

Article

Nonlinear Dynamic Modeling of Rotor-Bearing Systems with Combined Support Considering Coupled Motion: Numerical and Experimental Studies

Lei Li ^{1,2,3,4,*}, Tianyue Ma ¹, Feng Liang ¹ and Fei Xie ¹

¹ School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110819, China; 2370201@stu.neu.edu.cn (T.M.); liangf@mails.neu.edu.cn (F.L.); 20214725@stu.neu.edu.cn (F.X.)

² Guangxi Key Laboratory of Automobile Components and Vehicle Technology, Guangxi University of Science and Technology, Liuzhou 545006, China

³ Foshan Graduate School of Innovation, Northeastern University, Foshan 528312, China

⁴ National Key Laboratory of Particle Transport and Separation Technology, Tianjin 300180, China

* Correspondence: lilei2023@mail.neu.edu.cn; Tel.: +86-24-83680540

Abstract

The elastic support structure is widely employed in rotor systems and has an important influence on the nonlinear vibration of such systems. Nevertheless, coupled motion between elastic supports and bearings has not been taken into account, and the coupling effect of these two components on rotor dynamics remains insufficiently elucidated. Therefore, this work presents a bearing force model considering the motion of the elastic support. Subsequently, this work presents a new rotor-bearing dynamics model, in which the coupled motion between elastic supports and bearings is explicitly accounted for. Moreover, the coupling effect of elastic supports and bearings is systematically investigated through analyses of frequency–amplitude responses, waterfall plots, contact loads of bearings, operational deflection shape, and bifurcation diagrams. To further reveal this coupling effect, the nonlinear vibration behaviors of the rotor-bearing system with elastic support are analyzed under different bearing initial clearances. Finally, the experiments on rotor test rigs with and without elastic supports are conducted to validate the accuracy of the proposed dynamic model. Both simulation and experimental results indicate that elastic supports mitigate the nonlinear vibration of the rotor-bearing system; additionally, elastic support could reduce the bearing reaction forces and contact loads. Moreover, elastic supports alter the operational deflection shape of the rotor-bearing system.

Keywords: rotor system; elastic support; dynamic analysis; nonlinear vibration; bearing

MSC: 74H45; 74S05



Academic Editor: Yiu Yin Raymond Lee

Received: 3 October 2025

Revised: 7 November 2025

Accepted: 10 November 2025

Published: 18 November 2025

Citation: Li, L.; Ma, T.; Liang, F.; Xie, F. Nonlinear Dynamic Modeling of Rotor-Bearing Systems with Combined Support Considering Coupled Motion: Numerical and Experimental Studies. *Mathematics* **2025**, *13*, 3694. <https://doi.org/10.3390/math13223694>

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1. Introduction

Rotor-bearing systems (RBSs) serve as important dynamic components in a broad range of rotating machinery, including aero-engines, gas turbines, and steam turbines. During actual operation, rotor systems are subject to numerous nonlinear factors, which induce nonlinear vibration behaviors [1–5]. For instance, the rubbing force can be induced by rotor-casing contact, and the nonlinear bearing forces, etc. [6–9]. These nonlinear factors interact with each other, leading to complex nonlinear vibration behaviors in the rotor system [10–13]. Especially after the rotor adopts a combined support, the elastic support structure causes significant changes in the vibration of the rotor and the bearing. However,

existing research on elastic support-rotor-bearing systems is not in-depth, and the effect of elastic support on the nonlinearity of the bearing is also unclear. Therefore, analyzing rotor-bearing systems' nonlinear vibration response under elastic support has theoretical and practical significance.

In recent years, a number of scholars have studied nonlinear vibration behaviors of rotor-bearing systems. Taking nonlinear bearing forces into account, scholars have studied the vibration characteristics of Jeffcott rotors [14–16]. Considering misalignment and unbalance faults, the nonlinear vibration characteristics of rotor systems were investigated [17–19]. Kim et al. and Cao et al. have separately investigated the nonlinear dynamic behaviors of cracked rotor systems and blade-rotor assemblies, respectively [20–22]. Furthermore, many scholars have adopted the lumped mass method or the finite element method to establish rotor dynamics models [23–26]. These methods can not only better consider the influence of rotor weight and size on its elastic deformation but also bring in nonlinear factors, including bearings, cages, and squeeze film dampers, into the rotor system. Some scholars have established the static models of rolling bearings considering multiple degrees of freedom [27–29]. The bearing reaction forces acting on the shaft are imposed on the nonlinear rotor system to investigate the effects of variations in bearing parameters on the system's nonlinear dynamic behaviors. Building on the static model of the bearing, some other scholars have established quasi-static and dynamic models of rolling bearings [30–32]. The above-mentioned research has carried out meaningful work within the realm of nonlinear vibration of rotor-bearing systems. However, these studies have not focused on the changes in nonlinear vibration characteristics when the rotor system is influenced by elastic supports.

The support structure in the rotor system typically consists of bearings (i.e., rolling bearings and sliding bearings) and elastic support structures (e.g., squirrel cage, elastic ring, squeeze film damper, etc.). Firstly, for rolling bearings, they have nonlinear factors such as bearing clearance, Raceway defects, Hertz contact force, and variable flexibility vibration [28,33–36]. Subsequently, the nonlinearity of sliding bearings mainly comes from oil film force or air film force [37–41]. The studies conducted by the aforementioned researchers have fully considered the effect of the complex nonlinear factors of bearings on rotor response, while exploring the influence of nonlinear bearing forces on the periodic, quasi-periodic, and chaotic motion of the rotor system [42–45]. However, in order to deal with the complex and variable vibrations in rotating machinery, the support systems of rotors typically adopt a combined support consisting of bearings and elastic components, which increases the complexity of the mechanical structure. It also modifies the laws governing how bearings affect the nonlinear responses of the rotor system [46–50].

The modeling of elastic supports in rotor systems is predominantly addressed via two methodological approaches. The first approach neglects the mass of the support structure, idealizing it purely as a spring element, which is characterized by either linear [51–65] or nonlinear force–displacement relationships [66–81]. For instance, Ouyang [52] conducted research on the nonlinear dynamic characteristics of a dual-rotor system equipped with an active elastic support dry-friction damper. Wang [53] analyzed the vibration characteristics of a fan rotor considering the elastic deformation of the squirrel cage. Shao [54] established a dynamic model of a turbine rotor and explored the influence of the nonlinear oil film force induced by the SFD. In addition, some scholars consider the elastic ring [57–60] and the squirrel cage [61–65] as linear springs when investigating the vibration characteristics of rotor systems. Furthermore, Sun [72] proposed a variable stiffness model considering the contact deformation of the elastic ring. Tian [73] developed the equivalent stiffness and equivalent damping matrices of the squirrel cage to consider the influence on the SFD-bearing-rotor system. The second, more advanced approach models the elastic

support as a lumped mass point, thereby accounting for both its mass and stiffness properties [82,83]. A key advantage of this second category is its inherent capability to incorporate the degrees of freedom of the support structure itself, providing a more comprehensive dynamic representation.

Building upon this foundation, the modeling framework presented in this work introduces an improvement: the displacement of the squirrel cage is incorporated into the bearing force model. This improvement effectively couples the motion of the squirrel cage with that of the bearing, leading to a representation that more closely aligns with physical reality. This improvement is necessary for the accurate modeling of the combined support system, whereas previous studies have concentrated on other factors and neglected this coupling effect.

Through the above literature reviews, it can be found that scholars have carried out some meaningful and interesting studies on the nonlinear vibrations of the RBS. Building upon prior research, this study introduces enhancements. The specific contributions are outlined as follows:

1. A modeling framework is presented by introducing the displacement of the squirrel cage into the bearing force model, which aligns more closely with the actual scenario.
2. Interesting and novel vibration behaviors are found, which are not revealed in the existing studies. The action mechanism of elastic supports on the nonlinear responses of RBS is considered. Furthermore, the effect of elastic supports and the corresponding vibration behaviors have been verified through experimental tests, and the effectiveness of the proposed model and the findings from simulation-based analyses have been validated.

This paper is structured as follows. Section 2 introduces the mathematical simulation model developed in this work. In Section 3, the influence mechanism of elastic supports on the vibration of rolling bearings themselves and the nonlinear response of rotor systems is discussed. In Section 4, the experiments are conducted with different supports, and the proposed model and results are further verified. Finally, conclusions are given in Section 5.

