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Optimal Consumption and Investment Problem with Consumption Ratcheting in Luxury Goods

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Abstract

This paper investigates an infinite-horizon optimal consumption and investment problem for an agent who consumes two types of goods: necessities and luxuries. The agent derives utility from both goods but faces a ratcheting constraint on luxury consumption, which prohibits any decline in its level over time. This constraint captures the irreversible nature of high living standards or luxury habits often observed in real economies. We formulate the problem in a complete financial market with a risk-free asset and a risky stock and solve it analytically using the dual–martingale method. The dual problem is shown to reduce to a family of optimal stopping problems, from which we derive explicit closed-form solutions for the value function and optimal policies. Our results reveal that the ratcheting constraint generates asymmetric consumption dynamics: necessities adjust freely, whereas luxuries exhibit downward rigidity. As a consequence, the marginal propensity to consume necessities declines with wealth, while luxury consumption and portfolio risk exposure increase more sharply compared to the benchmark case without ratcheting. The model provides a continuous-time microfoundation for persistent high consumption levels and greater risk-taking among wealthy individuals.

Keywords: consumption ratcheting; luxury goods; duality; optimal stopping; portfolio choice; marginal propensity to consume; risk-taking

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1. Introduction

Understanding how individuals adjust their consumption and investment behavior when faced with changing wealth and lifestyle standards is a central question in financial economics. While the classical greenstudies of Merton [1,2] provide the foundation for continuous-time consumption and portfolio choice, these models assume that consumption is fully flexible and can both rise and fall smoothly over time. However, empirical evidence suggests that many households exhibit strong resistance to lowering their standard of living once it has been raised, particularly with respect to luxury consumption. This phenomenon—often referred to as the *ratcheting effect*—implies that the consumption of certain goods may be irreversible or downward-rigid.

The idea of consumption ratcheting goes back to the classic work of Duesenberry [3], and was first formalized in a continuous-time framework by Dybvig [4], who modeled an agent with intolerance for any decline in the overall standard of living. Subsequent

research, including Campbell and Cochrane [5] and Carroll [6], emphasized the behavioral and macroeconomic implications of habit persistence and relative consumption concerns. In parallel, empirical studies such as Aït-Sahalia et al. [7] and Wachter and Yogo [8] highlighted that luxury goods exhibit high income elasticity and are disproportionately consumed by wealthy households, suggesting that maintaining luxury consumption is an important determinant of portfolio behavior and wealth accumulation.

In this paper, we focus on consumption ratcheting in luxury goods. Unlike Dybvig's model, where the entire consumption level is subject to ratcheting, we distinguish between two categories of goods—necessities and luxuries—and impose the ratcheting constraint only on the latter. This specification captures the stylized fact that individuals can easily reduce spending on necessities but are reluctant to reduce luxury consumption once a certain social or lifestyle benchmark is reached. Economically, such a constraint can be interpreted as reflecting reputational concerns, habit persistence in luxury preferences, or psychological loss aversion associated with downward adjustments in status consumption.

Relative to the seminal ratcheting model of Dybvig [4], where the entire consumption stream is subject to a non-decreasing constraint, our framework differs in two key respects. First, we distinguish between a flexible “necessity” component and a ratcheted “luxury” component, which allows us to capture Engel-type patterns in a tractable continuous-time setting. Second, we show that, even in this two-good environment, the dual problem reduces to an optimal stopping problem that admits closed-form policy functions and sharp comparative statics. To the best of our knowledge, such an explicit analytical characterization with luxury-specific ratcheting has not been derived in the existing continuous-time literature on habit formation or consumption ratcheting.

Typical examples include private-school tuition, club memberships, luxury automobiles, and high-end housing. Once a household has committed to such items, scaling them down is often perceived as a social or psychological loss, and households instead adjust more flexible categories (such as generic consumption or savings). Qualitative evidence in household finance and marketing suggests that even during downturns, affluent households tend to preserve these “visible” luxury expenditures while cutting back on necessities or precautionary savings, which is precisely the kind of behavior our luxury-specific ratcheting constraint is intended to capture.

We formulate the problem in an infinite-horizon continuous-time economy with a risk-free asset and a risky stock, where the agent allocates wealth optimally across consumption of both goods and portfolio investment. By employing the dual-martingale method of Cox and Huang [9] and following the approach of Peskir [10], we show that the dual problem can be reduced to a family of optimal stopping problems. This structure allows us to derive explicit closed-form solutions for the value function, optimal consumption policies, and portfolio allocations under the ratcheting constraint.

The rest of the paper is organized as follows. Section 2 introduces the model setup and preference specification with ratcheting in luxury goods. Section 3 analyzes the benchmark model without ratcheting and then derives the optimal strategies for the ratcheting model using the dual-martingale approach. We also investigate the asymptotic behavior of the optimal solutions and provide their economic interpretations. Section 4 concludes.

2. The Economy

We consider a consumption and portfolio selection problem of an infinitely lived agent with two consumption goods: a necessity good and a luxury good, denoted by c_1 and c_2 , respectively. Following the theory in Dybvig [4], we assume that the agent experiences strong aversion to any decline in the consumption of luxury goods. Hence, her/his consumption process for luxury goods, c_{2t} , must satisfy the ratcheting constraint

$c_{2t} \geq c_{2t-}$, where c_{2t-} represents the consumption level just prior to time t . In other words, c_2 must be a non-decreasing, right-continuous process. This constraint captures the notion of “habit persistence in luxury consumption,” meaning that once the agent’s standard of luxury rises, it cannot be lowered without incurring psychological discomfort or social loss. We assume that the initial luxury consumption level $c_{20-} > 0$ is given.

The agent’s intertemporal preference is represented by the expected utility function

$$U := \mathbb{E} \left[\int_0^\infty e^{-\beta t} \left\{ \frac{c_{1t}^{1-\gamma_1}}{1-\gamma_1} + \alpha \frac{(c_{2t} + L)^{1-\gamma_2}}{1-\gamma_2} \right\} dt \right], \quad \gamma_1 > \gamma_2, L \geq 0, \quad (1)$$

where $\beta > 0$ is the subjective discount factor, $\alpha > 0$ is the weight placed on utility derived from the luxury good, and $L \geq 0$ is a shift parameter capturing a baseline level of luxury services (for instance, access to a minimum status good or a stock of durable luxury items). The term L ensures that the marginal utility from the luxury good remains finite even when $c_2 = 0$ and helps generate interior solutions in the optimal consumption problem.

The inequality $\gamma_1 > \gamma_2$ implies that the utility from necessity consumption is more curved than that from luxury consumption, so the agent behaves as more risk-averse with respect to necessities than luxuries. The shift parameter L makes the felicity from luxury goods of a modified Stone–Geary form, which belongs to the HARA family [2], and ensures that marginal utility from luxury goods remains finite at $c_2 = 0$. This specification captures the idea that necessities are essential, whereas luxury goods provide additional satisfaction above a baseline level and can be postponed when resources are scarce.

The elasticity of marginal utility of a utility function $u(c)$ is defined as $-\frac{d \log u'(c)}{d \log c}$. For the necessity good, this elasticity is constant and equal to γ_1 , while for the luxury good it is $\gamma_2 \frac{c_2}{c_2 + L}$, which increases with c_2 . This implies that as the level of luxury consumption rises, the agent becomes relatively more risk-averse with respect to that good, reflecting satiation effects and diminishing marginal pleasure from luxury consumption. Moreover, the luxury good exhibits an income elasticity greater than one for sufficiently high income levels in a static optimization problem, consistent with Engel’s law and the empirical classification of luxury goods.

Blinder [11] considers a similar utility specification for consumption and bequest with $\gamma = 0$, while Carroll [6] adopts the same specification as Equation (1) for consumption and wealth. The felicity function for the luxury good takes a modified Stone–Geary form and thus belongs to the class of hyperbolic absolute risk aversion (HARA) utility functions [2]. Note that the marginal utility of the necessity good is infinite when $c_1 = 0$, whereas when $L > 0$, the marginal utility of the luxury good remains finite even if $c_2 = 0$. This ensures interiority of the solution and guarantees that the agent will always consume a positive amount of necessity goods before allocating wealth to luxury goods.

