (6)$$

where $c_i(x_c, x_e) = \text{sign}(h_{c,i}^T x_c + h_{e,i}^T x_e)$ represents the noise-free quantized output corresponding to the transmitted symbol pair (x_c, x_e) .

The conditional probability of each quantized bit q_i depends on whether it matches the corresponding noise-free decision bit c_i . Specifically, when $q_i = c_i$, the quantizer output is consistent with the ideal decision; otherwise, an error occurs due to noise perturbation. Accordingly, the probability of observing q_i given (x_c, x_e) is expressed in [19] as

$$\mathbb{P}\{q_i | c_i(x_c, x_e)\} = \begin{cases} Q(\sqrt{2} |h_{c,i}^T x_c + h_{e,i}^T x_e|), & q_i \neq c_i(x_c, x_e), \\ 1 - Q(\sqrt{2} |h_{c,i}^T x_c + h_{e,i}^T x_e|), & q_i = c_i(x_c, x_e). \end{cases} \quad (7)$$

This expression reveals that the likelihood of a particular quantized observation depends exponentially on the inner product between the quantized channel vectors and transmitted symbols. Hence, the detection problem can be interpreted as minimizing the

weighted Hamming distance between the observed quantized vector $\mathbf{q}$ and its noise-free counterpart $\mathbf{c}(x_c, x_e)$, where the weights reflect the channel reliability of each quantization bit. Consequently, the JML detector can be formulated as

$$\{x_c^*, x_e^*\} = \underset{x_c, x_e \in \mathcal{A}}{\operatorname{argmax}} \mathbb{P}\{\mathbf{q} | x_c, x_e\}, \quad (8)$$

where $\mathcal{A}$ denotes the joint symbol set formed by the Cartesian product of user modulation alphabets.

3. Performance Analysis

In this section, the BER performance of the uplink NOMA system using one-bit ADCs at the receiver is analyzed. To this end, the PEP is first derived for the system model. Subsequently, using the derived PEP, an analytical upper bound on the BER is obtained to characterize system performance. For the special case of BPSK modulation, a closed-form expression of the BER is also derived, which provides valuable insights into the impact of one-bit quantization on detection accuracy.

3.1. PEP Analysis

Specifically, Jensen's inequality is used to handle the square-root function involved in the likelihood metric, whereas the Chernoff bound is applied to approximate the Gaussian Q -function. This combination of bounding techniques has been widely adopted in communication theory to derive tight analytical expressions for mutual information and bit error rates. By leveraging these tools, the PEP between the transmitted symbol pair $[x_c, x_e]$ and the erroneous decision $[\tilde{x}_c, \tilde{x}_e]$ can be expressed as

$$\mathcal{P}\{[x_c, x_e] \rightarrow [\tilde{x}_c, \tilde{x}_e]\} = \mathbb{E}[\mathbb{P}\{[x_c, x_e] \rightarrow [\tilde{x}_c, \tilde{x}_e] | \mathbf{h}_c, \mathbf{h}_e\}], \quad (9)$$

where the expectation is taken over all possible channel realizations of $\mathbf{h}_c$ and $\mathbf{h}_e$.

For a given channel realization, the conditional PEP is expressed as

$$\begin{aligned} & \mathbb{P}\{[x_c, x_e] \rightarrow [\tilde{x}_c, \tilde{x}_e] | \mathbf{h}_c, \mathbf{h}_e\} \\ &= \sum_{\mathbf{q} \in \{-1, 1\}^{2N}} \mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \mathbb{1}\{\mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \leq \mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}\}. \end{aligned} \quad (10)$$

The indicator function $\mathbb{1}\{\cdot\}$ can be upper bounded by the inequality $\mathbb{1}\{A \leq B\} \leq \left(\frac{B}{A}\right)^s$, where $s \in [0, 1]$. By setting $s = \frac{1}{2}$ to obtain a symmetric bound, the conditional PEP can be upper bounded as

$$\begin{aligned} \mathbb{P}\{[x_c, x_e] \rightarrow [\tilde{x}_c, \tilde{x}_e] | \mathbf{h}_c, \mathbf{h}_e\} &\leq \sum_{\mathbf{q} \in \{-1, 1\}^{2N}} \mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \left(\frac{\mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}}{\mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, x_c, x_e\}} \right)^{\frac{1}{2}} \\ &= \sum_{\mathbf{q} \in \{-1, 1\}^{2N}} \sqrt{\mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}}. \end{aligned} \quad (11)$$

The right-hand side of (11) represents a Bhattacharyya-type bound on the PEP, which provides a tight analytical upper bound on the detection error probability. This formulation will be further exploited to examine the asymptotic behavior of the PEP and to identify the achievable diversity order of the system under one-bit quantization constraints.

By applying Jensen's inequality to the expectation term in (11), a tractable upper bound can be obtained for the average pairwise error probability. The following lemma provides an analytical expression for the expectation over the channel realization, which is a key step toward the BER analysis.

Lemma 1. In uplink NOMA system with one-bit ADCs, the expectation of the Bhattacharyya-type expression in (11) is upper bounded as

$$\begin{aligned} & \mathcal{P}\{[x_c, x_e] \rightarrow [\tilde{x}_c, \tilde{x}_e]\} \\ & \leq \left(\left(1 - \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_c \|x_c + \tilde{x}_c\|^2 + \gamma_e \|x_e + \tilde{x}_e\|^2}{\gamma_c \|x_c - \tilde{x}_c\|^2 + \gamma_e \|x_e - \tilde{x}_e\|^2}} \right) \right) \right. \\ & \quad \times \left(\left(\sqrt{\frac{2}{2 + \gamma_c \|x_c\|^2 + \gamma_e \|x_e\|^2}} + \sqrt{\frac{2}{2 + \gamma_c \|\tilde{x}_c\|^2 + \gamma_e \|\tilde{x}_e\|^2}} \right) \right. \\ & \quad \left. \left. + \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_c \|x_c + \tilde{x}_c\|^2 + \gamma_e \|x_e + \tilde{x}_e\|^2}{\gamma_c \|x_c - \tilde{x}_c\|^2 + \gamma_e \|x_e - \tilde{x}_e\|^2}} \right) \right)^{2N} \right) \end{aligned} \quad (12)$$

where $\gamma_c = P_c d_c^{-\alpha}$, $\gamma_e = P_e d_e^{-\alpha}$ denote the average signal-to-noise (SNR) of the system, P is the transmit power, d is the user distance from the BS, and α represents the path-loss exponent.