2. Mathematical Modeling

In this section, the governing equations describing the combined support-rotor system with squirrel cage elastic elements taken into account are derived, where the combined support includes a squirrel cage (referred to as SC) elastic support and bearings (see Figure 1a). To more clearly reveal the responses of the squirrel cage-bearing combined support-rotor system, a comparative study on the vibration characteristics of the rotor system with and without the squirrel cage has been conducted, and the sketch of the rotor system without SC can be seen in Figure 1b. A point to note is that the motions of the bearing outer race and SC are incorporated into the combined support-rotor system model (see Figure 1c), and subscript n represents the index of the combined support, which is equal to 1 or 2. Figure 1d illustrates the movement of bearing races with combined support. See Figure 1e, where the shaft is dynamically modeled using four-degrees-of-freedom Timoshenko beam elements.

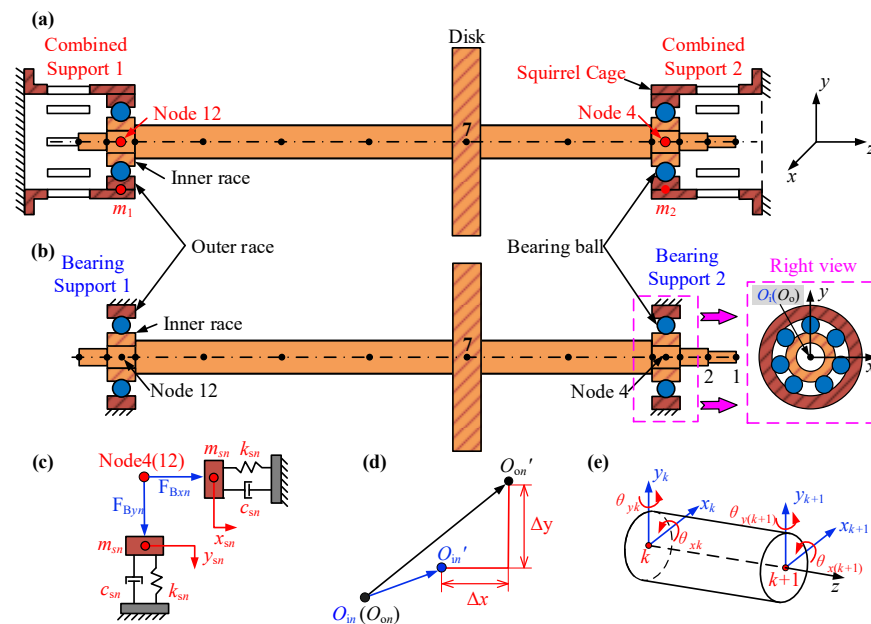


Figure 1. Sketch of a combined support-rotor system. (a) The rotor system with squirrel cages. (b) The rotor system without squirrel cages (SCs). (c) The combined support model. (d) Schematic diagram of the movement of bearing races with combined support. (e) Timoshenko beam element.

2.1. Bearing Force Model Considering the Movement of Elastic Supports

This subsection presents a bearing force model that considers the movement of the bearing outer race and the SC. Each combined support includes a rolling bearing, which is modeled by Hertz contact theory. Figure 2 illustrates the schematic diagram of the bearing, whose components include the inner race, outer race, and rolling balls. An interference fit is adopted for the connection between the inner races and the shaft, while the outer race of the bearing is mounted in the squirrel cage elastic support. Since the movement of the squirrel cage is considered, the outer races of bearings are not fixed.

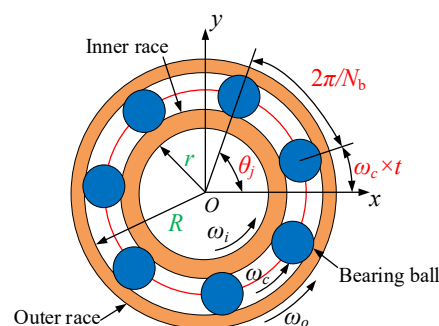


Figure 2. Schematic diagram of the simplified rolling bearing model.

The tangential velocities of the bearings' outer (v_o) and inner races (v_i) are

$$\begin{cases} v_i = \omega_i \times r \\ v_o = \omega_o \times R \end{cases} \quad (1)$$

where ω_i represents the angular velocity of the inner race, which is equal to the rotational speed of the shaft; ω_o represents the angular velocity of the outer race, and its value is 0; r and R denote the radius of the inner and outer races, respectively.

The angular velocity of the bearing cage can be obtained as

$$\omega_c = \frac{(v_i + v_o)/2}{(r + R)/2} = \frac{\omega_i \times r + \omega_o \times R}{r + R} \quad (2)$$

Furthermore, the angle of j -th rolling element at time t may be computed as follows:

$$\theta_j = \frac{2\pi(j-1)}{N} + \omega_c t, \quad (j = 1, 2, \dots, N) \quad (3)$$

in which N denotes the number of rolling balls.

For the combined support rotor system in practical scenarios, the outer ring of the bearing is mounted on the SC, and the SC is prone to deformation. Consequently, the outer ring of the bearing is not stationary; rather, its displacement is identical to that of the SC. More specifically, when taking into account the coupled motion between the bearing and the SC, the displacement of the squirrel cage is incorporated into the analysis of bearing deformation. The deformation of the n -th combined support and the mathematical expression for the j -th rolling ball is

$$\delta_{nj} = (x_{sn} - x_m) \cos \theta_j + (y_{sn} - y_m) \sin \theta_j - r_0 \quad (4)$$

where x_{sn} and y_{sn} represent the vibration displacements of the n -th squirrel cage. x_m and y_m represent the vibration displacements of the m -th node, and m is equal to 4 or 12. The initial clearance of bearing is represented by r_0 .

For the n -th combined support, its bearing forces in the x - and y -directions can then be formulated as

$$\begin{bmatrix} F_{Bxn} \\ F_{Byn} \end{bmatrix} = k_b \sum_{j=1}^{N_b} (\delta_{nj})^{3/2} H(\delta_{nj}) \begin{bmatrix} \cos \theta_j \\ \sin \theta_j \end{bmatrix} \quad (5)$$

where k_b represents the Hertz contact stiffness. $H(\cdot)$ is a Heaviside function.

2.2. Dynamic Modeling of SC Elastic Support

In this subsection, the dynamic model of the squirrel cage is presented, considering the motion of the squirrel cage and the outer race of bearings, and this better conforms to the actual situation. Figure 1a shows that the bearing outer race is installed in the squirrel cage elastic support instead of a rigid bearing housing, which results in the bearing outer race not being fixed.

The vibration originating from the rotating shaft is transmitted to the outer race of the bearing and the squirrel cage via the inner race in the form of bearing reaction forces ($-F_B$). At this time, the bearing outer race and the squirrel cage as a whole are regarded as a concentrated mass point. Under the excitation of bearing reaction force, the squirrel cage, together with the bearing outer race, will generate vibration displacements x_s and y_s . Assuming that the squirrel cage is symmetrical in the x and y directions, the mass, stiffness, and damping in the x and y directions are the same. As shown in Figure 1c, the combined support can be simplified as a 2-degree-of-freedom (DOFs) forced vibration system, and the motion equations are as follows:

$$\begin{cases} m_{sn} \ddot{x}_{sn} + c_{sn} (\dot{x}_{sn} - \dot{x}_m) + k_{sn} (x_{sn} - x_m) = -F_{Bxn} \\ m_{sn} \ddot{y}_{sn} + c_{sn} (\dot{y}_{sn} - \dot{y}_m) + k_{sn} (y_{sn} - y_m) = -F_{Byn} \end{cases} \quad (6)$$

where the subscript n represents the index of combined support. m_{sn} , c_{sn} and k_{sn} are the mass, damping, and stiffness of the squirrel cage. It is important to note that while the mass distribution of the squirrel cage demonstrates negligible influence on the linear vibration

response of the rotor system, the current study operates under the assumption that this minimal influence extends to nonlinear vibration responses as well. This constitutes the rationale behind employing a 2-degrees-of-freedom lumped mass model for the squirrel cage, while also representing a recognized limitation of this work.

