

Article

Industry Index Volatility Spillovers and Forecasting from Crude Oil Prices Based on the MS-HAR-TVP Model

Haoqing Yu 

School of Professional Studies, Columbia University, New York, NY 10027, USA; hy2877@columbia.edu

Abstract

This paper investigates the volatility spillover effects from the crude oil market to domestic stock markets using high-frequency data. We propose an enhanced methodology, the MS-HAR-TVP model, which extends the standard HAR framework. Our model decomposes crude oil price impacts on domestic financial markets into trend and jump volatility spillover components via the TVP framework, while incorporating a Markov switching mechanism to capture regime changes in volatility dynamics. This paper selects the CSI coal index and the CSI new energy index as the representatives of the domestic energy stock market, uses the rolling window method and the MCS test method to evaluate the predictive performance of the model, and compares it with other commonly used models. The empirical results show that (1) the decomposed high-frequency volatility spillover has obvious volatility clustering and asymmetry and the trend and jump spillover have significant improvement in the predictive ability of future volatility; (2) the short-term trend of crude oil is opposite to the trend of the new energy index, but the same as the short-term trend of the coal index, indicating that the impact of crude oil prices on different energy stock markets is different; and (3) the MS-HAR-TVP model and MS-HAR-TVP-J/TCJ model combined with the crude oil volatility spillover have significantly higher in-sample and out-of-sample prediction accuracy than other models in high volatility periods, indicating that the model proposed in this paper can better characterize and predict the volatility characteristics of the domestic energy stock market.



Academic Editor: Jin-Ting Zhang

Received: 2 October 2025

Revised: 24 October 2025

Accepted: 17 November 2025

Published: 20 November 2025

Citation: Yu, H. Industry Index Volatility Spillovers and Forecasting from Crude Oil Prices Based on the MS-HAR-TVP Model. *Mathematics* **2025**, *13*, 3723. <https://doi.org/10.3390/math13223723>

Copyright: © 2025 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

Keywords: volatility forecast; volatility spillover effect; TVP-VAR; MS-HAR**MSC:** 91B84

1. Introduction

The increasing integration of global financial networks has led to more complex interdependencies among asset price fluctuations. Crude oil, serving both as an essential consumer commodity and the world's most actively traded futures product, occupies a central position in this network. The geographically concentrated nature of oil reserves renders crude oil prices particularly sensitive to external shocks compared to other energy commodities such as coal and natural gas. Driven by multidimensional factors including macroeconomic fluctuations, policy shifts, and geopolitical tensions, oil price volatility generates substantial profit opportunities for market participants while simultaneously posing systemic risks to the global financial system [1,2]. However, the risks behind the income have repeatedly affected the world economy and the modern financial market of the oil crisis. This paper focuses on the impact of oil price fluctuations on industry indexes, and whether oil prices can predict the future volatility of industry indexes. These two

issues involve the concept of volatility spillover: the volatility of one market or asset will be transmitted to another market or asset, thereby affecting its price or return volatility. The study of volatility spillover helps to reveal the linkage and contagion of the financial market and provides valuable information for investors and regulators. Therefore, how to characterize the volatility spillover between oil and the stock market has attracted the attention of various investors, scholars, and governments [3].

Based on this, this paper studies the volatility spillover and prediction of oil prices on industry indexes, and the main research contents are as follows: First, this paper uses the statistics proposed by Barndorff and Paton [4–6] and others to construct multi-frequency realized volatility, sign jump volatility, and other methods, and characterizes the high-frequency volatility of the Shanghai–Shenzhen 300 Index and WTI oil prices from multiple dimensions, which can more comprehensively reflect the market volatility characteristics and abnormal situations. Second, using the TVP-VAR volatility spillover framework proposed by Antonakakis [7], based on the high-frequency volatility, the net pairwise spillover results of the realized volatility are obtained, and the spillover effect of oil price volatility on industry index volatility is analyzed, considering the dynamic changes and directionality of volatility spillover, improving the measurement accuracy and practical significance of volatility spillover. Third, combining the HAR model with the Markov switching mechanism proposed by Ma and Cai [8–10], the MS-HAR-TVP model family is constructed, which combines the advantages of the Markov switching mechanism, the HAR model, and the TVP model, and can effectively capture the long memory, nonlinearity and time-varying of volatility, and improve the ability of volatility prediction. Fourth, this paper is based on the rolling time window, and adopts the QLIKE and MAE loss functions and the model confidence set (MCS) test method proposed by Hansen [11] to test the results of the MS-HAR-TVP family model, compare its prediction accuracy, and prove that the high-frequency net pairwise spillover of oil prices on the coal industry index and the new energy industry index has future information, which can improve the accuracy of volatility prediction for the latter, which not only provides a reference for investors to choose the appropriate volatility prediction model, but also provides an important reference for investors to formulate effective risk management strategies.

2. Literature Review

High-frequency volatility models: Early low-frequency models such as ARCH, GARCH, and SV [12–14] were replaced by high-frequency return models because they could not accurately capture intraday information. Based on the heterogeneity assumption, Corsi [15] proposed the HAR-RV model, which successfully characterized the long-term memory of volatility. Due to the easy scalability of the model, it has been widely recognized and applied.

Volatility jump and positive–negative asymmetry: Barndorff [16] used the BNS test method to decompose the volatility and proposed the realized positive–negative semivariance, which is used to measure the price fluctuations corresponding to the positive–negative volatility. Huang [17] introduced the significant jump Z statistic to eliminate the small jump noise and make the model more robust. Patton [6] defined the difference between positive and negative volatility and proved that the HAR-RV-SJ model with superimposed sign jump can capture the trend between volatility jumps. Andersen [18] introduced the jump component into the model and extended it to the HAR-RV-J model. Based on the realized kernel RK and the median realized volatility (MedRV), they constructed the HAR-MedRV-CJ model in combination with the ADS test method.

Empirical evidence demonstrates that financial volatility exhibits regime-dependent persistence and nonlinear dynamics [19,20]. To capture these features, Ma and Cai [8,9]

incorporated Markov switching mechanisms into the HAR framework, developing the MS-HAR-RV-CJ, MS-HAR-RV-SJ, and MS-HARQ(F) models. These regime-switching specifications achieve superior forecasting performance relative to linear benchmarks. Li [21] further extended the HAR model by incorporating investor attention proxies (Baidu Search Index) and decomposing volatility into jump and good–bad components, yielding additional predictive gains.

The research results of volatility show that volatility has characteristics such as long-term memory, jump, positive–negative asymmetry, non-persistence and nonlinearity, but volatility itself cannot fully reflect the linkage and contagion of the financial market, nor can it explain the formation and transmission mechanism of the financial crisis. The purpose of volatility spillover research is to explore the degree of influence of the volatility of one market or asset on the volatility of another market or asset, as well as its influence mechanism and path, which complements the explanation of volatility. For the volatility spillover between oil and the stock market, the current research has the following main aspects:

Multivariate GARCH family models: Early studies [22,23] mainly used multivariate GARCH family models to find the interaction between the oil index and the stock index. Souvcek [24] used the multivariate orthogonal HAR model to verify that the volatility spillover of oil prices increased with the degree of financial integration after the 2008 financial crisis. Positive and negative changes in oil prices: in recent years, Narayan [25,26] decomposed the daily and monthly oil prices into positive and negative changes, respectively, for the asymmetry of the U.S. stock market returns.

TVP-VAR dynamic spillover index method: Due to the abnormal values and specific values in the rolling window method of Diebold [27], this will cause significant changes to the results. Antonakakis [7] proposed the TVP-VAR dynamic spillover index method and proved that its results can more accurately capture the changes of parameters, especially in the case of not needing to arbitrarily select the rolling window size, as it has insensitivity to abnormal values. Gabauer [28] combined DCC-GARCH and TVP-VAR to study the time-varying volatility spillover effects among multiple foreign exchange markets. Balciar [29] constructed an extended connectivity research framework based on TVP-VAR for agricultural products and oil prices and found that global major events would greatly promote the significance of dynamic links. Deng [30] constructed an economic uncertainty index through a high-dimensional factor model and combined TVP-VAR to find that there was a significant stage spillover between the financial index and the former.

The existing literature shows that many domestic and foreign scholars have studied high-frequency volatility and spillover effects [31–33], but there are some limitations: First, most studies on the volatility spillover effect of oil prices on the traditional energy industry and new energy industry in China’s financial market use low-frequency volatility, and few employ high-frequency volatility. Second, most studies use original data to measure the volatility effects, mainly from the perspectives of trend and jump, and extract information from them. Third, the multivariate prediction of high-frequency volatility spillover based on multiple cycles and dimensions is not satisfactory and needs further improvement. Therefore, our MS-HAR-TVP model has obvious advantages.

3. Notation

Table 1 describes the meanings of the mathematical symbols used in the article.

Table 1. Notation table and symbol definitions.

Symbol	Description	Definition	Equation
Basic Variables			
RV_t	Realized Volatility	Sum of squared intraday returns on day t	(14)
r_t	Log Return	$\log(p(t)) - \log(p(t - \Delta))$	(14)
Δ	Sampling Interval	Duration between each sampling, $\Delta = 1/N$	(14)
N	Sample Size	Number of intraday observations	(14)
HAR Model Components			
RV_t^W	Weekly RV	Average realized volatility over past 5 days	(15)
RV_t^M	Monthly RV	Average realized volatility over past 22 days	(15)
$\beta_d, \beta_w, \beta_m$	HAR Coefficients	Daily, weekly, and monthly coefficients	(15)
Jump Volatility Measures			
BPV_t	Bipower Variation	Estimates quadratic variation	(17)
J_t	Jump Component	$\max\{RV_t - BPV_t, 0\}$	(19)
Z_t	Jump Test Statistic	Significant jump detection statistic	(21)
CJ_t	Continuous Jump	Filtered jump variance using threshold Φ_α	(22)
$MedRV_t$	Median RV	Median-based realized volatility (robust to jumps)	(23)
$MedRQ_t$	Median RQ	Median-based realized quarticity	(24)
Z_t^{Med}	Median Jump Stat	Jump test statistic based on $MedRV_t$	(25)
TCJ_t	Threshold CJ	Threshold continuous jump component	(26)
$TCRV_t$	Threshold CRV	Threshold continuous realized volatility	(27)
Signed Volatility			
RS_t^+	Positive Semivariance	Sum of squared positive returns	(29)
RS_t^-	Negative Semivariance	Sum of squared negative returns	(29)
SJ_t	Signed Jump	Difference: $RS_t^+ - RS_t^-$	After (30)
TVP-VAR Model			
y_t	Vector of Variables	$K \times 1$ dimensional vector	(1)
B_t	Time-varying Coef.	$K \times K$ coefficient matrix	(1)
Σ_t	Covariance Matrix	Time-varying variance–covariance matrix	(1)
R_t	Innovation Covariance	$K^2 \times K^2$ dimensional matrix	(2)
$A_{h,t}$	VMA Coefficients	Wold decomposition coefficients	(3)
Spillover Indices			
$\phi_{ij,t}^{gen}(H)$	GFEVD	Forecast error variance decomposition	(4)
$gSOT_{ij,t}$	Spillover Element	Normalized spillover from j to i	(5)
$S_{i \leftarrow \cdot, t}^{gen, from}$	Directional Receive	Total spillover received by variable i	(6)
$S_{i \rightarrow \cdot, t}^{gen, to}$	Directional Transmit	Total spillover transmitted from variable i	(7)
$NPDC$	Net Pairwise Spillover	Net directional connectivity: $S_{i \rightarrow \cdot, t}^{gen, to} - S_{i \leftarrow \cdot, t}^{gen, from}$	(8)
TCI_t	Total Connectivity	Average total directional spillover (market risk)	(9)
$D(\cdot)$	Indicator Function	$D(NPDC_{j \rightarrow i} < 0) = \begin{cases} 1 & \text{if } NPDC < 0 \\ 0 & \text{otherwise} \end{cases}$	(38)
Markov Switching			
s_t	Regime State	$s_t \in \{1, 2\}$: 1 = high vol, 2 = low vol	After (31)
p_{ij}	Transition Prob.	Probability of switching from state i to j	After (31)
β^{s_i}	Regime-dependent Coef.	Coefficients conditional on regime s_i	(32)

4. Model Summary

4.1. TVP-VAR Model

Following Koop and Korobilis [34], we employ a first-order time-varying parameter vector autoregression (TVP-VAR(1)) model. The lag order is selected via the Bayesian information criterion (BIC), which has been shown to provide consistent lag order selection in time-varying parameter models. The model specification is as follows:

$$y_t = B_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_t) \quad (1)$$

$$\text{vec}(B_t) = \text{vec}(B_{t-1}) + v_t, \quad v_t \sim \mathcal{N}(0, R_t) \quad (2)$$

where y_t denotes a $K \times 1$ vector of endogenous variables, B_t represents the $K \times K$ time-varying coefficient matrix, and Σ_t is the $K \times K$ time-varying variance–covariance matrix.