Financial Market. The financial market consists of a risk-free asset and a risky asset. The risk-free asset yields a constant rate of return $r > 0$. The price process S_t of the risky asset evolves according to

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t, \quad (2)$$

where μ denotes the expected continuously compounded rate of return on the risky asset, and $\sigma > 0$ is its volatility. The parameter $\theta := \frac{\mu-r}{\sigma}$ is the market price of risk (Sharpe ratio) associated with holding the risky asset, and B_t is a standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ endowed with the augmented filtration $\{\mathcal{F}_t\}_{t \geq 0}$ generated by B_t (see Karatzas and Shreve [12] for details on the mathematical and probabilistic foundations). This market structure corresponds to the classical complete market setting of Merton [2], allowing the agent to continuously trade to smooth marginal utilities across goods and over time, subject to the ratcheting constraint on luxury consumption.

Wealth Dynamics. The agent's wealth process X_t evolves according to

$$dX_t = [rX_t + \pi_t(\mu - r) - c_{1t} - c_{2t}]dt + \sigma\pi_t dB_t, \quad (3)$$

where π_t denotes the dollar amount invested in the risky asset at time t , and c_{1t} and c_{2t} represent the consumption rates of necessity and luxury goods, respectively. These processes satisfy the standard integrability conditions: for all $t \geq 0$,

$$\int_0^t \pi_s^2 ds < \infty, \quad \int_0^t c_{1s} ds < \infty, \quad \int_0^t c_{2s} ds < \infty, \quad \text{a.s.}$$

We denote by $X_t = x > 0$ the initial wealth, and impose the transversality condition $\lim_{t \rightarrow \infty} X_t \geq 0$.

Following Cox and Huang [9], we can rewrite the dynamic budget constraint Equation (3) in its static (martingale) form as

$$\mathbb{E} \left[\int_t^\infty H_s^t (c_{1s} + c_{2s}) ds \right] \leq x, \quad (4)$$

where

$$H_t := e^{-(r + \frac{1}{2}\theta^2)t - \theta B_t}, \quad H_s^t := \frac{H_s}{H_t}, \quad \theta := \frac{\mu - r}{\sigma}.$$

Here, H_t is the stochastic discount factor (or state-price density), and θ is the market price of risk. The static form Equation (4) highlights that optimal consumption and investment choices can be interpreted as intertemporal allocations of wealth across states and goods, discounted by state prices. This representation will be particularly useful for applying duality methods in the subsequent analysis.

3. Optimization Problem

3.1. Benchmark Model

The agent's optimization problem can be stated as follows:

Problem 1.

$$V(x) := \max_{(c_1, c_2, \pi)} \mathbb{E} \left[\int_t^\infty e^{-\beta(s-t)} \left\{ \frac{c_{1s}^{1-\gamma_1}}{1-\gamma_1} + \alpha \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} \right\} ds \right]$$

subject to the static budget constraint Equation (4).

We make the following assumption which guarantees that a solution to Problem 1 exists.

Assumption 1. We assume that

$$K_i := r + \frac{\beta - r}{\gamma_i} + \frac{\gamma_i - 1}{2\gamma_i^2} \theta^2 > 0, \quad i = 1, 2.$$

Economically, K_i can be interpreted as a risk-adjusted effective discount rate for good i . The first term r is the risk-free rate, the second term $(\beta - r)/\gamma_i$ reflects time preference and intertemporal substitution, and the last term $\frac{\gamma_i - 1}{2\gamma_i^2} \theta^2$ captures the impact of exposure to the risky asset through the Sharpe ratio θ . The condition $K_i > 0$ ensures that the present value of optimal consumption is finite and that the agent's demand for the risky asset is

well defined. If $K_i \leq 0$, the objective would not be finite and the model would fail to admit an economically meaningful solution.

We now state our result in the following theorem.

Theorem 1.

(1) The optimal policies are given by

$$c_{1s}^* = (y_s^*)^{-\frac{1}{\gamma_1}}, \quad c_{2s}^* = \begin{cases} \left(\frac{y_s^*}{\alpha}\right)^{-\frac{1}{\gamma_2}} - L, & \text{if } 0 < y_s^* \leq \alpha L^{-\gamma_2}, \\ 0, & \text{if } y_s^* > \alpha L^{-\gamma_2}. \end{cases}$$

and

$$\pi_s^* = \begin{cases} \frac{\theta}{\sigma} \left[m_+(m_+ - 1)C(y_s^*)^{m_+ - 1} + \frac{1}{\gamma_1 K_1} (y_s^*)^{-\frac{1}{\gamma_1}} + \frac{\alpha^{\frac{1}{\gamma_2}}}{\gamma_2 K_2} (y_s^*)^{-\frac{1}{\gamma_2}} \right], & 0 < y_s^* \leq \alpha L^{-\gamma_2}, \\ \frac{\theta}{\sigma} \left[m_-(m_- - 1)D(y_s^*)^{m_- - 1} + \frac{1}{\gamma_1 K_1} (y_s^*)^{-\frac{1}{\gamma_1}} \right], & y_s^* > \alpha L^{-\gamma_2}, \end{cases} \quad (5)$$

where

$$y_s^* := y^* e^{\beta(s-t)} H_s^t, \\ C = L(\alpha L^{-\gamma_2})^{1-m_+} \frac{\left(\frac{m_- \gamma_2}{1-\gamma_2} + 1\right) \frac{1}{K_2} + \frac{m_- - 1}{r} - \frac{m_-}{\beta(1-\gamma_2)}}{m_+ - m_-}, \quad (6)$$

$$D = L(\alpha L^{-\gamma_2})^{1-m_-} \frac{\left(\frac{m_+ \gamma_2}{1-\gamma_2} + 1\right) \frac{1}{K_2} + \frac{m_+ - 1}{r} - \frac{m_+}{\beta(1-\gamma_2)}}{m_+ - m_-}, \quad (7)$$

$m_+ > 1$ and $m_- < 0$ are two roots to

$$\frac{1}{2}\theta^2 m^2 + \left(\beta - r - \frac{1}{2}\theta^2\right)m - \beta = 0, \quad (8)$$

and y^* is determined by

$$x = \begin{cases} -m_+ C(y^*)^{m_+ - 1} + \frac{1}{K_1} (y^*)^{-\frac{1}{\gamma_1}} + \frac{\alpha^{\frac{1}{\gamma_2}}}{K_2} (y^*)^{-\frac{1}{\gamma_2}} - \frac{L}{r}, & 0 < y^* \leq \alpha L^{-\gamma_2}, \\ -m_- D(y^*)^{m_- - 1} + \frac{1}{K_1} (y^*)^{-\frac{1}{\gamma_1}}, & y^* > \alpha L^{-\gamma_2}. \end{cases} \quad (9)$$

(2) The agent's wealth under the optimal policies is

$$X_s^* = \begin{cases} -m_+ C(y_s^*)^{m_+ - 1} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}} + \frac{\alpha^{\frac{1}{\gamma_2}}}{K_2} (y_s^*)^{-\frac{1}{\gamma_2}} - \frac{L}{r}, & 0 < y_s^* \leq \alpha L^{-\gamma_2}, \\ -m_- D(y_s^*)^{m_- - 1} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}}, & y_s^* > \alpha L^{-\gamma_2}. \end{cases} \quad (10)$$

(3) The value function $V(\cdot)$ is

$$V(x) = \begin{cases} (1 - m_+)C(y^*)^{m_+} + \frac{1}{1-\gamma_1} \frac{1}{K_1} (y^*)^{-\frac{1-\gamma_1}{\gamma_1}} + \frac{1}{1-\gamma_2} \frac{\alpha^{\frac{1}{\gamma_2}}}{K_2} (y^*)^{-\frac{1-\gamma_2}{\gamma_2}}, & 0 < y^* \leq \alpha L^{-\gamma_2}, \\ (1 - m_-)D(y^*)^{m_-} + \frac{1}{1-\gamma_1} \frac{1}{K_1} (y^*)^{-\frac{1-\gamma_1}{\gamma_1}} + \frac{\alpha}{\beta} \frac{L^{1-\gamma_2}}{1-\gamma_2}, & y^* > \alpha L^{-\gamma_2}. \end{cases}$$