The schematic illustration and dimensional parameters of SC investigated in this paper are presented in Figure 3. The dynamic stiffness of SC is considered, i.e., the stiffness k_{sn} varies with the excitation frequency (rotational speed). The dynamic stiffness of SC is obtained through a hammering experiment, and the experimental setup is shown in Figure 4. One end of the SC is fixed, which is in line with the constraints during its operation. The impact hammer (PCB 086C01) is used to excite the other end of SC, and the excitation force $F(\omega)$ is measured by the impact hammer. The accelerometer (PCB 352C22) and LMS SCADAS mobile front end are employed to acquire the acceleration $\ddot{x}(\omega)$. The acceleration can be converted to displacement in the frequency domain as follows:

$$\ddot{x}(\omega) = -\omega^2 x(\omega) \quad (7)$$

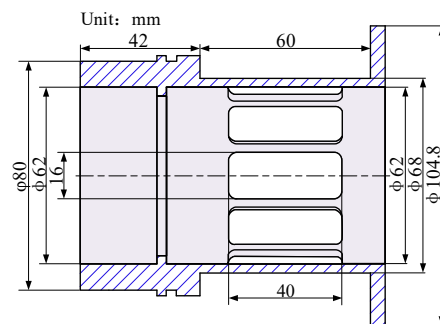


Figure 3. Schematic illustration and dimensional parameters of squirrel cage.

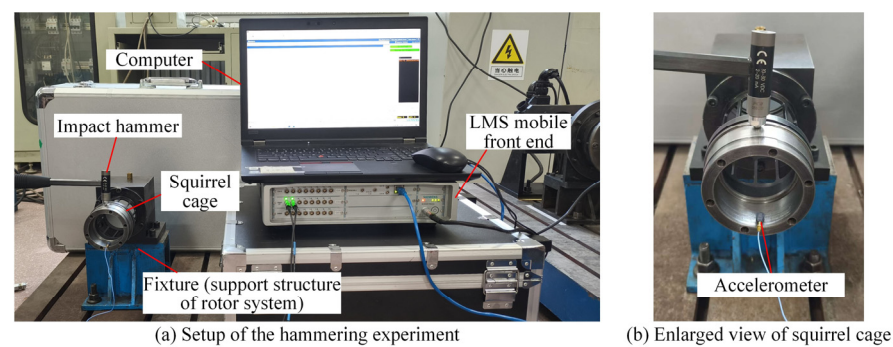


Figure 4. Setup of the hammering experiment and its enlarged view.

The dynamic stiffness k_{sn} can be obtained as follows:

$$K_{sn} = \frac{F(\omega)}{x(\omega)} = \frac{-\omega^2 F(\omega)}{\ddot{x}(\omega)} \quad (8)$$

Therefore, the dynamic stiffness of SC can be calculated by the experimental results, as shown in Figure 5. To ensure the validity of the experimental results, the experiment is replicated three times. It can be seen that as the excitation frequency varies, the stiffness exhibits a trend of first decreasing and then increasing. The natural frequency ω_{ns} of SC is 284.4 Hz, and the stiffness reaches its minimum value at the natural frequency. Subsequently, the dynamic stiffness is incorporated into Equation (6), and its value varies in accordance with the rotational speed.

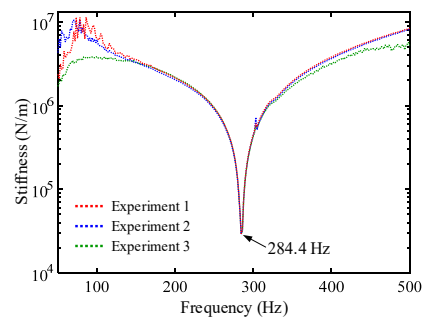


Figure 5. Dynamic stiffness of the squirrel cage, calculated from the experimental results.

The damping ratio of SC is measured through the LMS SCADAS mobile front end of the hammering experiment. The measured damping ratio ζ_{sn} is 0.015, and the damping coefficient c_{sn} can be obtained as follows [84]:

$$c_{sn} = 2m_{sn}\zeta_{sn}\omega_{ns} \quad (9)$$

The mass m_{sn} of SC is 1.04 kg, which is measured by weighing, as shown in Figure 6. Therefore, the damping coefficient c_{sn} of SC can be calculated and is listed in Table 1 along with other parameters. Note that the dynamic stiffness k_{sn} of SC is shown in Figure 4. It is assumed that SC is symmetric in the x - and y - directions. Thus, the mass, stiffness, and damping in the x - and y - directions are identical. The subscript n represents the index of combined support, which is equal to 1 or 2. The SCs of combined support 1 and 2 are identical.

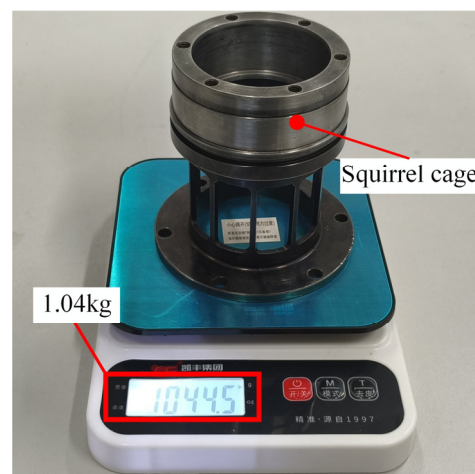


Figure 6. Measurement of the mass of squirrel cage.

Table 1. Physical parameters of the SC.

Physical Parameter	Value
k_{sn} (N/m)	See Figure 5
c_{sn} (Ns/m)	56
m_{sn} (kg)	1.04

2.3. Governing Equations for the Combined Support-Rotor System

The rotor system is constructed using 8-degrees-of-freedom Timoshenko beam elements, and each node of the finite element contains 2 translational degrees of freedom as well as 2 rotational degrees of freedom. The elastic rotating shaft is divided into 13 Timoshenko beam elements, with a total of 14 nodes. The shear, tension, bending deformations,

gyroscopic torques, rotational inertia of the rotor, and the coupling with elastic supports supports through nonlinear forces are considered. For the beam element, its generalized displacement vector is expressed as

$$\mathbf{q}^e = \left[x_k, y_k, \theta_{xk}, \theta_{yk}, x_{(k+1)}, y_{(k+1)}, \theta_{x(k+1)}, \theta_{y(k+1)} \right]^T \quad (k = 1, 2, \dots, 13) \quad (10)$$

The mass, stiffness, and gyroscopic matrices can be expressed as

$$\mathbf{M}_T^e = \frac{\rho A l}{(1 + \varphi_s)^2} \begin{bmatrix} T_1 & & & & & & & \\ 0 & T_1 & & & & & & \\ 0 & -T_4 & T_2 & & & & & \\ T_4 & 0 & 0 & T_2 & & & & \\ T_3 & 0 & 0 & T_5 & T_1 & & & \\ 0 & T_3 & -T_5 & 0 & 0 & T_1 & & \\ 0 & T_5 & T_6 & 0 & 0 & T_4 & T_2 & \\ -T_5 & 0 & 0 & T_6 & -T_4 & 0 & 0 & T_2 \end{bmatrix} + \frac{\rho I}{l(1 + \varphi_s)^2} \begin{bmatrix} R_1 & & & & & & & \\ 0 & R_1 & & & & & & \\ 0 & -R_4 & R_2 & & & & & \\ R_4 & 0 & 0 & R_2 & & & & \\ -R_1 & 0 & 0 & -R_4 & R_1 & & & \\ 0 & -R_1 & R_4 & 0 & 0 & R_1 & & \\ 0 & -R_4 & R_3 & 0 & 0 & R_4 & -R_2 & \\ R_4 & 0 & 0 & R_3 & -R_4 & 0 & 0 & R_2 \end{bmatrix} \quad (11)$$

$$\mathbf{K}_B^e = \frac{EI}{l^3(1 + \varphi_s)} \begin{bmatrix} B_1 & & & & & & & \\ 0 & B_1 & & & & & & \\ 0 & -B_4 & B_2 & & & & & \\ B_4 & 0 & 0 & B_2 & & & & \\ -B_1 & 0 & 0 & -B_4 & B_1 & & & \\ 0 & -B_1 & B_4 & 0 & 0 & B_1 & & \\ 0 & -B_4 & B_3 & 0 & 0 & B_4 & B_2 & \\ B_4 & 0 & 0 & B_3 & -B_4 & 0 & 0 & B_2 \end{bmatrix} \quad (12)$$

$$\mathbf{G}^e = \frac{\rho I}{15l(1 + \varphi_s)^2} \begin{bmatrix} 0 & & & & & & & \\ G_1 & 0 & & & & & & \\ -G_2 & 0 & 0 & & & & & \\ 0 & -G_2 & G_4 & 0 & & & & \\ 0 & G_1 & -G_2 & 0 & 0 & & & \\ -G_1 & 0 & 0 & -G_2 & G_1 & 0 & & \\ -G_2 & 0 & 0 & G_3 & G_2 & 0 & 0 & \\ 0 & -G_2 & -G_3 & 0 & 0 & G_2 & G_4 & 0 \end{bmatrix} \quad (13)$$