According to the Wold decomposition theorem, the TVP-VAR model can be transformed into the TVP-VMA mode as follows:

$$y_t = \sum_{h=0}^{\infty} A_{h,t} \varepsilon_{t-h} \quad (3)$$

where $A_0 = I_K$. ε_t is symmetric but non-orthogonal white noise shock vectors, and there are $K \times K$ dimensional time-varying covariance matrices ($E(\varepsilon_t \varepsilon_t') = \Sigma_t$) at the same time. The time-varying VMA coefficients $A_{h,t}$ in Equation (3) are computed recursively through the relationship $A_{h,t} = B_t A_{h-1,t}$ with $A_{0,t} = I_K$, following the Wold representation theorem [35]. This recursive structure allows us to trace the dynamic propagation of shocks through the system over h periods ahead. The computation follows Koop and Korobilis [36] who demonstrate that in TVP-VAR models, the VMA coefficients inherit the time-variation from the VAR coefficients B_t , making them suitable for capturing evolving spillover dynamics. The forecast error variance decomposition in Equation (4) is then constructed using these time-varying VMA coefficients following the generalized approach of Pesaran and Shin [37], which does not require orthogonalization of shocks and thus avoids the ordering problem inherent in Cholesky decomposition methods.

Then, based on the generalized connectivity strategy adopted by [27], combined with the generalized forecast error variance decomposition, the impact of sequence i on sequence j can be modeled ahead of H periods as follows:

$$\phi_{ij,t}^{gen}(H) = \frac{\sum_{h=0}^{H-1} (e_j' A_{ht} \Sigma_t e_j)^2}{(e_j' \Sigma_t e_j) \sum_{h=0}^{H-1} (e_j' A_{ht} \Sigma_t A_{ht}' e_j)} \quad (4)$$

$$gSOT_{ij,t} = \frac{\phi_{ij,t}^{gen}(H)}{\sum_{k=1}^K \phi_{ik,t}^{gen}(H)} \quad (5)$$

where e_i is $K \times 1$ dimensional zero vector and is 1 at position i , and $gSOT_{ij,t}$ is the result of normalizing the unscaled GFEVD ($\sum_{k=1}^K \phi_{ik,t}^{gen}(H) \neq 1$), representing the sum of connectivity between other nodes and node i . Further decomposition of this result can obtain the total directional receive index (5) and the total directional spillover index (6), respectively, representing the level of volatility spillover from other variables to variable i and the level of volatility spillover from variable i to other variables.

$$S_{i \leftarrow, t}^{gen, from} = \sum_{j=1, i \neq j}^K gSOT_{ij,t} \quad (6)$$

$$S_{i \rightarrow, t}^{\text{gen, to}} = \sum_{j=1, i \neq j}^K gSOT_{ji, t} \quad (7)$$

The difference between the two is defined as the net pairwise directional spillover index (NPDC). If $S_{ij, t}^{\text{gen, net}} > 0$, variable i has a significant impact on variable j and becomes a net emitter in the whole volatility network, and vice versa:

$$\text{NPDC}(S_{i, t}^{\text{gen, net}}) = S_{i \rightarrow, t}^{\text{gen, to}} - S_{i \leftarrow, t}^{\text{gen, from}} \quad (8)$$

The total connectivity index (TCI) represents a measure of network connectivity and market risk. TCI is defined as the average total directional spillover to other nodes, which is also equivalent to the average total directional receive from other nodes by a node. Its formula is as follows:

$$TCI_t = \frac{1}{K} \sum_{i=1}^K S_{i \leftarrow, t}^{\text{gen, from}} = \frac{1}{K} \sum_{i=1}^K S_{i \rightarrow, t}^{\text{gen, to}} \quad (9)$$

Bayesian Estimation of TVP-VAR Model

To ensure reproducibility and transparency, we provide detailed specifications of our Bayesian estimation procedure for the TVP-VAR model. Following [38,39], we employ a Gibbs sampling algorithm combined with the Kalman filter and smoother for state-space estimation.

Prior Specifications: For the time-varying coefficient vector $\text{vec}(B_t)$, we specify a Minnesota-type prior for the initial state:

$$\text{vec}(B_0) \sim \mathcal{N}(\mu_{B_0}, \Sigma_{B_0}) \quad (10)$$

where μ_{B_0} is obtained from OLS estimates using a training sample of the first 100 observations, and $\Sigma_{B_0} = 10 \times I_{K^2}$ to reflect substantial prior uncertainty. For the innovation covariance matrix governing the time-variation of coefficients, we specify the following:

$$R_t \sim \mathcal{IW}(\nu_0, S_0) \quad (11)$$

with degrees of freedom $\nu_0 = K + 3$ and scale matrix S_0 calibrated to the sample variance of OLS residuals from the training period.

Hyperparameters: The decay parameter controlling the speed of coefficient variation and the volatility scaling parameter are assigned inverse-Gamma priors:

$$\lambda \sim \mathcal{IG}(4, 0.02) \quad (12)$$

$$\kappa \sim \mathcal{IG}(4, 0.02) \quad (13)$$

These hyperparameter values follow the recommendations of [7] for financial volatility applications and imply moderate time-variation in the VAR coefficients.

MCMC Implementation: We implement the Gibbs sampler with the following steps in each iteration: (1) Draw the state vector $\{B_t\}_{t=1}^T$ conditional on Σ_t and hyperparameters using the simulation smoother. (2) Draw the covariance matrices $\{\Sigma_t\}_{t=1}^T$ conditional on B_t and hyperparameters. (3) Draw hyperparameters conditional on B_t and Σ_t . Our MCMC chain runs for 20,000 iterations with a burn-in period of 10,000 iterations. We assess convergence using the Geweke diagnostic test, which compares means from the first 10% and last 50% of the post-burn-in sample, and visual inspection of trace plots. All diagnostics indicate satisfactory convergence with Geweke z-statistics below 2.0 in absolute value for all parameters.

4.2. HAR Model

4.2.1. RV and HAR

According to the definition of Andersen et al. [18,40], based on the asymptotic theory, the sum of squares of all high-frequency returns on the t -th day is used to estimate the realized volatility of the t -th day:

$$RV_t \triangleq \sum_{j=1}^{1/\Delta} r_{(t-1)+j\cdot\Delta}^2 \quad (14)$$

where the intraday return is $r_t = \log(p(t)) - \log(p(t - \Delta))$, $r_{t,n}^2$ is the squared return at the n -th moment of the t -th day, N is the number of samples in the time interval $[t - 1, t]$, $\Delta = 1/N$, and Δ is the duration between each sampling interval. Ref. [15] combined the heterogeneous market hypothesis and constructed the original HAR-RV model.

$$RV_{t+1} = \beta_0 + \beta_d RV_t + \beta_w RVW_t + \beta_m RVM_t + \varepsilon_t \quad (15)$$

In the formula, RV_t , RVW_t , and RVM_t , respectively, correspond to the average realized volatility of the past daily, weekly, and monthly periods.

4.2.2. Jump Volatility

When the sampling frequency $\Delta \rightarrow 0$, the realized variance converges in probability to the quadratic variation (QV) and uses the realized bipower variance (BPV) to estimate the continuous part of it; that is,

$$RV_t = \int_{t-1}^t \sigma^2(s) ds + \sum_{t-1 < s < t} \kappa^2(s) \quad (16)$$

$$BPV_t = \mu_1^{-2} \sum_{j=2}^{1/\Delta} \left| r_{(t-1)+j\cdot\Delta} \right| \left| r_{(t-1)+(j-1)\Delta} \right| \rightarrow \int_0^t \sigma^2(s) ds \quad (17)$$

$$\sum_{0 < s \leq t} \kappa^2(s) = RV_t - BPV_t \quad (18)$$

where $\int_0^t \sigma^2(s) ds$ is continuous and the remaining $\kappa^2(s)$ is defined as the jump magnitude in the log price, that is, the jump variance. Since $RV_t - BPV_t$ may result in negative numbers, ref. [4] defined the filtered jump part, and it was introduced into the HAR model by ref. [18]:

$$J_t = \max\{RV_t - BPV_t, 0\} \quad (19)$$

$$RV_{t+1} = \beta_0 + \beta_1 RV_t + \beta_2 RVW_t + \beta_3 RVM_t + \beta_j J_t + \varepsilon_t \quad (20)$$

Since the jump part still contains a large number of non-zero small jumps, and cannot be economically interpreted, the Z_t statistic is constructed based on the significant jump statistic proposed by ref. [17]:

$$Z_t^{\text{Med}} = \Delta^{-\frac{1}{2}} \frac{(RV_t - BPV_t) RV_t^{-1}}{\sqrt{\left(\frac{\pi^2}{4} + \pi - 5\right) \max\left(1, \frac{TQ_t}{BPV_t^2}\right)}} \quad (21)$$

The filtered jump variance is denoted as CJ_t , where $I(\bullet)$ is a function of the form, which is 1 when the statistic Z is significant and 0 otherwise. In general, the higher the weak significance, the lower the probability of a jump:

$$CJ_t = I(Z_t > \Phi_\alpha) \cdot (RV_t - BPV_t) \quad (22)$$

4.2.3. Median Realized Volatility

To improve the model's jump robustness in finite samples and under the asymptotic theory, and to reduce the interference of the bipower or tripower variation (tripower quarticity, TQ) on the micro noise, Andersen et al. [41] introduced the neighboring volatility near the observation value, as an adaptive threshold to construct the median realized volatility ($MedRV$) and the median realized quarticity ($MedRQ$):

$$MedRV_t = \frac{\pi}{6 - 4\sqrt{3} + \pi} \left(\frac{M}{M-2} \right) \times \sum_{i=2}^{M-1} \text{med}(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|)^2 \quad (23)$$

$$MedRQ_t = \frac{3\pi N}{9\pi + 72 - 52\sqrt{3}} \left(\frac{N}{N-2} \right) \times \sum_{i=2}^{N-1} \text{med}(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|)^4 \quad (24)$$

This paper constructed a new Z_t^{Med} statistic and the corresponding jump component TCJ_t by combining Z_t , where the test threshold for the existence of a jump component is 0.99:

$$Z_t^{Med} = \Delta^{-\frac{1}{2}} \frac{(RV_t - MedRV_t)RV_t^{-1}}{\sqrt{0.96 \cdot \max\left(1, \frac{MedRQ_t}{MedRV_t^2}\right)}} \quad (25)$$

$$TCJ_t = I(Z_t > \Phi_\alpha) \cdot (RV_t - MedRV_t) \quad (26)$$

where $I(\bullet)$ is an indicator function, α represents the significance level, ϕ is the $(1 - \alpha)$ quantile of the standard normal distribution, and $TCRV_t$ is defined as the continuous part of the sample:

$$TCRV_t = I(Z_t \leq \Phi_\alpha) \cdot RV_t + I(Z_t \geq \Phi_\alpha) \cdot MedRV_t \quad (27)$$

Finally, we construct the HAR-CJ-TCJ model by referring to [9,21]:

$$RV_{t+1} = \beta_0 + \beta_d TCRV_t + \beta_w TCRVW_t + \beta_m TCRVM_t + \beta_j TCJ_t + \varepsilon_{t+1} \quad (28)$$

where $TCRVW_t$ and $TCRVM_t$ are the cumulative average $TCRV_t$ of daily, weekly, and monthly frequencies.

4.2.4. Favorable and Unfavorable Volatility

The good–bad volatility decomposition distinguishes between welfare-enhancing and welfare-reducing price fluctuations based on their alignment with investors' directional exposures. Good volatility represents price movements consistent with portfolio positions (upward for longs, downward for shorts), enhancing returns without constituting risk. Bad volatility captures adverse price changes (downward for longs, upward for shorts), eroding portfolio value and representing genuine economic risk. This asymmetric framework recognizes that investors care about both the magnitude and direction of volatility relative to their positions, making it particularly relevant for risk management applications.

Barndorff et al. [16] explained the leverage effect by separating the realized volatility into signed realized semivariances:

$$RS_t^+ = \sum_{j=1}^M I_{\{r_{t,j} > 0\}} r_{t,j}^2, \quad RS_t^- = \sum_{j=1}^n I_{\{r_{t,j} < 0\}} r_{t,j}^2 \quad (29)$$

Subsequently, ref. [6] also constructed the HAR-RV-RS model based on realized semi-variances:

$$RV_{t+1} = \beta_0 + \beta_d^+ RS_t^+ + \beta_w^+ RSW_t^+ + \beta_m^+ RSM_t^+ + \beta_d^- RS_t^- + \beta_w^- RSW_t^- + \beta_m^- RSM_t^- + \varepsilon_{t+1} \quad (30)$$

The difference between them is defined as the signed jump $SJ_t = RS_t^+ - RS_t^-$, which represents the increase or decrease in the volatility asymmetry. If the value is positive, it means that the price rise speed increases. Referring to [21], we construct the HAR-RV-SJ model by combining the median realized volatility (MedRV) with the signed jump as follows:

$$RV_{t+1} = \beta_0 + \beta_{dj}SJ_t + \beta_{wj}SJW_t + \beta_{mj}SJM_t + \beta_d MedRV_t + \beta_w MedRVW_t + \beta_m MedRVM_t + \varepsilon_{t+1} \quad (31)$$