Proof. See Appendix A. $\square$

The closed-form bound in Lemma 1 reveals how the transmit powers (P_c, P_e) , path losses (d_c, d_e) , and antenna count N jointly influence the PEP. This result provides a key analytical foundation for the subsequent derivation of the BER upper bound and the optimal power allocation strategy in the presence of one-bit quantization.

3.2. Bit Error Rate

Based on the derived PEP expression, the BER can be approximated or upper bounded by averaging over all possible symbol pairs in the modulation set. For the general case, the BER upper bound is expressed in [20] as

$$P_{err} = \frac{1}{|\mathcal{A}|} \frac{1}{\log |\mathcal{M}_\alpha|} \sum_{\mathbf{x} \in \mathcal{A}} \sum_{\tilde{\mathbf{x}} \in \mathcal{A}} \mathcal{P}\{\mathbf{x} \rightarrow \tilde{\mathbf{x}}\}, \quad (13)$$

where $\mathcal{A}$ denotes the modulation alphabet and $\mathcal{M}_\alpha$ represents the modulation order.

For a two-user uplink NOMA system, the BER can be similarly written in terms of joint symbol pairs as

$$P_{err} = \frac{1}{|\mathcal{A}|} \frac{1}{\log |\mathcal{M}_\alpha|} \sum_{x_c, x_e \in \mathcal{A}} \sum_{\tilde{x}_c, \tilde{x}_e \in \mathcal{A}} \mathcal{P}\{[x_c, x_e] \rightarrow [\tilde{x}_c, \tilde{x}_e]\}. \quad (14)$$

The PEP presented in Lemma 1 is used to obtain the BER expression in (14) for a NOMA system equipped with one-bit ADCs. Although the derivation is applicable to arbitrary modulation orders, higher-order constellations such as 4-quadrature amplitude modulation (QAM) are known to exhibit degraded BER performance under one-bit quantization [21].

In particular, as shown in [21], full diversity in MIMO systems with one-bit ADCs can be achieved only when the PEP symbol pairs is antipodal, i.e., $\mathbf{x} = -\tilde{\mathbf{x}}$. This antipodal condition is essential for attaining the maximum diversity order. However, in uplink NOMA systems with more than one user, the received signal can be interpreted as originating from multiple effective transmit dimensions due to signal superposition. As a result, non-antipodal symbol pairs naturally arise and tend to dominate the error events, thereby limiting the achievable diversity order.

This limitation becomes more pronounced as the modulation order increases, since higher-order constellations introduce a larger number of non-antipodal symbol pairs. Consequently, the diversity loss is further exacerbated in one-bit quantized NOMA systems employing higher-order modulations. For these reasons, the analysis in this study primarily

focuses on BPSK modulation, which provides the most favorable conditions for clearly highlighting the impact of power allocation.

3.3. Special Case: BPSK

For the special case of BPSK modulation, where $x_u \in \{+1, -1\}$, the BER can be derived in closed form for both the cell-center and cell-edge users. Using the upper bound of the PEP derived in Lemma 1, the BER expressions are obtained as follows.

For the cell-center user:

$$\begin{aligned} P_{err,c} &= \frac{1}{2} (\mathcal{P}\{[1, 1] \rightarrow [-1, -1]\} + \mathcal{P}\{[1, 1] \rightarrow [-1, 1]\}) \\ &\leq \frac{1}{2} \left(\left(\sqrt{\frac{8}{2 + \gamma_c + \gamma_e}} \right)^{2N} + \left(\left(1 - \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_e}{\gamma_c}} \right) \right) \right. \right. \\ &\quad \left. \left. \times \sqrt{\frac{8}{2 + \gamma_c + \gamma_e}} + \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_e}{\gamma_c}} \right) \right)^{2N} \right). \end{aligned} \quad (15)$$

Similarly, for the cell-edge user:

$$\begin{aligned} P_{err,e} &= \frac{1}{2} (\mathcal{P}\{[1, 1] \rightarrow [1, -1]\} + \mathcal{P}\{[1, 1] \rightarrow [-1, -1]\}) \\ &\leq \frac{1}{2} \left(\left(\sqrt{\frac{8}{2 + \gamma_c + \gamma_e}} \right)^{2N} + \left(\left(1 - \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_c}{\gamma_e}} \right) \right) \right. \right. \\ &\quad \left. \left. \times \sqrt{\frac{8}{2 + \gamma_c + \gamma_e}} + \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_c}{\gamma_e}} \right) \right)^{2N} \right). \end{aligned} \quad (16)$$

These results show that, for BPSK modulation, the BER upper bound can be expressed in closed form as a function of the SNR γ and the number of BS antennas N . When $\gamma \rightarrow \infty$, neither $P_{err,c}$ nor $P_{err,e}$ continues to decrease with increasing N , indicating that the achievable diversity order is fundamentally limited in NOMA systems with one-bit ADCs. This behavior can be interpreted from the fact that the received superimposed signal effectively corresponds to a 4-QAM constellation at the BS. As reported in [21], higher-order constellations under one-bit quantization cannot realize full diversity. Thus, the same limitation appears in the NOMA case. The result is therefore consistent with prior findings in the literature.

Although full diversity gain cannot be achieved with one-bit quantization, the coding gain still influences the overall error performance, allowing a moderate improvement in BER. Therefore, properly allocating the transmit powers of the two users in a NOMA system remains effective when the goal is to enhance the coding gain. The next section examines how power allocation can be optimized to maximize this coding gain.