where

$$\begin{cases} T_1 = \frac{13}{35} + \frac{7}{10}\varphi_s + \frac{1}{3}\varphi_s^2, T_2 = \left(\frac{1}{105} + \frac{1}{60}\varphi_s + \frac{1}{120}\varphi_s^2\right)l^2 \\ T_3 = \frac{9}{70} + \frac{3}{10}\varphi_s + \frac{1}{6}\varphi_s^2, T_4 = \left(\frac{11}{210} + \frac{11}{120}\varphi_s + \frac{1}{24}\varphi_s^2\right)l \\ T_5 = \left(\frac{13}{420} + \frac{3}{40}\varphi_s + \frac{1}{24}\varphi_s^2\right)l, T_6 = -\left(\frac{1}{140} + \frac{1}{60}\varphi_s + \frac{1}{120}\varphi_s^2\right)l^2 \end{cases} \quad (14)$$

$$\begin{cases} R_1 = \frac{5}{6} \\ R_2 = \left(\frac{2}{15} + \frac{1}{6}\varphi_s + \frac{1}{3}\varphi_s^2 \right) l^2 \\ R_3 = -\left(\frac{1}{30} + \frac{1}{6}\varphi_s - \frac{1}{6}\varphi_s^2 \right) l^2 \\ R_4 = \left(\frac{1}{10} - \frac{1}{2}\varphi_s \right) l \end{cases} \quad (15)$$

$$\begin{cases} \varphi_s = \frac{12EI}{GA_s l^2} \\ A_s = \frac{6A(1+\mu)}{7+6\mu} \\ I = \frac{\pi}{64} (D^4 - d^4) \end{cases} \quad (16)$$

$$\begin{cases} B_1 = 12 \\ B_2 = (4 + \varphi_s) l^2 \\ B_3 = (2 - \varphi_s) l^2 \\ B_4 = 6l \end{cases}, \begin{cases} G_1 = 36 \\ G_2 = 3l - 15l\varphi_s \\ G_3 = l^2 + 5l^2\varphi_s - 5l^2\varphi_s^2 \\ G_4 = 4l^2 + 5l^2\varphi_s + 10l^2\varphi_s^2 \end{cases} \quad (17)$$

In addition, the rigid disk is considered as a 4 DOFs element and is added to the rotor system according to the corresponding node. The mass, stiffness, and unbalance force matrices of the disk can be written as follows:

$$M_d = \begin{bmatrix} m_d & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 \\ 0 & 0 & J_d & 0 \\ 0 & 0 & 0 & J_d \end{bmatrix} \quad (18)$$

$$G_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -J_p \\ 0 & 0 & -J_p & 0 \end{bmatrix} \quad (19)$$

$$F_d = m_d e \Omega^2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cos(\Omega t) + m_d e \Omega^2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \sin(\Omega t) \quad (20)$$

where e represents the eccentricity of the disk.

The bearing force vector F_B can be written as follows:

$$F_B = [\cdots F_{Bx1} F_{By1} \cdots F_{Bx2} F_{By2} \cdots]^T \quad (21)$$

For the rotor system without the combined support, its motion equation can be obtained by assembling the matrices of the disk and the shaft, which is expressed as follows:

$$M_r \ddot{q}_r + (C_r - \Omega G_r) \dot{q}_r + K_r q_r = F_d + F_B \quad (22)$$

in which M_r , C_r , K_r , and G_r denote the mass matrix, damping matrix, stiffness matrix, and gyroscopic matrix for the rotor system without the combined support, respectively. Note that the structural damping of the system is characterized by Rayleigh damping. The damping ratio is revised according to the experimental results (Section 4), and its value is 0.04.

The motion equation for combined supports is subsequently added to the matrices of the rotor system. Therefore, the dynamics equation of the combined support-rotor can be expressed as:

$$\begin{bmatrix} \mathbf{M}_r & 0 & 0 \\ 0 & \mathbf{M}_{s1} & 0 \\ 0 & 0 & \mathbf{M}_{s2} \end{bmatrix} \ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{C}_r - \Omega \mathbf{G}_r & 0 & 0 \\ 0 & \mathbf{C}_{s1} & 0 \\ 0 & 0 & \mathbf{C}_{s2} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{K}_r & 0 & 0 \\ 0 & \mathbf{K}_{s1} & 0 \\ 0 & 0 & \mathbf{K}_{s2} \end{bmatrix} \mathbf{q} = \mathbf{F}_d + \mathbf{F}_B \quad (23)$$

3. Discussions of Numerical Simulation Results

The key focus of this section is to investigate the dynamic behaviors of the support-rotor assembly system, with particular emphasis on the effect of the combined support on the rotor-bearing system. The physical parameters of the rotor and bearing are listed in Tables 2 and 3, respectively. Sections 3.1 and 3.2 comparatively analyze the effect of the combined support on the nonlinear vibration of the rotor system. In Section 3.3, this study examines how bearing clearance affects the response of rotor systems for both support forms, where the nonlinear vibration behaviors are solved by the Newmark method. New vibration behaviors of the combined support-rotor system are discovered through amplitude–frequency responses, frequency spectrums, time waveform, Poincaré map, and variation in contact load. It is important to note that damping is essential for ensuring the validity of the system response. In this study, Rayleigh damping is adopted to represent the structural damping of the system. The damping ratio is subsequently calibrated against experimental results, yielding a final value of 0.04. This calibrated value is applied in simulations to obtain the dynamic response.

Table 2. Geometric parameters of the rotor system.

Element	Length/mm	Outer Radius/mm	Element	Length/mm	Outer Radius/mm
1	25	21	5–7	144	28
2	25	23	8–10	144	28
3	10	25	11,12	10	25
4	10	25	13	13	23

Table 3. Physical parameters of the bearing.

Physical Parameter	Value
contact stiffness k_b (N/m)	13.34×10^9
Number of balls N	7
Radius of outer race R (mm)	62
Radius of inner race r (mm)	25
radial clearance r_0 (mm)	0.01

In order to validate the effectiveness of the solution obtained by the Newmark method, a convergence analysis is conducted through the evaluation of calculation results corresponding to different time-step sizes. For the comparative analysis, four time-step sizes are selected, specifically 2.5×10^{-5} s, 1.2×10^{-5} s, 0.6×10^{-5} s, and 0.3×10^{-5} s. The comparison of time-domain responses with different time steps is presented in Figure 7. Three rotational speeds of $0.63\omega_{n1}$ (before resonance), ω_{n1} (resonance), and $1.09\omega_{n1}$ (after resonance) are selected for comparison. As the time-step size diminishes, the vibration response ceases to vary starting from 1.2×10^{-5} s, which demonstrates the convergence of the calculation results. Therefore, to further guarantee the solution accuracy, a time step of 0.6×10^{-5} s is chosen for the computations in this study.