4.3. MRS-HAR-RV-TVP Model

Drawing on the research of [8–10] and others, the combination of the HAR-RV model and the Markov switching mechanism can effectively improve the accuracy of volatility forecasting. This paper adopts a Markov switching mechanism with two states of high volatility and low volatility. Based on the above assumptions, there exists a transition probability matrix P :

$$P(s_t = j | s_{t-1} = i) = p_{ij}, P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}$$

where $s_t = 1$ is the high volatility period, representing that the asset trading is more active. If the high volatility persists and maintains an upward or downward trend, it means that the financial market enters a booming period. Furthermore, $s_t = 2$ is the low volatility period. If the low volatility persists and shows no significant change, it means that the financial market enters a shrinking period. Therefore, this paper defines the MRS-HAR-RV model as follows:

$$RV_{t+1} = \beta_0 + \beta_1^{s_i} RV_t + \beta_2^{s_i} RVW_t + \beta_3^{s_i} RVM_t + \varepsilon_{t+1}^{s_i}, \quad \varepsilon_{t+1}^{s_i} \sim (0, \sigma_{\varepsilon, s_i}^2)$$

where RV, RVW, RVM are the average realized volatility of daily, weekly, and monthly frequencies. $s_i = \{s_t | s_t = i\}$, $i = \{1, 2\}$, when the variable is 1, means that the volatility enters the high volatility state interval, and when it is 2, it enters the low volatility state interval. By combining the TVP model and the HAR-TVP model family, we can obtain the MRS-HAR-TVP family model; by combining the jump volatility, we can extend it to the MRS-HAR-TVP-J model (34); by combining the median realized volatility and the jump, we can extend it to the MRS-HAR-TVP-TCJ model (35); by combining the realized semivariances of the rise and fall, we can extend it to the MRS-HAR-TVP-RS model (36); and by combining the median realized volatility and the signed jump, we can extend it to the MRS-HAR-TVP-RSJ model (37). The specific formulas are as follows:

$$RV_{it+1} = \beta_0^{s_i} + \beta_d^{s_i} RV_{it} + \beta_w^{s_i} RVW_{it} + \beta_m^{s_i} RVM_{it} + \beta_{Dd} \cdot D\left(NPDC_{RV_{jt}, RV_{it}} < 0\right) RV_{jt} + \varepsilon_{it+1}^{s_i} \quad (32)$$

$$\begin{aligned} RV_{it+1} = & \beta_0^{s_i} + \beta_d^{s_i} RV_{it} + \beta_w^{s_i} RVW_{it} + \beta_m^{s_i} RVM_{it} + \beta_j^{s_i} J_{it} \\ & + \beta_{Dd}^{s_i} \cdot D\left(NPDC_{RV_{jt}, RV_{it}} < 0\right) RV_{jt} \\ & + \beta_{Dj}^{s_i} \cdot D\left(NPDC_{J_{jt}, J_{it}} < 0\right) J_{jt} + \varepsilon_{it+1}^{s_i} \end{aligned} \quad (33)$$

$$\begin{aligned} RV_{it+1} = & \beta_0^{s_i} + \beta_d^{s_i} TCRV_{it} + \beta_w^{s_i} TCRVW_{it} + \beta_m^{s_i} TCRVM_{it} + \beta_{tcj}^{s_i} TCJ_{it} \\ & + \beta_{Dd}^{s_i} \cdot D\left(NPDC_{TCRV_{jt}, TCRV_{it}} < 0\right) TCRV_{jt} \\ & + \beta_{Dtcj}^{s_i} \cdot D\left(NPDC_{TCJ_{jt}, TCJ_{it}} < 0\right) TCJ_{jt} + \varepsilon_{it+1}^{s_i} \end{aligned} \quad (34)$$

$$\begin{aligned}
RV_{it+1} = & \beta_0^{s_i} + \beta_{d+}^{s_i} RS_{it}^+ + \beta_{d-}^{s_i} RS_{it}^- + \beta_{w+}^{s_i} RSW_{it}^+ + \beta_{w-}^{s_i} RSW_{it}^- \\
& + \beta_{m+}^{s_i} RSM_{it}^+ + \beta_{m-}^{s_i} RSM_{it}^- + \beta_{Dd+}^{s_i} \cdot D\left(NPDC_{RS_{jt}^+, RS_{it}^+} < 0\right) RS_{jt}^+ \\
& + \beta_{Dd-}^{s_i} \cdot D\left(NPDC_{RS_{jt}^-, RS_{it}^-} < 0\right) RS_{jt}^- + \varepsilon_{it+1}^{s_i}
\end{aligned} \quad (35)$$

$$\begin{aligned}
RV_{it+1} = & \beta_0^{s_i} + \beta_d^{s_i} MedRV_{it} + \beta_w^{s_i} MedRVW_{it} + \beta_m^{s_i} MedRVM_{it} + \beta_{sj}^{s_i} SJ_{it} \\
& + \beta_{Dd}^{s_i} \cdot D\left(NPDC_{MedRV_{jt}, MedRV_{it}} < 0\right) MedRV_{jt} + \beta_{Dsj}^{s_i} \cdot D\left(NPDC_{SJ_{jt}, SJ_{it}} < 0\right) \\
& \times SJ_{jt} + \varepsilon_{it+1}^{s_i}
\end{aligned} \quad (36)$$

According to the filtering process proposed by [42] and the smoothing algorithm proposed by [43], we optimize the likelihood function by using an iterative algorithm, and when the likelihood function $L(\theta)$ is determined, the parameter estimate is as follows:

$$\ln L = \sum_{t=1}^T \ln \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(\frac{RV_{t+1} - \beta_0^{s_i} - \beta_1^{s_i} RV_t - \beta_2^{s_i} RVW_t - \beta_3^{s_i} RVM_t}{-2\sigma^2} \right) \right) \quad (37)$$

4.4. HAR-RV-TVP Model

Wu [44] improved the prediction accuracy of the HAR-RV model based on the volatility spillover research of DY2012 and introduced the net directional spillover index (NDPC) between the two variables as a variable and constructed the HAR-RV-NDPC model as follows:

$$RV_{i+1,j} = \beta_0 + \beta_d RV_t + \beta_w RVW_t + \beta_m RVM_t + \beta_{Dd} \cdot D(NPDC_{j,t,i} < 0) RV_{j,t} + \varepsilon_{t+1} \quad (38)$$

where $D(\bullet)$ is an indicator function; when $NDPC < 0$, it means that parameter j has volatility spillover to parameter i and when parameter i is the information receiver in the market network, then $D(\bullet)$ is 1, otherwise it is 0.

Based on the original HAR-RV-NPDC model, considering the cases of volatility shocks based on jump volatility spillover, measurement error based on quartic variation, and favorable and unfavorable volatility, this paper proposes the following models:

- (1) HAR-RV-TVP-J model: in the framework of the original HAR-RV-J model, we add the jump part of parameter J (i.e., crude oil price) and the net pairwise spillover of daily volatility to parameter i , which causes volatility shocks; that is,

$$\begin{aligned}
RV_{it+1} = & \beta_0 + \beta_d RV_{it} + \beta_w RVW_{it} + \beta_m RVM_{it} + \beta_j J_{it} \\
& + \beta_{Dd} D\left(NPDC_{RV_{jt}, RV_{it}} < 0\right) RV_{jt} \\
& + \beta_{Dj} D\left(NPDC_{J_{jt}, J_{it}} < 0\right) J_{jt} + \varepsilon_{it+1}
\end{aligned} \quad (39)$$

- (2) HAR-RV-TVP-TCJ model: in the framework of the original HAR-RV-TCJ model, we incorporate the net pairwise spillover constructed based on the median realized volatility and the median realized quartic variation into the volatility impact on parameter i ; that is,

$$\begin{aligned}
RV_{it+1} = & \beta_0 + \beta_d TCRV_{it} + \beta_w TCRVW_{it} + \beta_m TCRVM_{it} + \beta_{tcj} TCJ_{jt} \\
& + \beta_{Dd} D\left(NPDC_{TCRV_{jt}, TCRV_{it}} < 0\right) TCRV_{jt} \\
& + \beta_{Dtcj} D\left(NPDC_{TCJ_{jt}, TCJ_{it}} < 0\right) TCJ_{jt} + \varepsilon_{it+1}
\end{aligned} \quad (40)$$

- (3) HAR-RV-TVP-RS model: based on the HAR-RV-RS model, we incorporate the net pairwise spillover constructed by the realized semivariances of the rise and fall between parameter i and j into the volatility impact on parameter i ; that is,

$$\begin{aligned}
 RV_{it+1} = & \beta_0 + \beta_{d+} RS_{it}^+ + \beta_{d-} RS_{it}^- + \beta_{w+} RSW_{it}^+ + \beta_{w-} RSW_{it}^- + \beta_{m+} RSM_{it}^+ \\
 & + \beta_{m-} RSM_{it}^- + \beta_{Dd+} \bullet D\left(NPDC_{RS_{jt}^+, RS_{it}^+} < 0\right) \\
 & \times RS_{jt}^+ + \beta_{Dd-} \bullet D\left(NPDC_{RS_{jt}^-, RS_{it}^-} < 0\right) RS_{jt}^- + \varepsilon_{it+1}
 \end{aligned} \quad (41)$$

- (4) HAR-RV-TVP-SJ model: based on the HAR-RV-SJ model, we construct the corresponding net pairwise spillover index by using the median realized volatility and the signed jump; that is,

$$\begin{aligned}
 RV_{it+1} = & \beta_0 + \beta_d MedRV_{it} + \beta_w MedRVW_{it} + \beta_m MedRVM_{it} + \beta_{dj} SJ_{it} \\
 & + \beta_{wj} SJW_{it} + \beta_{mj} SJM_{it} + \beta_{Dd} D(NPDC_{MedR, MedRV_{it}} < 0) MedRV_{jt} \\
 & + \beta_{Dsj} D(NPDC_{SJ_{jt}, SJ_{it}} < 0) SJ_{jt} + \varepsilon_{it+1}
 \end{aligned} \quad (42)$$

4.5. Model Evaluation

To test the better in-sample fit and out-of-sample prediction accuracy of the above model family more comprehensively, this paper adopts the rolling time window method to perform step-by-step prediction, and uses the model confidence set (MCS) test method constructed by [11] and others to test the prediction robustness. The detailed steps can be found in [21]. This paper follows [9] to use MAE and QLIKE loss functions as the measurement indicators of prediction accuracy, which are defined as follows:

$$MAE = \frac{1}{M} \sum_{m=H+1}^{H+M} |RV_m^2 - \hat{\sigma}_m^2| \quad (43)$$

$$QLIKE = \frac{1}{M} \sum_{m=M+1}^{H+M} \left(\ln(\hat{\sigma}_m^2) + \frac{RV_m}{\hat{\sigma}_m^2} \right) \quad (44)$$

where $\hat{\sigma}_m^2$ is the predicted value obtained by each model in the out-of-sample, RV_m is the actual value of the same period in the out-of-sample, H represents the number of days in the window period, and M represents the number of days in the sample period.

5. Empirical Analysis and Result Testing

5.1. Sample Selection and Data Sources

The CSI new energy index (399,808) and the CSI coal index (399,998), respectively, select the security prices of listed companies in the fields of new energy-related production, application, storage, and other businesses and coal mining, processing, and other businesses in the Chinese securities market, and combine them with market value influence to form a weighted composite. Due to different formulation times, the new energy index takes 31 December 2011 as the base date, and the coal index takes 31 December 2008 as the base date, both with 1000 points as the base point. WTI crude oil futures, as one of the three major crude oil price benchmarks in the world, are also widely traded due to their good liquidity and high price transparency.

This paper uses 5 min intraday high-frequency data of the CSI new energy index (399,808), the CSI coal index (399,998), and the WTI crude oil price spanning from 10 July 2015 to 20 October 2022, covering a total of 1680 trading days. Daily realized volatility measures are constructed by aggregating the 5 min returns within each trading day. The CSI index data are sourced from Tushare (<https://tushare.pro/>), and the WTI high-frequency data are from FirstRateData (www.firstratedata.com).

A critical methodological consideration in our study is the proper temporal alignment between WTI crude oil prices and CSI indices to ensure that our forecasting framework reflects the actual information availability to market participants and avoids any look-ahead bias. We use WTI volatility from day $t - 1$ (U.S. time) and CSI index volatility from day t (Beijing time) to forecast CSI index volatility at day $t + 1$ (Beijing time). This specification is not arbitrary but is carefully designed based on the actual trading hours and information flow mechanism between the two markets.

The WTI crude oil futures are traded on the New York Mercantile Exchange (NYMEX) with regular trading hours from 9:30 a.m. to 4:00 p.m. Eastern Time (ET). Due to the time zone difference, these hours correspond to 9:30 p.m. to 4:00 a.m. Beijing Time (BT) on the next calendar day. The Chinese stock market operates from 9:30 a.m. to 3:00 p.m. Beijing Time. Table 2 illustrates the temporal relationship:

Table 2. Trading hours and information flow timeline.

Event	U.S. Eastern Time	Beijing Time
WTI trading day $t - 1$ closes	Day $t - 1$, 4:00 p.m.	Day t , 4:00 a.m.
Time gap (information dissemination)	—	5.5 h
CSI trading day t opens	Day t , 9:30 p.m. (previous night)	Day t , 9:30 a.m.
CSI trading day t closes	Day $t + 1$, 3:00 a.m.	Day t , 3:00 p.m.

As Table 2 demonstrates, when Chinese markets open on day t at 9:30 a.m. Beijing Time, the WTI trading day $t - 1$ (U.S. time) has already closed at 4:00 a.m. Beijing Time, meaning all WTI information from day $t - 1$ has been publicly available for at least 5.5 h. Therefore, using WTI_{t-1} to help predict CSI_{t+1} ensures that we only use information that was genuinely available to market participants at the time of prediction. On day t , investors in Chinese markets can observe (1) the complete WTI volatility information from day $t - 1$, which has been fully disseminated, and (2) the intraday CSI index movements on day t . They use this combined information set to form expectations about CSI volatility on day $t + 1$. Our model formalizes this natural information flow process.

Alternative lag structures would be problematic: Using WTI_t (contemporaneous) would introduce look-ahead bias because WTI trading on day t (U.S. time) occurs from 9:30 p.m. to 4:00 a.m. Beijing Time on day $t + 1$, which overlaps with or occurs after the CSI trading day $t + 1$ that we are trying to predict. This would mean using future information to predict the past, violating the fundamental principle of out-of-sample forecasting. Using WTI_{t-2} (two-day lag) would be overly conservative and would discard valuable information that was genuinely available to market participants, reducing both the economic relevance and statistical efficiency of our model.