4. Power Allocation

In this section, a power allocation strategy is developed for an uplink NOMA system that employs one-bit ADCs. Unlike systems with full-resolution receivers, the coarse quantization introduced by one-bit ADCs induces a strong nonlinear distortion, making the detection performance highly dependent on the received SNR of each user. As a result, transmit power allocation plays a crucial role in not only enhancing overall reliability but also balancing the error performance between users experiencing different channel conditions. Motivated by this observation, a fairness-oriented max-min optimization problem is formulated to minimize the worst-user BER subject to a total transmit power constraint.

4.1. Proposed Power Allocation

This subsection presents a power allocation strategy aimed at ensuring fairness between the two NOMA users. Because the users share the same radio resources, the transmit power allocated to one user directly affects the interference imposed on the other user. When one-bit ADCs are employed, this effect becomes even more pronounced due to the nonlinear quantization error. Therefore, a power allocation scheme that minimizes the maximum BER among the users can effectively balance the detection performance despite the differences in channel gains.

4.1.1. Problem Definition

The fairness-oriented power allocation is formulated as the following max–min BER minimization problem.

$$\begin{aligned} P_c^*, P_e^* &= \underset{P_c, P_e}{\operatorname{argmin}} \max \{P_{\text{err},c}, P_{\text{err},e}\} \\ \text{s.t. } & 0 \leq P_c + P_e < P_{\text{tot}}, \end{aligned} \quad (17)$$

where P_{tot} denotes the total available transmit power.

This problem seeks a power split that equalizes and minimizes the BER of both users. Such a formulation inherently encourages fairness because the objective prevents one user from dominating the power allocation at the cost of the other.

4.1.2. Solution

In this subsection, a closed-form solution to (17) is derived. To achieve this, following lemma is first introduced to investigate the behavior of the BER.

Lemma 2. For fixed constants $\gamma > 0$ and $N \in \mathbb{N}$, consider

$$\psi(\gamma_u) = \frac{1}{2} \left[A^{2N} + ((1 - \alpha(\gamma_u))A + \alpha(\gamma_u))^{2N} \right], \quad (18)$$

where

$$A = \sqrt{\frac{8}{2 + \gamma + \gamma_u}}, \quad \alpha(\gamma_u) = \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma}{\gamma_u}} \right). \quad (19)$$

Then $\psi(\gamma_u)$ is strictly decreasing with respect to γ_u for all γ_u satisfying $\gamma + \gamma_u > 6$.

Proof. See Appendix B. $\square$

Note that the condition $\gamma_c + \gamma_e > 6$ corresponds to $\gamma_c + \gamma_e > 7.78$ dB. This SNR range lies in a moderate operating regime, and therefore the condition is typically satisfied in practical scenarios. Under this assumption of sufficiently large SNR (i.e., $\gamma_c + \gamma_e > 6$), the BER upper bound $\psi(\gamma_u)$ becomes strictly decreasing in the received SNR of each user. Consequently, minimizing the BER in (17) is equivalent to maximizing the minimum received SNR:

$$\begin{aligned} P_c^*, P_e^* &= \underset{P_c, P_e}{\operatorname{argmax}} \min \{P_c d_c^{-\alpha}, P_e d_e^{-\alpha}\} \\ \text{s.t. } & 0 \leq P_c + P_e < P_{\text{tot}}. \end{aligned} \quad (20)$$

Intuitively, the max–min objective forces the received SNRs of the two users to be as close as possible. Thus, at the optimal point, the users must achieve the same SNR:

$$\frac{P_c d_c^{-\alpha}}{2} = \frac{P_e d_e^{-\alpha}}{2} = P_{\text{tot}}. \quad (21)$$

This leads to the closed-form fairness solution:

$$P_c^* = \frac{P_{\text{tot}}}{2d_c^{-\alpha}}, \quad P_e^* = \frac{P_{\text{tot}}}{2d_e^{-\alpha}}. \quad (22)$$

These results show that, to equalize the BERs, more power is assigned to users located farther from the BS. This behavior is consistent with NOMA principles but becomes especially important with one-bit quantization, where insufficient power leads to severe degradation. Moreover, since the proposed power allocation is expressed in closed form, it can be computed with a constant number of arithmetic operations and does not involve any iterative procedure.

5. Simulation Results

Numerical results were obtained through Monte Carlo simulations, averaging over 10^4 independent realizations of the channel matrices and additive noise. As in the system model, a Rayleigh fading channel is assumed between the BS and each user with $\alpha = 3$. Perfect channel state information is assumed at the receiver so that the accuracy of the analytical expressions and the influence of the power allocation strategies can be clearly evaluated. Both users employ BPSK modulation to highlight the PEP-based error behavior because BPSK detection becomes particularly challenging in NOMA systems with one-bit ADCs due to severe quantization distortion. Each user was equipped with a single transmit antenna, and the BS was equipped with $N = 8$ receive antennas. Each transmission frame contains 512 data symbols. The SNR range is selected so that a BER level of 10^{-3} can be observed for both the cell-center user and the cell-edge user. Unless otherwise specified, the distances are set to $(d_c, d_e) = (1, 2)$, representing typical cell-center and cell-edge users, respectively. When the distance d_e is varied, the corresponding value is explicitly indicated in the figure caption.

Figure 2 compares the analytical BER upper bounds in (15) and (16) with the simulation results. The analytical expressions provide well upper bounds over the entire SNR range, which confirms the accuracy of the proposed PEP-based derivation. This bound results from the Bhattacharyya-type expression and Jensen's inequality, which were used to obtain a tractable form in the analysis. Figure 3 shows the results when the cell-edge distance increased to $d_e = 4$, producing a more pronounced performance difference between the two users. In all settings, the proposed analysis continued to upper-bound the simulation results, which demonstrates that the analytical expressions provided reliable approximations for subsequent power allocation design.

Figure 4 shows the BER performance of conventional power allocation method for uplink NOMA with one-bit ADCs. For reference, an equal-power allocation method was included in which both users transmit with identical power. The proposed power allocation consistently outperformed equal-power allocation. To achieve a BER of 10^{-3} , the proposed method requires approximately 7 dB, whereas the equal-power method fails to reach this BER level. This improvement reflects the ability of the max–min formulation to balance the error rates of both users to minimize the worst-user BER and improve the overall reliability of the system.