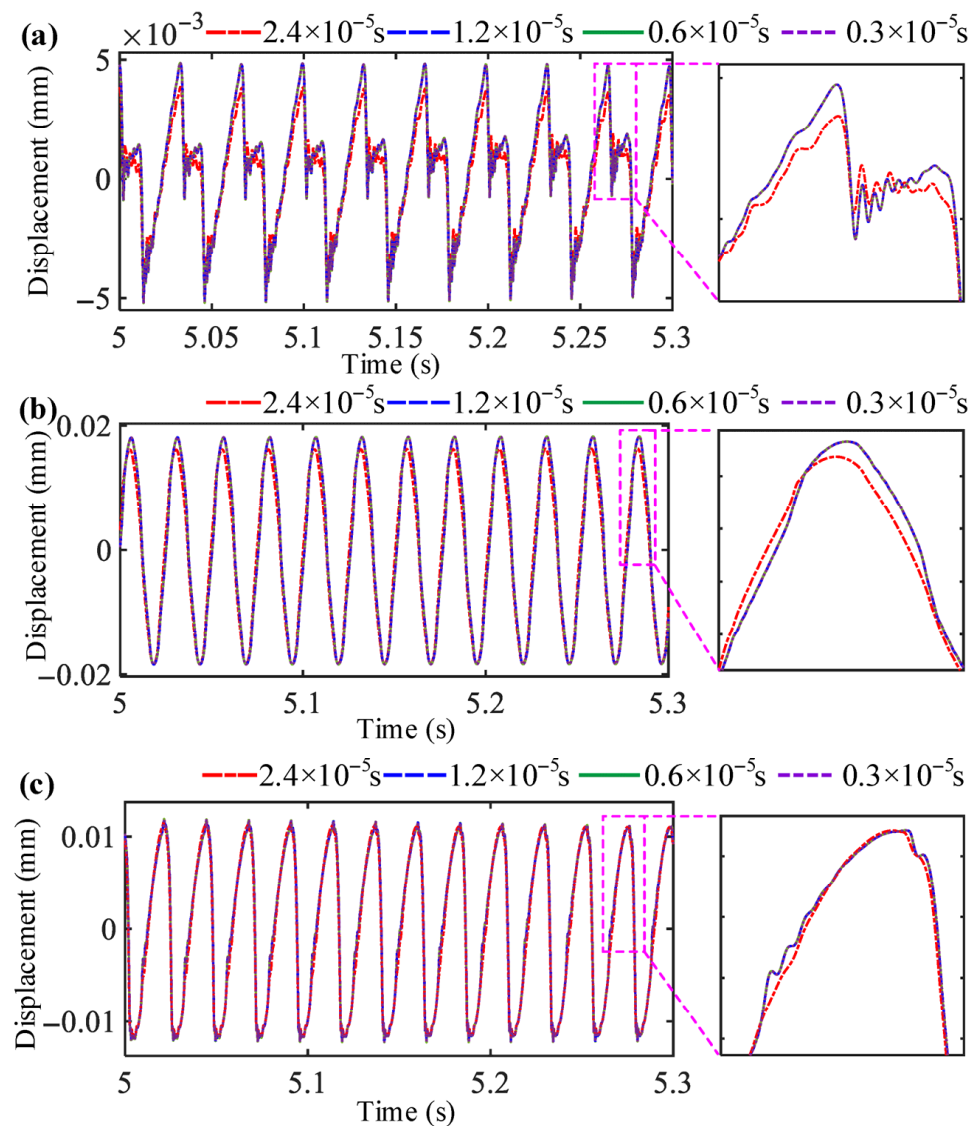


Figure 7. Time-domain responses of the rotor system under different time-step sizes and rotational speeds. (a) $0.63\omega_{n1}$. (b) ω_{n1} . (c) $1.09\omega_{n1}$.

3.1. Dynamic Analysis of Rotor-Bearing System Under Unbalance Excitation

Spectrum cascades under different speeds are seen in Figure 8a. The analysis reveals that obvious nonlinear vibration behaviors are present, with the main frequency components being f_r , $2f_r$, $3f_r$, $4f_r$, $5f_r$, etc., and the amplitudes gradually decreasing. Figure 8b illustrates the amplitude–frequency responses at node 4 (support) and 7 (disk), where the critical speed ω_{n1} of rotor systems supported only by bearings is 2375 r/min. The vibration amplitude at node 4 is much smaller than that at node 7. This is because the bearing has a strong constraint, and the deformation mainly occurs on the shaft. Figure 8c–e demonstrates the vibration response at speeds of $0.63\omega_{n1}$, ω_{n1} , and $1.09\omega_{n1}$, respectively. The obvious $2f_r$, $3f_r$, etc., before and after the critical speed emerge. Moreover, the shape of the Poincaré map is irregular, and the rotor’s motion behavior is close to a chaotic regime.

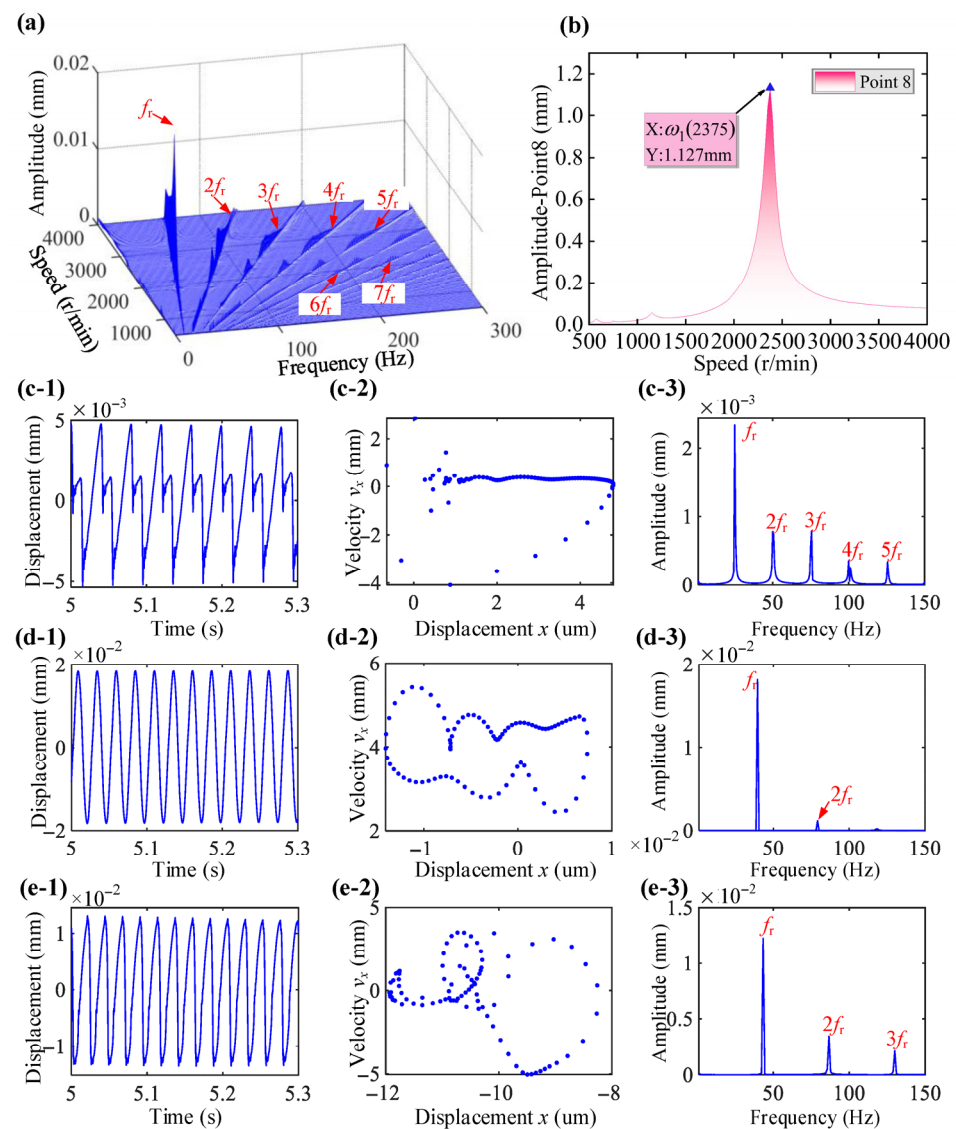


Figure 8. Vibration responses of bearing-only supported rotor systems. (a) Spectrum cascades of node 4 under different speeds. (b) Amplitude–frequency responses of node 8. (c) Vibration responses of node 4 at $0.63\omega_{n1}$, (c-1) is the time-domain waveform, (c-2) is Poincaré map, (c-3) is frequency spectrum. (d) Vibration responses of node 4 at ω_{n1} . (d-1) is the time-domain waveform, (d-2) is Poincaré map, (d-3) is frequency spectrum. (e) Vibration responses of node 4 at $1.09\omega_{n1}$. (e-1) is the time-domain waveform, (e-2) is Poincaré map, (e-3) is frequency spectrum.

3.2. Dynamic Analysis of RBS Considering Dynamic Stiffness

In this section, an analysis is conducted on the dynamic characteristics of RBS, where the effect of combined support is considered. Furthermore, the vibration responses under the conditions of combined support and bearing support only are compared.

Figure 9a illustrates the spectrum cascades for the rotor system supported by combined supports across a range of rotational speeds. Compared with Figure 8a, the vibration amplitudes of $2f_r$, $3f_r$, $4f_r$, $5f_r$, etc., are significantly reduced, which indicates that the nonlinearity of the rotor system is weakened by the combined support. It can be seen from Figure 9b that the critical speed ω_{n2} of the combined-supported rotor system is 2260 r/min. The critical speed ω_{n2} is reduced compared to the case with only bearing support (ω_{n1}) because the combined support is flexible. This also indicates that the combined support contributes to adjusting the critical speed of the rotor system.