Proof. For a Lagrange multiplier $y > 0$ we define the dual functional

$$\begin{aligned}\tilde{V}(y) &:= \max_{(c_1, c_2, \pi) \in \mathcal{A}(x)} \mathbb{E} \left[\int_t^\infty e^{-\beta(s-t)} \left\{ \frac{c_{1s}^{1-\gamma_1}}{1-\gamma_1} + \alpha \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} \right\} ds - y \int_t^\infty H_s^t (c_{1s} + c_{2s}) ds \right] \\ &= \mathbb{E} \left[\int_t^\infty e^{-\beta(s-t)} \left(\left\{ \frac{c_{1s}^{1-\gamma_1}}{1-\gamma_1} - y e^{\beta(s-t)} H_s^t c_{1s} \right\} + \alpha \left\{ \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} - \frac{y e^{\beta(s-t)} H_s^t}{\alpha} c_{2s} \right\} \right) ds \right] \\ &= \mathbb{E} \left[\int_t^\infty e^{-\beta(s-t)} \{ \tilde{u}_1(y_s) + \alpha \tilde{u}_2(y_s) \} ds \right],\end{aligned}$$

where $y_s = y e^{\beta(s-t)} H_s^t$,

$$\tilde{u}_1(y) := \max_{c_1 \geq 0} \left(\frac{c_1^{1-\gamma_1}}{1-\gamma_1} - c_1 y \right) = \frac{\gamma_1}{1-\gamma_1} y^{-\frac{1-\gamma_1}{\gamma_1}},$$

and

$$\begin{aligned}\tilde{u}_2(y) &:= \max_{c_2 \geq 0} \left(\frac{(c_2 + L)^{1-\gamma_2}}{1-\gamma_2} - \frac{y}{\alpha} c_2 \right) = \left[\frac{\gamma_2}{1-\gamma_2} \left(\frac{y}{\alpha} \right)^{-\frac{1-\gamma_2}{\gamma_2}} + \frac{Ly}{\alpha} \right] \mathbf{1}_{\{0 < y \leq \alpha L^{-\gamma_2}\}} \\ &\quad + \frac{L^{1-\gamma_2}}{1-\gamma_2} \mathbf{1}_{\{y > \alpha L^{-\gamma_2}\}}.\end{aligned}$$

The optimal c_1 and c_2 follow from the first-order conditions:

$$c_{1s}^* = (y_s^*)^{-\frac{1}{\gamma_1}}, \quad c_{2s}^* = \begin{cases} \left(\frac{y_s^*}{\alpha} \right)^{-\frac{1}{\gamma_2}} - L, & 0 < y_s^* \leq \alpha L^{-\gamma_2}, \\ 0, & y_s^* > \alpha L^{-\gamma_2}, \end{cases}$$

where $y_s^* := y^* e^{\beta(s-t)} H_s^t$ and y^* is determined by Equation (9).

Define

$$\begin{aligned}J(y) &:= \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\frac{\gamma_1}{1-\gamma_1} y_s^{-\frac{1-\gamma_1}{\gamma_1}} + \left\{ \frac{\gamma_2}{1-\gamma_2} \alpha^{\frac{1}{\gamma_2}} y_s^{-\frac{1-\gamma_2}{\gamma_2}} + L y_s \right\} \mathbf{1}_{\{0 < y_s \leq \alpha L^{-\gamma_2}\}} \right. \right. \\ &\quad \left. \left. + \alpha \frac{L^{1-\gamma_2}}{1-\gamma_2} \mathbf{1}_{\{y_s > \alpha L^{-\gamma_2}\}} \right) ds \right].\end{aligned}$$

By the Feynman–Kac Theorem,

$$\mathcal{L}J(y) + \left[\frac{\gamma_1}{1-\gamma_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \left\{ \frac{\gamma_2}{1-\gamma_2} \alpha^{\frac{1}{\gamma_2}} y^{-\frac{1-\gamma_2}{\gamma_2}} + L y \right\} \mathbf{1}_{\{0 < y \leq \alpha L^{-\gamma_2}\}} + \alpha \frac{L^{1-\gamma_2}}{1-\gamma_2} \mathbf{1}_{\{y > \alpha L^{-\gamma_2}\}} \right] = 0, \quad (11)$$

where

$$\mathcal{L} := (\beta - r) y \frac{d}{dy} + \frac{1}{2} \theta^2 y^2 \frac{d^2}{dy^2} - \beta. \quad (12)$$

Equivalently, J satisfies

$$\begin{aligned}\frac{1}{2} \theta^2 y^2 J''(y) + (\beta - r) y J'(y) - \beta J(y) + \frac{\gamma_1}{1-\gamma_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \left\{ \frac{\gamma_2}{1-\gamma_2} \alpha^{\frac{1}{\gamma_2}} y^{-\frac{1-\gamma_2}{\gamma_2}} + L y \right\} \mathbf{1}_{\{0 < y \leq \alpha L^{-\gamma_2}\}} \\ + \alpha \frac{L^{1-\gamma_2}}{1-\gamma_2} \mathbf{1}_{\{y > \alpha L^{-\gamma_2}\}} = 0.\end{aligned} \quad (13)$$

From Equation (13) we obtain

$$J(y) = \begin{cases} C y^{m_+} + \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \frac{\gamma_2}{1-\gamma_2} \frac{\alpha^{\frac{1}{\gamma_2}}}{K_2} y^{-\frac{1-\gamma_2}{\gamma_2}} + \frac{L}{r} y, & 0 < y \leq \alpha L^{-\gamma_2}, \\ D y^{m_-} + \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \frac{\alpha}{\beta} \frac{L^{1-\gamma_2}}{1-\gamma_2}, & y > \alpha L^{-\gamma_2}, \end{cases}$$

where $m_{\pm}$ solve Equation (8) and C, D are given by Equations (6) and (7). One can verify that J is strictly decreasing and strictly convex. Hence

$$V(x) = \min_{y>0} (J(y) + yx). \quad (14)$$

The first-order condition for Equation (14) yields

$$J'(y) = -x. \quad (15)$$

Thus the optimal y^* solves

$$x = -J'(y^*) = \begin{cases} -m_+ C (y^*)^{m_+-1} + \frac{1}{K_1} (y^*)^{-\frac{1}{\gamma_1}} + \frac{\alpha^{\frac{1}{\gamma_2}}}{K_2} (y^*)^{-\frac{1}{\gamma_2}} - \frac{L}{r}, & 0 < y^* \leq \alpha L^{-\gamma_2}, \\ -m_- D (y^*)^{m_- -1} + \frac{1}{K_1} (y^*)^{-\frac{1}{\gamma_1}}, & y^* > \alpha L^{-\gamma_2}. \end{cases} \quad (16)$$

Since $J'(y)$ is strictly increasing, y^* is unique. More generally,

$$J'(y_s^*) = -X_s^*,$$

which proves Equation (10). Applying Itô's formula to X_s^* gives

$$\pi_s^* = \frac{\theta}{\sigma} y_s^* J''(y_s^*),$$

which yields Equation (5). Finally, Equation (14) gives the stated form of $V(x)$. Verification proceeds as in Theorem 2.3 of Cox and Huang [9]. $\square$

Theorem 1 provides the agent's demand for necessity and luxury goods, as well as portfolio policies. We see from Equation (10) in Theorem 1 (2) that the agent optimally divides her wealth into two components: one for the consumption of the necessity good and the other for the consumption of the luxury good. The wealth allocated for the consumption of the necessity good, X_{1s}^* , is equal to $\frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}}$, and the wealth allocated for the consumption of the luxury good, X_{2s}^* , is the remaining part in the equation. That is,

$$\begin{aligned} X_{1s}^* &= \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}}, \\ X_{2s}^* &= \begin{cases} -m_+ C (y_s^*)^{m_+-1} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}} + \frac{\alpha^{\frac{1}{\gamma_2}}}{K_2} (y_s^*)^{-\frac{1}{\gamma_2}} - \frac{L}{r}, & 0 < y_s^* \leq \alpha L^{-\gamma_2}, \\ -m_- D (y_s^*)^{m_- -1} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}}, & y_s^* > \alpha L^{-\gamma_2}. \end{cases} \end{aligned} \quad (17)$$

Our analysis provides several novel insights. First, the ratcheting constraint introduces asymmetry in consumption dynamics: necessities adjust freely with wealth fluctuations, whereas luxuries exhibit strong downward rigidity. Second, this rigidity generates persistence in high consumption levels and makes the effective wealth available for adjustment smaller, leading to a sharper rise in the marginal propensity to consume luxuries as wealth increases. Third, the optimal risky share is higher than in the benchmark case without

ratcheting, suggesting that wealthy agents take greater financial risks to sustain their luxury standard. Collectively, these results provide a theoretical explanation for empirical patterns of consumption persistence and wealth-dependent risk-taking observed in household data [11,13,14].