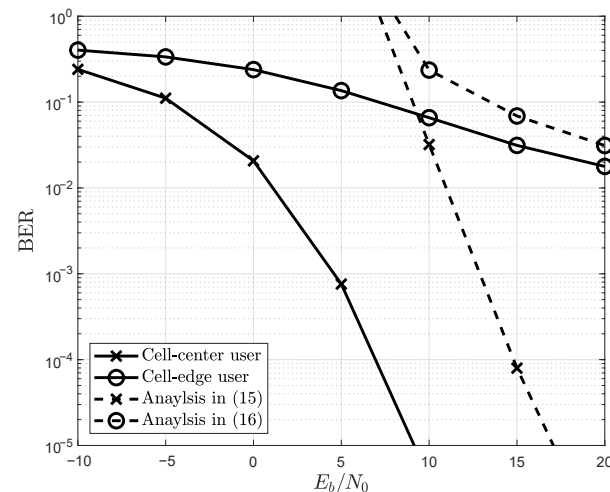


Figure 2. Verification of BER analysis results in (15) and (16) with $(d_c, d_e) = (1, 2)$ for NOMA systems using one-bit ADCs.

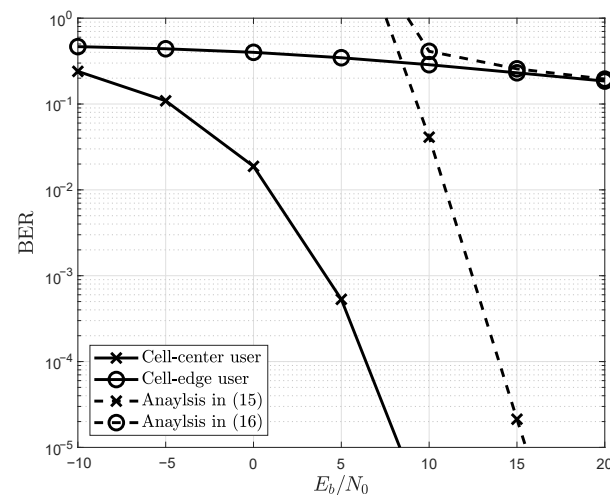


Figure 3. Verification of BER analysis results in (15) and (16) with $(d_c, d_e) = (1, 4)$ for NOMA systems using one-bit ADCs.

Figure 5 presents the average BER as a function of the cell-edge user distance d_e , while the cell-center user is fixed at unit distance. This result illustrates how the proposed power allocation manages system performance as the level of asymmetry between users becomes more pronounced. The BER was averaged over both users, to provide an overall measure of the performance of the NOMA system. The proposed method consistently achieved a lower average BER than the equal-power allocation, and the variation in BER across different distances remained small. This demonstrated that the proposed strategy provides enhanced robustness to user location and maintains stable performance even under highly imbalanced channel conditions.

Figure 6 shows the BER performance under QPSK modulation. Although Lemma 1 and (14) provide insight into the BER behavior, deriving an explicit BER expression for QPSK becomes considerably more involved due to the multi-dimensional constellation, making a direct analytical solution for the power allocation problem intractable. Nevertheless, Lemma 2 shows that the same monotonic property used in the BPSK analysis still holds for QPSK, implying that the proposed power allocation solution remains valid for higher-order modulations. As shown in the figure, the proposed scheme continued to

provide significant BER improvement over the equal-power allocation, which demonstrates its effectiveness regardless of the modulation order.

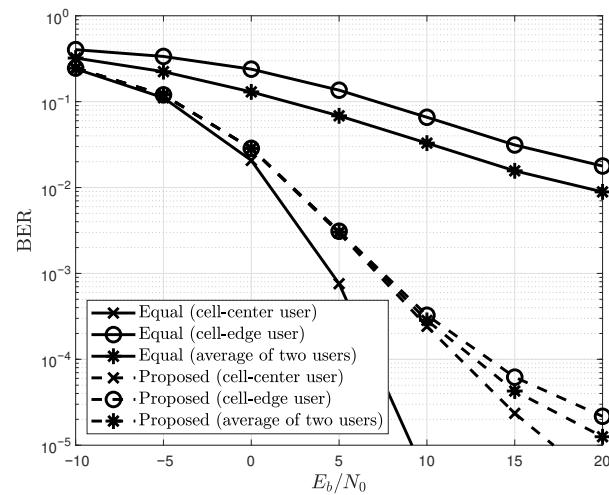


Figure 4. BER for different power allocations in NOMA system using one-bit ADCs.

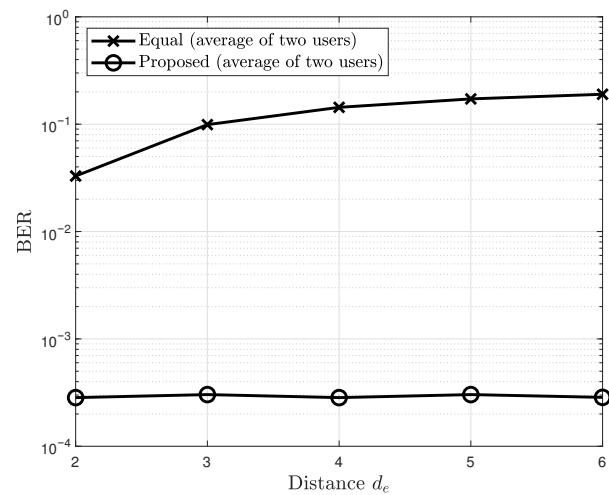


Figure 5. BER according to cell-edge distance d_e in NOMA system using one-bit ADCs.

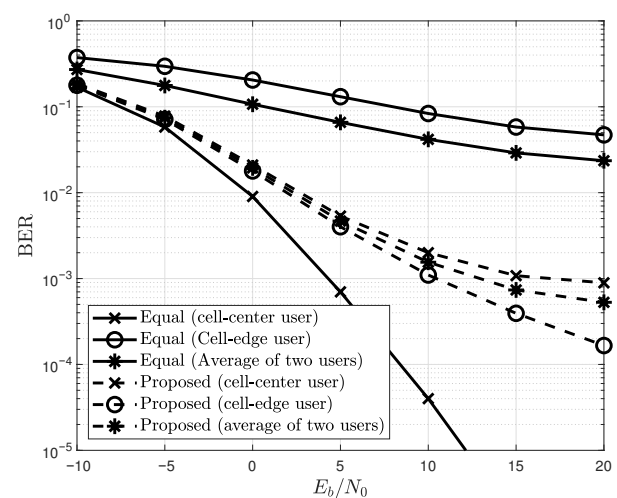


Figure 6. BER for QPSK modulation in NOMA system using one-bit ADCs.