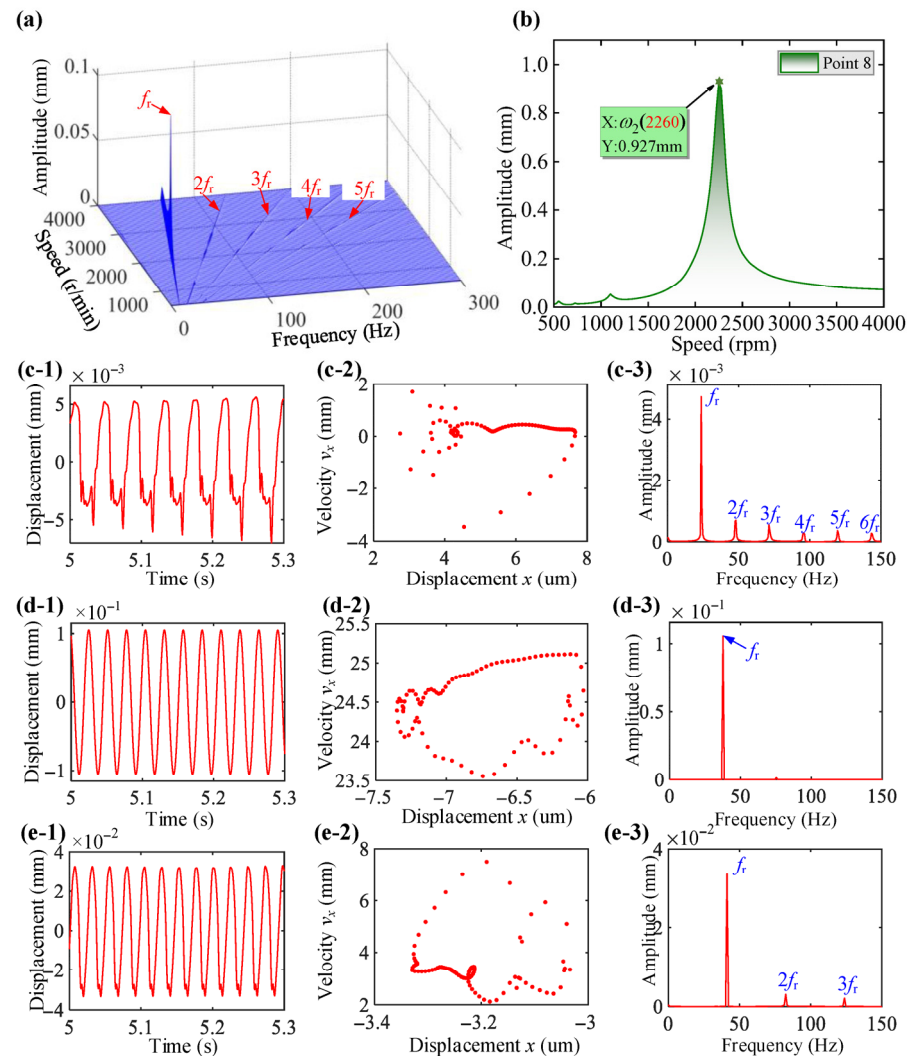


Figure 9. Vibration responses of the rotor system with combined supports. (a) Spectrum cascades of node 4 under different speeds. (b) Amplitude–frequency responses of node 8. (c) Vibration responses of node 4 at $0.63\omega_{n1}$, (c-1) is the time-domain waveform, (c-2) is Poincaré map, (c-3) is frequency spectrum. (d) Vibration responses of node 4 at ω_{n1} . (d-1) is the time-domain waveform, (d-2) is Poincaré map, (d-3) is frequency spectrum. (e) Vibration responses of node 4 at $1.09\omega_{n1}$. (e-1) is the time-domain waveform, (e-2) is Poincaré map, (e-3) is frequency spectrum.

Figure 9c–e show time-domain response curves of the system at different speeds. The shape of the Poincaré map is approximately circular, and it can be concluded that the rotor system is in a quasi-periodic motion state under the current conditions. Compared with Figure 9c–e, the rotor operates with greater stability. In addition, the amplitudes of ultraharmonics have also decreased.

For a thorough investigation into the influence of combined supports on the RBS, as illustrated in Figure 10, a comparison of the rotor orbit and vibration amplitude under two support forms is presented. It can be seen that combined support increases vibration near the rotor support and significantly reduces the vibration of the shaft. Because the squirrel cage absorbs the energy of the RBS vibration through the elastic deformation of the structure, the vibration of the shaft is greatly weakened, which is conducive to the stable operation of the RBS.

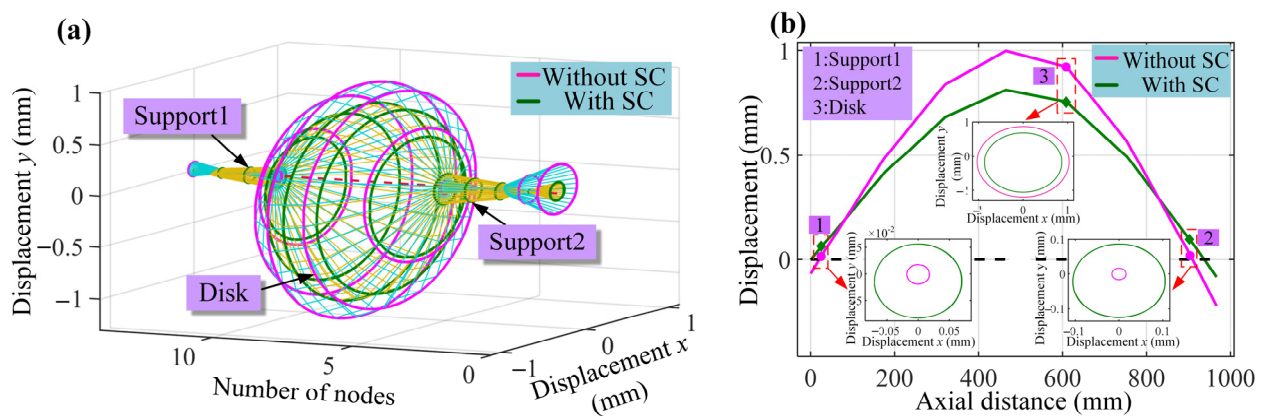


Figure 10. Comparison of rotor operational deflection shape under different supports. (a) Rotor orbits of all nodes. (b) Vibration amplitudes of all nodes.

Furthermore, the effect of the combined support on the reaction force is studied by comparing the accelerations under different supports, as shown in Figure 11. When the squirrel cage (SC) is considered, the amplitude of the vibration acceleration has decreased significantly. Thus, the SC lessens the rotor's reaction force while boosting the stability of both the rotor and its supporting structures.

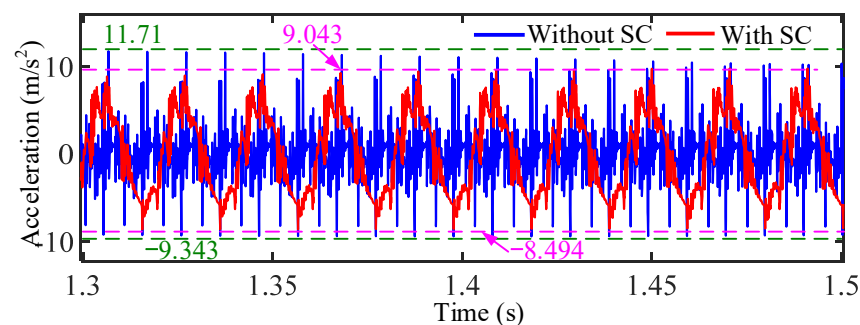


Figure 11. Comparison of vibration acceleration under different supports.

3.3. Effect of Bearing Clearance

This section focuses on the effect of different bearing initial clearances on the vibration characteristics of the RBS under combined support. Figures 12 and 13, respectively, show the waterfall plots under the bearing support and the combined support. As the initial bearing clearance increases, frequency components become increasingly complex and the amplitudes rise higher and higher. Figures 12 and 13 are analyzed comparatively to highlight their differences in dynamic response, and it is obvious that the combined support significantly reduces the frequency components in each clearance situation.