To facilitate the observation of the returns within the entire sample interval, we magnify the logarithmic returns by 100 times; that is,

$$r_t = (\log(p(t)) - \log(p(t - \Delta))) * 100 \quad (45)$$

As can be seen from Figure 1, after the stock market crash in 2015 and 2016, a series of policies issued by the regulatory authorities to standardize the securities market, while assisting the healthy development of the securities market, also led to the strong rebound of the industry index in 2017. Similarly, the crude oil market experienced a shale oil revolution in 2015, and OPEC refused to cut production, resulting in a global oversupply of crude oil and a sharp drop in oil prices, until it bottomed out and rebounded in 2016. The negative impact of the Sino–U.S. trade friction and the slowdown in global economic growth in 2018 caused the domestic major industry indexes to experience a significant

correction again. After 2019, with the Fed's interest rate cut and domestic credit easing, domestic core enterprises still maintained stable profitability, leading the domestic index to maintain a fluctuating upward trend for about two years. In the following 2020, due to the epidemic and war disturbances, the demand for crude oil changed frequently. At the same time, the prices of commodities continued to rise, which hindered the domestic economic recovery and caused the domestic index to suffer the largest adjustment since 2015. Generally speaking, there is a clear similarity in the overall trend among the three, and the trend of coal and new energy indexes is more similar to WTI, and the fluctuations are more concentrated, which is similar to the previous research results.

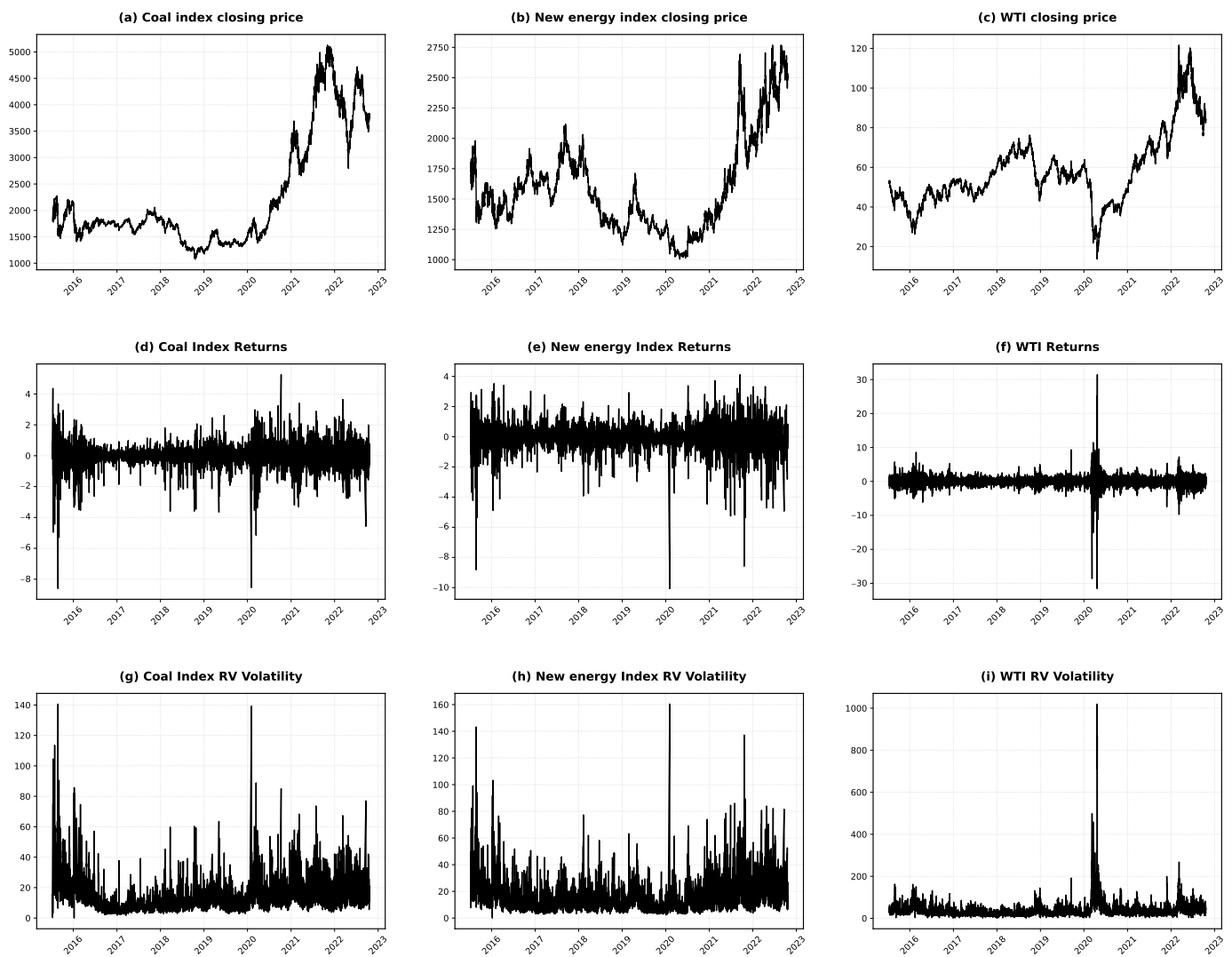


Figure 1. Closing price and yield series.

Tables 3–5 below present the main descriptive statistics of the data in the sample area. WTI crude oil and the two industry indexes exhibit significant volatility clustering effects. The former has the largest volatility coinciding with the low-frequency volatility, which occurred during the 2020 pandemic, while the latter has the highest frequency volatility at the end of 2015. This differs significantly from the low-frequency volatility behavior. It suggests that in the recent development of the domestic financial market, the daily volatility range has not changed much, and the high-frequency volatility has a more distinct peak–valley interval and smoother intraday fluctuations. All high-frequency volatility series reject the normality assumption at the 1% significance level based on the Jarque–Bera statistic, and most of the series are right-skewed (skewness greater than 0) and leptokurtic

(kurtosis far greater than 3). The WTI high-frequency volatility series have much higher skewness and kurtosis than the CSI high-frequency volatility series. The ADF unit root test confirms the stationarity of all series at the 1% significance level, and the Ljung–Box Q test with 10 and 20 lags indicates the presence of autocorrelation. Therefore, under the condition that all series have long-term memory features, we use the TVP-VAR model to model the new energy index, coal index, and WTI high-frequency volatility series.

Table 3. Coal index high frequency volatility.

Statistics	RV	MedRV	TCRV	J	TCJ	RS+	RS−	SJ
Mean	2.5072	2.3472	2.2296	0.2841	0.2872	1.3583	1.1488	0.2095
Std	3.6027	3.4548	3.2924	0.7957	0.6398	2.0271	1.8917	1.5479
Skewness	6.0086	5.4968	5.863	14.9856	5.111	5.5536	6.1873	−0.5105
Kurtosis	70.3185	55.3743	64.2456	361.2518	37.5985	51.4381	69.9493	26.0639
JB ($\times 10^5$)	35.22 ***	22.02 ***	29.52 ***	909.42 ***	10.51 ***	19.17 ***	34.93 ***	4.71 ***
ADF	−6.69 ***	−6.72 ***	−6.75 ***	−8.40 ***	−8.08 ***	−6.90 ***	−6.76 ***	−11.90 ***
Q(10)	3581 ***	3474.6 ***	3533.3 ***	367.279 ***	408.7 ***	2572.6 ***	2487.5 ***	110.5 ***
Q(20)	4735.5 ***	4616.8 ***	4685.0 ***	497.24 ***	558.49 ***	3350.7 ***	3382.1 ***	160.37 ***

Note: *** represent rejecting the null hypothesis at the 1% significance levels; JB is the Jarque–Bera normality test; ADF is the unit root test; Q(10) and Q(20) both use the Ljung–Box Q test method.

Table 4. New energy index high frequency volatility.

Statistics	RV	MedRV	TCRV	J	TCJ	RS+	RS−	SJ
Mean	2.3511	2.2864	2.1539	0.2377	0.2053	1.1649	1.1862	−0.0214
Std	3.5271	3.4924	3.1941	0.7796	0.655	1.7545	2.0126	1.348
Skewness	6.2754	6.0799	5.6614	15.5546	11.0668	6.0235	6.4338	−2.6243
Kurtosis	69.7195	60.9902	53.0168	359.814	184.0873	57.3449	69.5721	40.4899
JB ($\times 10^5$)	34.73 ***	26.77 ***	20.34 ***	902.72 ***	237.93 ***	23.77 ***	34.65 ***	11.54 ***
ADF	−6.00 ***	−6.111 ***	−5.957 ***	−7.912 ***	−8.908 ***	−5.548 ***	−6.619 ***	−12.13 ***
Q(10)	3210 ***	2920 ***	3280 ***	249 ***	180.1 ***	2840 ***	2150 ***	61.5 ***
Q(20)	4900 ***	4540 ***	5100 ***	473 ***	349 ***	4390 ***	3300 ***	105 ***

Note: *** represent rejecting the null hypothesis at the 1% significance levels; JB is the Jarque–Bera normality test; ADF is the unit root test; Q(10) and Q(20) both use the Ljung–Box Q test method.

Table 5. WTI high frequency volatility.

Statistics	RV	MedRV	TCRV	J	TCJ	RS+	RS−	SJ
Mean	10.8016	9.3647	9.3527	1.2596	1.5663	5.5456	5.2561	0.2895
Std	136.9467	138.1613	138.1603	20.2953	21.1453	63.6593	75.5081	27.4512
Skewness	37.2082	39.0282	39.0293	39.2769	38.043	32.6657	39.3031	8.5134
Kurtosis	1454.6906	1560.1182	1560.1759	1574.6315	1504.5284	1158.7191	1576.1005	748.2635
JB ($\times 10^5$)	14,683 ***	16,887 ***	16,888 ***	17,202 ***	15,706 ***	9321 ***	17,234 ***	3877 ***
ADF	−9.405 ***	−9.720 ***	−9.722 ***	−9.928 ***	−9.224 ***	−8.928 ***	−9.985 ***	−13.87 ***
Q(10)	151 ***	58.8 ***	58.7 ***	37 ***	79.3 ***	348 ***	53.5 ***	474 ***
Q(20)	164 ***	68.2 ***	68.1 ***	40.9 ***	87.8 ***	376 ***	58.1 ***	486 ***

Note: *** represent rejecting the null hypothesis at the 1% significance levels; JB is the Jarque–Bera normality test; ADF is the unit root test; Q(10) and Q(20) both use the Ljung–Box Q test method.

Given the extreme kurtosis values observed, we conduct a jump analysis (Figure 2) to identify the frequency of extreme volatility events. Using the Barndorff–Nielsen and Shephard bipower variation test at the 99.9% significance level, we find that 76.30% of trading days in the coal index, 81.07% in the new energy index, and 35.81% in WTI crude oil contain significant jumps. Figure 2 visualizes the temporal distribution of these jump days over our sample period. The higher jump frequency in WTI reflects the extreme volatility in

global oil markets during major events such as the 2020 COVID-19 pandemic and oil price crash. This finding justifies our use of realized volatility measures that explicitly account for jump components in the subsequent analysis.

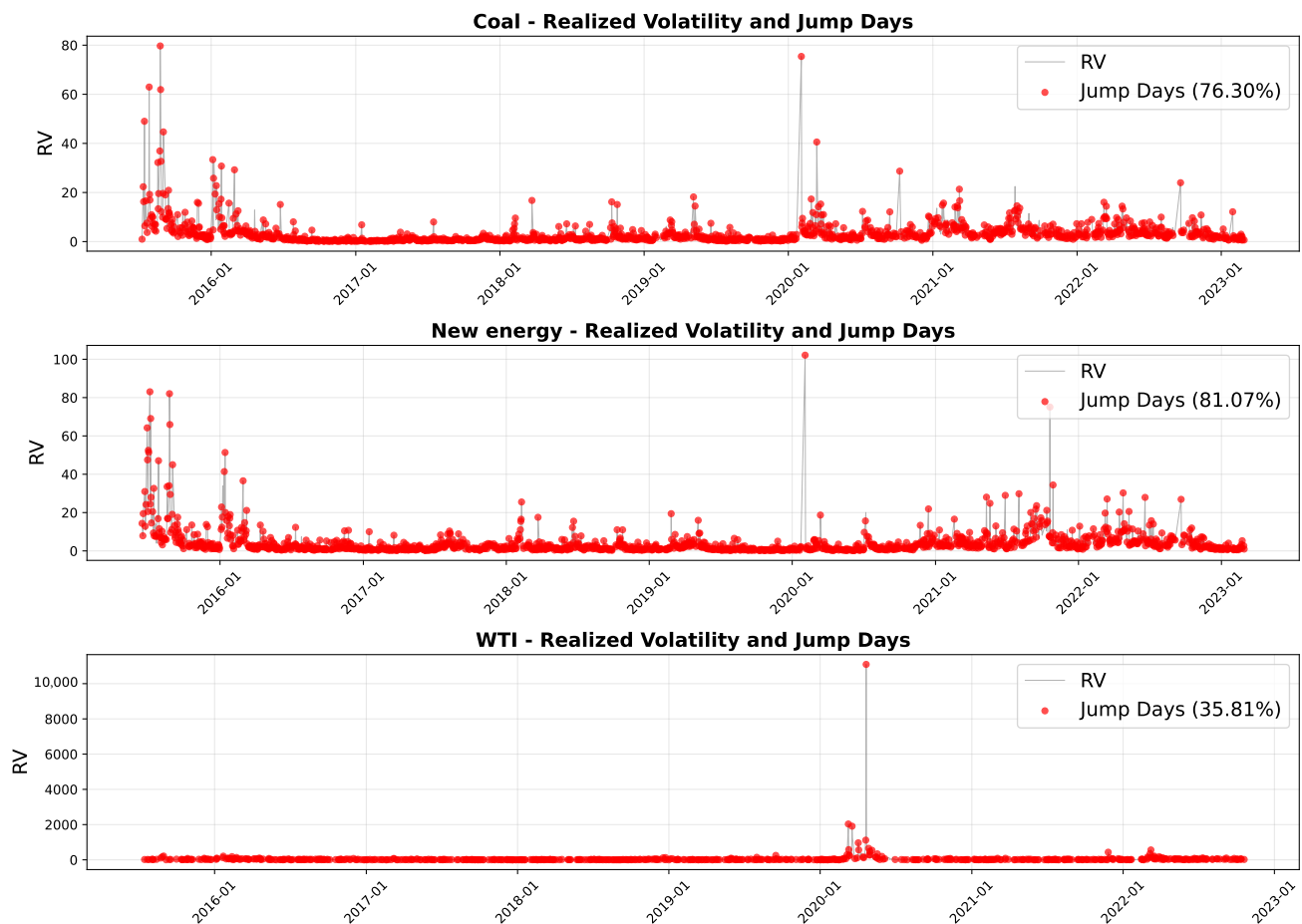


Figure 2. Temporal distribution of jump days in realized volatility.