Figure 7 shows the BER performance in a coded NOMA system. A rate- $\frac{1}{2}$ turbo code was applied and constructed with the conventional parallel concatenated convolutional encoder using the generator polynomials (15, 13) in octal notation. An exact analysis of the BER for coded systems with one-bit quantization is intractable; therefore, the power allocation result in (22) is applied directly. Even in the coded scenario, the proposed power allocation method achieved superior BER performance compared with equal-power allocation, which demonstrates its effectiveness in both coded and uncoded systems.

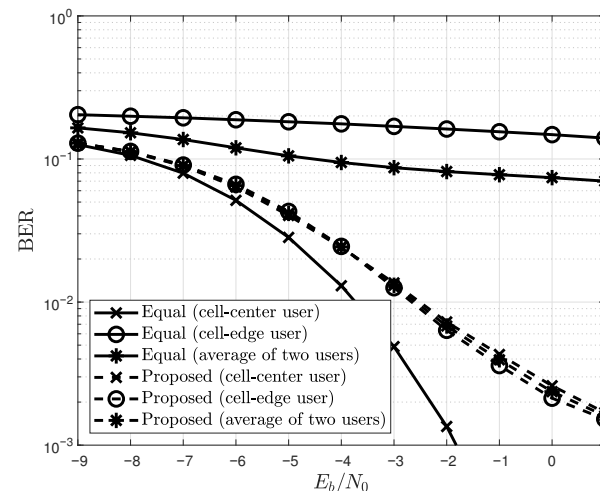


Figure 7. BER for coded NOMA system using one-bit ADCs.

6. Conclusions

In this study, the BER-oriented power allocation was investigated for uplink NOMA systems using one-bit ADCs. Although one-bit quantization offers substantial advantages in terms of power consumption and hardware simplicity, the resulting nonlinear distortion significantly complicates reliable multiuser detection and renders traditional power allocation rules ineffective. To address this challenge, a BER expression based on PEP was analytically derived for the JML detector under one-bit quantized observations to provide an accurate tool to assess error performance. Building upon this characterization, a max-min power allocation strategy based on fairness was formulated, and closed-form solutions were obtained for the considered BPSK signaling scenario. The derived expressions offer direct design insights to determine user transmit powers in practical quantized NOMA receivers. The results of the simulation demonstrated that the proposed approach substantially enhanced BER performance relative to conventional schemes. It also achieved either balanced user reliability or reduced transmit-power consumption depending on the chosen design objective. These results confirm the effectiveness of the analytical framework and highlight the importance of power allocation when severe quantization is present at the receiver.

In future research, the proposed framework can be extended in several important directions. First, extending the proposed framework to scenarios with arbitrary number of users is a natural and meaningful step. As the number of users increases, the superposition structure of NOMA becomes more complex, and the interaction between users through power allocation plays a more critical role. Second, the consideration of arbitrary modulation schemes is another promising extension. Although higher-order modulations suffer from more severe quantization distortion under one-bit ADCs, investigating their BER behavior and corresponding power allocation strategies would be valuable for supporting higher data rates. Third, the proposed approach can be generalized to NOMA systems using low-resolution ADCs with more than one bit. Low-resolution ADC architectures offer

a flexible trade-off between hardware complexity and performance. Finally, incorporating channel estimation errors into the proposed framework is an important and challenging research direction. Due to the strong nonlinearity introduced by one-bit quantization, accurate modeling of channel estimation errors is nontrivial, and their impact on BER-oriented power allocation has not yet been fully understood.

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Appendix A

Proof of Lemma 1. In this proof, a derivation framework similar to that used in [21] is followed which analyzed MIMO systems using one-bit ADCs. To prove Lemma 1, the PEP conditioned on a specific channel realization $\mathbf{h}_u$ is first investigated. Once the conditional expression is obtained, the expectation of the channel distribution is applied to obtain the final averaged result.

Using the likelihood expression in (11), the Bhattacharyya-type term appearing in (21) can be rewritten as

$$\begin{aligned} & \sum_{\mathbf{q} \in \{-1,1\}^{2N}} \sqrt{\mathbb{P}\{\mathbf{q}|\mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \mathbb{P}\{\mathbf{q}|\mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}} \\ & \leq \prod_{i=1}^{2N} \sum_{q_i \in \{-1,1\}} \left(\frac{\mathbb{P}\{q_i|c_i(x_c, x_e)\} + \mathbb{P}\{q_i|c_i(\tilde{x}_c, \tilde{x}_e)\}}{2} \right) \\ & \quad \times \sqrt{\frac{4 \mathbb{P}\{q_i|c_i(x_c, x_e)\} \mathbb{P}\{q_i|c_i(\tilde{x}_c, \tilde{x}_e)\}}{(\mathbb{P}\{q_i|c_i(x_c, x_e)\} + \mathbb{P}\{q_i|c_i(\tilde{x}_c, \tilde{x}_e)\})^2}}. \end{aligned} \quad (\text{A1})$$

Then, the term inside the product can be simplified further. Following an approach similar to that in [21], the above expression can be upper-bounded by using the minimum of the two probabilities.

$$\sum_{\mathbf{q} \in \{-1,1\}^{2N}} \sqrt{\mathbb{P}\{\mathbf{q}|\mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \mathbb{P}\{\mathbf{q}|\mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}} \leq \prod_{i=1}^{2N} 2\sqrt{A_i(1-A_i)}, \quad (\text{A2})$$

where

$$A_i \triangleq \sum_{q_i \in \{-1,1\}} \frac{\min\{\mathbb{P}(q_i|c_i(x_c, x_e)), \mathbb{P}(q_i|c_i(\tilde{x}_c, \tilde{x}_e))\}}{2}.$$

The inequality in (A2) follows from Jensen's inequality applied to $f(x) = \sqrt{x(1-x)}$.