The bifurcation diagrams under the bearing support and combined support are shown in Figure 14. It can be seen that the combined support largely weakens the nonlinear vibration. The rotor supported only by bearing exhibits chaotic motion state at many rotational speeds, while rotor systems supported by combined supports show 1-period or quasi-periodic motion at many rotational speeds. This indicates that the operational stability of the rotor with combined supports has been improved.

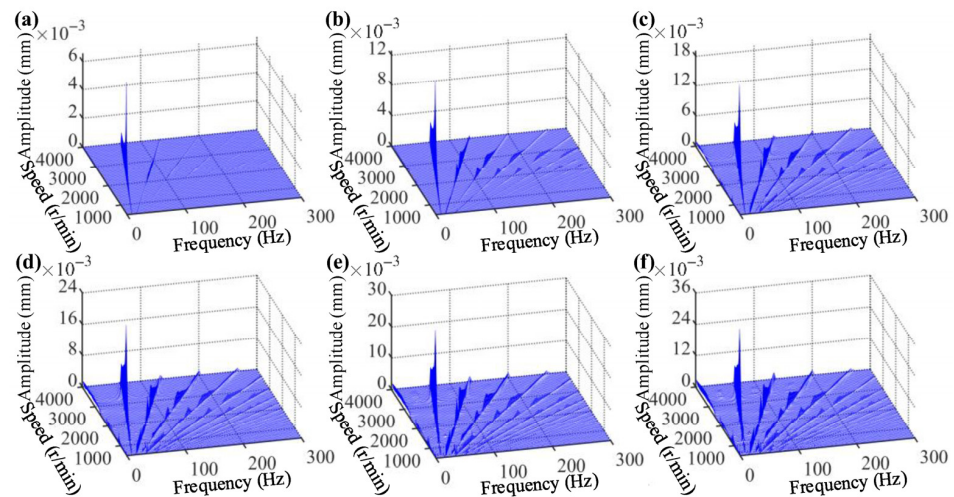


Figure 12. Spectrum cascades under different speeds and bearing initial clearances with only bearing support. (a) 0 μm . (b) 5 μm . (c) 10 μm . (d) 15 μm . (e) 20 μm . (f) 25 μm .

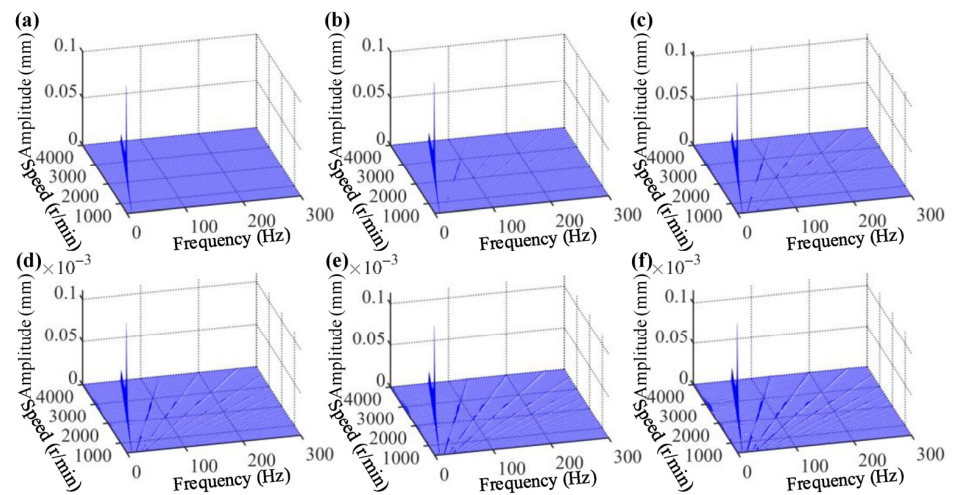


Figure 13. Spectrum cascades under different speeds and bearing initial clearances with combined supports. (a) 0 μm (b) 5 μm (c) 10 μm (d) 15 μm (e) 20 μm (f) 25 μm .

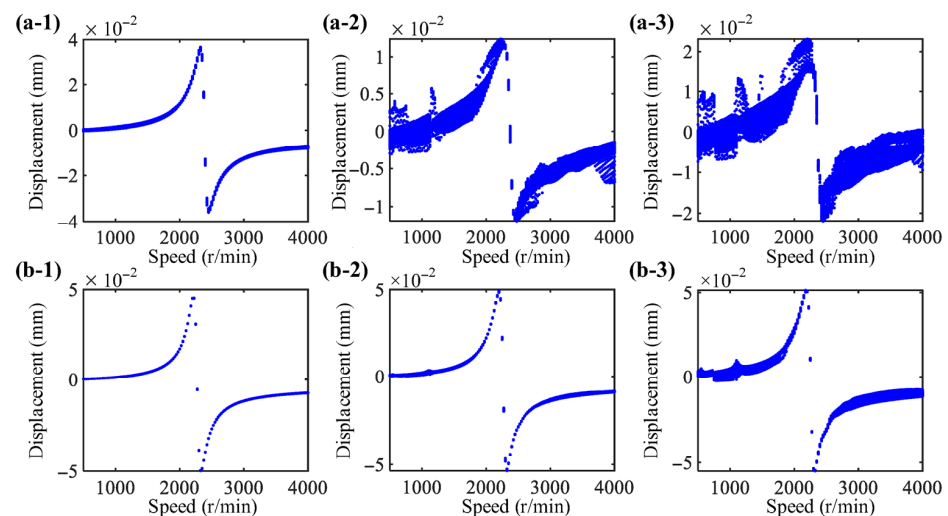


Figure 14. Bifurcation diagrams under different bearing initial clearances with bearing supports and combined supports. (a-1–a-3) Bearing support. (b-1–b-3) combined support. 1–3, respectively, represent the results when the bearing initial clearance is 0 μm , 10 μm , and 20 μm .

4. Experimental Study

To verify some typical dynamic behaviors of the RBS with combined supports, the test rigs with bearing supports (see Figure 15a) and with combined supports (see Figure 15b) are set up for validation and comparison. The shafts and disks of the two rotor test rigs are the same, with only the supports being different. For the combined support rotor test rig, the bearings are installed in SC, and then SC is fixed to the support structure. Eddy current sensors are employed to measure the vibration displacements at three positions of the rotating shaft. The sensitivity of eddy current sensors is 2000 mV/mm. The accelerometers (356A01, PCB Piezotronics, Inc., Depew, NY, USA) are utilized to measure the vibration acceleration by being attached to the support structure, with a sensitivity of 5.32 mV/g, as shown in Figure 15c. The vibration data acquired by the sensors are collected using an LMS mobile front end (SCM2E05, SIEMENS AG, Berlin, Germany) at a sampling frequency of 2048 Hz, as shown in Figure 15d.

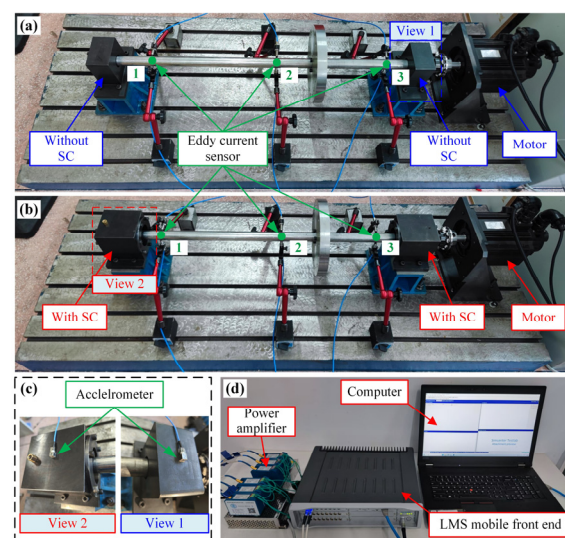


Figure 15. (a) Test rig of rotor system with bearing supports. (b) The test rig of the rotor system with combined supports. (c) Layout of test positions for the accelerometer. (d) Signal acquisition system.

4.1. Model Verification

In this section, a comparison between the simulation and experimental results is conducted to verify the effectiveness of the model. Firstly, the damping of the rotor system is identified by using the particle swarm optimization algorithm, and the optimization objective is the peak of the rotor system resonance. The objective function for the damping coefficients of the rotor system is as follows:

$$A(\xi) = \frac{|x^e - x^s(\xi)|}{x^e} \quad (24)$$

where x^e and x^s are the resonance amplitudes of the experimental and simulation results. ξ is the damping ratio that needs to be identified. After identification, the damping ratio is 0.04. The identified parameter is incorporated into the model developed in Section 2, and a comparison is made between the amplitude–frequency responses of the optimized simulation and experiment (sensor 2), as shown in Figure 16. The simulation results after optimization exhibit a high degree of consistency with the experimental results, thereby validating the effectiveness of the simulation model.