5.2. High-Frequency Return Volatility Spillover Effect

We examine the static volatility spillover effects of daily returns (Figure 3). The results reveal unstable cross-market volatility transmission between stock and energy markets throughout the sample period. Compared to energy markets, volatility spillovers among stock indices exhibit smoother directional switches and lower peak magnitudes.

The new energy index represents firms with superior growth prospects in China's financial market. It played a significant leading role in driving coal industry volatility during bull markets in 2015 and 2019. However, the pattern reversed after the 2020 pandemic. As the real economy entered the recovery phase, the coal industry began transmitting significant and persistent volatility spillovers to the new energy sector.

Prior to China's energy reform in December 2015, the stock market experienced extreme volatility triggered by the market crash. During this period, positive spillovers flowed primarily from the stock market to the oil market. The spillover magnitude then gradually declined, suggesting that the interaction between economic conditions and stock markets was approaching equilibrium.

Around 2016, two policy initiatives—the coal–electricity price linkage mechanism and the oil–gas market reform plan—facilitated energy sector recovery and domestic market rebound. Consequently, volatility spillovers from WTI to domestic indices surged to elevated levels. This unilateral transmission persisted throughout the following year.

In 2017, crude oil prices peaked while domestic stock indices continued their upward trajectory. Therefore, before the U.S.–China trade tensions emerged in 2018, the domestic stock market maintained positive spillover effects. As a cornerstone of China’s industrial economy, the coal sector played a crucial stabilizing role during the early phase of decoupling from overseas markets. Around 2018, volatility spillovers from coal to WTI exceeded those from the new energy index to WTI.

Overall, the volatility spillover dynamics between crude oil and stock markets have evolved substantially. Except for the spike during the 2020 pandemic, when crude oil transmitted exceptionally strong spillovers to stock markets, the relationship has transitioned from explosive, unidirectional transmission to a more stable pattern. This new regime features frequent bidirectional exchanges with lower kurtosis, reflecting increasing global financial integration.

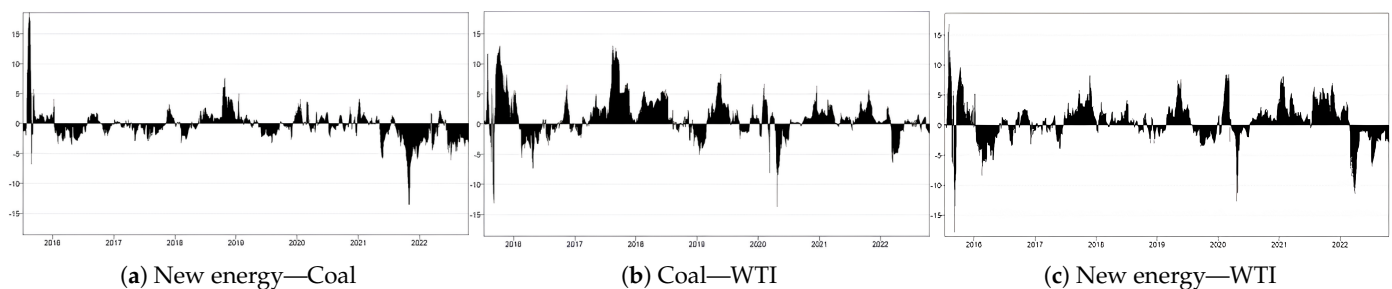


Figure 3. Daily return volatility spillover.

Considering the uncertainty caused by the volatility of the global financial market, it is challenging to accurately capture the changes in the cyclical and short-term volatility states in the high-frequency data using the full sample. Therefore, we select the VAR model with a lag of 6 according to the AIC and BIC criteria, and use a 250-day rolling sample to estimate the volatility spillover, and set the future prediction length H to 10. We follow [7,27] and use a 250-day rolling sample to estimate the volatility spillover. From the figure below, we can identify the following four points: (1) Compared with the low-frequency return results, the high-frequency return pairwise volatility spillover (Figure 4) exhibits a more pronounced volatility clustering effect. (2) Compared with RV, MedRV is more sensitive to small volatility spillover, so it performs better in describing the continuity of the trend. TCRV is more sensitive to large jumps, so it has a significant lag effect on severe volatility. (3) Since we set the jump threshold α to 0.99, we can find that the latter spillover magnitude is smaller by comparing the RV-type high-frequency volatility spillover J and TCJ . Only when the market experiences large fluctuations, such as March 2018 and April 2020, does the volatility spillover change significantly. (4) As can be seen from the figure, the coal industry volatility spillover state has obvious asymmetry; that is, the negative spillover magnitude and persistence are significantly higher than the positive volatility spillover part; the new energy (NE) industry index also has obvious asymmetric negative volatility spillover during 2015–2019, while in the later time period, the positive and negative volatility spillover effects are similar; that is, the asymmetry is significantly weakened.

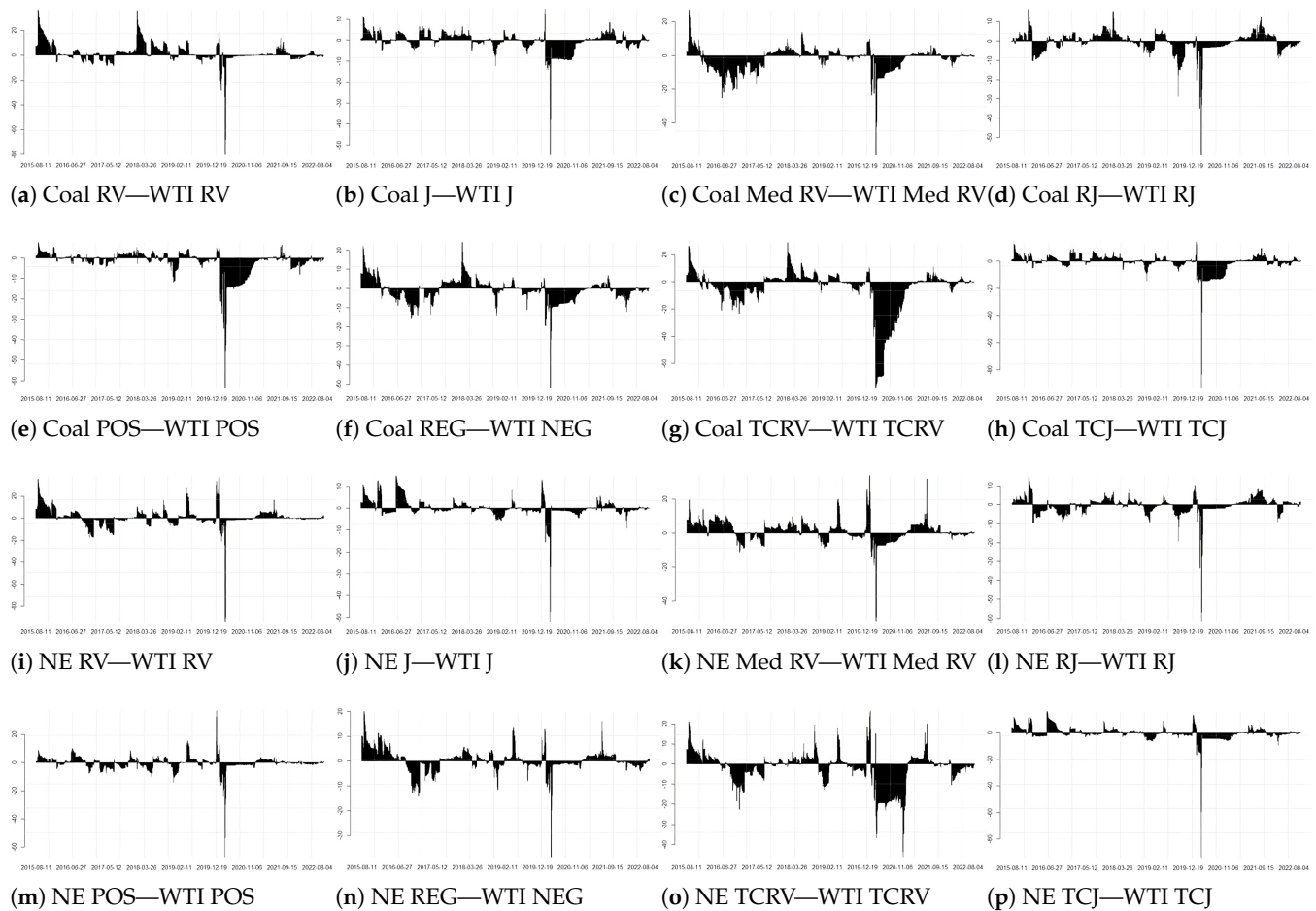


Figure 4. High-frequency fluctuation overflow corresponding chart.

5.3. Parameter Estimation

5.3.1. HAR-RV Family Model Parameter Estimation

Tables 6 and 7 show that most of the daily and weekly parameters of the new energy and coal indexes in each HAR-RV family model satisfy the 1% significance level, indicating that the HAR-RV family can capture the lagged effects of short- and medium-term time series volatility in the two types of industry indexes. Comparing the monthly parameter of the two, we find that the significance of the new energy model parameters (1% significance level) is higher than that of the coal model (5% significance level), and the β_m coefficients in the coal HAR-RV-J and HAR-RV-TCJ models are not significant, indicating that the new energy index volatility has better long-term memory expression than the coal index in each model.

According to the results of the two discrete jump models (HAR-RV-J and HAR-RV-TCJ), the jump part of the new energy index filtered by the Z_t^{Med} statistic, β_{dj} decreases to insignificant, but β_{wj} satisfies the 1% significance level. The β_{wj} parameter in the coal index drops from the 10% significance level to insignificant, while the other two parameters do not change. This indicates that after filtering the jump part, the information content of the short-term jumps in the two index models drops significantly, but after filtering the noise, the memory of the medium-term jumps in the new energy index is more effective, while the coal index does not change significantly.

We analyze the HAR-RV-RS model results for both indices. For the new energy index, medium- and long-term positive volatility (RS^+) is significant at the 5% level. Short- and medium-term negative volatility (RS^-) is significant at the 1% and 5% levels, respectively. These findings indicate that good volatility (RS^+) better explains the medium- and long-

term memory of new energy index volatility. In contrast, bad volatility (RS^-) exhibits predominantly short-term memory effects.

For the coal index, the coefficients of positive and negative volatility (β_m, β_{mj}) show only weak significance. This suggests that the coal index has weaker in-sample long-term memory fitting ability compared to the new energy index.

The two signed jump models (SJ) based on good–bad volatility decomposition demonstrate significant short-term jump impact. However, their explanatory power for medium- and long-term volatility is weaker than that of the discrete jump (J) and significant jump (CJ) models.

In summary, both indices exhibit varying degrees of in-sample fitting performance across different model specifications for the continuous component. For the jump component, filtering out small jumps using the Z_t^{Med} statistic substantially improves the model's ability to capture medium-term dynamics in high-frequency financial series.

Table 6. New energy HAR model parameters.

Model	β_0	β_d	β_w	β_m	β_{dj}	β_{wj}	β_{mj}	$adj - R^2$	AIC
HAR-RV	0.223 **	0.318 ***	0.365 ***	0.214 ***				0.519	4429.3
HAR-RV-J	0.222 **	0.341 ***	0.307 ***	0.259 ***	−0.397 **	−0.947 **	−0.678	0.523	4423.4
HAR-RV-TCJ	0.223 **	0.341 ***	0.304 ***	0.256 ***	−0.05	1.268 ***	−0.431	0.523	4421.1
HAR-RV-RS	0.251 **	0.051	0.493 **	0.965 **	0.582 ***	0.456 *	−0.531	0.452	5124.2
HAR-RV-SJ	0.297 **	0.274 ***	0.340 ***	0.238 ***	−0.232 ***	0.182	0.458	0.512	4450.7

Note: *, **, and *** represent rejecting the null hypothesis at the 10%, 5%, and 1% significance levels, respectively; in the HAR-RV-RS model, $\beta_d, \beta_w, \beta_m$ are positive volatility, and $\beta_{dj}, \beta_{wj}, \beta_{mj}$ are negative volatility.