- Case 1: $c_i(x_c, x_e) = c_i(\tilde{x}_c, \tilde{x}_e)$

When the ideal quantized signs match, $A_i(1-A_i)$ simplifies to

$$A_i(1-A_i) = \frac{1 - |p_i(x_c, x_e) - p_i(\tilde{x}_c, \tilde{x}_e)|^2}{4}, \quad (\text{A3})$$

where $p_i(x_c, x_e) = Q\left(\sqrt{2} \left| h_{c,i}^T x_c + h_{e,i}^T x_e \right| \right)$. This expression follows from the identity

$$\min\{\mathbb{P}(q_i|c_i(x_c, x_e)), \mathbb{P}(q_i|c_i(\tilde{x}_c, \tilde{x}_e))\} = 1 - |p_i(x_c, x_e) - p_i(\tilde{x}_c, \tilde{x}_e)|.$$

which holds when both hypotheses share the same sign pattern.

- Case 2: $c_i(x_c, x_e) \neq c_i(\tilde{x}_c, \tilde{x}_e)$

In this case,

$$A_i(1 - A_i) = \left(\frac{p_i(x_c, x_e) + p_i(\tilde{x}_c, \tilde{x}_e)}{2} \right) \left(1 - \frac{p_i(x_c, x_e) + p_i(\tilde{x}_c, \tilde{x}_e)}{2} \right). \quad (\text{A4})$$

Using (A3) and (A4) and bounding the second term by 1, it is given by

$$\begin{aligned} & \sum_{\mathbf{q} \in \{-1, 1\}^{2N}} \sqrt{\mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}} \\ & \leq \prod_{i: c_i(x_c, x_e) \neq c_i(\tilde{x}_c, \tilde{x}_e)} 2 \sqrt{\frac{p_i(x_c, x_e) + p_i(\tilde{x}_c, \tilde{x}_e)}{2}} \\ & \leq \prod_{i: c_i(x_c, x_e) \neq c_i(\tilde{x}_c, \tilde{x}_e)} \sqrt{e^{-|h_{c,i}^T x_c + h_{e,i}^T x_e|^2} + e^{-|h_{c,i}^T \tilde{x}_c + h_{e,i}^T \tilde{x}_e|^2}}, \end{aligned} \quad (\text{A5})$$

where the last bound follows from applying the Chernoff inequality $Q(t) \leq e^{-t^2/2}$.

Finally, (A5) is averaged over the Rayleigh fading channel of $\mathbf{h}_u$. Given the relevant term depends only on the magnitude of the channel,

$$\begin{aligned} & \mathbb{E} \left[\sum_{\mathbf{q} \in \{-1, 1\}^{2N}} \sqrt{\mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \mathbb{P}\{\mathbf{q} | \mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}} \right] \\ & \leq \sum_{n=1}^{2N} \mathbb{P}(d_H(\mathbf{c}(x_c, x_e), \mathbf{c}(\tilde{x}_c, \tilde{x}_e)) = n) \left(\mathbb{E} \left[\sqrt{e^{-|h_{c,i}^T x_c + h_{e,i}^T x_e|^2} + e^{-|h_{c,i}^T \tilde{x}_c + h_{e,i}^T \tilde{x}_e|^2}} \right] \right)^n. \end{aligned} \quad (\text{A6})$$

Using the fact that $|g_{u,i}^T x_u|^2$ follows a gamma distribution [22], the expectation can be evaluated in closed form as

$$\begin{aligned} \mathbb{E} \left[\sqrt{e^{-|h_{c,i}^T x_c + h_{e,i}^T x_e|^2}} \right] &= \mathbb{E} \left[\sqrt{e^{-|\sqrt{P_c d_c^{-\alpha}} g_{c,i}^T x_c + \sqrt{P_e d_e^{-\alpha}} g_{e,i}^T x_e|^2}} \right] \\ &\stackrel{(a)}{=} \sqrt{\frac{2}{2 + \gamma_c \|x_c\|^2 + \gamma_e \|x_e\|^2}}, \end{aligned} \quad (\text{A7})$$

where channels of center and edge user are independent in (a) and $\gamma_u = P_u d_u^{-\alpha}$. Furthermore, the probability mass function of the Hamming distance $d_H(\mathbf{c}(x_c, x_e), \mathbf{c}(\tilde{x}_c, \tilde{x}_e))$ is characterized in [21] as

$$\begin{aligned} \mathbb{P}(d_H = n) &= \binom{2N}{n} \left(1 - \frac{2}{\pi} \tan^{-1} \left(\frac{\|[\sqrt{\gamma_c} x_c, \sqrt{\gamma_e} x_e] + [\sqrt{\gamma_c} \tilde{x}_c, \sqrt{\gamma_e} \tilde{x}_e]\|}{\|[\sqrt{\gamma_c} x_c, \sqrt{\gamma_e} x_e] - [\sqrt{\gamma_c} \tilde{x}_c, \sqrt{\gamma_e} \tilde{x}_e]\|} \right) \right)^n \\ &\quad \times \left(\frac{2}{\pi} \tan^{-1} \left(\frac{\|[\sqrt{\gamma_c} x_c, \sqrt{\gamma_e} x_e] + [\sqrt{\gamma_c} \tilde{x}_c, \sqrt{\gamma_e} \tilde{x}_e]\|}{\|[\sqrt{\gamma_c} x_c, \sqrt{\gamma_e} x_e] - [\sqrt{\gamma_c} \tilde{x}_c, \sqrt{\gamma_e} \tilde{x}_e]\|} \right) \right)^{2N-n} \\ &= \binom{2N}{n} \left(1 - \frac{2}{\pi} \tan^{-1} \left(\frac{\sqrt{\gamma_c} \|x_c + \tilde{x}_c\|^2 + \gamma_e \|x_e + \tilde{x}_e\|^2}{\sqrt{\gamma_c} \|x_c - \tilde{x}_c\|^2 + \gamma_e \|x_e - \tilde{x}_e\|^2} \right) \right)^n \\ &\quad \times \left(\frac{2}{\pi} \tan^{-1} \left(\frac{\sqrt{\gamma_c} \|x_c + \tilde{x}_c\|^2 + \gamma_e \|x_e + \tilde{x}_e\|^2}{\sqrt{\gamma_c} \|x_c - \tilde{x}_c\|^2 + \gamma_e \|x_e - \tilde{x}_e\|^2} \right) \right)^{2N-n}. \end{aligned} \quad (\text{A8})$$