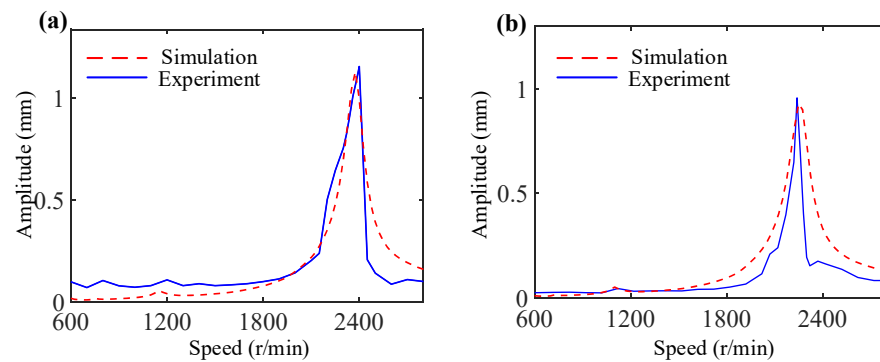


Figure 16. Comparison between the amplitude–frequency responses of the simulation and experiment (sensor 2). (a) Bearing supports. (b) Combined supports.

4.2. Experimental Results

Figure 17 presents the measured amplitude–frequency response of the bearing support and the combined support, respectively. The critical speed of bearing supports is 2400 r/min, and that of combined supports is 2240 r/min. Furthermore, the introduction of SC indeed reduces the vibration amplitude at the middle of the rotor, thereby effectively validating our simulation model.

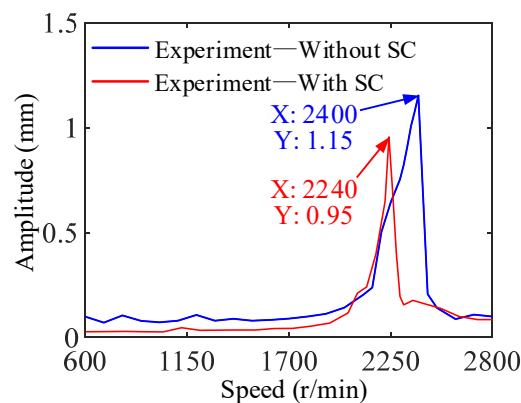


Figure 17. The measured amplitude–frequency responses of sensor 2.

Figure 18 shows the measured spectrum cascades under different speeds with different supports. For the rotor test rig with bearing supports, the frequency components include f_r , $2f_r$, $3f_r$, $4f_r$, etc. As for the case of combined supports, only f_r and $2f_r$ are obvious. The result of the combined support weakening nonlinearity is verified through experiments.

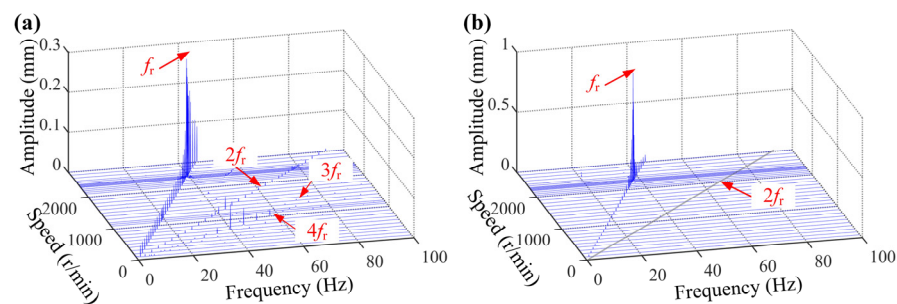


Figure 18. The measured spectrum cascades under different speeds with different supports (sensor 2). (a) Bearing supports. (b) Combined supports.

The measured three-dimensional rotor orbits are illustrated in Figure 19. By comparison, it is found that when the squirrel cage elastic support is introduced, the displacement

at both ends of the rotating shaft increases while that in the middle decreases, which is consistent with simulation results in Section 3.2. This is because the squirrel cage absorbs the vibration energy of the rotor through the elastic deformation of the structure, thereby reducing the vibration response in the middle of the rotating shaft.

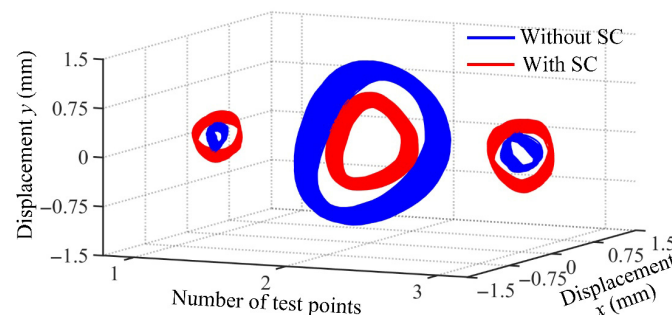


Figure 19. Comparison of measured rotor orbits under different supports.

The measured vibration accelerations with different supports are shown in Figure 20. Analysis shows that combined support reduces the vibration acceleration at the support location, consistent with simulation results, indicating that the squirrel cage elastic support can reduce the reaction force of the rotor.

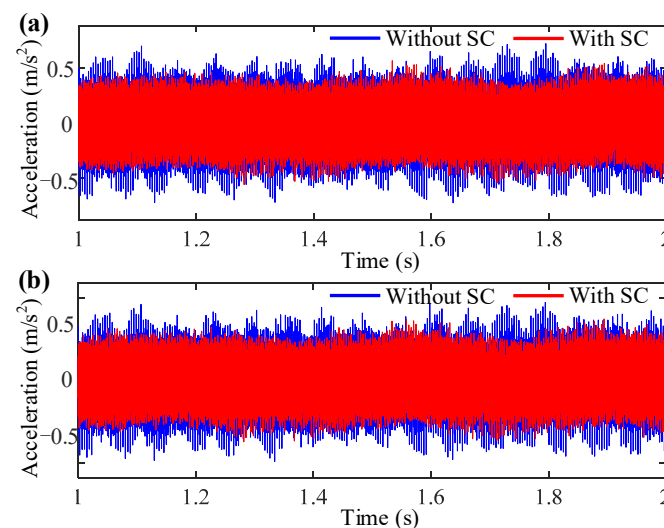


Figure 20. Comparison of measured vibration accelerations under different supports. (a) Horizontal direction. (b) Vertical direction.

5. Conclusions

In this work, we develop a dynamic model of the rotor-bearing system with SC. The effect of SC on the RBS is analyzed numerically and experimentally. New vibration behaviors and phenomena are discovered, which reveal the effect of SC on nonlinear rotor systems. The main conclusions are summarized as follows:

1. The nonlinear vibration behavior of the RBS is weakened by the SC elastic support, which is manifested as a reduction in frequency components and a stabilization of the bifurcation diagram. In addition, the SC elastic support can adjust the critical speed of the rotor.
2. The operational deflection shape of the RBS is changed by the squirrel cage elastic support, and vibration energy of the rotor is transformed into deformation of the SC, thereby reducing vibration displacement in the middle of the rotor.

3. For different bearing initial clearance conditions, the squirrel cage elastic support still has a strong weakening effect on the nonlinear vibration of RBS, and nonlinear vibration behavior changes are not significant under different initial bearing clearance conditions.

Author Contributions: Conceptualization, L.L.; methodology, L.L.; software, L.L., T.M., and F.X.; validation, L.L., T.M., and F.X.; formal analysis, F.L.; investigation, L.L. and T.M.; data curation, F.L. and F.X.; writing—original draft preparation, L.L.; writing—review and editing, F.L.; visualization, T.M. and F.X.; funding acquisition, L.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant number 52405095; the Guangdong Basic and Applied Basic Research Foundation, grant number 2023A1515110557; the Natural Science Foundation of Liaoning Province, grant number 2023-BSBA-102; the Fundamental Research Funds for the Central Universities of China, grant number N2403022; the Open Project of Guangxi Key Laboratory of Automobile Components and Vehicle Technology in 2024, grant number 2024GKLACVTKF07; the Basic Research Projects of Liaoning Provincial Department of Education, grant number JYTQN2023162; and the Open Fund of National Key Laboratory of Particle Transport and Separation Technology, grant number WZKF-2024-6.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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