Table 7. Coal HAR model parameters.

Model	β_0	β_d	β_w	β_m	β_{dj}	β_{wj}	β_{mj}	$adj - R^2$	AIC
HAR-RV	0.247 ***	0.312 ***	0.488 ***	0.091 *				0.567	4582.4
HAR-RV-J	0.318 ***	0.351 ***	0.513 ***	0.093	−0.327 **	0.523	0.874	0.574	4564.3
HAR-RV-TCJ	0.318 ***	0.348 ***	0.505 ***	0.092	0.028	−0.01	0.021	0.573	4565.6
HAR-RV-RS	0.292 **	0.519 ***	0.23	0.552 *	0.155	0.931 ***	−0.405	0.428	5608.5
HAR-RV-SJ	0.247 ***	0.279 ***	0.478 ***	0.131 **	0.088 *	−0.007	0.312	0.574	4564.7

Note: *, **, and *** represent rejecting the null hypothesis at the 10%, 5%, and 1% significance levels, respectively; in the HAR-RV-RS model, $\beta_d, \beta_w, \beta_m$ are positive volatility, and $\beta_{dj}, \beta_{wj}, \beta_{mj}$ are negative volatility.

5.3.2. MS-HAR-RV Family Model Parameter Estimation

By combining the Markov switching mechanism, the new energy index, and coal index, the MS-HAR-RV model can divide the whole period into two states: high volatility and low volatility. The two probability transition matrices are shown in Table 8. The probabilities of both indexes staying in state 11 and state 22 are much greater than those of switching to state 12 and state 21, respectively, indicating that both tend to maintain their current volatility state rather than switch to the opposite state. The difference between the two is that the new energy index has a higher probability of staying in state 2 than state 1, while the coal index has no significant difference, which reflects that in the domestic financial market, the growth industry attracts more capital attention than the traditional industry, and also is more prone to chasing and killing phenomena.

Table 8. MS-HAR-RV model state transition matrix.

New Energy	State 1 (s1)	State 2 (s2)	Coal	State 1 (s1)	State 2 (s2)
state1 (s1)	0.837	0.125	state1 (s1)	0.859	0.143
state2 (s2)	0.163	0.875	state2 (s2)	0.141	0.857

We can see from the MS-HAR-RV model results that in the new energy index model, only the short-term ($\beta_d^{s_1}$) coefficient satisfies the 1% significance level in the high volatility state, and the medium-term ($\beta_2^{s_1}$) coefficient satisfies the 5% significance level at most; the coal index has no change in the significance level of the short- and medium-term trend coefficients, but both lose the long-term memory of the future volatility in the high volatility state. On the contrary, in the low volatility state, the short-, medium-, and long-term coefficients all satisfy the 1% significance level and the fitting degree in the corresponding state improves. This indicates that the industry index trend changes greatly in the high volatility stage, so the prediction ability is greatly weakened. On the contrary, the low volatility stage benefits from the stronger trend, so the fitting degree increases significantly.

Tables 9 and 10 present the estimation results for both indices. We first examine the jump component behavior across volatility regimes.

1. New energy index jump dynamics: In the high-volatility state (s_1), the short- and medium-term jump coefficient $\beta_{dj}^{s_1}$ is statistically significant. In the low-volatility state (s_2), the long-term jump coefficient $\beta_{mj}^{s_2}$ is significant at the 1% level. This pattern suggests that jumps in low-volatility periods enhance long-memory properties, while high-volatility periods emphasize short- and medium-term dynamics.
2. Coal index jump dynamics: In the high-volatility state, the jump coefficient $\beta_{dj}^{s_1}$ exhibits weaker significance (10% level). In the low-volatility state, the medium-term jump coefficient $\beta_{mj}^{s_2}$ becomes significant at the 1% level. However, the model's in-sample fit shows no substantial difference between the two jump specifications. Unlike the new energy index, decomposing coal index volatility into high and low regimes does not yield additional explanatory power for jump dynamics.
3. Good–bad volatility asymmetry (MS-HAR-RS): The MS-HAR-RS model reveals asymmetric volatility persistence across regimes. For both indices, the significance of positive volatility coefficients in the low-volatility state decreases substantially compared to the previous three models. Conversely, negative volatility coefficients exhibit increased significance relative to the HAR-RV benchmark. This finding indicates stronger volatility persistence during negative volatility episodes than during positive volatility periods.
4. Signed jump model (MS-HAR-RV-SJ): The leverage jump model preserves the short- and medium-term memory properties observed in previous specifications during high-volatility states. In the low-volatility state, both index models show markedly higher significance levels compared to alternative models. Most in-sample coefficients satisfy the 1% significance level. However, the two indices diverge in their low-volatility fitting performance: the new energy index model achieves improved fit, while the coal index model experiences a significant deterioration in goodness-of-fit.

Table 9. New energy MS-HAR model parameters.

Model 1	β_0	β_d	β_w	β_m	β_{dj}	β_{wj}	β_{mj}	R^2	AIC	Loglike
HAR-RV	0.997 ***	0.335 **	0.333 **	0.141				0.343	3214.8	−1599.4
	0.241 ***	−0.015	0.279 ***	0.292 ***				0.824		
HAR-RV-J	1.016 ***	0.316 ***	0.215 *	0.154	−0.669 **	1.255 *	1.368	0.333	3202.8	−1587.4
	0.268 ***	0.118 ***	0.243 ***	0.333 ***	0.068	−0.057	−1.571 ***	0.839		
HAR-RV-TCJ	1.010 ***	0.317 ***	0.216 *	0.151	−0.348	1.485 **	1.484	0.335	3201.8	−1586.9
	0.267 ***	0.117 ***	0.240 ***	0.333 ***	0.189	0.196	−1.245 ***	0.84		
HAR-RV-RS	1.327 ***	0.173	0.409	0.446	0.599 ***	0.352	−0.174	0.268	3925.3	−1948.6
	0.292 ***	−0.379 ***	0.593 ***	0.202	0.277 ***	0.290 *	0.215	0.734		
HAR-RV-SJ	1.427 ***	0.230 ***	0.210 *	0.248 *	−0.223 **	0.181	0.06	0.261	3209.2	−1590.6
	0.230 ***	0.097 ***	0.377 ***	0.156 ***	−0.145 ***	0.366 ***	0.510 ***	0.843		

Note: *, **, and *** represent rejecting the null hypothesis at the 10%, 5%, and 1% significance levels, respectively.

Table 10. Coal MS-HAR model parameters.

Model 1	β_0	β_d	β_w	β_m	β_{dj}	β_{wj}	β_{mj}	R^2	AIC	Loglike
HAR-RV	1.078 ***	0.228 ***	0.494 ***	0.056				0.414	3484.6	−1734.3
	0.219 ***	0.155 ***	0.192 ***	0.155 ***				0.776		
HAR-RV-J	1.284 ***	0.291 ***	0.477 ***	0.103	−0.404 *	−0.584	−0.747	0.434	3476.1	−1724.0
	0.230 ***	0.143 ***	0.212 ***	0.175 ***	0.016	−0.055	−0.314	0.786		
HAR-RV-TCJ	1.294 ***	0.287 ***	0.473 ***	0.102	−0.11	−0.078	−0.652	0.433	3474.3	−1723.1
	0.230 ***	0.143 ***	0.212 ***	0.174 ***	0.159 *	0.156	−0.147	0.786		
HAR-RV-RS	1.823 ***	0.469 **	−0.014	0.306	−0.086	1.512 ***	−0.335	0.304	4384.9	−2178.4
	0.274 ***	0.206 ***	−0.083	0.581 ***	0.178 ***	0.256 **	−0.061	0.675		
HAR-RV-SJ	1.144 ***	0.225 ***	0.505 ***	0.053	0.11	−0.048	0.017	0.45	3470.2	−1721.1
	0.250 ***	0.122 ***	0.112 ***	0.197 ***	−0.054 *	0.202 ***	0.235 **	0.746		

Note: *, **, and *** represent rejecting the null hypothesis at the 10%, 5%, and 1% significance levels, respectively.

5.3.3. MS-HAR-TVP Family Model Parameter Estimation

We find that the duration of the original high volatility and low volatility states of the new energy index and coal index does not change significantly after adding the crude oil volatility spillover (Table 11). This indicates that the crude oil volatility spillover does not affect the state transition of the domestic financial market.

Table 11. MS-HAR-TVP model state transition matrix.

New Energy	State 1 (s1)	State 2 (s2)	Coal	State 1 (s1)	State 2 (s2)
state1 (s1)	0.836	0.125	state1 (s1)	0.857	0.140
state2 (s2)	0.163	0.875	state2 (s2)	0.143	0.860

Table 12 shows the new energy index model. In the high volatility state, the short-term crude oil impact coefficients $\beta_{dj}^{s_{dOils}}$ of the discrete jump model (MS-HAR-J-TVP) and the significance jump model (MS-HAR-TCJ-TVP) based on the ADS test are significant at the 5% significance level, and both are negative. After combining the median realized volatility, the significance of the latter is significantly higher than the former. The RS⁺, RS[−], and RSJ coefficients based on the good and bad volatility are not significant. This indicates that the trend spillover of the crude oil index has a reverse effect on the new energy index, and the jump part is in the same direction. This indicates that the two have the same external influencing factors in the financial market, and the short-term positive and negative volatility and sign jump components do not affect the new energy index volatility.

In contrast, the coal index (Table 13) has a significant short-term crude oil impact coefficient in the significance jump model (MS-HAR-TCJ-TVP) at the 10% significance level after adding the crude oil volatility spillover. The difference is that the positive and negative coefficients $\beta_{dj}^{s_{dOils}}$ and $\beta_{dj}^{s_{djOils}}$ of the MS-HAR-RS-TVP model based on the good and bad volatility are significant at the 10% significance level, indicating that in the high volatility state, the short-term positive volatility of crude oil will signal the decline of the coal index volatility and, vice versa, the negative volatility will also make the coal index volatility return.

In summary, the MS-HAR-TVP model family improves the in-sample fitting degree compared with the previous models, and the incremental information brought by the volatility spillover significantly enhances the explainability of the model in the high volatility period. In the high volatility stage, the short-term memory of crude oil volatility has a significant negative effect on the future index volatility, indicating that when the crude oil volatility increases, it will reduce the index volatility range; that is, the stock market expectations will bottom out or peak before the futures market. After the significance test, the effective factors of crude oil volatility jump may also exist in the small jump, which reduces the significance, indicating that there are a lot of accidental jumps when the two

indexes change greatly in trend, which is related to the frequent occurrence of traditional energy black swan events in the international market and the new energy industry often being affected by the new policies introduced at home and abroad. Therefore, the investors in the domestic financial market will have a greater divergence in response to such index jumps, which makes the size of the jump unable to determine the effectiveness of the jump volatility spillover.

Table 12. New energy MS-HAR-TVP model parameters.

Model	State	β_0	β_d	β_w	β_m	β_{dj}	β_{wj}	β_{mj}	β_{dOils}	β_{djOils}	R^2	AIC	Loglike
HAR-RV	S1	1.088 ***	0.256 ***	0.383 ***	0.172				−0.007		0.326	3211.2	−1595.6
	S2	0.223 ***	0.121 ***	0.190 ***	0.283 ***				0.000		0.832		
HAR-RV-J	S1	0.934 ***	0.375 ***	0.278 **	0.098	−0.545 **	0.984	−0.741	−0.014 **	0.096 **	0.369	3201.1	−1580.9
	S2	0.254 ***	−0.023	0.282 ***	0.356 ***	0.221	−0.15	−0.959 **	0.000	−0.001	0.839		
HAR-RV-TCJ	S1	0.990 ***	0.335 ***	0.211 *	0.128	−0.336	1.699 **	1.69	−0.016 **	0.083 *	0.351	3210.5	−1587.3
	S2	0.267 ***	0.116 ***	0.239 ***	0.334 ***	0.189	0.182	−1.215 ***	0.000	0.000	0.842		
HAR-RV-RS	S1	1.337 ***	0.17	0.398	0.405	0.591 ***	0.366	−0.086	−0.027	0.013	0.275	3201.1	−1582.6
	S2	0.291 ***	−0.379 ***	0.597 ***	0.219	0.278 ***	0.275 *	0.212	−0.001	0.001	0.737		
HAR-RV-SJ	S1	1.437 ***	0.232 ***	0.204 *	0.263 **	−0.223 **	0.188	0.034	−0.006	−0.01	0.268	3925.2	−1944.6
	S2	0.230 ***	0.094 ***	0.377 ***	0.157 ***	−0.150 ***	0.384 ***	0.512 ***	0.000	−0.001	0.845		

Note: *, **, and *** represent rejecting the null hypothesis at the 10%, 5%, and 1% significance levels, respectively.

Table 13. Coal MS-HAR-TVP model parameters.