Collectively, these results provide a theoretical explanation for empirical patterns of consumption persistence and wealth-dependent risk-taking observed in household data [11,13,14]. For instance, Aït-Sahalia et al. [7] show that luxury consumption is highly procyclical and strongly linked to stock market returns, while Wachter and Yogo [8] document that wealthier households hold a larger share of risky assets and maintain relatively stable expenditure on high-end consumption categories. These empirical patterns motivate our focus on the interaction between luxury ratcheting, consumption persistence, and portfolio risk-taking.

We now state the properties of the optimal policies.

Proposition 1.

- (1) The marginal propensity to consume necessities (luxuries, respectively), $\frac{dc_{1s}^*}{dX_s^*}$ ($\frac{dc_{2s}^*}{dX_s^*}$, respectively), is a decreasing (increasing, respectively) function of wealth for all sufficiently large wealth levels, and

$$\lim_{X_s^* \rightarrow \infty} \frac{c_{1s}^*}{X_s^*} = 0.$$

- (2) The ratio $\frac{X_{1s}^*}{X_s^*}$ ($\frac{X_{2s}^*}{X_s^*}$, respectively) is a decreasing (increasing, respectively) function of wealth, and

$$\lim_{X_s^* \rightarrow 0} \frac{X_{1s}^*}{X_s^*} = 1, \quad \lim_{X_s^* \rightarrow \infty} \frac{X_{2s}^*}{X_s^*} = 1.$$

- (3) The optimal proportion of investment in the risky asset is increasing in wealth, and

$$\lim_{X_s^* \rightarrow 0} \frac{\pi_s^*}{X_s^*} = \frac{\theta}{\gamma_1 \sigma}, \quad \lim_{X_s^* \rightarrow \infty} \frac{\pi_s^*}{X_s^*} = \frac{\theta}{\gamma_2 \sigma}.$$

Proof. We have

$$c_{1s}^* = (y_s^*)^{-\frac{1}{\gamma_1}}, \quad c_{2s}^* = \begin{cases} \left(\frac{y_s^*}{\alpha}\right)^{-\frac{1}{\gamma_2}} - L, & 0 < y_s^* \leq \alpha L^{-\gamma_2}, \\ 0, & y_s^* > \alpha L^{-\gamma_2}. \end{cases}$$

and

$$X_s^* = \begin{cases} -m_+ C(y_s^*)^{m_+ - 1} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}} + \frac{\alpha^{\frac{1}{\gamma_2}}}{K_2} (y_s^*)^{-\frac{1}{\gamma_2}} - \frac{L}{r}, & 0 < y_s^* \leq \alpha L^{-\gamma_2}, \\ -m_- D(y_s^*)^{m_- - 1} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}}, & y_s^* > \alpha L^{-\gamma_2}. \end{cases}$$

- (1) By Theorem 1, for $0 < y_s^* \leq \alpha L^{-\gamma_2}$ (i.e., $X_s^* > X_s^*(\alpha L^{-\gamma_2})$), differentiation yields

$$\frac{dc_{1s}^*}{dX_s^*} = \frac{dc_{1s}^*}{dy_s^*} \frac{dy_s^*}{dX_s^*} = \frac{dc_{1s}^*}{dy_s^*} \frac{1}{\frac{dX_s^*}{dy_s^*}} = \frac{\frac{1}{\gamma_1} (y_s^*)^{\frac{1}{\gamma_2} - \frac{1}{\gamma_1}}}{m_+ (m_+ - 1) C(y_s^*)^{m_+ + \frac{1-\gamma_2}{\gamma_2}} + \frac{1}{\gamma_1 K_1} (y_s^*)^{\frac{1}{\gamma_2} - \frac{1}{\gamma_1}} + \frac{1}{\gamma_2 K_2} \alpha^{\frac{1}{\gamma_2}}}.$$

From Theorem 1, y_s^* is a decreasing function of X_s^* , $\lim_{X_s^* \rightarrow 0} y_s^* = \infty$, and $\lim_{X_s^* \rightarrow \infty} y_s^* = 0$. Since $\frac{1}{\gamma_1} - \frac{1}{\gamma_2} < 0$, the above expression is an increasing function of y_s^* , and hence $\frac{dc_{1s}^*}{dX_s^*}$ is decreasing in X_s^* . Similarly, one can show that $\frac{dc_{2s}^*}{dX_s^*}$ is increasing in X_s^* .

Furthermore,

$$\frac{c_{1s}^*}{X_s^*} = \frac{(y_s^*)^{-1/\gamma_1}}{\frac{1}{K_1}(y_s^*)^{-1/\gamma_1} + \frac{1}{K_2}\left(\frac{y_s^*}{\alpha}\right)^{-1/\gamma_2}},$$

which is an increasing function of y_s^* and approaches 0 as $y_s^* \downarrow 0$, because $\frac{1}{\gamma_1} - \frac{1}{\gamma_2} < 0$. Thus $\frac{c_{1s}^*}{X_s^*}$ is decreasing in X_s^* and tends to zero as $X_s^* \rightarrow \infty$.

(2) We have

$$\frac{X_{1s}^*}{X_s^*} = \frac{\frac{1}{K_1}(y_s^*)^{-\frac{1}{\gamma_1}}}{\frac{1}{K_1}(y_s^*)^{-\frac{1}{\gamma_1}} + \frac{1}{K_2}\left(\frac{y_s^*}{\alpha}\right)^{-\frac{1}{\gamma_2}}} = \frac{\frac{1}{K_1}}{\frac{1}{K_1} + \frac{1}{K_2}\left(\frac{y_s^*}{\alpha}\right)^{\frac{1}{\gamma_1} - \frac{1}{\gamma_2}}}.$$

Since $\frac{1}{\gamma_1} - \frac{1}{\gamma_2} < 0$, $\frac{X_{1s}^*}{X_s^*}$ increases with y_s^* , $\lim_{y_s^* \rightarrow 0} \frac{X_{1s}^*}{X_s^*} = 0$, and $\lim_{y_s^* \rightarrow \infty} \frac{X_{1s}^*}{X_s^*} = 1$. Hence $\frac{X_{1s}^*}{X_s^*}$ decreases with X_s^* , while $\frac{X_{2s}^*}{X_s^*} = 1 - \frac{X_{1s}^*}{X_s^*}$ increases with X_s^* , satisfying the stated limits.

(3) We can write

$$\frac{\pi_s^*}{X_s^*} = \frac{\theta}{\sigma} \frac{\frac{1}{\gamma_1} X_{1s}^* + \frac{1}{\gamma_2} X_{2s}^*}{X_s^*} = \frac{\theta}{\sigma} \left[\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1} \right) \frac{X_{2s}^*}{X_s^*} + \frac{1}{\gamma_1} \right].$$

By part (2), $\frac{X_{2s}^*}{X_s^*}$ increases with X_s^* , and the limits yield

$$\lim_{X_s^* \rightarrow 0} \frac{\pi_s^*}{X_s^*} = \frac{\theta}{\gamma_1 \sigma}, \quad \lim_{X_s^* \rightarrow \infty} \frac{\pi_s^*}{X_s^*} = \frac{\theta}{\gamma_2 \sigma}.$$

□

Proposition 1 describes the benchmark case without the ratcheting constraint, where both necessity and luxury consumptions can adjust freely. It provides clear economic interpretations for how consumption and portfolio choices respond to changes in wealth.

Part (1) shows that the marginal propensity to consume necessities decreases with wealth, whereas that of luxuries increases. As the agent becomes wealthier, necessity consumption grows only slowly, while luxury consumption becomes increasingly sensitive to additional wealth. Consequently, the ratio c_{1s}^*/X_s^* converges to zero as $X_s^* \rightarrow \infty$, illustrating Engel's law: the share of necessities in total spending falls as income rises.

Part (2) shows that the share of wealth allocated to necessities, X_{1s}^*/X_s^* , decreases with total wealth, while the share devoted to luxuries, X_{2s}^*/X_s^* , increases. When wealth is low, almost all resources are devoted to basic consumption, but as wealth grows, the agent allocates an increasing portion of wealth to luxury goods. This represents a smooth shift in spending composition—from survival to comfort and status—as wealth expands.