Substituting (A7) and (A8) into (A6) yields

$$\begin{aligned} & \mathbb{E} \left[\sum_{\mathbf{q} \in \{-1,1\}^{2N}} \sqrt{\mathbb{P}\{\mathbf{q}|\mathbf{h}_c, \mathbf{h}_e, x_c, x_e\} \mathbb{P}\{\mathbf{q}|\mathbf{h}_c, \mathbf{h}_e, \tilde{x}_c, \tilde{x}_e\}} \right] \\ & \leq \left(\left(1 - \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_c \|x_c + \tilde{x}_c\|^2 + \gamma_e \|x_e + \tilde{x}_e\|^2}{\gamma_c \|x_c - \tilde{x}_c\|^2 + \gamma_e \|x_e - \tilde{x}_e\|^2}} \right) \right) \right. \\ & \quad \times \left(\left(\sqrt{\frac{2}{2 + \gamma_c \|x_c\|^2 + \gamma_e \|x_e\|^2}} + \sqrt{\frac{2}{2 + \gamma_c \|\tilde{x}_c\|^2 + \gamma_e \|\tilde{x}_e\|^2}} \right) \right. \\ & \quad \left. \left. + \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{\gamma_c \|x_c + \tilde{x}_c\|^2 + \gamma_e \|x_e + \tilde{x}_e\|^2}{\gamma_c \|x_c - \tilde{x}_c\|^2 + \gamma_e \|x_e - \tilde{x}_e\|^2}} \right) \right)^{2N} \right), \end{aligned} \quad (\text{A9})$$

which completes the proof. $\square$

Appendix B

Proof of Lemma 2. To show that $\psi(\gamma_u)$ is strictly decreasing in γ_u , each component of the expression in (18) is examined in terms of its dependence on γ_u . Note that γ_u appears only through

$$\alpha(\gamma_u) = \frac{2}{\pi} \tan^{-1}(t(\gamma_u)), \quad t(\gamma_u) \triangleq \sqrt{\frac{\gamma}{\gamma_u}}, \quad (\text{A10})$$

so the monotonicity of $\psi(\gamma_u)$ follows by analyzing how $\alpha(\gamma_u)$ changes with γ_u .

First, consider $t(\gamma_u) = \sqrt{\gamma/\gamma_u}$. Differentiating with respect to γ_u yields

$$\frac{dt(\gamma_u)}{d\gamma_u} = -\frac{1}{2} \gamma^{1/2} \gamma_u^{-3/2} < 0, \quad (\text{A11})$$

which implies that $t(\gamma_u)$ is strictly decreasing in γ_u .

Since $\tan^{-1}(\cdot)$ is strictly increasing, the derivative of $\alpha(\gamma_u)$ satisfies

$$\alpha'(\gamma_u) = \frac{2}{\pi} \frac{1}{1 + t^2(\gamma_u)} \frac{dt(\gamma_u)}{d\gamma_u} < 0, \quad (\text{A12})$$

so $\alpha(\gamma_u)$ is also strictly decreasing in γ_u .

Next, define

$$B(\gamma_u) = ((1 - \alpha(\gamma_u))A + \alpha(\gamma_u))^{2N} = (A + \alpha(\gamma_u)(1 - A))^{2N}, \quad (\text{A13})$$

where A is given by (19). From (19), this is given by

$$A = \sqrt{\frac{8}{2 + \gamma + \gamma_u}} \implies 0 < A \leq 2. \quad (\text{A14})$$

This bound is exact but not sufficiently restrictive to guarantee that $A < 1$, which is required in the subsequent monotonicity analysis. To obtain a more useful regime, the analysis is constrained to the practically relevant SNR region of interest, where

$$\gamma + \gamma_u > 6, \quad (\text{A15})$$

which guarantees that

$$0 < A < 1. \quad (\text{A16})$$

Under this condition, the denominator in (19) satisfies $2 + \gamma + \gamma_u > 8$, and hence

$$A = \sqrt{\frac{8}{2 + \gamma + \gamma_u}} < 1, \quad (\text{A17})$$

which enables the monotonic decrease of each term in $\psi(\gamma_u)$.

To study monotonicity, rewrite $B(\gamma_u)$ as

$$g(\alpha) = (A + \alpha(1 - A))^{2N}. \quad (\text{A18})$$

Differentiating gives

$$\frac{\partial g}{\partial \alpha} = 2N(A + \alpha(1 - A))^{2N-1}(1 - A). \quad (\text{A19})$$

Under the condition (A15), $1 - A > 0$ is satisfied, and hence

$$\frac{\partial g}{\partial \alpha} \geq 0. \quad (\text{A20})$$

Thus, $g(\alpha)$ is a non-decreasing function of α .

Applying the chain rule yields

$$\frac{dB(\gamma_u)}{d\gamma_u} = \frac{\partial g}{\partial \alpha} \alpha'(\gamma_u). \quad (\text{A21})$$

Since $\frac{\partial g}{\partial \alpha} \geq 0$ and $\alpha'(\gamma_u) < 0$ by (A12), it follows that

$$\frac{dB(\gamma_u)}{d\gamma_u} < 0, \quad (\text{A22})$$

so $B(\gamma_u)$ is strictly decreasing in γ_u .