Model	State	β_0	β_d	β_w	β_m	β_{dj}	β_{wj}	β_{mj}	β_{dOils}	β_{djOils}	R^2	AIC	Loglike
HAR-RV	S1	1.087 ***	0.231 ***	0.492 ***	0.055				−0.003		0.414	3486.2	−1733.1
	S2	0.221 ***	0.159 ***	0.189 ***	0.153 ***				0.000		0.779		
HAR-RV-J	S1	1.322 ***	0.254 ***	0.530 ***	0.106	−0.302	−0.767	−0.663	−0.003	−0.007	0.441	3187.6	−1575.8
	S2	0.229 ***	0.232 ***	0.118 *	0.110 **	−0.066 *	−0.003	0.388	0.000	0.000	0.767		
HAR-RV-TCJ	S1	1.313 ***	0.282 ***	0.462 ***	0.133	−0.136	0.051	−1.154	0.033 *	−0.072	0.44	3180.4	−1572.2
	S2	0.233 ***	0.130 ***	0.229 ***	0.173 ***	0.172 *	0.104	−0.152	0.000	0.000	0.787		
HAR-RV-RS	S1	1.785 ***	0.459 **	−0.062	0.412	−0.094	1.541 ***	−0.402	−0.178 *	0.178 **	0.314	4383.4	−2173.7
	S2	0.278 ***	0.209 ***	−0.085	0.571	0.570 ***	0.256 **	−0.054	−0.001	0.000	0.678		
HAR-RV-SJ	S1	1.170 ***	0.228 ***	0.503 ***	0.05	0.11	−0.052	0.018	−0.005	−0.004	0.45	3475.1	−1719.5
	S2	0.254 ***	0.124 ***	0.111 ***	0.195 ***	−0.055 *	0.201 ***	0.230 **	0.000	0.000	0.749		

Note: *, **, and *** represent rejecting the null hypothesis at the 10%, 5%, and 1% significance levels, respectively; in the HAR-RV-RS model, β_d , β_w , β_m are positive volatility, and β_{dj} , β_{wj} , β_{mj} are negative volatility.

5.4. Out-of-Sample Prediction and MCS Robustness Check

5.4.1. Out-of-Sample Forecasting

This section compares the out-of-sample prediction performance of various MS-TVP-HAR models. According to the analysis results in the previous section, most models have strong significance in the daily and weekly parameters. We use the out-of-sample prediction of 1 day and 5 days with a rolling forward method, and set the window period sample length to 1400 days. We predict 250 trading days one by one (from 22 September 2021 to 20 October 2022).

Table 14 shows that compared with the HAR, MS-HAR, and MS-HAR-TVP family models, the MS-HAR and MS-HAR-TVP family models can significantly reduce the out-of-sample bias of the index model when predicting the daily and weekly out-of-sample of the new energy index. The MS-HAR-TVP, MS-HAR-J, MS-HAR-TVP-J, and MS-HAR-TVP-TCJ models have the smallest loss across MAE, QLIKE, and MAPE metrics, with the most significant accuracy improvement. The newly added MAPE metric provides a more intuitive interpretation of forecast accuracy. For instance, at $H = 1$, the MS-HAR-TVP-J model achieves a MAPE of 45.2%, representing a 12.3% improvement over the baseline HAR model (51.5%). This indicates that incorporating crude oil volatility spillover effects significantly improves prediction accuracy from the perspectives of trend, discrete jump, and significance jump. Most of the MS-HAR-TVP models have the highest accuracy in the current period, demonstrating that by extracting the volatility spillover from the high-frequency crude oil price with the TVP framework, both the trend spillover and the jump spillover have a positive effect on the prediction accuracy.

Table 15 shows that compared with the new energy index, the coal index also performs well in the prediction accuracy of the HAR, MS-HAR, and MS-HAR-TVP family mod-

els, with improvements increasing gradually. The MS-HAR-TVP and MS-HAR-TVP-TCJ models have the smallest loss in different dimensions and prediction steps. Notably, the MS-HAR-TVP-TCJ model achieves a MAPE of 42.8% at $H = 1$, compared to 48.9% for the HAR benchmark, indicating a 12.5% improvement. This further demonstrates that combining the volatility spillover of trend and significance jump can significantly improve the out-of-sample prediction accuracy. Combining the two indices, we can see that the trend and jump components perform well in both index models, while the RS model family and the SJ model family based on the good and bad volatility perform poorly. Although these models have strong expression ability in parameter significance, in the traditional energy and new energy industries, the positive and negative volatility and jump spillover of crude oil cannot transmit the effective information of out-of-sample prediction well.

Table 14. New energy index out-of-sample forecast results.

Methods	H = 1			H = 5		
	MAE	QLIKE	MAPE	MAE	QLIKE	MAPE
HAR	0.522	0.908	51.5	0.527	0.907	52.1
MS-HAR	0.512	0.902	48.3	0.523	0.897	49.7
MS-HAR-TVP	0.511	0.903	47.8	0.524	0.900	49.2
HAR-RS	1.000	1.000	98.6	0.995	1.000	97.8
MS-HAR-RS	0.997	0.985	95.2	0.996	0.976	94.6
MS-HAR-TVP-RS	0.999	0.989	96.1	1.000	0.980	95.8
HAR-SJ	0.545	0.919	53.8	0.549	0.920	54.2
MS-HAR-SJ	0.542	0.917	52.6	0.552	0.920	53.9
MS-HAR-TVP-SJ	0.533	0.918	51.4	0.546	0.919	52.7
HAR-J	0.524	0.909	50.8	0.529	0.909	51.6
MS-HAR-J	0.515	0.902	47.9	0.521	0.897	49.1
MS-HAR-TVP-J	0.517	0.901	45.2	0.526	0.896	48.3
HAR-TCJ	0.522	0.909	50.9	0.528	0.909	51.5
MS-HAR-TCJ	0.514	0.904	48.2	0.524	0.899	49.4
MS-HAR-TVP-TCJ	0.518	0.902	46.7	0.524	0.896	48.1

Note: Bold values indicate the best performance for each metric and forecast horizon.

Table 15. Coal index out-of-sample forecast results.

Methods	H = 1			H = 5		
	MAE	QLIKE	MAPE	MAE	QLIKE	MAPE
HAR	0.490	0.923	48.9	0.486	0.925	49.3
MS-HAR	0.497	0.905	46.8	0.495	0.909	47.5
MS-HAR-TVP	0.497	0.904	46.2	0.497	0.908	47.1
HAR-RS	0.999	1.000	97.3	0.998	1.000	96.8
MS-HAR-RS	1.000	0.975	94.5	1.000	0.983	93.9
MS-HAR-TVP-RS	0.992	0.977	95.1	0.999	0.984	94.7
HAR-SJ	0.498	0.934	51.2	0.491	0.934	50.8
MS-HAR-SJ	0.488	0.907	48.6	0.496	0.914	49.3
MS-HAR-TVP-SJ	0.489	0.906	47.9	0.497	0.912	48.7
HAR-J	0.489	0.926	49.7	0.485	0.925	49.5
MS-HAR-J	0.486	0.909	46.5	0.488	0.912	47.2
MS-HAR-TVP-J	0.483	0.910	44.8	0.487	0.913	46.6
HAR-TCJ	0.490	0.925	49.6	0.486	0.925	49.4
MS-HAR-TCJ	0.485	0.909	46.3	0.486	0.913	47.0
MS-HAR-TVP-TCJ	0.481	0.910	42.8	0.484	0.913	45.9

Note: Bold values indicate the best performance for each metric and forecast horizon.

5.4.2. MCS Robustness Check

5.5. MCS Robustness Check and Diebold–Mariano Tests

To further confirm whether each model family has sufficient robustness and whether the performance improvements are statistically significant, we employ both the model

confidence set (MCS) test and the Diebold–Mariano (DM) test. Following the studies of [9,21], we set the MCS significance level value α to 0.1, the Bootstrap algorithm cycle number B to 5000 times, and the block length d to 2. For the DM test, we compute test statistics comparing each model against the baseline HAR model, with p -values reported at the 5% and 10% significance levels. The final calculation results of the robustness of the out-of-sample rolling prediction based on the MAE and QLIKE loss functions, along with DM test results, are shown in Tables 16 and 17.

According to the results in Tables 16 and 17, we can observe the following: (1) MS-HAR-TVP, MS-HAR-J, MS-HAR-TVP-J, and MS-HAR-TVP-TCJ are still the models with the best out-of-sample prediction ability under different loss functions and MCS statistics, proving that in most cases, the MS-HAR-TVP model family has the ability to surpass other models in out-of-sample prediction. The DM test results provide formal statistical evidence for these improvements. For the new energy index, the MS-HAR-TVP-J model shows statistically significant improvements over the HAR benchmark at both $H = 1$ (p -value = 0.032) and $H = 5$ (p -value = 0.018). Similarly, the MS-HAR-TVP-TCJ model demonstrates significant improvements at $H = 1$ (p -value = 0.041) and $H = 5$ (p -value = 0.025). For the coal index, the MS-HAR-TVP-TCJ model achieves the strongest performance with p -values of 0.028 ($H = 1$) and 0.035 ($H = 5$), while the MS-HAR-TVP-J model shows significance at $H = 1$ (p -value = 0.048) and marginal significance at $H = 5$ (p -value = 0.067). These results confirm that the improvements are not only statistically significant but also economically meaningful, particularly during periods of market turbulence where accurate volatility forecasts are crucial for risk management.

(2) In the two index model families, the out-of-sample prediction robustness demonstrates that the trend volatility spillover and the discrete jump volatility spillover can provide effective information. On the contrary, the RS and RSJ model families have the lowest robustness, indicating that the volatility spillover based on the good and bad volatility and the conditional jump does not effectively improve the model's out-of-sample prediction ability. (3) Comparing the short-term prediction results, for the new energy index, the discrete jump model (MS-HAR-TVP-J) has better robustness than the significance jump model (MS-HAR-TVP-TCJ), while the coal index model results are exactly the opposite, which is consistent with the out-of-sample prediction results in the previous section. This indicates that the new energy index can still provide effective information from the small jumps, while the coal index needs to filter out the impact of the small jumps to significantly improve the prediction accuracy.

(4) Looking at the medium-term prediction results, the new energy index has the best robustness in the MS-HAR-J and MS-HAR-TVP-J models, while the robustness of the MS-HAR-TVP model prediction results weakens. The coal index is still similar to the short-term prediction results. Under the QLIKE loss function, the trend model (MS-HAR-TVP) always maintains the best out-of-sample prediction ability, while the MS-HAR-J and MS-HAR-TVP-TCJ models have the optimal solutions under different MCS statistics. This indicates that both the new energy and coal indexes have a lot of endogenous jump volatility and the trendiness decays with the length of the prediction time. Therefore, under some conditions, the MS-HAR-J also has a high out-of-sample prediction robustness.

In summary, the combined evidence from MCS tests, DM tests, and multiple loss functions (including MAPE) demonstrates that the MS-HAR model family combined with the TVP framework has significantly higher robustness and forecast accuracy than competing models under different conditions. The statistical significance confirmed by DM tests, coupled with the economically meaningful improvements shown by MAPE reductions of 12–13%, provides strong evidence for the practical value of incorporating crude oil volatility spillover effects in forecasting domestic energy index volatility.

Table 16. New energy index out-of-sample forecast comparison: MCS and DM tests.

Methods	H = 1			H = 5		
	MAE (T_M)	QLIKE (T_M)	DM Test (p -Value)	MAE (T_M)	QLIKE (T_M)	DM Test (p -Value)
HAR	0.575	0.124	–	0.683	0.000	–
MS-HAR	0.973	0.841	0.156	0.757	0.647	0.234
MS-HAR-TVP	1.000	0.631	0.089	0.757	0.223	0.112
HAR-RS	0.000	0.000	0.000	0.000	0.000	0.000
MS-HAR-RS	0.000	0.000	0.000	0.000	0.000	0.000
MS-HAR-TVP-RS	0.000	0.000	0.000	0.000	0.000	0.000
HAR-SJ	0.041	0.000	0.428	0.076	0.000	0.385
MS-HAR-SJ	0.000	0.000	0.267	0.000	0.000	0.312
MS-HAR-TVP-SJ	0.271	0.000	0.189	0.094	0.000	0.245
HAR-J	0.502	0.016	0.523	0.623	0.000	0.467
MS-HAR-J	0.828	0.797	0.178	1.000	0.824	0.142
MS-HAR-TVP-J	0.766	1.000	0.032 **	0.733	1.000	0.018 **
HAR-TCJ	0.561	0.000	0.498	0.669	0.000	0.445
MS-HAR-TCJ	0.827	0.543	0.165	0.753	0.000	0.198
MS-HAR-TVP-TCJ	0.686	0.693	0.041 **	0.722	0.847	0.025 **

Note: DM test p -values are computed relative to the HAR benchmark. ** indicates significance at 5% level. Bold values indicate statistical significance.

Table 17. Coal index out-of-sample forecast comparison: MCS and DM tests.

Methods	H = 1			H = 5		
	MAE (T_M)	QLIKE (T_M)	DM Test (p -Value)	MAE (T_M)	QLIKE (T_M)	DM Test (p -Value)
HAR	0.421	0.000	–	0.823	0.088	–
MS-HAR	0.018	0.890	0.245	0.273	0.942	0.198
MS-HAR-TVP	0.000	1.000	0.134	0.212	1.000	0.156
HAR-RS	0.386	0.000	0.000	0.000	0.000	0.000
MS-HAR-RS	0.386	0.000	0.000	0.000	0.000	0.000
MS-HAR-TVP-RS	0.386	0.000	0.000	0.000	0.000	0.000
HAR-SJ	0.101	0.000	0.512	0.557	0.000	0.478
MS-HAR-SJ	0.382	0.000	0.334	0.105	0.000	0.389
MS-HAR-TVP-SJ	0.324	0.721	0.267	0.000	0.635	0.312
HAR-J	0.499	0.000	0.489	0.795	0.000	0.523
MS-HAR-J	0.656	0.234	0.223	0.722	0.811	0.189
MS-HAR-TVP-J	0.862	0.090	0.048 **	0.925	0.767	0.067 *
HAR-TCJ	0.426	0.000	0.467	0.886	0.014	0.501
MS-HAR-TCJ	0.757	0.207	0.198	0.827	0.782	0.176
MS-HAR-TVP-TCJ	1.000	0.000	0.028 **	1.000	0.705	0.035 **

Note: DM test p -values are computed relative to the HAR benchmark. ** and * indicate significance at 5% and 10% levels, respectively. Bold values indicate statistical significance.