Part (3) indicates that the optimal risky share π_s^*/X_s^* rises with wealth. At low wealth levels, the agent behaves as a risk-averse necessity consumer, holding a conservative portfolio proportional to $\theta/(\gamma_1\sigma)$. At high wealth levels, utility is dominated by the luxury component with lower risk aversion γ_2 , so the risky share approaches $\theta/(\gamma_2\sigma)$. Hence, richer agents optimally take more risk, reflecting decreasing relative risk aversion with wealth.

Overall, in the absence of the ratcheting constraint, the benchmark model captures three well-known empirical regularities in a continuous-time framework: (i) the declining share of necessities with wealth, (ii) the rising importance of luxury consumption, and (iii) the positive relation between wealth and risk-taking. These benchmark patterns serve as a natural reference for comparison when the ratcheting constraint is introduced in the next section.

3.2. Consumption Ratcheting in Luxury Goods

We now state the optimization problem of the agent at time t .

Problem 2. Given $c_{2t-} \geq 0$ and $x_t = x > c_{2t-}/r$, consider

$$V(x, c_{2t-}) = \max_{(c_1, c_2, \pi)} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left\{ \frac{c_{1s}^{1-\gamma_1}}{1-\gamma_1} + \alpha \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} \right\} ds \right]$$

subject to (1) the static budget constraint Equation (4) and (2) c_2 is a non-decreasing process, where the maximum is taken over all admissible (c_1, c_2, π) .

From Equation (4), consider the Lagrangian for Problem 2:

$$\begin{aligned} \mathbf{L} &\equiv \max_{c_1} \left\{ \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left\{ \frac{c_{1s}^{1-\gamma_1}}{1-\gamma_1} + \alpha \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} \right\} ds \right] + y \left(x - \mathbb{E}_t \left[\int_t^\infty H_s^t (c_{1s} + c_{2s}) ds \right] \right) \right\} \\ &= \max_{c_1} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\frac{c_{1s}^{1-\gamma_1}}{1-\gamma_1} - y e^{\beta(s-t)} H_s^t c_{1s} \right) ds \right] \\ &\quad + \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\alpha \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} - y e^{\beta(s-t)} H_s^t c_{2s} \right) ds \right] + y x \\ &= \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \tilde{u}_1(y_s) ds \right] + \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\alpha \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} - y_s c_{2s} \right) ds \right] + y x, \end{aligned}$$

where $y > 0$,

$$\tilde{u}_1(y) \equiv \max_{c_1 > 0} \left(\frac{c_1^{1-\gamma_1}}{1-\gamma_1} - y c_1 \right) = \frac{\gamma_1}{1-\gamma_1} y^{-\frac{1-\gamma_1}{\gamma_1}},$$

and $y_s \equiv y e^{\beta(s-t)} H_s^t$ is the state-price-scaled Lagrange multiplier at time s . The optimal necessity consumption is

$$c_{1s}^* = y_s^{-\frac{1}{\gamma_1}}, \quad s \geq t.$$

Problem 3 (Dual Problem). Define the dual value function $J(\cdot, \cdot)$ by

$$\begin{aligned} J(y, c_{2t-}) &\equiv \max_{c_2 \in \mathbf{ND}} \left\{ \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \tilde{u}_1(y_s) ds \right] + \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\alpha \frac{(c_{2s} + L)^{1-\gamma_2}}{1-\gamma_2} - y_s c_{2s} \right) ds \right] \right\} \\ &= J_1(y) + J_2(y, c_{2t-}), \end{aligned}$$

where

$$J_1(y) \equiv \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \tilde{u}_1(y_s) ds \right], \quad J_2(y, c_{2t-}) \equiv \max_{c_2 \in \mathbf{ND}} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} F(y_s, c_{2s}) ds \right], \quad (18)$$

$$F(y, c) \equiv \alpha \frac{(c + L)^{1-\gamma_2}}{1-\gamma_2} - y c,$$

and $\mathbf{ND}$ is the set of non-decreasing, nonnegative processes $(c_{2s})_{s \geq t}$ with $\int_t^\infty c_{2s} ds < \infty$.

Lemma 1. The dual value function $J(y, c_{2t-})$ can be written as

$$\begin{aligned} J(y, c_{2t-}) &= \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-} + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}}{r} y \\ &\quad + \int_{c_{2t-}}^\infty \max_{\tau_c} \mathbb{E}_t \left[e^{-\beta(\tau_c-t)} \left(\alpha \frac{(c + L)^{1-\gamma_2}}{\beta} - \frac{y \tau_c}{r} \right) \right] dc, \end{aligned}$$

where the maximum is over stopping times (τ_c) with respect to $\{\mathcal{F}_s\}_{s \geq t}$.

Proof. From Equation (18),

$$\begin{aligned} J_2(y, c_{2t-}) &= \max_{c_2 \in \mathbf{ND}} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} F(y_s, c_{2s}) ds \right] \\ &= \max_{c_2 \in \mathbf{ND}} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\int_{c_{2t-}}^{c_{2s}} \frac{\partial F}{\partial c}(y_s, c) dc + F(y_s, c_{2t-}) \right) ds \right] \\ &= \max_{c_2 \in \mathbf{ND}} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\int_{c_{2t-}}^\infty \frac{\partial F}{\partial c}(y_s, c) \mathbf{1}_{\{c \leq c_{2s}\}} dc \right) ds \right] \\ &\quad + \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} F(y_s, c_{2t-}) ds \right]. \end{aligned} \quad (19)$$

Define $\tau_c := \inf\{s \geq t : c_{2s} \geq c\}$. Since $(c_{2s})_{s \geq t}$ is non-decreasing,

$$\{\tau_c \leq s\} = \{c_{2s} \geq c\} \text{ a.e.}$$

Applying this and Fubini's theorem to the first term in Equation (19) yields

$$\begin{aligned} &\max_{c_2 \in \mathbf{ND}} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\int_{c_{2t-}}^\infty \frac{\partial F}{\partial c}(y_s, c) \mathbf{1}_{\{c \leq c_{2s}\}} dc \right) ds \right] \\ &= \int_{c_{2t-}}^\infty \max_{\tau_c} \mathbb{E}_t \left[e^{-\beta(\tau_c-t)} \left(\int_{\tau_c}^\infty e^{-\beta(s-\tau_c)} (\alpha(c+L)^{-\gamma_2} - y_s) ds \right) \right] dc \\ &= \int_{c_{2t-}}^\infty \max_{\tau_c} \mathbb{E}_t \left[e^{-\beta(\tau_c-t)} \left(\alpha \frac{(c+L)^{-\gamma_2}}{\beta} - \frac{y_{\tau_c}}{r} \right) \right] dc. \end{aligned}$$

The second term in Equation (19) equals

$$\alpha \frac{(c_{2t-} + L)^{1-\gamma_2}}{\beta(1-\gamma_2)} - \frac{c_{2t-}}{r} y.$$

Also,

$$J_1(y) = \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} \frac{\gamma_1}{1-\gamma_1} y_s^{-\frac{1-\gamma_1}{\gamma_1}} ds \right] = \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}}.$$

Combining these completes the proof. $\square$

Lemma 1 implies that the dual problem is equivalent to an infinite series of optimal stopping problems. By absorbing the factor $(c+L)^{-\gamma_2}$ into the initial condition for y , we reduce it to a single optimal stopping problem:

$$\begin{aligned} J(y, c_{2t-}) &= \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-} + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}}{r} y \\ &\quad + \int_{c_{2t-}}^\infty \alpha(c+L)^{-\gamma_2} \max_{\tau} \mathbb{E}_t \left[e^{-\beta(\tau-t)} \left(\frac{1}{\beta} - \frac{1}{r} \frac{y_{\tau}}{\alpha(c+L)^{-\gamma_2}} \right) \right] dc. \end{aligned}$$

Problem 4 (Optimal Stopping Problem). Define

$$Q(z) = \max_{\tau} \mathbb{E} \left[e^{-\beta(\tau-t)} \left(\frac{1}{\beta} - \frac{y_{\tau}}{r} \right) \middle| y_t = z \right], \quad (20)$$

where the maximum is over stopping times τ adapted to $\{\mathcal{F}_s\}_{s \geq t}$. Throughout this section we write $Q(\cdot)$ for the value function of the optimal stopping problem in Equation (20).

Recall that m_+ and m_- denote the two roots of the quadratic Equation (8), and let $z_\infty := \frac{r}{\beta} \frac{m_-}{m_- - 1}$.