The other term in $\psi(\gamma_u)$, namely A^{2N} , is constant with respect to γ_u . Therefore,

$$\frac{d\psi(\gamma_u)}{d\gamma_u} = \frac{1}{2} \frac{dB(\gamma_u)}{d\gamma_u} < 0, \quad (\text{A23})$$

which shows that $\psi(\gamma_u)$ is strictly decreasing on the SNR region $\gamma + \gamma_u > 6$, i.e., $A < 1$. This completes the proof. $\square$

References

1. Dai, L.; Wang, B.; Yuan, Y.; Han, S.; Chih-lin, I.; Wang, Z. Non-orthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends. *IEEE Commun. Mag.* **2015**, *53*, 74–81. [\[CrossRef\]](#)
2. Chen, Y.; Bayesteh, A.; Wu, Y.; Ren, B.; Kang, S.; Sun, S.; Xiong, Q.; Qian, C.; Yu, B.; Ding, Z.; et al. Toward the standardization of non-orthogonal multiple access for next generation wireless networks. *IEEE Commun. Mag.* **2018**, *56*, 19–27. [\[CrossRef\]](#)
3. Ahmed, A.; Wang, X.; Hawbani, A.; Yuan, W.; Tabassum, H.; Liu, Y. Unveiling the potential of NOMA: A journey to next-generation multiple access. *IEEE Commun. Surv. Tutor.* **2025**, *27*, 3099–3164. [\[CrossRef\]](#)
4. Islam, S.M.R.; Avazov, N.; Dobre, O.A.; Kwak, K.-S. Power domain non-orthogonal multiple access (NOMA) in 5G systems: Potentials and challenges. *IEEE Commun. Surv. Tutor.* **2017**, *19*, 721–742. [\[CrossRef\]](#)
5. Maraqa, O.; Rajasekaran, A.S.; Al-Ahmadi, S.; Yanikomeroglu, H.; Sait, S.M. A survey of rate-optimal power domain NOMA with enabling technologies of future wireless networks. *IEEE Commun. Surv. Tutor.* **2020**, *22*, 2192–2235. [\[CrossRef\]](#)

6. Kara, F.; Kaya, H. BER performances of downlink and uplink NOMA in the presence of SIC errors over fading channels. *IET Commun.* **2018**, *12*, 1834–1844. [[CrossRef](#)]
7. Yang, Z.; Ding, Z.; Fan, P.; Al-Dhahir, N.J. A general power allocation scheme to guarantee quality of service in downlink and uplink NOMA systems. *IEEE Trans. Wirel. Commun.* **2016**, *15*, 7244–7257. [[CrossRef](#)]
8. Sun, X.; Yang, N.; Yan, S.; Ding, Z.; Kwan, D.W.; Shen, C. Joint beamforming and power allocation in downlink NOMA multiuser MIMO networks. *IEEE Trans. Wirel. Commun.* **2018**, *17*, 5367–5381. [[CrossRef](#)]
9. Norouzi, S.; Champagne, B.; Cai, Y. Joint optimization framework for user clustering, downlink beamforming, and power allocation in MIMO NOMA systems. *IEEE Trans. Commun.* **2023**, *71*, 214–228. [[CrossRef](#)]
10. Yeom, J.S.; Jang, H.S.; Ko, K.S.; Jung, B.C. BER performance of uplink NOMA with joint maximum-likelihood detector. *IEEE Trans. Veh. Technol.* **2019**, *68*, 10295–10300. [[CrossRef](#)]
11. Yahya, H.; Alsusa, E.; Al-Dweik, A. Exact BER Analysis of NOMA with arbitrary number of users and modulation orders. *IEEE Trans. Commun.* **2021**, *69*, 6330–6344. [[CrossRef](#)]
12. Zhang, J.; Dai, L.; Li, X.; Liu, Y.; Hanzo, L. On low-resolution ADCs in practical 5G millimeter wave massive MIMO systems. *IEEE Commun. Mag.* **2018**, *56*, 205–211. [[CrossRef](#)]
13. Liu, J.; Luo, Z.; Xiong, X. Low-resolution ADCs for wireless communication: A comprehensive survey. *IEEE Access* **2019**, *7*, 91291–91324. [[CrossRef](#)]
14. Khalili, M.; Shirani, S.; Erkip, E.; Eldar, Y.C. MIMO networks with one-bit ADCs: Receiver design and communication strategies. *IEEE Trans. Signal Process.* **2021**, *69*, 6134–6150. [[CrossRef](#)]
15. Mo, J.; Heath, R.W., Jr. Capacity Analysis of One-Bit Quantized MIMO Systems with Transmitter Channel State Information. *IEEE Trans. Signal Process.* **2015**, *63*, 5498–5512. [[CrossRef](#)]
16. Mo, J.; Alkhateeb, A.; Abu-Surra, S.; Heath, R.W., Jr. Hybrid architectures with few-bit ADC receivers: Achievable rates and energy-rate tradeoffs. *IEEE Trans. Wirel. Commun.* **2017**, *16*, 2274–2287. [[CrossRef](#)]
17. Zhang, J.; Dai, L.; He, Z.; Ai, B.; Dobre, O.A. Mixed ADC/DAC multipair massive MIMO relaying systems: Performance analysis and power optimization. *IEEE Trans. Veh. Technol.* **2019**, *68*, 789–803. [[CrossRef](#)]
18. Min, M. Optimal power allocation for multiple channels with one-bit analog-to-digital converters. *IEEE Trans. Veh. Technol.* **2005**, *71*, 4438–4443. [[CrossRef](#)]
19. Jeon, Y.-S.; Lee, N.; Hong, S.-N.; Heath, R.W. One-bit sphere decoding for uplink massive MIMO systems with one-bit ADCs. *IEEE Trans. Wirel. Commun.* **2018**, *17*, 4509–4521. [[CrossRef](#)]
20. Tse, D.; Viswanath, P. *Fundamentals of Wireless Communication*; Cambridge University Press: Cambridge, UK, 2005.
21. Kim, T.K.; Min, M.; Jeon, Y.-S. Performance bound for MIMO systems using one-bit ADCs over Rayleigh fading channels. *IEEE Trans. Veh. Technol.* **2022**, *71*, 9067–9072. [[CrossRef](#)]
22. Papoulis, A.; Pillai, S.U. *Probability Random Variables and Stochastic Processes*; McGraw-Hill: New York City, NY, USA, 2002.

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