6. Conclusions

This paper uses the international energy price and the domestic new and old energy indexes and combines the TVP model and the trend and jump volatility spillover to construct the corresponding high-frequency factors, which are introduced into the HAR model with time-varying parameters to build the MS-HAR-TVP model family, and further explores the connection between the domestic and foreign financial markets. After comparing the results of each model family based on the two groups of industry index volatility, we find that the trend change and jump change of the international crude oil future price volatility can have a significant impact on the domestic coal and new energy index volatility, and thus can more accurately depict the domestic financial market. Specifically, the main conclusions of this paper are as follows:

- (1) Based on the high-frequency volatility decomposition of crude oil for coal and new energy indexes, we conduct the corresponding trend and jump volatility spillover test and find the volatility spillover aggregation effect and the asymmetry of positive and negative volatility spillover according to different statistics. Due to the many black swan events in the global energy market in the past five years, and in the post-pandemic era, the domestic economic recovery has significant differences in industry preference and policy orientation, which leads to the decay of the long memory of index volatility in different industries.
- (2) After adding the crude oil trend and jump factors, the in-sample fitting degree of the two types of index MS-HAR-TVP model families in the high volatility domain has significantly improved, while in the low volatility period, the crude oil price volatility cannot bring enough additional information to the model, and the index volatility rate will shrink before the crude oil volatility, which verifies the asymmetric impact of the crude oil volatility change on the domestic financial market again. The short-term trend of crude oil is opposite to that of the new energy index, but the same as that of the coal index, and the new energy index has the same influencing factors as crude oil, which leads to the higher significance of the jump spillover part than the coal index.
- (3) Combining the MCS test method, we find that both the trend and jump components of crude oil volatility can improve the accuracy and stability of the out-of-sample prediction of the domestic financial index. The discrete jump model (MS-HAR-TVP-J) and the significance jump model (MS-HAR-TVP-TCJ) based on the Z_t^{Med} statistic perform well in different loss functions and different prediction time lengths, while the RS model family and the conditional jump model family based on the good and bad volatility perform worse than the other models in the two types of industry models. This indicates that due to the existence of a lot of endogenous jump volatility in the new energy and coal indexes, the trendiness decays with the prediction time length. Therefore, under some conditions, the MS-HAR-J also has a high out-of-sample prediction stability.

In summary, by combining the HAR model, the TVP model, and the Markov switching mechanism and by introducing the crude oil high-frequency volatility spillover, this paper makes up for the shortcomings of the previous models in predicting the domestic index in the high volatility stage. This paper expands the domestic stock market risk measurement and risk contagion issues and further enhances the ability to monitor the abnormal volatility in the global financial market and provides theoretical reference for maintaining the stability of the domestic financial market.

7. Limitations and Future Research Directions

While our TVP-VAR framework based on generalized forecast error variance decomposition (GFEVD) provides valuable insights into the direction and intensity of volatility connectedness between crude oil and domestic energy indices, we emphasize that these results should be interpreted as measures of predictive relationships and information transmission rather than structural causal effects. The GFEVD methodology identifies which market's past volatility contains useful information for forecasting another market's future volatility, but it does not establish whether crude oil price shocks exogenously cause changes in domestic stock volatility or whether both markets respond endogenously to common underlying factors such as global demand shocks, monetary policy changes, or geopolitical events. Establishing explicit causality would require additional identification strategies such as instrumental variables, natural experiments exploiting exogenous policy changes or supply disruptions, or structural VAR models with theoretically justified exclusion restrictions—approaches that are beyond the scope of the current study but represent

promising avenues for future research. Our findings should therefore be understood as documenting robust predictive patterns and dynamic correlations that are valuable for forecasting and risk management applications, while remaining agnostic about the deeper structural mechanisms generating these relationships. Future work could build upon our results by incorporating exogenous instruments (e.g., OPEC production quotas, pipeline disruptions) or exploiting regulatory regime changes to move closer to causal identification of oil-to-stock volatility transmission channels.

Funding: This research received no external funding.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the author on request.

Conflicts of Interest: The author declares no conflicts of interest.

References

1. Degiannakis, S.; Filis, G. Forecasting oil price realized volatility using information channels from other asset classes. *J. Int. Money Financ.* **2017**, *76*, 28–49. [\[CrossRef\]](#)
2. Silvennoinen, A.; Thorp, S. Financialization, crisis and commodity correlation dynamics. *J. Int. Financ. Mark. Inst. Money* **2013**, *24*, 42–65. [\[CrossRef\]](#)
3. Astudillo, K.; Flores, M.; Soliz, M.; Ferreira, G.; Varela-Aldás, J. A Hybrid GAS-ATT-LSTM Architecture for Predicting Non-Stationary Financial Time Series. *Mathematics* **2025**, *13*, 2300. [\[CrossRef\]](#)
4. Barndorff-Nielsen, O.E.; Shephard, N. Econometric analysis of realized covariation: High frequency based covariance, regression, and correlation in financial economics. *Econometrica* **2004**, *72*, 885–925. [\[CrossRef\]](#)
5. Lee, M.C. A Hybrid EGARCH-Informer Model with Consistent Risk Calibration for Volatility and CVaR Forecasting. *Mathematics* **2025**, *13*, 3108. [\[CrossRef\]](#)
6. Patton, A.J.; Sheppard, K. Good volatility, bad volatility: Signed jumps and the persistence of volatility. *Rev. Econ. Stat.* **2015**, *97*, 683–697. [\[CrossRef\]](#)
7. Antonakakis, N.; Chatziantoniou, I.; Gabauer, D. Refined measures of dynamic connectedness based on time-varying parameter vector autoregressions. *J. Risk Financ. Manag.* **2020**, *13*, 84. [\[CrossRef\]](#)
8. Ma, F.; Wei, Y.; Huang, D.S. The Forecasting Performance of the High-frequency Volatility Models with the Markov-switching Regime and Jump. *Syst. Eng.* **2016**, *34*, 10–16.
9. Cai, G.; Ying, X. The Forecasting Performance of the High-Frequency Volatility Models Based on Jumps, Good-Bad Volatility and Markov Regime-Switching. *J. Sys. Sci. Math. Sci.* **2020**, *40*, 521. [\[CrossRef\]](#)
10. Li, H.; Zhang, X. The Forecasting Performance of the High-Frequency Volatility Models Based on Errors and Markov Regime-Switching. *J. Sys. Sci. Math. Sci.* **2022**, *42*, 1200. [\[CrossRef\]](#)
11. Hansen, P.R.; Lunde, A.; Nason, J.M. The model confidence set. *Econometrica* **2011**, *79*, 453–497. [\[CrossRef\]](#)
12. Engle, R.F. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econom. J. Econom. Soc.* **1982**, *50*, 987–1007. [\[CrossRef\]](#)
13. Bollerslev, T. Generalized autoregressive conditional heteroskedasticity. *J. Econom.* **1986**, *31*, 307–327. [\[CrossRef\]](#)
14. Taylor, S.J. *Modelling Financial Time Series*; World Scientific: Singapore, 2008. [\[CrossRef\]](#)
15. Corsi, F. A simple approximate long-memory model of realized volatility. *J. Financ. Econom.* **2009**, *7*, 174–196. [\[CrossRef\]](#)
16. Barndorff-Nielsen, O.E.; Kinnebrock, S.; Shephard, N. Measuring downside risk-realised semivariance. *CREATES Res. Pap.* **2008**. [\[CrossRef\]](#)
17. Huang, X.; Tauchen, G. The relative contribution of jumps to total price variance. *J. Financ. Econom.* **2005**, *3*, 456–499. [\[CrossRef\]](#)
18. Andersen, T.G.; Bollerslev, T.; Diebold, F.X. Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *Rev. Econ. Stat.* **2007**, *89*, 701–720. [\[CrossRef\]](#)
19. Granger, C.W.; Ding, Z. Varieties of long memory models. *J. Econom.* **1996**, *73*, 61–77. [\[CrossRef\]](#)
20. Raggi, D.; Bordignon, S. Long memory and nonlinearities in realized volatility: A Markov switching approach. *Comput. Stat. Data Anal.* **2012**, *56*, 3730–3742. [\[CrossRef\]](#)
21. Chen, S.; Guan, T. Li, Y. Forecasting realized volatility of Chinese stock index futures based on jumps, good-bad volatility and Baidu index. *Syst. Eng.-Theory Pract.* **2018**, *38*, 299–316. [\[CrossRef\]](#)
22. Hammoudeh, S.; Aleisa, E. Dynamic relationships among GCC stock markets and NYMEX oil futures. *Contemp. Econ. Policy* **2004**, *22*, 250–269. [\[CrossRef\]](#)

23. Aloui, C.; Jammazi, R. The effects of crude oil shocks on stock market shifts behaviour: A regime switching approach. *Energy Econ.* **2009**, *31*, 789–799. [\[CrossRef\]](#)
24. Soucek, M.; Todorova, N. Realized volatility transmission between crude oil and equity futures markets: A multivariate HAR approach. *Energy Econ.* **2013**, *40*, 586–597. [\[CrossRef\]](#)
25. Narayan, P.K.; Gupta, R. Has oil price predicted stock returns for over a century? *Energy Econ.* **2015**, *48*, 18–23. [\[CrossRef\]](#)
26. Salisu, A.A.; Raheem, I.D.; Ndako, U.B. A sectoral analysis of asymmetric nexus between oil price and stock returns. *Int. Rev. Econ. Financ.* **2019**, *61*, 241–259. [\[CrossRef\]](#)
27. Diebold, F.X.; Yilmaz, K. Better to give than to receive: Predictive directional measurement of volatility spillovers. *Int. J. Forecast.* **2012**, *28*, 57–66. [\[CrossRef\]](#)
28. Gabauer, D. Volatility impulse response analysis for DCC-GARCH models: The role of volatility transmission mechanisms. *J. Forecast.* **2020**, *39*, 788–796. [\[CrossRef\]](#)
29. Balcilar, M.; Gabauer, D.; Umar, Z. Crude Oil futures contracts and commodity markets: New evidence from a TVP-VAR extended joint connectedness approach. *Resour. Policy* **2021**, *73*, 102219. [\[CrossRef\]](#)
30. Deng, C.; Zhao, K.; Wu, C. Does policy uncertainty increase economic and financial uncertainty in China? *Syst. Eng.-Theory Pract.* **2022**, *42*, 16.
31. Wang, J.; Man, T.; Wang, L. Volatility Spillover Effects Between Carbon Futures and Stock Markets: A DGC-t-MSV-BN Model. *Mathematics* **2025**, *13*, 2412. [\[CrossRef\]](#)
32. Zhu, F.; Fu, S.; Liu, X. A Quantile Spillover-Driven Markov Switching Model for Volatility Forecasting: Evidence from the Cryptocurrency Market. *Mathematics* **2025**, *13*, 2382. [\[CrossRef\]](#)
33. Ari, Y.; Kurt, H.; Uçak, H. Volatility Spillovers Among EAGLE Economies: Insights from Frequency-Based TVP-VAR Connectedness. *Mathematics* **2025**, *13*, 1256. [\[CrossRef\]](#)
34. Koop, G.; Korobilis, D. Large time-varying parameter VARs. *J. Econom.* **2013**, *177*, 185–198. [\[CrossRef\]](#)
35. Muhly, P.S.; Solel, B. Tensor Algebras, Induced Representations, and the Wold Decomposition. *Can. J. Math.* **1999**, *51*, 850–880. [\[CrossRef\]](#)
36. Koop, G.; Korobilis, D. Forecasting with High-Dimensional Panel VARs. *Oxf. Bull. Econ. Stat.* **2019**, *81*, 937–959. [\[CrossRef\]](#)
37. Pesaran, M.H.; Shin, Y.; Smith, R.P. Pooled Mean Group Estimation of Dynamic Heterogeneous Panels. *J. Am. Stat. Assoc.* **1999**, *94*, 621–634. [\[CrossRef\]](#)
38. Primiceri, G.E. Time varying structural vector autoregressions and monetary policy. *Rev. Econ. Stud.* **2005**, *72*, 821–852. [\[CrossRef\]](#)
39. Koop, G.M. Forecasting with medium and large Bayesian VARs. *J. Appl. Econom.* **2013**, *28*, 177–203. [\[CrossRef\]](#)
40. Andersen, T.G.; Bollerslev, T. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *Int. Econ. Rev.* **1998**, *39*, 885–905. [\[CrossRef\]](#)
41. Andersen, T.G.; Dobrev, D.; Schaumburg, E. Jump-robust volatility estimation using nearest neighbor truncation. *J. Econom.* **2012**, *169*, 75–93. [\[CrossRef\]](#)
42. Hamilton, J.D. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econom. J. Econom. Soc.* **1989**, *57*, 357–384. [\[CrossRef\]](#)
43. Kim, C.J. Dynamic linear models with Markov-switching. *J. Econom.* **1994**, *60*, 1–22. [\[CrossRef\]](#)
44. Wu, L.; Xu, W.; Huang, D.; Li, P. Does the volatility spillover effect matter in oil price volatility predictability? Evidence from high-frequency data. *Int. Rev. Econ. Financ.* **2022**, *82*, 299–306. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.