Lemma 2. The solution $Q(z)$ of Problem 4 is

$$Q(z) = \begin{cases} \frac{1}{\beta} \cdot \frac{1}{1 - m_-} \left(\frac{z}{z^\infty} \right)^{m_-}, & z > z^\infty, \\ \frac{1}{\beta} - \frac{z}{r}, & z \leq z^\infty, \end{cases}$$

where

$$z^\infty \equiv \frac{r}{\beta} \cdot \frac{m_-}{m_- - 1},$$

and m_- is as in Equation (8).

Proof. Since $y_s = ye^{\beta(s-t)} H_s^t$, Itô's formula gives

$$\frac{dy_s}{y_s} = (\beta - r) ds - \theta dB_s, \quad \text{i.e.,} \quad dy_s = (\beta - r) y_s ds - \theta y_s dB_s.$$

Hence $(y_s)_{s \geq t}$ is a strong Markov diffusion with generator $\mathcal{L}$ in Equation (12).

By standard optimal stopping arguments (see Peskir [10]), Q satisfies the variational inequality

$$\begin{cases} \frac{\theta^2}{2} z^2 Q''(z) + (\beta - r) z Q'(z) - \beta Q(z) = 0, & \text{if } Q(z) > \frac{1}{\beta} - \frac{z}{r}, \\ \frac{\theta^2}{2} z^2 Q''(z) + (\beta - r) z Q'(z) - \beta Q(z) \leq 0, & \text{if } Q(z) = \frac{1}{\beta} - \frac{z}{r}. \end{cases}$$

We impose the growth condition

$$\lim_{z \rightarrow \infty} \frac{Q(z)}{z} = 0, \quad (21)$$

which rules out explosive solutions. There exists a free boundary z^∞ such that the stopping time

$$\tau_z = \inf\{s \geq t \mid y_s \leq z^\infty\}$$

is optimal in Equation (20). Thus the stopping (adjustment) region and continuation region are

$$\mathbf{AR} = \{0 < z \leq z^\infty\}, \quad \mathbf{CR} = \{z > z^\infty\}.$$

On $\mathbf{CR}$,

$$\frac{\theta^2}{2} z^2 Q''(z) + (\beta - r) z Q'(z) - \beta Q(z) = 0, \quad z > z^\infty,$$

whose homogeneous solutions are z^{m_+} and z^{m_-} . Since $m_+ > 1$ and by Equation (21), the z^{m_+} term must vanish, so $Q(z) = A_2 z^{m_-}$ for $z > z^\infty$. The smooth-pasting conditions at z^∞ ,

$$Q(z^\infty) = \frac{1}{\beta} - \frac{z^\infty}{r}, \quad Q'(z^\infty) = -\frac{1}{r},$$

imply

$$z^\infty = \frac{r}{\beta} \cdot \frac{m_-}{m_- - 1}, \quad Q(z) = \frac{1}{1 - m_-} \cdot \frac{1}{\beta} \left(\frac{z}{z^\infty} \right)^{m_-}, \quad z > z^\infty.$$

On $\mathbf{AR}$, $Q(z) = \frac{1}{\beta} - \frac{z}{r}$. This completes the proof. $\square$

By Problem 4 and Lemma 2, we obtain:

Proposition 2.

(1) For $c > c_{2t-}$, the optimal stopping time τ_c in Lemma 1 is

$$\tau_c \equiv \inf\{s \geq t \mid y_s \leq z^\infty \alpha(c+L)^{-\gamma_2}\}.$$

Moreover, the optimal luxury-consumption process is

$$c_{2s}^* = \max\left\{c_{2t-}, \left(\frac{1}{\alpha} \min_{t \leq \gamma \leq s} \frac{y_\gamma}{z^\infty}\right)^{-\frac{1}{\gamma_2}} - L\right\}.$$

(2) The dual value function $J(y, c_{2t-})$ is

$$J(y, c_{2t-}) = \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-}^* + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}^*}{r} y + \int_{c_{2t-}}^{\infty} \alpha(c+L)^{-\gamma_2} Q\left(\frac{y}{\alpha(c+L)^{-\gamma_2}}\right) dc.$$

Proof. (1) Since we absorb $\alpha(c+L)^{-\gamma_2}$ into the initial condition for y in Lemma 1, we directly have

$$\tau_c = \inf\{s \geq t \mid \frac{y_s}{\alpha(c+L)^{-\gamma_2}} \leq z^\infty\}.$$

As τ_c is increasing in c , for any $s \geq t$,

$$\{c_{2s}^* > c\} = \{\tau_c < s\} = \left\{ \min_{t \leq \gamma \leq s} \frac{y_\gamma}{z^\infty} < \alpha(c+L)^{-\gamma_2} \right\} = \left\{ \left(\frac{1}{\alpha} \min_{t \leq \gamma \leq s} \frac{y_\gamma}{z^\infty} \right)^{-\frac{1}{\gamma_2}} - L > c \right\},$$

which gives the stated representation of c_{2s}^* .

(2) If $\frac{y}{\alpha(c_{2t-}+L)^{-\gamma_2}} > z^\infty$ (equivalently, $(\frac{y}{\alpha z^\infty})^{-\frac{1}{\gamma_2}} - L < c_{2t-}$), then $c_{2t}^* = c_{2t-}$ and

$$J(y, c_{2t-}) = \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-} + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}}{r} y + \int_{c_{2t-}}^{\infty} \alpha(c+L)^{-\gamma_2} Q\left(\frac{y}{\alpha(c+L)^{-\gamma_2}}\right) dc.$$

If $\frac{y}{\alpha(c_{2t-}+L)^{-\gamma_2}} < z^\infty$ (equivalently, $(\frac{y}{\alpha z^\infty})^{-\frac{1}{\gamma_2}} - L > c_{2t-}$), then

$$c_{2t}^* = \left(\frac{y}{\alpha z^\infty}\right)^{-\frac{1}{\gamma_2}} - L,$$

and the same decomposition of the integral together with

$$\alpha \frac{(c_{2t}^* + L)^{1-\gamma_2}}{1-\gamma_2} - \alpha \frac{(c_{2t-} + L)^{1-\gamma_2}}{1-\gamma_2} - \frac{y}{r} (c_{2t}^* - c_{2t-})$$

leads to

$$J(y, c_{2t-}) = \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-}^* + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}^*}{r} y + \int_{c_{2t-}}^{\infty} \alpha(c+L)^{-\gamma_2} Q\left(\frac{y}{\alpha(c+L)^{-\gamma_2}}\right) dc.$$

By the minimax theorem (see Rockafellar [15]), the value function $V(x, c_{2t-})$ and the dual value function $J(y, c_{2t-})$ satisfy the following duality relationship:

$$V(x, c_{2t-}) = \min_{y>0} (J(y, c_{2t-}) + yx).$$

Theorem 2 (Main Theorem).

(1) The minimization problem

$$V(x, c_{2t-}) = \min_{y>0} (J(y, c_{2t-}) + yx) \quad (22)$$

has a unique solution $y^* > 0$.

(2) The optimal consumption processes of necessity and luxury goods are

$$c_{1s}^* = (y_s^*)^{-\frac{1}{\gamma_1}}, \quad c_{2s}^* = \max \left\{ c_{2t-}, \left(\frac{1}{\alpha} \min_{t \leq \gamma \leq s} \frac{y_\gamma^*}{z^\infty} \right)^{-\frac{1}{\gamma_2}} - L \right\},$$

where $y_s^* = y^* e^{\beta(s-t)} H_s^t$.

(3) The optimal wealth process is

$$X_s^{*,t} = \frac{c_{2s}^*}{r} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}} - \frac{1}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty} \right)^{m_- - 1} > \frac{c_{2s-}}{r}.$$

(4) The optimal portfolio process is

$$\pi_s^{*,t} = \frac{\theta}{\sigma} \left(\frac{1}{K_1 \gamma_1} (y_s^*)^{-\frac{1}{\gamma_1}} - \frac{(1-m_-)}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty} \right)^{m_- - 1} \right).$$

The agent optimally divides her wealth into two parts. Define

$$X_{1s}^{*,t} = \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}}, \quad X_{2s}^{*,t} = -\frac{1}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty} \right)^{m_- - 1}.$$

Then

$$\left(\pi_s^{*,t} - \frac{\theta}{\sigma \gamma_1} X_{1s}^{*,t} \right) = \frac{\theta(1-m_-)}{\sigma} \left(X_s^{*,t} - X_{1s}^{*,t} - \frac{c_{2s}^*}{r} \right).$$

Proof. (1) Using Lemma 1 and Q from Lemma 2,

$$\begin{aligned} J(y, c_{2t-}) &= \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-}^* + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}^*}{r} y \\ &\quad + \int_{c_{2t-}^*}^{\infty} \alpha (c+L)^{-\gamma_2} Q\left(\frac{y}{\alpha(c+L)^{-\gamma_2}}\right) dc \\ &= \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-}^* + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}^*}{r} y \\ &\quad + \int_{c_{2t-}^*}^{\infty} \frac{1}{\beta} \frac{1}{1-m_-} \alpha (c+L)^{-\gamma_2} \left(\frac{y}{\alpha(c+L)^{-\gamma_2} z^\infty} \right)^{m_-} dc. \end{aligned}$$

Let $f(m) = \frac{\theta^2}{2} m^2 + (\beta - r - \frac{\theta^2}{2})m - \beta$. Since

$$f\left(-\frac{1-\gamma_2}{\gamma_2}\right) = -K_2 < 0,$$

it follows that

$$m_- < -\frac{1-\gamma_2}{\gamma_2} < m_+ \quad \Rightarrow \quad 1 - \gamma_2 + \gamma_2 m_- < 0.$$

Hence the last integral evaluates to

$$\frac{1}{\beta} \frac{\alpha^{1-m_-}}{m_- - 1} \frac{(c_{2t}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y}{z^\infty} \right)^{m_-}.$$

Therefore

$$J(y, c_{2t-}) = \frac{\gamma_1}{1-\gamma_1} \frac{1}{K_1} y^{-\frac{1-\gamma_1}{\gamma_1}} + \alpha \frac{(c_{2t-}^* + L)^{1-\gamma_2}}{(1-\gamma_2)\beta} - \frac{c_{2t-}^*}{r} y + \frac{1}{\beta} \frac{\alpha^{1-m_-}}{m_- - 1} \frac{(c_{2t-}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y}{z^\infty}\right)^{m_-}.$$

The first-order condition for Equation (22) is

$$x = -\frac{\partial J}{\partial y}(y^*) = \frac{c_{2t}^*}{r} + \frac{1}{K_1} (y^*)^{-\frac{1}{\gamma_1}} - \frac{1}{r} \frac{(c_{2t}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y^*}{\alpha z^\infty}\right)^{m_- - 1} =: w(y^*).$$

It is straightforward that

$$\lim_{y \downarrow 0} w(y) = +\infty, \quad \lim_{y \uparrow \infty} w(y) = \frac{c_{2t}^*}{r}, \quad w'(y) < 0 \text{ on } (0, \infty),$$

hence for $x > c_{2t}^*/r$ there exists a unique y^* .

(2) Follows directly from Proposition 2 by substituting y^* .

(3) Since Problem 3 is time-consistent, the optimal Lagrange multiplier process is

$$y_s^* = y^* e^{\beta(s-t)} H_s^t, \quad s \geq t.$$

Hence, substituting y_s^* into $w(\cdot)$ yields the optimal wealth process

$$X_s^{*,t} = w(y_s^*) = \frac{c_{2s}^*}{r} + \frac{1}{K_1} (y_s^*)^{-\frac{1}{\gamma_1}} - \frac{1}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty}\right)^{m_- - 1} > \frac{c_{2s-}}{r}.$$

(4) Since c_{2s}^* evolves only when y_s reaches its lower bound (i.e., it is constant almost everywhere), we have $dc_{2s}^* = 0$ except at discrete adjustment times. Applying Itô's lemma to $X_s^{*,t} = w(y_s^*)$, we get

$$dX_s^{*,t} = -\frac{\partial^2 J(y_s^*, c_{2s}^*)}{\partial y^2} dy_s^* - \frac{\partial^2 J(y_s^*, c_{2s}^*)}{\partial c \partial y} dc_{2s}^* - \frac{1}{2} \frac{\partial^3 J(y_s^*, c_{2s}^*)}{\partial y^3} (dy_s^*)^2. \quad (23)$$

Since $dy_s^* = (\beta - r)y_s^* ds - \theta y_s^* dB_s$, comparison between the wealth dynamics Equation (23) and the standard form of the wealth equation Equation (3) gives

$$\pi_s^* = \frac{\theta}{\sigma} \frac{\partial^2 J(y_s^*, c_{2s}^*)}{\partial y^2} y_s^*.$$

Substituting $\partial^2 J / \partial y^2$ from the explicit form of J in Lemma 1 yields

$$\pi_s^{*,t} = \frac{\theta}{\sigma} \left(\frac{1}{K_1 \gamma_1} (y_s^*)^{-\frac{1}{\gamma_1}} - \frac{(1-m_-)}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty}\right)^{m_- - 1} \right).$$

This establishes part (4) and completes the proof. $\square$

Theorem 3.

(1) The marginal propensity to consume necessities, $\frac{dc_{1s}^*}{dX_s^{*,t}}$, is decreasing in wealth, and

$$\lim_{X_s^* \rightarrow \infty} \frac{c_{1s}^*}{X_s^* - \frac{c_{2s}^*}{r}} = 0.$$

(2) The ratio $\frac{X_{1s}^*}{X_s^*}$ ($\frac{X_{2s}^*}{X_s^*}$, respectively) is decreasing (increasing, respectively) in wealth, and

$$\lim_{X_s^* \rightarrow 0} \frac{X_{1s}^*}{X_s^* - \frac{c_{2s}^*}{r}} = 1, \quad \lim_{X_s^* \rightarrow \infty} \frac{X_{2s}^*}{X_s^* - \frac{c_{2s}^*}{r}} = 1.$$

(3) The optimal proportion invested in the risky asset increases with wealth, and

$$\lim_{X_s^* \rightarrow 0} \frac{\pi_s^*}{X_s^* - \frac{c_{2s}^*}{r}} = \frac{\theta}{\gamma_1 \sigma}, \quad \lim_{X_s^* \rightarrow \infty} \frac{\pi_s^*}{X_s^* - \frac{c_{2s}^*}{r}} = \frac{\theta(1 - m_-)}{\sigma} > \frac{\theta}{\gamma_2 \sigma}.$$

Proof. (1) As $y_s^* \downarrow 0$ (i.e., $X_s^* \rightarrow \infty$),

$$\frac{c_{1s}^*}{X_s^* - \frac{c_{2s}^*}{r}} = \frac{(y_s^*)^{-\frac{1}{\gamma_1}}}{\frac{1}{K_1}(y_s^*)^{-\frac{1}{\gamma_1}} - \frac{1}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty}\right)^{m_- - 1}} \rightarrow 0,$$

since $1 - \gamma_1 + \gamma_1 m_- < 0$.

(2) Similarly,

$$\lim_{X_s^* \rightarrow 0} \frac{X_{1s}^*}{X_s^* - \frac{c_{2s}^*}{r}} = 1, \quad \lim_{X_s^* \rightarrow \infty} \frac{X_{2s}^*}{X_s^* - \frac{c_{2s}^*}{r}} = 1.$$

(3) Using the explicit form of π_s^* and X_s^* ,

$$\begin{aligned} \frac{\pi_s^*}{X_s^* - \frac{c_{2s}^*}{r}} &= \frac{\frac{\theta}{\sigma} \left(\frac{1}{K_1 \gamma_1} (y_s^*)^{-\frac{1}{\gamma_1}} - \frac{(1-m_-)}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty}\right)^{m_- - 1} \right)}{\frac{1}{K_1}(y_s^*)^{-\frac{1}{\gamma_1}} - \frac{1}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty}\right)^{m_- - 1}} \\ &= \frac{\theta}{\gamma_1 \sigma} + \frac{\frac{\theta}{\sigma} \left(\frac{1}{\gamma_1} - 1 + m_- \right) \frac{1}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty}\right)^{m_- - 1}}{\frac{1}{K_1}(y_s^*)^{-\frac{1}{\gamma_1}} - \frac{1}{r} \frac{(c_{2s}^* + L)^{1-\gamma_2+\gamma_2 m_-}}{1-\gamma_2+\gamma_2 m_-} \left(\frac{y_s^*}{\alpha z^\infty}\right)^{m_- - 1}}. \end{aligned}$$

Since $1 - \gamma_2 + \gamma_1 m_- < 0$, it follows that

$$\lim_{X_s^* \rightarrow 0} \frac{\pi_s^*}{X_s^* - \frac{c_{2s}^*}{r}} = \frac{\theta}{\gamma_1 \sigma}, \quad \lim_{X_s^* \rightarrow \infty} \frac{\pi_s^*}{X_s^* - \frac{c_{2s}^*}{r}} = \frac{\theta(1 - m_-)}{\sigma},$$

and $\frac{\pi_s^*}{X_s^* - \frac{c_{2s}^*}{r}}$ increases with X_s^* . Moreover, since $1 - \gamma_2 + \gamma_2 m_- < 0$, we have

$$\frac{\theta(1 - m_-)}{\sigma} > \frac{\theta}{\gamma_2 \sigma}.$$

This completes the proof. $\square$

This theorem characterizes how the presence of the ratcheting constraint on luxury consumption alters the agent's optimal behavior relative to the benchmark case. The ratcheting condition $c_{2t} \geq c_{2t-}$ prevents any decline in luxury consumption, introducing consumption rigidity and path dependence. This irreversibility modifies marginal propensities to consume, wealth allocation, and portfolio risk-taking as follows.

Part (1) shows that the marginal propensity to consume necessities, $\frac{dc_{1s}^*}{dX_s^{*,t}}$, remains decreasing in wealth, as in the benchmark case. However, because a fraction of wealth is now "locked in" to sustaining past luxury consumption, the relevant comparison is between c_{1s}^* and the *adjusted wealth* ($X_s^{*,t} - c_{2s}^*/r$). The limit result

$$\lim_{X_s^* \rightarrow \infty} \frac{c_{1s}^*}{X_s^* - \frac{c_{2s}^*}{r}} = 0$$

indicates that the share of necessities in discretionary wealth still vanishes asymptotically, but the effective disposable wealth is smaller due to the ratcheting constraint. Hence, consumption flexibility declines, and a larger portion of wealth is committed to maintaining the luxury standard.

Part (2) shows that the wealth allocation pattern remains qualitatively similar to the benchmark: X_{1s}^*/X_s^* decreases with wealth while X_{2s}^*/X_s^* increases. Yet, under ratcheting, this transition becomes asymmetric. When wealth falls, luxury consumption cannot be reduced, so X_{2s}^*/X_s^* stays high, and the necessity component bears the adjustment. This implies a form of “consumption inertia”: the agent smooths necessities rather than luxuries in response to negative shocks. Economically, luxury habits become quasi-durable commitments, consistent with observed downward rigidity in high living standards.

Part (3) demonstrates that the optimal risky share still increases with wealth, but the limiting behavior differs from the benchmark. At low wealth, the risky share approaches $\theta/(\gamma_1\sigma)$, identical to the benchmark, since the agent focuses on necessities. At high wealth, however, the limiting share rises to $\theta(1 - m_-)/\sigma$, which exceeds $\theta/(\gamma_2\sigma)$ from the benchmark case. This means that the ratcheting constraint induces *greater risk-taking* than in the frictionless model: because luxury consumption cannot decline, the agent takes on more financial risk to sustain or raise her luxury level. The effective risk aversion thus falls below γ_2 , producing stronger wealth–risk sensitivity.

In summary, compared with the benchmark economy, the ratcheting constraint introduces asymmetry and persistence in consumption behavior. Necessity spending remains flexible, while luxury spending becomes sticky and wealth-dependent. This rigidity amplifies the sensitivity of portfolio risk-taking to wealth, providing a behavioral explanation for why high-income agents maintain both higher luxury consumption and higher financial risk exposure.

Figure 1a shows that, as wealth increases, the share of luxury consumption in total spending becomes larger when the luxury consumption level is locked in by the ratcheting constraint. Figure 1b illustrates how the optimal risky share varies with wealth under the same constraint, reflecting the interaction between luxury ratcheting and portfolio risk-taking.

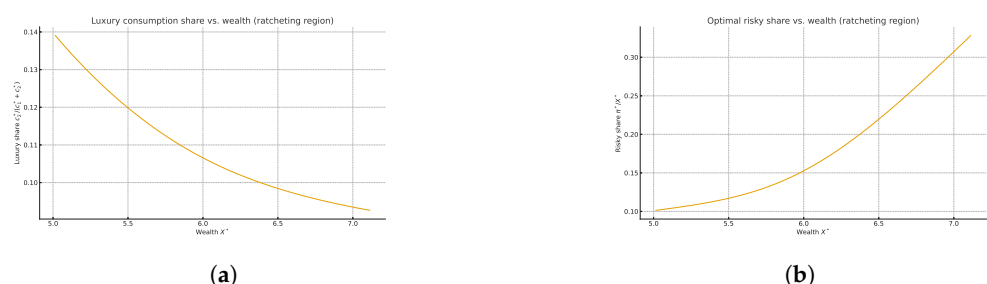


Figure 1. Effects of luxury-specific ratcheting on consumption composition and portfolio choice. (a) Luxury consumption share $c_{2s}^*/(c_{1s}^* + c_{2s}^*)$ as a function of wealth X^* in the region where the luxury ratcheting constraint is binding. (b) Optimal risky share π^*/X^* as a function of wealth X^* in the same ratcheting region.

4. Conclusions

This paper studied an infinite-horizon optimal consumption and investment problem for an agent who consumes two types of goods—necessities and luxuries—under a ratcheting constraint on luxury consumption. The constraint prohibits any downward

adjustment in luxury spending, capturing the behavioral feature that once a high living standard is achieved, it becomes psychologically or socially costly to reduce it. Using the dual-martingale approach, we transformed the primal problem into a dual formulation that further reduces to a sequence of optimal stopping problems. This structure enabled us to derive closed-form solutions for the value function, optimal consumption, and portfolio policies.

Our analysis shows that the ratcheting constraint generates asymmetric consumption dynamics. Necessity consumption adjusts smoothly in response to wealth changes, while luxury consumption exhibits strong downward rigidity. Consequently, the marginal propensity to consume necessities declines with wealth, and luxury spending dominates in the long run. The portfolio choice results demonstrate that the optimal risky share increases with wealth and exceeds the benchmark level without ratcheting, implying stronger risk-taking behavior among high-wealth individuals. These results provide an economic explanation for the persistence of high consumption levels and elevated risk exposure observed in wealthy households.

The model can be extended in several directions. One promising avenue is to introduce a finite planning horizon or incorporate stochastic labor income to analyze life-cycle implications of luxury ratcheting. Another extension is to relax the complete-markets assumption and study how limited risk sharing affects the interaction between luxury ratcheting and portfolio choice; we conjecture that in incomplete markets the ratcheting friction would further amplify precautionary saving and reduce effective risk sharing. A further direction is to consider general equilibrium effects when heterogeneous agents with different ratcheting intensities interact in financial markets. Such extensions would enhance our understanding of the interplay between consumption habits, inequality, and risk-taking behavior in dynamic economic environments.

From an empirical perspective, our model generates several testable implications. For instance, conditional on wealth, households with a higher past peak in luxury consumption should display stronger downward rigidity in luxury spending after negative shocks, and they should hold a larger share of risky assets to sustain their luxury standard. These predictions could be examined using panel data that track detailed consumption categories and portfolio allocations over time (such as CEX- or PSID-type data), and we leave a systematic empirical investigation to future work.

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Abbreviations

The following abbreviations are used in this manuscript:

X_t	Agent's financial wealth at time t
$c_{1,t}$	Consumption rate of the necessity good at time t
$c_{2,t}$	Consumption rate of the luxury good at time t
$c_{2,t-}$	Luxury consumption level just before t (ratcheting reference)
L	Shift parameter in the utility from luxury goods
β	Subjective discount rate
γ_1	Risk aversion parameter for the necessity good
γ_2	Risk aversion parameter for the luxury good

α	Utility weight on the luxury good
r	Risk-free interest rate
S_t	Price of the risky asset at time t
μ	Expected continuously compounded return on the risky asset
σ	Volatility of the risky asset
θ	Market price of risk, $\theta = (\mu - r)/\sigma$
π_t	Dollar amount invested in the risky asset at time t
H_t	State-price density (stochastic discount factor)
K_i	Risk-adjusted discount rate for good i , $i = 1, 2$
m_+, m_-	Roots of the characteristic equation in Equation (8)
z_∞	Free-boundary level in the optimal stopping problem
$V(\cdot)$	Primal value function
$J(\cdot)$	Dual value function
$Q(\cdot)$	Value function of the optimal stopping problem

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